

To: LSE Growth Commission
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Subject: Distributionally-sensitive measures of national income and income growth

Summary: This note discusses ‘distributionally sensitive’ measures of national income and of income growth, and suggests that these measures can be produced in a relatively timely fashion using data of the type that are currently made available. The material is a supplement to that presented at the session of the LSE Growth Commission on 2 May 2012, especially in the presentation by A. B. Atkinson.

Contents: Introduction, Measures, Empirical issues, Illustrations, Charts, References.

Introduction

1. GDP or GDP per capita is the mostly commonly-used yardstick of progress in the material well-being of societies. But, as has been long known, and was recently reiterated by the Stiglitz-Sen-Fitoussi Commission’s Report (Stiglitz et al. 2009), this measure does not take account of who within a society benefits more or, indeed, less than the average. Distributional aspects are ignored. Taking account of these accords with the SSF Commission’s Recommendations 2 and 3 to ‘Emphasise the household perspective’ and to ‘Give more prominence to the distribution of income, consumption and wealth’. (Of course, SSF also discuss many other critiques of GDP-based measures.)
2. Similar arguments have long been made about measures of income growth (rather than income levels). For example, Ahluwalia and Chenery (1974) made the case that the aggregate growth rate gives greater weight to proportionate income growth among those who are already rich than to proportionate income growth among the already poor. (See also Klasen 1994.)

Measures

3. Distributionally-sensitive measures of national income, and of income growth, have been developed. I discuss them in turn.
4. The most well-developed distributionally-sensitive indices of national income are those of Sen (1976, 1979) and Atkinson (1970). (See also Kolm 1969.) Both indices can be written as the product of real mean income and an index of income equality that lies between zero and one. In the case of Sen’s index of ‘real national income’, the equality index is one minus the Gini coefficient. In the case of Atkinson’s index, the equality index is one minus the Atkinson inequality index $A(\varepsilon)$, where $\varepsilon \geq 0$ is the inequality-aversion parameter. Larger values mean that greater weight in $A(\varepsilon)$ is given to income differences towards the bottom of the income distribution relative to those in the middle or top; $\varepsilon = 0$ is the case in which no distributional adjustment is made. Atkinson’s real

income measure is an increasing function of what statisticians refer to as a generalised mean. For both the Sen and Atkinson measures, mean income is adjusted downwards by a factor that depends on how much inequality there is and, how much inequality there is, is a matter of social judgement reflected by the choice of (in)equality index. Observe that both measures give positive weights to the incomes of everyone in the population whether rich or poor – but lower incomes are given greater weight than higher incomes, other things being equal. In the Sen index, the weights depend on income rank; in the Atkinson index, the weights depend on income per se.

5. There are a number of variations on the indices. For example, Jenkins (1997) proposes an increasing transformation of the Atkinson measure, showing that it is additively-decomposable by population subgroup. The advantage is that one can then write the income measure for the population as a whole as a size-weighted sum of the income measures for each population subgroup, thereby having a consistent accounting framework for summarising who gains and who loses. (Subgroups could be partitions by e.g. age or household type.) For a review of distributionally-sensitive measures of national income, see Lambert (1993), especially Chapter 7 on ‘abbreviated social welfare functions’.
6. Pairs of distributions can be compared using dominance methods, as well as by using indices of social welfare. Saposnik (1981) showed that non-intersection of cumulative distribution functions for income, or equivalently of quantile functions (‘Pen’s Parades’), is equivalent to dominance by all social welfare functions $W(x_1, x_2, \dots, x_N)$ that are non-decreasing in each income x_i . Shorrocks (1983) showed that non-intersection of generalised Lorenz curves is equivalent to dominance by all social welfare functions $W(x_1, x_2, \dots, x_N)$ that are non-decreasing and concave. These two properties are built into the Sen and Atkinson measures cited earlier. Using dominance methods has the advantage that comparisons are made using fewer assumptions about the nature of real income measure (notably which income equality measure is used). The same property is its Achilles heel: if there is non-dominance, additional assumptions are required for ranking distributions, which takes us back to the scalar indices.
7. There is a smaller literature on indices of distributionally-weighted income growth: see e.g. Klasen (1994) for a review. He refers to the conventional calculation of an aggregate growth rate as the ‘income weighted’ growth rate between a base and later year – the overall rate can be written as the weighted sum of income growth for each person in the distribution, where the weights are incomes in the base year. Alternative measures are ‘equally-weighted’ income growth (each person in the base year receives equal weight regardless of income) or ‘poverty-weighted’ income growth (weights are higher the lower is base-year income). His ‘Sen 1’ measure of income growth is the proportionate change in the Sen measure of real national income cited earlier. Distributional perspectives on income growth can also be summarised graphically for pairs of years. The Growth Incidence Curve (GIC) shows the proportionate income growth of each percentile of the distribution plotted against base year percentile: see e.g. Chen and Ravallion (2003). The GIC doesn’t take account of the income mobility accompanying income growth. The poor in the base year are not necessarily the same people who are poor in the later year. More generally, progressive income growth need not reduce inequality if there is sufficient

