Lobbying, Inside and Out: How Special Interest Groups Influence Policy Choices*

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Abstract

Scholars have long recognized two classes of special interest group (SIG) expenditures: inside lobbying, which is intended to influence the content of a bill; and outside lobbying, which is intended to influence the likelihood a bill is enacted into law. This paper juxtaposes both lobbying activities within a single model. Policy choices are a function of the decision-maker's assessment of SIGs' willingness to engage in outside lobbying. Importantly, inside lobbying expenditures do not always reflect SIGs' outside lobbying capacities and therefore cannot adequately measure SIG influence. Empirical studies which exclusively consider inside lobbying expenditures–as nearly all existing tests do–are likely to underestimate both the extent and strength of SIG influence. This paper establishes that SIGs have strong influence on policy choices in spite of results that highlight small effects of inside lobbying activities.

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Introduction

Copious news articles and polls reflect the popular wisdom that Special Interest Groups (SIGs) exert disproportionate influence on political decisions by reason of their financial resources.[1] Despite concrete examples of SIG influence, such as banks deciding the wording of a legislation softening financial regulation (Lipton and Protess, 2013), the popular wisdom has so far received little empirical validation. Politicians do not seem to accord too much importance to contributions (i.e., monetary transfers) or lobbying expenditures (i.e., transfers of information). Out of the 25 most powerful SIGs in 1999 according to members of Congress and their staff, only 12 belong to the top 25 for contributions and only 4 to the top 25 for lobbying expenditures.[2] Following multiple studies documenting a small effect of SIG's spending on policy-makers' decisions (for a review, see Ansolabehere et al., 2003), the new scholarly consensus is that money does not buy policies (Azari, 2015).

This new consensus, however, relies on analyses focusing exclusively on one class of SIGs' expenditures: inside lobbying expenditures—contributions and lobbying spending—meant generally to shape the content of a bill. While inside lobbying expenditures are undoubtedly important, SIGs have other means to influence policies. One prominent example, generally ignored by empirical studies, is outside lobbying—issue advocacy advertising or grass-roots mobilization—intended to affect the likelihood a bill is enacted into law.

In this paper, I study SIG influence—measured by the difference in policy choices with and without SIG presence—in a game-theoretic framework which incorporates both inside and outside lobbying activities. I show that empirical tests which consider exclusively inside lobbying expenditures underestimate the extent of SIG influence (that is, the circumstances under which SIGs affect policy choices) and the strength of SIG influence (that is, the change in policies induced by SIG presence). The empirically documented small effect of inside lobbying expenditures on political decisions is thus consistent with the popular wisdom that SIGs exerts strong influence. This influence arises through their ability to engage in outside lobbying. As such, this paper suggests a novel unbiased measure of the extent of SIG influence: outside lobbying expenditures.

[1]See, for example, Teachout's op ed in the *New York Times* in January 2015 and the CBS/New York Times' poll on Americans' View on Money in Politics in June 2015.

[2]Source for the power ranking: Birndaum (1999). Source for SIGs' money: Center for Responsive Politics (http://www.opensecrets.org/).

I study a model in which SIGs with opposing interests seek to influence a decision-maker (to whom I reserve the pronoun 'she'). SIGs decide how much inside lobbying expenditures to incur and whether to engage in outside lobbying activities. Lobbying activities are costly and this cost (assumed to be the same for inside and outside lobbying) depends on the SIG's strength, which is its private information. A weak SIG faces a high lobbying cost, whereas this cost is low for a strong SIG.

An SIG can use inside lobbying to signal its strength. Outside lobbying expenditures serve a very different purpose. Outside lobbying by an SIG opposed to the policy change proposed by the decision-maker (henceforth, anti-change SIG) reduces the probability that the decision-maker's bill is enacted into law. This probability is one absent outside lobbying activities and strictly less than one otherwise. The probability that the bill passes declines to zero if the decision-maker fails to defend her proposal against outside pressure. The decision-maker's cost of defending her proposal depends on whether the SIG favorable to policy change (henceforth, pro-change SIG) engages in outside lobbying and is strictly lower when it does. Only a strong pro-change SIG, however, is capable of bearing the cost of outside lobbying activities.

As in any set-up in which inside lobbying expenditures have an informative role, these expenditures capture influence only when SIGs play a separating strategy (i.e., the two types send different signals). Empirical analyses which exclusively focus on inside lobbying expenditures then yield an unbiased estimate of the extent and strength of SIG influence only when three conditions are met. First, a separating equilibrium always exists. Second, absent inside lobbying expenditures, SIGs have no influence (the decision-maker's choice is the same as when she faces no SIG). Third, inside lobbying expenditures reflect SIGs' ability to tilt the decision-maker's choice towards their preferred policy. In the environment considered in this paper, none of these three conditions is met.

Even though a strong SIG's lobbying cost is lower than a weak one (i.e., the single-crossing condition holds), a separating strategy is not incentive compatible for all parameter values. A strong anti-change SIG does not always have sufficient incentive to reveal its type with inside lobbying expenditures as it has the option to engage in outside lobbying and reduce the probability the bill is enacted into law. A strong pro-change SIG sometimes prefers imitating a weak type so as to avoid incurring outside lobbying expenditures to subsidize the defence of the decision-maker's proposal.

Further, SIGs can influence policy choices even absent inside lobbying expenditures. In a separating equilibrium, when its lobbying cost is very low, a strong pro-change SIG is willing to reveal its type with cheap talk messages. The gain from the decision-maker proposing a policy closer to its preferred position more than compensates for the risk that the bill is not passed due to the anti-change SIG's outside lobbying activities and the cost of defending the decision-maker's proposal. An anti-change SIG can also have strong influence without incurring inside lobbying expenditures in a pooling equilibrium. In this case, the decision-maker bases her policy choice on her assessment of the threat of outside lobbying activities. If she believes this threat to be high, the decision-maker compromises with the anti-change SIG by proposing a moderate policy change. As a result, the focus on inside lobbying expenditures underestimates the extent of SIG influence.

The model also highlights that inside lobbying expenditures only imperfectly capture SIG influence. When the pro-change plays a separating strategy, upon learning that it is strong, the decision-maker asks the SIG to subsidize the defence of her proposal. Consequently, a strong pro-change SIG has no incentive to incur inside lobbying expenditures to signal it is willing to engage in costly outside lobbying activity. In a separating equilibrium, only a weak pro-change SIG uses inside lobbying expenditures to credibly "plead poverty." Consequently, empirical analyses using inside lobbying expenditures are likely to yield estimates of pro-change SIG influence of the wrong sign.

In addition, inside lobbying expenditures may occur in pooling equilibria.[3] But these expenditures have no effect on the decision-maker's policy choice, which depends only on her prior about the SIG's strength. Further, for some parameter values, pooling equilibria differing only in the level of inside lobbying expenditures coexist with a separating equilibrium. Empirical researchers cannot use inside lobbying expenditures to distinguish which equilibrium is being played. Due to this equilibrium selection problem, empirical tests of SIG influence considering exclusively inside lobbying expenditures are likely to suffer from attenuation bias: they underestimate the strength of SIG influence.

In contrast, empirical researchers can recover unbiased estimates of SIG influence by considering outside lobbying expenditures (e.g., issue advocacy advertising). A pro-change SIG engages in

[3]This result is driven by the decision-maker's out-of-equilibrium and survives equilibrium refinement such as the Intuitive Criterion.

outside lobbying only if the decision-maker chooses a bill close to its preferred policy. Further, as the pro-change SIG incurs outside lobbying expenditures only when strong and it plays a separating equilibrium, this class of spending constitutes an unbiased estimate of the extent and strength of pro-change SIG influence. An anti-change SIG engages in outside lobbying only when the decision-maker proposes a bill far from the status quo. Consequently, outside lobbying reveals failure to influence policy choice. It does not inform researchers about the means (inside lobbying expenditures or threat) and strength of SIG influence. This paper thus suggests that there exist limits to our ability to understand anti-change SIG influence due to their strategic use of both inside and outside lobbying.

I conclude this introduction by connecting this paper to the most closely related theoretical literature. Copious studies investigate SIG influence under the assumption that contributions buy political favors (e.g., Denzau and Munger, 1986; Grossman and Helpman, 1996 and 2001; Besley and Coate, 2001). Other papers suppose that SIG money buys access in order to transmit information about the impact of a proposed policy change on constituents' welfare (e.g., Potters and Van Widen, 1992; Austen-Smith, 1995; Ball, 1995; Lohmann, 1995; Cotton, 2011).[4] The present paper differs from this literature by assuming that inside lobbying expenditures serve as a signal of an SIG's willingness to engage in costly activities that influence the fate of a bill.[5]

Some papers assume that SIGs use some sort of outside lobbying activities to influence political decisions. Yu (2005) studies a model where SIGs can raise the salience of an issue before engaging in quid pro quo contributions. Kollman (1998) supposes that outside lobbying activities can change a policy-maker's legislative agenda. Bombadini and Trebbi (2011) assume that firms can use money or promise votes by mobilizing their employees in exchange for public subsidies. They find that only intermediary-sized firms use money as small firms find it too costly to enter politics and large firms can promise votes to get their preferred policy. In contrast, I assume that outside lobbying activities occur after the decision-maker's policy choice and affect the likelihood her bill is enacted into law. Outside lobbying (by the anti-change SIG) can thus be interpreted as a threat.

Several works analyze how threats affect the political process. Ellman and Wantchekon (2000)

[4] See also Groll and Prummer (2015) who study informative lobbying in a political network.

[5] Gordon and Hafer (2005, 2007) also consider a set-up in which inside lobbying expenditures signal firms' willingness to contest an agency's regulatory decision. The SIG's choice to effectively contest regulation, however, is left unmodeled.

study how the threat of civil war biases electoral platforms in favor of the party backed by potential rebels. Scartascini and Tommasi (2012) adapt this set-up to legislative bargaining. Wolton (2015) investigates how threats by the rich induce a governing party to compromise on taxation and shows that the presence of an opposition party can be Pareto improving for political actors and interest groups alike. Dal Bó and Di Tella (2003) and Dal Bó et al. (2006) analyze how threats increase the effectiveness of bribes. Dahm and Porteiro (2008) suppose that an SIG can perform a test to reveal information about an ex-ante unknown state of the world prior to engaging in political pressure which affects the probability a bill is enacted into law. They show that the SIG generally prefers a public test (i.e., results are publicly observable) to a private test (i.e., results are the SIG's private information), but do not consider how political pressure affects estimates of SIG influence. None of these papers assumes that SIGs can use money to transmit private information and so have little to say about how outside lobbying activities affect the informativeness of inside lobbying expenditures.

Evidence on inside and outside lobbying

Most empirical studies of SIG influence consider how campaign contributions affect politicians' decisions, especially a legislator's vote. A common finding in this literature is that the effect of contributions is statistically insignificant when legislators' ideology is controlled for (see Ansolabehere et al., 2003 Table 1). Three major issues, however, plague this approach.

First, bills scheduled for a vote are not exogenous. They are the result of equilibrium plays anticipating legislators' and SIGs' actions at the voting stage. It is thus somewhat unsurprising that contributions have little effect so late in the political game. Empirical papers considering policies rather than votes have generally found a small, but positive impact of contributions. Examples include Goldberg and Maggi (1999) who find some support for Grossman and Helpman's (1994) theory of trade protection and Bombardini and Trebbi (2011) for firm subsidies.[6]

Second, contributions represent only a small portion of SIGs' inside lobbying expenditures (20% during the 2011-12 electoral cycle as documented by the Center for Responsive Politics). SIGs

[6] In a related fashion, Hall and Wayman (1990) find that contributions increase committee members' efforts. Firms' behavior is consistent with the idea that inside lobbying expenditures are meant to influence policy choices. Fouirnaies and Hall (2016a) show that SIGs seek access to legislators in charge of committee assignments. In addition, Fouirnaies and Hall (2016b) establish that firms' contributions vary with their degree of exposure to regulation.

invest substantially more in the transmission of information (i.e., hiring lobbyists). Empirically, informative lobbying seems to have a positive effect on the content of bills when it comes to academic earmarks (de Figueiredo and Silverman, 2006) and corporate taxes (Richter et al., 2009), or on the success of energy policy proposals (Kang, 2015).[7]

Third, most empirical studies consider exclusively inside lobbying expenditures, assuming implicitly that other SIG activities are of little consequences for their results. A long scholarly tradition, however, stresses the importance of other SIG activities, especially outside lobbying (e.g., Blaisdell, 1957; Wright, 1996, page 90; Kollman, 1998, page 103; Hojnacki and Kimball, 1999, page 1005-6; Baumgartner et al., 2009, page 150-7).[8] Due to the absence of empirical evaluation of outside lobbying, there is, however, no consensus on the purpose(s) of these activities.[9] Nonetheless, it is well documented that SIGs have used issue advocacy advertising to affect the fate of prominent legislative reforms—for example, Clinton's 1993 health care reform (West et al., 1996; Goldstein, 1999), the 1998 Senate tobacco bill (Jamieson, 2000; Derthick, 2012), Obama's 2010 Affordable Care Act (Hall and Anderson, 2012; LaPira, 2012). Further, as documented by Lord (2000), lobbyists and members of Congress report that inside lobbying expenditures are more effective to shape the content of a bill, whereas outside lobbying (constituency building) has greater impact on the legislative success of a policy proposal.[10] Using this (partial) evidence, this paper provides a theoretical framework to understand SIG influence on political decisions when outside lobbying activities (understood in what follows as issue advocacy advertising) are meant to affect the likelihood a bill is enacted into law.[11]

[7]See de Figueiredo and Richter (2014) for a review of the empirical literature on informative lobbying.

[8]Falk et al. (2006) estimate that issue advocacy advertising amounted to more than $400 million in the Washington DC media market alone during the 108th Congress. In comparison, SIGs contributed approximately $570m to Members of Congress (excluding presidential candidate John Kerry), and spent $4bn on lobbying during the 2003-2004 electoral cycle (source: Center for Responsive Politics).

[9]An exception is Hall and Reynold (2012) who study the targets of issue advocacy advertising, but do not analyze the impact of this type of advertising on voting decisions.

[10]This is in line with the findings of Mian et al. (2010; 2013) that constituency interests play a strong role (sometimes stronger than contributions) in a legislator's voting choice on bills affecting the mortgage industry.

[11]Notice that this paper does not consider important outside lobbying activities such as grass-roots mobilization, which, for example, played a role in President Obama's decision to reject the Keystone XL oil pipeline (Davenport, 2015).

The model

I study a one-period three-player game with a decision-maker (superscript D), a pro-change SIG (P), and an anti-change SIG (A). The decision-maker and the pro-change (anti-change) SIG have similar (opposite) policy preferences. The game has three parts. In the first stage, SIGs observe their strength and decide whether to reveal it to the decision-maker via their inside lobbying strategy. In the second stage, the decision-maker decides the content of a bill $b \in [0, 1]$, where 0 represents the status quo and 1 the preferred policy of the decision-making and pro-change SIG. In the third part ('outside lobbying' subgame), SIGs decide whether to engage in outside lobbying ($l_o^J \in \{0, 1\}$ for $J \in \{A, P\}$).

Outside lobbying activities have an impact on the outcome of the game denoted $y \in \{0, b\}$. When the anti-change SIG does not engage in outside lobbying ($l_o^A = 0$), the decision-maker's bill is always enacted into law: $y = b$. Otherwise ($l_o^A = 1$), the outcome depends on the decision-maker and the pro-change SIG's choices. The decision-maker chooses the intensity of her response to the anti-change SIG's outside lobbying activities, $d \in \{0, 1, 2\}$. When the decision-maker chooses $d = 0$, she backs down and the status quo prevails: $y = 0$. When the decision-maker chooses $d = 2$, she defends the bill on her own and the bill passes ($y = b$) with probability p; with probability $1 - p$, the status quo holds ($y = 0$). Finally, when the decision-maker chooses $d = 1$, she asks for the pro-change SIG's support and the outcome then depends on the pro-change SIG's decision whether to engage in outside lobbying. If the pro-change SIG engages in outside lobbying ($l_o^P = 1$), the bill passes with probability p; if not ($l_o^P = 0$), the bill fails and the status quo prevails ($y = 0$) with probability 1.

Outside lobbying is costly for all players. For the decision-maker, the cost of a response with intensity d is: $\frac{k}{2} \times d$. Importantly, the decision-maker faces a lower cost when she asks for support ($d = 1$) than when she defends the bill on her own ($d = 2$). This cost is common knowledge. On the other hand, an SIG's cost of outside lobbying is its private information and depends on its strength (type) $\tau \in \{s, w\}$, where s denotes strong and w denotes weak. A strong SIG $J \in \{A, P\}$ faces a lower cost of outside lobbying activities: $c_w^J > c_s^J$. It is common knowledge that the SIG's types are uncorrelated and the proportion of strong SIG $J \in \{A, P\}$ is $Pr(\tau^J = s) = q^J \in [0, 1]$.

In the first stage, SIGs can reveal their type by engaging in inside lobbying. An SIG's inside lobbying activities take the form of a costless message $m \in \{s, w\}$ and costly expenditures $l_i^J \geq 0$, $J \in \{A, P\}$. Denote $\zeta^J := (m, l_i^J)$ the signal of SIG J. The cost of inside lobbying expenditures depends on the SIG's strength and, for ease of exposition, is equal to the cost of outside lobbying activities: $c_\tau^J,\ J \in \{A, P\},\ \tau \in \{s, w\}$. Henceforth, I refer to c_τ^J as 'the lobbying cost.' Observe that since types are drawn independently, the anti-change SIG's signal reveals no information about the pro-change SIG's strength and vice versa.

As noted above, the decision-maker's preferred outcome (y) is 1. Incorporating the cost of responding to the anti-change SIG's outside lobbying activities, her utility function can be expressed as:

$$u^D(y, d) = y - \frac{kd}{2} \tag{1}$$

The pro-change SIG's preferred outcome is the same as the decision-maker's. Its utility function also includes the cost of both inside lobbying expenditures ($l_i^P \geq 0$) and outside lobbying activities ($l_o^P \in \{0, 1\}$) and thus assumes the following form:

$$u^P(y, l_i^P, l_o^P; \tau) = y - c_\tau^P(l_o^P + l_i^P),\ \tau \in \{s, w\} \tag{2}$$

The anti-change SIG prefers the status quo ($y = 0$), any change imposes a payoff loss. Adding the cost of inside ($l_i^A \geq 0$) and outside lobbying ($l_o^A \in \{0, 1\}$), its utility function is:

$$u^A(y, l_i^A, l_o^A; \tau) = -y - c_\tau^A(l_o^A + l_i^A),\ \tau \in \{s, w\} \tag{3}$$

To summarize the timing of the game is:

1. Nature draws SIGs' types independently: $\tau^J \in \{s, w\},\ J \in \{A, P\}$;

2. Both SIGs privately observe their type and send simultaneously a signal: $\zeta^J = (m, l_i^J) \in \{s, w\} \times \mathbb{R}_+$;

3. The decision-maker chooses the content of the bill: $b \in [0, 1]$;

4. The anti-change SIG decides whether to engage in outside lobbying: $l_o^A \in \{0, 1\}$;

5. The decision-maker then decides whether to back down, ask for support, or defend her pro-

posal: $d \in \{0, 1, 2\}$;

6. The pro-change SIG decides whether to engage in outside lobbying: $l_o^P \in \{0, 1\}$.

Outcome:

1. If the anti-change SIG does not engage in outside lobbying ($l_o^A = 0$): $y = b$

2. If the anti-change SIG engages in outside lobbying ($l_o^A = 1$):

$$\begin{cases} y = b \text{ with probability } \mathbb{I}_{\{d+l_o^P \geq 2\}}(1-p) \\ y = 0 \text{ with probability } 1 - \mathbb{I}_{\{d+l_o^P \geq 2\}}(1-p) \end{cases},$$

where $\mathbb{I}_{\{d+l_o^P \geq 2\}}$ is the indicator function equals to 1 if $d + l_o^P \geq 2$, and 0, otherwise.

