Forecasting German mortality via panel data procedures

(with Eckart Bomsdorf und Rafael Schmidt)
Structure

1) Introduction

2) The model

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1. Introduction

- Life expectancy at birth increased about 10 years since 1950.
- The precise assessment of its probabilistic future behavior is important for the financial stability of social security systems, life insurance companies, and related industries.
1. Introduction

Life expectancy at birth from 1950 to 2002.

Life expectancy at birth (in years)

- Female
- Male

Life expectancy at birth from 1950 to 2002.
Mortality models

• There are a lot of models in this context.
• The German Institute of Actuaries (DAV) came to the conclusion that a log-linear approach is most suitable (see also Lee/Carter 1992).
• Our model is based on the log-linear approach of Bomsdorf/Trimborn (1992).
Model by Bomsdorf/Trimborn

\[ m_x(t) = m_x(t_0) \cdot e^{\beta_x \cdot (t-t_0)} \]

- \( m_x(t) \): one-year mortality rates depending on age \( x \) and year \( t \)
- \( m_x(t_0) \): current mortality rate at time \( t_0 \)
- \( e^{\beta_x \cdot (t-t_0)} \): growth factor, change in death rates until year \( t \)
Interpretation

Interpretation of the growth factors:

\[ e^{\beta_x} - 1 \approx \beta_x \quad \text{(for } \beta_x \text{ near } 0) \]

Annual percentage change in death rates for x-year old persons.

- \( \beta_x > 0 \) : Increase of \( m_x \)
- \( \beta_x < 0 \) : Decrease of \( m_x \)
Goodness of fit

• Period life tables created by the German Federal Statistical Office, yearly published since 1962
• One year forecasts
• Goodness of fit measured by a measure which has been utilized in Lee and Carter (1992)

\[ 1 - \frac{\text{VAR}[\hat{m}_x(t) - m_x(t)]}{\text{VAR}[m_x(t)]} \]
# Goodness of fit

<table>
<thead>
<tr>
<th>Age group</th>
<th>male (in %)</th>
<th>female (in %)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-19</td>
<td>98.8</td>
<td>97.9</td>
</tr>
<tr>
<td>20-39</td>
<td>97.9</td>
<td>98.3</td>
</tr>
<tr>
<td>40-59</td>
<td>98.0</td>
<td>98.7</td>
</tr>
<tr>
<td>60-79</td>
<td>99.0</td>
<td>99.5</td>
</tr>
<tr>
<td>80-89</td>
<td>98.6</td>
<td>99.1</td>
</tr>
<tr>
<td>∅</td>
<td>98.6</td>
<td>98.7</td>
</tr>
<tr>
<td>min / max</td>
<td>96.5 / 99.5</td>
<td>95.3 / 99.6</td>
</tr>
</tbody>
</table>
2. The model

- Extension of the deterministic model in a stochastic context $\rightarrow$ confidence intervals

- The annual growth rates $\beta_x$ are modeled as a family of random variables depending on age $x$ and year $t$:

$$m_x(t) = m_x(t-1) \cdot e^{\beta(x,t)}$$
Panel Data Model

\[ \beta(x, t) = u_t + \mu_x + \sigma_x \varepsilon_{x,t} \]

Two effects are distinguished:
1. a common time effect \( u_t \) over all ages
2. an age specific effect \( \mu_x \) (independent of \( u_t \))

\( \varepsilon_{x,t} \) is an error term with age dependent volatility \( \sigma_x \).
Modeling $u_t$

- Period life tables created by the German Federal Statistical Office, yearly published since 1962
- Estimation of $\hat{u}_t$ as empirical median of $\beta_{x,t}$
- The estimation yields an empirical mean of -0.021 for $u_t$ for women and -0.018 for men, that is an average yearly decline of $m_x$ of 2.1% respectively 1.8%.
- $u_t$ tended back to its mean; we identified a mean reverting autoregressive process.
Modeling $u_t$

Discrete-time model:

$$u_{t+1} = u_t + r \cdot (s - u_t) + \gamma_t$$

with

- $s$: empirical mean of $u_t$
- $r$: degree of the backwards drift to the mean (special cases $r=1$ white noise and $r=0$ random walk)
- $\gamma_t$: normally distributed error term
Modeling $u_t$

Realized path of $u_t$ from 1963 to 2002 and example path for women
Modeling $\mu_x$

- $\mu_x$ : age specific differences from $u_t$.

\[ \hat{\mu}_x = \beta_x - \bar{u} \]

- Median (over all ages $x$) of $\mu_x$ is equal to 0.
- For men $\mu_x$ fluctuates between -0.032 for age 0 and 0.009 for age 46.
Modeling $\mu_x$

$\mu_x$ depending on age for men
Estimates for $\sigma_x$

$\sigma_x$ depending on age for men

Bernhard Babel

BSPS Conference 2006
3. Results

$m_x$ 2002 and 2050 for women
### Life expectancy 2050 (period)

<table>
<thead>
<tr>
<th></th>
<th>male</th>
<th>female</th>
</tr>
</thead>
<tbody>
<tr>
<td>5% quantil</td>
<td>79.5</td>
<td>86.1</td>
</tr>
<tr>
<td>median</td>
<td>82.4</td>
<td>88.3</td>
</tr>
<tr>
<td>95% quantil</td>
<td>85.3</td>
<td>90.5</td>
</tr>
<tr>
<td>Interval width</td>
<td>5.8</td>
<td>4.4</td>
</tr>
</tbody>
</table>

Life expectancy at birth 2050 in period view (in years)

(2003: m: 75.9; f: 81.6)
## Life expectancy 2050 (cohort)

<table>
<thead>
<tr>
<th></th>
<th>male</th>
<th>female</th>
</tr>
</thead>
<tbody>
<tr>
<td>5%-quantil</td>
<td>85.9</td>
<td>92.7</td>
</tr>
<tr>
<td>Median</td>
<td>90.1</td>
<td>96.2</td>
</tr>
<tr>
<td>95%-quantil</td>
<td>94.0</td>
<td>99.6</td>
</tr>
<tr>
<td>Intervallbreite</td>
<td>8.1</td>
<td>6.9</td>
</tr>
</tbody>
</table>

Life expectancy at birth 2050 in cohort view (in years)

\[(m: 84.8; w: 90.8)\]
Results for other countries

- Projections for 17 countries respectively 12 regions
- Death rates from the Human Mortality Database
- Here: 5 countries: England, Germany, Italy, Japan, USA
- Goodness of fit is satisfying: between 90% (Italy) and 99% (Japan)
- Median results for life expectancy
Life expectancy 2050 (period)
Life expectancy 2050 (cohort)

- ENG
- GER
- ITA
- JAP
- USA

- Male (2002)
- Male (2050)
- Female (2002)
- Female (2050)
Conclusion

• Basis: log-linear approach by Bomsdorf/Trimborn (1992)

• Stochastic modeling of the growth factors

• Two effects: a common time effect $u_t$ over all ages and an age specific effect $\mu_x$

• Plausible confidence intervals for death rates and life expectancy
Bibliography