income reranking (Jenkins and Van Kerm 2006). A graphical device and also indices for the case when one wants to account for both income growth and reranking are developed by Jenkins and Van Kerm (2011).

Empirical issues

8. From a practical point of view, one advantage of real income indices over dominance comparisons is that it is easier to calculate them when doing multiyear comparisons, because the measures are derived using only two pieces of information per year: estimates of mean income and of an inequality index.
9. Thus, distributionally-sensitive indices can be readily calculated with available data. Selected examples illustrating this are discussed below.
10. From a practical point of view, the choice of index or indices depends on several criteria. (The issues are the same as those discussed by Atkinson et al. 2002, in their commentary on the development of social indicators for assessing social progress in the EU.) Inter alia, an index needs to be statistically robust and reliable (in addition to being conceptually valid), transparent and understandable by the audiences to whom it is presented (which include the ‘intelligent layperson’), and available in a timely fashion.
11. The criterion of statistical robustness narrows the choice of measure because it is well-known that some inequality indices are very sensitive to outlier values: specifically the coefficient of variation is unduly sensitive to high-income outliers, and Atkinson inequality indices with aversion parameters of two or more are unduly sensitive to low-income outliers. Jenkins (1997) illustrates the problem with the Atkinson measure with $\epsilon = 2$. (It is not simply an outlier issue, however; measurement error in incomes is generally thought to be greater at the extremes of the income distribution.) One possible way to address these issues is to employ consistent bottom- and top-coding to the unit record data on incomes, though the price paid for greater robustness is then the inability to document the changes at the very top or the very bottom of distributions – groups that may be of special interest. These sorts of measurement issues are likely to be of much greater importance than issues of sampling variability, such as summarised by conventional calculations of standard errors.
12. The criterion of transparency and understandability suggests using measures that incorporate inequality indices that are already commonly-used in official statistics and public discourse about the income distribution. This points to primary use of the Gini-based measure of Sen. The added advantage of the measure is that the Gini coefficient is less sensitive than many other inequality indices to the robustness issues discussed earlier.
13. Timeliness is an issue. Distributionally-sensitive indices require estimates of inequality from household surveys. (Administrative record data with suitable measures of ‘income’ for households are not yet available for the UK, unlike in the Nordic countries.) Currently, the lag between data collection and publication of the UK’s official income distribution statistics is just over a year. (The *Household Below Average Income* statistics for financial year 2010-11, which include the Gini coefficient, are due to be published in early June 2012.) By contrast, preliminary estimates of GDP are available within months (and rely on modelling and imputation for their derivation). If having more timely income distribution measures becomes a greater priority than as currently, a greater investment of

resources (e.g. on government statisticians) to speed up the current procedures will be required. At the same time, more use could be made of tax-benefit microsimulation models based on less up-to-date data in order to ‘nowcast’ the contemporary income distribution before the survey-based estimates became available – just as modelling and imputation are employed for deriving timely GDP estimates. Illustrations of how such models may be used to ‘nowcast’ are provided in the assessment of the impact of the Great Recession on household income by Jenkins et al. (2012).

