Nonparametric equivalence scales with application to Poland

Gianni Betti

Department of Statistics
London School of Economics
Houghton Street
London WC2A 2AE, UK

January 21, 1999

Abstract

Equivalence scales are by one consent considered a necessary tool in poverty analysis, welfare comparison and income distribution analysis; but there is no unanimity in the methodology to be utilised to calculate them. This paper shows the usefulness of nonparametric regression analysis for specification of household Engel curves, in order to estimate equivalence scales. The regression is based on the Naradaya-Watson kernel estimator and a Cross-Validation procedure is used to evaluate the optimal bandwidth parameter. The method is applied to the 1995 data set of Living Conditions of Households in Poland.

Keywords: *equivalence scales, nonparametric regression, Engel curves, Poland.*

1. Introduction

In this paper we aim at calculating equivalence scales based on a food ratio applied as a welfare measure; we use nonparametric regression for the estimate of the Engel curves.

Equivalence scales represent a prerequisite in every study of well-being carried out using measures of income distribution, inequality and poverty; moreover, they constitute a suitable economic tool to incorporate the impact of demographic changes into models of spending allocation for aggregated consumption.

The calculation of equivalence scales based on a food ratio applied as a welfare measure is well known as the Engel's method. It traces its origins to the last century when Engel (1895) compared regularities in the spending behaviour of the families of Belgian factory workers. Engel's method has several limitations (Deaton and Muellbauer, 1986), but it is largely utilised for its simplicity and easy interpretation; although we are aware that more complex models could partially solve these limitations (Betti, 1998), this goes beyond the scope of this paper.

Traditionally, parametric approaches have been utilised for the estimation of the Engel curves. This could be inconvenient in the sense that the functional form of the equations must be specified in advance, directly or indirectly, through the specification of the analytical formula of the utility or cost functions. An erroneous specification of the microeconomic model can have serious consequences for the results.

We alternatively propose to estimate Engel curves through nonparametric regression; in particular we utilise the Naradaya-Watson kernel estimator. This consists of a local weighted average

of the observations for the response variable found in a band around the point at which the value should be calculated. The weight system depends slightly on the shape of the kernel function, and strongly on the bandwidth parameter of that function; this work gives more emphasis to the evaluation of the width parameter, using a Cross-Validation procedure.

The paper is composed of six sections; section two illustrates the principal methodologies for setting equivalence scales: the purpose is to allow for a comparison among the scales. However we have to bear in mind that differences in approaches make a *full comparison* not possible. Section three introduces the nonparametric kernel regression theory and explains why the kernel function choice problem is secondary to the choice of the bandwidth parameter. This section examines closely the nonparametric regression theory because it constitutes a new tool in Engel curves estimation.

After a brief description of 1995 data set of Living Conditions of Polish Households in section four, in section five Engel curves are estimated and equivalence scales are calculated for several types of household, by household size and composition. Concluding remarks end the paper.

2. Equivalence scales: an overview

Three different methodologies for setting equivalence scales, can be distinguished according to Buhmann *et al*. (1988) and Hagenaars *et al*. (1994):

a. normative and social security equivalence scales,

b. equivalence scales based on consumption or expenditure,

c. equivalence scales based on direct welfare measurement.

Although equivalence scales evaluated in this paper are based on expenditure, a wider set of definitions is need for the comparison in section five.

a. Normative and social security equivalence scales.

Normative equivalence scales are based on some norms set by experts in defining a minimum level of consumption or basket of goods for households of different composition and size.

Sometimes these norms define directly the scale values. The OECD-scale uses weights equal to one for the first adult, 0.7 for each of the following adults and 0.5 for each child younger than 14 years.

Hagenaars *et al.* (1994) introduces a modified OECD-scale, which presents lower elasticity of family size: this scale gives value 0.5 for each adult but the first and 0.3 for each child.

Other sets of scales can be calculated implicitly by social security regulations. For example in the United Kingdom the scale implicit in the Social Benefit Scale (for family with head below 65 years) is equal to 1 for the first adult, 0.6 for any additional adult and between 0.33 and 0.5 for any child according to age.

b. Equivalence scales based on consumption or expenditure.

This is the most used methodology in economic literature; equivalence scales are derived using data sets on household expenditures.

Engel (1895) presents the first important work on equivalence scales, based on the assumption that the household welfare, or standard of living of adults, is strongly related to the share of the budget devoted to food. For a fixed characteristic household set the food share is inversely related to total expenditure (Engel's law)

and, for a fixed level of total expenditure, the food-ratio is a direct function of the number of children. To restore the food share after the birth of a child the reference household (couple) would need to reach a higher level of total expenditure or income.

