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D12.1 Model Adopted
Methodology for analysing a Digital Business Ecosystem

Maria Petrou, Kostas Giannoutakis and Duminda Thilakawardana
CVSSP, School of Electronics and Physical Sciences,
University of Surrey,
Guildford, GU2 7XH, UK

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Abstract

A digital business ecosystem is treated here as a closed system of SMEs which interact with each other through buyer-seller relationships. As we are interested in modelling the business-to-business interactions (B2B), we present first an extensive introduction to the role of SMEs in the business world and the relationships they sustain with each other as well as with larger corporations through supply chains. We propose a way of modelling such B2B interactions and formulate a global cost function expressing the state of the system. We present methodology which may be used to find the states of the system, namely the use of Langevin equation and the use of a biologically inspired self-organising map.

Key words: Langevin equation, B2B interactions, self-organising maps

1 Introduction

The digital business ecosystem envisaged in this project is assumed to consist of SME businesses which will come together in cyberspace the same way companies gather in a business park in the physical world. Although no economy nowadays may be considered closed, especially under the globalisation ideas that have been catching up more and more, and especially as a business park, in cyberspace or whatever space, is not even an economy, treating the collection of SMEs in a digital business park as an ecosystem, ie a closed system, offers the opportunity of some interesting studies in the area of physico-economics. In this introductory section we discuss SMEs from the perspective of their position in the business world.

1.1 What is an SME nowadays?

According to the Organisation's for Economic Co-operation and Development (OECD) definition, a Small and Medium Enterprise (SME) is a firm that occupies less than 500 employees [23]. This definition is used to distinguish such firms from the larger ones, which are generally known as Translational Corporations (TNCs) or Multinational Corporations (MNCs). However, several classifications,

highly dependent on a country's labour force and economic status, have been introduced to make a distinction between MNCs and SMEs. Hoselitz [18] states that a firm, which in the United States might be considered as being an SME regarding its capital investment and number of employees, in some countries of Asia might be considered as a large-scale enterprise. Moreover, the Japanese SME policy uses the amount of capital invested as a criterion in defining what an SME is [22].

Europe has its own definition of what an SME is [12]: "The category of micro, small and medium-sized enterprises (SMEs) is made up of enterprises which employ fewer than 250 persons and which have an annual turnover not exceeding 50 million EUROS, and/or an annual balance sheet total not exceeding 43 million EUROS." Within this category, a small enterprise is defined as an enterprise which employs fewer than 50 persons and whose annual turnover and/or balance sheet total does not exceed 10 million EUROS. A microenterprise is defined as an enterprise which employs fewer than 10 persons and whose annual turnover and/or annual balance sheet total does not exceed 2 million EUROS.

SMEs used to be small industries focused on the domestic production of their countries, with a low proportion of labour employed and relatively small returns of capital. After World War II, however, SMEs started to undertake a more active role in their economic environment, as it has been demonstrated by Hoselitz's work, an aspect more obvious in the European countries and in Japan. Although they were of minor importance to begin with, the SMEs which managed to survive the competition of larger enterprises were gradually strengthened. During the development of Germany and Japan, firms with 6-49 employees were increased until the 1950's in Germany, but then they started to decline. In Japan, however, until 1955 small firms grew more rapidly than the big ones and that is the exceptional characteristic of the Japanese development, since in Europe that happened only during economic recessions.

Nowadays, SMEs constitute entities of vital importance for the global economic transactions. Not only do they expand the range of their markets in their host countries, but they also deal with international trade. This feature is stressed by many surveys conducted, such as that of Knight in 2001 [23]. Knight presented some OECD data of 1997 illustrating that 95% of the businesses worldwide are SMEs and they yield 50% of the total value added. They also provide employment for 60%-90% of the global work-force, depending on the country, and currently they account for about a quarter of exports in most industrialised countries. In agreement with the above, Hvolby and Trienekens [21], write that SMEs are the major kind of firms in the European industry, not only in number of enterprises, but in number of employees occupied as well. More than that, they seem to be independent from big industries, as they collaborate mainly with agencies and the academia and are supported by industrial development programs. It is very interesting, however, to make a comment about the Japanese SMEs [22]. SMEs in Japan, provide employment for 81% of the country's labour force and they constitute 99% of the enterprises in the country (62% wholesale industry and 93% retail). Japan's development and wealth was stimulated by the SMEs as it is easily inferred by the Japan SMEs Corporation's data. The same conclusion is reached by studying El-Agraa's research [9]. That is the reason the Japanese government strongly supports the SMEs by allocating resources among industries, by promoting industrial restructuring and by assisting SMEs.

However, this is not the same in less developed countries (LDCs). As stated by Hoselitz [18], after the second World War the lion's share of capital and revenues were flowing to big industries, which are more easily found in Western countries. On the other hand, small industries generally appeared in Asian densely populated countries, due to the relative abundance of labour. The establishment of small industries in LDCs created two branches: One of them included firms that tended to be modernised, and the other one, included the industries that kept on working as agricultural firms. The production between these two branches did not have significant differences. Some firms in the former category, however, have been turning to medium-sized industries. However, in general terms, small industries in LDCs base their presence more on the interaction among themselves, and less on their ability to sustain the competition with large industries. Although more SMEs from Asia are currently involved in international trade [23], these firms require a lot of effort in order to catch up with SMEs from developed countries. One of the developing countries' ambitious plans for SMEs is the Asian-Pacific Economic Corporation (APEC) [25]. This Corporation includes 21 Asia-Pacific economies and wishes to establish a financial network among these countries.

1.2 Business Models for SMEs

According to Petrovic et al [40] "a business model describes the logic of a business system for creating value, that lies behind the actual processes.... It is a description of the value a company offers to one or several segments of customers and the architecture of the firm and its network of partners for creating, marketing and delivering its value and relationship capital, in order to generate profitable and sustainable revenue streams". Linder and Cantrell say [30] that "most people speak about business models, but they actually mean parts of a business model". That happens because, "business model" is a term rather difficult to define explicitly. A business model must be regarded as the mind and the soul of a business. It is the framework in which the business tries to accomplish its tasks, ie selling goods and services to customers, and it is characterised by assembling processes that lead to the final achievement. Some of these processes are presented below.

1.2.1 Strategy plan

Strategy is the initial business's step in the race of market competition. The better the strategy planned, the better the firm's position in the market. It is the guideline that an SME has to follow, while implementing its tasks. White summarises in his paper [49] a collection of ideas about strategic planning: According to Webb's and Sayer's opinion [48], SMEs should make the most of their web-site services, while this seems to be one main reason why they have quite unsatisfactory evaluation of their objectives. Day et al [7] share the view that marketing and entrepreneurship should constitute different goals, while Tonge et al [45] distinguish firms into either "high" or "super growth" companies, characterised by seven aspects: flexibility, product diversification, employment of quality staff, profitable marketing, early entry into growth markets, quality, and frequent innovation. Day et al [7] reach a similar conclusion and say that SMEs pay a lot of attention to customers' needs and offer after sales service. Peel and Bridge [39] consider SMEs to be high or low level planners. The former category tends to take up more complicated investments, such as re-investing the profit into

the company, control a greater proportion of the market transactions and are believed to be more successful than firms which prefer to be reactive and intuitive. Strategic planning is the backbone for the SMEs and specialists tend to push firms to adopt the most appropriate strategy.

1.2.2 Business to Consumer (B2C) relationships

As mentioned above, every business's final objective is to attract consumers and sell its products and/or services. In economic terms, this means that a firm is interested in maximising its profit. Profit maximisation is a twofold task. On the one hand, an SME has to expand its market aiming at earning as much revenues as it can, but on the other hand, it is supposed to do so at the least possible cost. Profit is a relative term, defined as the difference between revenues and total cost. So, profit maximisation is achieved by maximising revenues, but with the assumption that the total cost remains unchanged. Consequently, establishing profitable relationships with customers is a source of life for an SME. Economic variables and financial statements are nothing more than the illustration of such a relationship: they show how good or bad it is. A B2C relationship is created and affected by several factors, some of them listed below.

One very important aspect of the interaction between business and consumer is the way in which a business comes in touch with the customer. Not only is it necessary for a business to contact the target customer, but it is also important to provide the most useful information to them [38] in order to persuade them that it sells exactly what they need. According to Osterwalder et al [37], the B2C relationship is composed of three elements: a)**The Feel & Serve element**, b) **the Information Strategy element** and c) **the Trust & Loyalty element**. The first one, has to do with the image an SME projects for itself in the market and how it contacts its potential consumers. Does it have to follow a more or a less assertive marketing plan? Will it get in touch with customers directly, or indirectly through launching an advertising campaign? Will it sell its commodities through the traditional market channels or on the web-site? Questions of this type are involved in the Feel & Serve element. The Information Strategy element aims at accumulating information about customers' needs, which can make the B2C relationship stronger. By creating a feedback from past experiences, SMEs are able to modify their strategies towards customers, they can direct to their needs more precisely and they can figure out a consumption profile for every particular customer. Basic prerequisite for all the above is the use of advanced technology. Last but not least, the Trust & Loyalty element contributes to ensuring trust in the (e–)business networks and therefore, trust from the consumer's point of view.

One other necessary characteristic of the B2C relationship is the way that customer's orders are received and executed by a business. In other words, how direct the communication between business and customer is and how quickly the customer's needs are fulfilled. As it is stated by Hvolby and Barfod [20], a customers' order passes though different departments, until it is executed[1]. To make matters worse, when product customisation according to a specific customer's needs takes place, the required procedures become more cumbersome and costly. So, SMEs always ought to bear in mind that satisfying customers' needs is not related to one only department, but it is a process

[1]Sales department, design department, production preparation department etc.

based on a schedule deadline, limited by the available resources. One of the models proposed by Hvolby and Barfod is the **Activity Chain Model**. The core of this model is to create activities and chains of activities that take into consideration the customers' opinion impact on the development and customisation of products and operations. The model's purpose is to strengthen the interaction between tasks and organisational functions and is divided into four segments: The Product Development Chain, which is focused on producing new products, the Stock Chain, which deals with controlling the purchasing and production of the essential materials used in the production process, the Customer Order Chain, which copes with functions handling customer orders (eg adaptation, purchase, delivery etc) and finally, the Shop Floor Chain, which has to do with processes related to the production and assembly of basic components and customer specific products in the production facilities (units, factories).

1.2.3 Patent Protection

Patents are generally recognised as one efficient method for protecting a firm's innovative products from being copied by other firms. It is quite interesting to concentrate on the European SMEs, since it is claimed that they do not treat patents as they should do. A survey conducted by the European Patent Office (EPO) [10] shows that the European SMEs do not make the most effective use of their patent rights. SMEs from Japan and other countries protect their products much better. In the USA for instance, in 1987 68,000 SMEs filled patent applications and this number had been increased at 100,000 in 1993 (47% increase). This is in keeping with the efforts made by the US government to revitalise research and development (R&D) and to encourage firms to follow the patent system. A similar trend exists in Japan, and the Japanese government tried to limit the patent applications. Nevertheless, in 1987 there were 310,000 applications and in 1993 the applications were 332,000. In contrast to these figures, there is a stagnation in the number of patent applications in the 27 EPO members[2]. Although European industry is well protected, it is relatively weak in the fields of new technology and is below the world average. But why European SMEs are so timid about adopting a patent policy? There are several reasons for that: Many SMEs regard patents as a non-economical and risky type of investment. Also, patents do not ensure one firm's success in the market. More than that, patents do not prevent imitators from producing close substitutes to the protected commodities. Finally, SMEs use to bank on their own know-how and have the tendency of collaborating with each other in the pursuit of innovation. Some other factors are also referred, such as the lack of information about the patent system, the lack of funds and experienced staff, etc. Regarding the sector of production, the biggest protection seems to exist in the manufacturing of metal products, and the smallest one in the precision mechanics and optics. However, there is no great difference in the protection of other sectors, such as energy, chemicals, vehicle building, electrical engineering and data processing equipment, food and luxury food, textiles and clothing. The rates of patent protection in all of these sectors fluctuate between 23%-40%. In addition, the use of patents is not uniform among the member-countries, with Germany, France, United Kingdom,

[2]They are : Austria, Belgium, Bulgaria, Cyprus, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Ireland, Italy, Lichtenstein, Luxembourg, Monaco, the Netherlands, Portugal, Rumania, Slovakia, Slovenia, Spain, Sweden, Switzerland, Turkey and the United Kingdom [11].