14. There are additional empirical issues that would need to be resolved, including the choice of price index used to make comparisons across time within a country, or comparisons between countries, e.g. the UK and other EU nations. (On the former, there is also the issue of whether the same price index should be applied to all households, or whether price indices should also differ by income group, region, etc.) On cross-national price comparisons, Purchasing Power Parities, etc., see inter alia, Deaton (2010) and the references therein, and the Stiglitz, Sen, and Fitoussi (2009) report. In the UK case, the price indices currently used to adjust incomes when preparing the *Household Below Average Income* (HBAI) statistics – which include estimates of real mean household income – are obvious candidates (and appear to be relatively noncontroversial).
15. An additional issue is the source of the estimate of ‘mean income’, where the leading candidates are GDP per capita or total household sector income per capita (both derived from national accounts) or a survey-based measure of mean household income. As is well-known, income concepts in the national accounts and household surveys differ, e.g. they include different income sources, and have different coverage (e.g. the total economy versus private households). See e.g. the LSE GC presentation by A. B. Atkinson or, for further discussion with reference to a developing country context, see Deaton (2005) and references therein, and the Stiglitz, Sen, and Fitoussi (2009) report. Use of survey data also allows one to use a different equivalence scale than the ‘per capita’ scale used to adjust GDP when deriving GDP per capita. (The HBAI statistics currently use the ‘modified OECD’ equivalence scale, as do official EU statistics on income.) Since the inequality measure refers to households, it seems preferable to use the survey-based measure. The main exception is for very long term comparisons, for which survey based measures may be unavailable, but GDP based measures are. See the illustration for Italy below.

Illustrations

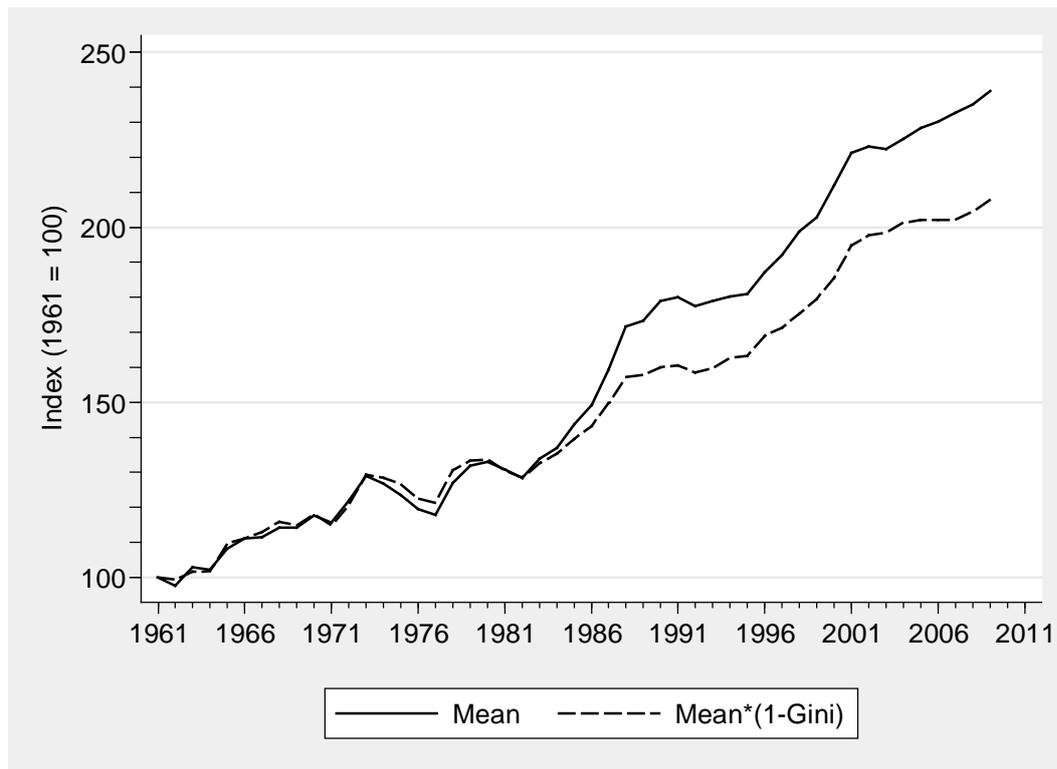
16. This section provides a small number of examples to illustrate the point that distributionally-sensitive measures of national income and its growth may be calculated relatively straightforwardly, and that estimates based on these measures do provide a different perspective from the one based on estimates of mean income.
17. Figure 1 compares growth rates between 1961 and 1991 in the incomes of UK households according to mean income and the Sen measure of ‘real national income’. (A. B. Atkinson presented a picture derived from the same data source at the LSE GC meeting on 2 May 2012.) From the late 1980s onwards, the growth rate in the Sen measure has been lower than the growth in real mean income, reflecting the substantial rise in inequality during the 1980s in particular. Up to the late 1970s, it is the growth rate for the