The equilibrium concept is Perfect Bayesian Equilibrium (PBE) in pure strategies (see Appendix A for a formal definition).[12] A PBE requires that a) each player's choices be sequentially rational given her belief at the time of choice and other players' strategies; b) beliefs satisfy Bayes' rule on the equilibrium path (are consistent with priors and equilibrium strategies). As it is common in signaling games, multiple PBE can emerge absent additional restrictions. I therefore impose the Intuitive Criterion (Cho and Kreps, 1987), which states that there should not exist a deviating signaling strategy satisfying (i) the voter anticipates only one type of SIG deviates and, given the voter's belief, (ii) only this specific type finds it profitable to deviate. To restrict further the number of outcome-equivalent equilibria and facilitate the exposition, I also impose that a pro-change SIG's signal as a function of its strength—denoted $\zeta^P(\tau)$, $\tau \in \{s, w\}$—satisfies $\zeta^P(s) \neq \zeta^P(w)$ only if its inside lobbying activity influences the decision-maker and anti-change SIG's strategies on the equilibrium path.[13] In what follows, the term 'equilibrium' refers to this class of equilibria.

[12]In Appendix B, I show that the main results hold when I allow for mixed strategies.

[13]Define the decision-maker's policy choice b as a function of the pro-change SIG's signal ζ^P and the anti-change SIG's signal ζ^A; and the intensity of her response d as a function of ζ^P, ζ^A, the content of the bill b, and the anti-change SIG's outside lobbying activities l_o^A. This restriction imposes that if $b(\zeta^P, \zeta^A) = b(\zeta^{P'}, \zeta^A)$ and $d(\zeta^P, \zeta^A, b(\zeta^P, \zeta^A), l_o^A) = d(\zeta^{P'}, \zeta^A, b(\zeta^{P'}, \zeta^A), l_o^A)$ for all $\zeta^P \neq \zeta^{P'}$ and for all $\zeta^A \in \{s, w\} \times \mathbb{R}_+$, $b(\cdot) \in [0, 1]$, and $l_o^A \in \{0, 1\}$ on the equilibrium path (i..e, the pro-change SIG's strategy has no impact on equilibrium outcome), then the pro-change SIG plays a pooling strategy ($\zeta^P(s) = \zeta^P(w)$). Footnote 19 details the role played by this restriction in the analysis.

Assumptions

To limit the number of cases to be considered and ensure both SIGs' signals play a role in the analysis, I impose some conditions on the model parameters. For the anti-change SIG, I assume that the following inequalities hold:

Assumption 1. *A weak anti-change SIG's lobbying cost satisfies:* $\max\left\{1-p-k, \frac{k}{1-p}\right\} < \frac{c_w^A}{p} < \min\left\{\frac{1-p-k}{p}, 1\right\}$

The lower bound guarantees that the decision-maker prefers to avoid outside lobbying activities by a weak anti-change SIG. The upper bound guarantees that the weak type does not have a strictly dominated strategy and that the decision-maker is not willing to compromise if she cannot defend her proposal.[14] The upper bound is for exposition purposes only.

Regarding the pro-change SIG, I impose the following conditions.

Assumption 2. *The following inequalities hold: i.* $(1-p)\frac{c_s^A}{p} < c_s^P < (1-p) < c_w^P$ *and ii.* $q^P \leq 1/2$

Point i. implies that engaging in outside lobbying is a strictly dominated strategy for a weak pro-change SIG (the inequality $(1-p)\frac{c_w^A}{p}$ is meant to simplify the analysis, but does not affect the main results). Point ii. is a sufficient condition such that when the pro-change SIG's inside lobbying activities reveal no information about its strength, the decision-maker does not ask for support ($d = 1$).

Discussion

This game shares similarities with traditional signaling games, with one important twist. In traditional signaling games, the sender sends a signal, the receiver decides what action to take after observing the signal, and the game ends. In the present set-up, the game does not end after the receiver's (decision-maker's) policy choice. One of the senders (the anti-change SIG) has the opportunity to act again (engage in outside lobbying) to affect the final outcome of the game.

[14] When the decision-maker cannot defend her proposal b, a type $\tau \in \{s, w\}$ anti-change SIG gets $-b$ if it does not engage in outside lobbying and $-c_\tau^A$ if $l_o^A(\tau) = 1$. To make the SIG indifferent, the decision-maker must propose $b = c_\tau^A$. The upper bound in Assumption 1 guarantees she then prefers proposing $b = 1$ and obtaining in expectation $1 - p - k$.

This assumption, which corresponds to the idea that outside lobbying is intended to influence the likelihood that a bill is enacted into law, is the key force behind the results below.[15]

The anti-change SIG's outside lobbying activities in this set-up can be thought as airing ads attacking the decision-maker in a future (unmodeled) election. It can also be understood as issue advocacy advertising to inform the public of the consequences of the decision-maker's proposal.[16] In Appendix D, I show how outside lobbying activities can be understood as a war of information, in the sense of Gül and Pesendorfer (2012). The pro-change SIG's outside lobbying activities can be understood as ads defending the decision-maker's proposal and thus as a form of subsidy. As a minimum number of ads is required to inform and persuade the public, outside lobbying activities are modeled as a binary variable rather than a continuous variable.[17]

Another important assumption is that the cost of inside lobbying expenditures depends on an SIG's strength: the single-crossing condition holds. Both inside and outside lobbying activities require money and an SIG's strength captures its capacity to collect funds from its members or usual donors. As documented by Kingdon (1989, page 152) in his study of Members of Congress, legislators are particularly concerned about interest groups mobilizing constituencies against them. But policy-makers have little information about SIGs' ability to mobilize their member on particular issues (e.g., Ainsworth, 2000 page 122). The SIG's private information about his lobbying cost captures this uncertainty. Committee hearings then serve to verify and publicize SIGs' preferences (Wright, 1996 pages 41-43).[18]

Other technical assumptions are simply for ease of exposition. The assumption that the decision-maker's bill is always enacted into law absent outside lobbying activities can be relaxed as long as it is strictly higher than the probability of enactment when the anti-change SIG engages in outside

[15]Models of signaling in the shadow of war exhibit a similar feature (Fearon, 1997; Arena, 2013). However, signaling in these models is a binary choice variable, the single-crossing condition does not hold, and there is no equivalent to a pro-change SIG. As such, the present paper is substantively and technically different.

[16]Commenting on the success of the Harry and Louise campaign in the debate on Clinton's Health Care Reform in 1993, Bill McInturff, who helped in the campaign, explains, "In terms of the questions raised about the "public policy process," if the White House cannot build majority support faced with "soft" advertising that raised simple and fundamental questions, it suggests to our firm that we have materially made a contribution to the process by not allowing such a substantial piece of legislation to pass without a full airing of its consequences." (cited in Brodie, 2001).

[17]All the results hold with a continuous level of outside lobbying effort (at the cost of complicating the analysis) as long as there is a fixed (entry) cost of engaging in outside lobbying.

[18]This can be related to the idea that who you know is as important as what you know when it comes to informative lobbying (Bertrand et al., 2014).

lobbying. The same holds true for the assumption that the bill is never enacted into law when the decision-maker does not respond to the anti-change SIG's attacks. The key insights are unchanged when players are risk-adverse or when an SIG's strength affects differently the cost of inside and outside lobbying expenditures. Inside lobbying expenditures correspond to burning money in the present set-up, but part of it could be a transfer to the decision-maker (to incorporate contributions) without affecting the main results. Imposing that SIGs can only engage in outside lobbying after the decision-maker has made her proposal is without loss of generality. Since SIGs correctly anticipate future actions, their equilibrium behavior would remain the same if they can start an outside lobbying campaign before the decision-maker decides on the content of the bill. Finally, the main conclusions hold for any finite type-space (but the analysis becomes significantly more complicated).

Pro-change SIG influence

I first analyze how the pro-change SIG influences policy choices. To this end, I assume that it is common knowledge that the anti-change SIG is strong: $q^A = 1$. Consequently, an anti-change SIG's signal ζ^A reveals no information and an anti-change SIG's inside lobbying strategy can be ignored without loss of generality.

At the policy-making stage, the decision-maker faces a choice between proposing her preferred policy $b = 1$ or finding a compromise with the anti-change SIG. A compromise takes the form of a bill which leaves the anti-change SIG indifferent between $l_o^A = 1$ (engaging in outside lobbying) and $l_o^A = 0$. Denote b_s this 'compromise bill.' Simple algebra yields $b_s := \frac{c_s^A}{p} < 1$.

To measure pro-change SIG influence, Lemma 1 establishes the decision-maker's equilibrium policy choice (denoted $b^*(\emptyset, \zeta^A)$) absent a pro-change SIG.

Lemma 1. *Absent a pro-change SIG, the decision-maker's equilibrium policy choice is* $b^*(\emptyset, \zeta^A) = b_s$ *if and only if* $b_s \geq \max\{1 - p - k, \frac{k}{1-p}\}$ *and* $b^*(\emptyset, \zeta^A) = 1$*, otherwise.*

Compromising with the anti-change SIG is the decision-maker's equilibrium strategy only if the compromise bill is not too moderate. When $b_s < 1 - p - k$, the decision-maker prefers to propose $b = 1$ and face the costly lottery induced by the anti-change SIG's outside lobbying activities than to compromise. When $b_s < \frac{k}{1-p} \Leftrightarrow (1 - p)b_s - k < 0$, the decision-maker is not credible when she proposes $b = b_s$ since she is not willing to defend her proposal if the SIG engages in outside lobbying. By choosing $l_o^A = 1$, the SIG then gets a payoff of $-c_s^A$ as the bill fails with certainty ($y = 0$). By choosing $l_o^A = 0$, it gets $-b_s = -c_s^A/p$ since the compromise bill passes with probability 1. Obviously, a strong anti-change SIG strictly prefers to engage in outside lobbying and, anticipating this, the decision-maker proposes her preferred bill ($b = 1$) by Assumption 1.

Lemma 1 indicates that when $b_s < \max\{1 - p - k, \frac{k}{1-p}\}$, the decision-maker chooses the pro-change SIG's preferred policy even when it is inactive. A pro-change SIG has no influence on policy choice then (by Assumption 2, it can be verified that the decision-maker never chooses $b = b_s$ when a pro-change SIG is present). To make the problem interesting, I thus assume in the remainder of this section that $b_s \geq \max\{1-p-k, \frac{k}{1-p}\}$, and the pro-change SIG influences policy choice whenever the decision-maker chooses $b = 1$. By Assumption 2, this can occur only when the pro-change SIG plays a separating strategy (i.e., $\zeta^P(s) \neq \zeta^P(w)$). But a strong pro-change SIG also has some incentive to hide its type so as not to pay the cost of defending the decision-maker's proposal. The next lemma establishes conditions under which a separating equilibrium exists.

Lemma 2. *There exists a unique* $\overline{c^P} : [0,1]^2 \to (0, (1-p))$ *such that a separating equilibrium exists if and only if i. the compromise bill* b_s *satisfies* $k/2 \leq 1 - p - b_s$*; and ii. a strong pro-change SIG's lobbying cost satisfies:* $c_s^P \leq \overline{c^P}(b_s, c_w^P)$.

A separating equilibrium exists when a strong pro-change SIG's benefit from differentiation is greater than the associated cost. The benefit from differentiation is positive only if the decision-maker chooses her preferred bill ($b = 1$) after learning the pro-change SIG is strong. The decision-maker must thus prefer the lottery induced by choosing $b = 1$, anticipating the anti-change SIG's outside lobbying activities and the strong pro-change SIG's help, to the certain payoff from proposing the compromise bill $b = b_s$. That is, a first necessary condition is: $b_s \leq 1 - p - k/2$ (Condition i.).[19] The cost from differentiation corresponds to the cost of outside lobbying activities since the decision-maker asks for support ($d = 1$) after choosing $b = 1$. A second necessary condition is

[19]When this condition is not satisfied, the decision-maker chooses $b = b_s$, the anti-change SIG chooses $l_o^A = 0$ on the equilibrium path independently of the pro-change SIG's signal. The restriction on the pro-change SIG's equilibrium behavior then implies that the pro-change SIG plays a pooling strategy (i.e., $\zeta^P(s) = \zeta^P(w)$). Absent this restriction, the pro-change SIG might be willing to truthfully reveal its type, but this would not affect other player's equilibrium strategies or the equilibrium outcome. As such, the equilibrium restriction simply guarantees that the pro-change SIG has influence on policy choice if and only if it plays a separating strategy.

then that the cost of outside lobbying activities (measured by the lobbying cost) is not too large: $c_s^P \leq \overline{c^P}(b_s, c_w^P)$ (Condition ii.).

Denote $b^*(\zeta^P, \zeta^A)$ the equilibrium bill as a function of the pro-change SIG's signal. The next remark establishes that in a separating equilibrium, a strong pro-change SIG obtains a more favorable bill.[20]

Remark 1. *In a separating equilibrium,* $b^*(\zeta^{P*}(s), \zeta^A) = 1 > b^*(\zeta^{P*}(w), \zeta^A) = b_w$.

The next proposition characterizes the pro-change SIG's strategy in a separating equilibrium—$\zeta^{P*}(\tau)$, $l_o^{P*}(\tau)$, $\tau \in \{s, w\}$—assuming without loss of generality that the SIG announces its type ($m^*(\tau) = \tau$). To this end, it is useful to define the following quantity: $\overline{l_i^P}(c_s^P, b_s) := \frac{c_s^P - (1-p-b_s)}{c_s^P}$.

Proposition 1. *In a separating equilibrium, the pro-change SIG's equilibrium strategy satisfies:*
1. $\zeta^{P*}(s) = (s, 0)$ *and* $\zeta^{P*}(w) = (w, l_i^{P*}(w))$*, with* $l_i^{P*}(w) = \max\left\{0, \overline{l_i^P}(c_s^P, b_w)\right\}$*;*
2. $l_o^{P*}(s) = 1$ *and* $l_o^{P*}(w) = 0$.

In a separating equilibrium, a strong pro-change SIG incurs no inside lobbying expenditures: $l_i^{P*}(s) = 0$. To understand this result, notice that the decision-maker wants the SIG to subsidize the cost of defending her proposal, whereas the pro-change SIG wants the decision-maker to pay this cost in full. A strong pro-change SIG has no incentive to pay a cost at the inside lobbying stage to reveal it is willing to engage in costly outside lobbying. Consequently, only a weak pro-change SIG sometimes incurs inside lobbying expenditures to credibly signal it is *not* able to bear the cost of outside lobbying activities. Inside lobbying expenditures serve to credibly "plead poverty." Costly signaling, however, is not always necessary. When a strong pro-change SIG's lobbying cost is low ($c_s^P \leq 1 - p - b_s$), it strictly prefers the risky bill $b = 1$ to the compromise bill $b = b_s$. Hence, a cheap talk message by the strong pro-change SIG is credible and a separating equilibrium exists absent any inside lobbying expenditure.[21]

Proposition 1 considers separately inside and outside lobbying expenditures and thus cannot characterize the relationship between money and influence. The next remark shows that a strong pro-change SIG's total observed expenditures $(l_i^{P*}(s) + l_o^{P*}(s))$ are always higher than a weak one's.

[20]The pro-change SIG also obtains a more favorable outcome in expectation (despite the lottery induced by the anti-change SIG's outside lobbying activities).

[21]Observe that imposing the Intuitive Criterion guarantees that the separating equilibrium with the minimum level of expenditures is selected.

Remark 2. *In a separating equilibrium, the following inequality always holds:*

$$l_i^{P*}(s) + l_o^{P*}(s) > l_i^{P*}(w) + l_o^{P*}(w)$$

As the separating equilibrium exists only under specific conditions, it is essential to characterize conditions for existence of a pooling equilibrium. As the decision-maker always chooses $b = b_s$ in a pooling equilibrium, this equilibrium exists whenever a strong pro-change SIG has no profitable signaling deviation satisfying the Intuitive Criterion. As only a a weak pro-change SIG incurs inside lobbying expenditures in a separating equilibrium, a profitable signaling deviation can only occur when cheap talk messages credibly reveal an SIG's strength. By Lemma 2 and Proposition 1, a pooling equilibrium therefore exists whenever $\min\{c_s^P, k/2\} \geq 1 - p - b_s$.

When this condition is satisfied, there might exist (infinitely) many pooling equilibria for given parameter values. These equilibria differ only in the level of inside lobbying expenditures determined by the decision-maker's out-of-equilibrium belief. To see this, suppose that the decision-maker believes that the pro-change SIG is strong when she observes no inside lobbying expenditures (an out-of-equilibrium event). Whenever $k/2 \leq 1 - p - b_s$, she would choose $b = 1$ and asks for help following $l_i^P = 0$. As this would impose a high cost on both strong and weak types, the pro-change SIG is willing to incur inside lobbying expenditures so that the decision-maker compromises with the anti-change SIG.[22] In contrast, a pro-change SIG never engages in outside lobbying in a pooling equilibrium since the decision-maker always compromises.

Proposition 2. *A pooling equilibrium exists if and only if* $\max\{c_s^P, k/2\} \geq 1-p-b_s$. *The decision-maker always chooses* $b^*(\zeta^{P*}(\tau), \zeta^A) = b_s,\ \tau \in \{s, w\}$. *The pro-change SIG's equilibrium strategy satisfies for* $\tau \in \{s, w\}$*:*

1. $\zeta^{P*}(\tau) = (m(\tau), l_i^{P*}(\tau))$*, with* $m(\tau) \in \{s, w\}$ *and* $l_i^{P*}(\tau) = 0$ *if* $k/2 > 1 - p - b_s$ *and* $l_i^{P*}(\tau) \in \left[0, \min\left\{\overline{l_i^P}(c_s^P, b_w), \frac{b_s}{c_w^P}\right\}\right]$ *otherwise;*
2. $l_o^{P*}(\tau) = 0$.

Table 1 summarizes this section's main theoretical findings by distinguishing between four different cases (omitting boundary cases and ignoring arguments for ease of exposition). The first

[22]Importantly, while the Intuitive Criterion restricts signaling (inside lobbying) expenditures in a separating equilibrium, it imposes no restriction on signaling expenditures in a pooling equilibrium.

case corresponds to $k/2 > 1 - p - b_s$ when only a pooling equilibrium with no lobbying expenditure exists. The second case corresponds to $k/2 < 1 - p - b_s$ and $c_s^P > \overline{c^P}$ when only a pooling equilibrium with (possibly) positive inside lobbying expenditures exist. The third case corresponds to $k/2 < 1 - p - b_s < c_s^P \leq \overline{c^P}$ when the separating equilibrium and pooling equilibria can arise. The separating equilibrium exhibits a positive level of inside lobbying expenditures as do some pooling equilibria. The last case corresponds to $\max\{c_s^P, k/2\} < 1 - p - b_s$ when only a separating equilibrium with no inside lobbying expenditure exists.