Italy and Spain being the most frequent users. On the opposite side are countries like Greece, Portugal and the Eastern European countries.

1.3 Interaction among SMEs

Globalisation is a new stimulant for SMEs to revise their business models, so they are suitable for the modern requirements that the upheaval in global economies and in information technology imposes. Such subjects are presented in the OECD's magazine, "The Observer", and are also available on the web site [46]. Globalisation gives to businesses the opportunity to create new goods and to sell them worldwide. In a previous section we made a few comments about the necessity of international trade, which accounts for 1/3 of goods exports from Japan and the USA, a similar proportion of all US goods imports and 1/4 of all Japanese goods imports (OECD data). Much attention is also paid nowadays to the intra-firm trade, which mainly deals with nearly finished goods which require little additional processing. One other characteristic of trade between firms of OECD countries is trade between different firms of the same industry (intra-industry trade), ie the import and export of similar goods by the same country. Intra-industry trade involves manufactured goods and especially the most sophisticated ones, like chemicals, machinery and transport equipment, electrical equipment and electronics. It has shown significant increase in most OECD countries and continues to rise[3], while in eight OECD countries[4] it accounts for more than half of their GDP (Gross Domestic Product). This characteristic generates not only advantages, but disadvantages as well, as an economic crisis in the domestic market of a country may trigger a chain of economic crises in many countries. All the above suggest that globalisation introduces a new concept of running a business, a concept that inevitably affects academicians, specialists and policy makers.

1.3.1 Business to Business (B2B)

Profit maximisation is the ultimate task of a business, but so far only the interaction between business and customer has been discussed. Of equal necessity is the interaction of an SME with other SMEs, a mutual co-existence that influences their activities. It is usually referred to in the economic bibliography as "competitiveness". Man et al. [32] study the concept of competitiveness in three different levels: The **individual firm** level, the **micro-economic** level and the **macro-economic** level. The individual firm level is concerned with the activities taken by a firm in order to be competitive, the micro-economic level has to do with the results of an SME's competitiveness in the industrial sector it belongs to, and finally, the macro-economic level examines the same results in the whole economic system of one country. They also make another distinction by regarding competitiveness as being a dependent, independent or intermediary variable[5]. More specifically, Horne et al [17] define an SME's competitiveness as "the interaction of the scope for action or growth in the business environment, the degree of access to capital resources and the intrinsic ability

[3] Some statistical data: Mexico 73%, USA 69%, Austria, France and UK 70%-75%

[4] Austria, Belgium, Czech Republic, Hungary, Ireland, Luxembourg, Netherlands and Slovakia

[5] In the same report they present Corbett's and Wassenhove's opinions [6], who claim that competitiveness is a multidimensional concept with dimensions like price, place etc.

of the firm to act as represented in entrepreneurship". In short, we may say that competitiveness is the firm's ability to meet its potential.

One other kind of B2B relationship does not approach businesses from the scope of competitive markets, but in contrast, it describes businesses as cooperators in the same production process. This trend is relatively new, even though it could be found in post-war Japan, where firms followed the "boss-henchman" system. The "boss-henchman" system is a type of industrial organisation where the production of one firm is continued by another. Thus, the survival of every industry is dependent on other firms. One good example is the bicycle industry in Sakei City, near Osaka, where different manufacturers, each producing a different part of the bicycle, like handlebars, wheel rims, hubs etc, were co-operating in producing the final commodity [18]. Something similar to that is nowadays the "Supply Chain Planning" [19], a supply network that appeared due to SMEs' inabilities to supply their products in the most effective way. The traditional view of a supply chain consists of one single supplier, one manufacturer and one specific customer (or group of customers). The interaction in this case is one way: The supplier renders products to the manufacturer, who in turn, sells them to the customer. In a more advanced version of this supply model, we can find more than one supplier and more than one customer, but the interaction is one and the same. According to the Supply Chain Planning, not only can the manufacturer provide feedback information to the supplier, but also the customers can do so. Moreover, customers give to the manufacturer useful information about their products. Both the supplier and the manufacturer maintain a database with the feedback they receive and are able to interchange planning information for better production and customer attraction.

1.3.2 Information Technology (IT) adoption

Quite relevant to the above subsection, is the aspect of IT. The more advanced the IT used by the SMEs is, the better and more direct the interactions among them. According to a report written by Vassell [47], special attention is paid to the Computer Integrated Manufacturing (CIM). SMEs are struck as being a very dynamic part of modern industry and CIMs can offer to them some very useful tools. SMEs should regard CIMs as a new type of investment and there is evidence that SMEs which purchase CIMs turn to be more competitive than many big industries. To be without CIMs is a considerable disadvantage for an SME, unless it is financially incapable of doing so. However, for many SMEs, an objective like using CIM may be a tall order, as it requires very careful implementation, a well-defined strategy and a supportive system of alliances.

According to Fink's research [13], IT adoption by SMEs was first introduced in the 1980s when microcomputers dominated the technology market and enabled firms to be equipped with low-cost computer hardware and package systems. However, small firms at the beginning were not very willing to adopt this technology. The main reasons for that was the high environmental uncertainty that SMEs faced and their scarce human, financial and material resources. SMEs tend to be highly dependent on external sources that provide them with scientific and technological information. That is why governments strive to make it easier for information technology transfer to SMEs. It is observed that organisations which have already used IT in the past, are more likely to adopt IT.

In spite of the fact that the more people involved in IT development, the faster the IT adoption is, adapting business practices to use IT often turns out to be time consuming and discourages firms from doing so. Most of these facts are conclusions of Fink's research, focused on Western Australia's SMEs.

1.4 Project objective

The objective of this work-package is: Given a business ecosystem, in particular a closed system of SMEs, identify:

1. The optimal supply/demand partnerships for the system to function efficiently.
2. The self-organisation of the system into clusters of companies which are mutually dependent and which are only weakly dependent on the rest.

To satisfy this objective, we first examine the relationships between companies (B2B) from the point of view of supply/demand chains. In particular we examine in the next section the characteristics that influence this type of relationship.

In section 3 we present a model that we propose to adopt for expressing these relationships. In sections 4 and 5 we present the tools that have been chosen to optimise that model, and our conclusions are presented in section 6.

2 Supply chain in a business ecosystem

Since we are interested in the way companies interact with companies, in this section we discuss some aspects of the interactions between SMEs through the supply/demand chain.

2.1 What supply chain management is

Supply chain management is the part of a business's administration that deals with the supply policy of the firm. A clear definition is given by Simchi-Levi et al in [42]. According to them,

...supply chain management is a set of approaches utilised to efficiently integrate suppliers, manufacturers, warehouses and stores, so that merchandise is produced and distributed at the right quantities, to the right locations, and at the right time, in order to minimise system-wide costs while satisfying service level requirements...

The same authors make three observations regarding the definition given above:

- Supply chain management considers every factor that affects cost.
- Supply chain management must be efficient and cost-effective across the entire system.

- Supply chain management encompasses the firm's activities at many levels, from the strategic level through the tactical to the operational level.[6]

It is a common phenomenon in the managerial literature to use the term "logistics" instead of supply chain management and vice versa. Simchi–Levi et al in [42] and Ballou in [1] give the definition of Logistics Management, as it is defined by the Council of Logistics Management:

...the process of planning, implementing and controlling the efficient, cost–effective flow and storage of raw materials, in–process inventory, finished goods, and related information from point–of–origin to point–of–consumption for the purpose of conforming to customer requirements...

After reading the two definitions above concerning supply chain management and logistics, one realizes that they refer to the same processes and so, we can also use them interchangeably in the remaining parts of this report.

2.2 Key issues in a supply chain

Fisher in [14] identifies a number of issues that characterise a firm's ideal supply chain. He starts first with the observation that In spite of the facilities offered by the technology nowadays, the performance of many supply chains is getting worse. The most usual symptoms of this deterioration are increase in cost, adversarial relations between supply chain partners and dysfunctional industry practices. The main causes, to his view, are the inability of many companies to predict demand, and managers' lack of skills to decide what the best for their companies is.

So, the first thing firms have to do is to recognise the nature of demand for their products, which means that they must take into consideration several matters, such as product life cycle, demand predictability, product variety, etc. In other words, managers must cope with a structural problem that lies on the mismatch between the type of product and the type of the supply chain. It is useful to mention that this is very related to the nature of demand, which is an economic factor that no company can control. The formulation of demand takes place in the economic environment and every firm can just adapt its policy and strategy to the current demand patterns.

Fisher [14] states that one of the key issues in a successful supply chain is for a company to define the type of its products. He distinguishes the products in two types: **Functional** products, which include products that people buy in retail shops. Functional products satisfy basic needs that do not change very often. Therefore, the demand for them does not fluctuate a lot and is predictable. Because of this stability, many companies decide to produce them, and that leads to great competition and low profit margins. On the other hand, one can find the so called **innovative** products, which embody innovations some companies introduce in fashion or technology to give customers an additional reason to buy them. Good examples of innovative products are fashion

[6] The strategic level refers to decisions that have a long-lasting effect for the company, eg number, location and capacities of warehouses. The tactical level has to do with decisions that are updated once every year, eg purchasing and production decisions. Finally, the operational level refers to decisions that concern the day–to–day activities of the firm, eg scheduling, routing, truck loading, etc.

apparel and personal computers. We can find innovative products even in the food industry, such as McDonald's products, Starbucks, Pizza Hut, etc, which may differ from their odd substitutes because of their appearance, their shape or their taste. In spite of giving the opportunity for higher profit margins, innovative products cannot avoid some handicaps: because they are very new, it is not easy to predict the demand for them. Moreover, they have short life cycles (only a few months) as quite soon imitators emerge in the market, and consequently, companies have to introduce new innovative products. Their short life cycle increases unpredictability for their demand.

The reasons above create the need for a more special supply chain for each type of product. To achieve that, managers have to recognise that a supply chain performs two distinct types of function: a **physical function** and a **market mediation function**. The physical function concerns concepts such as transformation of raw materials into parts, components and finished goods, and the transportation of all of them from one point of the supply chain to the next. The market mediation's function is to ensure that the variety of products reaching the marketplace are in keeping with the consumers' preferences. More precisely, functional products require an efficient process, that is to say, fulfil demand at the lower possible cost, yield high turns and minimise inventory through the chain, shorten lead times in a cost–effective way, select suppliers with cost and quality criteria and make the product design as good as possible. Innovative products, however, require a responsive process, which means, respond quickly to unpredictable demand to minimise stock-outs and inventory, invest aggressively to reduce lead time, select the best, the most flexible and the fastest suppliers and postpone the product differentiation for as long as possible. Finally, Fisher mentions that products that are physically the same can be either functional or innovative.

2.3 Determinants

Various factors influence the commitment in the buyer-seller relationship between businesses. Some of them constitute economic determinants and some other non-economic. It is very interesting to regard them according to the importance agents pose on them. Empirical research has shown that they influence to a different degree the supply chain commitment, not only when included in a buyer-seller relationship, but also when they appear in the relationship between a manufacturer and a distributor.

2.3.1 Buyer-seller relationship

Kalafatis [24] drew the conclusion that in the buyer-seller relationship, interactions between the agents involved have changed from adversarial to relationship building and more attention is being focused on the relational exchanges. He supports that the organisation of a distribution channel requires two main elements: a) the implementation of an efficient governance structure and b) the development of relationships between the channel intermediaries.