Sen index that is slightly higher: this was a period when income inequality was falling. Using the same data source, but for the period 1961–1991, Jenkins (1997) calculated a larger number of measures, showing how estimates of income growth were smaller during the 1980s, the greater degree of aversion to inequality.

18. Cross-national comparisons of cross-country comparisons of aggregate welfare for the 1970 to 2000 period are undertaken by Gruen and Klasen (2008), who summarise their findings as showing that their ‘welfare measures ... drastically change the impression of levels of welfare, significantly affect the welfare ranking of countries in different benchmark years, affect changes in ranking over time, and affect convergence between industrialized and developing countries’ (2008: 212).
19. Trends in real national income over 150 years in Italy are shown in Figure 2. This is a case in which estimates of GDP per capita is used for the ‘mean’, and the inequality indices are compiled from a range of sources. Over the long-term, the distributionally-sensitive indices show greater growth than does the mean, with the greatest growth for the most distributionally-sensitive index (based on an Atkinson inequality aversion parameter of 2) and the least growth for the mean.
20. Income growth rates are shown explicitly in Figure 3 – these are examples of Growth Incidence Curves calculated from data published in the UK’s official income distribution statistics (the *HBAI*). Also shown are the growth rates in mean income for each period. The chart clearly shows a striking change in the pattern of income growth in the late 1990s (when it was relatively pro-poor) and the mid-1990s (when income growth was greater, the richer the base-year income group). Examples of multi-country calculations are the estimates of income growth rates by quintile group for two periods, mid-1980s to mid-1990s, and mid-1990s to mid-2000s, by the OECD (2008) in its *Growing Unequal?* report. (The numbers are reproduced in Table 3 of Stiglitz, Sen, and Fitoussi (2009: 118).) The tables reveal not only that income growth rates were greater for the richest fifth in each country than the poorest fifth or the middle-three-fifths, in both periods, and for most countries, but also that real income growth was actually negative for the poorest fifth in a number of countries. This diverse experience is not revealed by a focus on the growth in the overall mean.
21. Distributionally-weighted growth rates for the USA, for periods corresponding to the ‘Kennedy and Johnson’ and ‘Reagan’ administrations are shown in Figure 4, taken from Klasen (1994). Regardless of the real income measure used, the growth rate was greater in the earlier period than the later one. But also observe that growth rates in ‘real national income’ are lower in the Reagan period, the more distributionally-sensitive the index is (i.e. as one moves left to right across the chart) The estimate based on the mean is adjusted downwards because inequality had risen. By contrast, lower inequality in the Kennedy period means that the distributionally-sensitive measures of growth rates are somewhat higher than those for mean income.

