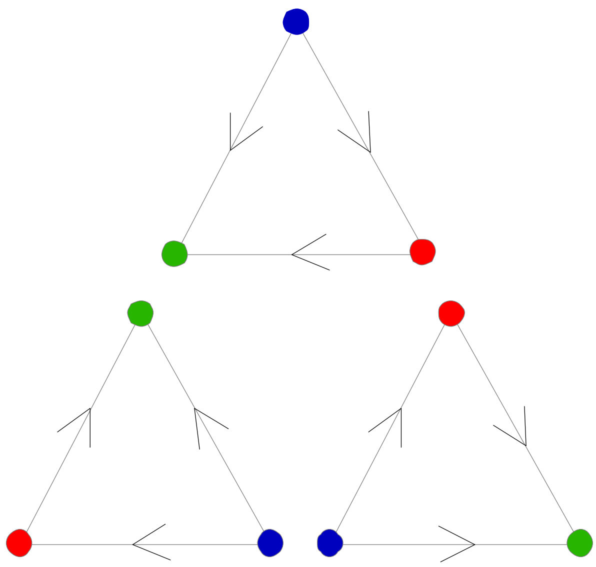
THE LONDON SCHOOL OF ECONOMICS AND POLITICAL SCIENCE

Queen Mary University of London

Queen Mary, University of London

The London School of Economics and Political Science

One Day Colloquia in Combinatorics



19th & 20th May 2010

Queen Mary
University of London

WEDNESDAY 19th MAY 2010

The first of the two linked colloquia in combinatorics will be held at Queen Mary, University of London on Wednesday 19th May, starting at 10.40. Everyone interested is welcome to attend any part of the event. All the talks will be held in the Clinical Medical Lecture Theatre (CMLT) which can be found in the Francis Bancroft Building.

10.00 - 10.30	*Coffee available in Francis Bancroft Building Foyer.*
10.40 - 11.20	**I. Bárány** (UCL)
	Extremal problems for convex lattice polytopes
11.25 - 12.05	**D. Lockett** (Leeds)
	Homogeneous coloured multipartite graphs
12.05 - 13.30	***Lunch** (own arrangements - options on campus and nearby)*
13.30 - 14.10	**E. Long** (Cambridge)
	Long paths in subgraphs of the cube
14.15 - 14.55	**K. Markström** (Umeå, Sweden)
	Minimum degree conditions for perfect matchings in uniform hypergraphs
14.55 - 15.35	***Coffee break** – Francis Bancroft Building Foyer*
15.35 - 16.15	**J. van den Heuvel** (LSE)
	Graph Colouring with Distances
16.20 - 17.00	**B. Bukh** (Cambridge)
	Sum-product estimates for rational functions

Support for this event by the London Mathematical Society and the British Combinatorial Committee is gratefully acknowledged.

Extremal problems for convex lattice polytopes

Imre Bárány

In this survey talk I will present several extremal problems, and some solutions, concerning convex lattice polytopes. A typical example is to determine the minimal volume that a convex lattice polytope can have if it has exactly n vertices. Other examples are the minimal surface area, and the minimal lattice width in the same class of polytopes. These problems are related to a question of V I Arnold from 1980 asking for the number of (equivalence classes of) lattice polytopes of volume V in d-dimensional space, where two convex lattice polytopes are equivalent if one can be carried to the other by a lattice preserving affine transformation.

Homogeneous coloured multipartite graphs

Deborah Lockett

A relational structure is homogeneous if any isomorphism between finite substructures can be extended to an automorphism of the whole structure. Homogeneity of countable structures has been widely studied, and classifications have been found for various types of structures including graphs, partially ordered sets, tournaments, permutations, and digraphs.

Cherlin posed the problem of classifying all the countable homogeneous n-graphs. Here, for n a positive integer, an n-graph is a structure whose domain is partitioned into n parts, each of which has a graph structure, and such that between each pair of parts there are finitely many possible edge types (which we think of as colours). We investigate part of this problem, in the special case of coloured multipartite graphs, that is, where each of the parts is a null graph. Using Fraïssé's Theorem, this problem reduces to that of classifying amalgamation classes of finite coloured multipartite graphs, and I will present the results obtained so far. This is joint work with John Truss.

