

ON THE BOUNDARY BEHAVIOUR OF DIFFUSIONS AND THE MARTINGALE PROPERTY OF THE ASSOCIATED LOCAL MARTINGALES

MIKHAIL URUSOV

Abstract. Let ψ_r denote the increasing r -harmonic function ($r > 0$) associated with a regular one-dimensional diffusion X with the state space $[\alpha, \beta] \subseteq [-\infty, \infty]$. We give several different necessary and sufficient conditions for the local martingale $(e^{-rt}\psi_r(X_t))_{t \geq 0}$ to be a true martingale when the boundary β is inaccessible (if β is accessible, there is nothing to study). One of them is that β should be a natural boundary. In particular, the property of interest does not depend on $r > 0$, and the boundary behaviour of X at α does not play any role. Clearly, the result has its analogue for the decreasing r -harmonic function φ_r . The functions ψ_r and φ_r and their properties are important in optimal stopping. The counterpart of the property of interest for $r = 0$, namely the property of the local martingale $(p(X_{t \wedge \zeta}))_{t \geq 0}$ to be a true martingale (p is the scale function, ζ is the hitting time of $\{\alpha, \beta\}$), turns out to have a different characterisation, where the boundary behaviour of X both at α and at β plays a role.

This is a joint work with Mihail Zervos.

UNIVERSITÄT DUISBURG-ESSEN

E-mail address: `mikhail.urusov@uni-due.de`