More detailed results are presented by Leahy et al in [26]. Their research deals with the importance of third–party logistics that intermediate in a supply chain, facilitating the flow of products between the businesses. The most important benefit of third party logistics involvement is the reduction of cost. However, there is no extensive research related to this type of company, as they constitute a

Factor	Rate of response (%)
Customer orientation	3.57
Dependability	3.54
Change orientation	3.38
Timeliness	3.32
Convenience	3.30
Control and performance appraisal	3.30
Improved service	3.27
Mutual trust and consideration	3.27
Focus on core competency	3.24
Total organisational involvement	3.24
Knowledge of customer operations	3.22
Cost savings	3.19
Long term relationships	3.14
Management expertise	3.11
Sharing relevant information	3.11
Access to latest technology	3.08
Financial strength	3.05

Table 1: Determining factors of great importance (taken from [26])

Factor	Rate of response (%)
Channel perspective	2.89
Sharing of common goals	2.89
Guidelines exist to resolve disputes	2.81
Number of services offered	2.78
Sharing of benefits and risks	2.76
Provider's knowledge of external environment	2.56
Exit provision	2.41

Table 2: Determining factors of moderate importance (taken from [26])

young industry. Such companies first appeared in the USA. Most of them were founded after 1980 after the deregulation of the US air freight, truck and rail industries. Their collaborations with firms are mostly based on the long–term relationships with users of third–party services.

Leahy et al [26] present a list of twenty–five factors considered by third–party logistics to be influential to successful relationship with firms. Their importance differs and they are classified in three separate categories. However, all of them have a significant impact in the buyer–supplier commitment. The survey was conducted by a mail questionnaire sent to well established third–party firms. They were asked about the importance they impose on some factors that influence their relationships with firms. The first category includes factors with great importance and they are shown in Table 1. The second category includes 7 factors of moderate importance and they are shown in Table 2. Finally, the third category includes the factors of slight importance and it contains only one determinant, namely **sharing facilities and human resources** (1.87%).

The most important factor is customer orientation (ie responsiveness to customer needs), which

Factor	Rate of response (%)
Customer orientation	17.1
Cost savings	11.4
Total organisational involvement	11.4

Table 3: Single most important factor (taken from [26])

refers to the concept that customer service is a process that results in value added to the services exchanged. That includes the seller's ability to customise its products according to the buyer's preferences. The second most important determinant is dependability, in which services are provided in a consistent and reliable manner. The third one is change orientation, which means that a provider can modify its performance in a changing business environment to eliminate as much as possible system drawbacks.

Regarding the **single most important factor**, the results are slightly different and they are shown in Table 3.

2.3.2 Distributor-manufacturer relationship

A second type of supply chain commitment found in the literature is the relationship that exists between a manufacturer and a distributor. In fact, it is another type of buyer–seller relationship, but in this case the manufacturer can be regarded as the seller and the distributor as the buyer. Most of the distributors are real industries that mediate in the logistics network. Many of them do not aim at producing a specific type of product, but at providing services to their customers, which usually are other industries.

Research upon the distributor-manufacturer interactions has been conducted by Goodman and Dion [16]. In their article, they enumerate the most noticeable changes that have occurred in the supply chain recently:

- The distributors' sales as a percentage of industrial sales have grown higher for all manufacturers, except the largest industrial ones.
- Distributors continued to expand in size and activities.
- Industrial distributors have become more specialised in the products they offer.
- Manufacturers tend to work with distributors whose marketing strategy is similar to their own.
- Manufacturers tend to increase the use of distributors, and so the percentage of sales that move through distributors to the end user has been on the increase.

Nowadays, as it is mentioned in [16], it is not easy for a manufacturer to establish a long–term collaboration with a distributor, because distributors act more and more frequently as independent business entities, and their management rarely matches that of the manufacturer. This may cause conflicts, since manufacturers try to exert power over their distributors. Goodman and Dion refer to several factors that influence the distributor-manufacturer relationship. They divide them into two

Behavioural determinants	Marketing determinants
Trust Power Continuity Communications	Dependence Idiosyncratic investments Product salability

Table 4: Determinants of distributor-manufacturer relationship (taken from [16])

categories: **the behavioural determinants** and **the marketing determinants**, which are listed in Table 4.

Trust is defined as the state when "a partner has confidence in another parter, because the latter provides expertise, dependability and direction". Trust implies that both partners work in a manner that on the one hand generates mutual benefits for them, and on the other hand, they avoid actions that would result in undesirable effects. Power is further divided into five forms: **Legitimate power** that refers to the manufacturer's ability of imposing power over their manufacturer due to its position in the hierarchy. **Expert power**, which refers to the power held by the manufacturer due to their expertise in a particular field, and thus, the distributor counts on this expertise. **Referent power**, which is the manufacturer's ability of developing a strong bond because of their managerial performance and it is based on emotional ties. **Reward power** that is the ability held by the manufacturer to reward their distributor, and **coercive power**, which is based on the fear the manufacturer causes to the distributor, by threatening them with discount cuts and lesser support. Continuity is expressed by the total number of years of partnership, and finally, communications is the interchange of ideas and information in the business network.

Regarding the marketing determinants, dependence is how difficult will be for the distributor to survive in the business environment if they do not have access to the manufacturer's products. Idiosyncratic investments are investments directed at marketing initiatives for a specific manufacturer's product offering. Examples of idiosyncratic investments are product training, promotional programs, advertising campaigns, etc. Product salability is the value that the distributor believes one product includes, eg how useful it is, what its quality is, what the services it provides are, etc. Product salability is a factor usually neglected by the theorists when they examine business interactions.

Empirical results demonstrate that manufacturers are less able to use coercive power over their distributors. Thus, non-coercive power (all other forms of power except coercive power) has more importance. What is weird, is that coercive power was expected to have a negative impact on the distributor–manufacturer relationship. However, research data correlation analysis showed a positive relationship. One explanation is that if there are not many options for a distributor, they have to go along with the manufacturer's plans. Trust seems to have a positive effect on the commitment, since it provides a relief for both partners who do not have to waste time and effort in counteracting negative effects. Continuity does not seem to have any significant importance in the commitment, according to Goodman's and Dion's research, but it possibly appeared in their study due to the distribution of their data. Communications is a necessary determinant and has a quite strong correlation with the commitment ($r = 0.49593$). The significance of dependence appears too small and its connection

with commitment is not as strong as the other variables. Idiosyncratic investment seems to be very important and it is the second strongest predictor of commitment. That means, distributors regard idiosyncratic investment as beneficial. Finally, product salability demonstrates the highest level of correlation to commitment ($r = 0.63309$) and this indication is also supported by the regression results.

2.4 The role of SMEs

In the recent years SMEs tend to get more involved in the supply chain management. However, the picture of SMEs is less clear, as Mudambi and Schründer support in [36]. They conduced a survey of over 600 firms and they concluded that SMEs do not have great partnerships in the supply chain. Partnering is found to be very popular in high-tech and fast-moving industries. In most cases SMEs appear to have a weak position in competing or collaborating with larger industries with no great observable advantage for SMEs in such relationships. The most relevant factors that determine their partnering are the long-term relationships and just–in–time production. They tend to be providers or producing materials for larger industries or senior/junior partners. Little development has been towards more cooperative buyer-seller relationships.

3 Modelling the interactions between companies

In this section we are developing a crude model that tries to capture as many as possible of the B2B relationship characteristics of SMEs. In particular, in view of the analysis of the previous two sections, we may say that

- European SMEs tend not to protect their intellectual property using the patent system. This may have implications as to what services a digital business ecosystem may offer to attract subscribers. For example an on-line service for patent application in DIY style, may be a plus.
- SMEs are rather slow in adopting information technology, unless they start from the beginning as information technology based businesses. This has implications as to what type of companies one expects to find in a business ecosystem in cyber-space.
- SMEs tend to be more benefited by interacting with each other than by interacting with big enterprises. This goes some way in justifying the assumption that a set of SME businesses may be approximately considered as closed with respect to large corporations.
- SMEs interact with each other through supply/demand relationships and occasionally through cooperating in putting a finished product to the market.
- For the SME interactions trust and historical ties are very important.

The notation we use in developing our model is summarised in table 5:

C	:	The set of all companies in the ecosystem
N	:	The total number of companies in the ecosystem
M	:	The total number of non-empty subsets of C
C_i	:	A particular company in the ecosystem (ie $C_i \in C$)
$D(C_i)$	:	The set of all services/products C_i requires
$S(C_i)$	:	The set of all services/products C_i may supply
d_k^i	:	A particular demand company C_i has (ie $d_k^i \in D(C_i)$)
s_k^i	:	A particular service/product C_i may supply (ie $s_k^i \in S(C_i)$)
$x_l^{d;ki}$	:	A particular characteristic of product/service d_k^i of company C_i needs. Examples: $l = 0$ indicates amount, ie by how much the capacity of the supplier will be reduced if this service is provided $l = 1$ indicates price C_i is willing to pay $l = 2$ indicates time C_i is willing to wait $l = 3$ indicates quality of service C_i requires etc
$x_l^{s;ki}$	:	A particular characteristic of product/service s_k^i company C_i offers. Examples: $l = 0$ indicates the total capacity C_i has to supply service/product s_k^i $l = 1$ indicates price C_i expects to receive $l = 2$ indicates time C_i takes to deliver $l = 3$ indicates quality of service C_i offers etc
$R_2(C_i, C_j)$	:	Direct relationship company C_i has with company C_j
$T_2(C_i, C_j)$	:	Indirect relationship company C_i has with company C_j (via third parties)
R_{ij}	:	Abbreviation of $R_2(C_i, C_j)$
T_{ij}	:	Abbreviation of $T_2(C_i, C_j)$
$R_3(C_i, C_j, C_k)$	:	Direct relationship companies C_i, C_j and C_k have with each other
$T_3(C_i, C_j, C_k)$	:	Indirect relationship companies C_i, C_j and C_k have with each other
$R_n(C_{i_1}, C_{i_2}, \ldots, C_{i_n})$	:	Direct relationship companies $C_i, \ldots, C_{i_n}$ have with each other
$T_n(C_{i_1}, C_{i_2}, \ldots, C_{i_n})$	:	Indirect relationship companies $C_i, \ldots, C_{i_n}$ have with each other
P_t	:	The set of all sub-sets of C consisting of t distinct companies
$R_t = R_t(C_{i_1}, C_{i_2}, \ldots, C_{i_t})$	:	Abbreviation of $R_t(C_{i_1}, C_{i_2}, \ldots, C_{i_t})$ when there is no danger not to know which t companies it is referring to
$T_t = T_t(C_{i_1}, C_{i_2}, \ldots, C_{i_t})$	:	Abbreviation of $T_t(C_{i_1}, C_{i_2}, \ldots, C_{i_t})$ when there is no danger not to know which t companies it is referring to
$C_i \leftarrow C_j$	:	Company C_i buys from company C_j
$p(C_i, C_j)$	:	Probability that company C_i buys from company C_j
N_0	:	Set of indices $\{1, 2, \ldots, N\}$
N_i	:	Set of indices $\{1, 2, \ldots, i-1, i+1, \ldots, N\}$

Table 5: Summary of notation

3.1 Modelling the initial probability of pairing

Let us count the number of services a company C_j offers the characteristics of which are within a certain tolerance of the demands a company C_i has. Let us call this number v_{ij}. For example, say that by comparing sets $D(C_i)$ and $S(C_j)$ we found that there were only two elements we could pair such that:

$$
\begin{array}{llll}
d_2^i \leftrightarrow s_5^j & |x_1^{d2i} - x_1^{s5j}| < V_1 & |x_2^{d2i} - x_2^{s5j}| < V_2 & |x_3^{d2i} - x_3^{s5j}| < V_3 \\
d_4^i \leftrightarrow s_1^j & |x_1^{d4i} - x_1^{s1j}| < V_1 & |x_2^{d4i} - x_2^{s1j}| < V_2 & |x_3^{d4i} - x_3^{s1j}| < V_3
\end{array}
$$

where V_1, V_2 and V_3 are some thresholds. Then $v_{ij} = 2$.

