The Three A's of Government Formation: Appointment, Allocation, and Assignment

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Abstract. How does the Prime Minister organize her government so that she can implement her policy agenda? In our model, a popularly elected Prime Minister appoints individuals to her cabinet, allocates their portfolios, and assigns their policy tasks - that is, she decides the relevant jurisdiction of departments. Upon appointment ministers obtain expertise on policies specific to their jurisdiction and strategically communicate this information to the Prime Minister before a policy is implemented. Assignment allows the Prime Minister to implement her agenda even when she is constrained to appoint ministers whose policy preferences are far from her own. There are normative implications: an open leadership contest for the position of Prime Minister produces policies that are Pareto efficient and when a Condorcet winning policy exists it is implemented with certainty. We explore the minimal costs of re-assigning policy tasks and show that in a reasonably large government these are negligible. We provide comparative statics on the relationship between cabinet personnel and the assignment of their policy tasks.

1. Introduction

The standard view of relations in parliamentary democracy, certainly under the Westminster model, is of a dominant Prime Minister whose power is nevertheless constrained by cabinet government. But what are the sources of the Prime Minister’s influence? How effective are the instruments at her disposal in allowing her to implement her policy agenda? And does a diverse cabinet act as an effective constraint on the exercise of Prime Ministerial power? In this paper we develop a formal model that provides answers to these questions and that builds on key structural features of parliamentary government, which are most developed in the Westminster system - though certainly not unique to that system.

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In the Westminster model, the Prime Minister is leader of the majority party and so ultimately requires the support of the electorate. Upon taking up her position, the Prime Minister appoints her ministers and decides which government departments they will run. Permanent civil servants provide advice to ministers on policies that fall within the jurisdiction of their department. Ministers then bring policy proposals to the full cabinet, or to a cabinet committee, that includes the Prime Minister. Finally, once a policy decision is reached and is ratified by Parliament it is then implemented. We capture key elements of these institutional procedures in a simple model that serves as a useful metaphor for exploring parliamentary democracy: a leader (a Prime Minister) is elected, the leader organizes her government, and finally a policy is implemented.

Our focus is on the way that a Prime Minister organizes her government. We highlight three instruments at her disposal: (i) the appointment of her ministers - the Prime Minister chooses who will serve under her and who will remain on the back-benches; (ii) the allocation of portfolios - the Prime Minister decides which ministers will be assigned to which government departments; and (iii) the assignment of ministerial tasks - once her cabinet is in place the Prime Minister decides which policies will be allocated to which portfolios.

The appointment of ministers is perhaps the most basic and well understood element of these instruments at the Prime Minister’s disposal, and has been analyzed by Huber and Martinez-Gallardo (2008), Dewan and Myatt (2007), and Thies (2001), amongst others. The allocation of ministerial portfolios is central to the seminal models by Laver and Shepsle (1996), Austen-Smith and Banks (1990), and the model of cabinet reshuffles by Indridason and Kam (2008). Little attention has been given, however, to analyzing how the Prime Minister assigns policy tasks to her ministers once portfolios are allocated. Indeed all models that we are aware of treat the jurisdiction of government departments as fixed.

This is surprising. The organization of policy functions is a core element of a Prime Minister’s responsibilities and there have been several critical changes in departmental responsibilities in the UK in the post war era. For example, Aneurin Bevan, the post war Minister of Health was also presided over a huge growth in public sector housing. A more recent example is the overhaul of the Home Office, that included the setting up of a new Ministry of Justice and a new Office for Security and Counter-Terrorism within the Home Office.2

Such changes, which amount to a major reorganization of the policy responsibilities and functions of British government, fall under the Royal Prerogative which is exercised by the Prime Minister.3 No primary legislation is required and changes are made in the absence of


3 Relatedly, in the German Federal Republic the reassignment of policy competencies ultimately falls under the Richtlinienkompetenz of the Chancellor according to Article 65 of the Basic Law.
an objection from either House. Indeed a recent report of the House of Commons Public Administration Select Committee, stated that “it is anomalous that it is so procedurally straightforward for the Prime Minister to reorganize the Civil Service by amending the functions of the ministers it serves, when reorganizing other public services may often involve statutory consultation, parliamentary approval, or even primary legislation.”

We start from the premiss that the organization of the machinery of government is a critical element of a Prime Minister’s strategic plan and should be considered alongside the other instruments at her disposal. Our focus highlights a key difference between congressional and presidential systems: in the former the tasks of the bureaucracy are decided by legislation; in the latter, tasks are assigned to ministers as part of the government formation process and civil service reorganization is a consequence. As former cabinet secretary Lord Butler states: “when a new Prime Minister has to make appointments, the structure of the departments has to be decided so that it is clear what ministerial positions there are to be filled.”

Following standard models of delegation we assume that the Prime Minister cannot formulate policy on all dimensions. The consequent division of labor involves the strategic appointment of a cabinet of ministers, the allocation of their portfolios, and the assignment of their tasks. Our focus is on asymmetric information between a minister and the Prime Minister. We ask how does a Prime Minister organize her government to minimize agency problems and ensure successful implementation of her agenda?

We assume that ministers have commonly known idiosyncratic policy biases and are perfectly informed about underlying fundamentals relevant to policy-making in their jurisdiction. Their expertise extends only to those areas where the Prime Minister has assigned them jurisdiction. More precisely, we assume that upon being allocated to a portfolio and assigned their policy tasks, ministers can extract all relevant information from the civil servants who form the permanent staff of government departments (and are thus experts in the relevant jurisdictions). The fact that ministers are specialists, and thus are informed only on policies falling within their jurisdiction, has an interesting implication: in allocating a

\footnotesize{In response to the question made on February 6th 2006 by Lord Stoddart of Swindon, who asked Her Majesty’s Government “whether they will issue a Green Paper on the proposed reorganisation and splitting of the Home Office, and allow for a period of public debate and consultation and the issuing of a White Paper before any Bill to implement such reorganization is presented to Parliament”, the Minister of State for the Home Office, Baroness Scotland of Asthal, replied unequivocally: “questions of changes to the machinery of government are decided by the Prime Minister.” By contrast, it was Congress that set up the Homeland Security Department in response to the Sept. 11, 2001, terrorist attacks.


This relates to a more general distinction between policy-making in presidential as opposed to parliamentary systems that is highlighted by Persson, Roland, and Tabellini (2000).

Report to the Public Administration Select Committee, 19th April 2007.}
minister to a portfolio and assigning his tasks, the Prime Minister decides that minister’s area of specialism.

To explore our question we develop a multi-dimensional model of policy formation. A prime-ministerial candidate decides whether to take part in an electoral contest, determined by plurality rule. The winner of the contest (the Prime Minister) then sets about organizing her government- appointing ministers, allocating their portfolios, and assigning their tasks. Upon taking up her post, each minister (defined by an ideal point in policy space) learns about the underlying fundamentals relevant to policy making though unobserved by the Prime Minister. The minister then strategically communicates this information to the Prime Minister. Finally, a policy is implemented. As we show, whether policy is implemented by the minister or the Prime Minister is irrelevant to the final outcome.

This basic set-up allows us to explore the effectiveness of different institutional sources of prime ministerial power. First, fixing the jurisdiction of each department, we explore to what extent the power to appoint ministers and allocate them to specific portfolios allows the Prime Minister to implement her policy agenda. Second, taking the composition of cabinet as given, but allowing the Prime Minister to reallocate and reassign tasks, we explore how these instruments of cabinet management affect the policies that are implemented.

We initially explore a simple two minister example and show that, in this setting, the allocation of portfolios and assignment of tasks is critical toward attainment of a Prime Minister’s policy goals. More precisely we show that in any situation where the preferences of the cabinet are heterogenous, that is where the Prime Minister cannot appoint ministers whose policy preferences are direct reflections of her own, then she can implement her full policy agenda by assigning competencies optimally (from her perspective). By contrast, when the only instrument she wields is appointment of her cabinet personnel, to portfolios with predefined and unalterable jurisdictions, then she most likely will fall short of full implementation.

A surprising implication of this result, is that a Prime Minister is not deterred from implementing her agenda by the appointments that she makes: even when the Prime Minister must appoint ministers whose policy preferences are very far from her own, she can organize her government in such a way that she is able to fully implement her policy agenda. Indeed, as we show, the Prime Minister can benefit from a diversity of preferences over policy amongst members of her cabinet.

