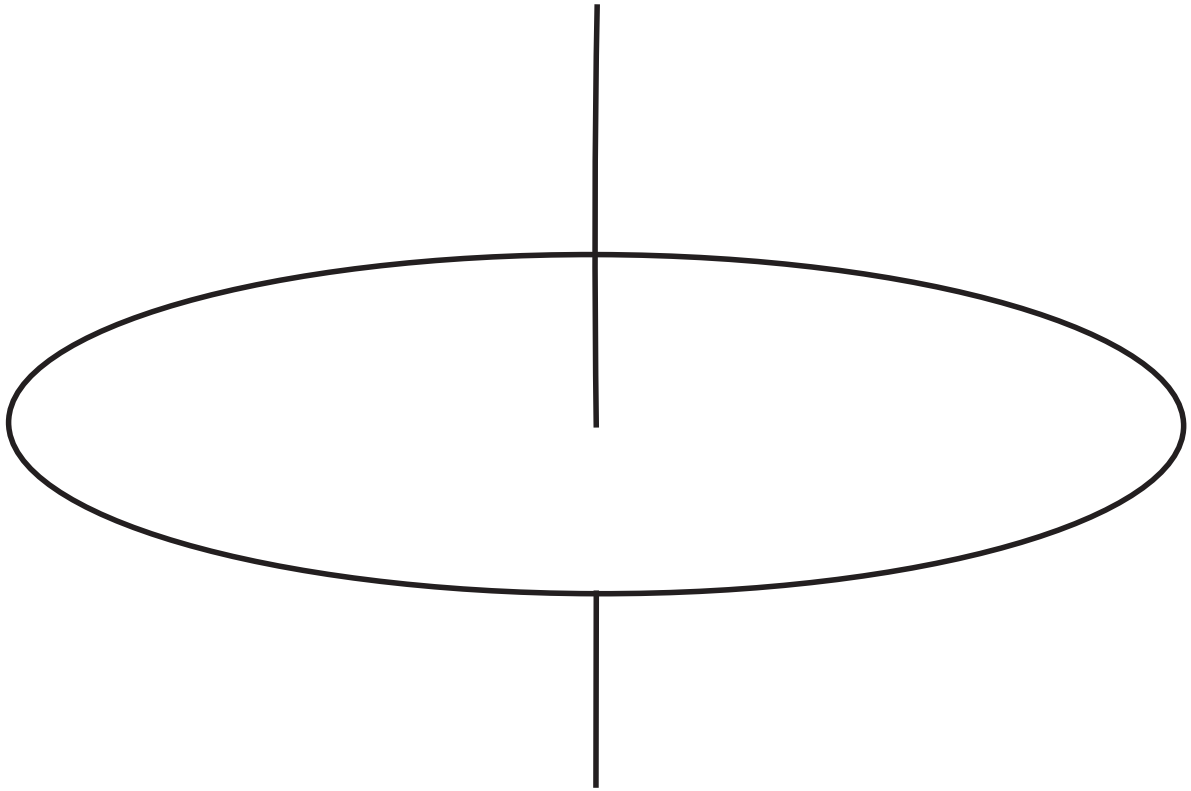
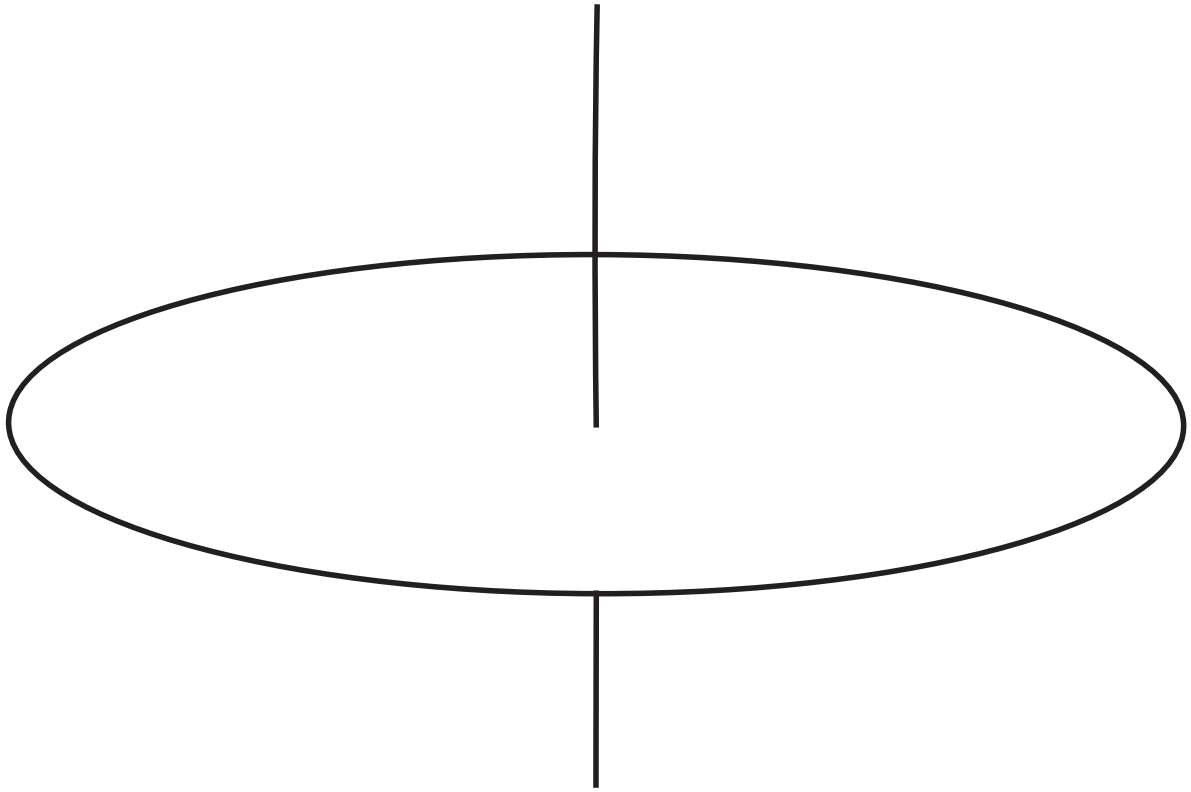
On the Concept of "Rotation" in Relativity Theory

David B. Malament





What does it mean to say that the ring is *not-rotating* about the axis?

Principal Claims:

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In some circumstances allowed by relativity theory (not all) ...

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(b) None of these criteria fully answers to our classical intuitions.

(c) It is possible to capture (b) in the form of a "no-go theorem".

Three criteria of non-rotation:

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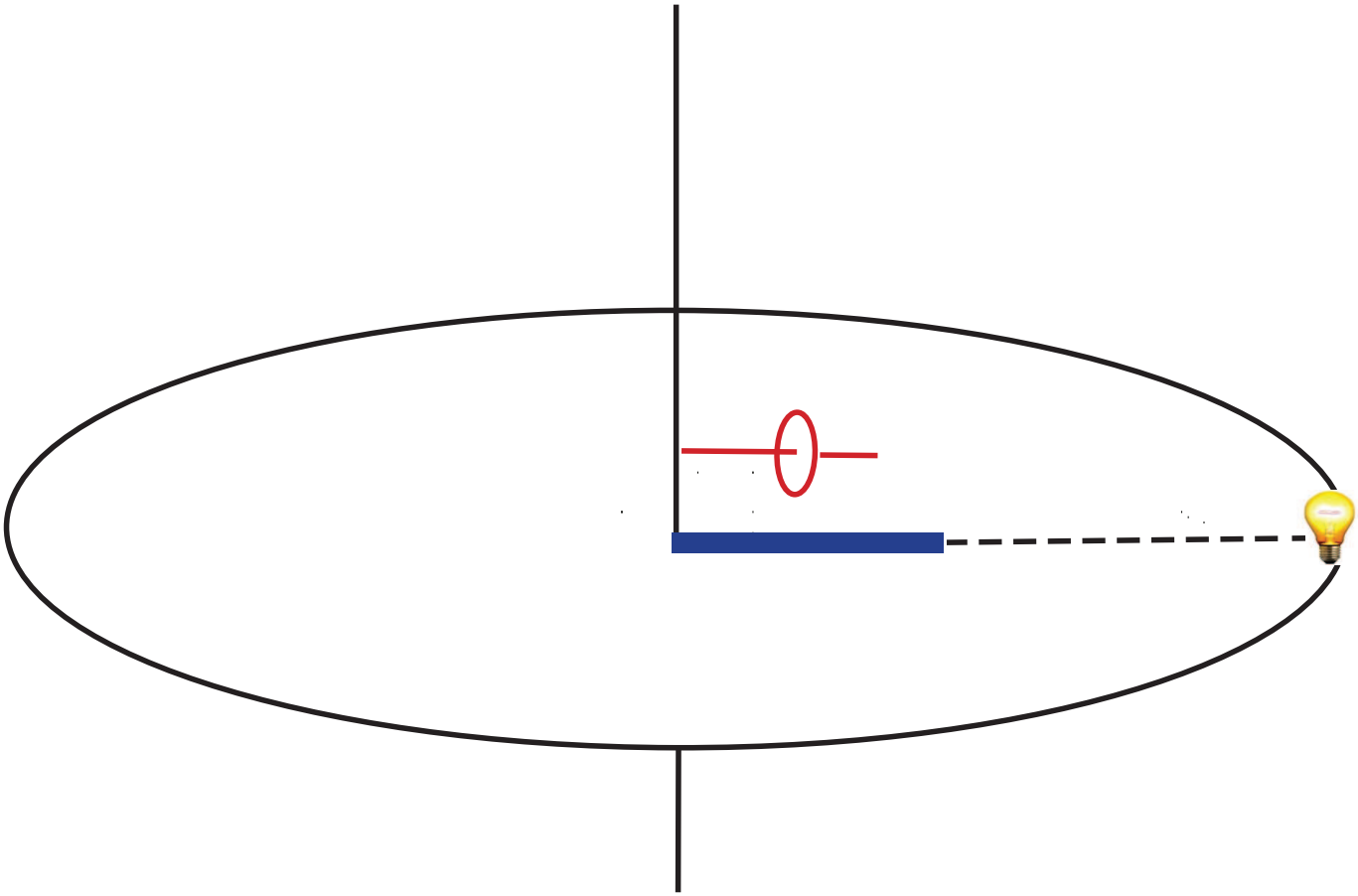
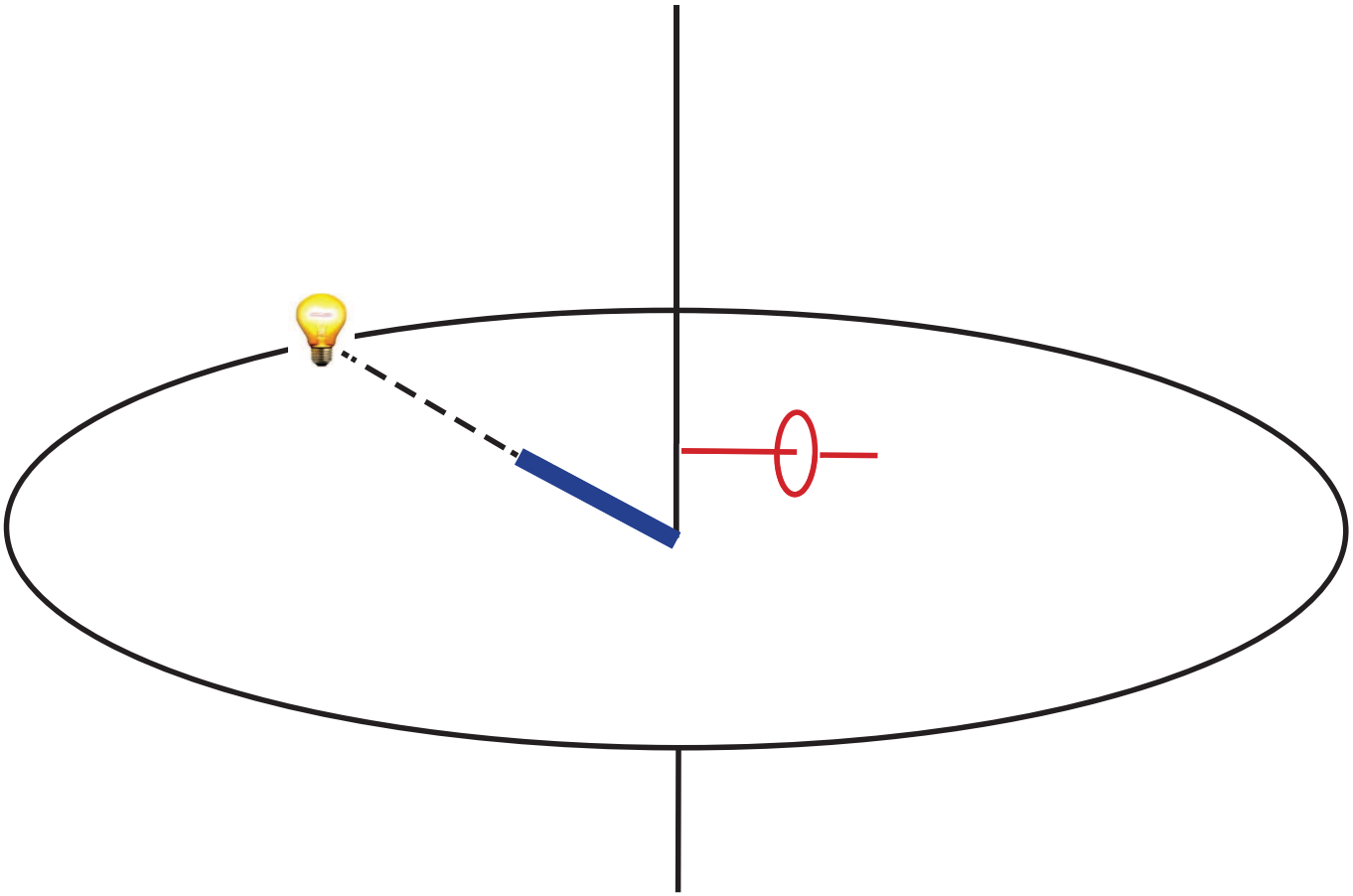
(3) zero angular momentum (ZAM)