The development in constructing models suitable for equivalence scales calculation, has been mostly focused on the introduction of demographic variables into demand systems.

Barten (1964) considers the utility function associated with the household demographic characteristics to be:

$$u = v\left(\frac{q_1}{m_1(\mathbf{z})}, \frac{q_2}{m_2(\mathbf{z})}, \ldots, \frac{q_n}{m_n(\mathbf{z})}\right) \tag{1}$$

where $m_i(\mathbf{z})$ is the equivalence scale for the particular good i q_i is the corresponding consumption, $\mathbf{z}$ is the vector of household characteristics and n the number of consumer goods; all $m_i(\mathbf{z})$ are equal to unity in the case of the reference household. Although the model is more general than the previous one, there is a drawback due to the evaluation of equivalence scales for goods that are not consumed in the reference household (for example child food).

Gorman (1976) presents a modification pointed out in the cost function:

$$x = c\left[u, p_1 m_1(\mathbf{z}), p_2 m_2(\mathbf{z}), \ldots, p_n m_n(\mathbf{z})\right] + \sum_i p_i n_i(\mathbf{z}) \tag{2}$$

where p_i is the price of good i and $n_i(\mathbf{z})$ is the corresponding fixed consumption, so that the added term on the right hand side represents the fixed cost associated with the demographic characteristic vector $\mathbf{z}$. In the last decade many authors have pointed out the necessity of setting up new models incorporating demographic variables as Pollak and Wales (1981) and Lewbel (1985).

c. Equivalence scales based on direct welfare measurement.

A different approach which is explicitly based on welfare measurement has been developed at Leyden University, in the Netherlands: the subjective approach on poverty lines and equivalence scales by Van Praag (1968) and Kapteyn and Van Praag (1976). Households are asked some evaluation questions with respect to income levels (IEQ):

"Please try to indicate what you consider to be an appropriate amount of money for total income, for each of the follow cases:

about .... monetary units		*very bad*
about	"	*bad*
about	"	*insufficient*
about	"	*sufficient*
about	"	*good*
about	"	*very good".*

On the basis of the IEQ an individual welfare function of income is calculated for each household; the derivation of the subjective scales is well described in Kapteyn and Van Praag (1976).

3. Theory of nonparametric regression

This section examines closely the nonparametric regression theory, because it constitutes a new tool in Engel curves estimation.

Even though the techniques of nonparametric regression have a long tradition, new theories and methodologies have been developed and expanded only in the last fifteen years. The growing interest is owed essentially to two factors: first of all, scholars of statistics have found that the purely parametric philosophy of the estimation of curves is not always sufficiently flexible for the needs of the data analysis. Moreover the development of hardware, and therefore the availability of computers, has cleared the path for nonparametric

estimates that, in the past, met with insurmountable computational obstacles.

The first contributor in estimating Engel curves by nonparametric regression is Bierens and Pott-Buter (1990); successively Banks *et al.* (1997) and Betti (1999) have utilised this tool in explaining the non-linearities in the curves.

Some theory of nonparametric regression is introduced here. Let $\{(x_i ; y_i)\}_{i=1}^n$ be the values of the independent variable X and the response variable Y, observed on a set of n units; the usual regression function is:

$$y_i = m(x_i) + e_i \tag{3}$$

The nonparametric regression estimator is defined as a local average of the observations for the response variable found in a band around the point x in which the value should be estimated:

$$\hat{m}(x) = \frac{1}{n}\sum_{i=1}^{n} w_{ni}(\mathbf{x}) y_i \tag{4}$$

where $\{w_{ni}(\mathbf{x})\}_{i=1}^n$ denotes a sequence of weights that depends on the independent variable vector **x**.

The nonparametric approach for the estimate of $m(x)$ has four main characteristics:

i) it is a very versatile method for exploring a general relationship between two variables;

ii) it provides predictions of observations without reference to particular or fixed parametric models;

iii) it constitutes an excellent means for analysing the effects of isolated points or outliers;

iv) it turns out to be a flexible method for imputing missing data

through interpolation with adjacent points.

Among the more important smoothing techniques (the manner in which succession of weights is calculated) one can list the kernel, the k_{th} closest point, the orthogonal series and the "spline smoothing" (Hardle, 1990). The most utilised of these is the kernel technique, adopted also in the present work.