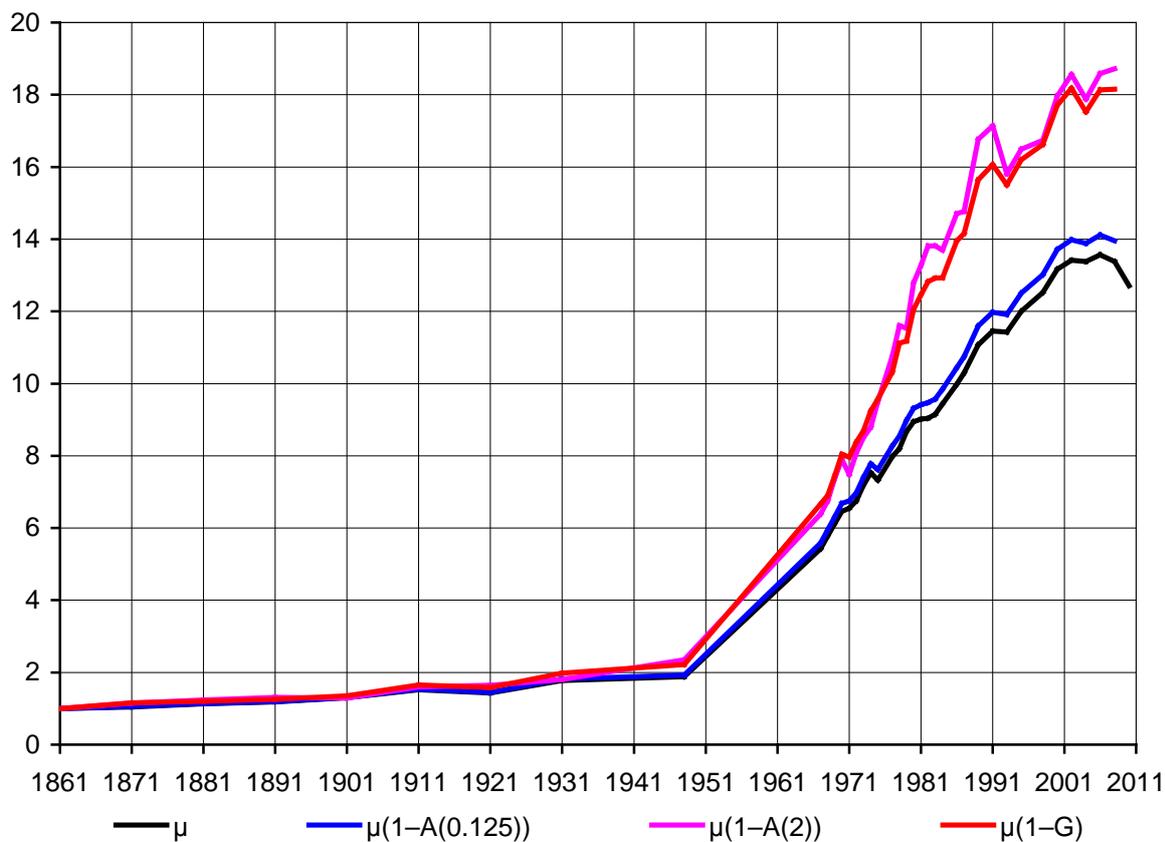
Acknowledgements. I wish to thank Andrea Brandolini for helpful discussions, and for sharing Figure 2 in particular.

Figure 1. Trends in mean income and inequality-adjusted mean income, UK, 1961–2009/10



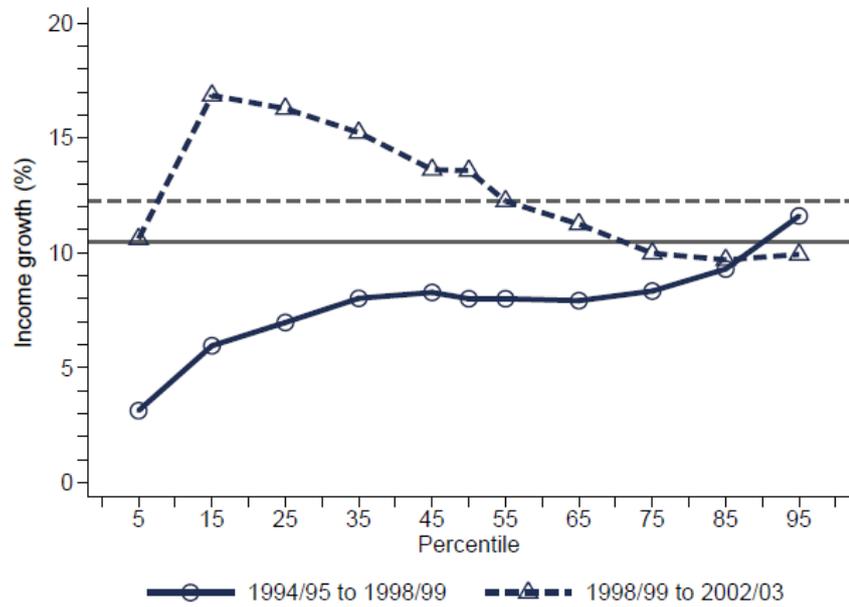
Source: www.ifs.org.uk/bns/bn19figs.xls, Spreadsheet to accompany IFS Commentary 118, *Poverty and Inequality in the UK: 2011*. Years refer to financial years from 1994/95. Data sources are the FES, and the FRS from 1994/95. The data refer to GB, and to the UK from 2002/03 ('2002' in the chart). Income is real equivalized net household income before the deduction of housing costs.