CIA criterion of non-rotation



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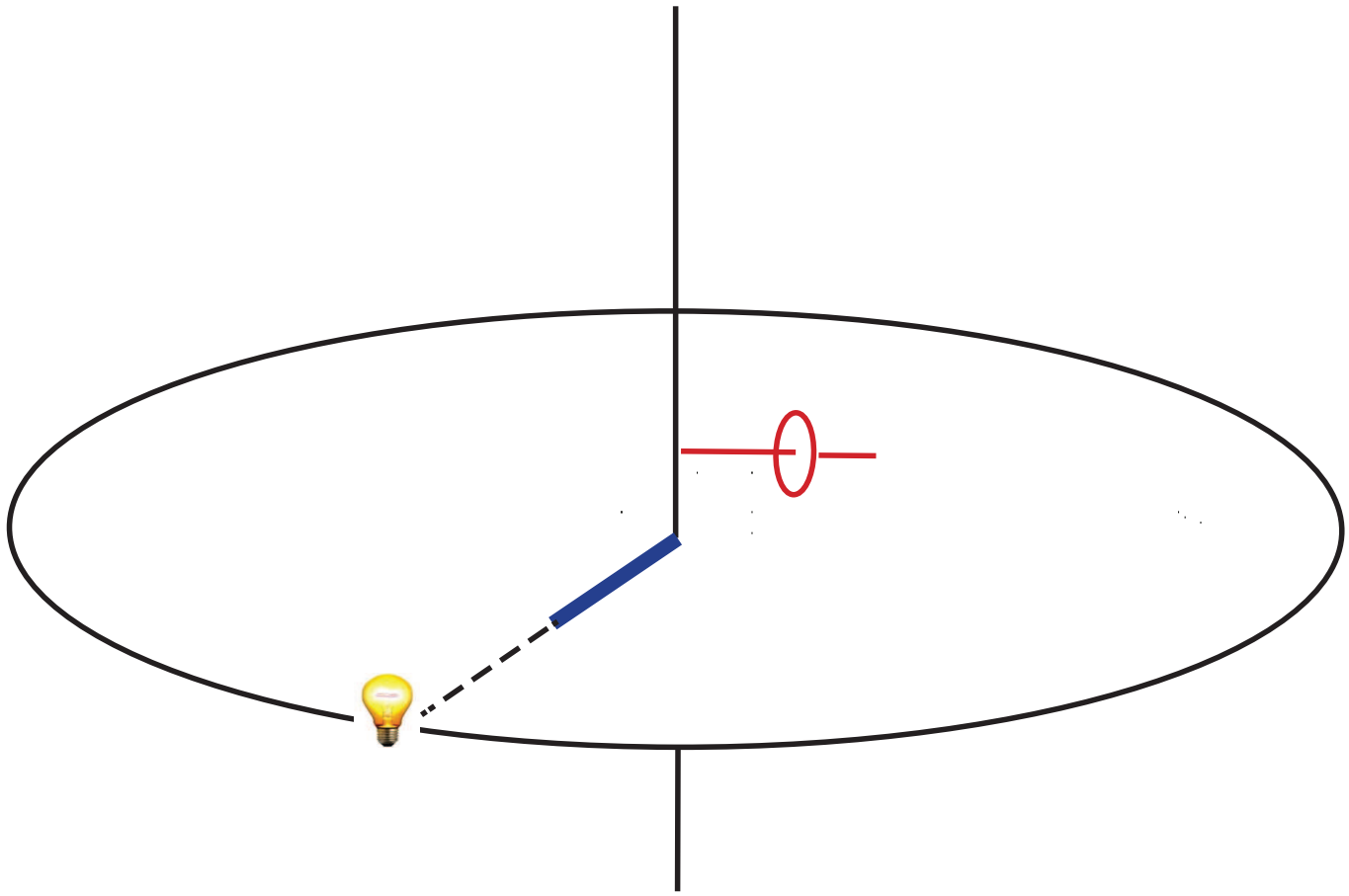

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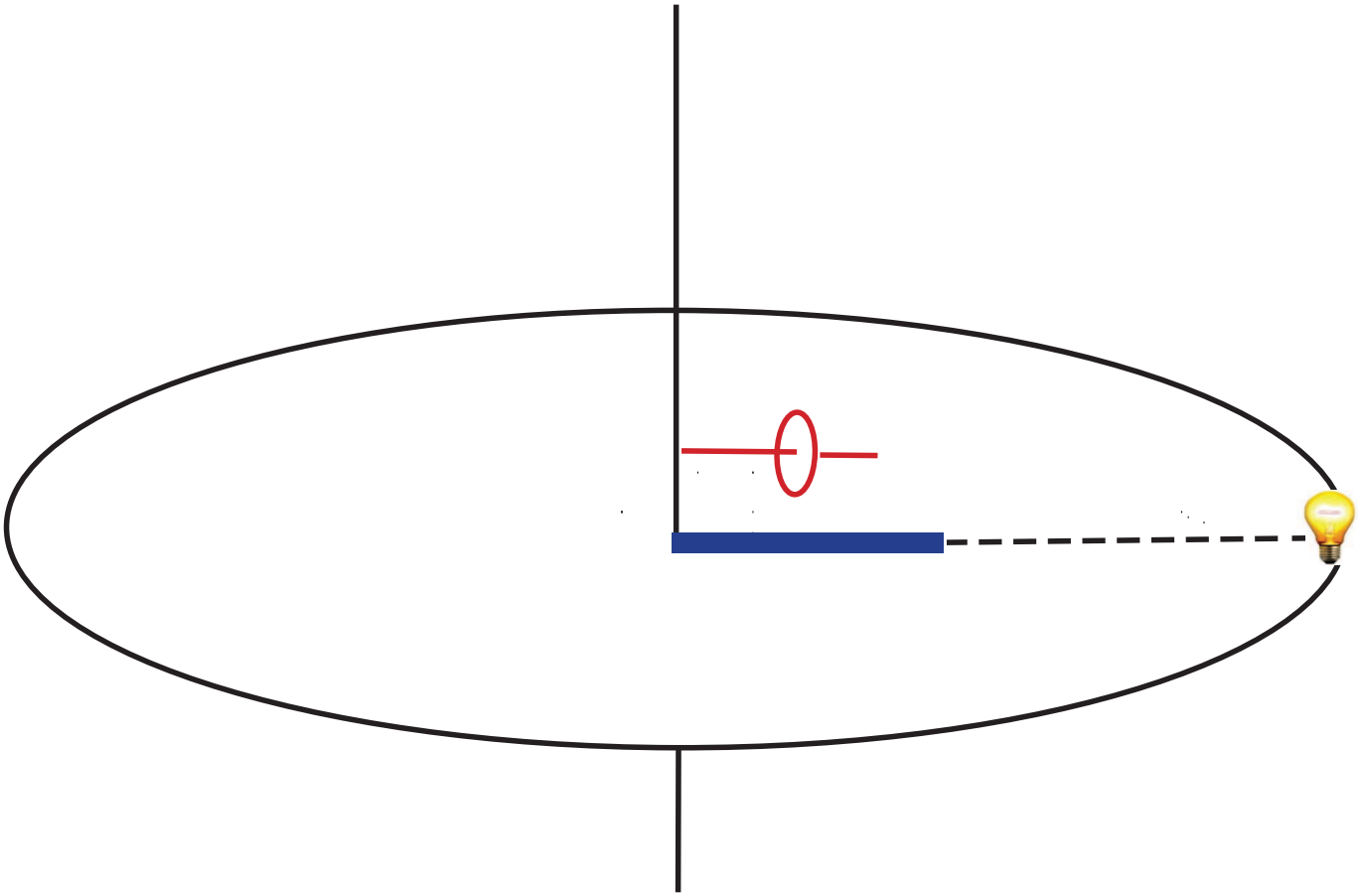

