A Stochastic Ramsey Theorem

We present and prove a stochastic extension of Ramsey's theorem. For any Markov chain, we consider any 2-valued colour function defined on every pair consisting of a bounded stopping time and a finite partial history of the chain truncated before this stopping time. For any infinite history ω, let $\omega|\theta$ denote the finite partial history contained in ω up to and including the stopping time $\theta(\omega)$. We prove that for every $\epsilon > 0$, there is an increasing sequence $\theta_1 < \theta_2 < ...$ of bounded stopping times having the property that, with probability greater than $1 - \epsilon$, the history ω is such that the values assigned to all pairs $(\omega|\theta_i,\ \theta_j)$, with $i < j$, are the same. Just as for the classical Ramsey theorem, we also obtain a finitary stochastic Ramsey theorem: for any finite l and for long enough partial histories there exists an increasing sequence of bounded stopping times $< \theta_1, ..., \theta_l >$ and, with appropriate finiteness assumptions, we find that the time one must wait for the last θ_l is uniformly bounded, independently of the probability transitions. We generalise the results to any finite number of colours.