In kernel smoothing the sequence of weights is defined as:

$$w_{ni} = \frac{\frac{1}{h} K\left(\frac{x - x_i}{h}\right)}{\frac{1}{n}\sum_{i=1}^{n} \frac{1}{h} K\left(\frac{x - x_i}{h}\right)} \qquad (5)$$

here K(.) is the kernel, a symmetric, limited, continuous function whose integral is equal to one on the interval for which it is defined; h is the bandwidth or *smoothing parameter*.

This parameter regulates the width of the interval around x. A local average for an interval too wide can lead to the consideration of observations that have little in common with x. On the other hand, consideration of a low number of observations can make the estimate $m(x)$ too irregular and can inflate the variability too much.

The shape of the kernel function regulates the way in which weights diminish as we move away from x. The denominator in formula (5) is set up to guarantee that the weights add up to one.

Substituting the weight formula (5) into the smoothing (4) one gets:

$$\hat{m}(x) = \frac{\frac{1}{n}\sum_{i=1}^{n} \frac{1}{h} K\left(\frac{x - x_i}{h}\right) y_i}{\frac{1}{n}\sum_{i=1}^{n} \frac{1}{h} K\left(\frac{x - x_i}{h}\right)} \qquad (6)$$

that is defined as the Nadaraya-Watson estimator.

The choice of the kernel function and the band parameter is intended to minimise the distortion and variability of the estimate of the function $m(x)$. For this purpose two precision measures are considered: the mean integrated squared error (MISE) and the Kullback-Liebler distance, which will be defined later on.

Under the simple assumptions contained in Hardle (1990) and the additional assumption that the function $m(x)$ has continuous derivatives of at least the second order, one can approximate the bias and the variance of $\hat{m}(x)$ and thus calculate the MISE:

$$\mathrm{MISE}\left(\hat{m}(x)\right) = \frac{1}{4} h^4 K_2{}^2 \int_{D(x)} m''(x)^2 dx + \frac{1}{nh} \int_{-\infty}^{+\infty} K(t)^2 dt \qquad (7)$$

where $K_2 = \int_{-\infty}^{+\infty} t^2 K(t) dt$, and $m''(x)$ is the second order derivative of $m(x)$. The optimal value of h derived by minimisation of (7) is:

$$h = K_2{}^{-2/5} \left\{ \int_{-\infty}^{+\infty} K(t)^2 dt \right\}^{1/5} \left\{ \int_{D(x)} m''(x)^2 dx \right\}^{-1/5} n^{-1/5} \qquad (8)$$

This value cannot be calculated for the unknown function $m(x)$; the way to obtain it is shown below.

Substituting the previous formula in the equation for MISE one gets: $\mathrm{MISE} = (5/4) C(K) \{\int m''(x)^2 dx\}^{1/5} n^{-4/5}$, where the function $C(K)$ is $C(K) = K_2^{2/5} \{\int K(t)^2 dt\}^{4/5}$. By analysing this formula, we observe that the kernel function minimising the MISE, holding other parameters constant, is the same as the one minimising the function C(K). On the basis of this kernel the efficiency of a generic kernel is defined as $\mathrm{eff}(K) = \{C(K_e)/C(K)\}^{5/4}$. The efficiency of the

most common kernels is almost equal to one; the consequence is that the choice of kernel is not crucial for the convergence of the estimate $\hat{m}(x)$ to $m(x)$, while the bandwidth h parameter is. In the present work, the kernel with a normal distribution has been utilised for its analytical properties. The study of the minimisation of the MISE is thus focused on the optimisation of the smoothing parameter, which however cannot be calculated directly from (8).

In the smoothing parameter choice there is a trade-off between the mean and the variance of $\hat{m}(x)$; a value for h too small makes the estimate jagged, with minimum distortion and high variance. In contrast a high value of h makes the estimate homogenous, with low variance but strong distortion.

Among the more common techniques for the choice of the parameter h, is the plug-in method. The functional form of $m(x)$ is hypothesised a priori, from which the second derivative is substituted in (8); this is what the present paper is meant to avoid.

Alternatively, instead of considering the MISE as an index of the approximation of $\hat{m}(x)$ to $m(x)$, one can take into account the Kullback-Liebler *information distance*:

$$KL = I(m,\hat{m}) = \int m(x) \log\left\{\frac{m(x)}{\hat{m}(x)}\right\} dx \tag{9}$$

Silverman (1986) shows that the minimisation of KL respect to h is equivalent to the minimisation of the cross-validation function $CV(h) = \frac{1}{n}\sum_{i=1}^{n} \log[\hat{m}_{-i}(x_i)]$ where $\hat{m}_{-i}(x_i) = \frac{1}{(n-1)h}\sum_{j \neq i} K\left(\frac{x - x_j}{h}\right)$.

