THE MULTI-COLOURED RAMSEY NUMBERS OF ODD CYCLES.

ABSTRACT. For a graph G and an integer $k \geq 2$, $R_k(G)$ denotes the smallest integer N for which any edge-coloring of the complete graph K_N by k colors contains a monochromatic copy of G. In 1973 Bondy and Erdős conjectured that, for an odd cycle C_n on $n \geq 3$ vertices,

$$R_k(C_n) = 2^{k-1}(n-1) + 1 \quad \text{for} \quad n > 3.$$

Recenlty, Kohayakawa, Simonovits and Skokan resolved the $k = 3$ case of this conjecture for large n. For $k \geq 4$, the conjecture remains open. Łuczak, Simonovits and Skokan provide the upper bound

$$R_k(C_n) \leq k2^k n + o(n) \qquad \text{as } n \to \infty.$$

In this talk we discuss recent improvements to this bound. We will build on the ideas of Łuczak, Simonovits and Skokan, and import techniques from the theory of non-linear programming. We also discuss progress towards an asymptotic verification of the Bondy-Erdős conjecture.