There may however be other constraints on the Prime Minister. For example, reassigning policy jurisdictions may involve a costly transfer of staff and materials between departments. If such costs are prohibitive, assignment can yield a potentially powerful but effectively blunt instrument of control. To explore this issue we ask how the cost of reassignment responds
to changes in the institutional environment. We show that as the size of the government - defined as the number of policy jurisdictions - grows large, the organizational costs of optimal reassignment become arbitrarily small. Moreover, even in a moderately sized government, the costs of reorganization required for the Prime Minister to implement her programme are small (they converge rapidly to zero). Critically, we thus show that even when a Prime Minister is able to make only marginal changes in government jurisdictions by reassigning tasks, she can implement her agenda.

The main implication of our model is that when the Prime Minister can maneuver the instruments at her disposal, then she can implement her preferred policy. However, control over all three instruments is not necessary. For example, when she has no control on who sits in her cabinet, nor on which departments they are allocated, then the assignment of tasks allows her to implement her agenda completely.

Our investigation has normative implications for the understanding of parliamentary governance. Since the Prime Minister has the instruments to ensure implementation of her agenda, any promise of policy concessions in the leadership contest is not enforceable. A striking implication is that the initial election of the Prime Minister is driven solely by expectations over government policy: although she relies on her minister’s reports, once elected a Prime Minister implements her ideal policies and cannot credibly commit to doing otherwise. Since the policies that will be implemented are foreseeable, prime ministerial candidates can do no better than standing on their preferred policies. A direct implication is that, at the stage of electing the Prime Minister, Pareto dominated outcomes are always avoided and, moreover, if a Condorcet winner exists, it will surely be chosen.

Our model also allows us to explore the interconnection between the organization of government departments and the choice of cabinet personnel. More specifically, we show how the assignment of a minister’s policy tasks responds to reshuffles and to shifts in the policy position of a minister. We also provide insights into the relationship between the Prime Minister’s use of her instruments - appointment, allocation, and assignment - and the diversity of her cabinet. When appointment is the only instrument available, the Prime Minister prefers to choose ministers who are most closely aligned with her preferences. By contrast, when a Prime minister can select ministerial tasks she (weakly) prefers diversity and can attain her policy goals even in the presence of (very) large ideological differences within the cabinet.

We next provide a brief overview of the main literature. In section 3 we provide details of our model, and the main results of our investigation into the Prime Minister’s strategies are in sections 4 and 5. Section 6 extends our main results to a large cabinet, whilst in section 7 we discuss the implications of costly government reorganization. Section 9 provides some comparative statics analysis. Finally, section 10 concludes.
Our model relates to a growing literature that uses principal agent theory to understand the multiple relations in parliamentary democracies (Strøm, Muller, and Bergstrom, 2003; Thies, 2001; Martin and Vanberg, 2004). We sketch all of the key agency relationships identified by Strøm (2000) within our model of parliamentary democracy, and capture many of its inherent tensions. Whilst our model abstracts from issues of party competition, coalition formation, and agency problems between ministers and civil servants we nevertheless believe it serves as a useful framework for inclusion of such analysis. Our analysis of aggregation of dispersed information by cabinet members is circumscribed in the cheap talk literature (Crawford and Sobel, 1982) that analyzes strategic communication by an agent to a principal who implements policy. Whilst this literature has been used to explore information transmission in presidential democracies (Gilligan and Krehbiel, 1987), to our knowledge our work is the first to apply this machinery to parliamentary democracies.

Our focus on allocation relates our work to that of Laver and Shepsle (1996) who model a government organized into mutually exclusive jurisdictions with a ministerial head exercising complete control over policy. A clear difference, amongst others, between their model and ours is that, whilst they take jurisdictions as fixed, we treat jurisdiction as a strategic choice variable of the Prime Minister. Another somewhat more subtle, yet important, difference is that in their model a minister implements his ideal point whilst in ours the minister reports to the Prime Minister who, on the basis of the minister’s policy recommendation, then implements policy. This raises the question: would the Prime Minister ever choose to fully delegate decision-making to her minister? Surprisingly, once we take into account that the Prime Minister organizes her government optimally, the same policy outcomes prevail regardless of who implements policy - so the Prime Minister is strictly indifferent between implementing policy directly or delegating the job to her minister.

The analysis of jurisdictional assignments as equilibrium phenomena has been studied in a legislative setting by Ting (2002). He analyzes optimal jurisdictional assignments in the presence of moral hazard when the legislature controls the agency budget and contractual rewards. He shows conditions under which a legislature would wish to consolidate bureaucratic tasks in a single agency. In our model, a Prime Minister can exploit the policy differences between her ministers to align the incentives of her ministers with her own. A similar mechanism is analyzed by Indridason and Kam (2008) in their model of reshuffles.

The initial stage of our model, in which the Prime Minister is chosen, encompasses a citizen-candidate model in which voters may act strategically (Besley and Coate, 1997). In the citizen candidate model, as in ours, a candidate for election has no commitment ability and
so can credibly commit only to implementing her ideal policy. Whilst some extensions of the citizen candidate model take account of possible uncertainty over candidates characteristics, a key difference with existing models is that ours includes a stochastic policy shock unobserved by voters. However, because a Prime Minister can organize her government in such a way that she receives all the relevant information about this shock, she can, despite the inherent uncertainty in the policy environment, credibly commit to implementing her ideal policies.

Some of our results (propositions 1 and 2) rely heavily on those published by Battaglini (2002) on multi-dimensional cheap talk. Although these results are well known amongst economic and political theorists, to our knowledge they have not yet been applied in a political setting. We embed the structure of Battaglini’s analysis in a full model of political competition in which a polity selects a leader, a leader organizes her government, and a policy is implemented. Our application highlights the importance of these results toward understanding aspects of parliamentary democracies. Whereas Battaglini shows that in a multi-dimensional cheap talk setting there is a fully revealing truth-telling equilibrium where jurisdictions are orthogonal to the biases of senders, we show that in our world this is true of any truth-telling equilibrium. Moreover we develop this technology to develop comparative static results that help us understand the inter-relationship between appointment and assignment in parliamentary democracies.

The classic unidimensional model of information transmission in legislatures (Gilligan and Krehbiel, 1987) needs to be revised when applied to cabinet design. The power of the Prime Minister to reassign policy tasks, combined with the fact that departmental jurisdictions include several policy areas, means that a multi-dimensional model is more appropriate. Some of our results can clearly be distinguished from those of the unidimensional model: whereas in that model information transmission improves when the (median) preference of the committee and the parent body are not far apart, in our multi-dimensional model of parliamentary government diversity does not undermine the power of the Prime Minister.

Finally, a critical question that we address is the optimal assignment of policy tasks when changing jurisdictions is costly. This is very relevant in our application, as reassignment of policy tasks involves the costly transfer of personnel and materials. The political situation we analyze is a special case of the assignment problem studied in mathematics (see Burkhard, Dell’Amico, and Martello (2009)). In their classical form, these combinatorial optimization problems address the optimal way to assign a number of agents to a number of tasks when agents have different abilities for each task. In our analysis a Prime Minister wishes to implement her agenda given the policy bias of those she appoints; the optimal assignment of jurisdictions achieves this goal subject to minimizing the average cost of reallocating tasks between government departments.
3. A formal analysis of the Westminster model

We develop a formal model that explores a situation in which a polity elects a leader who forms a government, which consists of the leader and the people she appoints to serve as ministers in her government. A policy is then implemented. The polity is composed of a finite number of \( I \) vote-holding citizens each labeled \( i \). The preferences of members of the polity are defined over policy outcomes \( x \in \mathbb{R}^2 \) and are single peaked and quadratic around each citizens bliss point in \( \mathbb{R}^2 \). Our definition of a polity corresponds to a number of different scenarios ranging from the entire citizenry to a subset of a party’s elected representatives.

We label the leader in our model as the Prime Minister who is directly elected by all members of the polity. To be elected, a leader must first stand for election at cost \( c > 0 \) which captures the costs to the individual of mounting a campaign. As in Besley and Coate (1997) any member \( i \) of the polity may decide to run for office so long as he is willing to bear the cost of doing so. Members of the polity play mixed strategies thus with some probability each member enters the race to become leader of the party. Members of the polity correctly anticipate the policies that will be implemented by the winning candidate and vote strategically, with the candidate receiving a plurality of the votes becoming the Prime Minister. All our results extend easily to a situation where the leader of the party becomes Prime Minister with some probability; for example she may become the leader of the opposition, before being elected to office.