Case	Type	Sep.	Pool.
1	s		0
	w		0
2	s		$[0, \frac{b_s}{c_w^P}]$
	w		$[0, \frac{b_s}{c_w^P}]$
3	s	0	$[0, \overline{l_i^P}]$
	w	$\overline{l_i^P}$	$[0, \overline{l_i^P}]$
4	s	0	
	w	0	

(a) Inside lobbying

Case	Type	Sep.	Pool.
1	s		0
	w		0
2	s		0
	w		0
3	s	1	0
	w	0	0
4	s	1	
	w	0	

(b) Outside lobbying

Case	Type	Sep.	Pool.
1	s		b_s
	w		b_s
2	s		b_s
	w		b_s
3	s	1	b_s
	w	b_s	b_s
4	s	1	
	w	b_s	

(c) Policy choice

Table 1: Equilibrium strategies

'Sep.' stands for separating equilibrium; 'Pool.' for pooling equilibria. The different cases are described in the text. In Tables 1a and 1b, for each case, the first line s (second line w) describes a strong (weak) pro-change SIG's strategy. In Table 1c, for each case, the first (second) line corresponds to the decision-maker's bill choice after observing a strong (weak) pro-change SIG's signal: $\zeta^{P*}(s)$ ($\zeta^{P*}(w)$).

To understand the empirical implications of these results, consider an empirical researcher interested in assessing a pro-change SIG's ability to tilt policy choice b in its favor. For simplicity, assume that the researcher's data set includes the full population of pro-change SIGs.[23] A researcher using exclusively inside lobbying expenditures as a proxy for influence would then run the following regression (with b the content of a bill, X^P a set of controls, and ϵ^P the residual):

$$b = \beta_0^P + \beta_1^P l_i^P + \beta_2^{P'} X^P + \epsilon^P \tag{4}$$

[23] In particular, assume that all the cases described in Table 1 are included in the data set and for each c_s^P, c_w^P, b_w, k, p, there are $N > 1$ SIGs, q^P of them being strong and $N' < N$ playing a separating strategy profile when separating and pooling equilibria can arise.

This section's results suggest that the resulting $\widehat{\beta_1^P}$ is a biased estimate of SIG influence even under the (arguably unrealistic) assumption that an empirical researcher can distinguish between the four cases detailed in Table 1.

The researcher would under-estimate the extent of the pro-change SIG influence as regression 4) yields a non-zero coefficient only in case 3. The researcher would correctly identify the absence of pro-change SIG influence in cases 1 and 2: the decision-maker ($b = b_s$) does not depend on the presence of an active pro-change SIG. The researcher would, however, fail to identify the pro-change SIG influence in case 4 when cheap talk messages are enough to bias the decision-maker's policy choice.

Further, the researcher would under-estimate the strength of the pro-change SIG influence in case 3. Regression (5) would yield a negative point estimate for $\widehat{\beta_1^P}$ and the researcher risks mistakenly concluding that a pro-change SIG is better off when it does not engage in inside lobbying. Importantly, the empirical researcher cannot just correct for the sign. Fixing parameter values, multiple pooling equilibria exist differing only in the amount of inside lobbying expenditures. Since these expenditures have no effect on policy choice, the estimate $\widehat{\beta_1^P}$ suffers from attenuation bias. The risk of attenuation bias does not require that inside lobbying expenditures be positive in (some) pooling equilbria. It is also present when the researcher can control for the pro-change SIG's type. The attenuation bias is eliminated only if the researcher can control for equilibrium selection.[24]

To recover an unbiased estimate of the extent and strength of the pro-change SIG influence, the researcher can leverage the pro-change SIG's outside lobbying activities. The present theoretical framework suggests that the correct specification to measure pro-change SIG influence is:

$$b = \alpha_0^P + \alpha_1^P l_o^P + \alpha_2^{P'} X + \nu^P \tag{5}$$

As Table 1 describes, only a strong pro-change SIG incurs outside lobbying expenditures. Hence, regression (5) would yield $\widehat{\alpha_1^P} > 0$. Further, a strong SIG engages in outside lobbying only in a separating equilibrium, thus controlling for equilibrium selection and guaranteeing that $\widehat{\alpha_1^P}$ is unbi-

[24]To see that observing types is not sufficient, consider the following regression: $b = \gamma_0^P + \gamma_1^P \mathbb{I}_{\{\tau=s\}} + \gamma_2^{P'} X^P + \eta^P$ with $\mathbb{I}_{\{\tau=s\}}$ an indicator function equals to 1 when $\tau = s$. The researcher would correctly obtain $\gamma_1^P = 0$ in cases 1 and 2 and $\gamma_1^P > 0$ in case 4. However, in case 3, the estimate would suffer from attenuation bias since an SIG's type does not provide information about the equilibrium being played.

ased. Empirical researchers would, therefore, do well to consider SIGs' outside lobbying activities rather than (as it is common) their inside lobbying expenditures.

Anti-change SIG influence

In this section, I focus on the anti-change SIG. To this end, I assume that the pro-change SIG is known to be weak ($q^P = 0$). Consequently, the pro-change SIG's signal (ζ^P) has no impact on the decision-maker's policy choice and the decision-maker never asks for help under Assumption 2. Further, I now assume $q^A \in (0, 1)$ so the decision-maker is uncertain ex-ante about the anti-change SIG's strength.

The decision-maker now chooses between a radical change $b = 1$, a compromise bill b_w which makes a weak type indifferent between engaging in outside lobbying or not, and the compromise bill b_s, defined above.[25]

To measure anti-change SIG influence, Lemma 3 establishes the decision-maker's equilibrium policy choice absent an anti-change SIG (denoted $b^*(\zeta^P, \emptyset)$). Since she does not fear outside lobbying activities, unsurprisingly, the decision-maker always proposes its preferred policy ($b = 1$) which passes with probability 1.

Lemma 3. *Absent an anti-change SIG, the decision-maker's equilibrium policy choice is* $b^*(\zeta^P, \emptyset) = 1$.

The anti-change SIG thus influences policy choice whenever $b \neq 1$. I say that influence is limited when $b = b_w$ (as only a weak anti-change SIG is made indifferent) and complete when $b = b_s$ (as both types obtain a favorable policy). Influence can take two distinct forms. First, the threat of outside lobbying may induce the decision-maker to compromise. Second, a strong anti-change SIG credibly reveals its strength at the inside lobbying stage to induce the decision-maker to propose $b = b_s$; a weak type, in turn, obtaining $b = b_w$. Importantly, inside lobbying expenditures are correlated with influence only when a separating equilibrium exists ($\zeta^A(s) \neq \zeta^A(w)$). Lemma 4 characterizes necessary and sufficient conditions for a separating equilibrium to exist.

[25] It can be checked that any other proposal can only reduce the decision-maker's policy payoff if the bill is enacted without diminishing the likelihood of outside lobbying activities.

Lemma 4. *A separating equilibrium exists if and only if:*

$$\max\left\{1-p-k, \frac{k}{1-p}\right\} \leq b_s \leq (1-p)b_w$$

A separating equilibrium exists only if a strong anti-change SIG's benefit from differentiation is greater than the associated cost. The benefit from differentiation is positive only if the decision-maker chooses b_s after learning the anti-change SIG is strong. As explained in the previous section (see Lemma 1), compromising with a strong type, however, is the decision-maker's best response only if $b_s \geq \max\left\{1-p-k, \frac{k}{1-p}\right\}$. A second necessary condition for the existence of a separating equilibrium is that a strong anti-change SIG is willing to reveal its type. Even though the single-crossing condition holds ($c_s^A < c_w^A$), this is not always guaranteed. To understand this result, notice that in a separating assessment, a weak anti-change SIG's gain from imitating a strong type is high since it always obtains a bill closer to the status quo. Indeed, the benefit from imitation equals $b_w - b_s$. In contrast, the gain from revealing its strength is low relatively for a strong SIG. While it faces a bill further away from the status quo by pretending to be weak, it then engages in outside lobbying and reduces the probability the bill is enacted into law. The benefit from differentiation is equal to: $-b_s - (-(1-p)b_w - c_s^A = (1-p)(b_w - b_s)$. Consequently, a weak type's benefit from imitation is always strictly greater than a strong type's benefit from differentiation. This implies that absent a lower lobbying cost for the strong type, a separating equilibrium does not exist. By continuity, a strong anti-change SIG is not willing to bear the necessary lobbying expenditures to credibly reveal its type when its lobbying cost is relatively close to a weak SIG's.

The next proposition characterizes the anti-change SIG's strategy in a separating equilibrium: $\zeta_i^{A*}(\tau),\ l_o^{A*}(\tau),\ \tau \in \{s, w\}$ assuming without loss of generality that the anti-change SIG announces its type ($m^*(\tau) = \tau$). Only a strong anti-SIG incurs strictly positive inside lobbying expenditures since it always obtains the more favorable bill b_s. Since the decision-maker always compromises with the SIG on the equilibrium path, no type engages in outside lobbying. Denoting $\overline{l_i^A}^{sep}(b_s) := \frac{b_w - b_s}{c_w^P}$, I obtain:

Proposition 3. *In a separating equilibrium, the anti-change SIG's equilibrium strategy satisfies:*

1. $\zeta^{A*}(s) = (s, l_i^{A*}(s))$ *and* $\zeta^{A*}(w) = (w, l_i^{A*}(w))$*, with* $l_i^{A*}(s) = \overline{l_i^A}^{sep}(b_s)$ *and* $l_i^{P*}(w) = 0$*;*
2. $l_o^{A*}(\tau) = 0,\ \tau \in \{s, w\}$.

Proposition 3 indicates that in a separating equilibrium, inside lobbying expenditures are perfectly positively correlated with influence. However, as established in Lemma 4, a separating equilibrium does not always exist. Therefore, it is critical to study the extent of the anti-change SIG influence absent informative inside lobbying expenditures.

A pooling equilibrium exists unless a strong anti-change SIG has a profitable signaling deviation satisfying the Intuitive Criterion. This is the case only when the conditions such that Lemma 4 holds are satisfied and absent any information at the inside lobbying stage, the decision-maker prefers b_w to b_s. A strong anti-change SIG would then be better off by incurring inside lobbying expenditures $\overline{l_i^A}^{sep}(s)$, which would credibly reveals its type and induces the decision-maker to propose b_s.

When a pooling equilibrium exists ($\zeta^{A*}(s) = \zeta^{A*}(w) = \zeta^A$), the decision-maker's choice between b_s, b_w, and 1 depends on two factors: (i) her assessment of the threat of outside lobbying activity as measured by her prior q^A that the SIG is strong and (ii) the strong anti-change SIG's lobbying cost. When a strong anti-change SIG's lobbying cost is low ($b_s < \max\{1-p-k, k/(1-p)\}$), the decision-maker never wants to or cannot credibly compromise with a strong SIG. Therefore she always chooses between $b = b_w$ and $b = 1$. The decision-maker gains little from compromising with the weak anti-change SIG when the probability the SIG is strong (q^A) is high since she faces a high risk of outside lobbying activities. Hence, she proposes b_w only if q^A is sufficiently low. When a strong anti-change SIG's lobbying cost is relatively high ($b_s \geq \max\{1-p-k, k/(1-p)\}$), b_s always dominates $b = 1$. Whenever the risk of outside lobbying is high, the decision-maker prefers compromising with a strong anti-change SIG. Hence, as before, she proposes b_w if and only if q^A is low. Denoting $b^*(\zeta^A, \zeta^P)$ the decision-maker's policy choice in a pooling equilibrium, Lemma 5 summarizes the above reasoning after imposing $(1-p)b_w > \max\left\{1-p-k, \frac{k}{1-p}\right\}$ to limit the number of cases (an amended statement holds when the inequality is reversed).[26]

Lemma 5. *Suppose* $(1-p)b_w > \max\left\{1-p-k, \frac{k}{1-p}\right\}$. *There exists a unique* $\overline{q^A} : (0,1) \to (0,1)$ *such that a pooling equilibrium exists if and only if* $b_s \notin \left[\max\left\{1-p-k, \frac{k}{1-p}\right\}, (1-p)b_w\right)$ *or* $q^A > \overline{q^A}(b_s)$. *Furthermore, there exists a unique* $\underline{q^A} \in (0,1)$ *such that in a pooling equilibrium, the decision-maker's policy choice satisfies:*

1. *When* $b_s \leq \max\left\{1-p-k, \frac{k}{1-p}\right\}$, $b^*(\zeta^P, \zeta^A) = b_w$ *if and only if* $q^A \leq \underline{q^A}$, *and* $b^*(\zeta^P, \zeta^A) = 1$

[26] Notice that it is impossible to uniquely characterize the decision-maker's equilibrium strategy when $b_s = \max\left\{1-p-k, \frac{k}{1-p}\right\}$.

otherwise;

2. When $b_s \geq \max\left\{1-p-k, \frac{k}{1-p}\right\}$, $b^*(\zeta^P, \zeta^A) = b_w$ *if and only if* $q^A \leq \overline{q^A}(b_s)$, *and* $b^*(\zeta^P, \zeta^A) = b_s$ *otherwise.*

The next proposition characterizes the anti-change SIG's equilibrium strategy in a pooling equilibrium. First, as the decision-maker does not always propose $b = b_s$, the anti-change SIG sometimes engages in outside lobbying on the equilibrium path. Furthermore, the anti-change SIG may incur inside lobbying expenditures even though they have no impact on the decision-maker's policy choice. This result is again driven by the decision-maker's out-of-equilibrium belief. Absent inside lobbying expenditures (an out-of-equilibrium event), the decision-maker would choose a more extreme policy than when she observes inside lobbying expenditures. The anti-change SIG then incurs inside lobbying expenditures to induce a compromise. To state the next proposition, it is useful to define $\overline{l_i^A}^{po} := \frac{(1-p)(1-b_w)}{c_w^A}$.

Proposition 4. *Suppose* $(1-p)b_w > \max\left\{1-p-k, \frac{k}{1-p}\right\}$. *In a pooling equilibrium, the anti-change SIG's equilibrium strategy satisfies* $m(s) = m(w) \in \{s, w\}$ *and:*

1. When $b_s \leq \max\left\{1-p-k, \frac{k}{1-p}\right\}$,

i. If $q^A \leq \underline{q^A}$, $l_i^{A*}(s) = l_i^{A*}(w) \in \left[0, \overline{l_i^A}^{po}\right]$ *and* $l_o^{A*}(s) = 1,\ l_o^{A*}(w) = 0$;

ii. If $q^A > \underline{q^A}$, $l_i^{A*}(s) = l_i^{A*}(w) = 0$ *and* $l_o^{A*}(s) = l_o^{A*}(w) = 1$;

2. When $b_s \geq \max\left\{1-p-k, \frac{k}{1-p}\right\}$,

i. If $q^A \leq \overline{q^A}(b_s)$, $l_i^{A*}(s) = l_i^{A*}(w) = 0$ *and* $l_o^{A*}(s) = 1,\ l_o^{A*}(w) = 0$

ii. If $q^A > \overline{q^A}(b_s)$, $l_i^{A*}(s) = l_i^{A*}(w) \in \left[0, \overline{l_i^A}^{sep}(b_s)\right]$ *and* $l_o^{A*}(s) = l_o^{A*}(w) = 0$.

Table 2 summarizes this section's main findings under the assumption that $(1-p)b_w > \max\{1-p-k, \frac{k}{1-p}\}$ (omitting arguments and boundary cases for ease of exposition). Cases 1 and 2 correspond to $b_s < \max\left\{1-p-k, \frac{k}{1-p}\right\}$ when only a pooling equilibrium exists and are differentiated by the value of the decision-maker's posterior ($q^A < \underline{q^A}$ and $q^A > \underline{q^A}$, respectively). Cases 5 and 6 correspond to $b_s > (1-p)b_w$ when again only a pooling equilibrium exists and are differentiated by the value of the decision-maker's prior ($q^A < \overline{q^A}(b_s)$ and $q^A > \overline{q^A}(b_s)$, respectively). Finally, cases 3 and 4 correspond to $b_s \in \left(\max\left\{1-p-k, \frac{k}{1-p}\right\}, (1-p)b_w\right)$ when a separating equilibrium exists. In case 3, the decision-maker's prior satisfies $q^A < \overline{q^A}(b_s)$ so a pooling equilibrium does not exist.

Case	Type	Sep.	Pool.
1	s		$\left[0, \overline{l_i^A}^{po}\right]$
	w		$\left[0, \overline{l_i^A}^{po}\right]$
2	s		0
	w		0
3	s	$\overline{l_i^A}^{sep}$	
	w	0	
4	s	$\overline{l_i^A}^{sep}$	$\left[0, \overline{l_i^A}^{sep}\right]$
	w	0	$\left[0, \overline{l_i^A}^{sep}\right]$
5	s		0
	w		0
6	s		$\left[0, \overline{l_i^A}^{sep}\right]$
	w		$\left[0, \overline{l_i^A}^{sep}\right]$

(a) Inside lobbying

Case	Type	Sep.	Pool.
1	s		1
	w		0
2	s		1
	w		1
3	s	0	
	w	0	
4	s	0	0
	w	0	0
5	s		1
	w		0
6	s		0
	w		0

(b) Outside lobbying

Case	Type	Sep.	Pool.
1	s		b_w
	w		b_w
2	s		1
	w		1
3	s	b_s	
	w	b_w	
4	s	b_s	b_s
	w	b_w	b_s
5	s		b_w
	w		b_w
6	s		b_s
	w		b_s

(c) Policy choice

Table 2: Equilibrium strategies

'Sep.' stands for separating equilibrium; 'Pool.' to pooling equilibrium. The different cases are described in the text. In Tables 2a and 2b, for each case, the first line s (second line w) describes a strong (weak) anti-change SIG's strategy. In Table 2c, for each case, the first (second) line corresponds to the decision-maker's bill choice after observing a strong (weak) anti-change SIG's signal: $\zeta^{A*}(s)$ ($\zeta^{A*}(w)$).

To study the empirical implications of this section's results, consider an empirical researcher who seeks to measure anti-change SIGs' influence on policy choice b. For simplicity, I assume again that the empirical researcher's data set contains the full population of SIGs with multiple observations for each parameter value and a vector of independent variables X^A to control for the anti-change SIGs' (potential) lobbying costs. To show the issues associated with using inside lobbying expenditures as a measure of influence, assume the researcher runs the following regression (with ϵ^A the residual):

$$b = \beta_0^A + \beta_1^A l_i^A + \beta_2^{A'} X^A + \epsilon^A \tag{6}$$

Regression (6) would correctly identify that inside lobbying expenditures are positively correlated with influence. That is, it would yield $\widehat{\beta_1^A} \leq 0$. However, the regression would again under-estimate the extent and strength of the SIG influence even if the empirical researcher can control for the decision-maker's prior and therefore can fully distinguish between the six cases described in Table 2.

The researcher would under-estimate the extent of the SIG influence since in cases 1 and 6, regression (6) yields a null coefficient. However, in both cases, the anti-change SIG is able to bias policy choice in its favor (the equilibrium policy choice is different than $b = 1$). The researcher would also under-estimate the strength of the SIG influence in case 4. The obtained estimate would suffer from attenuation bias because the researcher faces an equilibrium selection problem: (s)he cannot distinguish whether the anti-change SIG plays a separating or pooling strategy. Notice that in this case, the anti-change SIG influence is complete (even if weak) due to the threat of outside lobbying activities. As such, the downward bias of $\widehat{\beta_1^A}$ might be severe. The present theoretical framework thus suggests that estimates regarding the impact of lobbying on effective tax rate (Richter et al., 2009) or contributions on trade protection (Maggi and Goldberg, 1999), where SIGs arguably oppose changes, should be understood as lower bounds on SIG influence.