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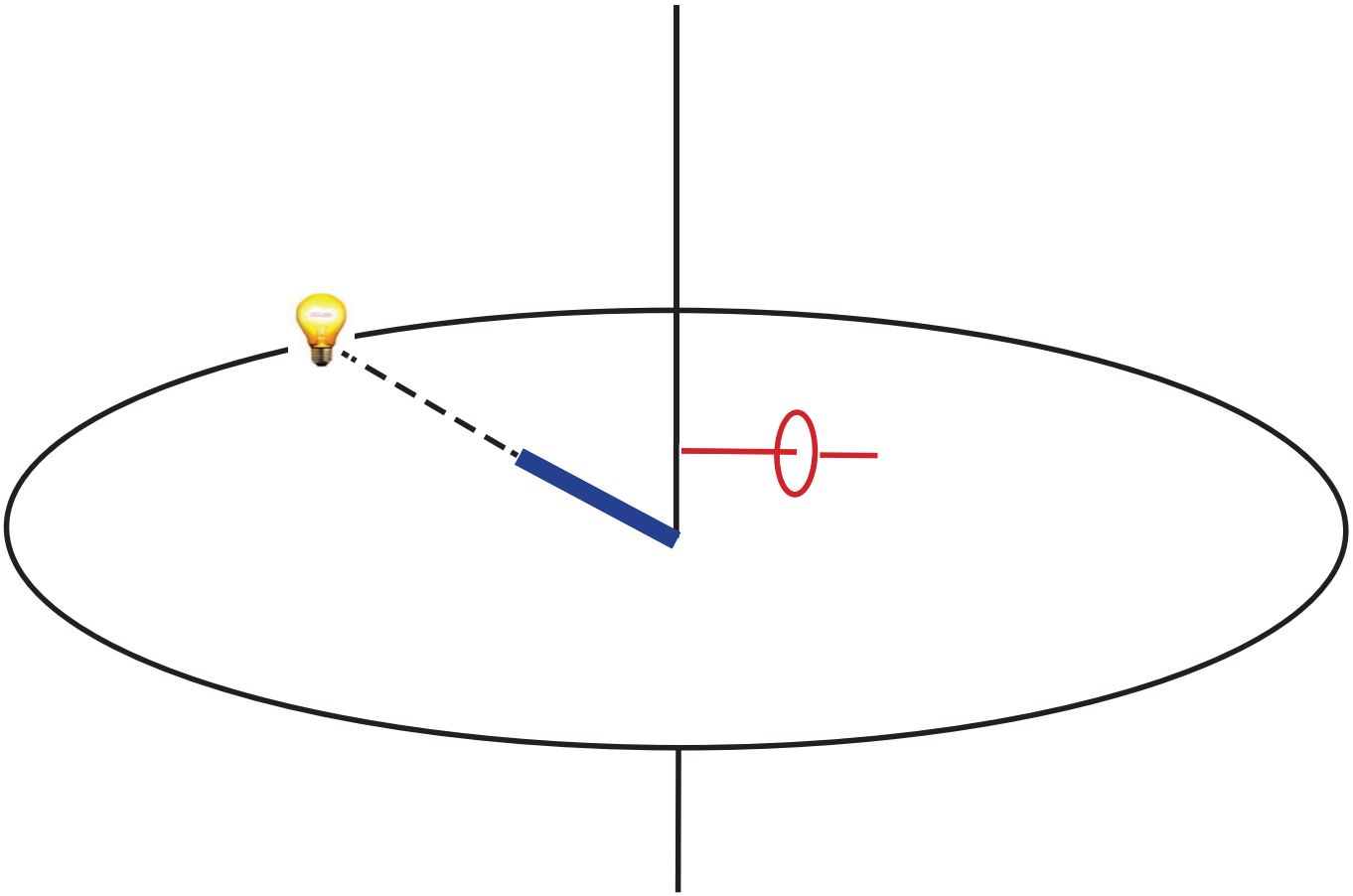

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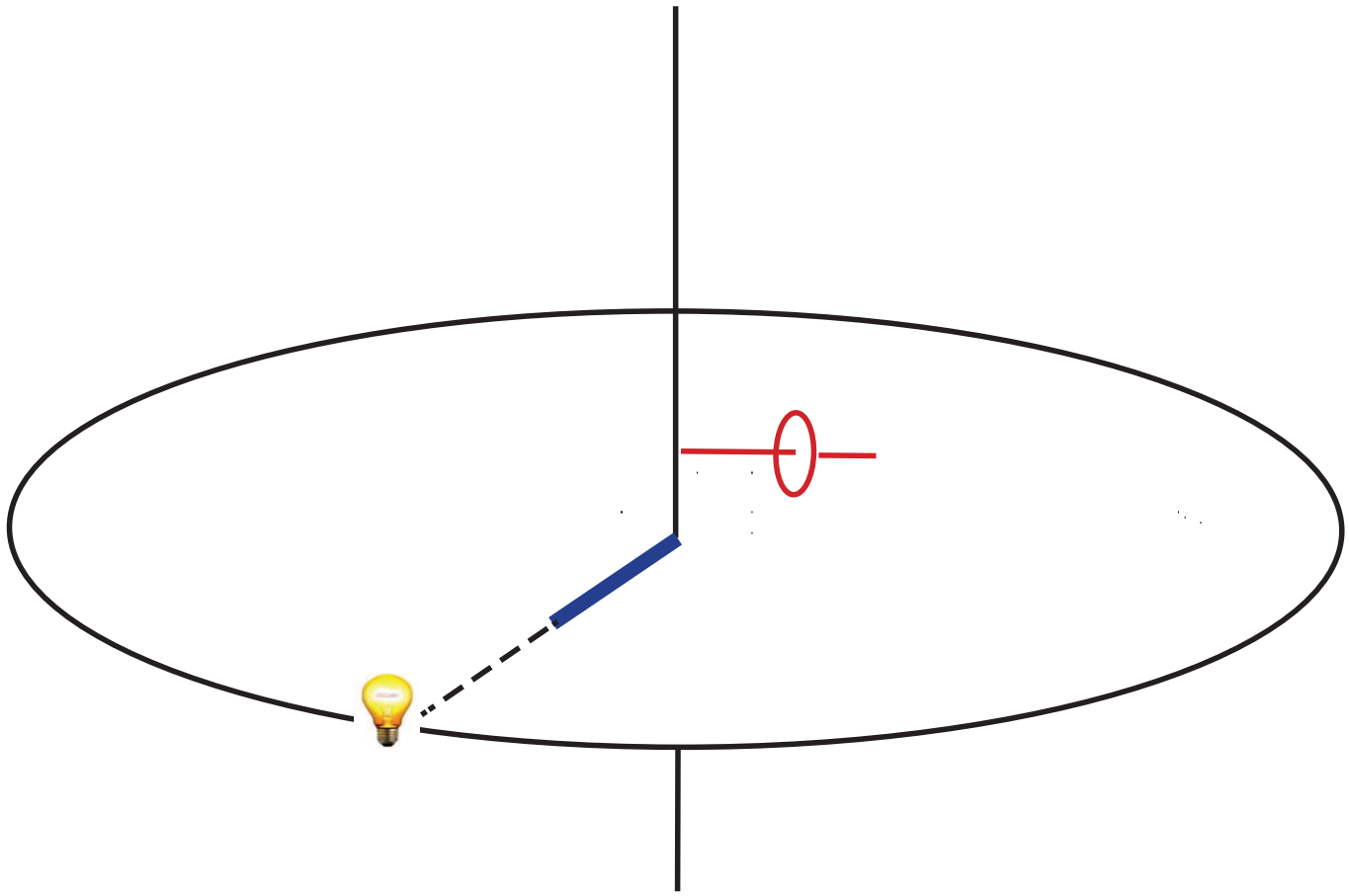