In the second stage of the game, the newly elected Prime Minister organizes her cabinet. Her organizational strategy has two elements. First, she selects the personnel who will serve in her cabinets. Without loss of generality we restrict to a government which consists of two ministerial posts though we later extend our results to a larger cabinet. As with all members of the polity, the preferences of ministers are defined over policy outcomes \( x \in \mathbb{R}^2 \) and are single peaked and quadratic with bliss points at \( m_i \) and \( i \in \{1, 2\} \) so that payoffs are defined as \( u_i(x) = \sum_{n=1}^{2}(x^n - m^n_i)^2 \). We write the Prime Minister’s ideal point as \( pm^* \) and, for notational simplicity, we assume that once elected her ideal point is located at the origin so that \( pm^* = (0, 0) \). This implies that the ministers’ bias with respect to the Prime Minister is \( (m_i - pm^*) = m_i \).

Upon appointing her cabinet the Prime Minister defines the portfolio of her ministers. We describe these as a minister’s jurisdiction. More formally, jurisdictions are single dimensions over which ministers report to the Prime Minister. Our model may appear simplistic in assuming that ministers’ report are unidimensional, but we thereby capture that ministers
are unable (or do not have the resources) to report on all policy dimensions. In some cases we might think of the jurisdictions as equivalent to policy dimensions. For example one minister might be assigned to report over foreign policy, whilst the other reports only on domestic issues. Whilst in such situations the sphere of influence of ministers is clear cut, there are other possible allocations we would want to consider. Jurisdictions may not always contain decisions on the same policy decisions. We capture this in our two dimensional model by assuming that jurisdictions are two directions $J_i \in \mathbb{R}^2$, $i = \{1, 2\}$ that span the whole policy space (i.e. $J_1 \neq \lambda J_2, \forall \lambda \in \mathbb{R}$).

We assume that there are underlying social, economic and political fundamentals that are not directly observed by either the Prime Minister or her ministers. We capture these fundamentals via the vector $\theta \in \Theta \subseteq \mathbb{R}^2$. Once a minister has been allocated a department and its jurisdiction is assigned, he acquires all information relevant to policy-making in that jurisdiction: thus when minister 1 receives the jurisdiction $J_1$, upon taking up her position, she learns the initially unknown fundamentals on that subset of the policy space; likewise when $m_2$ takes up her position, and is assigned $J_2$, she learns all there is to know on the fundamentals in her sphere of influence. A critical feature of this set-up is that, in assigning ministers to different posts and allocating their tasks, the Prime Minister decides which set of policies they will become informed about. Thus whilst expertise is held by civil servants it is also assigned endogenously by the Prime Minister who decides which of her minister will become informed, and on which set of issues. Upon receiving the policy-relevant information in their jurisdiction, ministers then report a single dimensional variable to the PM, $s_i : \Theta \to \mathbb{R}$, in the third stage of the game.

The motivation for our assumption of information asymmetry between the Prime Minister and her ministers is that ministers are chiefs of departments that house permanent civil servants who provide policy relevant expertise. In the chain of command, ministers are directly responsible for their departments and civil servants are accountable to them. As ministers are the first point of contact in the government for civil servants, it is reasonable to assume that they are better informed than the Prime Minister. Nevertheless, our assumption that, upon taking up their positions, ministers learn everything there is to know about their brief, is questionable. In particular it takes no account of possible agency conflicts between the minister and his staff. We address this issue in the concluding section.

In the final stage of the game the Prime Minister chooses a policy which we denote as $y \in \mathbb{R}^2$. Her chosen policy may depend on the declarations made by ministers so that $y((s_1(\theta), s_2(\theta)))$.

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8If that were the case the Prime Minister could gather this information and not rely on his cabinet to implement policies. Later in section 5 we show the robustness of our results to the case where ministers have general rather than policy specific expertise (i.e. when ministers observe and report on all policy dimensions).
We write the policy outcomes as $x$, which depends on both the chosen policy $y$ and the underlying fundamentals $\theta$. Specifically the final outcome satisfies $x = y + \theta$. The policy that the Prime Minister chooses depends upon her beliefs about these fundamentals given the declarations of her ministers. We write the posterior belief of the PM on the possible states of the world as $\mu : \mathbb{R} \times \mathbb{R} \rightarrow P(\Theta)$.

Our solution concept is Perfect Bayesian Equilibrium. Loosely speaking, this requires that the actions of the Prime Minister and her ministers be sequentially rational given their beliefs, and that their beliefs be consistent with rational play and Bayes rule along the equilibrium path. In addition, our solution to the second and third stages of the game adopts the terminology of a truthful and fully revealing equilibrium. In this equilibrium the ministers adopt a truthful strategy, that is they report the true state of the world in their jurisdiction and, in combining her ministers’ joint declarations, the Prime Minister learns all there is to know about the true state of the world. In game theoretic terms a fully revealing equilibrium is defined by posterior beliefs cumulated on the true state of the world, $\mu(s_1(\theta), s_2(\theta)(\theta)) = 1$. Of course an equilibrium may not be truthful and yet still be fully revealing. This occurs, for example, if the ministers systematically misreport: it may be common knowledge that a minister exaggerates by adding a bias to his report and that the Prime Minister takes this into account by discounting such reports. Applying reasoning akin to the revelation principle, it can be shown that there is no loss of generality in restricting attention to fully revealing equilibria that are truthful and have degenerate out of equilibrium beliefs.\footnote{See Lemma 1 of Battaglini (2002).} The intuition behind this result is that cabinet members only care about final outcomes thus when a fully revealing equilibrium exists they may as well report truthfully (they need never construct complicated out-of-equilibrium beliefs, as the preferred policy of the Prime Minister is an equilibrium outcome in either case).

We develop our results by looking at two distinct and extreme cases. In the first, we treat jurisdictions as fixed. The only instrument the Prime Minister can use in order to implement her policy agenda is to choose which individuals sit in her cabinet and to assign a ministry to each of them. In the second case, we treat the members of the cabinet as exogenously fixed, but Prime Minister allocates a portfolio to each minister and assigns its tasks.

4. APPOINTING MINISTERS

We begin our analysis by focussing on the simplest scenario in which departmental jurisdictions are fixed, but the Prime Minister has complete discretion to appoint her ministers. To
make things interesting we make a slight restriction on the type of ministers that are available assuming that there will always be some ideological conflict. That is, a Prime Minister is unable to appoint ministers who share her ideal point.

As a toy example that helps illustrate some of the main ideas and introduces our basic notation, consider the situation depicted in Figure 1 where the ideal points satisfy $pm^* = (0, 0), m_1 = (0, 1)$ and $m_2 = (1, 0)$ respectively (the first coordinate represents the ideal point on policy $X$, and the second coordinate the ideal point with respect to policy $Y$). The figure explores a situation where the jurisdictions correspond to the policy axis and so a minister’s influence extends to a single policy dimension. In this situation the Prime Minister has perfectly aligned preferences with $m_1$ on policy $X$ in that both the Prime Minister and her minister would like the policy outcome to be as close as possible to the origin. Analogously, on policy $Y$, the Prime Minister and $m_2$ wish to implement the same policy.

Under these circumstances, and given any underlying value of $\theta$, the Prime Minister can strategically appoint her ministers so that, given their relative policy expertise, ministers reveal their information truthfully. Consider a situation where $m_1$ is appointed to the department that has full jurisdiction over policy $X$, so that his jurisdiction $J_1 = (1, 0)$, or any proportional vector to this one, and agent 2 has jurisdiction on the second policy dimension so that $J_2 = (0, 1)$. Now imagine that $m_2$ truthfully reveals her information to the Prime Minister (so that $s_2(\theta) = \theta_2$). Armed with this information the Prime Minister will implement a policy ($y_2 = -\theta_2$) such that $x_2 = 0$. Figure 1 depicts minister 1’s best response to truthful revelation by minister 2. His indifference curve is tangent with $x_2 = 0$ at the origin which coincides with the ideal point of the Prime Minister; thus he delivers a report that yields a policy outcome $x_1 = 0$. In a truthful equilibrium, the minister would be reporting $s_1(\theta) = \theta_1$ and the Prime Minister would implement $y_1 = -s_1(\theta)$. Thus, in this example, the Prime Minster elicits all the information relevant to full implementation of her agenda.

Figure 1. Appointing Ministers with Germain Jurisdictions
This example highlights that truthful revelation, $s_i(θ) = θ_i$, for $m_i$ is a best response when the other minister reveals her information truthfully. But it remains to be shown that this is in fact part of (perfect Bayesian) equilibrium. Before moving on we state the salient features of this equilibrium in the following proposition.

Proposition 1. When jurisdictions coincide with policy dimensions and each minister is allocated to a jurisdiction orthogonal to his bias, the Prime Minister implements her ideal policy. In such case, there is an equilibrium where each minister reports truthfully the state of the world on his jurisdiction, the Prime Minister believes these statements, and the policy that is implemented yields an outcome coinciding with the Prime Ministers ideal point.

