Cutting Polygons, Configuration Spaces, and Binomial Coefficients

Günter Ziegler

We describe a combinatorial model for the configuration space of n distinct labeled points in the plane: a cell complex whose $n!$ vertices are indexed by permutations, and whose $n!$ facets are permutahedra. Based on this, we treat the conjecture by Nandakumar and Ramana Rao that every polygon can be partitioned into n convex parts of equal area and perimeter. Equivariant obstruction theory reduces everything to the question which rows of Pascal's triangle have a common divisor.

This is joint work with Pavle V. M. Blagojević.

The Evolution of Achlioptas Processes

Lutz Warnke

In the *Erdős–Rényi random graph process*, starting from an empty graph, in each step a new random edge is added to the evolving graph. One of its most interesting features, both mathematically and in terms of applications, is the 'percolation phase transition': as the ratio of the number of edges to vertices increases past a certain critical point, the global structure changes radically, from only small components to a single macroscopic ('giant') component plus small ones.

In this talk we discuss *Achlioptas processes*, which are widely studied variations of the classical Erdős–Rényi process. Starting from an empty graph these proceed as follows: in each step *two* potential edges are chosen uniformly at random, and using some rule *one* of them is selected and added to the evolving graph. Many simulations suggested that for certain Achlioptas rules the percolation phase transition is particularly radical: more or less as soon as the macroscopic component appears, it is already extremely large; this phenomenon is known as 'explosive percolation'. We shall briefly explain this striking and unusual phenomenon, and discuss some recent progress in our mathematical understanding of Achlioptas processes.

This is joint work with Oliver Riordan.

Applications of Discrete Harmonic Analysis, Probabilistic Method and Linear Algebra in Fixed-Parameter Algorithmics

Gregory Z. Gutin

Let P be a decision problem (answers are yes or no). A parameterized problem Π is a set of pairs (x, k) where x is an instance of P and k (usually an integer) is the parameter. One example is the k-Vertex Cover problem, where for a given graph G we are to decide whether there is a set of k vertices covering all edges of G.

A parameterized problem Π is fixed-parameter tractable (FPT) if it can be solved in time $O(f(k)|x|^c)$ for some function f in k and an absolute constant c. For example, k-Vertex Cover is FPT; in particular, there is a k-Vertex Cover algorithm with runtime $O(1.2738^k + kn)$ (Chen et al., 2010).

In 2006, several open questions were published on whether some problems parameterized above average (AA) are FPT (the problem Betweenness AA in a monograph of Niedermeier and Max-r-Sat AA, MaxLin2 AA and Max Subdigraph AA in a conference paper of Mahajan, Raman and Sikdar).

It turned out that whilst traditional combinatorial methods seemed not appropriate for solving the questions, approaches based on Discrete Fourier Analysis, Probabilistic Method and Linear Algebra allowed us to solve the questions. The main aim of the talk is to describe the successful approaches, which are of interest not only in fixed-parameter algorithmics, but also in combinatorics including graph theory.

A Multipartite Hajnal-Szemerédi Theorem

Richard Mycroft

The celebrated Hajnal-Szemerédi Theorem states that if G is a graph on rn vertices with minimum degree at least $(r-1)n$, then G contains a perfect K_r-packing. That is, we can find vertex-disjoint cliques of size r in G which together cover every vertex of G. Fischer conjectured that an analogous result holds for r-partite graphs; however, a single family of counterexamples to this conjecture was identified by Magyar and Martin. The 'modified Fischer conjecture' states that the graphs of this family are the only counterexamples to Fischer's conjecture; this modified conjecture was proved for $r = 3$ by Magyar and Martin and for $r = 4$ by Martin and Szemerédi. In this talk I will outline a proof of the 'modified Fischer conjecture' for any r, which proceeds by proving a more general result on K_r-packings in r'-partite graphs for $r \leq r'$.

This is joint work with Peter Keevash.

An Approximate Version of the Tree Packing Conjecture for Trees with Bounded Maximum Degree

Julia Böttcher

An old conjecture of Gyárfás and Lehel states that every family of trees $T_2, T_3, ..., T_n$ such that T_i is a tree on i vertices for each i, can be packed into the complete graph K_n. We show that this is true for families of trees with bounded maximum degree if K_n is replaced by $K_{(1+\epsilon)n}$. Our proof uses a random embedding approach and combines a nibble method with suitable error correction techniques.

This is joint work with Jan Hladký, Diana Piguet and Anusch Taraz.

Flag Algebra Method in Extremal Combinatorics

Daniel Král'

Razborov [J. Symbolic Logic (2007), 1239-1282] developed a formal algebraic framework for deriving true relations among densities of substructures of combinatorial objects (e.g., subgraphs of a graph). There are now many successful applications of this method to problems from extremal combinatorics.