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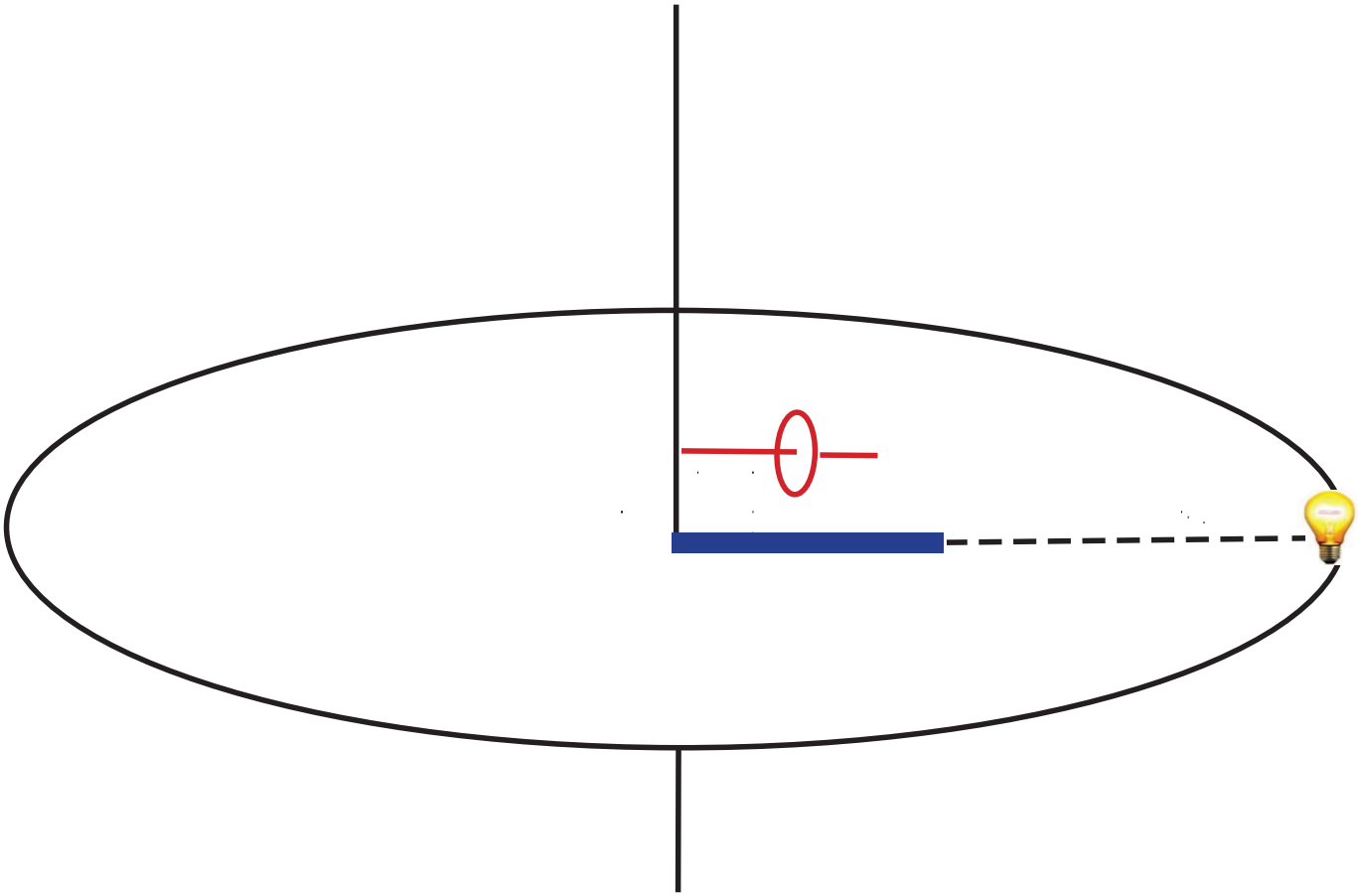
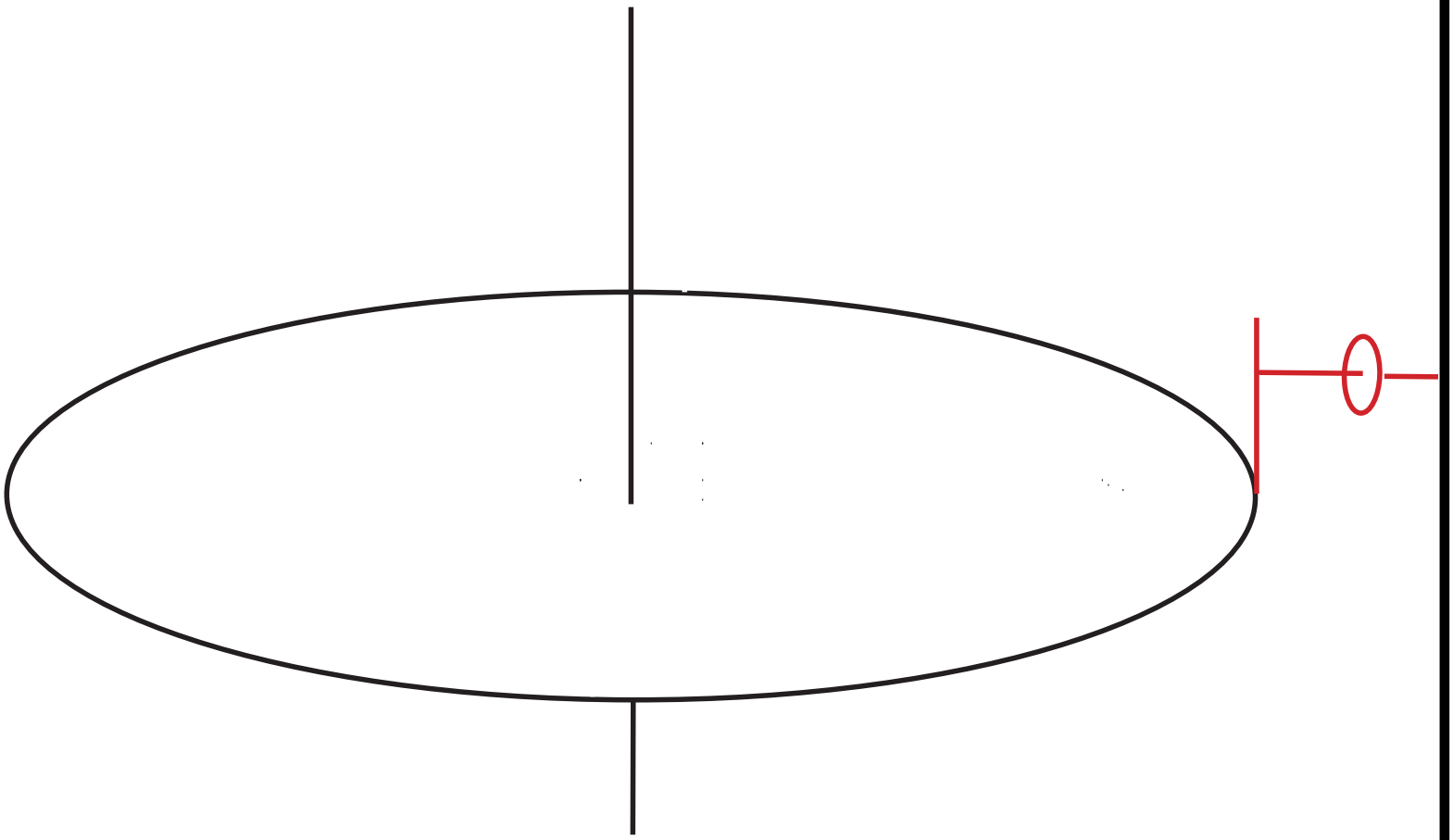
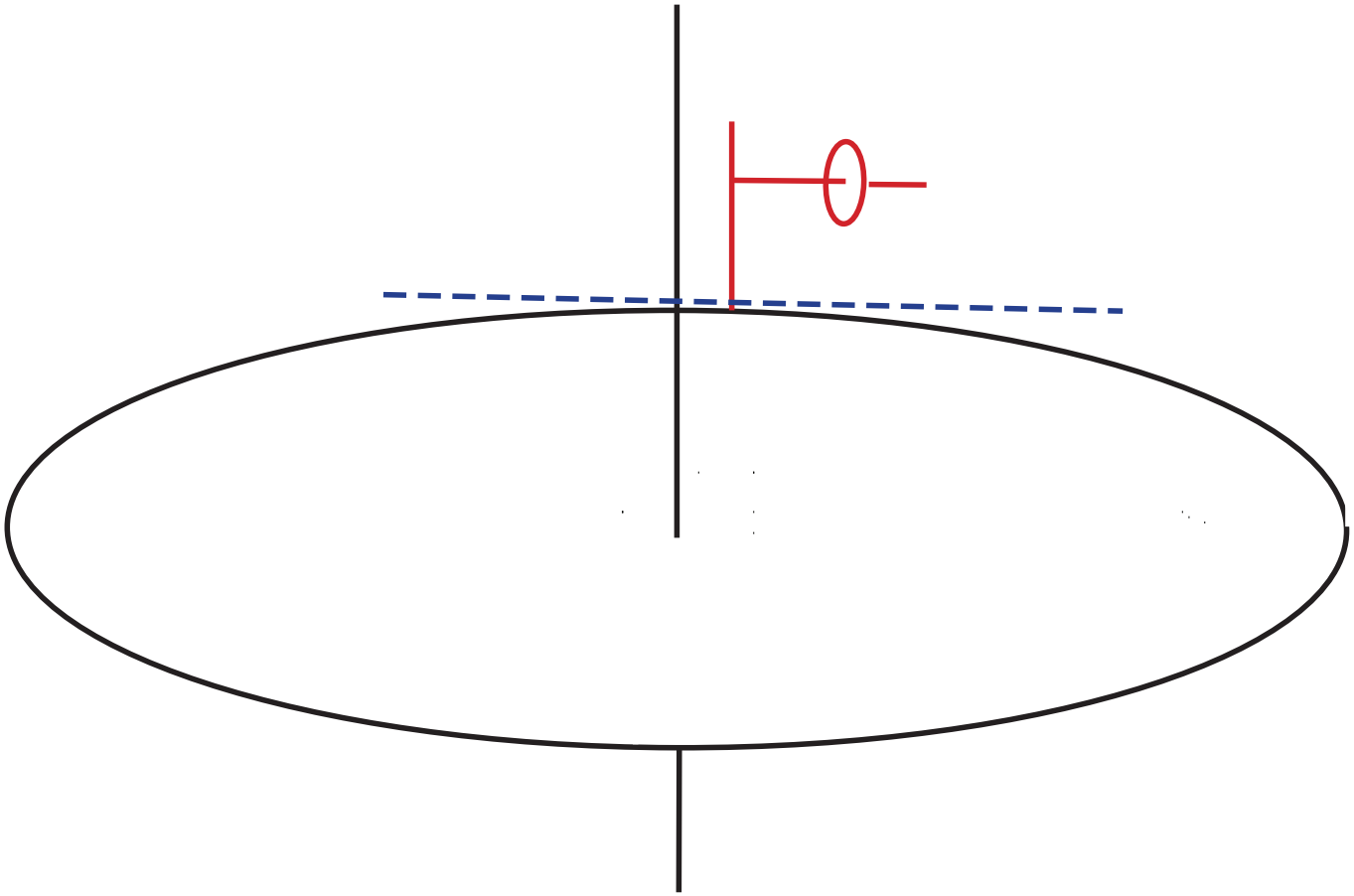


CIA criterion of non-rotation

We could also set this up with a water bucket.



CIR criterion of non-rotation



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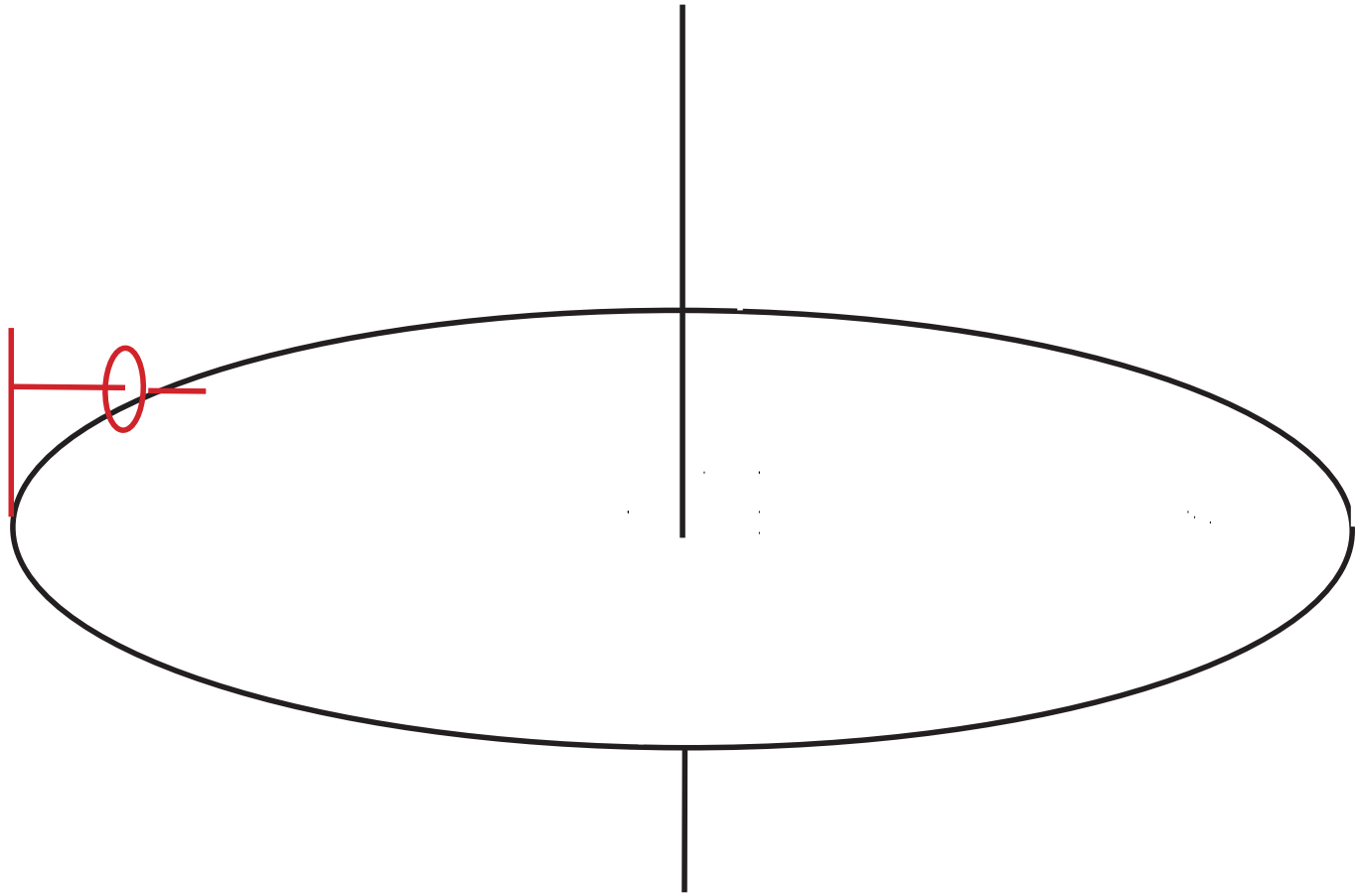