This first result shows that, without specifying the agents’ messages nor the response of the Prime Minister to these messages, there is an equilibrium in which the preferred policy of the Prime Minister is implemented.\textsuperscript{10} One such equilibrium is both truthful and fully revealing. Of course, there may be other fully revealing equilibria (not truthful) where ministers systematically overstate the true state of the world ($s_i(θ) = θ_i + δ$), but where the Prime Minister, taking such behavior into account, implements the policy ($y(s) = −(s_1 − δ, s_2 − δ)$). When the Prime Minister is able to appoint a minister whose bias is orthogonal to the jurisdiction, then she can obtain full information and so implement her preferred policy.

5. Allocating Portfolios and Assigning Tasks

The results of the previous section show the importance of cabinet selection as an instrument allowing the Prime Minister to implement her agenda. But there are limitations to the use of this tool as the Prime Minister may be forced to select on some individual trait other than the political preferences of her appointee - for example, talent, experience, or following in the party - and may find that those best able to serve under her do not share her political opinions.\textsuperscript{11} This might suggest that the Prime Minister’s position is weakened and indeed the view that a Prime Minister’s power is both limited and contextual due to restrictions on her ability to appoint is commonly held.

This viewpoint overlooks the fact that the Prime Minister has other instruments at her disposal, and that their use may also influence the policies that are implemented. In particular,

\textsuperscript{10}As usual in these environments there are many more equilibria (e.g. a babbling equilibrium where no information is revealed and the messages are never used by the Prime Minister).

\textsuperscript{11}An alternative explanation is that the Prime Minister prefers to have her opponents in the cabinet. John Major was unwilling to sack three staunch opponents of his policies arguing that “we don’t want another three bastards out there. What’s Lyndon Johnson’s maxim?...” Johnson had famously declined to sack FBI director Edgar Hoover on the basis that “it’s probably better to have him inside the tent pissing out than outside pissing in”.
although the Prime Minister may be constrained in choosing her ministers, the Prime Minister maintains discretion over which portfolio they are allocated. We can see the effect of this instrument immediately by referring back to Figure 1. In our discussion of that scenario we looked at the selection of ministers as the Prime Minister’s choice variable. Of course we could equally have asked, if the Prime Minister is constrained to appointing ministers with ideal points $m_1 = (1,0)$ and $m_2 = (0,1)$, which portfolios should they be allocated? The obvious answer is that they be provided with jurisdictions orthogonal to their biases, since only then would their preferences be perfectly aligned with the Prime Minister on policies that fall in their sphere of influence.

We can go one step further. Recall that in our set up a Prime Minister can not only allocate portfolios but has discretion over which policy tasks are assigned to which portfolio. In the previous section we looked at a situation where jurisdictions are aligned with the policy axis and so each department has complete jurisdiction in a given policy area. Although analytically convenient, such a neat configuration is rarely found in political decision-making, as usually there is some overlap in the jurisdictions of different departments. Figure 2 depicts a situation where the jurisdiction $J_1$ involves both policy $X$ and policy $Y$ and the minister’s policy bias relative to that of the Prime Minister is given by $m_1$. Recall that the Prime Minister’s ideal policy lies at the origin and so in this example the minister’s bliss point is to the north-east of that point. Ideally, from his perspective, the minister would recommend a policy that is greater than $pm^*$ on both policy dimensions. However, when he is forced to report on $J_1$ then he must acknowledge a trade-off in which he can increase $X$ only by decreasing $Y$.  

![Figure 2. Appointing Ministers with Non-germane Jurisdictions](image)
A natural way to think of this trade-off regards policies over distribution, where $X$ and $Y$ involve spending on particular policies and/or targeted distribution to specific constituencies. For example, a report on $J_1$ to the northwest of the origin ties an increase in spending on policy $Y$ to a decrease in spending on policy $X$, relative to the Prime Minster's ideal point; the opposite spending pattern is implied by a report to the southeast of the origin. Thus, although relative to the Prime Minster, the minister would like to spend more on both policies (constituencies), he is forced into a trade-off between the two. The Prime Minster has the power to determine this trade-off by choosing the slope of $J_1$, and will do so optimally given her knowledge of the minister's bias.

There is another possible interpretation of our jurisdictions. Taking into account that directions are determined up to a scalar transformation, we can normalize each jurisdiction so that its components add up to 100. Then each component of a jurisdiction can be interpreted as the percentage of decisions on each policy dimension reported by that minister.\footnote{The mathematical interpretation is not exact. In our model each minister’s decision is a single scalar, and it is not clear how all decisions in the various decisions aggregate into a single scalar. This definition also requires that the Prime Minster has full agenda control by deciding how much attention his cabinet invests in each policy dimension.}

We are now ready to show that in any fully revealing equilibrium (in which the Prime Minster implements her preferred policy) each minister’s jurisdiction needs to be orthogonal to his own bias.

**Lemma 1.** Fix the ideal points of ministers and allow the Prime Minster to choose their jurisdiction. A minister’s jurisdiction is orthogonal to his bias and is determined solely by his own bias and not by the ideal point of the remaining cabinet ministers. Multiplying a minister’s bias $m_i$ by a constant $\lambda$ does not change the jurisdiction $J_i$.

The implication of this result is that, when the Prime Minster is constrained to appoint a minister with a given and known bias then she can always allocate policy tasks in such a way that the minister will truthfully reveal the information she obtains in her jurisdiction. This result is not affected by the size of the minister’s bias relative to the Prime Minster’s ideal point, nor the distribution of ideal points in the cabinet.

Building on Lemma 1, we can explore what the allocation of jurisdictions would look like with a cabinet of two ministers with given biases $m_1$ and $m_2$ respectively. Figure 3 illustrates a scenario where both ministers would ideally like to spend more on both policies $X$ and $Y$ relative to $pm^*$, but they have different preferences over the trade-off involving these policies. Although in such a situation the Prime Minster may appear vulnerable she is able to exploit the tension in ministers’ preferences in assigning their jurisdictions. The minister with ideal point $m_1$ is given the jurisdiction $J_1$ that is orthogonal to his bias; likewise the minister...
with the bias $m_2$ is given jurisdiction over $J_2$. Note that along their allocated jurisdictions each minister’s preferences are perfectly aligned with the Prime Minister as their respective indifference curves are tangent to their jurisdictions at the origin.

The following proposition now shows that an allocation of these jurisdictions is part of a perfect-Bayesian equilibrium.

**Proposition 2.** Fixing the ideal points of the ministers, allow the Prime Minister to choose their jurisdictions, and assume that the biases of the ministers with respect to the ideal point of the Prime Minister are linearly independent. The Prime Minister elicits full information from her agents and so can fully implement her policy agenda.

A technicality in the wording of the proposition deserves further attention: the biases of the ministers with respect to the ideal point of the Prime Minister need to be linearly independent. By Lemma 1 we know that in a fully revealing equilibrium each jurisdiction needs to be orthogonal to its minister’s bias. If biases are linearly dependant, orthogonal jurisdictions would coincide and the Prime Minister would no longer be able to infer the true state of the world in our two-dimensional policy space. Instead, linear independence ensures that both orthogonal jurisdictions span the whole policy space and the Prime Minister is then able to illicit all information she needs to implement her preferred policy.

According to one prominent and widely held view, the Prime Minister’s control over policy is limited by the need to include ministers who (i) do not share her policy preferences and (ii) are either too senior, talented, or well supported in the party, to be overlooked.\(^{13}\) Our analysis suggests that these are necessary though not sufficient conditions. According to our

\(^{13}\text{King (1994) argued that a few “big beasts of the jungle” maintained such stature as to be able to impose their views on policy outcomes in their departments (see also Laver and Shepsle (2000)).} \)
view the Prime Minister is constrained, that is she is unable to fully implement her agenda, only when each of the following conditions hold with respect to a particular minister: the Prime Minister is forced to appoint the minister even though that ministers policy preferences are not aligned with her own; the minister has veto power over his/her appointment to a particular ministry; the minister has veto-power over any changes in his jurisdiction; in such circumstances, the minister’s report cannot be used towards the implementation of the Prime Minister’s preferred policy.

In all other situations, even when the Prime Minister is constrained to appoint ministers whose preferred policies are very far from her own, she nevertheless is able to implement her desired policies. Moreover, and perhaps surprisingly, this does not depend on the assumption that the Prime Minister implements policy as we show in the following corollary to proposition 2.