The inclusion of outside lobbying expenditures does not help correct for the identified attenuation bias. This does not imply that outside lobbying activities are of no use for the empirical researcher. They play a critical role when the (arguably unrealistic) assumption that the researcher can control for q^A is relaxed; that is, the researcher cannot distinguish between cases 1 and 2, between cases 3 and 4, and between cases 5 and 6. Outside lobbying expenditures can then be used as a proxy for the decision-maker's prior. As Table 2a illustrates, an anti-change SIG engages in outside lobbying whenever its influence on policy choice is limited (i.e., the decision-maker chooses $b = b_w$) or null (i.e., the decision-maker chooses $b = 1$). Incorporating outside lobbying expenditures in the regression therefore allows the researcher to identify situations when the anti-change SIG does *not* completely influence policy choices. If one is interested in SIG complete (rather than limited) influence, this implies that the empirical researcher can recover the extent of the anti-change SIG influence with outside lobbying expenditures. These expenditures, however, provide no information regarding how these SIGs tilt policy choices in their favor (threat of outside lobbying activities or inside lobbying expenditures). This paper thus suggests there exist limits to our ability to understand anti-change SIG influence due to their strategic use of both inside and outside lobbying.

Policy choices with competing SIGs

In this section, I discuss the case when both SIGs' strength is unknown to the decision-maker (i.e., $q^J \in (0,1),\ J \in \{A, P\}$) and relegate the formal analysis of this set-up to Appendix C. Both SIGs now have an opportunity to affect the decision-maker's policy choice: there is competition for influence. This competition, however, does not change the key insights from the previous sections.

When a pro-change SIG plays a separating strategy, a weak type uses inside lobbying expenditures to credibly plead poverty. There also exist pooling equilibria with positive level of inside lobbying expenditures. Further, a separating and pooling equilibria can arise for some parameter values. Inside lobbying expenditures, therefore, are still a poor proxy for pro-change SIG influence. Outside lobbying expenditures, on the other hand, remain an unbiased measure of influence.

An anti-change SIG plays a separating strategy on the equilibrium path if and only if the compromise bill b_s takes intermediary values. The decision-maker, however, has less incentive to compromise since the pro-change SIG may be able to help. The decision-maker therefore sometimes proposes its preferred policy $b = 1$ even after learning the anti-change SIG is strong. As a result, the expected benefit from differentiation is lower, and the set of parameter values such that the anti-change SIG plays a separating strategy profile is lower (in the sense of set inclusion) than absent competition for influence. Furthermore, even when it plays a separating strategy, the anti-change SIG may engage in outside lobbying. This means that inside lobbying expenditures are no longer necessarily positively correlated with influence and regression (6) might yield an estimate of the wrong sign. Since the strength of anti-change SIG influence is measurable only when it plays a separating strategy, the analysis in the previous section should thus be understood as an upper bound on empirical researchers' ability to quantify SIG influence. Outside lobbying expenditures, in contrast, provide an unbiased estimate of the extent of anti-change SIG (complete) influence even when it plays a separating strategy.

Conclusion

In this paper, I consider a model in which SIGs can use both inside lobbying—to shape the content of a bill—and outside lobbying—to affect the fate of a bill. I show that empirical analyses which exclusively consider inside lobbying expenditures are likely to produce under-estimate both the strength and extent of SIG influence. The small impact of money (understood as contributions) on political decisions generally documented is thus not inconsistent with the widely held belief that SIGs bias the political process in their favor. To evaluate the empirical validity of this popular wisdom, one needs to consider the right type of SIG expenditures. In the environment considered in this paper, only outside lobbying expenditures permit to correctly identify the extent of SIG influence.

Several interesting extensions would help refine our understanding of SIGs' ability to bias policy choices in their favor. The present manuscript supposes that outside lobbying activities only affect the likelihood a bill is enacted into law. However, outside lobbying can also change the salience of an issue and thus the decision-maker's policy agenda. Future theoretical work would do well to consider the strategic interactions between pre- and post-policy choice outside lobbying. This paper also abstains from studying the economic consequences of SIG influence. Analyzing the impact of outside lobbying activities in important policy domains such as income redistribution, trade protection, or environmental regulation constitutes promising avenues for future research.

Proofs

I first introduce some notation. Recall $\zeta^J(\tau) \in \{s, w\} \times \mathbb{R}_+,\ J \in \{A, P\}$, is an SIG's signal as a function of her type $\tau \in \{s, w\}$. Throughout, I assume without loss of generality that when an SIG plays a separating strategy, it announces its type: $m(\tau) = \tau,\ \tau \in \{s, w\}$. The decision-maker's posterior that a pro-change (resp., anti-change) SIG is strong following its signal is $\mu^P(\zeta^P)$ (resp., $\mu^A(\zeta^A)$). As I restrict attention to pure strategy in the main text, posterior always satisfies $\mu^J(\zeta^J) \in \{0, q^J, 1\},\ J \in \{A, P\}$. Denote $b(\zeta^P, \zeta^A) \in [0, 1]$ and $d(\zeta^P, \zeta^A, b, l_o^A) \in \{0, 1, 2\}$ the decision-maker's strategy (resp. policy choice and defense of her proposal) as a function of SIGs' signals, her policy choice, and the anti-change SIG's outside lobbying activity. Denote $l_o^A(b, \zeta^P; \tau) \in \{0, 1\}$ the anti-change SIG's outside lobbying strategy as a function of the decision-maker's proposal, pro-change SIG's signal, and its own type. Similarly, denote $l_o^P(b, l_o^A, d; \tau) \in \{0, 1\}$ the pro-change SIG's outside lobbying strategy as a function of the decision-maker's proposal, anti-change SIG's outside lobbying activities, decision-maker's defense strategy, and its own type. Starred strategies denote equilibrium strategies.

In the proofs, I focus on the SIGs' inside lobbying strategy with players playing their best response down the game tree. This implies in particular (assuming $(1 - p)b - k \geq 0$ so $d^*(\cdot, l_o^A = 1) \neq 0$): i) decision-maker's defense strategy satisfies: $d^*(\zeta^P, \zeta^A, 1) = 1$ if and only if $\mu^P(\zeta^P) = 1$ and $(1 - p)b - c_s^P \geq 0$ (so a strong pro-change SIG has incentive to engage in outside lobbying), and $d^*(\zeta^P, \zeta^A, 1) = 2$ otherwise; ii) the anti-change SIG chooses $l_o^{A*}(b, \zeta^P; \tau) = 1$ if and only if $-(1 - p)b - c_\tau^A > -b,\ \forall b \in [0, 1]$. Notice as well that the decision-maker's equilibrium proposal satisfies $b^*(\cdot) \in \{b_s, b_w, 1\}$, with $b_\tau = \frac{c_\tau^A}{p},\ \tau \in \{s, w\}$. Any other choice decreases her payoff conditional on the bill being enacted and weakly reduces the likelihood the proposal passes.

I now prove the results regarding the pro-change SIG influence (with ζ^A the anti-change SIG's uninformative signal). The decision-maker's equilibrium strategy satisfies $b^*(\cdot) \in \{b_s, 1\}$.

Proof of Lemma 1. When $1 - p - k > b_s$ so the decision-maker strictly prefers 1 to b_s, the decision-maker's best response is then $b(\emptyset, \zeta^A) = 1$. When $(1 - p)b_s - k < 0$, the anti-change SIG's outside lobbying best response is $l_o^A(b_s, \emptyset; s) = 1$ since $d(\emptyset, \zeta^A(s), b_s, 1) = 0$ so the decision-maker gets 0 by choosing b_s and $(1 - p) - k > 0$ by choosing $b = 1$. Her best response is again $b(\emptyset, \zeta^A) = 1$. □

When $b_s \geq \max\{1 - p - k, \frac{k}{1-p}$, by Assumptions 1 and 2, when $\zeta^P(s) = \zeta^P(w) = \zeta^P$ (so $\mu^P(\zeta^P) =$

q^P), the decision-maker's equilibrium strategy satisfies: $b^*(\zeta^P, \zeta^A) = b_w$ and $d^*(\zeta^P, \zeta^A, b_w, 0) = 0$, $d(\zeta^P, \zeta^A, b_w, 1) = 2$. In turn, the anti-change SIG's strategy is (slightly abusing notation) $l_o^{A*}(b_s, \zeta^P; \tau) = 0,\ \tau \in \{s, w\}$.

Lemma 6. *The pro-change SIG plays a separating strategy on the equilibrium path (i.e. $\zeta^P(s) \neq \zeta^P(w)$) only if: $b(\zeta^P(s), \zeta^A) = 1$ and $b(\zeta^P(w), \zeta^A) = b_s$.*

Proof. The proof is by contradiction. Suppose $b^*(\zeta^P(s), \zeta^A) = 1 = b^*(\zeta^P(w), \zeta^A)$ so $l_o^{A*}(1, \zeta^P; \tau) = 1$ for all ζ^P, τ. The decision-maker's best response is then $d^*(\zeta^P(s), \zeta^A, 1, 1) = 1$ and $d^*(\zeta^P(w), \zeta^A, 1, 1) = 2$. The strong and weak types' incentive compatibility constraints (IC) are then, respectively: $(1-p) - c_s^P - c_s^P l_i^P(s) \geq (1-p) - c_s^P l_i^P(w)$ and $(1-p) - c_w^P l_i^P(w) \geq 0 - c_w^P l_i^P(s)$. A necessary condition for existence of such equilibrium is $1 \leq l_i^P(w) - l_i^P(s) \leq \frac{(1-p)}{c_w^P}$, which is never satisfied by Assumption 2. A similar reasoning directly implies that $b(\zeta^P(s), \zeta^A) = b_s$ and $b(\zeta^P(w), \zeta^A) = 1$ cannot be an equilibrium strategy. Finally, when $b(\zeta^P(s), \zeta^A) = b_s = b(\zeta^P(w), \zeta^A)$, the pro-change SIG's signal has no effect on the decision-maker's strategy on the equilibrium path (since $l_o^{A*}(b_s, \zeta^P; \tau) = 0$ and $d^*(\zeta^P, \zeta^A, b_s, 0) = 0$ for $\zeta^P \in \{\zeta^P(s), \zeta^P(w)\}$). The equilibrium restriction then imposes the pro-change SIG plays a pooling strategy. □

A direct consequence of Lemma 6 is that whenever the pro-change SIG plays a separating strategy, a strong type's equilibrium outside lobbying strategy satisfies $l_o^{P*}(b^*, l_o^{A*}, d^*; s) = 1$ since $b^*(\zeta^P(s), \zeta^A) = 1$ and $\mu^P(\zeta^P(s)) = 1$ so $l_o^{A*}(\cdot) = 1$ and $d^*(\zeta^P(s), \zeta^A, 1, 1) = 1$.

Lemma 7. *The pro-change SIG plays a separating strategy on the equilibrium path only if: $l_i^P(s) = 0$.*

Proof. The proof is by contradiction. Suppose $\zeta^P(s) = (s, l_i^P(s))$ with $l_i^P(s) > 0$. By the Intuitive Criterion, this implies $l_i^P(w) = 0$. By Lemma 6, a weak pro-change SIG's expected policy payoff is b_s when she sends signal $\zeta^P(w)$ and 0 when she sends signal $\zeta^P(s)$. Consequently, a weak pro-change SIG has no incentive to send a signal ζ^P satisfying $l_i^P > 0$. By the Intuitive Criterion, the strong type has a profitable deviation to $\widehat{l_i^P} \in (0, l_i^P(s))$, a contradiction. □

Proof of Lemma 2. Necessity. Suppose $\zeta^P(s) \neq \zeta^P(w)$. When $b_s > 1-p-k/2$, the decision-maker's best response is: $b(\zeta^P, \zeta^A) = b_w$ for all ζ^P. By Lemma 6, a separating equilibrium does not exist. Suppose $b_s \leq 1 - p - k/2$ so the decision-maker's best response satisfies $b(\zeta^P(s), \zeta^A) = 1$ and

$b(\zeta^P(w),\zeta^A) = b_s$. Using Lemma 7, a strong pro-change SIG's incentive compatibility constraint (IC) is: $(1-p) - c_s^P \geq b_s - c_s^P l_i^P(w)$. The weak type's (IC) is: $b_s - c_w^P l_i^P(w) \geq 0$. By the Intuitive Criterion, $l_i^P(w) = \max\left\{\frac{c_s^P + b_s - (1-p)}{c_s^P}, 0\right\}$. Therefore, both (IC) are automatically satisfied whenever $c_s^P \leq (1-p) - b_s$ (so $l_i^P(w) = 0$). When $c_s^P > (1-p) - b_s$, a weak type's (IC) is satisfied if and only if $c_s^P \leq \overline{c^P}(b_s, c_w^P)$, with $\overline{c^P}(b_s, c_w^P) := c_w^P \frac{(1-p)-b_s}{c_w^P - b_s}$, with $\overline{c^P}(b_s, c_w^P) < (1-p)$ as claimed.

Sufficiency. Suppose $b_s \leq 1 - p - k/2$ and $c_s^P \leq \overline{c^P}(b_s, c_w^P)$, and consider the following assessment: i) A weak (strong) pro-change SIG's signal is $\zeta^P(w) = (w, l_i^P(w))$ ($\zeta^P(s) = (s, 0)$), with $l_i^P(w) = \max\left\{\frac{c_s^P + b_s - (1-p)}{c_s^P}, 0\right\}$; ii) The decision-maker's posterior is: $\mu^P(\zeta^P) = 0$ if $\zeta^P = (w, l_i^P)$, with $l_i^P \geq l_i^P(w)$, and 1 otherwise; iii) the decision-maker's policy choice is: $b(\zeta^P,\zeta^A) = b_s$ if $\zeta^P = (w, l_i^P)$, with $l_i^P \geq l_i^P(w)$ and $b(\zeta^P,\zeta^A) = 1$, otherwise; (iv) all players play their best response down the game tree (see above). It can be checked that beliefs satisfy Bayes' rule, the decision-maker's policy choice is a best response given her belief, and the pro-change SIG's (IC) hold. Hence, the assessment described above is an equilibrium. □

Proof of Proposition 1. Direct from the proof of Lemma 2. □

The proof of Remarks 1 follows directly from the proof of Lemma 2. The proof of Remark 2 follows from noticing that outside lobbying expenditures are normalized to 1 and by Lemma 2, $l_i^P(w) < 1$.

Proof of Proposition 2. The proof of existence follows directly from Lemma 7 and noticing that a weak type has no incentive to imitate a strong type when $b(\zeta^P(s),\zeta^A) = 1$ and $d(\zeta^P(s),\zeta^A, b, 1) = 1$. Therefore, when $c_s^P < 1 - p - b_s$ and $b_s < 1 - p - k/2$, only a strong type has incentive to send signal $\widehat{\zeta^P(s)} = (s, 0)$ when $\mu^P(\widehat{\zeta^P(s)}) = 1$. By the Intuitive Criterion, a pooling equilibrium does not exist then. It exists for all other parameter values.

Suppose $1-p-b_s < k/2$. The decision-maker's best responses are $b^*(\zeta^P,\zeta^A) = b_s$ and $d^*(\zeta^P,\zeta^A, b_s, 0) = 0$, $d(\zeta^P,\zeta^A, b_s, 1) = 2$ for all ζ^P.[27] The only equilibrium signal is then $\zeta^P(s) = \zeta^P(w) = (m, 0)$ for some $m \in \{s, w\}$. Suppose $k/2 \leq 1 - p - b_s$ and consider the following belief structure for the decision-maker: $\mu^P(\zeta^P) = 1$, when $\zeta^P = (m, l_i^P)$ for $m \in \{s, w\}$ and $l_i^P \in [0, \widehat{l_i^P})$, with $\widehat{l_i^P} > 0$, and $\mu^P(\zeta^P) = q^P$, otherwise. Given this belief structure, the decision-maker's best response is: $(b^*(\zeta^P,\zeta^A) = 1; d^*(\zeta^P,\zeta^A, 1, 0) = 0, d^*(\zeta^P,\zeta^A, 1, 1) = 1)$, $\forall \zeta^P \in \{s, w\} \times [0, \widehat{l_i^P})$ and

[27]By Assumptions 1 and 2, $d^*(\zeta^P,\zeta^A, b_s, 1) = 2$ for all ζ^P (since $c_s^P > (1-p)b_s$). This directly implies that it is never a best response for the decision-maker to propose $b < b_s$. Suppose now there exists $\widehat{\zeta^P}$ such that $b^*(\widehat{\zeta^P},\zeta^A) = 1$. It must be that $\mu^P(\widehat{\zeta^P})$ satisfies $\mu^P(\widehat{\zeta^P})(1 - p - k/2) \geq b_s$, which contradicts $1 - p - b_s < k/2$.

$(b^*(\zeta^P, \zeta^A) = b_s; d^*(\zeta^P, \zeta^A, b_s, 0) = 0, d^*(\zeta^P, \zeta^A, b_s, 1) = 2)$, $\forall \zeta^P \in \{s, w\} \times [\widehat{l_i^P}, \infty)$. The strong type's (IC) is: $b_s - c_s^P \widehat{l_i^P} \geq (1-p) - c_s^P$. The weak type's (IC) is: $b_s - c_w^P \widehat{l_i^P} \geq 0$. Both (IC) are satisfied whenever $\widehat{l_i^P} \leq \min\left\{\overline{l_i^P}(c_s^P, b_w), \frac{b_s}{c_w^P}\right\}$. Consequently, when $c_s^P > 1 - p - b_s \geq k/2$, any signaling strategy satisfying $\zeta^P(s) = \zeta^P(w) = (m, l_i^P)$, with $l_i^P \leq \min\left\{\overline{l_i^P}(c_s^P, b_s), \frac{b_s}{c_w^P}\right\}$ can be part of a pooling equilibrium.

Since $b^*(\zeta^P, \zeta^A) = b_s$ and $d^*(\zeta^P, \zeta^A, b_s, 0) = 0, d^*(\zeta^P, \zeta^A, b_s, 1) = 2$ in a pooling equilibrium, we directly obtain $l_o^{P*}(\tau) = 0$, $\tau \in \{s, w\}$. □

I now prove the results regarding the anti-change SIG influence (denoting ζ^P the pro-change SIG's uninformative signal). As $q^P = 0$ and $(1-p) < c_w^P$, $d(\cdot) = 1$ is a strictly dominated strategy. I focus on the effect of anti-change SIG's inside lobbying strategy on policy choices with all players playing their best response down the game tree.