We may define the probability of these two companies to become business partners as

$$p_0(C_i \leftarrow C_j) = 1 - e^{-\alpha v_{ij}} \tag{1}$$

where $\alpha > 0$ is a parameter. Note that this function has the right properties to be treated as a probability:

- $p_0(C_i \leftarrow C_j)$ takes values between 0 and 1, since $0 \leq v_{ij} < \infty$, and thus $0 < e^{-\alpha v_{ij}} \leq 1$.
- When two companies have a large number of matched supply-demand pairs, v_{ij} is large, $e^{-\alpha v_{ij}}$ is small and $p_0(C_i \leftarrow C_j)$ tends to 1, ie the probability of these companies to be paired is high.
- If the two companies have very few matched supply-demand pairs, v_{ij} is small, tending to 0, $e^{-\alpha v_{ij}}$ tends to 1, and $p_0(C_i \leftarrow C_j)$ tends to 0, ie the probability of these companies to be paired is very low.

3.2 Modelling the direct relationship between two companies

Let us say that two companies C_i and C_j have had u_{ij} successful transactions so far. We may define the value of their relationship as

$$R_2(C_i, C_j) = 1 - e^{-\beta u_{ij}} \tag{2}$$

where $\beta > 0$ is a parameter. Note that this function has the right properties to be treated as a probability:

- $R_2(C_i, C_j)$ takes values between 0 and 1, since $0 \leq u_{ij} < \infty$, and thus $0 < e^{-\beta u_{ij}} \leq 1$.
- When two companies have a large number of previous successful interactions, u_{ij} is large, $e^{-\beta u_{ij}}$ is small and $R_2(C_i, C_j)$ tends to 1.
- If the two companies have had no integration in the past, $u_{ij} = 0$, $e^{-\beta u_{ij}} = 1$ and $R_2(C_i, C_j) = 0$, ie there is no historical trust in the relationship.

3.3 Modelling the indirect second order relationship between two companies

The indirect relationship between pairs of companies are those that exist through a third party, or through a chain of demand/supply route. For simplicity, we shall consider first the indirect relationship between a pair of companies through a single intermediary. So, let us consider a triplet of companies C_i, C_j and C_k. Let us consider all possible demand/supply routes through them:

$$
\begin{array}{l}
C_i \leftarrow C_j \leftarrow C_k \\
C_i \leftarrow C_k \leftarrow C_j \\
C_j \leftarrow C_i \leftarrow C_k \\
C_j \leftarrow C_k \leftarrow C_i \\
C_k \leftarrow C_i \leftarrow C_j \\
C_k \leftarrow C_j \leftarrow C_i
\end{array}
$$

Let us call the value of a chain $Chain(C_{i_1} \leftarrow C_{i_2} \leftarrow C_{i_3})$ and define it to be:

$$Chain(C_{i_1} \leftarrow C_{i_2} \leftarrow C_{i_3}) \equiv p_0(C_{i_1} \leftarrow C_{i_2})p_0(C_{i_2} \leftarrow C_{i_3}) \tag{3}$$

where $p_0(C_i \leftarrow C_j)$ is defined by equation (1). We define the strength of the relationship between C_i, C_j and C_k as

$$
\begin{aligned}
Rel(C_i, C_j, C_k) \equiv & \frac{1}{6}[Chain(C_i \leftarrow C_j \leftarrow C_k) + Chain(C_i \leftarrow C_k \leftarrow C_j) \\
& +Chain(C_j \leftarrow C_i \leftarrow C_k) + Chain(C_j \leftarrow C_k \leftarrow C_i) \\
& +Chain(C_k \leftarrow C_i \leftarrow C_j) + Chain(C_k \leftarrow C_j \leftarrow C_i)]
\end{aligned} \tag{4}
$$

Then we define the strength of the indirect relationship between C_i and C_j, as the sum of $Rel(C_{i_1} \leftarrow C_{i_2} \leftarrow C_{i_3})$ over all C_k companies in the ecosystem:

$$T_2(C_i, C_j) \equiv \sum_{C_k} Rel(C_i, C_j, C_k) \tag{5}$$

In a similar way we may model the indirect second order relations via two intermediate companies, then via three etc.

4 Seeking the solution

4.1 Using a probabilistic relaxation approach

Problems which have been successfully solved by the method of probabilistic relaxation, are separable. Separable in this context means: the pairing of two companies depends on their characteristics and their prior relationship, and on nothing else. Then we may find the perfect partner for each company if we maximise the posterior probability of pairing. If there are N companies in the ecosystem, the prior probability of company C_i to buy from company C_j is $1/(N-1)$. This case obviously is of no interest. We are really interested in the posterior probability that company C_i will buy from company C_j, ie the conditional probability of $C_i \leftarrow C_j$ given the needs each company has, what each company

may offer, the value of their relationship, direct as well as indirect, etc. We call this probability $p(C_i, C_j)$ for brevity, and we understand it as the probability of pairing these two companies, given all the characteristics of all the companies in the ecosystem, and the relations company C_i has with every other company:

$$p(C_i, C_j) \equiv p(C_i \leftarrow C_j | D(C_i), S(C_m), R_2(C_i, C_n), T_2(C_i, C_n), \ldots, R_t, T_t; \forall m, t \in N_0, \forall n \in N_i) \quad (6)$$

Let us say that company C_i will buy from company C_0 if:

$$C_0 = arg\left\{\max_{C_j}\{p(C_i, C_j)\}\right\} \quad (7)$$

Our problem is for a given company C_i to calculate $p(C_i, C_j)$ for all other companies C_j in the ecosystem, and choose the company C_0 that maximises it.

If we omit any indirect relations in the above expression and also any higher order relations, then we may choose C_0 to be the company that maximises

$$C_0 = arg\left\{\max_{j}\left\{\left(1 - e^{-\alpha v_{ij}}\right)\left(1 - e^{-\beta u_{ij}}\right)\right\}\right\} \quad (8)$$

This formulation is very easy, because it is very simplistic. There is no relaxation involved, as there are no constraints that need to be satisfied and propagated. If, however, we start taking into consideration the limited capacity a company has, then, we may have to restrict the matching into, say one-to-one: a company may not be able to act as supplier to two companies, offering the same service or product, due to its limited capacity. In the extreme case, when a company may act as a supplier to only one other company, we have the case that one has to choose the pairings so that the joint probability $p(C_i \leftarrow C_{n_i}, \forall i \in N_0)$ has to be maximised. Then, pairing C_i with C_j has implications on the other pairings in the system. An iterative system is needed, where these constraints propagate through a relaxation process: Initially, every company is paired with every other company with some initial probability which, could for example, be given by (1). But then each one of these probabilities is updated, taking into consideration the effect of the other pairings in the system. The process is repeated until for each company the probability of being paired with another one approaches 1, while the probability of being paired with all others approaches 0. This would have been an ideal situation where the system converges to a solution. However, it is far from certain that such an ideal solution will exist. The key point in this approach is on the modelling of how a particular transaction affects other transactions.

4.2 Using a global optimisation approach

Probabilistic relaxation approaches are based on the assumption of separability and independence. This assumption allows one to write the joint probability $p(C_i \leftarrow C_{n_i}, \forall i \in N_0)$ as the product of individual probabilities $p(C_i \leftarrow C_{n_i})$, each conditioned on different conditioning variables. The imposed iterations then serve to balance the errors made by this assumption [4]. It has been shown [43] that the probabilistic relaxation approach implicitly minimises a global cost function which characterises the solution of the problem, which however is truncated, ie it does not take into consideration all

interactions of the system. A better formulation of the problem is to start from the beginning with a cost function which incorporates all problem characteristics, and try to minimise that. The minimisation of a cost function is equivalent to the maximisation of the joint probability density function of all transactions to occur. For the trivial case of (7), such a global cost function can easily be written by first writing each probability in the form $e^{-U_{ij}}$, where obviously $U_{ij} \equiv -\ln p(C_i \leftarrow C_{n_i})$, and then multiplying all such probabilities to form the joint probability (this is justified because we consider each transaction independent from the others, and the joint probability of two independent events A and B to occur is $p(A, B) = p(A)p(B)$). The multiplication of such probabilities, written in exponential form, is equivalent to the addition of their exponents, and the maximum then of the joint probability occurs at the minimum of the sum of the exponents. This sum of the exponents is the cost function we are minimising, and it is given by:

$$F \equiv -\sum_{ij} \left\{ \ln \left(1 - e^{-\alpha v_{ij}} \right) + \ln \left(1 - e^{-\beta u_{ij}} \right) \right\} \tag{9}$$

This function is trivial because it does not incorporate the effects one transaction may have on the other transactions. Minimising this cost function does not require any sophisticated approach. If, however, interactions between transactions are incorporated in the model, the cost function may become much more complicated. Then it will have to be optimised by a genetic algorithm, simulated annealing or some other optimisation method. Alternatively, it may be optimised with the help of Langevin equation which could be designed to be equivalent to a simulated annealing optimisation. However, the Langevin equation has also the capacity to be modified to incorporate stochastic variations of the system parameters themselves, by including an extra stochastic term in the equation, in addition to the one used to escape from local optima (which makes it equivalent to simulated annealing). That is why we decided to adopt this approach here.

Next, we shall try to define a cost function which goes some way in incorporating inter-dependences of the variables of the problem.

First, we start by observing that an ecosystem is a closed system. It may be assumed to correspond to an endogenous economy. In an ecosystem, all the needs of all members have to be satisfied by what the other members have to offer. Clearly, what the members have to offer is limited, and therefore one may view the problem as one of sharing supplies and maintain relations. So, for example, according to (8), C_i may get all it needs from C_0. However, C_0 may not wish to sell all its products to C_i because it has other good customers it wishes to satisfy as well. So, our variables in the optimisation problem should not be probabilities of partnership, but rather fractions of supply that go to each potential customer, so that all are satisfied within the constraints of the system. We may think of the ecosystem as a primitive closed community where people have to do good business by selling and buying their products, but at the same time they have to maintain good relations with their friends and supply them too, even if those deals are not, for example, the best in terms of revenue.

So, let us start by assuming that C_i has a need for product $d^i_{k_i}$ in quantity $x_0^{d;k_i i}$. This product may be bought from several suppliers and let us say that from supplier C_j, C_i will buy quantity $x_{0j}^{d;k_i i} \leq x_0^{d;k_i i}$. One constraint is that

$$\sum_{j \in N_i} x_{0j}^{d;k_i i} = x_0^{d;k_i i} \tag{10}$$

As every company will want to meet this target over all the products it needs, a global constraint the solution we seek has to satisfy is

$$U_1 \equiv \sum_{i\in N_0} \sum_{k_i \in D(C_i)} \left(\sum_{j\in N_i} x_{0j}^{d;k_i i} - x_0^{d;k_i i} \right)^2 \tag{11}$$

We call this constraint the **fully supplied** constraint.