**Corollary to Proposition 2:** *When the configuration of tasks is optimally designed from the Prime Minister’s perspective, the Prime Minister is strictly indifferent between choosing policy herself or fully delegating the task to her minister.*

The implication of this results is stark. As long as the Prime Minister has full control over jurisdiction, her influence is undiminished even when she allows policy to be chosen and implemented by her ministers. Thus what Lupia (2003) has referred to as the “perils of delegation” in parliamentary democracies are avoided so long as the Prime Minister has full control over the allocation of ministerial tasks. When the policy decisions are delegated to the ministers, they will implement precisely the same policy as the Prime Minister would implement in the event that she, (the Prime Minister), had full and perfect information.

One objection to our analysis, with its focus on allocation and assignment, is that a Prime Minister might not need such powerful instruments at her disposal in order to implement her agenda. Instead she could use cabinet deliberations as a mechanism for learning the true state of the world before making policy decisions. One way she could do this is to glean information by requiring her ministers to report on all policy dimensions. As an example, we might imagine a world where, instead of being policy specialists as in our model, ministers are generalists perfectly informed on all policy dimensions. Now if the Prime Minister asked each of her ministers to report to cabinet, any discrepancy between their reports would immediately reveal that at least one minister did not report truthfully. In such a world, Ambrus and Takahashi (2008) show that there is a fully revealing equilibrium, so long as off the equilibrium path actions can be appropriately punished.

Applied to our world the intuition is straightforward: if the Prime Minister can commit to implementing a policy commonly disliked by her ministers in the event their statements
do not match, this limits ministers’ willingness to conceal information. This logic yields an outcome - full implementation of the Prime Minister’s agenda- that is observationally equivalent to ours. Which model then provides a better account of Prime Ministerial power? There are a number of reasons why we believe our account to be more compelling.

The first set of arguments rely on the specific application at hand. An equilibrium that relies on the machinery developed by Ambrus and Takahashi (2008) holds only in a world where ministers are policy generalists- they know the true state of the world on all policy dimensions. By contrast, in our model ministers have specific expertise corresponding to the jurisdiction assigned to them by the Prime Minister - arguably this is a more compelling account of ministerial expertise. Indeed, if members in cabinet where policy generalist nothing would prevent us from assuming that the Prime Minister is herself a policy generalist and does need to form a cabinet to obtain expertise in each policy dimension.

The second set of arguments are more theoretical. Firstly, our equilibrium is more compelling due to its robustness. Although it is possible to devise truthful and fully-revealing equilibria with particular out of equilibrium beliefs, these equilibria do not survive straightforward refinements. Secondly, our results hold with very simple extensions to the basic set-up of the model. Suppose that each minister observes $\theta_i$ with some noise. As all reports contain error the Prime Minister is unable to draw comparisons between them and so the truthful and fully-revealing equilibria of the sort discussed above will fall away. However, as long as noise is independent and symmetrically distributed across the various policy dimensions, equilibria of the sort we have constructed in Proposition 2 remain (see Battaglini (2004)). This equilibrium is no longer fully informative, but ministers still have perfect incentives to reveal what they know and the Prime Minister can implement a policy that minimizes her loss; she can’t always get what she wants, but on average she is able to implement her policies. Moreover, when the noise becomes arbitrarily small, the equilibria approaches the fully revealing solution.14

A final theoretical concern is collusion-proofness. Asking ministers to report on the whole dimensionality introduces scope for collusion between ministers. They may jointly agree to report a message that yields a final outcome closer to their bliss point. The Prime Minister cannot discern such deviations when both ministers report precisely the same message. It is

14Levy and Razin (2007) show the limitations of the fully revealing equilibrium when the signals the ministers receive on each dimension are not independent. In that case, the declaration of a minister contains information about jurisdictions other than his own and so the Prime Minister can not credibly commit to ignore this fact. The ministers’ strategic reaction to the behavior of the Prime Minister may then prevent the existence of fully revealing equilibria.
It can be easily shown that Pareto improvements from the ministers’ perspective are always possible but they do never constitute an equilibrium. It remains an open question whether ministers are able to forge an agreement that avoids unilateral deviations.
without being subject to blocking vetoes. Note that we do not need to adopt a strict definition of the polity for this result to hold. Whereas, in most situations we might think of the leader as emerging from amongst a party’s elite set of ministerial prospects or ministrables, our result holds also when we allow any citizen to become the Prime Minister.

7. Choosing Jurisdictions in Large Cabinets

In the remainder of the paper we consider how robust our findings are to different assumptions about the institutional environment. Thus far our analysis has relied on a two-member cabinet. It is natural to consider the implications of analyzing a fully-fledged cabinet consisting of an arbitrary number of ministers deciding over multiple policy issues. We extend our model to consider a multi-member cabinet with $n > 2$ distinct policy issues related to the same number of government departments. All other elements of our model are otherwise as before: the Prime Minister has full control over the organization of her government; and once appointed to a position each minister receives all information relevant to her jurisdiction and files a report $s_i(\theta)$ that is observed by the Prime Minister. As we show in the following proposition, all of the results from the two-minister example carry over to the general case.

**Proposition 4.** Consider a situation where the number of policy issues decided by the government is $n > 2$ so that the cabinet consists of $n$ ministers with fixed ideal points. Allowing the Prime Minister to choose the jurisdiction of each of her ministers, and assuming that the biases of at least two ministers with respect to the ideal point of the Prime Minister are linearly independent, the Prime Minister elicits full information from her agents and so can fully implement her policy agenda.
Perhaps surprisingly, when moving to the general case we can use less restrictive assumptions about the ideal points of the ministers. In particular, for $n > 2$, we only need two ministers’ ideal points to be linearly independent (i.e. the orthogonal hyperplanes to their biases span the whole policy space). This can be shown with a three minister example as illustrated in Figure 4. Here note that the ideal points of $m_1$ and $m_2$ are linearly dependent: both ministers agree with the Prime Minister on two of the three policy dimensions. Following our earlier logic, it is straightforward to see that the Prime Minister can elicit all of the relevant information when assigning jurisdiction over policy $X$ to $m_3$, jurisdiction over policy $Y$ to $m_1$, and finally jurisdiction over policy $Z$ to $m_2$. This is not the only way in which the Prime Minister can organize her government and be in a position to implement her agenda: she could for instance obtain the same outcome by switching the jurisdictions of $m_1$ and $m_2$.

8. **Minimizing Organizational Costs**

Thus far we have assumed that the Prime Minister has complete discretion to reorganize departmental tasks in any way she chooses fit. Whilst true that in many parliamentary democracies this is a prerogative enjoyed by the Prime Minister, in reality the ability to change jurisdictions may be limited. In the United Kingdom, for example, there have been few major overhauls of the government machinery- the recent dismantling of the Home Office is one of them- and those that have occurred have received such prominence precisely because of their rarity. There are significant operational and organizational costs that the Prime Minister needs to consider before taking such actions which involve the transfer of personnel and materials between departments.\(^{16}\)

In this section we treat the costs of reorganization as part of a Prime Minister’s strategic plan and explore how these costs respond to changes in the institutional and policy environment. We assume that the Prime Minister has no discretion in who she appoints to her cabinet, but that she has some discretion in allocating and assigning the jurisdictions to those appointed. Further, we assume the cost of reorganization of each jurisdiction is increasing in its extent in a way we make precise below.

Of course, the Prime Minister might be fortunate enough to find herself in a situation, where for a given status quo allocation of policy tasks, the ideal points of her minister are aligned precisely along the given jurisdictions. In such cases the Prime Minister need not engage in any reorganization of her government at all.

\(^{16}\)When asked how expensive such reorganizations were former cabinet secretary Lord Butler replied “the answer is, more than you think. They are very expensive.” Report to the Public Administration Select Committee, 19th April 2007.
In any fully revealing equilibrium, the Prime Minister will reassign jurisdictions so that \( J_i \) is orthogonal to \( m_i \) as illustrated. We consider a Prime Minister who wants to reach his preferred policy outcome, whilst minimizing the organizational cost of shifting jurisdictions. We define costs in terms of the angular move that is necessary to make a jurisdiction orthogonal to a minister’s bias, though our results extend trivially to a situation where costs are a function of this angle. Figure 5 illustrates the costs of moving the status quo jurisdiction on the horizontal axis to \( J_i \), which is defined as \( \alpha \). When \( m_i = (1, 1) \), this move is 45°. The worst possible situation exists with two policy dimensions, status quo jurisdictions that coincide with the policy axis, and each minister’s bias is 45° away from either jurisdiction; then the Prime Minister needs to rotate each jurisdiction by 45° around the origin. From this simple exercise, we can see that the upper bound on the degree of government reorganization when there are two policy dimensions is twice 45° equal to 90°.