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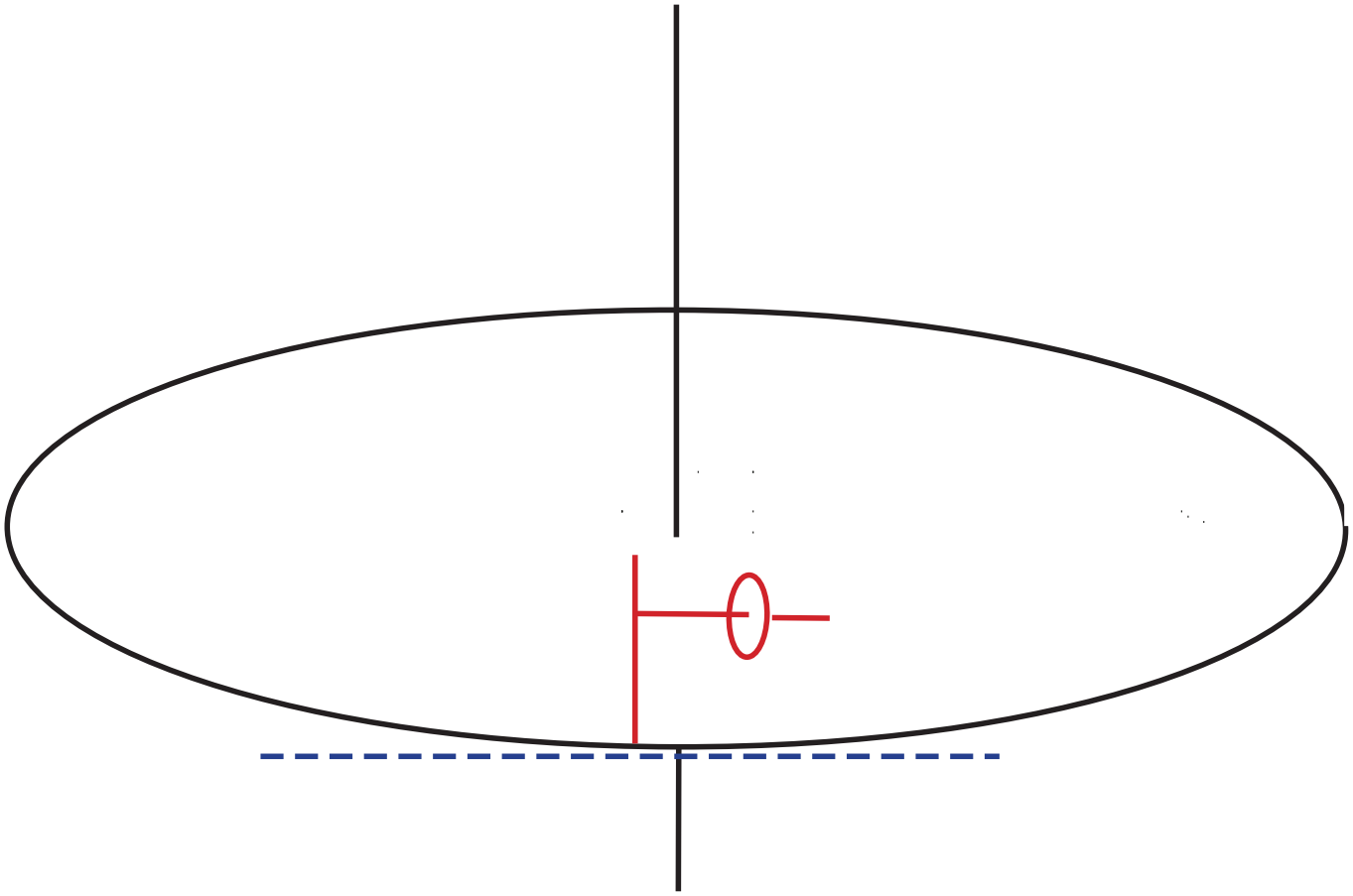

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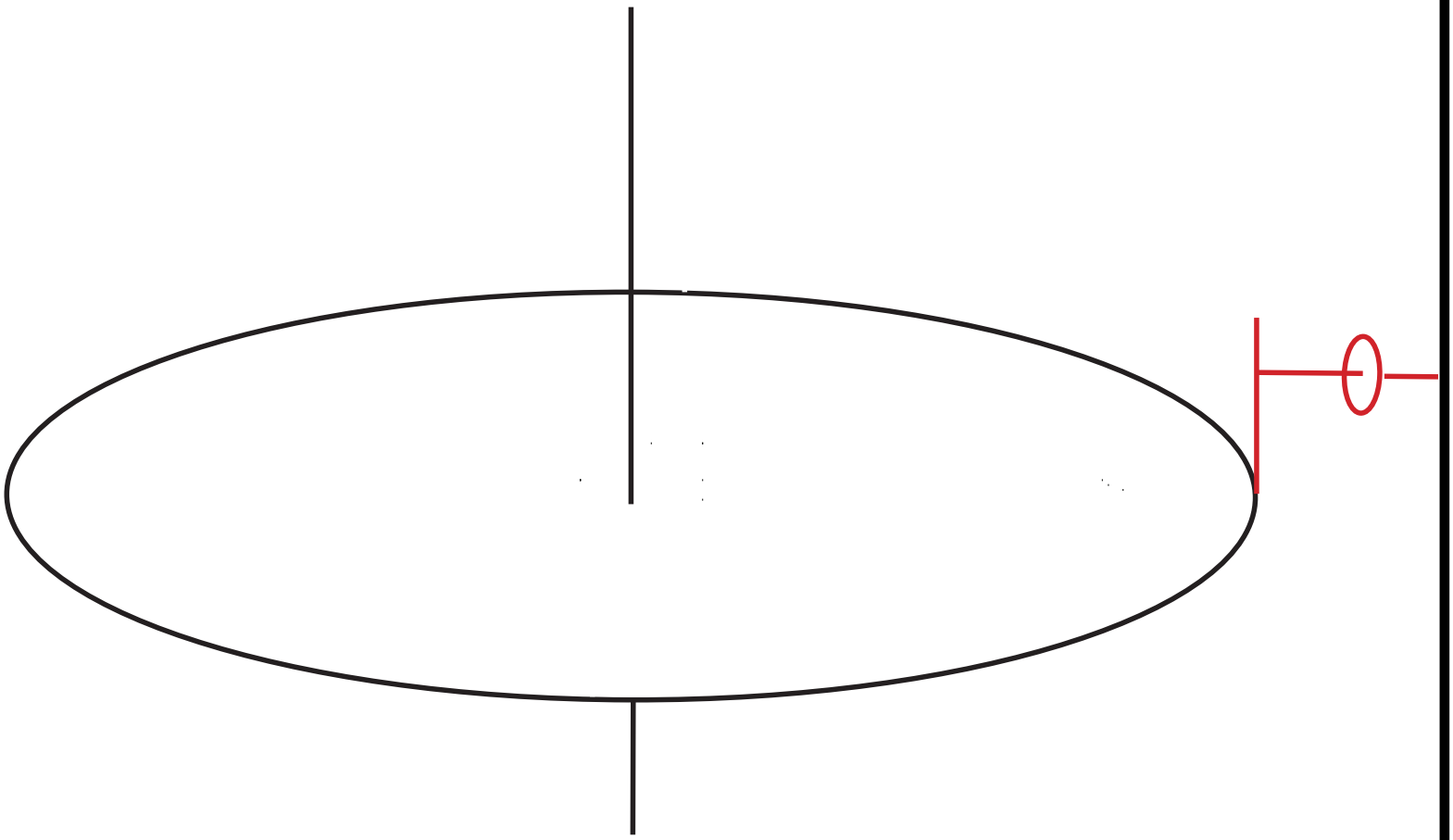

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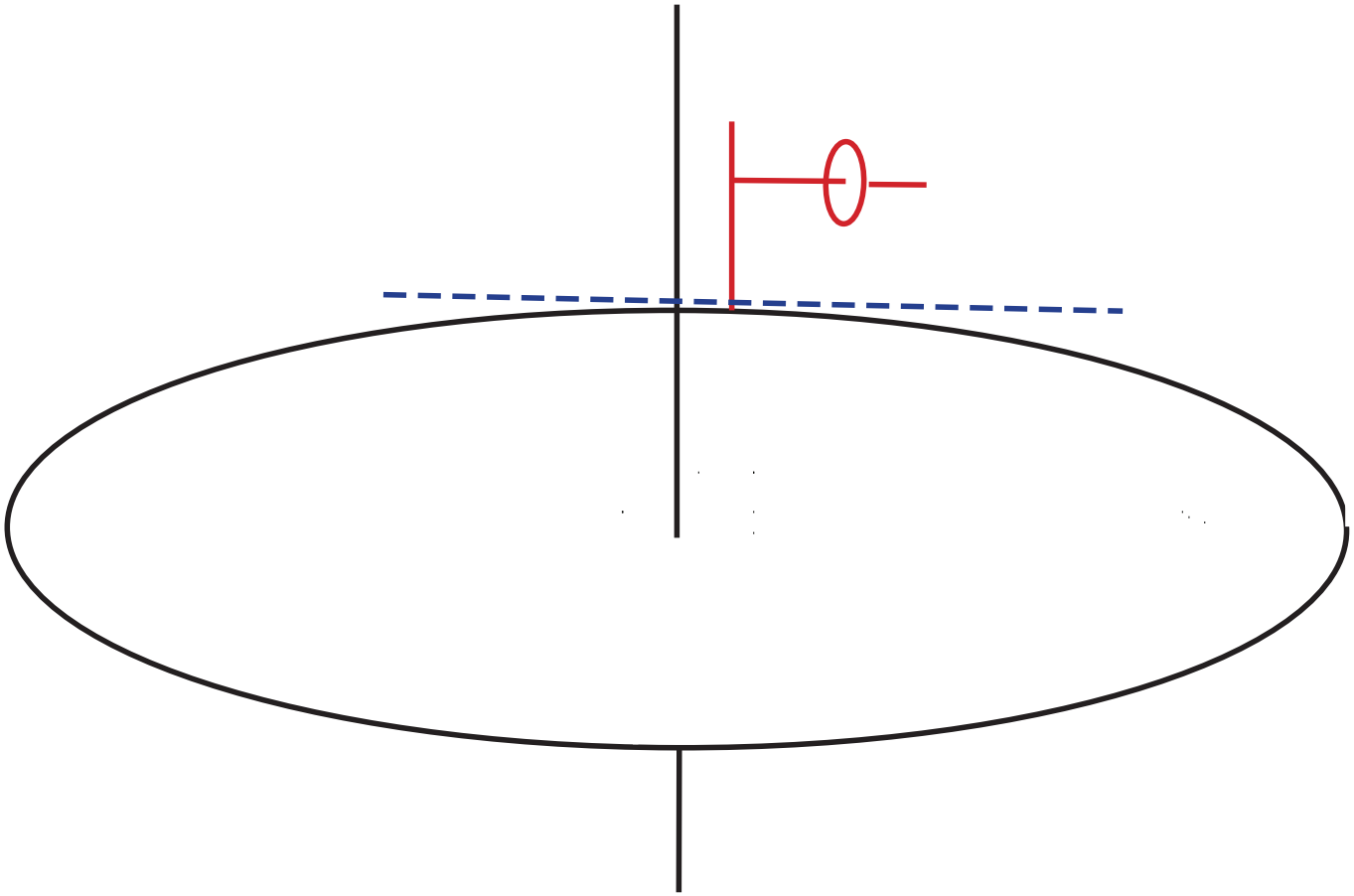