This paper proposes to use the bandwidth parameter minimising the cross-validation function for every Engel curve $m(x)$.

4. The data set

The data on which we are working is the 1995 Living Conditions of Polish Households conducted by the Warsaw School of Economics, in co-operation with the Central Statistical Office, the Ministry of Labour and Social Policy, the Polish Statistical Society and the University of Siena.

This survey consists of the first wave of a panel survey started in May 1995, which encompasses 2,696 households selected by using the two-stage stratified sampling scheme. The units selected in the first stage were regions or groups of regions encompassing at least 250 dwellings, stratified by *voivodship*[1] and urban or rural type of community. Proportional selection was applied so that the sample would automatically be balanced. During the second stage dwellings were selected systematically from a randomly generated list, independently inside each of the strata created during the first stage.

From the original sample a few households with zero total expenditure were excluded.

5. Empirical analysis

In this section we present an empirical analysis of equivalence scales based on both parametric and nonparametric regression. Some normative and subjective scales are also reported with the aim of a general comparison; however it is our conviction that a full comparison among scales is not possible because of the remarkable differences in the approaches utilised.

The estimate of the equivalence scales with a nonparametric regression is firstly calculated considering the Engel curves for six

family typologies characterised by household size only; families with six people or more are combined in a single family typology because of the low sample size in each individual typology.

The analysis begins with an exploratory phase of sample data regarding the food ratio and the total spending for each of the six typologies. Regressing food ratio on total consumption leads to a calibration problem[2]; for this reason we consider food ratio as independent variable and total consumption (for each household size) as the response variable.

All the data processing is done with programs in GAUSS, divided into the main program ENGEL and the CVH procedure; this procedure aims at calculating the bandwidth parameter value h using the cross-validation method described in section three; the values corresponding to the six curves are reported in the Appendix. The program performs point estimation and 95% bootstrap confidence intervals; the bootstrap procedure is based on 1000 replications.

Once the six Engel curves are obtained, it is possible to calculate an infinite number of equivalence scales, each of which corresponds to a different food share and therefore to a different level of economic well-being. Very small variations in the food share correspond to very small variations in the scale. We have chosen the specific food ratio value 0.3 corresponding to its average for two-component families, or more precisely to the reference units for this analysis. This food ratio value is projected onto the Engel curves for families with a different number of people in order to obtain the expenditure amounts equivalent to the value of two-component families. Moreover we have calculated equivalence scales for other two levels of food share or economic utility; these values are 0.25

and 0.35, and correspond to, respectively, richer and poorer households.

In order to compare the scales obtained by nonparametric regression, a parametric model based on food expenditure is considered. This model refers to Van Ginneken (1982):

$$\ln F_i = a + b \ln C_i + c \ln N_i + e_i \tag{10}$$

where F is the food expenditure, C is the total consumption expenditure and N is the family size. Considering the same assumption as in the nonparametric approach - the household's level of living varies inversely with the food ratio – it is possible to derive the economies of scale e (for d(F/C) = 0):

$$e = \frac{\partial \ln C}{\partial \ln N} = \frac{c}{1-b} \tag{11}$$

which is equal to e = 0.5875 from the regression on the entire sample (see Appendix).

Table 1: Equivalence scales according with family size.

Persons		1	2	3	4	5	6**
Nonparametric / ratio							
Expenditure*	0.25	632.76	1096.63	1301.70	1549.54	1809.44	1983.80
Scale		0.58	1.00	1.19	1.41	1.65	1.81
Expenditure*	0.30	530.59	962.95	1158.43	1371.24	1611.62	1772.79
Scale		0.55	1.00	1.20	1.42	1.67	1.84
Expenditure*	0.35	467.29	880.01	1082.41	1273.37	1507.48	1679.06
Scale		0.53	1.00	1.23	1.45	1.71	1.91
Parametric scale		0.63	1.00	1.29	1.55	1.79	1.91
Subjective scale***		0.84	1.00	1.11	1.19	1.26	1.32

* In zloties per month. ** This value refers to six or more persons for nonparametric scales. *** The subjective scale is due to Podgórski (1991).