In the talk, we provide a brief but self-contained introduction to the method and we then focus on the following question, attributed to Graham, which is related to quasirandomness of permutations: "Does there exists k such that the following holds? If the density of every k-point permutation in a permutation is $1/k! + o(1)$, then the density of every k'-point permutation is $1/k'! + o(1)$." We show that the answer is positive for $k = 4$ and it is negative for $k = 3$.

The talk is based on joint work with Oleg Pikhurko.

Random Geometric Graphs on Non-Euclidean Spaces

Nikolaos Fountoulakis

Random geometric graphs have been well studied over the last 50 years or so. These are graphs that are formed between points randomly allocated on a Euclidean space and any two of them are joined if they are close enough. However, all this theory has been developed when the underlying space is equipped with the Euclidean metric. But, what if the underlying space is curved?

The aim of this talk is to initiate the study of such random graphs. Our focus will be on the case where the underlying space is a hyperbolic space. We will discuss some typical structural features of these random graphs as well as some applications, related to their potential as a model for networks that emerge in social life or in biological sciences.

Szemerédi Strikes Back

János Pach

By an argument reminiscent of Furstenberg's original ergodic theoretic proof for Szemerédi's Theorem on arithmetic progressions, Furstenberg and Weiss (2003) proved the following result. For every k and l, there exists an integer $n(k, l)$ such that no matter how we color the vertices of a complete binary tree of depth $n > n(k, l)$ with k colors, it always contains a monochromatic equispaced complete binary subtree T' of depth l; that is, a complete binary subtree T' of depth l which has the property that all of its vertices are of the same color and every vertex at level i of T' lies at level $j + id$ in T. (Here j and d are suitable integers and $0 \leq i \leq l$.) Moreover, the two children of any vertex v of T' are descendants of different children of v in T. Furstenberg and Weiss also established several density versions of the above results, generalizing Szemerédi's Theorem.

We show that all of these results can be obtained by elementary combinatorial arguments, using Szemerédi's classical theorem itself.

This is joint work with J. Solymosi and G. Tardos.

Fast and Slow Mixing in the Ferromagnetic Potts Model

Magnus Bordewich

The Potts model is a statistical physics model of magnetism closely related to the Tutte polynomial of a graph in combinatorics. One element of interest is the Glauber dynamics of the model - a Markov chain process on a vertex colourings of the underlying graph. Each step of the Markov chain involves recolouring a single vertex. The state of the chain converges to a stationary distribution which is a weighted distribution on all colourings of the graph. This convergence can either happen in polynomial time in the number of vertices of the graph (rapid mixing), or it can take an exponential number of steps (torpid mixing). In this talk we will explore what properties of the graph and the parameters of the model lead to fast or slow mixing.

Finding Cycles and Trees in Sublinear Time

Artur Czumaj

We study the complexity of one-sided error property testing algorithms for finding subgraphs in the bounded-degree graphs model. Our main focus is on the problem of finding simple cycles and tree-minors in such graphs.

We first show that one can test (with one-sided error) within time complexity $O*(poly(1/\epsilon) \cdot N^{1/2})$ if a given N-vertex graph is cycle-free or an ϵ-fraction of the edges must be deleted to make the graph cycle-free. This matches the known $\Omega(N^{1/2})$ query lower bound for one-sided error cycle-freeness testing, and contrasts with the fact that any minor-free property admits a two-sided error tester of query complexity that only depends on ϵ. Furthermore, the same upper bound holds for testing whether the input graph has a simple cycle of length at least k, for any $k \geq 3$. On the other hand, for any fixed tree T, we show that T-minor freeness has a one-sided error tester of query complexity that only depends on the proximity parameter ϵ. These time complexity bounds are optimal up to polylogarithmic factors.

Joint work with Oded Goldreich, Dana Ron, C. Seshadhri, Asaf Shapira, and Christian Sohler.

Regularity Method in Sparse Graphs

Mathias Schacht

Over the last two decades sparse versions of Szemerédi's regularity lemma for subgraphs of sparse random and pseudorandom graphs were systematically studied. In this talk we focus on recent results for subgraphs of the random graph $G(n,p)$. In particular, we show that an accompanying probabilistic embedding lemma for a fixed graph F is true for the best possible condition on p, as it was conjectured by Kohayakawa, Łuczak, and Rödl. We present a few applications of this result. This joint work with D. Conlon, W. T. Gowers, and W. Samotij. Similar results based on a differnt approach were also obtained recently by J. Balogh, R. Morris, and W. Samotij.

"The Norman Biggs Lecture"

Levi Graphs and Concurrence Graphs as Tools to Evaluate Designs

Rosemary Bailey

Each incomplete-block design for v treatments in b blocks gives rise to a Levi graph (with $b+v$ vertices) and a concurrence graph (with v vertices). The various criteria that statisticians use to assess the quality of the design are related to familiar graph concepts: number of spanning trees, electrical resistance, edge-connectivity, and so on. Sometimes calculations are easier for the Levi graph than for the concurrence graph. Sometimes nice classes of graphs, such as distance-regular graphs, give mathematically satisfying results but not necessarily the best block design to use in practice.