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## **The Declining Talent Pool of Government**

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# The Declining Talent Pool of Government

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**Abstract.** We consider a government for which success requires high performance by talented ministers. A leader provides incentives to her ministers by firing those who fail. However, the consequent turnover drains a finite talent pool of potential appointees. The severity of the optimal firing rule and ministerial performances decline over time: the lifetime of an effective government is limited. We relate this lifetime to various factors including external shocks; the replenishment of the talent pool; and the leader's reputation. Some results are surprising: an increase in the stability of government and the exogenous imposition of stricter performance standards can both shorten the era of effective government, and an increase in the replenishment of the talent pool can reduce incumbent ministers' performance.

Commentators often highlight differences in the performances of governments. What accounts for these differences? At a basic level, performance depends upon the qualities of the ministers who form the executive and the actions they take during their time in office: we might expect high performance whenever talented individuals use their skills to pursue the collective goals of the government, rather than their private ambitions.

The inherent talents of office-holders have been apparent in many governments of note. For example, the remarkable legislative achievements of the United Kingdom's Liberal government of 1908–14 were arguably related to the combination of talent in Herbert Asquith's cabinet which included such notable figures as David Lloyd George and Winston Churchill. Similarly if we consider George Washington's first cabinet, or that of Abraham Lincoln, what is striking is the number of talented individuals in each. According to one view, the problem of enhancing performance is resolved by attracting high-calibre individuals to serve in office (Caselli and Morelli, 2004; Besley, 2006); once the talented have been induced to serve then they should be retained and not discarded. In describing his view, Besley (2006, p. 37) traced his influences to Key (1956, p. 10):

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“The nature of the workings of government depends ultimately on the men who run it. The men we elect to office and the circumstances we create that affect their work determine the nature of popular government.”

Yet to consider government performance only in the light of the talents of office-holders is to miss a critical piece of the puzzle; it is important to recognize “the circumstances we create that affect their work.” Even able ministers require the correct incentives to perform well. Absent such incentives a minister may pursue his private ambitions rather than the collective objectives of the government. It is the role of the executive’s leader to tackle this agency problem by harnessing the ambitions of ministers and focusing their energies on responding to the challenges of government. As Kearns Goodwin (2005) has argued recently, such was the genius of Abraham Lincoln who formed an effective cabinet that included his rivals for the Republican presidential candidacy in 1860.

So, to understand government performance we must consider the ministerial management strategy adopted by the executive’s leader. In so doing, we suggest that political agency problems differ from those in economic settings: the instruments available to an executive’s leader are blunter than those available in a private-sector organization. In the business world, incentive contracts can allow financial compensation to vary with performance measures. Even if a sophisticated incentive contract is not feasible, an agent may be offered a high “efficiency wage” (Akerlof, 1982; Shapiro and Stiglitz, 1984). These mechanisms are often absent in a political world: typically, ministerial salaries are not chosen by the executive’s leader and performance-related pay schemes are impossible.

The absence of performance-related pay does not eliminate the incentives to perform. Ministers value their positions and are motivated by a desire to keep their jobs. In using her prerogative to hire and fire the executive’s leader can align the incentives of her ministers with her own. This mechanism is central to the classic account of the development of cabinet government (Cox, 1987) as well as to more recent formal work: Indridason and Kam (2006) have argued that cabinet reshuffles can be used to bring departmental spending under control, while Dewan and Myatt (2007) have suggested that the judicious use of a firing rule can provide incentives for ministers to pursue radical policies even when, by so doing, they come under attack from interest groups. In the context of our paper, an executive’s leader encourages better performance by firing those who fail.

Alas, the blunt instrument of firing those who fail has unintended consequences. Even if ministers respond to their incentives by performing well, there will always be occasions when, despite their efforts, they fail. Fired ministers must be replaced and new talent must be found. So long as the pool of ministerial talent is deep, this presents no problem.

However, in many systems, particularly those involving parliamentary governance, the holders of ministerial office must be drawn from a shallower pool. In the United Kingdom, for instance, most ministers are drawn from the House of Commons; others are members of the House of Lords. The importance of having a large stable of talented ministers has been acknowledged elsewhere: for instance Laver and Shepsle (2000) argued that the size of the talent pool is an important determinant of government formation.