Some justification may be necessary for the use of the power of 2 in the above expression. It is impossible for the solution we are seeking to satisfy all constraints, as they may be contradictory. For example, selling to the best buyer may be in contradition to satisfying the needs of one's best friend. So, the solution we are seeking is going to be a compromise between all the constraints. A compromise means that (10) will not be exactly true and $\sum_{j\in N_i} x_{0j}^{d;k_i i} - x_0^{d;k_i i}$ will deviate from zero by some quantity $\epsilon_{k_i i}$. If we assume that the values of $\epsilon_{k_i i}$ are drawn from a Gaussian distribution with mean $\mu_{k_i i}$ and standard deviation $\sigma_{k_i i}$, then the probability of such an epsilon to occur is

$$p(\epsilon_{k_i i}) = \frac{1}{\sqrt{2\pi}\sigma_{k_i i}} e^{-\frac{(\epsilon_{k_i i}-\mu_{k_i i})^2}{2\sigma_{k_i i}^2}} \tag{12}$$

If the violation of (10) is not biased, the values of $\epsilon_{k_i i}$ will be distributed with mean value 0, ie $\mu_{k_i i} = 0$ for all companies i and all products k_i. In addition, if the standard deviation of this distribution is the same for all company-product pairs, we may drop the indices from σ in (12). Assuming that constraint (10) will be violated independently for each company-product pair, the joint probability of all corresponding values of epsilons to arise will be given by the product of the individual such probabilities:

$$\begin{aligned}
p(\epsilon_{k_i i}; \forall i \in N_0, k_i \in D(C_i)) &= \left(\frac{1}{\sqrt{2\pi}\sigma}\right)^H \prod_{i\in N_0, k_i \in D(C_i)} e^{-\frac{\epsilon_{k_i i}^2}{2\sigma^2}} \\
&= \left(\frac{1}{\sqrt{2\pi}\sigma}\right)^H \prod_{i\in N_0, k_i \in D(C_i)} e^{-\frac{\left(\sum_{j\in N_i} x_{0j}^{d;k_i i} - x_0^{d;k_i i}\right)^2}{2\sigma^2}} \\
&= \left(\frac{1}{\sqrt{2\pi}\sigma}\right)^H e^{-\frac{1}{2\sigma^2}\sum_{i\in N_0}\sum_{k_i\in D(C_i)}\left(\sum_{j\in N_i} x_{0j}^{d;k_i i} - x_0^{d;k_i i}\right)^2} \\
&= \left(\frac{1}{\sqrt{2\pi}\sigma}\right)^H e^{-\frac{1}{2\sigma^2}U_1}
\end{aligned}$$

where H is the total number of company-product pairs. We note that apart from some constants which do not play role in the optimisation process, minimising U_1 given by (11) is equivalent to maximising $p(\epsilon_{k_i i}; \forall i \in N_0, k_i \in D(C_i))$, ie it is equivalent to assuming that the errors of constraint violation constitute Gaussian iid noise (Gaussian idependent identically distributed noise), and trying to minimize them. With this understanding, we proceed to formulate the remaining constraints of the problem.

At the same time, a supplier C_i, has a product $s_{k_i}^i$ in quantity $x_0^{s;k_i i}$. It sells to customer C_j only an amount $x_{0j}^{s;k_i i} \leq x_0^{s;k_i i}$. C_i will be happy if it sells all its stock, so the second constraint we have is

$$\sum_{j\in N_i} x_{0j}^{s;k_i i} = x_0^{s;k_i i} \tag{13}$$

As every company will want to sell all its stocks, a global constraint is to integrate this over all the products each company sells and over all companies:

$$U_2 \equiv \sum_{i \in N_0} \sum_{k_i \in S(C_i)} \left(\sum_{j \in N_i} x_{0j}^{s;k_i i} - x_0^{s;k_i i} \right)^2 \tag{14}$$

We call this constraint the **sell out** constraint.

The unknowns of our problem are all values $x_{0j}^{d;ki}$ and $x_{0j}^{s;ki}$ for all products k a company buys or sells. To simplify notation, we call x_{ijk} the quantity of product k company i buys from company j. We can start now building the global cost function that will have to be minimised in order to tell us who should be buying what from whom for maximum harmony in the closed society.

First of all, every body wants to satisfy to the full their demands. Each product has certain characteristics, and we saw in section 3.1 how to take them into consideration to create a number that expresses how much a particular product one needs fits with a corresponding product one supplies. Given that a supplier puts different emphasis to different aspects of supply-demand matching, this weight of corresponding products will be different for the supplier than for the buyer. So, a product k_i company i wants, when compared with a product k_j company j supplies, from the point of view of the buyer has matching value $v^d_{k_i k_j ij}$ but from the point of view of the supplier has matching value $v^s_{k_i k_j ij}$. The buyer wants to satisfy their needs in full, with the best possible deal, so overall, the following quantity must be minimised:

$$U_3 \equiv \sum_{i \in N_0} \sum_{j \in N_i} \sum_{k_j \in S(C_j)} \sum_{k_i \in D(C_i)} \left(x_{ijk_j} - v^d_{k_i k_j ij} x_0^{d;k_i i} \right)^2 \tag{15}$$

Weight $v^d_{k_i k_j ij}$ expresses how well a product k_j of company C_j matches the product k_i company C_i needs, from the point of view of the buyer. If there is no match, $v^d_{k_i k_j ij} = 0$, and the corresponding term in U_3 will be minimal when $x_{ijk_j} = 0$, ie C_i does not try to satisfy its need in k_i by buying k_j from C_j. This check has to be performed for all products on offer, by all companies and of course for all products C_i needs. We call this constraint the **buy only what fits** constraint.

Now every company wants to sell in full all its stock, with the best possible deals, so for a particular product k_i it sells to C_j to satisfy its need in product k_j, it wants to minimise

$$\left(x_{jik_j} - v^s_{k_j k_i ji} x_0^{s;k_i i} \right)^2 \tag{16}$$

If selling to C_j is a good deal from the point of view of the supplier, $v^s_{k_j k_i ji}$ will be near 1, and this term will be minimised when the quantity x_{jik_j} sold to C_j is as large as possible. Each company wants to do that for all its products, and all companies want to do it. So, the term that must be minimised is:

$$U_4 \equiv \sum_{i \in N_0} \sum_{j \in N_i} \sum_{k_i \in S(C_i)} \sum_{k_j \in D(C_j)} \left(x_{jik_j} - v^s_{k_j k_i ji} x_0^{s;k_i i} \right)^2 \tag{17}$$

We call this constraint the **sell to the best buyer** constraint.

At the same time, companies want to honour past good relations, so they may feel they have some obligation to buy as much as possible from old friends. If R^d_{ij} is the historical relationship between companies i and j, defined as in section 3.2, so that $R^d_{ij} = 0$ if there is no love lost between them

and $R^d_{ij} = 1$ if they have very close ties, with all possible values in between, then for a particular product k_i company i needs, it may try to buy as much as possible from the most similar product from its old time partner. So, it will first choose the most similar product from company j, ie the product that maximises $v^d_{k_i k_j ij}$ and then try to satisfy as much of its needs as possible by buying it. The quantity that has to be minimised is

$$R^d_{ij}\left(x_{ij\ \arg\max_{k_j}\{v^d_{k_i k_j ij}\}} - x_0^{d;k_i i}\right)^2 \tag{18}$$

Note that if $R^d_{ij} \to 0$, this term will not contribute to the whole cost of the solution we are seeking, so we may sum over all suppliers j. Further, we may sum over all products k_i, and finally over all companies i since all of them want to do that:

$$U_5 \equiv \sum_{i\in N_0}\sum_{k_i\in D(C_i)}\sum_{j\in N_i} R^d_{ij}\left(x_{ij\ \arg\max_{k_j}\{v^d_{k_i k_j ij}\}} - x_0^{d;k_i i}\right)^2 \tag{19}$$

We call this the **loyalty** constraint.

One may reverse the roles in the above argument, since a supplier also wishes to satisfy customers as much as possible so they remain faithful in the long term. Therefore, a supplier will try to meet as much as possible the demands of good old friends, ie minimise

$$R^s_{ji}\left(x_{ji\ \arg\max_{k_j}\{v^s_{k_j k_i ji}\}} - x_0^{s;k_i i}\right)^2 \tag{20}$$

C_i will try to do this for all products it supplies, k_i, and all customers j, and of course all companies will try to do that:

$$U_6 \equiv \sum_{i\in N_0}\sum_{k_i\in S(C_i)}\sum_{j\in N_i} R^s_{ji}\left(x_{ji\ \arg\max_{k_j}\{v^s_{k_j k_i ji}\}} - x_0^{s;k_i i}\right)^2 \tag{21}$$

We call this the **not letting down** constraint.

Note that we used a superscript in the relation expression, d or s, because the feelings between two companies may not be mutual: a buyer may consider as a trusty supplier somebody who over the years has delivered on time, while a supplier may consider as a good customer somebody who over the years complaint least.

We may say, therefore, that the optimisation problem we have to solve is: Choose values for all x_{ijk} to minimise

$$U \equiv U_1 + \alpha_2 U_2 + \alpha_3 U_3 + \alpha_4 U_4 + \alpha_5 U_5 + \alpha_6 U_6 \tag{22}$$

where α_2, α_3, α_4, α_5 and α_6 are parameters that may be used to control the relative importance of the individual terms.

In summary, the optimisation problem we have to solve may be expressed as follows: We have a set of companies which have comodities to sell and buy. We have a set of products that are on offer by these companies. We also have a set of products that these companies wish to purchase. This second set is a virtual set: these products exist only in the imagination of the customer. One may assume that in a digital business ecosystem a company that wishes to buy something fills in a form on line with the specifications of the product it is looking for. This product with exactly these

specifications does not exist in reality. It exists only in the head of the customer. All these filled forms with the required products form the **virtual product database**. The first issue is to identify for each product in the virtual product database the closest product in the **database of products on offer**. This may be done by a process like the one described in section 3.1. The next problem is to try to match products and companies taking into consideration all factors that led us to write expression (22). In other words, find values for variables x_{ijk} for all comanies i, j in the ecosystem, (such that $j \neq i$, since a company does not buy from itself), and for all products k in the database of products on offer, so that U defined by (22) is minimised. We remind the reader that x_{ijk} indicates the quantity of product k sold by company j to company i. Function U is called the cost function because it measures the "cost" of each possible solution, not in terms of money, but in terms of how badly the imposed constraints are violated. If it is written in inverse (ie $1/U$), it may be thought of as a fitness function which expresses how good a solition is. (A solution is understood to mean any combination of values x_{ijk} for all possible triplets of subscripts ijk.)

The cost function defined in (22) does not encode the possibility that some transaction may be affected by unforeseen events due to third parties. One may multiply each transaction in the equations that define U_1, U_2, U_3, U_4, U_5 and U_6 with a random factor that depends on the indirect relationship T_{ij} between the two companies. This random factor is not easy to express in this formulation, but if we try to optimise this function using the Langevin approach, the mechanism is easy: In the Langevin approach, function U is treated like a potential function, and the solution like a particle that moves in the solution space under the influence of the force created by this potential function. The solution space has as many axes as we have different triplets ijk. Along each axis of the solution space we measure the values of one of the unknowns x_{ijk}. In this space, a solution is represented by a point. We can think of this point as wandering in the solution space, like a particle under the influence of some forces. Our job is to affect the wandering of this particle so that it stops at a place where U is globally minimum. The force component along each axis this particle feels is $-\frac{\partial U}{\partial x_{ijk}}$. At a point where U has an extremum, this force is zero and the particle will not move away from it unless another, external or new force starts influencing it. At this stage we shall exclude any such external force. However, we may try to take into consideration some random events that may call off a transaction, and model them as an extra force which is a random impulse with strength drawn from a Gaussian distribution with mean 0 and standard deviation proportional to T_{ij}: the higher the inter-dependence of the two transacting partners through other parties, the more likely it is that something might go wrong somewhere and influence their behaviour away from the expected one.

All the above is nothing more than a crude attempt to model a closed business system. There must be much better and more sophisticated models available, developed by experts. We wish to stress that the reason we developed it is to make the connection between the optimisation methodology used by Physicists and the dynamics of a business ecosystem. We are prepared to replace it or adapt it according to the input of expert partners.

One of the points the above model has not addressed is its relevance to the digital aspect of the ecosystem we are discussing. The first thing that comes to mind, is that in a digital business

ecosystem relationships of trust are perhaps more important than in a real endogenous economy, because of lack of face-to-face communication with all its connotations of messages conveyed by the body language. In a world deprived of this aspect of communication (reputed to constitute 80% of human communication), trust build by previous successful transactions becomes a very important factor. So, coefficients α_5 and α_6 in (22) may be given higher values than the other coefficients in the same equation. Other factors in a digital business ecosystem which distinguish it from a physical one may be identified later during the project, and they may include the different nature of products exchanged and the particular requirements that a buyer has from a digital supplier or the concessions a supplier is prepared to offer to a digital customer.

4.3 Using a neural network

Optimisation may also be achieved with the help of a neural network. This is a different way of seeing the probabilistic relaxation approach (which is effectively a parallel algorithm). What characterises the problem we are dealing with, is the asymmetry in the interactions between the units: company C_i may act as a supplier to C_j, in which case C_j is a customer of C_i. Effects of limited supply etc may be modelled as suppressing influences to similar companies approaching the same supplier, while excitatory influences may be exerted by a customer to other potential customers requiring products of different nature. This asymmetry in the interactions and even relations of trust is very obvious in the attempting modelling of the previous section. A network, with asymmetric interactions, may be based on the networks implemented to model biological phenomena. So, we also started work on the architecture of such a self-organising network used to identify the states of the ecosystem and its natural organisation into sub-clusters.

