Approximable Discounted Stochastic Games and Stationary Markov Equilibria

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Current Version, September 2, 2013

While the existence of Nash equilibria in stationary Markov strategies for m-player, discounted stochastic games with *countable* state spaces and compact metric action spaces has long been established (e.g., see Federgruen, 1978), the existence of such equilibria for the *uncountable* case remains an open question.[2] The objective of this paper is to resolve this long-standing open question by establishing just such an existence result. Our approach has two parts. First, we show that all approximable stochastic games have stationary Markov equilibria.[3] Our proof is based on upon a new selection result for measurable selection valued correspondences. We establish this selection result via a novel application of Caratheodory approximation. Second, we establish that all discounted stochastic games - in the class of game models treated by Mertens and Parthasarathy (1987, 1991), Nowak and Raghavan (1992), Duffie, Geanakoplos, Mas-Colell, and McLennan (1994), Salon (1998), and Maitra and Sudderth (2007) - are approximable. We will refer

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[2] We will often refer to a finite-player, nonzero-sum discounted stochastic game in which players' strategy sets are compact metric spaces and the state space is uncountable as an *uncountable-compact* discounted stochastic game.

[3] For example, all noisy stochastic games - a class recently studied by Duggan (2012) - are approximable. But the class of approximable discounted stochastic games is strictly larger than the class of noisy discounted stochastic games.

to this class of discounted stochastic game models as the *Mertens class*. Our proof of this fact rests on a new continuous approximation result for *minimal* Nash mapping - and this result, in turn, rests upon our result showing that all such minimal Nash mappings take values which are nonempty, compact, connected, arcwise connected, R_δ, and hereditarily unicoherent. The fact that minimal Nash mappings take hereditarily unicoherent values means that these values - these minimal sets of Nash equilibria - contain no closed curves (i.e., are not homeomorphic to the unit circle). It is precisely this hereditary unicoherence property of minimal Nash equilibria sets that puts the Mertens class of discounted stochastic game models outside the set of game models to which the counterexamples recently constructed by Levy (2012a and b) applies.

JEL Classification C73, E23

Key words. stationary Markov equilibrium, discounted stochastic games, approximable Nash correspondences, approximate Caratheodory selections, USCO maps, minimal USCO maps, quasi-minimal USCO maps, locally connected spaces, M-convex metric spaces, Peano continua, continuous selections, measurable selection valued correspondences, Komlos' Theorem, K-Convergence, approximate fixed points, Lyapunov's Theorem, Aumann Integrals, Brouwer-Schauder-Tychonoff Fixed Point Theorem