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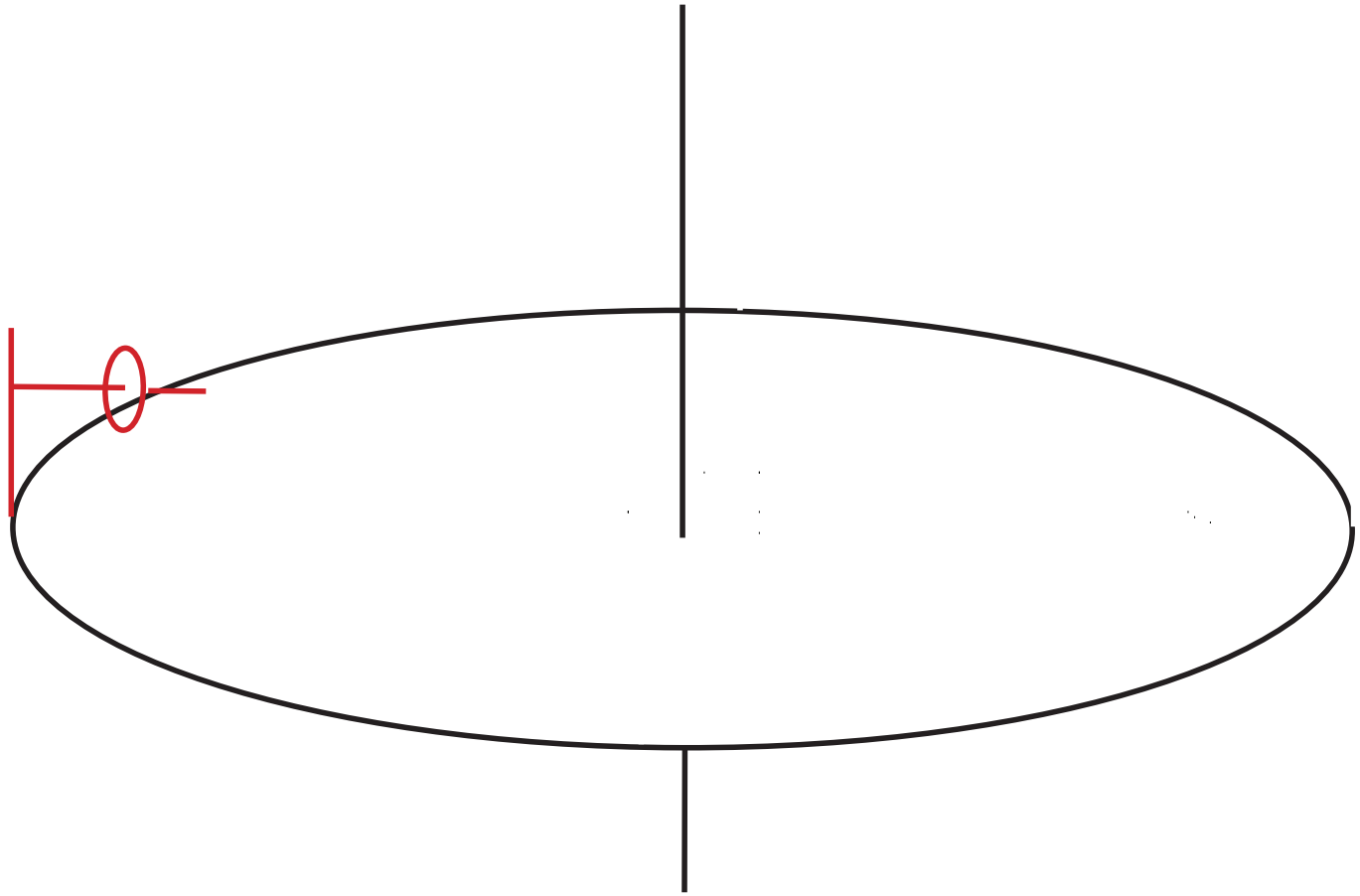

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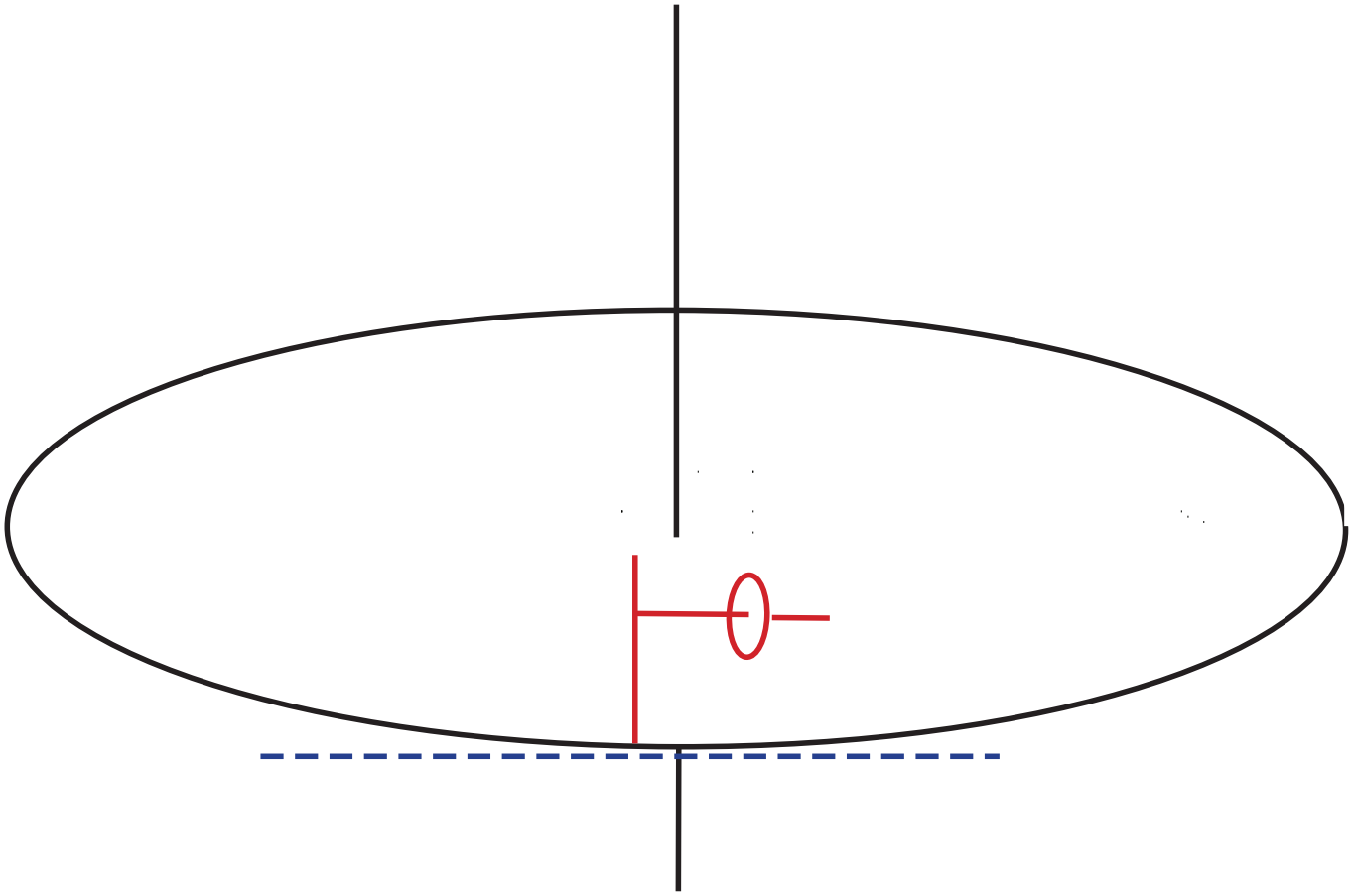

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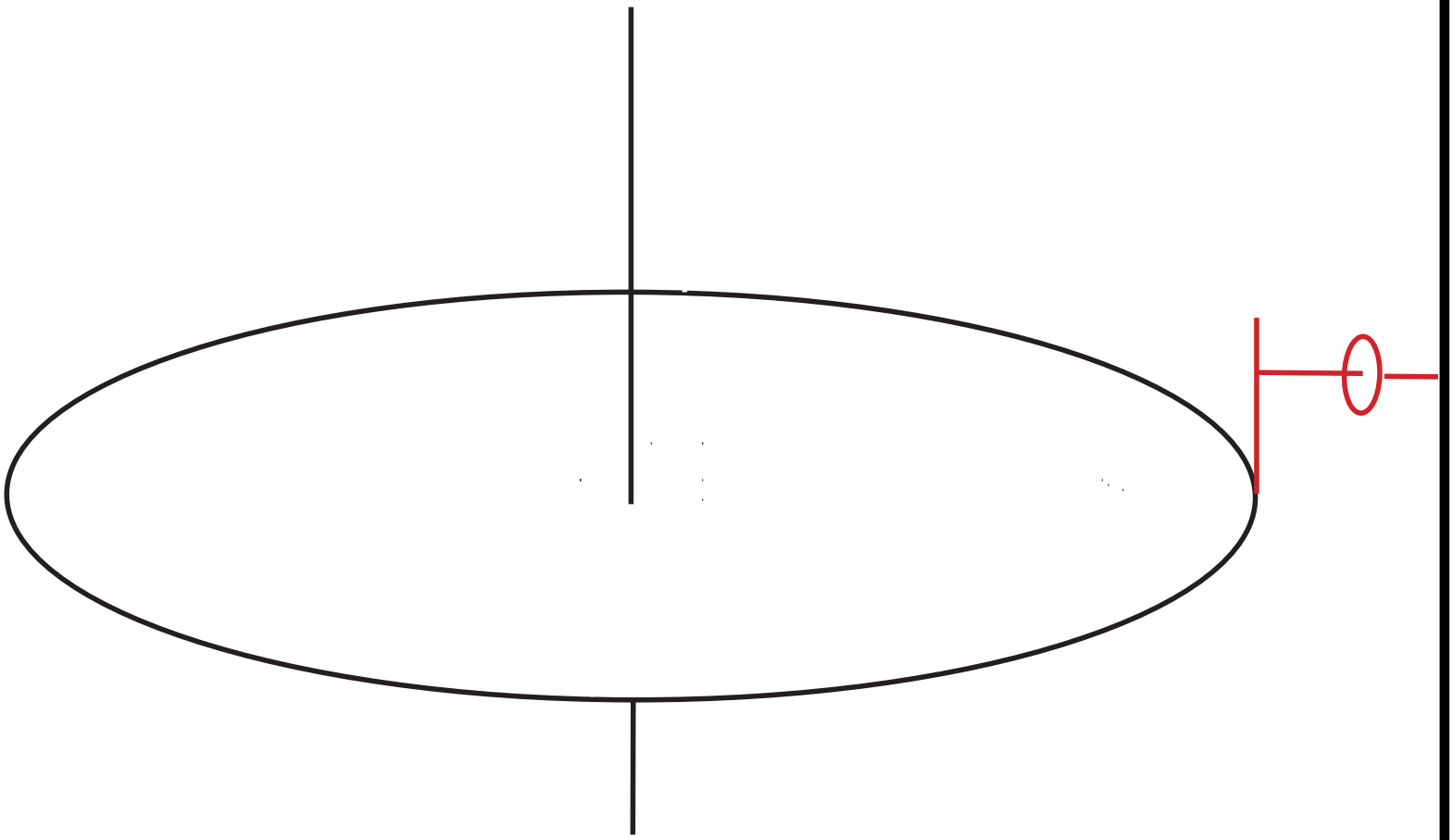