Lemma 8. *The anti-change SIG plays a separating strategy (i.e., $\zeta^A(s) \neq \zeta^A(w)$) on the equilibrium path only if $b(\zeta^P, \zeta^A(s)) < b(\zeta^P, \zeta^A(w))$.*

Proof. First, notice that by Assumption 1, $(1-p)b_w - k > 0$ (so $d(\zeta^P, \zeta^A, b_w, 1) = 2$ is a best response) and $b_w > 1-p-k$. The decision-maker's best response after observing $\zeta^P(w)$ is thus $b(\zeta^P, \zeta^A(w)) = b_w$. The rest of the proof proceeds by contradiction. Suppose $b(\zeta^P, \zeta^A(s)) > b(\zeta^P, \zeta^A(w)) = b_w$. The strong type's (IC) is: $-(1-p)b(\zeta^P, \zeta^A(s)) - c_S^A - c_S^A l_i^A(s) \geq -(1-p)b_w - c_s^A - c_S^A l_i^A(w)$ since $l_o^A(b_w, \zeta^P; s) = 1$. The weak type's (IC) is: $-b_w - c_w^A l_i^A(w) \geq -(1-p)b(\zeta^P, \zeta^A(s)) - c_w^A - c_w^A l_i^A(s)$. Using the definition of b_w, the two constraints are equivalent to: $c_w^A(l_i^A(w) - l_i^A(s)) \leq (1-p)(b(\zeta^P, \zeta^A(s)) - b_w) \leq c_s^A(l_i^A(w) - l_i^A(s))$. The first inequality implies $l_i^A(w) > l_i^A(s)$. Since $c_w^A > c_s^A$, we have reached a contradiction. □

Lemma 9. *The anti-change SIG plays a separating strategy on the equilibrium path only if: $l_i^A(w) = 0$ and $l_i^A(s) > 0$.*

Proof. By Lemma 8 (i.e., $b(\zeta^P, \zeta^A(s)) < b_w$), a weak type's (IC) is satisfied only if $l_i^A(s) > 0$. $l_i^A(w) = 0$ follows by the Intuitive Criterion. □

Proof of Lemma 4. Necessity. Suppose $\zeta^A(s) = (s, l_i^A(s)) \neq \zeta^A(w) = (w, l_i^A(w))$. By the same logic as in Lemma 1, the decision-maker's best response is $b(\zeta^P, \zeta^A(s)) = 1$ when $b_s < \max\{1-p-k, \frac{k}{1-p}\}$. By Lemma 8, a separating equilibrium cannot exist then.

Assume $b_s \geq \max\{1-p-k, \frac{k}{1-p}\}$ so $b(\zeta^P, \zeta^A(s)) = b_s$. Using Lemma 9, the reasoning in the text implies the weak type's (IC) is: $-b_w \geq -b_s - c_w^A l_i^A(s)$. By the Intuitive Criterion, $l_i^A(s) = \frac{b_w - b_s}{c_w^A} = \overline{l_i^A}^{sep}(b_s)$. The strong type's (IC) is: $-b_s - c_s^A l_i^A(s) \geq -(1-p)b_w - c_s^A$. Plugging in $\overline{l_i^A}^{sep}(b_s)$ and simple algebra yield that a necessary condition is $c_s^A \leq (1-p)c_w^A$ as claimed.

Sufficiency. Same logic as in the proof of Lemma 2. □

Proof of Proposition 3. Follows directly from the proof of Lemma 4. □

Proof of Lemma 5. I just prove necessity (sufficiency follows from the usual argument). A pooling equilibrium does not exist when $\max\{1-p-k, k/(1-p)\} < b_s < (1-p)b_w$ and $q^A < \overline{q^A}(b_s)$. Consider the out-of-equilibrium signal $\widehat{\zeta^A(s)} = (s, \overline{l_i^A}^{sep}(b_s))$ and suppose decision-maker's out-of-equilibrium belief satisfies: $\mu^A(\widehat{\zeta^A(s)}) = 1$ so $b(\zeta^P, \widehat{\zeta^A(s)}) = b_s$. Only a strong type has an incentive to send signal $\widehat{\zeta^A(s)}$. By the Intuitive Criterion, the anti-change SIG does not play a pooling strategy on the equilibrium path. It can be checked that a pooling equilibrium exists for all other parameter values.

To characterize the decision-maker's policy choice, denote the anti-change SIG's signal $\zeta^A(\tau) := \zeta^A = (m, l_i^A)$ for $\tau \in \{s, w\}$ and some $m \in \{s, w\}$ and $l_i^A \geq 0$ (to be determined). By Bayes' rule, $\mu^A(\zeta^A) = q^A$. When the decision-maker chooses $b = 1$, her expected utility is $1-p-k$; when $b = b_w$, her expected utility is: $q^A[(1-p)b_w - k] + (1-q^A)b_w$ (as $l_o^A(b_w, \zeta^P; s) = 1$ and $l_o^A(b_w, \zeta^P; w) = 0$); when $b = b_s$, her expected utility is b_s if $(1-p)b_s - k \geq 0$ and 0 otherwise. When $b_s < \max\{1-p-k, k/(1-p)\}$, $b = b_s$ is strictly dominated by $b = 1$. Simple computations yield the decision-maker's best response is $b(\zeta^P, \zeta^A) = b_w$ if and only if $q^A \leq \underline{q^A} := \frac{b_w - (1-p-k)}{pb_w + k}$. When $b_s > \max\{(1-p)b_w, k/(1-p)\}$, $b = 1$ is strictly dominated by $b = b_s$. Simple algebra yields $b(\zeta^P, \zeta^A) = b_w$ if and only if : $q^A \leq \overline{q^A}(b_s) := \frac{b_w - b_s}{pb_w + k}$. □

Proof of Proposition 4. Point 1. Suppose $q^A \leq \underline{q^A}$. Consider the following belief structure: $\mu^A(\zeta^A) = 1$ when $\zeta^A = (m, l_i^A)$ for $m \in \{s, w\}$ and $l_i^A \in [0, \widehat{l_i^A})$ with $\widehat{l_i^A} > 0$, and $\mu^A(\zeta^A) = q^A$, otherwise. Given this belief structure, the decision-maker's best response is: $(b^*(\zeta^P, \zeta^A) = 1; d^*(\zeta^P, \zeta^A, 1, 0) = 0, d^*(\zeta^P, \zeta^A, 1, 1) = 2)$, $\forall \zeta^A \in \{s, w\} \times [0, \widehat{l_i^A})$ and $(b^*(\zeta^P, \zeta^A) = b_w; d^*(\zeta^P, \zeta^A, b_w, 0) = 0, d^*(\zeta^P, \zeta^A, b_w, 1) = 2)$, $\forall \zeta^A \in \{s, w\} \times [\widehat{l_i^A}, \infty)$. The strong type's (IC) is: $-(1-p)b_w - c_s^A - c_s^A \widehat{l_i^A} \geq -(1-p) - c_s^A$. The weak type's (IC) is: $-b_w - c_w^A \widehat{l_i^A} \geq -(1-p) - c_w^A$. Both (IC) are satisfied whenever $\widehat{l_i^A} \leq \frac{(1-p)(1-b_w)}{c_w^A} = \overline{l_i^A}^{po}$. So any signaling strategy satisfying $\zeta^A = (m, l_i^A)$ with $l_i^A \leq \frac{(1-p)(1-b_w)}{c_w^A}$ can be part of a pooling equilibrium. For outside lobbying, notice that $l_o^{A*}(b_w, \zeta^P; s) = 1$ and $l_o^{A*}(b_w, \zeta^P; w) = 0$. Suppose $q^A > \underline{q}^A$ so $b^*(\zeta^P, \zeta^A) = 1$ for all ζ^A and $l_i^{A*} = 0$. Obviously, $l_o^{A*}(1, \zeta^P; \tau) = 1$ for all $\tau \in \{s, w\}$.

Point 2. Similar logic as point 1.

$\square$

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A Equilibrium definition

To define the Perfect Bayesian Equilibrium of the game, denote $U^P(\zeta^P, l_o^P; \tau, l_o^A, b, d)$ the pro-change SIG's utility as a function of the decision-maker's policy choice b and defense strategy d, the strategy of the anti-change SIG, and its own type. Denote $U^A(\zeta^A, l_o^A; \tau, b)$ the anti-change SIG's utility as a function of the decision-maker's policy choice b, the strategy of the pro-change SIG , and its own type. Denote $U^D(b, d; l_o^A)$ the decision-maker's utility as a function of the anti-change SIG's outside lobbying strategy.

A PBE in pure strategies consists of: 1) the pro-change SIG's decision to engage in outside lobbying: $l_o^{P*}(b, l_o^A, d; \tau) \in \{0, 1\}$; 2) the decision-maker's defense strategy: $d^*(\zeta^P, \zeta^A, b, l_o^A) \in \{0, 1, 2\}$; 3) the anti-change SIG's decision to engage in outside lobbying: $l_o^{A*}(b, \zeta^P; \tau) \in \{0, 1\}$; 4) the decision-maker's policy choice: $b^*(\zeta^P, \zeta^A) \in [0, 1]$, 5) the SIGs' signaling strategy: $\zeta^{J*}(\tau) \in \{s, w\} \times \mathbb{R}_+$ for all $\tau \in \{s, w\}$, $J \in \{A, P\}$; 6) and beliefs $\mu^J(\zeta^J)$ that SIG $J \in \{A, P\}$ is strong, which together satisfy the following conditions:

- C1: $l_o^{P*}(b, l_o^A, d; \tau) = 1$ if and only if (iff): $EU^P(\zeta^P, 1; \tau, l_o^A, b, d) \geq EU^P(\zeta^P, 0; \tau, l_o^A, b, d)$ for all $\zeta^A \in \{s, w\} \times \mathbb{R}_+$, $\tau \in \{s, w\}$, $\zeta^P \in \{s, w\} \times \mathbb{R}_+$, $l_o^{-J} \in \{0, 1\}$, $b \in [0, 1]$, $d \in \{0, 1, 2\}$ (where the expectation is over outcomes).

- C2: $d^*(\zeta^P, \zeta^A, b, l_o^A) \in arg\max_{d \in \{0,1,2\}} E(U^D(b, d; l_o^A)|\zeta^P)$ for all $\zeta^P \in \{s, w\} \times \mathbb{R}_+$, $l_o^{-J} \in \{0, 1\}$, $b \in [0, 1]$ (where the expectation is over outcomes and $l_o^{P*}(b, l_o^A, d; \tau)$ conditional on ζ^P).

- C3: $l_o^{A*}(b, \zeta^P; \tau) = 1$ iff: $E(U^A(\zeta^A, 1; \tau, b)|\zeta^P) \geq E(U^A(\zeta^A, 0; \tau, b)|\zeta^P)$ for all $\zeta^A \in \{s, w\} \times \mathbb{R}_+$, $\tau \in \{s, w\}$, $\zeta^P \in \{s, w\} \times \mathbb{R}_+$, $b \in [0, 1]$ (where the expectation is over outcomes and $l_o^{P*}(b, l_o^A, d; \tau)$ conditional on ζ^P).

- C4: $b^*(\zeta^P, \zeta^A) \in arg\max_{b \in [0,1]} E(U^d(b, d^*; l_o^{A*})|\zeta^A, \zeta^P)$ for all $\zeta^A \in \{s, w\} \times \mathbb{R}_+$, $\tau \in \{s, w\}$, $\zeta^P \in \{s, w\} \times \mathbb{R}_+$ (where the expectation is over outcomes, $l_o^{A*}(b, \zeta^P; \tau)$ conditional on ζ^A, and $l_o^{P*}(b, l_o^A, d; \tau)$ conditional on ζ^P).

- C5: $\zeta^A(\tau) \in arg\max_{\hat{\tau} \in \{s,w\},\ l_i^A \geq 0} EU^A(\zeta^A, l_o^{A*}; \tau, b^*)$ and $\zeta^P(\tau) \in arg\max_{\hat{\tau} \in \{s,w\},\ l_i^A \geq 0} EU^P(\zeta^P, l_o^{P*}; \tau, b^*, d^*)$ (where the expectation is over outcomes, b^*, d^*, l_o^{-J*}, τ^{-J}, $-J$ denotes the opposite SIG).

- C6: Beliefs $\mu^A(\zeta^A)$ and $\mu^P(\zeta^P)$ satisfy Bayes' rule whenever possible.

B Allowing for mixed strategies

In this section, I allow for mixed strategies. I start with the pro-change SIG under the conditions of the "Pro-change SIG influence" section and then consider the anti-change SIG under the conditions of the "Anti-change SIG influence" section. For simplicity, I only consider the case when one type plays a non-degenerate mixed strategy. Focusing on this type of 'semi-separating assessment' is without significant loss of generality as the reasoning can easily be extended to more general mixed strategies.

Pro-change SIG

When it comes to the pro-change SIG, the conditions for existence of a semi-separating equilibrium is more restrictive (in the sense of set inclusion) than for a separating equilibrium except for a set of measure zero of parameter values (see Lemma 10). For the same reason as in the main text, a semi-separating equilibrium does not exist when $b_s < \max\{1-p-k, \frac{k}{1-p}\}$. A semi-separating equilibrium cannot exist when a strong type's cost of lobbying is too low since a strong pro-change SIG strictly prefers to reveal its type (therefore, we can never satisfy the indifference condition). As in the main text, a semi-separating equilibrium does not exist when the cost of lobbying is relatively large: $c_s^P > \overline{c^P}(b_w, c_w^P)$. The reason is that the decision-maker's best-response set at the policy stage is $\{b_s, 1\}$. Any other policy reduces the decision-maker's policy payoff conditional on being passed. Since the pro-change SIG does not change the decision-maker's best-response set when it plays a mixed strategy, it faces the same type of incentives as in a separating assessment. Therefore, all the results described in the main text hold when mixed strategies are allowed.

Proposition 5. *Suppose $b_s \geq \frac{k}{1-p}$ and $b_s > 1-p-k$. A semi-separating equilibrium exists if and only if: i. $k/2 \leq 1-p-b_s$; and ii. $(1-p)-b_s \leq c_s^P \leq \overline{c^P}(b_s, c_w^P)$.*

Proof. I only prove necessity. Sufficiency follows from the usual argument.

Point i. follows from a similar reasoning as in the proof of Lemma 2. I thus focus on point ii. First, notice that Lemma 7 still holds when the pro-change SIG plays a semi-separating strategy: a strong type never incurs inside lobbying expenditures.

Suppose that the weak type always sends signal $\zeta^P(w) = (w, l_i^P(w))$ for some $l_i^P(w) \geq 0$ to be determined in equilibrium. The strong type randomizes between $\zeta^P(s) = (s, 0)$ and $\zeta^P(w)$. The decision-maker's posterior satisfies $\mu^P(\zeta^P(w)) < q^P$ and $\mu^P(\zeta^P(s)) = 1$. Under Assumption 2, the decision-maker's policy choice and defense strategy $d(\cdot)$ after signals $\zeta^P(w)$ and $\zeta^P(s)$ is exactly the same as in a separating assessment. By a similar reasoning as in Lemma 2, a strong and weak pro-change SIG's incentive compatibility constraints (IC) are then respectively:

$$1 - p - c_s^P = b_s - c_s^P l_i^P(w) \tag{7}$$

$$b_s - c_w^P l_i^P(w) \geq 0 \tag{8}$$

Notice that the constraint $l_i^P(w) \geq 0$ implies that (7) is never satisfied when $(1-p) - b_s > c_s^P$. Therefore, a first necessary condition is $(1-p) - b_s \leq c_s^P$. Since $l_i^P(w) = \frac{b_s - (1-p-c_s^P)}{c_s^P}$ as in a separating equilibrium, a second necessary condition is $c_s^P \leq \overline{c^P}(b_s, c_w^P)$,

Suppose instead that the strong type always sends signal $\zeta^P(s) = (s, 0)$, whereas the weak type randomizes between $\zeta^P(w) = (w, l_i^P(w))$ for some $l_i^P(w) \geq 0$ and $\zeta^P(s) = (s, 0)$. I claim that after receiving signal $\zeta^P(s)$, the decision-maker must satisfy: $\mu^P(\zeta^P(s))(1 - p - k/2) + (1 - \mu^P(\zeta^P(s)))(-k/2) = b_s$. That is, the decision-maker must be indifferent (anticipating equilibrium play down the game tree) between $(b(\zeta^A, \zeta^P(s)) = 1; d(\zeta^P(s), \zeta^A, 1, 1) = 1)$ and $(b_s, 0)$. The claim is formally shown in Lemma 10 below. Denote α the probability that the decision-maker chooses $(1, 1)$ after signal $\zeta^P(s)$. By a similar reasoning as in Lemma 2, a strong and weak pro-change SIG's (IC) are then respectively:

$$\alpha(1 - p - c_s^P) + (1 - \alpha)b_s \geq b_s - c_s^P l_i^P(w) \tag{9}$$

$$b_s - c_w^P l_i^P(w) = (1 - \alpha)b_s \tag{10}$$

As above, it can easily be checked that $c_s^P \geq 1 - p - b_s$ is a necessary condition for existence. Suppose it holds in what follows. From (14), we obtain $l_i^P(w) = \alpha \frac{b_s}{c_w^P}$. Plugging this into (9), we get after some simple algebra that a second necessary condition is: $c_s^P \leq c_w^P \frac{1-p-b_s}{c_w^P - b_s} = \overline{c^P}(b_s, c_w^P)$. □

Lemma 10. *There exists semi-separating equilibrium in which the decision-maker randomizes between $d = 1$ and $d = 2$ on the equilibrium path only when $b_s \geq \frac{k}{1-p}$ and $b_s = 1 - p - k$.*

Proof. Suppose $b_s \geq \frac{k}{1-p}$ and $b_s > 1 - p - k$. The proof proceed by contradiction. Suppose there is such equilibrium. By the reasoning in the proof of Proposition 5, the decision-maker can be indifferent between $d = 1$ and $d = 2$ after choosing $b = 1$ only if the weak type randomizes between $\zeta^P(s) = (s, 0)$ and $\zeta^P(w) = (w, l_i^P(w))$ for some $l_i^P(w) \geq 0$. This occurs if and only if her posterior $\mu^P(\zeta^P(s))$ satisfies: $\mu^P(\zeta^P(s))(1-p-k/2)+(1-\mu^P(\zeta^P(s)))(-k/2) = 1-p-k$. For such randomization to occur on the equilibrium path, it must be that the decision-maker prefers $(1, 2)$ to $(b_s, 0)$. This is the case only if $1 - p - k \geq b_s$. But this contradicts the assumption.

Suppose $b_s < \max\{1 - p - k, \frac{k}{1-p}\}$ so the decision-maker never chooses $b = b_s$. As above, the decision-maker can be indifferent between $d = 1$ and $d = 2$ after choosing $b = 1$ only if the weak type randomizes between $\zeta^P(s) = (s, 0)$ and $\zeta^P(w) = (w, l_i^P(w))$ for some $l_i^P(w) \geq 0$. A strong and weak pro-change SIG's (IC) are then, respectively (denoting γ the probability that $d = 2$):

$$\gamma(1-p) + (1-\gamma)(1-p-c_s^P) \geq 1-p-c_s^P l_i^P(w) \tag{11}$$

$$1-p-c_w^P l_i^P(w) = \gamma(1-p) \tag{12}$$

Given $c_s^P < c_w^P$, it can be checked that both conditions cannot be satisfied simultaneously.