Long paths in subgraphs of the cube

Eoin Long

Let Q_n denote the graph of the n-dimensional cube with vertex set $\{0,1\}^n$ in which two vertices are adjacent if they differ in exactly one coordinate. Suppose G is a subgraph of Q_n with average degree at least d. How long a path can we guarantee to find in G?

My aim in this talk is to show that G must contain an exponentially long path. In fact, if G has minimum degree at least d then G must contain a path of length $2^d - 1$. Note that this bound is tight, as shown by a d-dimensional subcube of Q_n. I hope to give an overview of the proof of this result and to discuss some generalizations.

Minimum degree conditions for perfect matchings in uniform hypergraphs

Klas Markström

In recent years a number of authors have considered Dirac-type conditions for different properties of k-uniform hypergraphs. One of the classical problems in the area is to determine the minimum vertex degree of a hypergraph G which implies the existence of a perfect matching, when the number of vertices in G is divisible by k. For general k the best bound has been that of Daykin and Häggkvist, from 1982. Similarly the degree of l-tuples with $1 \leq l \leq k$ has been considered. For $k/2 \leq l$ and $k = 3$ the exact values have been found.

In this talk I will survey some of these results and present a new bound for $l < k/2$ which improves the known bounds in that range of l, and $k > 3$, including the Daykin-Häggkvist bound. This is joint work with Andrzej Rucinski.

Graph Colouring with Distances

Jan van den Heuvel

For a graph G and a positive integer k, a *distance-k colouring* of G is a colouring of the vertices of G in which vertices at distance most k must get different colours. Another way to look at this problem is by considering the *kth power* G^k of G; the graph with the same vertex set as G and with an edge between any two different vertices that have distance at most k in G. So finding the minimum number of colours required in a distance-k colouring is the same as finding the chromatic number $\chi(G^k)$ of the kth power G^k.

A major part of research on graph colouring with distances has been for the case $k = 2$, and for specific graphs like planar graphs. In this talk we aim to give some idea of results (and open problems) for larger k.

We will also discuss some work on variants of the problem where the distance requirement is changed. For instance, what happens if we require that only vertices at distance **exactly** k need to receive different colours?

Sum-product estimates for rational functions

Boris Bukh

Suppose A is a set of numbers and $f(x, y)$ is a polynomial, how small can $f(A, A)$ be? If $f(x, y) = x + y$ or $f(x, y) = xy$, then $f(A, A)$ can be very small indeed if A is a progression. However, Erdős and Szemerédi proved that $A+A$ and AA cannot be simultaneously small when A is a set of real numbers. Their results has been generalised to other rings, and have found numerous applications in number theory, combinatorics, theoretical computer science, and other fields.

In this talk, I will give a gentle introduction to the combinatorial ideas behind the sum-product estimates, and will discuss several new results for other rational functions f. Joint work with Jacob Tsimerman.

THURSDAY 20th MAY 2010

The second of the two linked colloquia in combinatorics will be held at the London School of Economics on Thursday 20th May, starting at 10.00am. Everyone interested is welcome to attend any part of the event. The talks will be held in the Wolfson Theatre (NABLG01), in the New Academic Building (NAB). All visitors are asked to report to the main entrance of the NAB, opposite Lincoln's Inn Fields and behind Kingsway, where they will be asked to sign in before entering the building.