Here our focus is on the impact of a finite talent pool on the executive's leader's actions. When firing a transgressor, the executive's leader recognizes that she is depleting a finite reserve of ministerial talent. A tension emerges: in order to govern successfully, an executive must comprise talented ministers who perform well; however, providing the incentives to perform necessarily drains the reservoir of talent. The difficulty of filling ministerial positions when the pool of talent has evaporated has been subject to commentary. Paxman (2003, p. 209) recounted that Tristan Garel-Jones, a whip in a United Kingdom government of the 1980s and a close confidante of Prime Minister John Major, recalled scanning a list of fifteen candidates for a junior ministerial post and thinking

“I wouldn't employ a single one of them. The problem was that, if you include all the various ranks of ministers, you have to find maybe ninety people to form a government. You have perhaps 350 or so people to choose from. Once you've eliminated the bad, mad, drunk and over-the-hill, you've got rid of a hundred. You then have to pick ninety people out of a pool of 250. Is it any wonder that the calibre is so low?”

Our new account of government performance begins with this observation that talent can be scarce amongst the elected representatives from whom ministers are often chosen. A shallow talent pool constrains the resolution of agency problems: to provide incentives a leader must be prepared to reshuffle her cabinet, but in doing so she depletes her talent pool. The leader's firing rule—her response to a failure or scandal attributable to a minister—must balance the conflicting needs of incentive provision and talent retention. We use our analysis to explore the response of an optimal firing rule to various factors including the size of the talent pool, the arrival of external shocks, the possibility of talent-pool replenishment, and the need for a leader to maintain her reputation.

Emerging from our results is the conclusion that effective governments must eventually fail. Providing incentives necessitates ministerial resignations and the eventual evaporation of talent; the fall of a government and subsequent election provide an opportunity for a new government to enter with a fresh talent pool. Our conclusion resonates with the

views of Sartori (1997) who was perhaps the first to question (and dismiss) the view that long-lasting and effective government go hand in hand. He asked (Sartori, 1997, p. 113)

“Why is it important that governments should not fall? The answer generally is that stable government indicates effective government. Alas, no. Government stability stands for a mere duration; and government can be both long lived and impotent: their duration over time is by no means an indicator even less an activator of efficiency or efficacy.”

We explore this issue, focusing on the length of time that a government remains effective. Commentators often highlight the “first hundred days” of government as being its most important. But some governments maintain high performance for longer, whilst some fall short before a hundred days have passed. We examine how exogenous factors combine with an optimal firing rule to determine the lifetime of an effective government.

We also explore the effect of institutional procedures intended to enhance accountability of ministers and standards in public life and thus increase the effectiveness of government. For instance, in the United Kingdom a Committee into Standards in Public Life was set up by Prime Minister John Major (in October, 1994) to examine “concerns about standards of conduct of all holders of public office” and to make recommendations “to ensure the highest standards of propriety in public life.” Such standards impose a lower bound to the severity of the leader’s firing rule; a range of failures and scandals inevitably lead to a resignation. We explore the effects of an exogenous increase in the standards imposed on ministers; such an increase can reduce the lifetime of an effective government by shortening the era during which higher standards are endogenously used.

In an extension to our model we consider institutional features that might influence the ministerial talent pool. In many parliamentary systems selection into government is restricted to those who serve in the legislature. By contrast, in presidential systems ministers may be chosen from all walks of life (though if a president is constrained to selecting only those who are aligned with her policy goals then the problem of a finite pool remains). This feature appears attractive since it deepens the reservoir of talent. This is one reason why many parliamentary systems have seen attempts to build more inclusive cabinets. For example, the Italian cabinets led by Prime Ministers Giuliano Amato (1992–93) and Carlo Azeglio Ciampi (1993–94) contained so-called technocrats. Similarly, Prime Minister Gordon Brown of the United Kingdom recently called for a “cabinet of all the talents” including ministers selected for their professional expertise.<sup>2</sup> We explore the

<sup>2</sup>In Spain, Ireland, and several countries in Middle and Eastern Europe ministers can also be drawn from outside the parliament. Recent work by Dowding and Dumont (2008) provides details.

effects of introducing new blood into the talent pool. Perhaps surprisingly, a replenishing talent pool can reduce rather than enhance the performance of incumbent ministers.

We conclude our analysis by assessing the credibility of the Prime Minister's firing rule. The promise to fire a failing minister provides incentives, but when such a failure is observed the Prime Minister would prefer not to carry out her threat and so preserve her pool of ministerial talent. To maintain credibility she must carry out her promises. When the ministerial talent pool is fixed, this is impossible: eventually her talent pool dries up and she will be unwilling to fire her last remaining talented minister. An unravelling argument then leads to the collapse of her entire ministerial management strategy. However, talent-pool replenishment can restore credibility by creating a future in which the Prime Minister's reputation matters, and this can induce her to carry out her threats.

#### A MODEL OF MINISTERIAL PERFORMANCE, SCANDALS, AND RESIGNATIONS

We begin our formal analysis by describing a simple model of ministerial performance.