Table 1 reports equivalence scales estimated with the parametric and nonparametric regression approach. A subjective scale for Poland calculated by Podgórski (1991) is also considered, but it

appears to be too “flat” in the sense that this scale presents too strong economies of scale.

Let us consider the three nonparametric scales; it is possible to see that lower is the food ratio (higher is the utility level), flatter is the scale. This is coherent with the welfare theory: for example let us consider a couple with a child; a richer couple has relative child costs (the scale) smaller than a poorer one (obviously the absolute child costs are larger).

Next, let us consider the parametric scale. Model (10) endogenously incorporates the family size effect and, by construction, the fixed economies of scale lead to a smooth and proportional increment in the scale. This effect is misleading because the household structure is highly variable according with the family size. One-component households have a higher percentage of elderly people than other households; on the other hand, three and four-component households present a higher number of children than others. Elderly people and children clearly contribute less than adults to household costs. Therefore the parametric scale overestimates the values for one, three and four persons.

One important feature has emerged from the comparison among the scales calculated in the first part of the analysis: there is a need to construct equivalence scales according also to household composition. The second part of the empirical analysis is thus based on nonparametric estimation of Engel curves according to the following household composition characteristics:

number of children (aged 0 – 15);

number of adults (aged 16 – 64);

number of elderly people (aged over 64).

Using exactly the same procedure as in the analysis above, we have estimated the nonparametric Engel curves for 11 household categories, which are reported in table 2. Subsample sizes of other categories are not sufficient to guarantee consistent curve estimates.

Table 2: Equivalence scales according with household composition.

	Ch	Ad	Eld	Total expend.	Nonpar. scale	OECD 70-50*	OECD 50-30*	Nonpar. scale**	Param. scale**
A	0	0	1	510.71	0.52	0.59	0.67	0.55	0.63
B	0	1	0	608.12	0.62	0.59	0.67	0.55	0.63
C	0	0	2	869.03	0.89	1.00	1.00	1.00	1.00
D	0	1	1	901.08	0.92	1.00	1.00	1.00	1.00
E	0	2	0	979.43	1.00	1.00	1.00	1.00	1.00
F	1	2	0	1167.30	1.19	1.29	1.20	1.20	1.29
G	0	3	0	1213.67	1.24	1.41	1.33	1.20	1.29
H	2	2	0	1329.94	1.36	1.59	1.40	1.42	1.55
I	1	3	0	1369.51	1.40	1.71	1.53	1.42	1.55
J	0	4	0	1441.27	1.47	1.82	1.67	1.42	1.55
K	3	2	0	1531.81	1.55	1.88	1.60	1.67	1.79

*These scales are obtained considering the couple of adults as the reference household. **These scales are reported from table 1.

The nonparametric total expenditures correspond to the food-ratio value 0.3, as in the previous analysis. Table 2 reports the obtained nonparametric scales; here OECD 70 – 50 and OECD 50 – 30 scales are included for comparison. OECD 50 – 30 scale has been officially introduced at Eurostat by Hagenaars *et al.* (1994); this is because "…most comparative research (Whiteford 1985, Buhmann *et al.* 1988, Atkinson 1991) finds OECD scale (70 – 50) to put relative much weight to additional persons".

Although a full comparison between nonparametric and OECD scales is not possible, table 2 clearly shows that the nonparametric scale is more similar to the OECD 50 – 30 than to the OECD 70 – 50. This seems to show that Polish consumer expenditure pattern and household welfare have became more and more similar to those

of western European countries during the first half of the nineties (at least according to the food non-food classification). In fact, according to Szulc (1993) "…the *steepness* of equivalence scales may be also a useful tool for the indirect comparison of the welfare in the international perspective". This seems to be confirmed by the fact that the budget share devoted to food has fallen from 55% in 1990 to 50% in 1991 and to 30% in 1995 (according to different sample sources).

Finally, a further interesting comparison would be the one between nonparametric scales and *quasi-exact* scales reported in Szulc (1993). Unfortunately, the limited sample size does not allow a subsample partition corresponding to five age classes as in Szulc. We intend to make this comparison in the future, using the cross-section Polish Household Budget Survey composed of about 30,000 households.

6. Concluding remarks

This paper presents a nonparametric approach for estimating Engel curves and equivalence scales. Nonparametric kernel regression curves are estimated for several types of households, distinct in accord with both household size and composition; the application has been conducted utilising the 1995 data set of Living Conditions of Polish Households.