A natural step is to ask how the Prime Minister’s cost of reorganization vary with the size of government.\(^{17}\) The political implications of increasing the size of the government are the following: on the one hand, an increase in the size of government implies an increase in the number of possible assignments and this is associated with an increase in the complexity of the Prime Minister’s assignment problem; on the other, an increase in the size of government increases the possibility, that for a given set of ministers, and on any particular assignment, the Prime Minister can find a match that induces very low costs of reorganization.

We look at the extreme case where the Prime Minister has no appointment power, thus we treat each minister’s ideal point as random draw from a known distribution. Upon obtaining \(^{17}\)Recall that we need as many ministers as policy dimensions so that in our analysis an increase in the number of policy dimensions is equivalent to an increase in the size of government.
In the limit, as the size of the government grows large, the organizational costs necessary for the Prime Minister to implement her agenda go to zero. Of course our main interest is analyzing costs in commonly sized cabinets (i.e. $n$ just below 30). An immediate concern is how fast the average cost $AC_n$ converges to zero and whether, in commonly sized cabinets these costs are also negligible. We analyze this question numerically by assuming that there are $n$ ministers whose bias with respect to the Prime Minister on each policy dimension is drawn from a normal distribution with zero mean and unit variance. As for our our earlier results, status quo jurisdictions are the coordinate axis. We compute the minimum angular move so that each jurisdiction is orthogonal to the bias of each minister. For each of the 500 simulations we run, we find the optimal assignment that minimizes the costs of assigning jurisdictions to ministers and compute the average cost among all our simulations. Full
details of our algorithm can be found in the appendix, here we concentrate instead on our
substantive results represented by figure 6.

The upper (red) line in figure 6 shows the rate of convergence when the Prime Minister has no
freedom to appoint who sits in her cabinet, but can allocate portfolios and assign jurisdictions
in such a way that ministers will report truthfully. We see that, as the number of jurisdictions
increases, the average cost per minister falls rapidly toward zero.\textsuperscript{18} When \( n = 2 \) the (average)
necessary angular move away from the status quo jurisdictions is \( \alpha = 29.832^\circ. \) This falls to
\( \alpha = 0.722^\circ \) as the government size increases to \( n = 29. \) The lower (gray) line in figure 6 shows
a situation where the Prime Minister has some discretion over appointments and has twice as
many ministerial options as jurisdictions. As might be expected, convergence is much faster
as under these circumstances the Prime Minister can seat in her cabinet those ministers
with the smallest biases. These graphs provide an indication as to how the instruments at
the Prime Minister’s disposal, namely her ability to appoint ministers, allocate portfolios,
and assign tasks, interact to allow her to implement her agenda. Moreover they show that
for these particular parameters, the costs of changing jurisdictions in a reasonably sized
government are not prohibitive. In fact, our results suggest that when the government is
reasonably large, so that the costs of reassignment are small, then the key instrument that
the Prime Minister has available to her is the allocation of portfolios.

9. Diversity and Reassignment

We have shown that, even when the Prime Minister is constrained in her ability to appoint
ministers, she is able to implement her policy agenda by optimally assigning jurisdiction over
policy. A Prime Minister can be more or less constrained with regard to who she appoints
to sit in her government and this may in turn depend on the size of the government majority
or the balance of party factions. Although an analysis of the overall mix of instruments
that the Prime Minister will adopt is beyond the scope of this paper, a direct implication
of our analysis is that a Prime Minister who has little room for manoeuvre with regard to
appointments will engage in more manipulation of the machinery of government relative to
a Prime Minister with a less constrained choice of ministers.

An immediate empirical question that we can shed light on is how the allocation of tasks
varies with the policy preferences (biases) of those ministers that are appointed. A situation
might arise where, due to unforeseen circumstances, the biases of ministers with respect to
the Prime Minister changes so that, in order to implement her agenda, a Prime Minister
may then need to appoint new members to her cabinet, or reshuffle the existing allocation
of portfolios, but she may also wish to reassign tasks.

\textsuperscript{18}Here we depict the average results from 500 simulations.
How does a change in a ministers bias affect her jurisdiction? With regard to some of the cases we have analyzed the answer is straightforward. For example, in the particular case depicted in Figure 1, where jurisdictions are aligned with the policy axes, the answer is immediate: increasing the policy bias of either minister has no effect on their jurisdiction. More generally, however, this is not true as we show in Figure 7.

Here the minister starts at position $m_1$ with a corresponding jurisdiction $J_1$. Fixing the first coordinate of the bias whilst increasing the second one leads to a shift in jurisdiction to $\hat{J}_1$ providing the minister with relatively more influence on $X$ and less on $Y$. Thus in this example a minister is given (relatively) less influence on a policy issue when he becomes more (relatively) more biased on that issue.

Building on this exercise we can provide clear predictions about the relationship between a minister’s jurisdiction and his policy preferences relative to those of the Prime Minister. We suppose that a Prime Minister is unable to bring new faces into her cabinet, or reallocate portfolios, but that at the margin she maintains control over assignment of tasks.

**Proposition 6.** Suppose that the Prime Minister is unable to appoint new members of cabinet or reallocate existing portfolios. When the bias of minister $i$ changes the Prime Minister will reassign her tasks. The extent of this reassignment depend on the change in the minister’s relative bias on dimensions $X$ and $Y$. When there is an increase in the minister’s bias on $Y(X)$ relative to his bias on $X(Y)$ then his jurisdiction on $Y(X)$ decreases whilst it increases on $X(Y)$. This effect is more pronounced the more initial influence the minister has on $Y(X)$. When the minister’s ideal point is closer to the Prime Minister’s then an increase in relative bias on a dimension leads to larger change in jurisdictions.
Thus in some situations an increase in a minister’s bias will leave his jurisdiction unaltered, whereas in others the extent of the change in his jurisdiction responds to the change in his bias in a straightforward manner. Whilst these results are intuitive they provide, to the best of our knowledge, the first clear predictions concerning the interconnection between the organization of government and policy preferences of cabinet personnel that could be tested given suitable data.\(^{19}\)

10. Concluding Comments

We analyzed a model in which a polity elects a leader who then organizes her government before a policy is implemented. Organization of government consists of appointing leaders, allocating portfolios, and finally assigning policy tasks to those portfolios. Our model serves as a useful metaphor for analyzing agency relationships in parliamentary regimes and within this framework we asked how effective are the instruments at the Prime Minister’s disposal in allowing her to implement her policy agenda? Our key finding is that full control over assignment suffices for the Prime Minister to be able to implement her preferred policies. Our contribution, is not the claim that a Prime Minister can implement her preferred policy. Rather we provide a framework within which to explore different elements of a Prime Minister’s organizational strategy, and emphasize that the assignment of tasks is a powerful tool at the Prime Minister’s disposal that political analysts have so far overlooked.

We asked whether a diverse cabinet acts as an effective constraint on the exercise of Prime Ministerial power? We found that even when a Prime Minister is constrained to appoint ministers whose preferences are far from her own, she can implement her ideal policies so long as she is able to allocate ministerial portfolios and assign policy tasks. When the only instrument at the Prime Minister’s disposal is the selection of ministers then she (weakly) prefers a cabinet consisting of ministers closely aligned with her. Instead, when she is constrained in her ability to appoint ministers, the Prime Minister (weakly) prefers a diverse cabinet. More precisely, when the Prime Minister relies on the power to assign jurisdictions then a diverse cabinet is both necessary and sufficient for her to implement her agenda.

Although our model captures some of the critical features of parliamentary democracy, we use simplifying assumptions. In our basic model civil servants are both perfectly informed and perfect agents of ministers, and so we abstract from the agency problem between a Prime Minister and her ministers and the incentives required for a bureaucrat to obtain information (Bawn, 1995). However our model does extend to the basic agency problem between a

\(^{19}\)Although voting data for the British parliament exists, empirical estimation of ideal points from this data is not straightforward (see Spirling and McLean (2006)). Data on changes to British government are at http://www.nuffield.ox.ac.uk/politics/whitehall/.
minister and her civil servants. Indeed the permanency of official in most parliamentary systems means that the assignment of tasks is likely to be used by a ministerial principle to extract information from her agents.

For most of our results we assume that the ideological conflicts between the Prime Minister and her ministers cannot be resolved by full revelation of the agents information, and we assume that revelation of policy is costless so that our model falls within the class of “cheap talk” models. Moreover, the Prime Minister cannot commit to using information in a particular way. Put another way, once the Prime Minister has the information available she has complete discretion over policy choice. Critically, our model abstracts from issues of party competition and coalition formation and thus serves best as a metaphor for Westminster style democracies where single party majority government and the absence of checks and balances implies that elections are indeed leadership contests between two candidates.