**Ministerial Payoffs and Performance.** The Prime Minister appoints a minister to his post by selecting him from a talent pool of suitable candidates. During his time in office a minister enjoys a flow payoff of  $\bar{v} > 0$  which reflects his salary, the perquisites of office such as a ministerial car and research staff, and the personal benefits of being in a position of power and influence. Our model differs from principal-agent analyses of labor relations in one critical aspect: the material benefits of office are exogenously fixed, and so performance-related pay cannot be used to provide incentives.

While in post the minister controls a single variable: at each moment in continuous time he chooses his performance  $e \in [0, \bar{e}]$ , where the upper bound  $\bar{e}$  is one aspect of the minister's talent. Higher performance helps the government, but is costly for the minister; this divergence of interests is the source of an agency problem. Specifically, the minister incurs a flow cost of  $ce$ .<sup>3</sup> Rather than think of this as the cost of effort, we instead think of it as the opportunity cost to the minister of not pursuing his own personal agenda. The parameter  $c > 0$  is a second aspect of the minister's talent; when  $c$  is low the minister finds it relatively easy to follow the government's agenda.

Bringing the payoff components together, a minister enjoys a net flow payoff of  $\bar{v} - ce$  so long as he retains his job, and he is free to vary his performance over time if he wishes. He receives nothing if either he is fired or the government falls. If the minister's tenure in office were fixed and he faced no other incentives then he would choose  $e = 0$ .

<sup>3</sup>The linear functional form is without loss of generality; the property we use is the convexity of the relationship between the arrival rate of scandals (described in the next subsection) and the cost of performance.

**Failures and Scandals.** During her time in office, which ends only when her government falls, the Prime Minister (whose preferences perfectly reflect those of her government) enjoys a flow payoff of  $\bar{w} > 0$ . However, she suffers a penalty whenever her minister encounters a policy failure or becomes embroiled in a scandal. When a minister falls short in such ways he faces a resignation call, made by the opposition, by the media, or even by the wider membership of the governing party. We refer to such resignation calls as scandals. The magnitude of scandals varies: the penalty  $s$  suffered by the Prime Minister: is drawn from the cumulative distribution function  $F(\cdot)$  with expectation  $\bar{s} \equiv E[s]$ .<sup>4</sup>

Scandals arrive according to a Poisson process with arrival rate (or, equivalently, hazard rate) of  $\bar{\lambda} + \lambda(e)$ . This arrival rate of scandals comprises two components.

Firstly, there is an exogenous hazard rate  $\bar{\lambda} > 0$  which is unrelated to the minister's own actions. An increase in this term might reflect elements of the minister's portfolio which place him at particular risk. For example, the difficulty of the task facing him may increase his exposure or policies enacted by a predecessor may come back to haunt him.<sup>5</sup>

Secondly, there is an endogenous hazard rate  $\lambda(e)$  which depends upon the minister's performance. This rate is strictly decreasing, convex, and continuously differentiable in  $e$ . By increasing his performance a minister increases the quality of his policies and their execution and so, correspondingly, reduces the likelihood that resignation calls are made. For some of our results we highlight two illustrative functional forms for  $\lambda(e)$ .

**Definition.** *The arrival rate of scandals is inversely proportional to performance if  $\lambda(e) = 1/e$ . The arrival rate is linearly decreasing in performance if  $\lambda(e) = \hat{e} - e$  where  $\hat{e} \geq \bar{e}$ .*

Bringing the payoff components of the Prime Minister together, she enjoys a net expected flow payoff of  $\bar{w} - [\bar{\lambda} + \lambda(e)]\bar{s}$  when her minister devotes performance  $e$  to his portfolio of tasks. Notice that a conflict of interest arises: other things equal, the minister wishes to choose  $e = 0$  to minimize his cost of performance, whereas the Prime Minister would like him to choose  $e = \bar{e}$  and so slow down the arrival of damaging scandals.

**Ministerial and Government Termination.** So far, we have noted that the minister and his boss enjoy flow payoffs of  $\bar{v} - ce$  and  $\bar{w} - [\bar{\lambda} + \lambda(e)]\bar{s}$  respectively. These payoffs are enjoyed only while these actors survive in office. But what determines their survival?

<sup>4</sup>Here performance affects the arrival rate of scandals but not their severity. We could change our model to relate the severity of scandals to performance. Under this alternative specification, the tension we highlight between incentive provision and talent retention remains.

<sup>5</sup>In the United Kingdom during the 1960s and 70s successive ministers attempted to reform industrial relations and failed. The most famous, Barbara Castle, was undoubtedly a talented minister who gave her all. Arguably it was the nature of her task, rather than her poor performance, which led to her downfall.