10.00 - 10.45	**A. Thomason** (Cambridge) *The probability of hereditary properties*
10.45 - 11.05	***Coffee break –*** *Outside Wolfson Theatre*
11.05 - 11.50	**P. Allen** (Warwick) *Randomising extremal combinatorics*
11.55 - 12.40	**O. Pikhurko** (Carnegie Mellon) *All large trees are prime*
12.40 - 14.05	***Lunch*** *(own arrangements - options on campus and nearby)*
14.05 - 14.50	**M. Penrose** (Bath) *Strict inequalities of critical points in continuum percolation*
14.50 - 15.10	***Coffee break –*** *Outside Wolfson Theatre*
15.10 - 15.55	**D. Kühn** (Birmingham) *Sumner's universal tournament conjecture*
16.00 - 17.00	"The Norman Biggs Lecture" **T. Łuczak (**Poznan, Poland) *Colouring dense graphs via VC-dimension*

Support for this event by the London Mathematical Society, the British Combinatorial Committee and an anonymous donor, is gratefully acknowledged.

The probability of hereditary properties

Andrew Thomason

A hereditary property of graphs is one which is preserved under the removal of vertices - ie it is inherited by induced subgraphs. The number of graphs with a given property was studied by Prömel and Steger and by Alexseev, and Bollobás and Thomason studied the probability that a random graph has a given property.

Recent results on coloured and weighted graphs have shone light on the earlier work and allow much more precise statements and calculations to be made. We shall describe this work (which is joint with Ed Marchant) and its relationship with classical theories.

Randomising extremal combinatorics

Peter Allen

Turán's theorem is a typical theorem in extremal combinatorics. It states that whenever one is given an n-vertex graph G, satisfying the condition that no r-set of vertices of G form a clique of G, then there is a non-trivial upper bound on the number of edges in G. But how robust is Turán's theorem?

We propose a random model in which such questions can be answered, and give some results - in particular, we describe the behaviour of Turán's theorem in our model. This is joint work with Julia Böttcher, Jan Hladký, and Diana Piguet.

All large trees are prime

Oleg Pikhurko

Around 1980 Entringer conjectured that every tree is prime, that is, its vertices can be bijectively labeled with integers $1, \ldots, n$, where n is the order of the tree, so that every two adjacent vertices get coprime labels. We prove this conjecture for all sufficiently large n. This is joint work with Penny Haxell and Anusch Taraz.

Strict inequalities of critical points in continuum percolation

Mathew Penrose

For any infinite connected graph, the critical probabilities for bond percolation and for site percolation satisfy the inequality $p_c^{\text{bond}} \leq p_c^{\text{site}}$. Moreover, this is known to be a strict inequality on a large class of lattices. In this talk, we discuss the extension of the strict inequality to certain *random* graphs arising in continuum percolation, including the Gilbert graph in which each point of a homogeneous planar Poisson point process of supercritical intensity λ is connected by an edge to every other Poisson point within unit distance.

More generally, we consider the random connection model, in which each pair of Poisson points distant at most r apart is connected by an edge with probability p, so that the average node degree is $\lambda\pi r^2 p$. Given r and p there is a critical intensity $\lambda_c(p, r)$ above which the graph percolates. As well as the strict inequality $p_c^{\text{bond}} < p_c^{\text{site}}$ for this graph, we discuss a related result which says that $\lambda_c(r^{-2}, r)$ is strictly decreasing in r. That is, for a given average node degree it is easier to percolate in a graph with long-range connections than in a graph with only short-range connections.

Joint work with Massimo Franceschetti and Tom Rosoman.

Sumner's universal tournament conjecture

Daniela Kühn

Sumner's universal tournament conjecture states that any tournament on $2n-2$ vertices contains a copy of any directed tree on n vertices. In my talk I will discuss a proof of this conjecture for large n as well as related results and open problems. This is joint work with Richard Mycroft and Deryk Osthus.

"The Norman Biggs Lecture"