Nonparametric equivalence scales are initially calculated for households by family size only; this analysis has pointed out the need to differentiate by household composition as well. The resulting scales are compared with OECD 70 – 50 and 50 – 30 scales. Evidence shows that - according to the food non-food

classification, and compared with those in Szulc (1993) - Polish consumer expenditure pattern and household welfare have become more and more similar to those of western European countries during the first half of the nineties.

Acknowledgements:

The author would like to thank Maryl Faith Cohen, Elena Quercioli, Marcia Taylor, and particularly Jane Galbraith, for helpful comments.

Appendix

Table A1: Smoothing parameters for several Engel curves.

Persons	1	2	3	4	5	6+
Sub sample size	272	616	608	636	311	243
Smoothing h	0.0500	0.0394	0.0390	0.0389	0.0475	0.0564
Type	A	B	C	D	E	F
Sub sample size	133	138	110	126	343	262
Smoothing h	0.0581	0.0570	0.0557	0.0485	0.0447	0.0458
Type	G	H	I	J	K	
Sub sample size	237	322	152	106	116	
Smoothing h	0.0467	0.0442	0.0527	0.0561	0.0628	

Table A2: Parametric estimation*.

Parameter	a	b	c	e
	1.1638	0.6161	0.2255	0.5875
	(0.0949)	(0.0146)	(0.0163)	

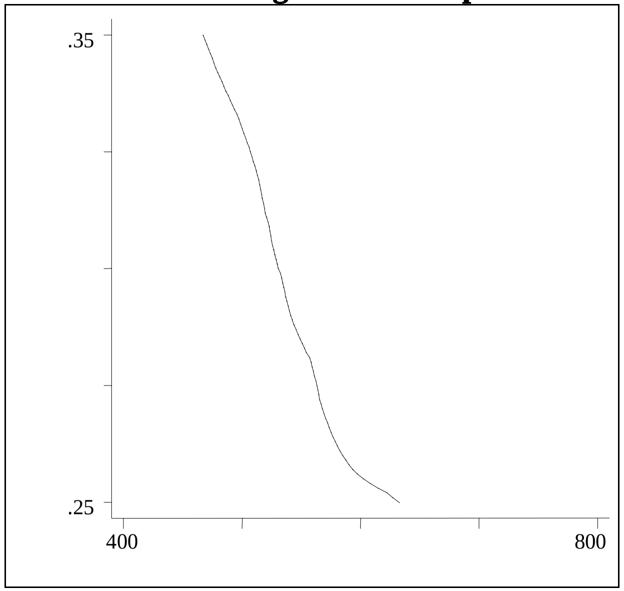
*Standard errors are in parentheses.

Six nonparametric Engel curves are reported below. In each curve, the y-axis corresponds to the food ratio, while the x-axis corresponds to the total expenditure in zloties p.m.

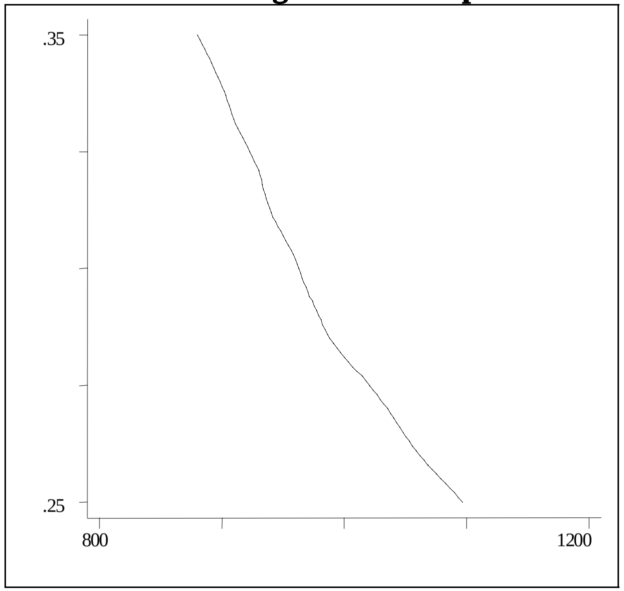
[1] A *voivod* is an administrative district or province; Poland is divided into 49 *voivod*s.

[2] Since the data are subject errors in both variables, calibration problem arises when we fix a value of food ratio (response variable) and we estimate the corresponding total consumption values by projection onto the Engel curves.