Figure 2. Distributionally-adjusted growth rates: Italy, 1861–2010 (indexed 1861 = 1)



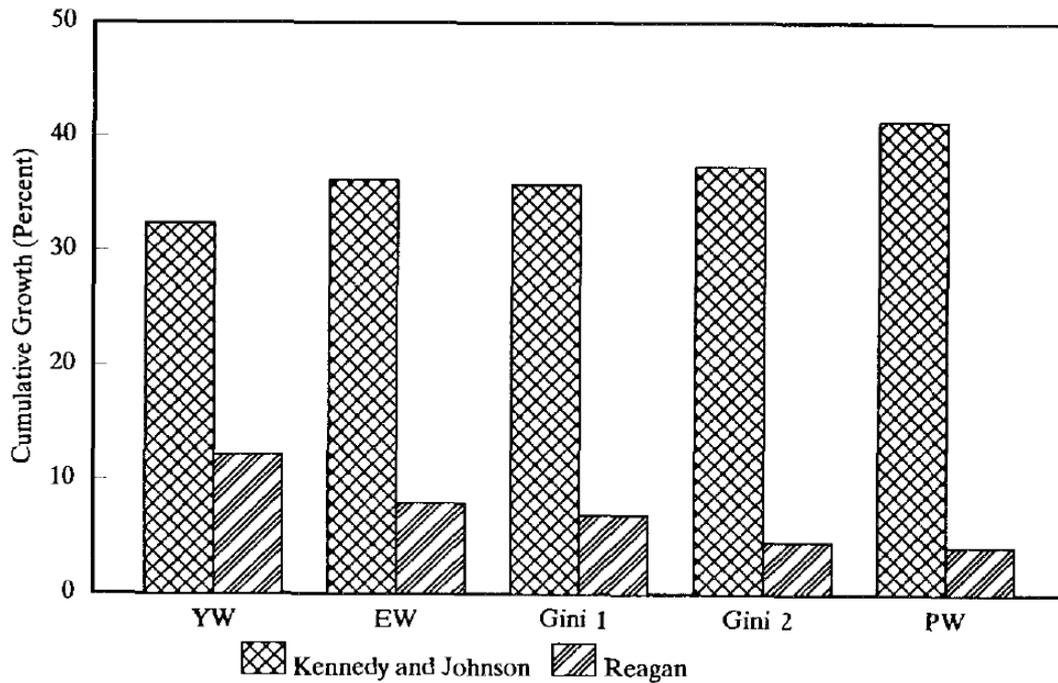
Source: Brandolini, A. and Vecchi, G. (forthcoming), 'Standards of living', in G. Toniolo (ed.), *The Oxford Handbook of the Italian Economy, 1861-2011*, Oxford University Press. μ is GDP per capita, G is the Gini index, and $A(\varepsilon)$ is the Atkinson index of inequality with inequality aversion parameter ε .

Figure 3. Income growth (%) of selected percentiles, UK, 1994/95–1998/99 and 1998/99–2002/03



Note: The horizontal lines show the growth rate of mean income in each period.
 Source: calculations by Jenkins and Van Kerm (2011) from DWP (2008, Table 2.1ts, *Households Below Average Income*).

Figure 4. Distributionally-weighted income growth rates (%), USA 1961–68 versus 1981–88



Source: Calculations based on Table 3.

Figure 4. Well-Being Growth, Kennedy–Johnson vs. Reagan: Cumulative Growth of Well-Being Indices 1961–68 vs. 1981–88

Source: Klasen (1994, Figure 4). YW is the income-weighted index; EW the equal-weights index; PW is the ‘poverty-weighted’ index; and ‘Gini 1’ is the proportionate change in the Sen measure of real national income.

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