Weighted admissibility with respect to the right shift on H^2

February 20, 2014

Let $\mathcal{H}$ be a Hilbert space, $T \in \mathcal{B}(\mathcal{H})$ and $C \in \mathcal{H}^*$. Now consider the discrete time control system

$$x_{n+1} = Tx_n,\ y_n = Cx_n, \qquad x_0 \in \mathcal{H},\ n = 0, 1, 2, \ldots.$$

We say that C is α-admissible ($\alpha \geq 0$) with respect to T if the output map $x_0 \mapsto ((1+n)^{\frac{\alpha}{2}} y_n)_0^\infty$ is bounded from $\mathcal{H}$ to $l^2(\mathbb{N})$. In the early 2000s Jacob, Partington and Weiss showed that 0-admissibility with respect to a contraction is characterized by a certain resolvent condition. Their proof relied heavily on understanding admissibility with respect to the shift on Hardy space H^2. The main result of this talk is a characterization of α-admissibility, with respect to the shift on H^2, in terms of a resolvent condition. This is obtained through new results about a type of generalized Hankel operators on H^2. The talk is based on a joint work with Birgit Jacob and Andrew Wynn.