Both actors are exposed to a background risk of losing office. Following the early empirical literature we specify a risk of critical events, such as a financial crisis or international conflict, that may topple a government (Browne, Frendreis, and Gleiber, 1984, 1986; Diermeier and Stevenson, 2000). We model these risks using a Poisson process with arrival rate  $\gamma > 0$ . Consistent with the later literature on government termination there may be institutional features that enhance a government's durability when shocks arrive. For example King, Alt, Burns, and Laver (1990) found that majority status and the existence of an investiture requirement are both positively related to government duration. Here, the existence of these features corresponds to a lower value of  $\gamma$ .<sup>6</sup> The parameter  $\gamma$  can also be interpreted as a continuous-time discount rate, and so can represent impatience.

A minister faces a further risk: a scandal prompts a call for his resignation. In response the Prime Minister can retain him or to fire him. This risk is endogenous since it depends upon the attitude of the Prime Minister to resignation calls. Furthermore, the minister can reduce his exposure via his performance. We suppose that some risk is inescapable: the Prime Minister is unable to protect her minister in the face of extreme scandals. Formally, there is a critical threshold  $s^\ddagger$  such that a minister is always fired whenever  $s > s^\ddagger$ . However, the Prime Minister retains her hiring-and-firing prerogative whenever  $s \leq s^\ddagger$ .

The Prime Minister also faces a further risk of losing office. We assume that she can only survive so long as she has talent available to her: if a minister is forced to resign and there is no suitably qualified replacement then the government is removed from office through some mechanism, constitutional or otherwise. We suppose that the government begins with a talent pool comprising  $n$  potential ministerial appointees. Each time a minister is removed from office, the pool loses a member. (In a later extension to our basic model, we allow the pool to become replenished by the arrival of new members.)

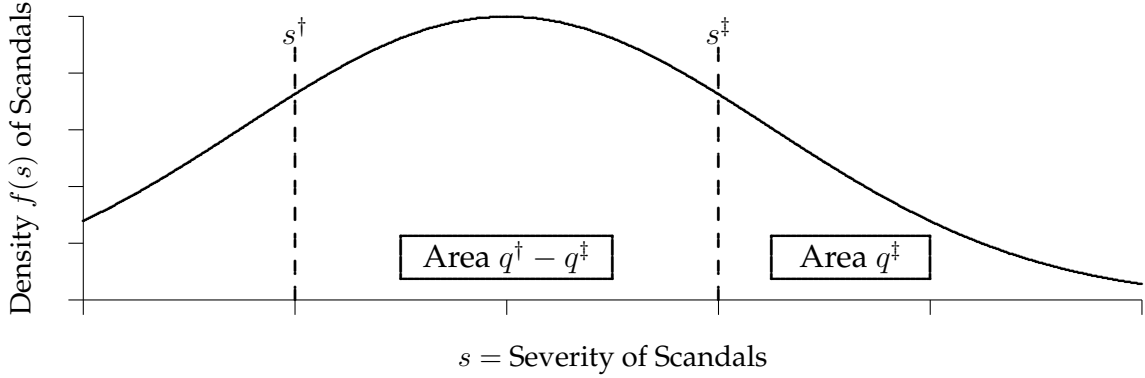
#### MINISTERIAL PERFORMANCE

Here we study the performance of a minister and his response to the hiring-and-firing stance of a Prime Minister (Propositions 1–2). Fixing the firing rule, we relate the expected length of the minister's career to factors including the value of office, the exogenous risk of government termination, and the exogenous risk of resignation calls (Proposition 3).

**The Prime Minister's Firing Rule.** We have already noted that the actors in our model differ in their preferred choice of performance. We assume that the performance of the minister cannot be directly observed by the Prime Minister, leading to a classic moral-hazard problem. However, there is an imperfect performance measure: the arrival of

<sup>6</sup>Similarly, and following Warwick (1992), a lower  $\gamma$  may capture relatively stable economic conditions.





*Notes.* The area  $q^\dagger \equiv 1 - F(s^\dagger)$  is the unavoidable risk of resignation following a scandal, while  $q^\dagger - q^\dagger = F(s^\dagger) - F(s^\dagger)$  is the additional risk to which a minister is exposed when the Prime Minister uses a stricter threshold  $s^\dagger < s^\dagger$ .

FIGURE 1. The Prime Minister's Firing Rule

scandals and subsequent resignation calls. Although the Prime Minister is unable to observe directly the performance of her ministers, she can respond to a scandal.