Finally, suppose that $b_s \geq \frac{k}{1-p}$ and $b_s = 1 - p - k$. A strong pro-change SIG sends signal $\zeta^P(s) = (s, 0)$. A weak anti-change SIG randomizes between $\zeta^P(s)$ and $\zeta^P(w) = (w, l_i^P(w)$ for some $l_i^P(w) \geq 0$. After signal $\zeta^P(s)$ (or any signal such that $l_i^P < l_i^P(w)$), the decision-maker chooses $b = 1$ and randomizes between $d = 1$ and $d = 2$ (with γ the probability that $d = 2$). After signal $\zeta^P(w)$ (or any signal such that $l_i^P \geq l_i^P(w)$), the decision-maker chooses $b = b_s$ (and $d = 2$ if $l_o^A = 2$, an out-of-equilibrium event). A strong and weak pro-change SIG's (IC) are then, respectively:

$$\gamma(1-p) + (1-\gamma)(1-p-c_s^P) \geq b_s - c_s^P l_i^P(w) \tag{13}$$

$$b_s - c_w^P l_i^P(w) = \gamma(1-p) \tag{14}$$

By the usual reasoning, we obtain that a semi-separating equilibrium exists only if $c_s^P \leq c_w^P \frac{(1-p)-b_s}{(1-\gamma)c_w^P+\gamma(1-p)-b_s}$. Notice that $\frac{(1-p)-b_s}{(1-\gamma)c_w^P+\gamma(1-p)-b_s} > \overline{c^P}(b_s, c_w^P)$ for all $\gamma > 0$. $\square$

Anti-change SIG

As for the pro-change SIG, I obtain the same result as in the main text when mixed strategies are allowed for the anti-change SIG. As above, the intuition behind this result is that the decision-maker never proposes a policy different than b_s, b_w, or 1 when the anti-change SIG randomizes. Any other policy choice only reduces the decision-maker's policy payoff conditional on being passed.[28] Consequently, the anti-change SIG faces the same incentive as in a separating assessment and all the results described in the main text carry through when I allow for mixed strategies.

Proposition 6. *A semi-separating equilibrium exists if and only if:*

$$\max\left\{1-p-k, \frac{k}{1-p}\right\} \leq b_s \leq (1-p)b_w$$

Proof. I only prove necessity. Sufficiency proceeds from the usual argument.

I consider an assessment in which a type $\tau \in \{s, w\}$ sends signal $\zeta^A(\tau) = (\tau, l_i^A(\tau))$ for some $l_i^A(\tau) \geq 0$ with positive probability. Using a similar reasoning as in Lemma 8, it can be checked that a semi-separating equilibrium exists only if $b(\zeta^A(s), \zeta^P) < b(\zeta^A(w), \zeta^P)$. As a consequence, Lemma 9 still holds in this setting so $l_i^A(s) > 0 = l_i^A(w)$. Using a similar reasoning as in Lemma 4, this directly implies that a necessary condition for existence of a semi-separating equilibrium is $\max\left\{1-p-k, \frac{k}{1-p}\right\} \leq b_s$ so the decision-maker's best response to $\zeta^A(s)$ is $b(\zeta^A(s), \zeta^P) = b_s$. Suppose that the weak type randomizes (so the strong type plays $\zeta^A(s)$ with probability 1). After signal $\zeta^A(s)$, the decision-maker's posterior must satisfy: $\mu^A(\zeta^A(s))((1-p)b_w - k) + (1 - \mu^A(\zeta^A(s)))b_w = b_s$ so she is indifferent between b_w and b_s.[29] Consequently, a necessary condition for this equilibrium to exist is $q^A \leq \frac{b_w - b_s}{pb_w + k} = \overline{q^A}(b_s)$ (otherwise, $\mu^A(\zeta^P(s)) > \overline{q^A}(b_s)$ and the decision-maker cannot be made indifferent). Denote β the probability that the decision-maker chooses b_s after signal $\zeta^A(s)$. Using a similar reasoning as in Lemma 4, the strong and weak types' (IC)

[28]For example, assuming $(1-p)b_s - k \geq 0$, any policy $b \in (b_s, b_w)$ is associated with outside lobbying by a strong anti-change SIG and no outside lobbying activity by a weak type. As such, the probability that any $b \in (b_s, b_w)$ passes is the same as b_w which provides a strictly higher policy payoff (when passed) to the decision-maker.

[29]If the decision-maker strictly prefers b_w to b_s, then the necessary condition $b(\zeta^A(s), \zeta^P) < b(\zeta^A(w), \zeta^P)$ is violated. If the decision-maker strictly prefers b_s, the results are exactly the same as Lemma 4.

constraints are respectively:

$$\beta(-(1-p)b_w - c_s^A) + (1-\beta)(-b_s) - c_s^A l_i^A(s) \geq -(1-p)b_w - c_s^A \tag{15}$$

$$\beta(-b_w) + (1-\beta)(-b_s) - c_w^A l_i^A(s) = -b_w \tag{16}$$

From (16), we obtain $l_i^A(s) = (1-\beta)\frac{b_w - b_s}{c_w^A}$. Plugging the inside lobbying expenditures into (15, a necessary condition is (after simple algebra): $b_s \leq (1-p)b_w$.

Suppose that the strong type randomizes (so the weak type plays $\zeta^A(w)$ with probability 1). After signal $\zeta^A(w)$, the decision-maker's posterior must satisfy: $\mu^A(\zeta^A(w))((1-p)b_w - k) + (1 - \mu^A(\zeta^A(w)))b_w = b_s$ so she is indifferent between b_w and b_s. Consequently, a necessary condition for this equilibrium to exist is $q^A \geq \overline{q^A}(b_s)$. Denote γ the probability that the decision-maker chooses b_s after signal $\zeta^A(w)$. Using a similar reasoning as in Lemma 4, the strong and weak types' (IC) constraints are respectively:

$$-b_s - c_s^A l_i^A(s) = \gamma(-b_s) + (1-\gamma)(-(1-p)b_w - c_s^A) \tag{17}$$

$$\gamma(-b_s) + (1-\gamma)(-b_w) \geq -b_s - c_w^A l_i^A(s) \tag{18}$$

From (17), we obtain $l_i^A(s) = (1-\gamma)\frac{(1-p)(b_w - b_s)}{c_s^A}$ (using $b_s = \frac{c_s^A}{p}$). Plugging the inside lobbying expenditures into (18, a necessary condition is (after simple algebra): $b_s \leq (1-p)b_w$. □

C Policy choices with competing SIGs

In this section, I analyze formally the case when the anti-change and pro-change SIGs compete to influence political decisions. Throughout, I assume that Assumptions 1 and 2 hold. In addition, I also impose the following equilibrium restriction. When indifferent, the decision-maker prefers to compromise with the anti-change SIG and thereby avoid paying the cost to counter her outside lobbying activities.[30]

Pro-change SIG and policy choices

I first study how the pro-change SIG's signal influences the decision-maker's policy choice when it plays a separating strategy on the equilibrium path: $\zeta^{P*}(s) \neq \zeta^{P*}(w)$.[31] First, as in the main text, the pro-change SIG obtains more favorable political decision. Unlike the main text, one needs to compare the expected policy choice.

Lemma 11. *When the pro-change SIG plays a separating strategy on the equilibrium path, $E(b(\zeta^A(\tau), \zeta^P(s))) > E(b(\zeta^A, \zeta^P(w))$ (where the expectation is over A's type).*

The pro-change SIG's strength is positively correlated with policy outcomes whether the anti-change SIG's fund-raising cost is unknown or known (as in the main text). I still find that in a separating equilibrium, when inside lobbying expenditures are strictly positive, they are negatively correlated with policy choice.

Proposition 7. *When the pro-change SIG plays a separating strategy on the equilibrium path, there exists $\underline{c^P} : [0,1]^2 \to (0,(1-p))$ such that:*

1. $l_i^{P*}(s) = 0$ and $l_i^{P*}(w) > 0$ if and only if $c_s^P > \underline{c^P}(b_s, b_w)$;

2. $E(l_o^{P*}(s)) > 0$ and $l_o^{P*}(w) = 0$.

The reasoning is exactly the same as in the main text. A strong pro-change SIG does not want to incur inside lobbying expenditures to reveal that it is willing to engage in costly outside

[30]This restriction does not drive any result except in one knife-edge case: when the anti-change SIG does not separate and the decision-maker is indifferent between a) compromising with a weak anti-change SIG by proposing b_w and choosing $d = 2$ when $l_o^A = 1$ or b) compromising with both types by proposing b_s (see Lemma 14 for more details). Without this equilibrium criterion, Proposition 7 is generically true (i.e., everywhere, but for a set of measure 0 of parameter values).

[31]Lemma 18 shows that a separating strategy is an equilibrium strategy for some parameter values.

lobbying. As such, only a weak type uses inside lobbying expenditures to credibly "plead poverty" and push the decision-maker to compromise. On the equilibrium path, a strong pro-change SIG's total lobbying expenditures (outside plus inside lobbying) are always higher than a weak type's.

Remark 3. *When the pro-change SIG plays a separating strategy on the equilibrium path,* $l_i^{P*}(s) + l_o^{P*}(s) > l_i^{P*}(w) + l_o^{P*}(w)$.

Finally, as in the main text, when the pro-change SIG plays a pooling strategy on the equilibrium path ($\zeta^P(s) = \zeta^P(w)$), it can still incur inside lobbying expenditures. Again, this is driven by the decision-maker's out-of-equilibrium belief (see the main text for more details).

Proposition 8. *There exists a non-empty open set of fund-raising costs such that the pro-change SIG plays a pooling strategy in equilibrium and on the equilibrium path, incurs strictly positive inside lobbying expenditures and never engages in outside lobbying.*

Propositions 7 and 8 indicate that the empirical implications discussed in the main text are unaffected by uncertainty about the anti-SIG's strength. Empirical analyses of pro-change SIG influence focusing exclusively on inside lobbying expenditures are likely to produce wrongly signed and downwardly biased estimates. Using outside lobbying expenditures, however, can help the researcher recover pro-change SIG influence.

Anti-change SIG and policy choices

I now study how uncertainty about the pro-change SIG's strength (and so the possibility that the pro-change SIG can help the decision-maker counter the anti-change SIG's attacks) changes the anti-change SIG's incentives to play a separating strategy. Notice that when the pro-change SIG plays a pooling strategy, the results described in the main text hold. I thus assume that the pro-change SIG plays a separating strategy in what follows. When the decision-maker learns that the pro-change SIG is strong, the expected payoff from proposing $b = 1$ increases (since the cost of defending her proposal is lower). Consequently, there is a risk that the decision-maker does not compromise with a strong anti-change SIG. This reduces a strong anti-change SIG's benefit from differentiation. Therefore, the set of parameter values such that a separating equilibrium (with both SIGs playing a separating strategy) is lower (in the sense of set inclusion) than in the main text.

Lemma 12. *i. When the pro-change SIG plays a pooling strategy on the equilibrium path, the anti-change SIG plays a separating strategy on the equilibrium path if and only if:*

$$\max\left\{1-p-k, \frac{k}{1-p}\right\} \leq b_s \leq (1-p)b_w.$$

ii. When the pro-change SIG plays a separating strategy on the equilibrium path, there exists $\bar{b}_{se} \leq (1-p)b_w$ *such that the anti-change SIG plays a separating strategy on the equilibrium path if and only if:*

$$\max\left\{1-p-k, \frac{k}{1-p}\right\} \leq b_s \leq \bar{b}_{se}.$$

As in the main text, only a strong anti-change SIG incurs inside lobbying expenditures in exchange for a more favorable political decision (closer to the status quo) in expectation.

Proposition 9. *When the anti-change SIG plays a separating strategy on the equilibrium path, equilibrium strategies satisfy:*

1. $l_i^{A*}(s) > 0$ *and* $l_i^{A*}(w) = 0$*;*

2. $E(b^*(\zeta^{A*}(s), \zeta^{P*}) > E(b^*(\zeta^{A*}(w), \zeta^{P*})$ *(where the expectation is over the pro-change SIG's type).*

The next proposition establishes conditions under which both the anti-change and pro-change SIGs play a separating strategy on the equilibrium path and engage in both types of activities. Even though both types of lobbying are *strategic substitutes*, outside lobbying might complement inside lobbying on some issues.

Proposition 10. *There exists a non-empty open set of parameter values such that on the equilibrium path, both SIGs play a separating strategy and the anti-change SIG engages in both inside and outside lobbying.*

When both SIGs compete for influence at the inside lobbying stage, the anti-change SIG's inside lobbying expenditures still tilt political decisions in its favor (Proposition 9). However, inside lobbying expenditures do not guarantee the anti-change SIG can bias policy choice towards its preferred policy due to the possible presence of a strong pro-change SIG. The anti-change SIG might fail to influence the content of a bill and be forced to resort to outside lobbying activities. This reinforces the empirical implications discussed in the main text. Proposition 10 suggests that the attenuation bias of estimates of influence based exclusively on inside lobbying expenditures is even more severe when there is competition for influence.[32] It also highlights that using outside lobbying expenditures can help the researcher identify when the anti-change SIG fails to influence policy even, in particular it plays a separating strategy.

Proofs

I first study when the pro-change SIG's best response is to play a separating strategy: $\zeta^P(w) \neq \zeta^P(s)$. The following lemmas provide the key elements to prove Lemma 11 and Proposition 7.

Lemma 13. *The anti-change and pro-change SIGs play a separating strategy on the equilibrium path only if:* $b(\zeta^A(s), \zeta^P(s)) = 1$ *and* $b(\zeta^A(s), \zeta^P(w)) = b_s$.

Proof. The proof is by contradiction. Suppose $b(\zeta^A(s), \zeta^P(s)) = 1$ and $b(\zeta^A(s), \zeta^P(w)) = 1$. This implies $b(\zeta^A(w), \zeta^P(s)) = 1$ and $b(\zeta^A(w), \zeta^P(w)) = b_w$ or else either the anti-change SIG or the pro-change SIG does not separate. But this implies that $b(\zeta^A(s), \zeta^P(w)) > b(\zeta^A(w), \zeta^P(w))$. A similar logic as in Lemma 8 implies that this cannot be an equilibrium.
Suppose $b(\zeta^A(s), \zeta^P(s)) = b_s$ and $b(\zeta^A(s), \zeta^P(w)) = b_s$. Then it must be that $b(\zeta^A(w), \zeta^P(s)) = b_w$ and $b(\zeta^A(w), \zeta^P(w)) = b_w$ (since $b_w > b_s$ and since Assumption 1 holds). But, in this case, the pro-change SIG does not play a separating strategy given our equilibrium restriction. Hence we have reached a contradiction. □

Notice that Lemma 13 directly implies: $b(\zeta^A(w), \zeta^P(w)) = b_w$.

When the pro-change SIG plays a separating strategy on the equilibrium path, denote $p(l_o^P = 1|\zeta^P(l))$ the probability that a type $l \in \{s, w\}$ pro-change SIG engages in outside lobbying after sending signal $\zeta^P(l)$.

Lemma 14. *The pro-change SIG play a separating strategy on the equilibrium path only if: i)* $p(l_o^P = 1|\zeta^P(s)) > 0$ *and ii)* $p(l_o^P = 1|\zeta^P(w)) = 0$.

[32] When the anti-change SIG does not separate, the decision-maker's policy choice depends on her evaluation of the threat of outside lobbying (q^A) and the pro-change SIG's strategy. The anti-change SIG might incur inside lobbying expenditures on the equilibrium path, but they have no impact on policy choices: Proposition 4 applies. So the attenuation bias identified in the main text is still present.

Proof. Point ii. follows directly from Assumption 2. Point i. is always satisfied when the anti-change SIG plays a separating strategy by Lemma 13. When the anti-change SIG does not separate ($\zeta^A(s) = \zeta^A(w) = \zeta^A$), the proof is by contradiction. Suppose $p(l_o^P = 1|\zeta^P(s)) = 0$. This implies: $E(U^d(b, d^*; l_o^{A*})|\zeta^A, \zeta^P(s)) = E(U^d(b, d^*; l_o^{A*})|\zeta^A, \zeta^P(w))$ since a strong pro-change SIG does not subsidize part of the cost of defending the decision-maker's proposal, which is the only way the pro-change SIG affects the decision-maker's payoff. So it must be that $b^*(\zeta^A, \zeta^P(s)) = b^*(\zeta^A, \zeta^P(w))$ (since we assume that the decision-maker avoids outside lobbying activities when indifferent) and $d^*(\zeta^A, \zeta^P(s), b, l_o^{A*}) = d^*(\zeta^A, \zeta^P(w), b, l_o^{A*})$. But then the pro-change SIG does not play a separating strategy under our equilibrium restriction.[33] $\square$

Denote $Y^e(\zeta^P(l))$, $l \in \{s, w\}$ a type l pro-change SIG's expected policy payoff from revealing its type and $\hat{Y}^e(\zeta^P(-l), l)$ its expected policy payoff from pretending to be type $-l \neq l$ when the pro-change SIG plays a separating strategy on the equilibrium path.

Lemma 15. *The pro-change SIG plays a separating strategy on the equilibrium path only if:* $Y^e(\zeta^P(w)) < Y^e(\zeta^P(s))$

Proof. Since $p(l_o^P = 1|\zeta^P(w)) = 0$ (Lemma 14), $\hat{Y}^e(\zeta^P(w), s) = Y^e(\zeta^P(w))$. Since $p(l_o^P = 1|\zeta^P(s)) > 0$, it must be that: $\hat{Y}^e(\zeta^P(s), w) > Y^e(\zeta^P(s)) - p(l_o^P = 1|\zeta^P(s))c_w^P$ (a weak type never helps as it is a strictly dominated strategy by Assumption 2). The proof now proceeds by contradiction. Suppose $Y^e(\zeta^P(s)) \leq Y^e(\zeta^P(w))$. A separating strategy is an equilibrium strategy only if the incentive compatibility constraints (IC) are satisfied. The strong and weak types' (IC) are, respectively:

$$Y^e(\zeta^P(s)) - p(l_o^P = 1|\zeta^P(s))c_s^P - c_s^P l_i^P(s) \geq Y^e(\zeta^P(w)) - c_s^P l_i^P(w) \tag{19}$$

$$Y^e(\zeta^P(w)) - c_w^P l_i^P(w) \geq \hat{Y}^e(\zeta^P(s), c_w^P) - c_w^P l_i^P(s) \tag{20}$$

The first inequality is never satisfied when $l_i^P(w) = 0$ since $Y^e(\zeta^P(s)) \leq Y^e(\zeta^P(w))$. So $l_i^P(s) = 0$

[33] The assumption that the decision-maker avoids outside lobbying activities when indifferent excludes the following case. Suppose that i) $(1-p)b_s - k \geq 0$, ii) $E(U^d(b_w, d^*; l_o^{A*})|\zeta^A, \zeta^P) = (1 - q^A)b_w + q^A((1-p)b_w - k) = b_s = E(U^d(b_s, d^*; l_o^{A*})|\zeta^A, \zeta^P)$ for $\zeta^P \in \{\zeta^P(s), \zeta^P(w)\}$, and iii) $E(U^d(b_s, d^*; l_o^{A*})|\zeta^A, \zeta^P) \geq E(U^d(1, d^*; l_o^{A*})|\zeta^A, \zeta^P(s)) = 1 - p - k/2$. For these parameter values, the following strategies constitute an equilibrium: $\zeta^P(s) \neq \zeta^P(w)$, $b(\zeta^A, \zeta^P(s)) = b_w$ and $b(\zeta^A, \zeta^P(w)) = b_s$ and $p(l_o^P = 1|\zeta^P(s)) = p(l_o^P = 1|\zeta^P(w)) = 0$. This equilibrium relies strongly on the assumption that the decision-maker chooses b_w (b_s) after observing $\zeta^P(s)$ ($\zeta^P(w)$) when indifferent between both policy choices (which can only happen for a set of parameter values with measure 0). Observe that, in this particular case, a strong the pro-change SIG makes strictly positive signaling expenditures ($l_i^P(s) > 0$ and $l_i^P(w) = 0$).

and $l_i^P(w) > 0$, which imply: $l_i^P(w) = \frac{(Y^e(\zeta^P(w)) - Y^e(\zeta^P(s))) + p(l_o^P = 1|\zeta^P(s))c_s^P}{c_s^P}$ by the Intuitive Criterion. Plugging $l_i^P(w)$ in (20) yields the following necessary condition:

$$[(Y^e(\zeta^P(w)) - \hat{Y}^e(\zeta^P(s), c_w^P)) - p(l_o^P = 1|\zeta^P(s))c_w^P]c_s^P \geq (Y^e(\zeta^P(w)) - Y^e(\zeta^P(s)))c_w^P \tag{21}$$

From the reasoning above, $(Y^e(\zeta^P(w)) - \hat{Y}^e(\zeta^P(s), c_w^P)) - p(l_o^P = 1|\zeta^P(s))c_w^P < (Y^e(\zeta^P(w)) - Y^e(\zeta^P(s)))$. Since $0 < c_s^A < c_w^A$, (21) is never satisfied. Hence, we reached a contradiction. □

Lemma 16. *The pro-change SIG plays a separating strategy on the equilibrium path only if:* $l_i^P(s) = 0$.