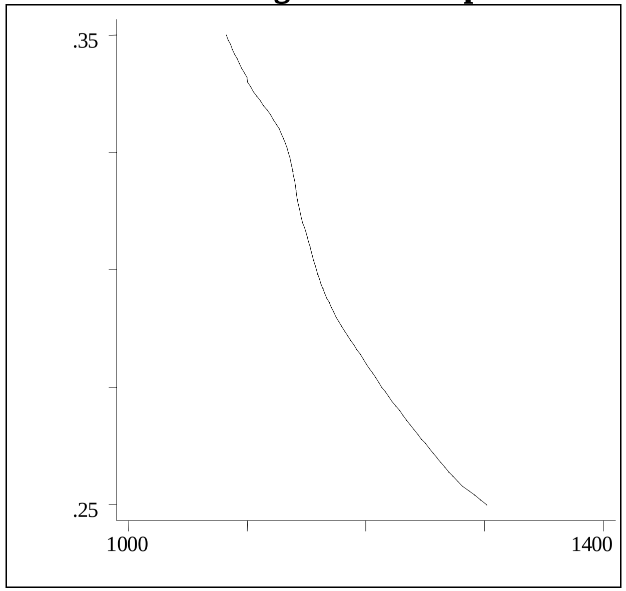
Picture A1: Engel curve 1 person.



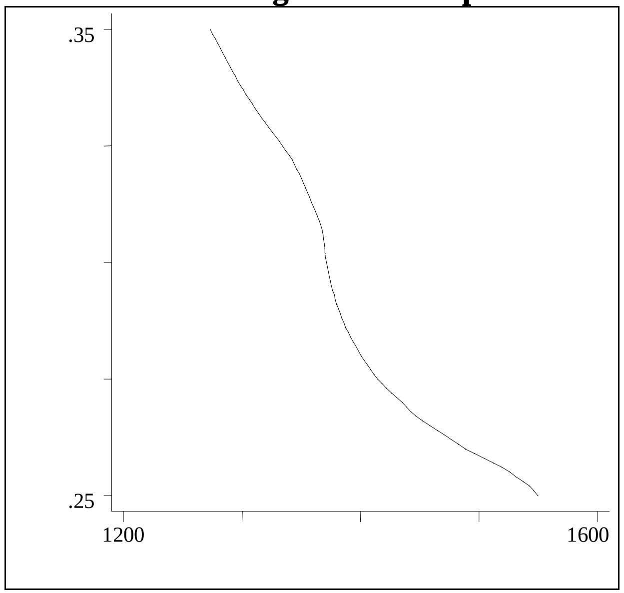
Picture A2: Engel curve 2 persons.



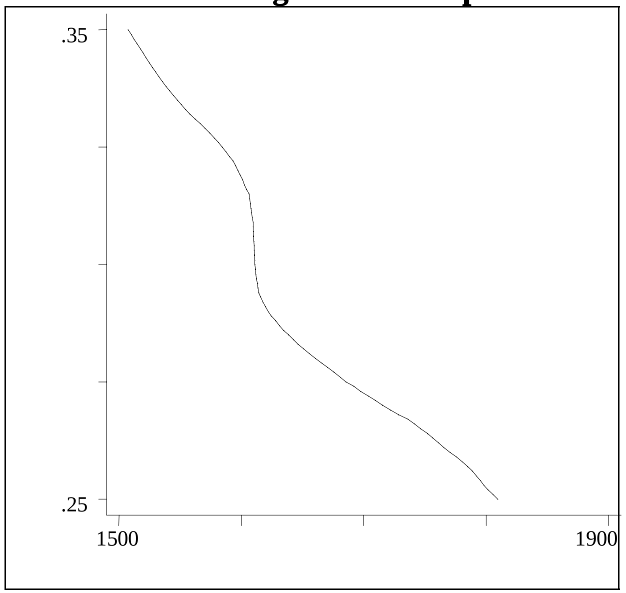
Picture A3: Engel curve 3 persons.