The basic framework developed here could be extended to include variations on this basic institutional architecture; for example, an investiture vote under majority rule. In these situations a sophisticated voter would need to anticipate the concessions and bargaining over appointments, and the impact on final policies, before casting her ballot. Extensions should relax some of these assumptions, and consider the implications of our analysis in light of some of the institutional detail that is missing.

11. Appendix

Proof of Proposition 1. Jurisdictions are the coordinate axis: $J_1 = (0,1)$ and $J_2 = (1,0)$. The Prime Minister chooses two ministers with preferences $m_1 = (m_1^1, 0)$ and $m_2 = (0, m_2^2)$. We can show that there exists a truthful fully revealing equilibrium where a minister with preferences $m_1$ ($m_2$) has jurisdiction on the first (second). This equilibrium has the following features: each minister reports the true state of the world in their jurisdiction so that $s_1(\theta) = \theta_2$ and $s_2(\theta) = \theta_1$; the Prime Minister follows the minister’s advice in implementing policy so that

$$y(s_1(\theta), s_2(\theta)) = -(J_1 \cdot s_1(\theta) + J_2 \cdot s_2(\theta));$$

and, finally, the Prime Minister’s beliefs put all mass on the jointly reported state of the world

$$\mu(s_1(\theta), s_2(\theta))(\theta) = 1.$$
We show that this set of strategies and beliefs constitutes an equilibrium. The interim utility of minister \( i \) is

\[
u_i(y(s_1(\theta), s_2(\theta))) = -(-s_2(\theta) + \theta_1 - m^1_i)^2 - (-s_1(\theta) + \theta_2 - m^2_i)^2.
\]

Ministers want to send the report that maximizes their interim utility. Taking as given the behavior of minister 2, the previous expression for minister 1 boils down to

\[
\max_{s_1(\theta)}[-(m^1_i)^2 - (-s_1(\theta) + \theta_2)^2],
\]

and so the optimal behavior of minister 1 is to report \( s_1(\theta) = \theta_2 \) (i.e. the prescribed behavior in a truthful and fully revealing equilibrium). Minister 2’s optimal behavior can analogously be proved. Beliefs are consistent with equilibrium behavior as they accumulate all mass on the true state of the world. Finally, the Prime Minister’s behavior is optimal given these beliefs: the Prime Minister implements a policy, \( y = -\theta \), that yields his preferred policy outcome \( x = (0,0) \), and so has no incentive to deviate.

\[\square\]

**Proof of Lemma 1.** Given minister \( j \)'s report, minister \( i \)'s optimal report should lead to \( pm^* \) when the equilibrium is fully revealing. As can be observed in Figure 2, this can only happen when minister \( i \)'s indifference curve over \( pm^* \), is tangent to the set of policies from which he can choose, i.e. his own jurisdiction. Indifference curves of quadratic utilities are circles, and any tangent line to a circle is orthogonal to the radius of the circle on the tangency point and this implies that the direction of the jurisdiction is orthogonal to the bias between the minister and the Prime Minister, \((m_i - pm^*) \cdot J_i = 0\). There consequences follow immediately: a jurisdiction is orthogonal to a minister’s bias and thus is invariant with respect to changes in another minister’s bias; and, multiplying a minister’s bias by a constant does not change the jurisdiction as the orthogonal direction of any vector coincides with the orthogonal direction of that vector multiplied by any non-zero scalar.

\[\square\]

**Proof of Proposition 2.** We show that when the Prime Minister chooses orthogonal jurisdictions to the ministers’ biases, agents report the true state of the world in the new coordinate system induced by their jurisdictions. The Prime Minister then holds beliefs that allow the implementation of her preferred policy. Consider two arbitrary bliss points \( m_1, m_2 \in \mathbb{R}^2 \). Lemma 1 implies that, without loss of generality, we can restrict attention to bliss points of the form: \( m_1 = (m^1_1, 1) \) and \( m_2 = (m^2_1, 1) \). Two possible orthogonal jurisdictions read as follows: \( J_1 = (-1, m^1_1) \) and \( J_2 = (-1, m^2_1) \). It is useful to first express the true state of the
world in the new set of coordinates given by jurisdictions $J_1$ and $J_2$. Doing so we need to find $x$ and $y$ to solve $x \cdot J_1 + y \cdot J_2 = \theta$ so that

$$-x - y = \theta_1 \quad \text{and} \quad x m_1^1 + y m_2^1 = \theta_2.$$ 

A necessary and sufficient condition for this set of equations to have a solution is that the biases need to be linearly independent. Solving these two equations yields:

$$x = \frac{\theta_2 + \theta_1 m_2^1}{m_1^1 - m_2^1} \quad \text{and} \quad y = \frac{-\theta_2 - \theta_1 m_1^1}{m_1^1 - m_2^1}.$$  \hspace{1cm} (3)

We can describe the truth-telling equilibrium strategies and beliefs. In this equilibrium each minister reports the coordinate of the true state of the world in their jurisdiction:

$$s_1(\theta) = \frac{\theta_2 + \theta_1 m_2^1}{m_1^1 - m_2^1} \quad \text{and} \quad s_2(\theta) = \frac{-\theta_2 - \theta_1 m_1^1}{m_1^1 - m_2^1}.$$ 

In a truthful and fully revealing equilibrium, the Prime Minister follows her ministers’ advice and so implements policy following equation 1 in the proof of Proposition 1, and, as in that earlier result, her beliefs put all weight on the jointly reported state of the world (equation 2). As above, in order to prove the optimality of the prescribed strategies we compute the interim utility of minister 1 given minister 2’s behaviour:

$$u_1(y(s_1, s_2(\theta))) = -(s_1 + s_2(\theta) + \theta_1 - m_1^1)^2 - (s_1 m_1^1 - s_2(\theta) \cdot m_2^1 + \theta_2 - 1)^2 =$$

$$= -(s_1 - \frac{\theta_2 + \theta_1 m_2^1}{m_1^1 - m_2^1} + \theta_1 - m_1^1)^2 - (s_1 m_1^1 + \frac{\theta_2 + \theta_1 m_2^1}{m_1^1 - m_2^1} \cdot m_2^1 + \theta_2 - 1)^2$$

which we use to find the optimal response of the first minister. The first order condition of the optimization problem is:

$$\frac{\partial u_1(y(s_1, s_2(\theta)))}{\partial s_1} = 2 \left( s_1 - \frac{\theta_2 + \theta_1 m_2^1}{m_1^1 - m_2^1} + \theta_1 - m_1^1 \right) + m_1^1 \left( -s_1 m_1^1 + \frac{\theta_2 + \theta_1 m_2^1}{m_1^1 - m_2^1} \cdot m_2^1 + \theta_2 - 1 \right)$$

and its solution yields

$$s_1 \left( 1 + \left( \frac{m_2^1}{m_2^1 - m_1^1} \right)^2 \right) = \frac{1}{m_1^1 - m_2^1} \left( 1 + \left( \frac{m_1^1}{m_1^1 - m_2^1} \right)^2 \right) \theta_2 + \left( \frac{m_1^1}{m_1^1 - m_2^1} \right) \theta_1 m_2^1$$

which is precisely the behavior prescribed above. Minister 2’s optimal behavior is analogously proved. Beliefs are consistent with equilibrium behavior as they accumulate all mass on the
true state of the world. Finally, the Prime Minister’s behavior is optimal given beliefs: the Prime Minister implements the policy $y = -\theta$ that yields her preferred policy outcome $x = (0,0)$, thus has no incentive to deviate. □

**Note on Proof to Proposition 2**: When jurisdictions do not depend on any other minister’s bias, the equilibrium declarations depend on the other minister’s jurisdictions or biases. Equilibrium declarations can be interpreted geometrically as the new coordinates of the true state of the world in the coordinate system formed by both jurisdictions. When jurisdictions are not orthogonal (which is generally the case), the new coordinates will depend on the relative position of the new axis as can be shown in equations 3). This means that, in equilibrium, a minister takes into account the messages reported by other ministers so that the aggregation of reports does not lead to a final policy different to the one he expected.