Proof. It is sufficient to show that $Y^e(\zeta^P(w)) > \hat{Y}^e(\zeta^P(s), w)$, $\forall \zeta^A$. The Intuitive Criterion then implies $l_i^P(s) = 0$.

Suppose the anti-change SIG plays a separating strategy on the equilibrium path ($\zeta^A(s) \neq \zeta^A(w)$). Lemma 13 implies $b(\zeta^A(s), \zeta^P(w)) = b_s$ and $b(\zeta^A(w), \zeta^P(w)) = b_w$. So $Y^e(\zeta^P(w)) = q^A b_s + (1-q^A)b_w$. By Lemma 13 again, $b(\zeta^A(s), \zeta^P(s)) = 1$ and $b(\zeta^A(w), \zeta^P(s)) \in \{b_w, 1\}$. As the decision-maker chooses $d^*(\zeta^P(s), \zeta^A(s), 1, 1) = 1$, it must be that $\hat{Y}^e(\zeta^P(s), w) \leq (1-q^A)b_w < Y^e(\zeta^P(w))$.

Suppose the anti-change SIG does not separate ($\zeta^A(s) = \zeta^A(w) = \zeta^A$). By Assumption 1 (i.e., $(1-p)b_w < c_w^P$) and Lemma 15, it must be that $b(\zeta^A, \zeta^P(s)) = 1$ (otherwise, $p(l_o^P = 1|\zeta^P(s)) = 0$). This implies that $\hat{Y}^e(\zeta^P(s), c_w^P) = 0$ since $d^*(\zeta^P(s), \zeta^A(s), 1, 1) = 1$ and $l_o^{P*}(\cdot; w) = 0$ so $y = 0$ with probability 1. In turn, $Y^e(\zeta^P(w)) = (1 - q^A p)b_w > 0$ if $b(\zeta^A, \zeta^P(w)) = b_w$ since for all $\zeta^P \in \{\zeta^P(w), \zeta^P(s)\}$, $l^{A*}(b_w, \zeta^P; \tau) = 1$ if and only if $\tau = s$, $d^*(\zeta^P(s), \zeta^A(s), b_w, 1) = 2$ by Assumptions 2 and 1), and the bill is passed with probability $1-p$. Further, $Y^e(\zeta^P(w)) = b_s > 0$ if $b(\zeta^A, \zeta^P(w)) = b_s$ as it can be checked that this would occur only if $(1-p)b_s - k \geq 0$ so $l^{A*}(b_w, \zeta^P; \tau) = 0$ for all τ. Putting the result together, the claim holds. □

Lemma 17. *There exists a non-empty open set of anti-change SIG's lobbying costs such that a separating strategy is a best response for the pro-change SIG. Further, there exists* $\underline{c^P} : [0,1]^2 \rightarrow (0, (1-p))$ *and* $\overline{c^P} : [0,1]^3 \rightarrow (\underline{c^P}(b_s, b_w), (1-p))$ *such that* $l_i^P(w) = 0$ $\forall c_s^P \in (0, \underline{c^P}(b_s, b_w)]$ *and* $l_i^P(w) > 0$, $\forall c_s^P \in (\underline{c^P}(b_s, b_w), \overline{c^P}(b_s, b_w, c_w^P)]$.

Proof. Whenever $Y^e(\zeta^P(s)) - p(l_o^P = 1|\zeta^P(s))c_s^P \geq Y^e(\zeta^P(w)) \Leftrightarrow c_s^P \leq \frac{Y^e(\zeta^P(s)) - Y^e(\zeta^P(w))}{p(l_o^P = 1|\zeta^P(s))} \equiv \underline{c^P}(b_s, b_w)$, a strong pro-change SIG never pretends to be weak and by the Intuitive Criterion,

$l_i^P(w) = 0$.

Suppose $c_s^P > \underline{c^P}(b_s, b_w)$. To satisfy the strong type's (IC), it is necessary that $l_i^P(w)$ satisfies (by the Intuitive Criterion): $Y^e(\zeta^P(s)) - p(l_o^P = 1|\zeta^P(s))c_s^P = Y^e(\zeta^P(w)) - c_s^P l_i^P(w)$. In turn, to satisfy a weak pro-change SIG's (IC) (see 20), it is necessary that: $c_s^P \leq c_w^P \frac{(Y^e(\zeta^P(s)) - Y^e(\zeta^P(w)))}{p(l_o^P = 1|\zeta^P(s))c_w^P - (Y^e(\zeta^P(w)) - \hat{Y}^e(\zeta^P(s), w)} \equiv \overline{c^P}(b_s, b_w, c_w^P)$.

It is easy to check that $\overline{c^P}(b_s, b_w, c_w^P) > \underline{c^P}(b_s, b_w)$. Further, $\overline{c^P}(b_s, b_s, c_w^P) < (1-p)$ whenever $c_w^P(Y^e(\zeta^P(s)) - Y^e(\zeta^P(w)) - (1-p)p(l_o^P = 1|\zeta^P(s))) < (1-p)((\hat{Y}^e(\zeta^P(s), w) - Y^e(\zeta^P(w)))$. Given $c_s^P < (1-p) < c_w^P$ and $c_s^P > \underline{c^P}(b_s, b_w)$, it is enough to show that: $Y^e(\zeta^P(s)) - (1-p)p(l_o^P = 1|\zeta^P(s)) \leq \hat{Y}^e(\zeta^P(s), w)$, which holds since $Y^e(\zeta^P(s)) \leq (1-p)p(l_o^P = 1|\zeta^P(s)) + \hat{Y}^e(\zeta^P(s), w)$. Hence, the claim is verified.

To see the lemma holds for an open set of parameter values, suppose $b_w < 1 - p - k/2$ and $b_s > \max\{1 - p - k, k/(1-p)\}$ so $b(\zeta^A, \zeta^P(s)) = 1$, $\forall \zeta^A$ and $b(\zeta^A, \zeta^P(w)) < 1$, $\forall \zeta^A$. It is easy to check that both a strong and weak the pro-change SIG's (IC) can be satisfied and a separating strategy can be a best response to other players' actions (notice uniqueness is not satisfied) for some c_s^P and c_w^P. The results hold for other combination of parameter values (see Lemma 18 for more details). $\square$

I now show that a separating strategy is the pro-change SIG's best response to other players' actions for some parameter values

Lemma 18. *i. When the anti-change SIG does not separate, there exists a unique* $\Xi \subset [0,1]^2 \times (0, (1-p))$ *such that a pro-change SIG's separating strategy is a best response to other players' actions if and only if* $(b_s, b_w, c_s^P) \in \Xi$*;*

ii. When the anti-change SIG separates, there exists a unique $\bar{c}_{se}^P : [0,1]^2 \to (0, (1-p))$ *such that the pro-change SIG's separating strategy is a best response to other players' actions if and only if:* $\max\{k/(1-p), 1-p-k\} \leq b_s \leq 1 - p - k/2$ [34] *and* $0 \leq c_s^P \leq \bar{c}_{se}^P(b_s, b_w)$*.*

Proof. Suppose the anti-change SIG does not separate. We first prove necessity. Under Assumption 2, a strong pro-change SIG helps if and only if the decision-maker chooses $b = b_w$. Therefore, the pro-change SIG separates only if the decision-maker's equilibrium policy choice satisfies: $b^*(\zeta^P(s), \zeta^A) =$

[34] Note this condition might not be binding, see Proposition 12.

1.[35] We can therefore define the following sets (which differ in the decision-maker's best response after signal $\zeta^P(w)$):

$$\Upsilon_1 := \left\{\begin{array}{l} (b_s, b_w, c_s^P) \in [0,1]^2 \times (0,(1-p)) \ s.t. \\ i.\ E[u^D(b_s, d^*(\zeta^P(w), \zeta^A, b_s, l_o^A))|\zeta^A, \zeta^P(w)] \\ \quad \geq \max\{E[u^D(b_w, d^*(\zeta^P(w), \zeta^A, b_w, l_o^A))|\zeta^A, \zeta^P(w)], E[u^D(1, d^*(\zeta^P(w), \zeta^A, 1, l_o^A))|\zeta^A, \zeta^P(w)]\} \\ ii.\ E[u^D(1, d^*(\zeta^P(s), \zeta^A, 1, l_o^A))|\zeta^A, \zeta^P(s)] \\ \quad > \max\{E[u^D(b_s, d^*(\zeta^P(s), \zeta^A, b_s, l_o^A))|\zeta^A, \zeta^P(s)], E[u^D(b_w, d^*(\zeta^P(s), \zeta^A, b_w, l_o^A))|\zeta^A, \zeta^P(s)]\} \end{array}\right\}$$

$$\Upsilon_2 := \left\{\begin{array}{l} (b_s, b_w, c_s^P) \in [0,1]^2 \times (0,(1-p)) \ s.t. \\ i.\ E[u^D(b_w, d^*(\zeta^P(w), \zeta^A, b_w, l_o^A))|\zeta^A, \zeta^P(w)] \\ \quad > \max\{E[u^D(b_s, d^*(\zeta^P(w), \zeta^A, b_s, l_o^A))|\zeta^A, \zeta^P(w)], E[u^D(1, d^*(\zeta^P(w), \zeta^A, 1, l_o^A))|\zeta^A, \zeta^P(w)]\} \\ ii.\ E[u^D(1, d^*(\zeta^P(s), \zeta^A, 1, l_o^A))|\zeta^A, \zeta^P(s)] \\ \quad > \max\{E[u^D(b_s, d^*(\zeta^P(s), \zeta^A, b_s, l_o^A))|\zeta^A, \zeta^P(s)], E[u^D(b_w, d^*(\zeta^P(s), \zeta^A, b_w, l_o^A))|\zeta^A, \zeta^P(s)]\} \end{array}\right\}$$

As, by assumption, the decision-maker compromises when indifferent, $\forall (b_s, b_w, c_s^P) \in \Upsilon_1$, the decision-maker's best response satisfies: $b(\zeta^A, \zeta^P(w)) = b_s$ (and $d^*(\zeta^P(s), \zeta^A, b_s, 1) = 2$) and $b(\zeta^A, \zeta^P(s)) = 1$ (and $d^*(\zeta^P(s), \zeta^A, 1, 1) = 1$ by Lemma 14). $\forall (b_s, b_w, c_s^P) \in \Upsilon_2$, the decision-maker's best response satisfies: $b(\zeta^A, \zeta^P(w)) = b_w$ (and $d^*(\zeta^P(s), \zeta^A, b_w, 1) = 2$) and $b(\zeta^A, \zeta^P(s)) = 1$ (and $d^*(\zeta^P(s), \zeta^A, 1, 1) = 1$).

By Lemma 17, the pro-change SIG's (IC) are satisfied only if:

$$l_i^P(w) = \max\left\{\frac{(Y^e(\zeta^P(w)) - Y^e(\zeta^P(s))) + p(l_o^P = 1|\zeta^P(s))c_s^P}{c_s^P}, 0\right\}$$

Define $\vartheta(c_w^P) = c_w^P \frac{(Y^e(\zeta^P(s)) - Y^e(\zeta^P(w)))}{c_w^P p(l_o^P = 1|\zeta^P(s)) + (\hat{Y}^e(\zeta^P(s), c_w^P) - Y^e(\zeta^P(w)))}$. By Lemma 17, $\vartheta(c_w^P) \in (0,1)$. It is easy to check that $\vartheta'(c_w^P) < 0$. Furthermore, $\forall (b_s, b_w, c_w^P) \in \Upsilon_1$, $\vartheta(c_w^P) < (1-p)$. To see that, note that

[35] If $b^*(\zeta^P(s), \zeta^A) = b_w$ then $b^*(\zeta^P(w), \zeta^A) = b_w$ since $l_o^{P*}(b_w, \cdot; s) = l_o^{P*}(b_w, \cdot; w) = 0$. Hence, the pro-change SIG's signal has no effect on the decision-maker's equilibrium strategy and so it does not play an equilibrium strategy under the equilibrium restriction.

$\forall(b_s, b_w, c_w^P) \in \Upsilon_1$, $Y^e(\zeta^P(s)) = (1-p)$, $Y^e(\zeta^P(w)) = b_s$, $\hat{Y}^e(\zeta^P(s), c_w^P) = 0$, and $p(l_o^P = 1|\zeta^P(s)) = 1$. Simple computations yield: $\vartheta((1-p)) = (1-p)$. So $\vartheta(c_w^P) < (1-p)$.

Define $\Lambda := [0,1]^2 \times \left[0, c_w^P \frac{(Y^e(\zeta^P(s)) - Y^e(\zeta^P(w)))}{c_w^P p(l_o^P=1|\zeta^P(s)) + (\hat{Y}_W^e(b_s,b_w) - Y^e(\zeta^P(w)))}\right]$ and $\Xi := \Upsilon_1 \cup \Upsilon_2 \cap \Lambda$. the pro-change SIG plays a separating strategy only if $(b_s, b_w, c_s^P) \in \Xi$. The sets $\Upsilon_k,\ k \in \{1,2\}$ defines the decision-maker's best response at the policy stage after observing $\zeta^P(s)$ and $\zeta^P(w)$. The decision-maker's policy choice satisfies the necessary conditions described in Lemmas 13-16. By definition of the set Ξ and Lemma 17, a separating strategy is incentive compatible for the pro-change SIG.

We now show sufficiency. Suppose $(b_s, b_w, c_s^P) \in \Upsilon_1 \cap \Lambda$ and consider the following assessment. i) A weak (strong) the pro-change SIG sends signal $\zeta^P(w) = (w, l_i^P(w))$ ($\zeta^P(s) = (s, 0)$), with $l_i^P(w)$ defined above. ii) The decision-maker's belief is: $\mu^P(\zeta^P(s)) = 1$, and 0 otherwise. iii) The decision-maker's policy choice is: $b(\zeta^A, \zeta^P) = 1,\ \forall\zeta^P \neq \zeta^P(w)$ and $b(\zeta^A, \zeta^P(w)) = b_s$. The remaining strategies down the game tree satisfy the equilibrium definition (see above).

It can be checked that beliefs satisfy Bayes' rule, the decision-maker's policy choice is a best response given her belief, the pro-change SIG's incentive compatibility constraints hold. Hence, $(b_s, b_w, c_s^P) \in \Upsilon_1 \cap \Lambda$ is a sufficient condition for the pro-change SIG to play a separating strategy. Note that we have not proven that it is sufficient for the pro-change SIG to separate in a PBE since we have assumed, but not shown that the anti-change SIG does not separate.

A similar reasoning holds for $(b_s, b_w, c_s^P) \in \Upsilon_2 \cap \Lambda$.

Consider the case when the anti-change SIG separates. We just prove necessity. Sufficiency follows from a similar argument as above.

By Lemma 13, a necessary condition is: $b(\zeta^A(s), \zeta^P(s)) = 1$ and $b(\zeta^A(s), \zeta^P(w)) = b_s$. This is equivalent to: $\max\{1-p-k, k/(1-p)\} \leq b_s \leq 1-p-k/2$. The first inequality follows from Lemma 4, the second inequality from Lemma 2.

Using a similar reasoning as above, it can be checked that the pro-change SIG incentive compatibility constraints are satisfied only if: $0 < c_s^P \leq \bar{c}_{se}^P$, where $\bar{c}_{se}^P(b_s, b_w) := c_w^P \frac{(Y^e(\zeta^P(s)) - Y^e(\zeta^P(w)))}{c_w^P p(l_o^P=1|\zeta^P(s)) + (\hat{Y}^e(\zeta^P(s), c_w^P) - Y^e(\zeta^P(w)))}$, with:

$$Y^e(\zeta^P(s)) = \begin{cases} q^A(1-p) + (1-q^A)b_w \text{ if } b_w \geq 1-p-k/2 \\ 1-p \text{ if } b_w < 1-p-k/2 \end{cases}$$

$$Y^e(\zeta^P(w)) = q^A b_s + (1-q^A)b_w$$

$$\hat{Y}^e(\zeta^P(s), c_w^P) = \begin{cases} (1-q^A)b_w \text{ if } b_w \geq 1-p-k/2 \\ 0 \text{ if } b_w < 1-p-k/2 \end{cases}$$

$$p(l_o^P = 1|\zeta^P(s)) = \begin{cases} q^A \text{ if } b_w \geq 1-p-k/2 \\ 1 \text{ if } b_w < 1-p-k/2 \end{cases}$$

When both SIGs separate, the decision-maker's best response after observing $\zeta^A(w)$ and $\zeta^P(s)$ is: $b(\zeta^A(w), \zeta^P(s)) = b_w$ when $b_w \geq 1-p-k/2$, and $b(\zeta^A(w), \zeta^P(s)) = 1$, otherwise (with $g^*(\zeta^P(s), \zeta^P(w), 1, 1) = 1$).

Consequently, $b_s \in \left[\max\left\{\frac{k}{1-p}, 1-p-k\right\}, 1-p-\frac{k}{2}\right]$ and $c_s^P \in (0, \bar{c}_{se}^P(b_s, b_w)]$ are necessary conditions for the pro-change SIG to separate when the anti-change SIG separates as claimed. □

Proof of Lemma 11. The proof follows directly from Lemmas 15-18. □

Proof of Proposition 7. Point i. follows directly from Lemmas 16 and 17. Point ii. follows from Lemma 14. □

Proof of Remark 3. The econometrician only observes inside lobbying expenditures ($l_i^P(w)$) and outside lobbying expenditures (normalized to 1). The observed expenditures are then (in expectation): $l_i^P(w)$ and $p(l_o^P = 1|\zeta^P(s))$. Using Lemma 17, we have that $l_i^P(w) = p(l_o^P = 1|\zeta^P(s)) - \frac{(Y^e(\zeta^P(s)) - Y^e(\zeta^P(w)))}{c_s^P} < p(l_o^P = 1|\zeta^P(s))$ since $Y^e(\zeta^P(s)) > Y^e(\zeta^P(w))$ by Lemma 15. □

Proof of Proposition 8. Under Assumption 2.ii ($q^P \leq 1/2$), the decision-maker never asks for help ($d(\cdot) = 1$) when she does not know the pro-change SIG's type. Hence, the pro-change SIG never engages in outside lobbying in a pooling equilibrium.

