

Title: Some history and applications of generalized coloring numbers

Abstract: The notion of generalized coloring numbers arose from an idea of Chen and Schelp for extending a Ramsey theoretic result from the class of graphs with bounded degree to a larger class including planar graphs. Trotter, Zhu, and I, as well as other authors, developed their idea to bound the game chromatic number of various graph classes, including planar graphs. For a graph G , Yang and I formalized these ideas by introducing hierarchies of *weak k -coloring numbers* $wcol_k(G)$, *k -coloring numbers* $col_k(G)$, and *game k -colorings* $gcol_k(G)$ numbers, and showed each hierarchy is bounded in terms of each other hierarchy. Shortly after Nešetřil and Ossona de Mendez introduced the notion of *classes with bounded expansion*, and Zhu proved these are exactly the classes whose k -coloring numbers are all bounded. Grohe, Kreutzer, Rabinovich, Siebertz, and Stavropoulos proved that tree-width can be characterized in terms of *infinite* coloring number, and Nešetřil and Ossona de Mendez proved that tree-depth can be characterized in terms of *infinite* weak coloring number. I will give examples of applying these notions to graph theoretic problems concerning **coloring**, **games**, and **packing**.