We restrict attention (with little loss of generality) to the following (stationary) firing rule: if a minister faces a resignation call then the Prime Minister fires him if and only if  $s > s^\dagger$ . (Later in the paper we extend our analysis by allowing this threshold to depend upon the state of the ministerial talent pool.) The threshold  $s^\dagger$  must satisfy  $s^\dagger \leq s^\dagger$  since resignation calls are irresistible (by assumption) when  $s > s^\dagger$ . Given that the penalty of a scandal is drawn from  $F(\cdot)$ , the firing rule means that the minister is fired with probability  $q^\dagger \equiv 1 - F(s^\dagger)$ . This probability satisfies  $q^\dagger \geq q^\dagger$  where  $q^\dagger \equiv 1 - F(s^\dagger) > 0$ ; Figure 1 illustrates. Given this firing probability, the hazard rate of the minister's resignation is  $[\bar{\lambda} + \lambda(e)]q^\dagger$ . Fixing his performance, this rate is increasing in  $q^\dagger$ . However, the endogenous hazard rate  $\lambda(e)$  of scandals responds to  $q^\dagger$  via  $e$ , and so to understand the overall effect of  $q^\dagger$  we must consider the minister's performance choice.

**The Choice of Performance.** Facing the probability  $q^\dagger$  of losing his job following the arrival of a scandal, a minister is concerned to slow the arrival of resignation calls. In doing so he balances the direct flow cost of performance against the change in the hazard rate. The second factor is weighted by the value he places on his career.

To calculate this value explicitly we write  $V$  for the present value of a minister's career during his time in office, given that he chooses his performance optimally. His environment is stationary and so this value will be constant over time. Thus,

$$\underbrace{\bar{v} - ce}_{\text{flow benefits and costs}} = \underbrace{[\bar{\lambda} + \lambda(e)]q^\dagger V}_{\text{risk of resignation}} + \underbrace{\gamma V}_{\text{exogenous risk}}. \quad (1)$$

The first term comprises the flow benefits of holding office  $\bar{v}$  minus the cost  $ce$  of performance. The second term reflects the endogenous risk of a successful resignation call: scandals arrive according to the hazard rate  $\bar{\lambda} + \lambda(e)$ ; the minister is subsequently forced to resign with probability  $q^\dagger$ ; and following his resignation the minister loses the value  $V$  of his career. Finally, the third term stems from the exogenous risk of government failure.

Given the optimal choice of performance  $e$  we can use Equation (1) to solve for the value  $V$  of the minister's career. However, we need to characterize the optimal choice of  $e$ . When increasing his performance the minister balances the marginal flow cost  $c$  against the marginal impact on the expected flow penalty of resignation calls. Bringing these two effects together, the optimal choice of  $e$  satisfies the simple first-order condition

$$c = -\lambda'(e)q^\dagger V \quad (2)$$

so long as this solution satisfies  $0 < e < \bar{e}$ ; otherwise, performance either attains its upper bound (if  $c \leq -\lambda'(\bar{e})q^\dagger V$ ) or drops to its lower bound (if  $c \geq -\lambda'(0)q^\dagger V$ ). Equations (1) and (2) can be used to solve jointly for the minister's optimal performance  $e$  and the value of his career  $V$ . However, some immediate observations are apparent from an inspection of Equation (2). Since  $\lambda(e)$  is convex (there are decreasing returns to ministerial performance) the solution for  $e$  must, other things equal, be increasing in  $q^\dagger V$  (the penalty of a resignation call) and decreasing in  $c$  (the opportunity cost of performance).

**The Effect of the Firing Rule.** Although the Prime Minister cannot use monetary transfers to encourage performance she does control the tenure of her minister via her firing rule, which affects him via the resignation probability  $q^\dagger$ . There are two effects to consider.

Firstly, as  $q^\dagger$  increases it becomes more likely that a scandal terminates a ministerial career. A minister may respond by raising his performance, thereby minimizing the risk of scandal. This is a substitution effect: Equation (2) reveals that an increase in  $q^\dagger$  encourages a minister to substitute away from private concerns and toward the government's agenda.

Secondly, an increase in  $q^\dagger$  has an income effect. Conditional on his performance, a minister is more likely to lose his job when  $q^\dagger$  is large. This reduces the present value  $V$  of his career. He cares less about keeping his job and so is more willing to divert his efforts toward his own private projects. This income effect can weaken his performance.

Whereas these two effects of an increasingly severe firing rule conflict, their net effect is indeed to enhance performance as we confirm in our first formal result.

**Proposition 1.** *Ministerial performance increases with the severity  $q^\dagger$  of the firing rule, the benefits  $\bar{v}$  of office, and the minister's talent  $\bar{e}$ , but decreases with the opportunity cost  $c$  of performance, the exogenous risk  $\gamma$  of government termination, and the exogenous risk  $\bar{\lambda}$  of resignation calls. When the endogenous risk of scandals is inversely proportional to performance then*

$$e = \frac{\sqrt{1 + \frac{\bar{v}}{c}[\frac{\gamma}{q^\dagger} + \bar{\lambda}]} - 1}{\frac{\gamma}{q^\dagger} + \bar{\lambda}} \quad (3)$$

so long as this solution satisfies  $e < \bar{e}$ ; the minister chooses performance  $e = \bar{e}$  otherwise.