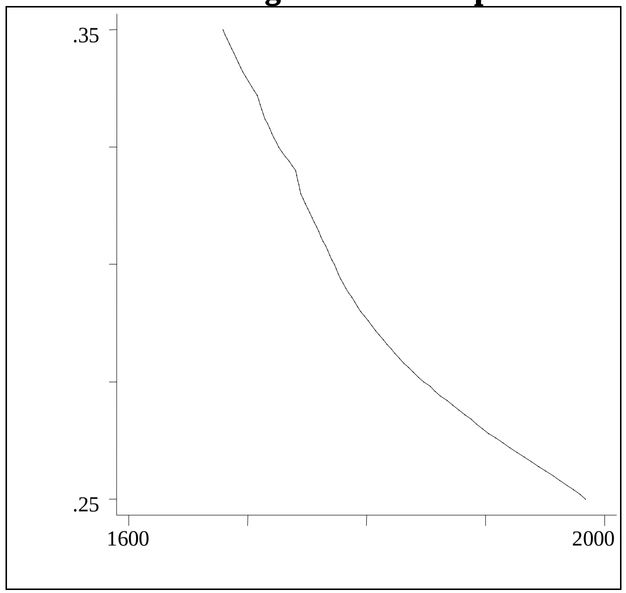
Picture A4: Engel curve 4 persons.



Picture A5: Engel curve 5 persons.



Picture A6: Engel curve 6+ persons.



References

Atkinson A.B. (1991), Comparing Poverty Rates Internationally: Lessons from Recent Studies in Developed Countries, *The World Bank Economic Review* **5**, pp. 3-21.

Banks J., Blundell R. and Lewbel A. (1997), Quadratic Engel Curves and Consumer Demand, *The Review of Economics and Statistics* **79**, pp. 527-539.

Barten A.P. (1964), Family Composition, Prices and Expenditure Patterns, in Hart P.E., Mills F. and Whitaker J.K. (eds.), *Econometric Analysis for National Economic Planning*. London: Butterworths.

Betti G. (1998), Intertemporal Equivalence Scales and Cost of Children Using BHPS, *Working papers of the ESRC Research Centre on Micro-social Change*. Paper 98-11, Colchester: University of Essex.

Betti G. (1999), Quadratic Engel Curves and Household Equivalence Scales: the Case of Italy 1985-1994, *London School of Economics, Statistics Research Report*, (to appear).

Bierens J.H. and Pott-Buter H.A. (1990), Specification of Household Engel Curves by Nonparametric Regression, *Econometric Reviews* **9**, pp. 123-184.

Buhmann B., Rainwater L., Schmaus G. and Smeeding T. (1988), Equivalent Scales, Well-Being, Inequality and Poverty: Sensitivity Estimates across Ten Countries Using the Luxembourg Income Study Database, *The Review of Income and Wealth* **32**, pp. 115-142.

Deaton A.S. and Muellbauer J. (1986), On measuring Child Cost: With Application to Poor Countries, *Journal of Political Economy* **94**, pp. 720-744.

Engel E. (1895), Die Lebenskosten Belgischer Arbeitfamilien fruher und jetzt, *International Statistical Institute Bulletin* **9**.

Gorman W. (1976), Tricks with Utility Functions, in Artis M. and Nobay A. (eds.), *Essays in Economic Analysis*, Cambridge University Press, Cambridge.

Hagenaars A.J.M., de Vos K. and Zaidi M.A. (1994), *Poverty statistics in the late 1980s: research based on micro-data*, Luxembourg: Official Publications of the European Communities.

Hardle W. (1990), *Applied Nonparametric Regression*, Cambridge University Press, Cambridge.

Kapteyn A. and Van Praag B.M.S. (1976), A New Approach to the Construction of Family Equivalence Scales, *European Economic Review* **7**, pp. 313-335.

Lewbel A. (1985), A Unifying Approach to Incorporating Demographic or Other Effects into Demand Analysis, *Review of Economic Studies* **52**, pp. 1-18.

Podgórski J. (1991), Subjective Poverty Line, in *Poverty Measurements for Economies in Transitions in Eastern European Countries*, International Scientific Conference, Warsaw.

Pollak R.A. and Wales T.J. (1981), Demographic Variables in Demand Analysis, *Econometrica* **49**, pp. 1533-1551.

Silverman B.W. (1986), *Density Estimation for Statistics and Data Analysis*, Chapman and Hall, London, New York.

Szulc A. (1993), Consumer Expenditure Patterns During the Transition Period in Poland, *Statistics in Transition* **1**, pp. 187-199.

Van Ginneken W. (1982), Generating Internationally Comparable Income Distribution Data: Evidence from the Federal Republic of Germany (1974), Mexico (1968) and the United Kingdom (1979), *The Review of Income and Wealth* **28**, pp. 365-379.

Van Praag B.M.S. (1968), *Individual Welfare Functions and Consumer Behaviour*, North Holland.

Whiteford P. (1985), A Family's needs: equivalence scales, poverty and social security, *Research Paper* n. 27, Development Division, Department of Social Security.