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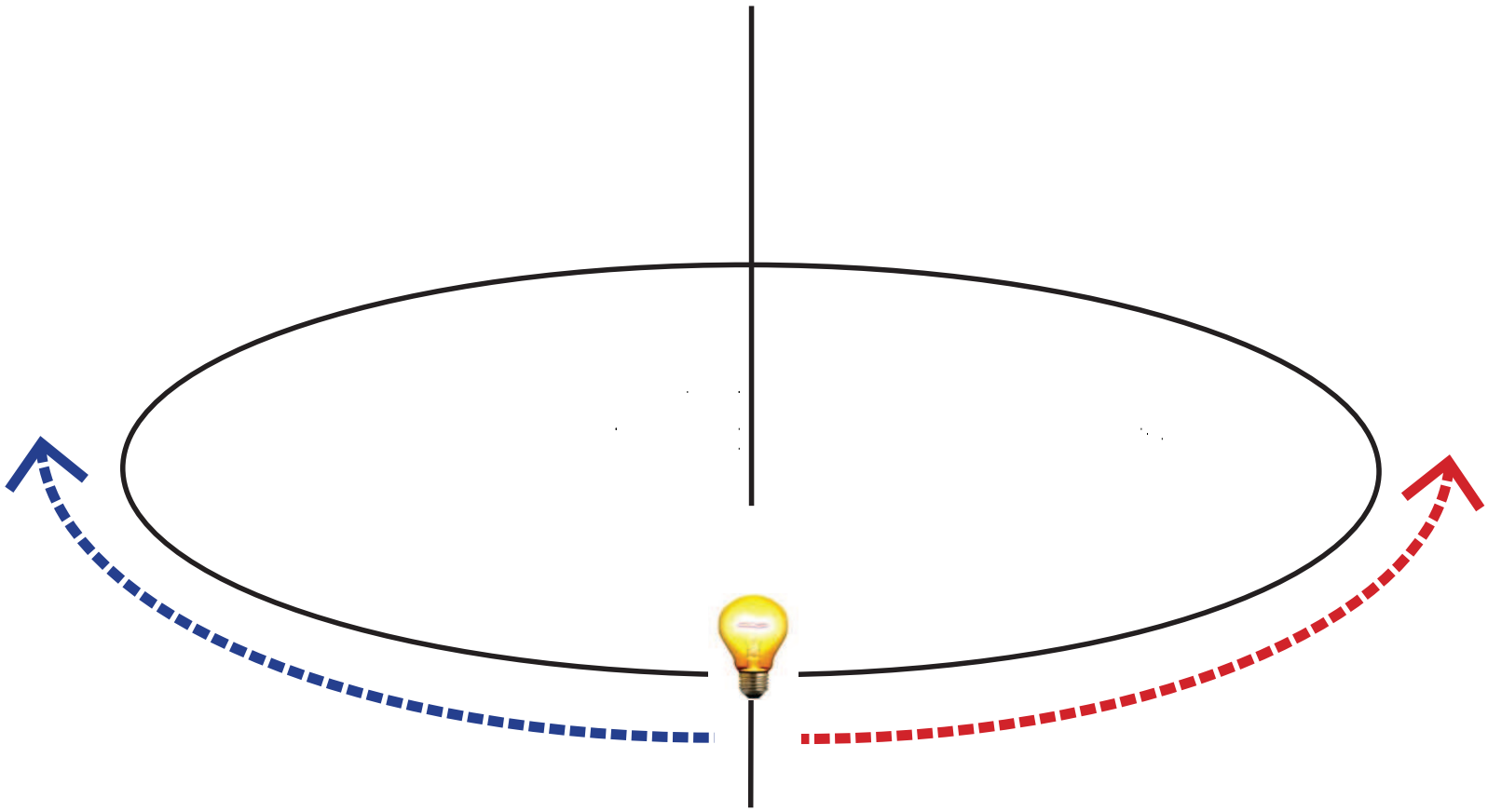
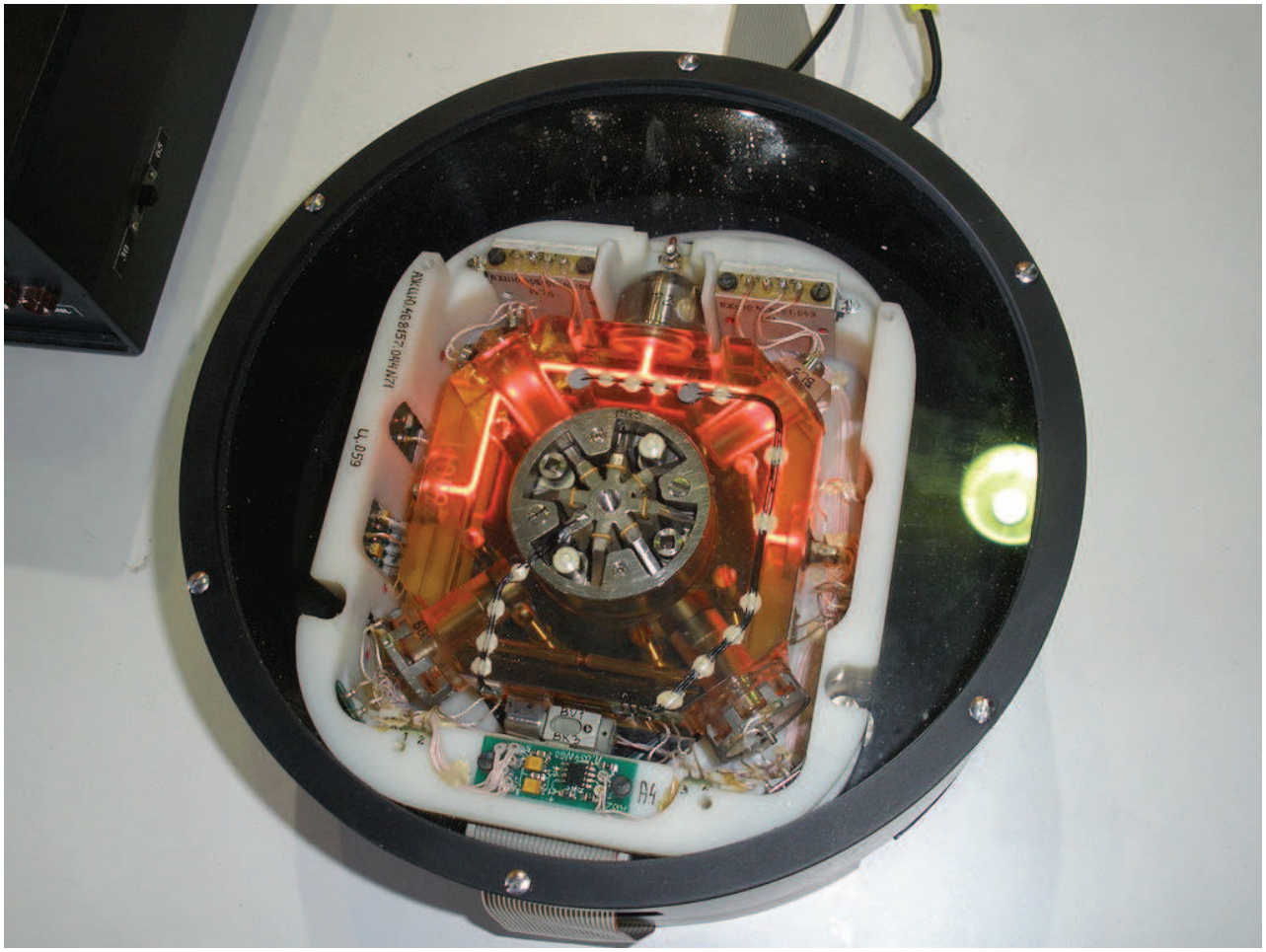


ZAM criterion of non-rotation



Ring Laser Gyroscope (courtesy of Wikipedia)

Do the three criteria (CIA, CIR, ZAM) agree?

First Point:

In some relativistic spacetime models – including ones that may well describe regions of our universe, e.g., the Kerr solution – no two of the three criteria agree.

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criteria of
non-rotation

conditions on
criteria of
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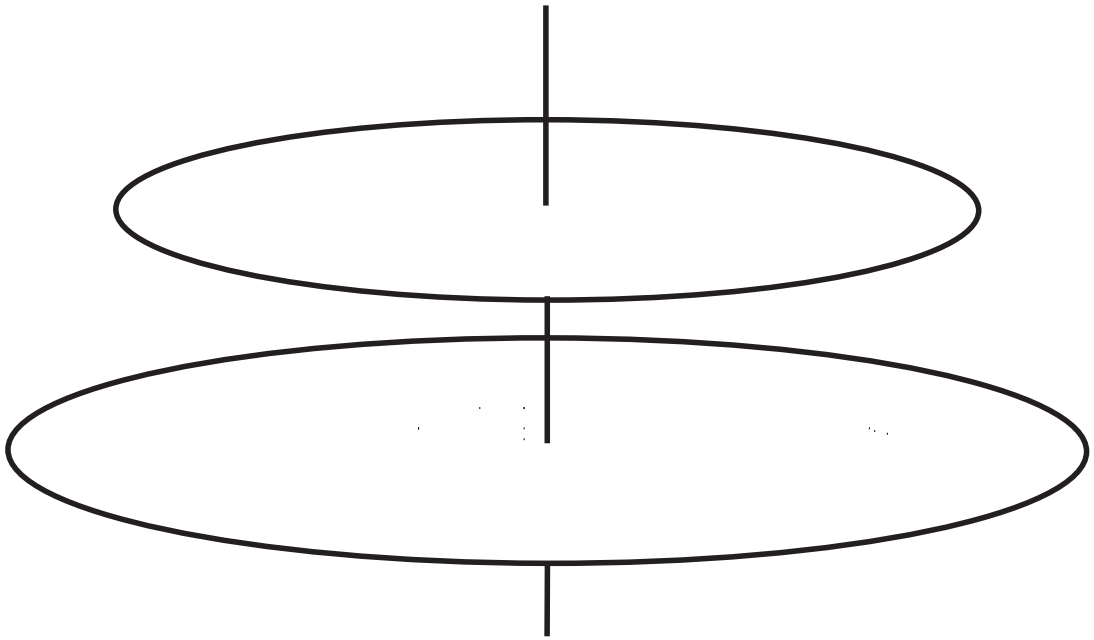

Relative Rotation Condition

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For all rings R_1 and R_2 (with the same axis), if