Suppose $\max\{b_s, (1-pq^A)b_w - q^A k\} < 1-p-k$ so that on the equilibrium path, the anti-change

SIG does not separate (see Lemma 12), the pro-change SIG does not separate (see Lemma 18),[36] and the decision-maker chooses $b(\zeta^A, \zeta^P) = 1$.

Consider the following belief structure for the decision-maker: $\mu^P(c_s^P|\zeta^P) = 1$ if $\zeta^P \in \{\hat{\tau}, l_i^P\}$, $\forall \hat{\tau} \in \{s, w\}$, $l_i^P < \widehat{l_i^P}$ for some $\widehat{l_i^P} > 0$, and $\mu(\zeta^P) = q^P$, otherwise. Given this belief structure, for given ζ^A, the decision-maker's best response is: $(b(\zeta^A, \zeta^P) = 1, d(\zeta^P, \zeta^A, 1, 1) = 1)$, $\forall \zeta^P \in \{s, w\} \times [0, \widehat{l_i^P})$, ζ^A, whereas $(b(\zeta^A, \zeta^P) = 1, d(\zeta^P, \zeta^A, 1, 1) = 2)$, $\forall \zeta^P$ satisfying $\zeta^P \in \{s, w\} \times [\widehat{l_i^P}, \infty)$. Using the weak and strong the pro-change SIG's incentive compatibility constraint (and a similar reasoning as in the proof of Proposition 2), $\forall \widehat{l_i^P} \leq \frac{1-p}{c_w^P}$, the pro-change SIG's (IC) are satisfied and a pooling equilibrium with strictly positive inside lobbying expenditures by the pro-change SIG exists.

For completeness, I show that Proposition 8 does not require that the anti-change SIG plays a pooling strategy on the equilibrium path. Suppose $1 - p - k/2 > b_w$ and $1 - p - k < b_s < (1 - p)b_w$. In this case, an equilibrium when the anti-change SIG separates exists (whether the pro-change SIG separates or not on the equilibrium path, see Lemma 12 below for more details). Suppose $c_s^P > \frac{c_w^P[(1-p)-b^e]}{c_w^P - b^e}$, with $b^e := q^A b_s + (1 - q^A)b_w$ so the pro-change SIG does not play a separating strategy on the equilibrium path when the anti-change SIG separates (Lemma 18).

Consider the following belief structure for the decision-maker: $\mu^P(\zeta^P) = 1$ $\forall \zeta^P = (\hat{\tau}, l_i^P)$ satisfying $(\hat{\tau}, l_i^P) \in \{s, w\} \times [0, \widehat{l_i^P})$ and $\mu^P(\zeta^P) = q^P$, otherwise. Given this belief structure, the decision-maker's best response is: $(b(\zeta^A(s), \zeta^P) = 1, d(\zeta^P, \zeta^A, 1, 1) = 1)$, $\forall \zeta^P \in \{s, w\} \times [0, \widehat{l_i^P})$. The weak and strong types' (IC) are, respectively:

$$0 \leq b^e - c_w^P \widehat{l_i^P}$$

$$(1 - p) - c_s^P \leq b^e - c_s^P \widehat{l_i^P}$$

It can be checked that $\forall \widehat{l_i^P} \leq \frac{b^e}{c_w^P}$, the two (IC) constraints are satisfied (given $c_s^P > \frac{c_w^P[(1-p)-b^e]}{c_w^P - b^e}$) and a pooling equilibrium with positive inside lobbying expenditures by the pro-change SIG exists. □

In what follows, I study conditions under which a separating strategy ($\zeta^A(s) \neq \zeta^A(w)$) is a best

[36]If the pro-change SIG were to play a separating strategy, given that the anti-change SIG does not separate, the decision-maker chooses $(b = 1, d = 2)$ when he know he works with a weak the pro-change SIG and $(b = 1, d = 1)$ when he knows he works with a strong pro-change SIG. Obviously, there is no benefit and only a cost (need to engage in costly outside lobbying) for a strong the pro-change SIG to reveal its type. And a separating strategy is not incentive compatible.

response for the anti-change SIG when there is uncertainty about the pro-change SIG's type. As before, I assume that all players play their best response down the game tree.

Proof of Lemma 12. Point i. of Lemma 12 follows directly from the proof of Lemma 4. We thus focus on point ii. By Lemma 4, a necessary condition for the anti-change SIG to play a separating strategy on the equilibrium path is that the decision-maker prefers compromising with a strong type and is credible doing so by Assumption 1. Hence, a necessary condition is (still) $b_s > \max\left\{1-p-k, \frac{k}{1-p}\right\}$.

A separating strategy must also be incentive compatible for the anti-change SIG. By Lemma 13, $b(\zeta^A(s), \zeta^P(s)) = 1$ and $b(\zeta^A(s), \zeta^P(w)) = b_s$ when both SIGs separate. Further, $b(\zeta^A(w), \zeta^P(w)) = b_w$. So we only need to consider two cases: case a): $b(\zeta^A(w), \zeta^P(s)) = 1$; case b): $b(\zeta^A(w), \zeta^P(s)) = b_w$.

We first consider case a). By a similar logic as in Lemma 9, it must be that $l_i^A(w) = 0$ and $l_i^A(s) > 0$. The weak the anti-change SIG's (IC) is:

$$q^P(-(1-p) - c_w^A) + (1-q^P)(-b_w) \geq q^P(-(1-p) - c_w^A) + (1-q^P)(-b_s) - c_W^A l_i^A(s)$$

When the pro-change SIG is strong, the decision-maker chooses $b(\zeta^A(w), \zeta^P(s)) = b(\zeta^A(s), \zeta^P(s)) = 1$ so a weak anti-change SIG always engages in outside lobbying. When the pro-change SIG is weak, the decision-maker chooses b_w when the anti-change SIG reveals its type, whereas she chooses b_s when the weak anti-change SIG pretends to be strong. By the Intuitive Criterion, we obtain: $l_i^A(s) = (1-q^P)\frac{(b_w - b_s)}{c_w^A}$.

Now consider the strong type's (IC):

$$q^P(-(1-p) - c_s^A) + (1-q^P)(-b_s) - c_S^A l_i^A(s) \geq q^P(-(1-p) - c_s^A) + (1-q^P)(-(1-p)b_w - c_s^A)$$

Substituting for $l_i^A(s)$ and rearranging, we get: $(1-p)(b_w - b_s) \geq c_S^A \frac{(b_w - b_s)}{c_w^A}$. This is exactly the same condition as in the proof of Lemma 4. Therefore, a necessary condition for a separating strategy to be incentive compatible is: $b_s \leq (1-p)b_w$.

We now consider case b). As before, $l_i^A(w) = 0$ and $l_i^A(s) > 0$. The weak the anti-change SIG's

(IC) is:

$$-b_w \geq q^P(-(1-p) - c_w^A) + (1-q^P)(-b_s) - c_w^A l_i^A(s)$$

When a weak the anti-change SIG reveals her type, the decision-maker chooses b_w ($b(\zeta^A(w), \zeta^P(l)) = b_w,\ l \in \{s, w\}$). When it pretends to be strong, it obtains $b = b_s$ when the pro-change SIG reveals to be weak, but needs to engage in outside lobbying expenditures when the pro-change SIG is strong as $b(\zeta^A(s), \zeta^P(s)) = 1$. By the Intuitive Criterion, we obtain: $l_i^A(s) = \frac{b_w - q^P[(1-p)-c_w^A]-(1-q^P)b_s}{c_w^A}$. I claim this last term is positive and verify the claim afterwards.

A strong type knows the decision-maker chooses $b = b_s$ only when the pro-change SIG is weak. Its (IC) is thus (using the definition of $l_i^A(s)$ and $b_w = \frac{c_w^A}{c_S^A} b_s$):

$$-q^P((1-p) + c_s^A) - (1-q^P)b_s - c_s^A l_i^A(s) \geq -(1-p)b_w - c_s^A$$
$$\Leftrightarrow b_s \leq \frac{(1-p)b_w - q^P(1-p)}{1-q^P}$$

Notice that when $b_s = \frac{(1-p)b_w - q^P(1-p)}{1-q^P}$, then $l_i^A(s) = 1 - q^P > 0$. Since $l_i^A(s)$ is decreasing with b_s, we have $l_i^A(s) > 0$ as claimed.
Note also that $\frac{(1-p)b_w - q^P(1-p)}{1-q^P} < (1-p)b_w$ since $b_w < 1$. Denote $b_{se} = (1-p)b_w$ or $\frac{(1-p)b_w - q^P(1-p)}{1-q^P}$ depending on whether the decision-maker's best response satisfies (respectively) case a) or b). The anti-change SIG separates only if $\max\{1-p-k, k/(1-p)\} \leq b_s \leq b_{se}$ and $b_{se} \leq (1-p)b_w$ as claimed.

Sufficiency follows from the usual argument. □

Proof of Proposition 10. Suppose an equilibrium with both SIGs separating exists. By Lemmas 12 and 13, a strong anti-change SIG incurs inside lobbying expenditures and engages in outside lobbying when the pro-change SIG is strong. We thus just need to show that such a separating equilibrium exists. Consider parameter values satisfying the following conditions: $b_w \leq 1 - p - k/2$ (so $b(\zeta^A(w), \zeta^P(s)) = 1$), $\min\{1-p-k, k/(1-p)\} \leq b_s \leq (1-p)b_w$ (so a separating strategy is incentive compatible for the anti-change SIG), and $c_s^P \leq c_w^P \frac{(1-p) - q^A b_s - (1-q^A)b_w}{c_w^P - [q^A b_s + (1-q^A)b_w]}$ (so a separating strategy is incentive compatible for the pro-change SIG). By Lemma 18 and Proposition 12, under these parameter values there exist an equilibrium when both SIGs play a separating strategy. Notice that the result holds for other sets of parameter values as well. □

D Micro-founding the impact of outside lobbying activities

In this section, I micro-found the influence of outside lobbying on public opinion. To do so, I use a simplified version of the War of Information (Gül and Pesendorfer, 2012). To simplify the exposition, I do not include the pro-change SIG. Further, since the anti-change SIG's type announcement is not credible without inside lobbying expenditures (see Lemma 9), I assume without loss of generality that the anti-change SIG's signal takes the form of inside lobbying expenditures (i.e., $\zeta^A := l_i^A$).

I consider a three-player game with the decision-maker, the anti-change SIG, and a representative voter. There are two states of the world: $\omega \in \{I, L\}$. In state I, the decision-maker's legislative proposal *increases* the voter's utility compared to the status quo. In state L, the decision-maker's proposal *lowers* the voter's utility. No player knows the state of the world at the beginning of the game. However, it is common knowledge that players' common prior is biased in favor of the decision-maker's proposal (see Assumption 3 below). This reflects the fact that the decision-maker has won an election.

As in Gül and Pesendorfer's (2012) war of information, the anti-change SIG's outside lobbying activities and the decision-maker's response to them reveal information to the representative voter. The voter receives no, one, or two signals of the state of the world depending on other players' actions. The voter then sides with the decision-maker or the SIG according to her belief regarding ω. Providing information to the voter is costly for the anti-change SIG and the decision-maker.[37]

To summarize, the timing of the augmented game is:

0. Nature draws the SIG's type ($\tau^A \in \{s, w\}$)

1. After observing its type, the SIG sends a signal: $l_i^A \geq 0$

2. The decision-maker chooses the content of the bill: $b \in [0, 1]$

3. The SIG decides whether to start a war of information: $l_o^A \in \{0, 1\}$

   If there is no war, the representative voter sides with the decision-maker or SIG according to her prior

[37] A major difference with Gül and Pesendorfer (2012) is that the anti-change SIG and the decision-maker can provide information only once to the voter.

4. If there is a conflict, the voter receives a signal of the state of the world: $w_1 \in \{i, l\}$. The decision-maker observes the signal

5. The decision-maker decides whether to continue the war of information: $d \in \{0, 1\}$

   i. If she stops the war of information ($d = 0$), the voter sides with the decision-maker or SIG according to her posterior after observing w_1;

   ii. If he continues the war of information ($d = 1$), the representative voter receives a second signal $w_2 \in \{i, l\}$;
   the voter sides with the decision-maker or the SIG according to her posterior after observing w_1 and w_2.

The outcome of the game y depends on the voter's choice. When the voter sides with the decision-maker, her bill passes: $y = b$. When the voter sides with the anti-change SIG, the bill fails and the status quo remains in place: $y = 0$.

In the context of a war of information, the representative voter receives a signal $w_t = i$ with probability $\rho > 1/2$ when the state of the world is I and with probability $1 - \rho$ when the state of the world is L, $t \in \{1, 2\}$. The voter receives a signal $w_t = l$ with probability $1 - \rho$ when the state of the world is I and with probability ρ when the state of the world is L, $t \in \{1, 2\}$. Signals are independent conditional on the state of the world. Denote π_0 players' common prior that the state of the world is L. $\pi_1(w_1)$ is the voter's and the decision-maker's posterior that the state is L after observing signal $w_1 \in \{i, l\}$. Finally, $\pi_2(w_1, w_2)$ is the voter's posterior that the state is L after observing w_1 and w_2 (when this occurs on the equilibrium path). As the equilibrium concept is PBE, the voter's posterior satisfies Bayes' Rule.

The decision-maker and the anti-change SIG's utility functions are respectively:

$$u^D(y, d) = y - c^D d \tag{22}$$

$$u^A_{si}(y, l^A_i, l^A_o; \tau) = -y - c^A_\tau (l^A_i + l^A_o) \tag{23}$$

The parameter $c^D > 0$ captures the cost of sending a signal to the voter for the decision-maker.

The utility function of the representative voter is:

$$u_v(y;\omega) = h(\omega)v(y) \tag{24}$$

with $v(\cdot)$ continuous and strictly increasing, and $h(\omega)$ a function with the following properties: $h(I) > 0$ and $h(L) < 0$. For example, the function $h(\cdot)$ can be of the following form:

$$h(\omega) = \begin{cases} 1 & \text{if } \omega = I \\ -1 & \text{if } \omega = L \end{cases}$$

To make the problem interesting, I impose some restrictions on the common prior and informativeness of the signals received by the voter:

Assumption 3. *ρ and π_0 satisfy:*

$$\pi_1(l) = \frac{\rho\pi_0}{\rho\pi_0 + (1-\rho)(1-\pi_0)} > \frac{-h(L)}{h(I) - h(L)} > \pi_0$$

The first inequality in Assumption 3 states that, after receiving a signal $w_1 = l$, the voter prefers the status quo to any bill b and so always side with the anti-change SIG absent additional information. This assumption is satisfied when the prior is not too biased in the decision-maker's direction (π_0 is not too low) and the signal is sufficiently informative (ρ is sufficiently high). When this inequality does not hold, the anti-change SIG never engages in outside lobbying and the decision-maker always chooses $b = 1$. The second inequality implies that absent additional information, the voter sides with the decision-maker.

Notice that if the anti-change SIG engages in outside lobbying (starts a war of information) and the voter receives signal $w_1 = i$, the decision-maker never continues the war of information. Indeed, $\pi_1(i) < \pi_0 < \frac{-h(L)}{h(I)-h(L)}$ so the voter always sides with the decision-maker absent additional information. Providing information to the voter is costly to the decision-maker and does not change the voter's decision.[38]

The following notations are useful for the analysis of this set-up. First, denote $p_0(NF) := \pi_0\rho + (1-\pi_0)(1-\rho)$ the probability that the voter receives a signal $w_1 = l$. This is also the ex-

[38] We have $\pi_2(i,i) < \pi_2(i,l) = \pi_0 < \frac{-h(L)}{h(I)-h(L)}$ so the voter always sides with the decision-maker.

ante probability that the anti-change SIG wins the war of information if the decision-maker stops the war of information after signal $w_1 = l$. The ex ante expected cost of a war of information for the decision-maker is: $k = p_0(NF)c_g$. Denote $p_0(F) := \pi_0\rho^2 + (1-\pi_0)(1-\rho)^2$ the ex ante probability (before the start of the war of information) that the anti-change SIG wins the war of information when the decision-maker continues the war of information after signal $w_1 = l$. Lastly, denote $p_1(l) := \pi_1(l)\rho + (1-\pi_1(l))(1-\rho)$ the probability that the anti-change SIG wins the war of information given that the voter has received a signal $w_1 = l$. In what follows, the following assumption holds:

Assumption 4. *Denote* $b_w = \frac{c_w^A}{p_0(F)}$. b_w *satisfies:* $b_w > \max\left\{\frac{k}{1-p_1(l)}, 1 - p_0(F) - k\right\}$

Assumption 4 is the equivalent to Assumption 1 in the context of the war of information. Importantly, a separating equilibrium can exist when Assumption 4 does not hold (since unlike in the main text, the probability that the bill fails is not 1 when the anti-change SIG engages in outside lobbying anticipating that the decision-maker does not respond to the anti-change SIG's attack). I simply impose Assumption 4 to facilitate comparison with the set-up discussed in the main text.

The following proposition establishes that like in the main text, a separating equilibrium does not always exist when the anti-change SIG's outside lobbying activities take the form of a war of information.

Proposition 11. *Denote:* $b_s^F = \frac{c_s^A}{p_0(F)}$ *and* $b_s^{NF} = \frac{c_s^A}{p_0(NF)}$. *A separating equilibrium exists if and only if:*

1. $1 - p_0(F) - k \leq b_s^F \leq (1 - p_0(F))b_w$ *if* $b_s^F \geq \frac{c^D}{1-p_1(l)}$
2. $1 - p_0(F) - k \leq b_s^{NF} \leq (1 - p_0(F))b_w$ *if* $b_s^F < \frac{c^D}{1-p_1(l)}$

Proof. I first show that the conditions are necessary.

First, under Assumption 4, the decision-maker always credibly compromises with a weak anti-change SIG by proposing $b_w = \frac{c_w^A}{p_0(F)}$. The decision-maker can also choose a radical bill and gets in expectation $1 - p_0(F) - k$.

When it comes to the strong SIG, the decision-maker does not necessarily decide to continue the war of information after a negative signal $w_1 = l$. When the decision-maker proposes $b = b_s^F$, the anti-change SIG is indifferent between engaging in outside lobbying activities and not starting a

war of information if and only if the decision-maker decides to continue the war of information after $w_1 = l$. That is, if and only if $b_s^F \geq \frac{c^D}{1-p_1(l)}$. In this case, a similar reasoning as in Lemma 4, the decision-maker's best response after learning the anti-change SIG is strong must be $b = b_s^F$. That is, a necessary condition is: $1 - p_0(F) - k \leq b_s^F$. Further, the anti-change SIG's (IC) constraints must be satisfied which is the case only if $b_s^F \leq (1 - p_0(F))b_w$. Hence, the conditions detailed in point 1. are necessary whenever $b_s^F \geq c^D/(1 - p_1(l))$.
When $b_s^F < c^D/(1 - p_1(l))$, the decision-maker does not continue the war of information after signal $w_1 = l$. A strong anti-change SIG is not indifferent between no outside lobbying and starting a war of information when the decision-maker proposes $b = b_s^F$. To guarantee indifference, the decision-maker must propose $b = b_s^{NF}$. The reasoning then proceeds as in the previous paragraph simply replacing b_s^F by b_s^{NF}.
Sufficiency proceeds by the usual argument. □

The main result described in the main text still holds when outside lobbying activities take the form of a war of information. A separating equilibrium exists only under some parameter values (a strong anti-change SIG's lobbying cost must be intermediate).