Our comparative-static results indicate the importance of income effects. Any factor that increases the value of a ministerial career increases performance and vice-versa. For example, performance is increasing in the level of office-holding benefits. These benefits may increase whenever the minister receives a more stately home, a larger limousine, or an increase in his entourage.<sup>7</sup> In response, his career becomes more desirable (relative to languishing on the backbenches) and so the minister devotes more effort to his tasks. This enhances the performance of his department and reduces the risk of resignation calls.

Similarly, as the exogenous hazard  $\bar{\lambda}$  of scandals increases it becomes more likely that, irrespective of performance, the minister's career will be curtailed. This lowers the value of his career; with less to lose his performance falls. One interpretation of a higher  $\bar{\lambda}$  is that the minister may be haunted by the scandals of his predecessor: a policy failure may have more to do with a previous incumbent than anything a minister has done since being in post. The anticipation of such a scandal lowers the value of a ministerial career. Accordingly, since he has less to lose he devotes less effort to his ministerial tasks and the performance of his department declines, thus reinforcing the likelihood of a scandal.

Finally, performance is decreasing in  $\gamma$ . Our interpretation of  $\gamma$  is the exogenous background risk of government termination. As the expected lifespan of the government shortens, the value of a ministerial career drops, and so the performance of the minister declines. The comparative-static effects of both  $\bar{\lambda}$  and  $\gamma$  reveal that the risks are self-reinforcing: whenever a minister or government is expected to fail the income effect depresses the incentives to perform and so accelerates the onset of further scandals.

**All-or-Nothing Performance.** To derive further insight we turn to our second illustrative specification in which the arrival rate of scandals is linearly decreasing in the minister's performance. This linearity ensures that a "bang bang" solution is optimal: the minister chooses either the maximum feasible performance or devotes no effort to his tasks.

<sup>7</sup>Such benefits correspond to the payment of an efficiency wage (Akerlof, 1982; Shapiro and Stiglitz, 1984).

**Proposition 2.** *When the arrival rate of ministerial scandals is linearly decreasing in performance the Prime Minister can only induce positive performance from her minister if  $\bar{v} \geq c[\gamma + \bar{\lambda} + \hat{e}]$ . If this is so, then the minister provides performance that satisfies  $e = \bar{e}$  if and only if  $q^\dagger \geq q^*$  where*

$$q^* = \frac{c\gamma}{\bar{v} - c[\bar{\lambda} + \hat{e}]} \quad (4)$$

*Otherwise, when either  $\bar{v} < c[\gamma + \bar{\lambda} + \hat{e}]$  or  $q^\dagger < q^*$ , the minister chooses  $e = 0$ .*

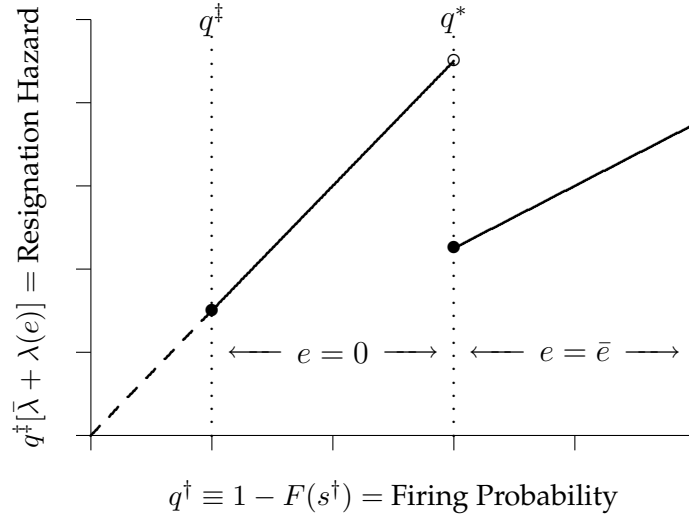
To induce high-performance the Prime Minister must threaten to sack a scandal-hit minister with sufficiently high probability. However, she can do no worse than respond to every resignation call. Hence  $q^* \leq 1$  or equivalently  $\bar{v} \geq c(\gamma + \bar{\lambda} + \hat{e})$ : to induce high performance, office benefits must be large. Others have argued that such benefits can attract higher quality politicians (Caselli and Morelli, 2004; Messner and Polborn, 2004). Our focus on income effects suggests that the same factor influences action choice. Turning to the right-hand side of the inequality, high performance becomes harder to induce as exogenous risks grow: such risks erode the value of a ministerial career.