(1) R_1 is "non-rotating,"

and

(2) R_2 is non-rotating relative to R_1,

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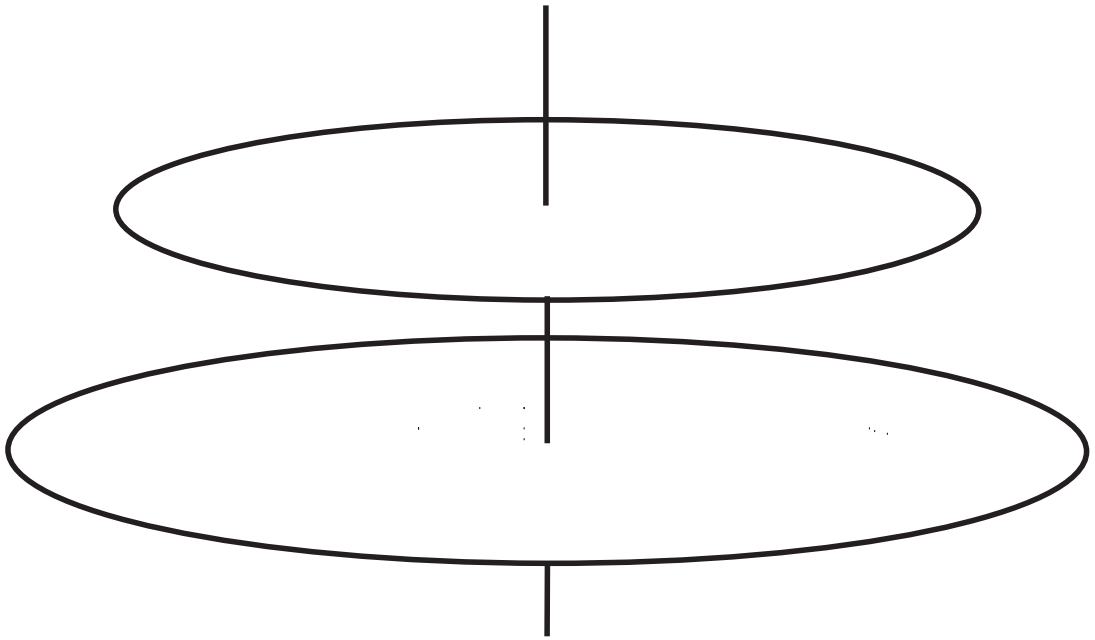

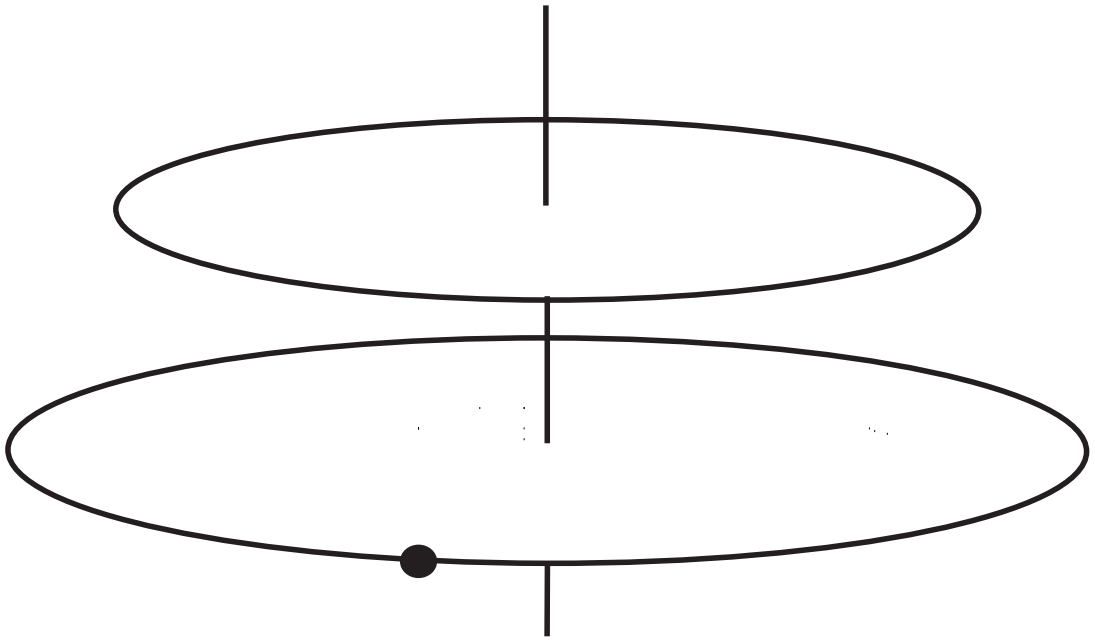

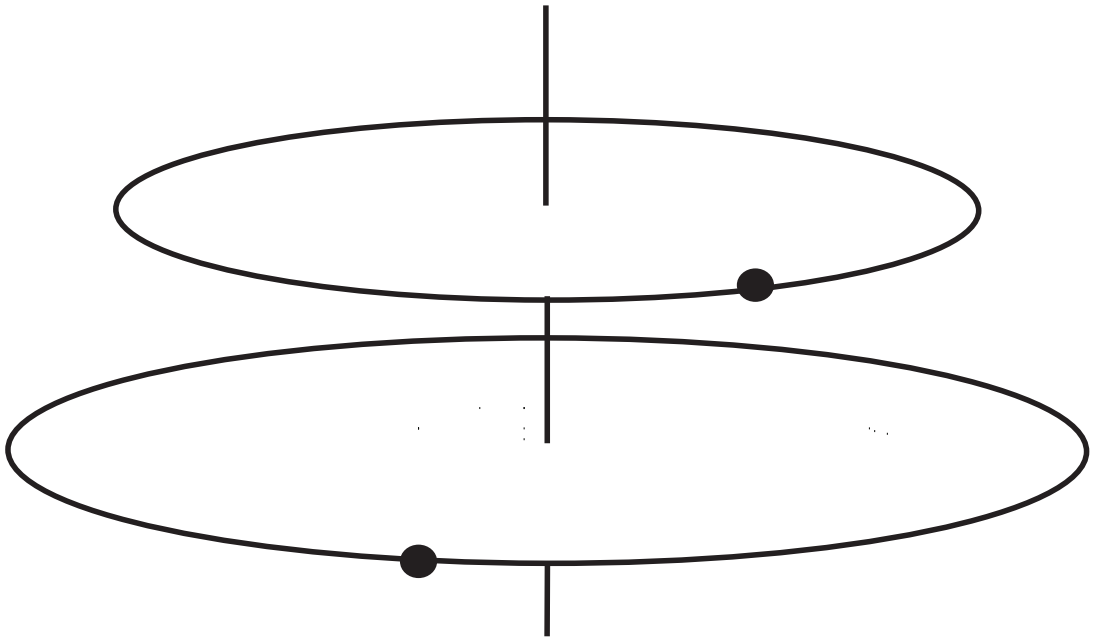

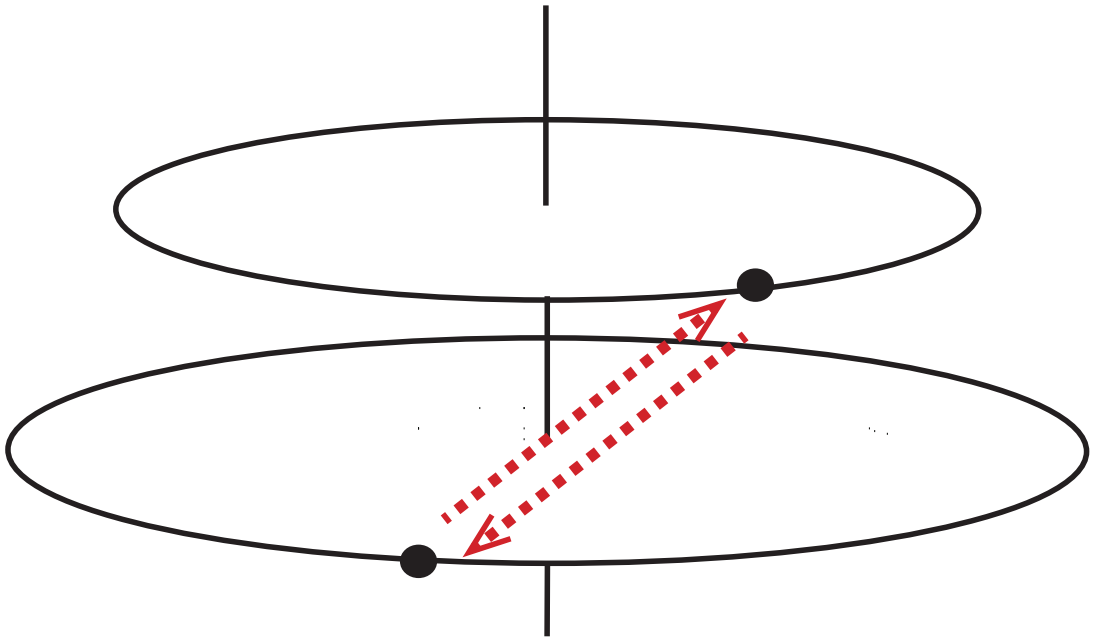

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Do the three criteria (CIA, CIR, ZAM) satisfy the relative rotation condition?

Second Point:

In the Kerr solution, for example, none of them satisfy the relative rotation condition.

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Are there *any* criteria of non-rotation that satisfy the relative rotation condition in the Kerr solution?

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Yes, but none are reasonable candidates.

Now we turn to two other conditions (that one might want a criterion of non-rotation to satisfy).

[relative rotation condition]

limit condition

non-vacuity condition

The three criteria do not agree in general, but they (always) agree "in the limit for infinitely small rings".

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The claim requires proof, but it is what we should expect.

rotation
at a point

rotation over
extended regions

Limit Condition:

Let $R_1, R_2, R_3, ...$ be a sequence of rings, each "non-rotating," that converges to a point on the axis. For all i, let ring R_i have angular velocity ω_i with respect to the CIA criterion. Then $\omega_i \to 0$.

Third Point:

In *all* relativistic spacetimes, including the Kerr solution, the CIR and ZAM criteria (and the CIA criterion) satisfy the limit condition.

Are there *any* criteria of non-rotation that satisfy both the relative rotation condition and the limit condition in the Kerr solution?

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Exactly one – the vacuous criterion according to which no ring ever qualifies as "non-rotating".

Non-Vacuity Condition:

Some ring, in some state of motion (or non-motion), qualifies as "non-rotating."

Fourth Point:

No-Go Theorem. There is no criterion of non-rotation that satisfies the following three conditions in the Kerr solution:

(1) the relative rotation condition

(2) the limit condition

(3) the non-vacuity condition.

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Given any candidate criterion of "non-rotation" in the Kerr solution, if it makes correct determinations of non-rotation in the "limit for infinitely small rings", and if it is non-vacuous,

Think about it this way:

Given any candidate criterion of "non-rotation" in the Kerr solution, if it makes correct determinations of non-rotation in the "limit for infinitely small rings", and if it is non-vacuous, then it *must* violate the relative rotation condition.

Does this mean we cannot talk about rotation in relativity theory?

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Not at all.

The End

Thank you for awarding me this wonderful prize.