In the sections that follow, we first review the Mathematical foundations of Langevin equation, and then we present a brief overview of the neural network system we propose to adapt and adopt for this study.

5 The Langevin equation and its applications to Economics

5.1 The Brownian motion

In 1826 Robert Brown, a Scottish cleric and botanist, observed with a microscope that small grains of pollen thrown into a viscous fluid performed a zigzag-like move [27], [2]. This phenomenon was later named after him as “Brownian motion”. A satisfactory explanation about it was not found until 1905, when Albert Einstein proved that it is caused by the collisions of specks of pollen with the molecules of the fluid. Einstein’s approach was a well-expressed explanation for the Brownian motion, but it was quite different from the traditional approaches of the physicists. In 1908 Paul Langevin, a French mathematician, was the first to apply Newton’s second law of motion to a particle that followed a Brownian motion. His theory was an expansion of the Newtonian law, in that he added a random term in it. Langevin’s approach was widely accepted, since it was simpler than Einstein’s one and closer to the Newtonian Physics. Approached rather differently, Langevin’s work can be understood as the move of a particle on a landscape towards the global minimum point. That

move is driven by gravity and a frictional force which gradually makes the particle stop at a local minimum. There is, however, a random force that pushes this particle out of the local minimum and makes it continue its move towards the global minimum.

As it is described in [27], Brownian motion can be regarded as a series of random and independent displacements of a particle that take place in a particular time interval. Though we cannot observe any separate displacement with the naked eye, we can see the final result at the end of the time interval. We assume that the Brownian particle starts from the position $x = 0$ and is free to move to any of the two directions along the x–axis. Consequently, each displacement X_i can take one of two values $X_i = +\Delta_x$, or, $X_i = -\Delta_x$ with probability $\frac{1}{2}$ each. By assuming that every random and independent displacement is characterised by "The Random Walk Model" then for each $i = 1, 2, ..., n$,

$$\langle X_i \rangle = \frac{1}{2} \times (+\Delta_x) + \frac{1}{2} \times (-\Delta_x) = 0 \tag{23}$$

so that

$$\langle X \rangle = \langle X_1 \rangle + \cdots + \langle X_n \rangle = 0 \tag{24}$$

and

$$\begin{aligned} \mathrm{var}(X_i) &= \langle X_i^2 \rangle - \langle X_i \rangle^2 = \langle X_i^2 \rangle \\ &= \frac{1}{2} \times (+\Delta_x^2) + \frac{1}{2} \times (-\Delta_x^2) = \Delta_x^2 \end{aligned} \tag{25}$$

so that

$$\langle X^2 \rangle = \sum_{i=1}^{n} \mathrm{var}(X_i) = n\Delta_x^2 \tag{26}$$

The total time interval is $t = n\Delta_t$, i.e. $n = \frac{t}{\Delta_t}$, which if substituted in equation (26), yields:

$$\langle X^2 \rangle = \frac{\Delta_x^2}{\Delta_t} t \tag{27}$$

Equation (27) describes the Brownian motion: the variance $\langle X^2 \rangle$ of the total displacement X is proportional to the time t during which the displacement takes place. Equation (27), however, is not the best way of describing the Brownian motion, because the variance of the total displacement seems to depend separately on the terms Δ_x^2 and Δ_t. We can tackle this problem by introducing "the Wiener process" in our explanation for the Brownian motion. In this case, Brownian motion is presented not as a discrete process, but as a continuous process.

We can define the Wiener process by using a Markov propagator defined as

$$F[X(t), dt] \equiv X(t + dt) - X(t) \tag{28}$$

This is the dynamical form of a Markov propagator. We assume that, when the Wiener process variable $X(t)$ takes a particular value $x(t)$ at time t, $X(t + dt)$ is a normally distributed random variable with mean $x(t)$ and variance $\delta^2 dt$. We assume that,

$$F[X(t), dt] = \sqrt{\delta^2 dt} N_t^{t+dt}(0, 1) \tag{29}$$

where $N_t^{t+dt}(0,1)$ is a unit standard deviation normal probability density function related to the time interval $[t, t+dt]$, and δ^2 is a parameter that characterises the process. This is the simplest continuous Markov process. Markov propagator is a random variable itself, with mean $X(t)$ and variance dt. By replacing (29) into (28) we get:

$$X(t+dt) - X(t) = \sqrt{\delta^2 dt} N_t^{t+dt}(0,1) \tag{30}$$

We shall use (30) to calculate the displacement of the Brownian motion in time. To do that, we first set $t = 0$

$$X(dt) = X(0) + \sqrt{\delta^2 dt} N_0^{dt}(0,1) \tag{31}$$

In a similar way, when $t = dt$

$$X(2dt) = X(dt) + \sqrt{\delta^2 dt} N_{dt}^{2dt}(0,1) \tag{32}$$

Substituting $X(dt)$ from (31) into (32) we get

$$X(2dt) = X(0) + \sqrt{\delta^2 dt} N_0^{dt}(0,1) + \sqrt{\delta^2 dt} N_{dt}^{2dt}(0,1) \tag{33}$$

$N_0^{dt}(0,1)$ and $N_{dt}^{2dt}(0,1)$ refer to disjoint time intervals, so they are statistically independent. If we apply the normal sum theorem to equation (33) (see Appendix A) we obtain:

$$X(2dt) = X(0) + \sqrt{\delta^2 dt}\Big(N_0^{dt}(0,1) + N_{dt}^{2dt}(0,1)\Big) = X(0) + \sqrt{\delta^2 dt} N_0^{2dt}(0,2) \tag{34}$$

If we apply the normal linear transform theorem to (34) (see Appendix A) we obtain:

$$X(2dt) = N_0^{2dt}(X(0), \delta^2 2dt) = X(0) + N_0^{2dt}(0, \delta^2 2dt) \tag{35}$$

Repeating this substitution and addition indefinitely yields:

$$X(t) = X(0) + N_0^t(0, \delta^2 t) \tag{36}$$

This is Einstein's solution for the Brownian motion. According to equation (36), $X(t) - X(0)$ is normally distributed with mean 0 and variance $\delta^2 t$. According to (27), the variance grows linearly with time t, but in (36) the ratio $\frac{\Delta_x^2}{\Delta_t}$ has been replaced by the single factor δ^2. More details about the transformation from the random step Brownian motion to the continuous Brownian motion can be found in [27].

5.2 From the Brownian motion to the Langevin equation

In 1908, Paul Langevin defined the Brownian motion, by using the second Newton' s law of motion. According to this law, the net Force F_{net} on a particle with mass m moving with acceleration α is given by:

$$F_{net} = m \times \alpha \tag{37}$$

What is original about Langevin's work, is that he added a stochastic term u_t in (37). Knowing that acceleration is the first derivative of the velocity and that there is a coefficient $\gamma \geq 0$ that refers to the frictional force, we may rewrite (37) as

$$m\frac{d}{dt}V(t) = -\gamma V(t) + u_t \tag{38}$$

This is Langevin equation. It is an extension of Newtonian law, because it includes the stochastic term u_t. In other words, the net force F consists of the frictional force $-\gamma V(t)$ (which reduces the particle's velocity and that is why it has a negative sign) that directs the particle back to the equilibrium state, and the "fluctuating", or "complementary" force u_t [27], [2], [33], which represents the random collisions of the particle with the molecules of the liquid that makes the particle fluctuate around the equilibrium point. So, Langevin equation is a stochastic differential equation, the first one introduced in the analysis of the Brownian motion. By replacing the differential $dV(t)$ with a finite difference, assuming that the stochastic term is equivalent to the Markov propagator in the Wiener process and by dividing by m we have

$$V(t+dt) - V(t) = -\frac{\gamma}{m}V(t)dt + \frac{1}{m}\sqrt{\delta^2 dt}N_t^{t+dt}(0,1) \tag{39}$$

In order to solve Langevin equation we make the following observations: Our final objective is to find an appropriate form for function $V(t)$. $V(t)$ is itself a random variable, i.e. it may take several different values. Each variable in the sequence of random variables $V(dt), V(2dt), ..., V(t)$ is a linear combination of the independent normal variables $N_0^{dt}(0,1), N_{dt}^{2dt}(0,1), ..., N_{t-dt}^{t}(0,1)$ and those are, in turn, random variables themselves. So, $V(t)$ may be written in the form $V(t) = N_0^t(meanV(t), varV(t))$. In the following, we shall try to find appropriate forms for variables $meanV(t)$ and $varV(t)$.

If we take the expectation value of both sides in (39) we have an ordinary differential equation:

$$\langle V(t+dt) - V(t)\rangle = -\frac{\gamma}{m}\langle V(t)\rangle dt + \frac{1}{m}\sqrt{\delta^2 dt}\langle N_t^{t+dt}(0,1)\rangle = -\frac{\gamma}{m}\langle V(t)\rangle dt \tag{40}$$

or

$$\frac{d\langle V(t)\rangle}{dt} = -\frac{\gamma}{m}\langle V(t)\rangle \tag{41}$$

Equation (41) is a first order linear homogeneous differential equation of the type $\frac{d}{dt}y + p(t)y(t) = 0$ with general solution

$$y(t) = Ce^{-\int p(t)dt} \tag{42}$$

where C is a constant. If we combine (41) and (42) we get

$$\langle V(t)\rangle = Ce^{-\int \frac{\gamma}{m}dt} = v_0 e^{-\frac{\gamma}{m}t} \tag{43}$$

with the initial condition $V(0) = v_0$. So, $meanV(t) = v_0 e^{-\frac{\gamma}{m}t}$.

The next step is to calculate the variance of $V(t)$, or the difference $\langle V(t)^2\rangle - [\langle V(t)\rangle]^2$. It is easy to deduce from equation (42) that $[\langle V(t)\rangle]^2 = v_0^2 e^{-2\frac{\gamma}{m}t}$. Consequently, we only have to calculate $\langle V(t)^2\rangle$. Obviously

$$d[V(t)^2] = [V(t+dt)]^2 - [V(t)]^2 \tag{44}$$

Replacing $V(t+dt)$ from equation (39) to equation (44):

$$d[V(t)^2] = \left[V(t)\left(1 - \frac{\gamma}{m}dt\right) + \sqrt{\delta^2 dt}N_t^{t+dt}(0,1)\right]^2 - [V(t)]^2 \tag{45}$$

or

$$d[V(t)^2] = V(t)^2\left(1-\frac{\gamma}{m}dt\right)^2 + 2V(t)\left(1-\frac{\gamma}{m}dt\right)\sqrt{\delta^2 dt}N_t^{t+dt}(0,1) + \\ + \delta^2 dt[N_t^{t+dt}(0,1)]^2 - V(t)^2 \tag{46}$$

or

$$d[V(t)^2] = -2V(t)^2\frac{\gamma}{m}dt + V(t)^2\frac{\gamma^2}{m^2}(dt)^2 + 2V(t)\sqrt{\delta^2 dt}N_t^{t+dt}(0,1) - \\ - 2V(t)\frac{\gamma}{m}\delta(dt)^{\frac{3}{2}}N_t^{t+dt}(0,1) + \delta^2 dt[N_t^{t+dt}(0,1)]^2 \tag{47}$$

We neglect terms of higher order than dt i.e. terms in dt^2 and $dt^{\frac{3}{2}}$ as too small and obtain:

$$d[V(t)^2] = -2V(t)^2\frac{\gamma}{m}dt + 2V(t)\sqrt{\delta^2 dt}N_t^{t+dt}(0,1) + \delta^2 dt[N_t^{t+dt}(0,1)]^2 \tag{48}$$

Then we take the expectation value of both sides of (48)

$$d\langle V(t)^2\rangle = -2\langle V(t)^2\rangle\frac{\gamma}{m}dt + 2\langle V(t)N_t^{t+dt}(0,1)\rangle\sqrt{\delta^2 dt} + \delta^2 dt \tag{49}$$

Because V(t) and $N_t^{t+dt}(0,1)$ are statistically independent, $\langle V(t)N_t^{t+dt}(0,1)\rangle = \langle V(t)\rangle\langle N_t^{t+dt}(0,1\rangle = 0$. After that (49) becomes

$$\frac{d}{dt}\langle V(t)^2\rangle = -2\frac{\gamma}{m}\langle V(t)^2\rangle + \delta^2 \tag{50}$$

Equation (50) is a first order linear inhomogeneous differential equation of the type $\frac{d}{dt}y + p(t)y(t) = q(t)$, and its general solution is given by:

$$y(t) = e^{-\int p(t)dt}\int q(t)e^{\int p(t)dt}dt + Ce^{-\int p(t)dt} \tag{51}$$

Combining (50) with (51)

$$\langle V(t)^2\rangle = v_0 e^{-\frac{\gamma}{m}t} + \left(\frac{\delta^2 m}{2\gamma}\right)(1-e^{-\frac{\gamma}{m}t}) \tag{52}$$

So, varV(t) $= \langle V(t)^2\rangle - [\langle V(t)\rangle]^2 = \left(\frac{\delta^2 m}{2\gamma}\right)(1-e^{-\frac{\gamma}{m}t})$. At last, we are able to define function $V(t)$ as

$$V(t) = N_0^t\left(v_0 e^{-\frac{\gamma}{m}t}, \left(\frac{\delta^2 m}{2\gamma}\right)(1-e^{-\frac{\gamma}{m}t})\right) \tag{53}$$

Equation (53) is the solution of Langevin equation for the Brownian particle.