**Proof of Proposition 3.** The proof of this proposition is an immediate consequence of Besley and Coate (1997). □

**Proof of Proposition 4.** The rationale of lemma 1 still applies when $n$ is larger than 2: in a fully revealing equilibrium jurisdictions need to be orthogonal to the biases. However, whereas when $n = 2$ the orthogonal direction is uniquely determined, when $n > 2$ this is no longer the case as there are now $n - 1$ orthogonal directions. This allows the Prime Minister to elicit full information when (at least) two ministers have linearly independent biases. To see this, consider the most extreme case where $n - 1$ ministers have exactly the same bias and 1 minister has a bias that is linearly independent to the rest. We can choose $n - 1$ directions in the orthogonal hyper-plane of the $n - 1$ ministers -one for each of them- and an orthogonal direction for the last minister that is not included in the previous hyper-plane. Given the initial linear independence of the biases this direction always exists. This set of directions is orthogonal to each minister’s biases and spans the whole $n$-dimensional policy space. Once we have determined this set of orthogonal jurisdictions we can follow the proof of proposition 2 and rewrite any point in the Euclidean space using the new set of coordinates. The coordinate in the direction of each jurisdiction is precisely the declaration of the minister that has authority on that jurisdiction. After receiving all ministers’ declarations, the posterior beliefs of the Prime Minister are concentrated on the true state of the world and so she implements a policy that yields her preferred policy outcome. □

**Proof of Proposition 5.** We begin by showing that the asymptotic behavior of the expected average cost in a random assignment problem is determined by that of the smallest order statistic when the costs are i.i.d. and their distribution function $F$ satisfies the following two conditions:
(1) \( \lim_{n \to \infty} F^{-1} \left( \frac{1}{n} \right) = 0 \)

(2) \( F \) is of positive decrease at 0 if there exists \( a \in (0, 1) \) such that \( \lim_{x \downarrow 0} \frac{F(x)}{F(ax)} > 1 \).

As we increase the number of policy dimensions, the greater number of possible assignments may increase the complexity of the Prime Minister’s problem but also increases the possibility of having particular assignments with arbitrarily low costs. Note that only low realizations of the costs matter and this is why we only need near zero conditions on the distribution function \( F \). More specifically, when \( AC_n \) is the average cost of the optimal assignment of \( n \) ministers to \( n \) jurisdictions, Frenk, van Houweninge, and Rinnoy Kan (1987) show that when conditions 1 and 2 above are satisfied the following condition is also satisfied:

\[
\limsup_{n \to \infty} E \{ AC_n \} F^{-1} \left( \frac{1}{n} \right) < \infty.
\]

Which, given condition (1) above implies that \( \lim_{n \to \infty} \{ AC_n \} = 0 \).

When there is a strictly positive probability that the costs of moving jurisdiction are arbitrarily small, \( F(x) > 0 \) for all \( x > 0 \) and the distribution is atomless in 0 we have that \( \lim_{n \to \infty} F^{-1} \left( \frac{1}{n} \right) = 0 \). Having a strictly positive probability of arbitrarily small realizations of the random variable implies that there exists \( r > 0 \) (possibly very small) where \( F'(x) = f(x) \) is bounded and \( f(x) > 0 \) for all \( x \in (0, r) \). Thus, \( \lim_{x \to 0} f(x) > 0 \) or \( \lim_{x \to 0} f'(x) > 0 \). In both cases, we can apply Hôpital’s rule at the limit in condition (2). That is, when \( \lim_{x \to 0} f(x) > 0 \), for all \( a < 1 \),

\[
\lim_{x \to 0} \frac{F(x)}{F(ax)} = \lim_{x \to 0} \frac{F'(x)}{aF'(ax)} = \lim_{x \to 0} \frac{f(x)}{af'(ax)} = \frac{1}{a} > 1.
\]

Instead, when \( \lim_{x \to 0} f(x) = 0 \) and \( \lim_{x \to 0} f'(x) > 0 \), we can further apply Hôpital’s rule to the last limit and obtain:

\[
\lim_{x \to 0} \frac{f(x)}{af'(ax)} = \lim_{x \to 0} \frac{f'(x)}{a^2f'(ax)} = \frac{1}{a^2} > 1.
\]

We have just shown that when there is a strictly positive and atomless probability that costs are arbitrarily small, conditions 1 and 2 above are satisfied. We can now use the results of Frenk, van Houweninge, and Rinnoy Kan (1987) and conclude that the expected average cost of the optimal assignment goes to zero as we increase the number of policy dimensions.

When there is a positive mass on 0, we can apply the results above simply by considering the costs to be \( \bar{x} = (x + \varepsilon) \) where \( x \) is distributed according to \( F \) and \( \varepsilon \) is uniformly distributed in \([0, r]\), where \( r > 0 \) is arbitrarily small. It follows that the average cost of the optimal assignment when costs are augmented by \( \varepsilon \) can only be higher. We are now in a situation where
there is a strictly positive probability that the costs of moving jurisdiction are arbitrarily small and where the distribution is atomless in 0. Thus the above proof applies. □

***Algorithm for Numerical Simulations***: In our simulation we assume there are \( n \) citizens whose bias with respect to the Prime Minister on each policy dimension is drawn from a normal distribution with zero mean and unit variance. Status quo jurisdictions are the coordinate axis (i.e. \( J_i = (0, ..., 0, 1, 0, ...0) \)). We define the minimum angle move that is required to make jurisdiction \( J_i \) orthogonal to the bias of citizen \( j \) as \( \alpha(m_j, J_i) \). We compute \( \alpha \) using the definition of the scalar product between two vectors, i.e.

\[
\alpha(m_j, J_i) = 90^\circ - \arccos \left( \frac{m_j^1}{\left( (m_j^1)^2 + \cdots + (m_j^n)^2 \right)^{0.5}} \right)
\]

Once we know the exact costs of assigning a citizen to a particular jurisdiction, we can apply the Hungarian algorithm (see Kuhn (1955)) to find the optimal assignment, i.e. the one that minimizes the average costs. We iterate our simulations 500 times and report the average results in figure 6 (the gray line in figure 6 corresponds to a situation with \( n \) policy dimensions and \( 2n \) citizens).

***Proof of Proposition 6***. The minister’s bliss point is \( m_i = (m_i^1, m_i^2) \) and his orthogonal jurisdiction is \( J_i = (-m_i^2, m_i^1) \). Jurisdictions are determined up to scalar transformations so it is useful to define the relative jurisdiction of \( J_i \) as \( RJ_i = \frac{m_i^1}{m_i^2} \). Note that this is the slope of the jurisdiction and a relative jurisdiction closer to 0 denotes more jurisdiction on policy \( x \) and a relative jurisdiction larger in absolute value denotes more jurisdiction on policy \( y \). It follows that an increase in jurisdiction in one policy dimension can only happen when there is a decrease in another.

An increase in the bias denotes a movement away from the Prime Minister’s ideal point. When \( pm^* = (0, 0) \), we should interpret an increase in the minister’s bias as an increase in absolute value. When \( m_i^2 \) is positive, an increase in the bias in the second dimension is an increase in \( m_i^2 \); instead, when \( m_i^2 \) is negative, an increase in the bias in the second dimension is a decrease in \( m_i^2 \).

We prove the proposition when \( m_i^2 > 0 \) and \( m_i^1 > 0 \) so that an increase in the bias in dimension 2 is an increase in \( m_i^2 \). Recall that an increase in the relative influence on policy \( x \) is captured by \( RJ_i \) being closer to 0. Given that \( m_i^1, m_i^2 > 0 \), the relative jurisdiction is negative thus an increase in the relative influence on policy \( x \) is an increase in \( RJ_i \). We need to show that \( \frac{\partial RJ_i}{\partial m_i^1} > 0 \). The partial derivative is \( \frac{m_i^1}{(m_i^2)^2} \) which is indeed positive when \( m_i^1 > 0 \).
In other words, as we increase the bias in the second dimension, the relative influence in the first dimension increases (whilst it decreases in the second).

In the expression of the partial derivative we observe that the rate of change is larger the lower the initial bias on policy $y$ (i.e. the lower $m_i^2$). This is equivalent to saying that the rate of change is larger, the higher the initial jurisdiction in policy $y$.

When the bliss point of the minister is closer to that of the Prime Minister’s, the rate of change is larger. To prove this, consider a minister with bias $m_j = \lambda m_i$ for $\lambda \in (0, 1)$. The orthogonal jurisdiction is invariant to $\lambda$ but the rate of change is now larger: $\frac{\partial R_j}{\partial m_j^2} = \frac{\partial R_i}{\partial m_i^2} \cdot \frac{1}{\lambda} > \frac{\partial R_j}{\partial m_j^2}$. The above results pertain to the case where there is an increase in the minister’s bias on one dimension only. By Lemma 1 we know that an increase in the magnitude of the bias with respect to the Prime Minister does not have any effect on the assigned jurisdiction. Building on this result we can replicate any change in the overall bias of a minister by an increase in the magnitude of his bias (which has no effect on his jurisdiction) and an increase or decrease in one component of the bias. □
REFERENCES