When  $\bar{v} > c(\gamma + \bar{\lambda} + \hat{e})$  (local) changes in the various parameters no longer influence the feasibility of high performance but instead change the severity of the firing rule required. From Equation (4),  $q^*$  is decreasing in the value of office benefits, whilst increasing in the opportunity cost of performance and the exogenous-risk parameters. Factors that encourage performance, then, may feed through into a more lenient response to scandals.

**Resignations.** Before concluding this section we consider the arrival of resignations, rather than scandals. This hazard rate is the product of two effects: the arrival rate  $\bar{\lambda} + \lambda(e)$  of scandals, and the proportion  $q^\dagger$  of scandals that result in the minister's resignation.

Fixing the other parameters, the effect of an increase in  $q^\dagger$  can go either way: its direct effect is to increase the hazard rate, while the consequent increase in performance (Proposition 1) reduces it. The conflicting effects are most easily seen when the arrival of scandals is linearly decreasing in performance. Local to  $q^*$  and starting from  $q^\dagger < q^*$  an increase in the firing probability to  $q^\dagger > q^*$  prompts a jump in performance from  $e = 0$  to  $e = \bar{e}$  and so a discrete fall in the arrival of resignations. However, local increases in  $q^\dagger$  away from  $q^*$  generate more resignations with no associated increase in performance (Figure 2.)

Fixing the firing rule, however, the effects of all other parameters are clear. An increase in the exogenous scandal risk  $\bar{\lambda}$  directly increases resignations whilst also increasing the arrival of scandals via reduced performance (Proposition 1). All other parameters feed via the minister's performance choice. The tenure of a minister in post is then inversely related to the hazard of resignations and the exogenous arrival  $\gamma$  of government termination. Assembling these simple observations we obtain the following formal result.



*Notes.* The relationship between the firing probability  $q^\dagger$  and the arrival of resignations can be non-monotonic. Here the hazard rate of scandals is linear in performance. For  $q^\dagger \leq q^\dagger < q^*$ , the resignation hazard is  $q^\dagger[\bar{\lambda} + \lambda(0)]$ , which is increasing in  $q^\dagger$ . However, at  $q^\dagger = q^*$  the minister's performance jumps up, and so the resignation hazard falls to  $q^\dagger[\bar{\lambda} + \lambda(\bar{e})]$ , before continuing to increase with  $q^\dagger$  once again.

FIGURE 2. Firing Rules and the Arrival Rate of Resignations

**Proposition 3.** *Given the choice of a particular firing rule, the expected length of a minister's career increases with the benefits  $\bar{v}$  of holding office and the minister's talent  $\bar{e}$ , but decreases with the opportunity cost  $c$  of performance, the exogenous hazard  $\gamma$  of government termination, and the exogenous hazard  $\bar{\lambda}$  of resignation calls. When the arrival rate of ministerial scandals is linearly decreasing in performance and the Prime Minister adjusts her firing rule to ensure that  $q^\dagger = q^*$  then the same set of comparative-static predictions continue to apply.*

The effect of the exogenous risk of government termination deserves further comment. A growing literature has considered the tenure of individual ministers rather than of their governments. So far we know little about how these aspects relate to each other. In their study of the French Fourth and Fifth Republics, Huber and Martinez-Gallardo (2004) questioned whether higher turnover of governments is positively related to higher turnover of ministers.<sup>8</sup> Here these features are connected, and so our analysis enhances our understanding of the political phenomena that are affected by exogenous shocks. Thus whilst Lupia and Strøm (1995) and Diermeier and Stevenson (2000) have focused on how an increase in the *ex ante* exogenous risk of termination affects renegotiation of the initial government bargain, here we show how it can affect individual ministers.

<sup>8</sup>Huber and Martinez-Gallardo (2004) analyzed an empirical situation which differs somewhat from the scenario studied here: they found that although governments may fall, individual ministers can enjoy longer lives. In our theoretical model a minister's life automatically ends with the demise of his government.

## THE OPTIMAL FIRING RULE

Having established the optimal behavior of an individual minister, we now turn our attention to the firing rule operated by the Prime Minister. We assume for now that she is able to commit fully to her desired firing rule; later in the paper we will revisit this assumption, and so ascertain the credibility of different firing rules.

**Firing Rules and the Size of the Talent Pool.** The basic ingredient of the Prime Minister's strategy is her firing rule: she operates a threshold  $s^\dagger \leq s^\ddagger$ , interpreted as the maximum severity of a scandal that a minister is permitted to experience while in post; this is equivalent to selecting the firing probability  $q^\dagger \equiv 1 - F(s^\dagger)$ .