5.3 The relationship between Physics and Economics

As mentioned above, the Langevin equation is a stochastic differential equation. The main point in this characterisation is that Langevin can be used in occasions were we deal with problems that cannot be formulated by a concrete mathematical model. Some unpredictable or unexpected disrupting factors keep our solution away from the equilibrium point. These factors enter our models in a random way. Processes governed by these phenomena are called "stochastic processes", as opposed to deterministic. Brownian motion is a stochastic process, since the collisions of the particle with the molecules occur in a random way and the latter fluctuates around the equilibrium point. Langevin was

the first one to attempt to simulate the second Newtonian law in a stochastic process, but inevitably one may wonder why we try to couple Langevin with economics. The gap between the theoretical fields of Newtonian Physics and economics is huge indeed, but there are some commonalities between the two.

The Scottish philosopher Adam Smith is considered to be "The father" of economic science. His most well-known book "The Wealth of Nations"[7], published in 1776, is regarded as the corner stone of political economy and it is so popular, that quite a lot of scientists are able to recall its author when asked about it. However, many of them seem to overlook that Smith wrote this book deeply affected and inspired by Newtonian Physics. According to Adam Smith, one economy is an evolving, self-organising and self-optimising system of agents (particles, corresponding to physics), and thus we can imagine it obeying the same natural laws, just like the laws discovered by physicists. Every agent in such an economic system behaves with the most rational manner, and the whole system itself, always reaches the best solutions for every agent. The Smithian equilibrium is achieved by those natural laws of the economy, as if "an invisible hand", as it was stated by Smith, pushes the economy to the equilibrium state. Nowadays, many of the above concepts have been proven to be inaccurate and inconsistent with what economists believe, but this is not the point. What matters in our approach is that, on the one hand, Adam Smith borrowed some ideas from Physics as a breath of life for his own newborn science, but on the other hand, this served as casting a bridge between the disjunct fields of physics and economics.

Adopting the same view with Liso and Filatrella in [31], we cannot deny that various economic theories developed according to the Smithian concept, turned out to be useless after a certain period of time. It does not mean though, that physics' techniques have nothing to say when applied to economics. The source of the problem lies in the fact that simulating human behaviour (the fundamental task of economics is the study of humans' economic behaviour) is not so easy as simulating physical processes. Unlike what happens with many natural phenomena, an individual's behaviour is not exclusively affected by some standard variables one tries to model. In contrast, it may be influenced by other factors, which invade into one economic model as random terms in the sense that, we cannot predict what one's reaction will be like. For instance, we cannot always know in advance what will happen in the market of a specific agricultural product, because if a random event, such as a flood, destroys some farmers' yield, the price of this specific product will rise unexpectedly and both producers and consumers will react unpredictably to this event. Here we find the interacting point between economics and another branch of physics: mathematics of chaos. Knowledge of the past is not always sufficient for making predictions. "An economy seems to be more of a dynamical system that evolves in time" [31]. By definition, economists are forced to simulate an economic phenomenon in terms of stochastic processes and Langevin equation has been a useful tool in this case.

There are several common characteristics between Economics and Physics that make Langevin equation desirable to the economists [31]. We can find many phenomena in dynamical economic systems that have random behaviour described by a function $x(t)$. Economists believe that the regular economic forces are not the only contributors to these phenomena, but there are also some

[7]The full title of this book is "An inquire into the nature and causes of the wealth of nations"

random forces that affect the behaviour of these phenomena.

$$\frac{d^2x}{dt^2} = \gamma F + Z(t) \tag{54}$$

The Langevin–like equation (54) could be a mathematical formulation for such cases, where F is the regular economic force, γ is a constant and $Z(t)$ is the random force. More reasons making Langevin equation familiar to economics are the following:

- Every system in Economics consists of agents (particles in Physics), whose collective behaviour can be considered as a dynamical system and presented as a set of equations which express the evolution of the state of the system in time, based on the knowledge of its previous history.

- An equation of motion, like Langevin equation, helps one to describe a system through numbers. Economics is a field where the object of investigation can be easily modelled by quantitative parameters.

- Economics seems to be a field where one can define a system with a great number of particles and thus, the laws of thermodynamics can be applied. Also, economic activities are human activities and may be regarded as "energy" consuming, and this is one more motive for economists to think of the laws of thermodynamics

- Macroscopic economic variables are caused by interaction of many microscopic degrees of freedom and their statistical equations will bear resemblance to the equations used in stochastic dynamics.

- Regarding the financial markets, they are supposed to be completely controlled by random forces and economists have to investigate a "financial signal", as Physicists investigate the noise.

The applications of Langevin equation into economics presented here refer to business cycles, supply and demand matching, financial time series analysis and financial risk.

5.4 Economic applications of the Langevin equation.

Since stochastic methods have been fostered by economists, it was bound for Langevin equation to become a tool for economic analysis. As it is mentioned by Cobb in [5], stochastic differential equations are more and more frequently used to express mathematical models of macro-economic systems.[8] Economists are encouraged to do so, by the fact that these techniques can be applied in economics quite easily and they are very powerful in the construction of social theories. Surprisingly, the first time that stochastic differential equations were used in mathematical models was when Bachelie conducted research about the behaviour of stock market prices. In his PhD Thesis in 1900, "Theory of speculation", Bachelie anticipated Einstein's work in all major respects. Even though

[8] Macroeconomics is the branch of economics that studies an economy as a whole, rather than its separate economic agents or entities, with which Microeconomics deals.

economic analysis is mainly based on econometrics and statistics, Langevin has been already used in treating economic problems, some of which we are going to present now.

One of Langevin's applications in economics has to do with the study of the so called "Business Cycles". Economists define as business cycles the seasonal oscillations of major macro-economic variables, such as the Gross Domestic Product (GDP), the Per Capita Income (PCI), the Price Indicator of products (PI), the total rate of private or governmental Investments (I_p, I_g) etc. Generally, economists focus their interest on the fluctuations of GDP (or PCI when divided by a country's population) and compare it with the contemporary fluctuations of other economic variables in order to find some evidence about a country's development. Chen in [3] mentions that in an informal conference paper 1933, Frisch said that damped harmonic cycles may be produced by persistent shocks. The Frisch model corresponds to the Brownian motion of harmonic oscillations and may be described by the Langevin equation. Not only did Langevin equation turned out to be applicable in that case, but also the results were different from those Frisch expected ("the harmonic oscillations will be dampened in an exponential way and persistent cycles cannot be maintained by random shocks"). A fallacy was dispelled.

Granger[9] [15] states that in Economics situations may be found where one variable is plotted against others, e.g. the price of tomatoes in two or more different places of the country, and the variables behave as if they wanted to be near some point in this 2-dimensional space. Such a point is called an **attractor**. The move towards the attractor is not straightforward. The greater the difference between the variables is, the stronger the attraction exerted by the attractor. This can be encoded by using a Langevin equation:

$$\Delta X_{t+1} = -P'(X_t) + \varepsilon_{t+1} \tag{55}$$

where X_t is the vector of variables, $P'(X_t)$ is the derivative with respect to vector X_t and ε_{t+1} is a stochastic term with zero mean. Let us assume that X_t is a single series, $P(0) = 0$ and $P(X)$ is monotonically non–decreasing (ie $P(X) \geq 0$ and $P'(X) \geq 0$). Function $P(X)$ is called the potential at X and is equivalent to the potential energy in Physics. The attractor is at $X = 0$. The regular step towards the attractor $-P'(X_t)$ may be interrupted by a shock ε_{t+1} and then, the variables tend away from the attractor, but they are still affected by an attracting force $-P'(X_t)$ towards the attractor. So, their values do not move directly to the attractor. This process is similar to the Brownian motion. In this case, the viscous fluid is replaced by sticky prices and the potential energy by potential profits.

Langevin equation is also related with Nash equilibrium in Game Theory. Marsili and Zhang introduce in [34] and [35] a class of models with a continuum spectrum of strategies x_i based on Langevin dynamics. It has the form

$$\partial_t x_t = \Gamma_i \frac{\partial u_i}{\partial x_i} + \eta_i \tag{56}$$

where $\partial_t x_t$ is the gradient according to which players adjust their strategies, u_i is the utility function of player i, Γ_i refers to the Gamma Distribution and η_i is a Gaussian stochastic term that models

[9] Clive Granger, Nobel prize in Economics for 2003.

deviation from perfect rationality. Equation (56) is a dynamic model which contains both the deterministic control each player exerts to increase his payoff and the effects of random events. The deterministic part assumes rationality of the agents: every agent knows how to increase their utility. The stochastic term represents all the obstacles that prevent a rational behaviour. These may affect only one player (e.g. illness) or affect more players (e.g. earthquake).

The greatest use of Langevin equation in economics has been in the branch of financial economics. Langevin equation has enabled econometricians and researchers of the stock markets to make predictions about the future values of the stock prices and to create models about the stocks' behaviour. Financial economics borrows a lot of concepts from statistical physics, such as the theories of the random walk and the white noise, Markov processes and Bayesian statistics, because many financial processes are stochastic. Information and experience from the current past are not always adequate to model a financial market. Investors seem to react asymmetrically in sudden events and the traditional economic approaches of Neoclassical and Keynesian advocates, which considered investors as rational agents, do not serve economists well any more. Dominated by these conditions, it was quite easy for financial economics to harbour Langevin equation.

Rosenow in [41] deals with the oscillations in the stock market. He mentions that large price fluctuations tend to cluster together in time and that is the reason why stock price returns were described by models of volatility changing in time. Time series are used in this analysis, in which the volatility at a given time depends on the magnitude of previous returns. In his work, Rosenow uses a different model. He investigates the price dynamics on an intermediate grained time scale Δ_t which is long enough to average over the details of market micro-structure, but too short to resolve the trading dynamics. This method bears resemblance to an observable at which energy is dissipated when it is pulled by an external force. The variance of position fluctuations is proportional to the temperature times the mobility of the particle, just like in equations (5) and (14). The model presented by Rosenow is similar to Langevin equation and is given by:

$$\tau \frac{d}{dt} g(t) = -r g(t) - \kappa g^3(t) + q(t) + \xi_i(t) \tag{57}$$

where g(t) is the the instantaneous return, τ is the number of trades and $q(t)$ is is the instantaneous order imbalance (i.e. the volume of buy orders minus the volume of sell orders). It describes relaxation, controls the strength of fluctuations and it is analogous to the magnetic field of spins. The terms r and $-\kappa$ enforce the soft spin constraint of finite and discrete price changes, $g^3(t)$ stabilises the theory in a regime of strong fluctuations and finally, $\xi_i(t)$ is a Gaussian white noise. It represents the influx of news not captured by the content of the order imbalance.