Colouring dense graphs via VC-dimension

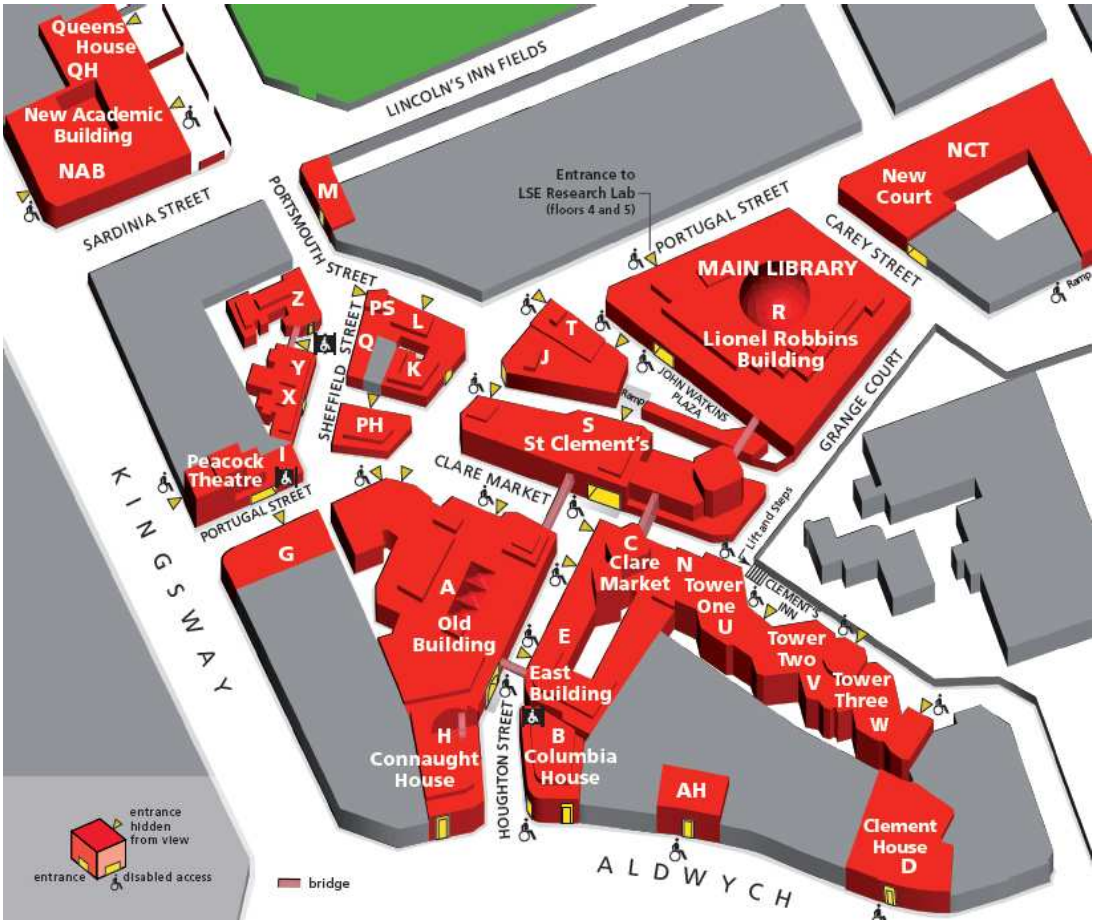
Tomasz Łuczak

In the talk we demonstrate how to use properties of Vapnik-Chervonenkis dimension and its generalization to study the structure of graphs with large minimum degree not containing a copy of a given small subgraph.

This is joint work with Stephan Thomassé.

Finding Your Way Around The LSE

The number indicates both the floor and the room. Room numbers in the basement begin with a zero, numbers 1-99 are on the ground floor, 100-199 are on the first floor, 200-299 on the second floor and so on. Some rooms are identified by name rather than number.



A – Old Building	N – The Anchorage
AH – Aldwych House	**NAB – New Academic Building**
B – Columbia House	PH – Parish Hall
C – Clare Market Building	PS – Portsmouth Street
D – Clement House	Q – Sheffield Street
E – East Building	R – Lionel Robbins Building, LSE Library
G – 20 Kingsway	S – St Clement's Building
H – Connaught House	T – The Lakatos Building
I – Peacock Theatre	U , V, W – Tower One, Two, Three
J – Cowdray House	X – St Philips Building – Health Centre
K – King's Chambers	Y – St Philips Building – South Block
L – Lincoln Chambers	Z – St Philips Building – North Block
M – 50 Lincoln's Inn Fields	