A final approach is presented by Takayasu et al in [44]. It has to do with tick[10] data analysis of foreign exchange rate. They show that the fat tails of the rate change distribution may be modelled by a Langevin equation with a random coefficient. This equation is of the type

$$\Delta r(t + \Delta t) = b(t) \Delta r(t) + f(t) \tag{58}$$

where the time t may be either tick time or physical time, $f(t)$ and $b(t)$ represent the random force term and the random coefficient respectively. The random coefficient term is dominated by the

[10] A tick in the financial jargon is a financial transaction (buying or selling).

different characters of the dealers (some are more rational than others, some are not very trustworthy etc), and their different predictions.

6 A biologically inspired self-organising neural network

From the analysis of the problem in section 4, it has become very clear that the interactions between the members of the ecosystem are asymmetric. Companies are exchanging inhibitory as well as excitatory signals. This is also evident from the observation of cycles in economic phenomena: systems with asymmetric interactions tend to oscillate, while systems with symmetric interactions hallucinate (ie converge to the wrong solution) [28, 29]. In order to model, therefore, a digital business ecosystem, we must use a network with characteristics that are encountered in biological systems. An example of such a network has been proposed in [8, 28, 29] for modelling the V1 part of the human visual cortex. We present here an abstracted version of it, and we intent to use it for solving the second problem identified in the work-package objectives section.

The network consists of three layers, input, middle and output which are organised into a pyramidal shape. The middle layer is composed of N excitatory and inhibitory (EI) neuron pairs connected with each other. A pair of excitatory and inhibitory neurons is called a segment column. Each excitatory neuron i is connected to k number of sensors to receive sensory inputs ($I_{i1}, I_{i2}, \ldots, I_{ik}$) which are somehow combined into a total input stimulus $I_i = f(I_{i1}, I_{i2}, \ldots, I_{ik})$. The output of such a neuron is projected to the output layer as shown in figure 1. The principal excitatory neurons are connected using intra-layer connections w_{ij} while the excitatory-inhibitory neurons are linked using inter-neuron connections h_{ij}. The excitatory and inhibitory neurons carry values x_i and y_i respectively, from which they generate their output values $g_x(x_i)$ and $g_y(y_i)$ respectively. These gain functions $g_x(\cdot)$ and $g_y(\cdot)$ are sigmoid-like, non-linear and non-decreasing functions, defined as:

$$g_x(x) = \begin{cases} 0 & \text{if } x < \Gamma, \\ (x - \Gamma) & \text{if } \Gamma \leq x \leq \Gamma + 1 \\ 1 & \text{if } x > \Gamma + 1 \end{cases} \tag{59}$$

$$g_y(y) = \begin{cases} 0 & \text{if } y < 0, \\ b & \text{if } 0 \leq y \leq \Lambda \\ b\Lambda + c(y - \Lambda) & \text{if } 0 < \Lambda \leq y \end{cases} \tag{60}$$

where Γ, Λ, b, and c are some constants.

Rather than connecting negative (inhibitory) signals directly to an excitatory neuron, all inhibitory signals are directed to an inhibitory neuron, and their total is then subtracted from the corresponding excitatory signal received directly by the corresponding excitatory neuron. So the inhibitory neuron in the segment column sends an output $g_y(y_i)$ to its corresponding excitatory neuron. An excitatory neuron in another segment column (j) can excite segment column i by sending an excitatory signal $w_{ij} g_x(x_j)$ to the excitatory neuron in segment i, or inhabit the segment by directing an inhibitory signal $h_{ij} g_x(x_j)$. The horizontal inter-layer weights, w_{ij} and h_{ij} primarily depend on two factors, the degree of similarity between sensory inputs and the grid distance d between two segment columns. The further away two neurons are, the weaker their mutual influence.

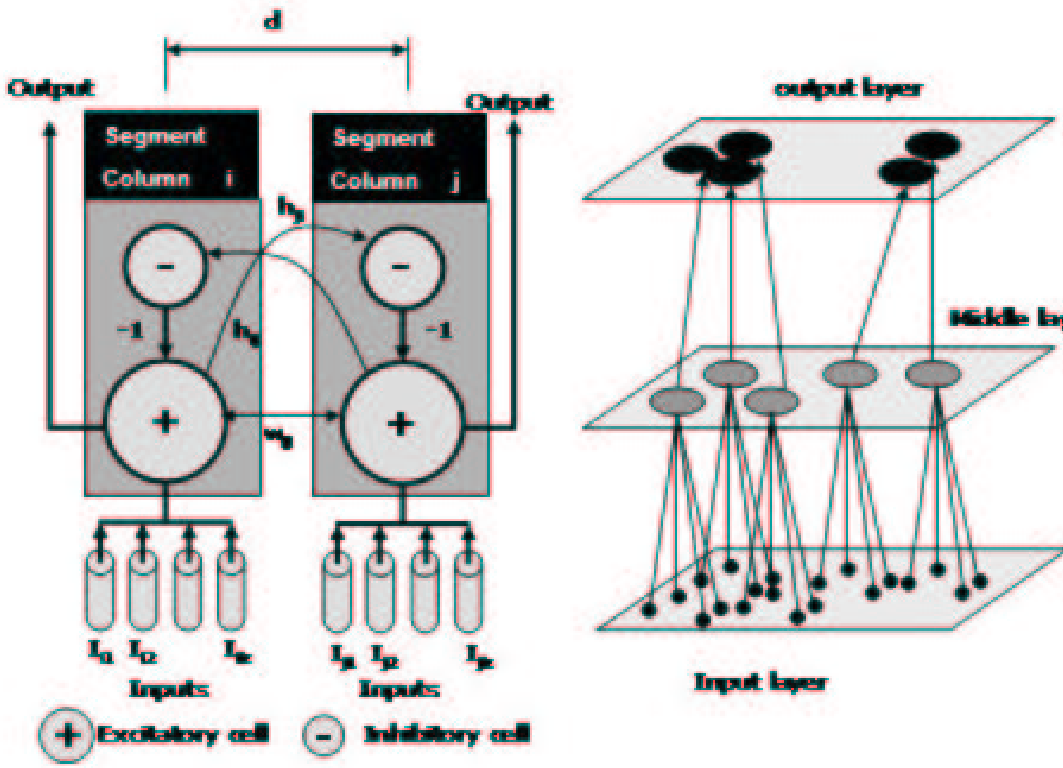

Figure 1: The neural model, its functions and layer architecture

We assume that function $s(f(I_{ir}), f(I_{jr}))$, where $r = 1, 2, \ldots, k$, determines the degree of similarity between the input signals to two neurons, and we define a similarity threshold (Δ), which decides whether the signal from the cell is excitatory or inhibitory. If $s(f(I_{ir}), f(I_{jr})) \geq \Delta$ then neuron j fires an excitatory signal or otherwise, that is if $s(f(I_{ir}), f(I_{jr})) < \Delta$, then it fires an inhibitory signal to suppress the activities of the corresponding neuron i. Therefore, we define the synaptic weighs as follows:

$$w_{ij} = \begin{cases} F(s(f(I_{ir}), f(I_{jr}), d) & \text{if } s(f(I_{ir}), f(I_{jr}) < \Delta \\ 0 & \text{otherwise} \end{cases} \tag{61}$$

$$h_{ij} = \begin{cases} 0 & \text{otherwise} \\ G(s(f(I_{ir}), f(I_{jr}) & \text{if } s(f(I_{ir}), f(I_{jr}) \geq \Delta \end{cases} \tag{62}$$

Where $F(\cdot)$ and $G(\cdot)$ are functions of $s(f(I_{ir}), f(I_{jr}))$ and of the grid distance d.

Since the outputs from the principal excitatory cells, $g_x(x_i)$ and $g_y(y_i)$, are continuous, the network dynamics of the model may be described by a set of simultaneous non-linear differential equations. For a given input pattern I the neural dynamics evolve according to [29]:

$$\dot{x}_i = -\alpha_x x_i - g_y(y_i) + \sum_{j \neq i} w_{ij} g_x(x_j) + \sum_k I_{ik} \tag{63}$$

$$\dot{y}_i = -\alpha_y y_i + \sum_{j \neq i,} h_{ij} g_x(x_j) \tag{64}$$

Here, $\dot{x}_i$ and $\dot{y}_i$ are the firing rates of the excitatory and inhibitory cells respectively. x_i is the value of an excitatory cell, which generates network output through the activation function $g_x(x_i)$. y_i is the value of an inhibitory cell, which inhibits the principal neuron through the activation function $g_y(x_i)$. Time constants, $1/\alpha_x$ and $1/\alpha_y$ describe how rapidly variables x_i and y_i respond to change in input. Both excitatory and inhibitory cells are assumed to be made infinitely fast and therefore α_x and α_y are assumed to be zero.

If we hold the stimulus (sensory inputs) constant over a time period, the system will reach the equilibrium state, where $\dot{x}_i = 0$ and $\dot{y}_i = 0$ and equations (63) and (64) will reduce to:

$$x_i = -g_y(y_i) + \sum_{j \neq i} w_{ij} g_x(x_j) + \sum_k I_{ik} \tag{65}$$

$$y_i = \sum_{j \neq i,} h_{ij} g_x(x_j) \tag{66}$$

We can see that when a pair of excitatory neurons are exciting each other, $h_{ij} = 0$ and $g_y(y_i) = 0$, the value of the excitatory cells will boost their input signals resulting in high gain outputs. Similarly, when they are inhibiting each other,($y_i > 0$ and $g_y(y_i) > 0$), their values are lowered significantly and therefore they are suppressed.

7 Conclusions

There are two very important aspects that characterise the problems we wish to solve in relation to a Digital Business Ecosystem:

- The interactions between the units are asymmetric.
- There are stochastic forces which affect the system.

The mathematical tools we decided to adopt to deal with these issues are asymmetric self-organising neural networks that are inspired from the Biological sciences, and the use of Langevin equation for optimising the objective (or cost) function that characterises each possible solution.

We presented a methodology for defining such a cost function. It takes into consideration several of the aspects discussed in sections 1 and 2, which were found in the economics/business literature. Some of these aspects are

- The increased importance placed in the trust relationship between buyer/supplier.
- The factors that make a product desirable or not to a customer.
- The fact that the desired product is never exactly identical with what is available in the market.
- The possibility of incorporating relative importance to the various factors that define the matching value of sought-supplied product.
- The possibility of modelling 3rd party-dependent indirect interactions between companies.
- The possibility of modelling stochastic effects influencing the solution.

We intent to proceed by implementing this model using an example endogenous economy which will play the role of a business ecosystem. The code that will be developed will be written in a way that allows the substitution of the input variables by the true parameters of the SMEs that will constitute the digital business ecosystem of the demonstrator. The danger is, of course that the SMEs that will subscribe, will not really form an ecosystem, ie a closed system. This issue is very real and in the second phase of this project we intent to simulate the effect of external factors, ie the fact that the system is not closed, as an external influencing field, borrowing again concepts from the Physical sciences.

APPENDIX A

According to the normal sum theorem, two statistically independent normal variables sum to another normal variable:

$$N_1(m_1, a_1^2) + N_2(m_2, a_2^2) = N(m_1 + m_2, a_1^2 + a_2^2) \tag{67}$$

According to the normal linear transform theorem, a linear function of a normal variable is another normal variable with appropriately modified mean and variance:

$$\alpha + \beta N(m, \alpha^2) = N(\alpha + \beta m, \beta^2 \alpha^2) \tag{68}$$

With $m = 0$ and $\alpha^2 = 1$

$$\alpha + \beta N(0,1) = N(\alpha, \beta^2) \tag{69}$$

So a normal variable $N(\alpha, \beta^2)$ is a linear transform of the unit normal $N(0,1)$.

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