Of course, the Prime Minister might wish to change her firing rule to suit her environment. The aspect of her environment that interests us is the size of her talent pool. Recall that she begins with  $n$  potential ministerial appointees. She begins by hiring one of them, and so the initial size of her talent pool is  $n - 1$ . Whenever a sufficiently severe scandal arrives she fires the incumbent minister and replaces him; the size of her talent pool declines. We write  $k \in \{0, 1, \dots, n - 1\}$  for the current size of the ministerial talent pool. If a minister is fired when  $k = 0$  then, since the talent pool has evaporated, the government falls. We note that, absent any replenishment of the talent pool (we will allow for the possibility of such replenishment later in our paper)  $k$  cannot grow. Furthermore, since  $\bar{\lambda} > 0$  and  $q^\ddagger > 0$  there is always the risk of a scandal that results in a resignation, and hence  $k$  will (in expectation) strictly fall over time. The rate of decline depends, of course, on the firing rule and the response of the ministers to whom it applies.

As  $k$  declines the Prime Minister may become more concerned with the end of her time in office and so may adjust her firing rule. We write  $s_k^\dagger \leq s^\ddagger$  for the threshold she operates when there are  $k$  remaining members of her talent pool. Equivalently,  $q_k^\dagger \equiv 1 - F(s_k^\dagger)$  is the resignation probability faced by a scandal-hit minister when there are  $k$  potential replacements. Corresponding to this probability is the performance  $e_k$  chosen by the incumbent. Of central interest to us is the relationship between the size  $k$  of the talent pool and the endogenous variables  $q_k^\dagger$  and  $e_k$ ; these variables then jointly determine the arrival rate of resignations and hence the expected length of each minister's career.

**Optimal Firing Rules and Declining Performance.** We have shown (Proposition 1) that a stricter firing rule enhances ministerial performance. If this were the only concern of the Prime Minister then she would be as strict as possible. However, firing ministers is costly: the talent pool dries up, and the end of the government draws nearer. Here we characterize the optimal firing rule in the presence of these conflicting pressures.

To do this we begin by calculating the present value of the Prime Minister's career (equivalent here to the value of her government). We write  $W_k$  for this value when there are  $k$  members of the talent pool. The flow payoffs accruing to the Prime Minister must balance the arrival of events which change the composition of the talent pool. For  $k \geq 1$  we have

$$\underbrace{\bar{w} - [\bar{\lambda} + \lambda(e_k)]\bar{s}}_{\text{flow benefits and costs}} = \underbrace{q_k^\dagger [\bar{\lambda} + \lambda(e_k)] [W_k - W_{k-1}]}_{\text{risk of talent depletion}} + \underbrace{\gamma W_k}_{\text{exogenous risk}}. \quad (5)$$

The first term comprises the flow benefits of holding power minus the expected inflow of costly scandals. The second term reflects the risk of successful resignation calls: scandals arrive at rate  $\bar{\lambda} + \lambda(e_k)$ ; the minister quits with probability  $q_k^\dagger$ ; following a resignation the loss  $W_k - W_{k-1}$  is the marginal value of the  $k$ th member of the talent pool. Finally, the third term stems from the exogenous risk of government termination. A variant of Equation (5) holds when  $k = 0$ : since the talent pool is empty, the Prime Minister loses  $W_0$  when her minister leaves office and so  $\bar{w} - [\bar{\lambda} + \lambda(e_0)]\bar{s} = ([\bar{\lambda} + \lambda(e_0)]q_0^\dagger + \gamma)W_0$ .

Once the Prime Minister has specified and committed to her strategy (a sequence of firing probabilities indexed by  $k$ ) we can use Equation (5) to calculate the value of her government. However, her strategy will be chosen optimally; the Prime Minister will balance the different effects of a change in  $q_k$ .

So what are these effects? Consider an increase in the firing probability  $q_k^\dagger$ . The most direct effect is higher performance and so lower penalties from the arrival of scandals. The change also influences the arrival of resignations: the increase in performance slows the arrival of scandals, while the higher firing probability increases the fraction of scandals which turn into resignations. As noted earlier in the paper, the net effect on  $[\bar{\lambda} + \lambda(e_k)]q_k^\dagger$  can take either sign. However, if the net effect is negative, so that an increase in the firing probability reduces the hazard of resignations, then the Prime Minister will always choose to increase  $q_k^\dagger$ . This, in turn, means that  $q_k^\dagger$  will always be raised to a point at which either the minister's performance satisfies  $e_k = \bar{e}$ , or a point at which  $[\bar{\lambda} + \lambda(e_k)]q_k^\dagger$  is increasing in  $q_k^\dagger$ . Furthermore, this logic implies that if different firing probabilities are employed for different talent-pool sizes, then higher choices of  $q_k^\dagger$  must be associated with an accelerated arrival of resignations. We record this observation as a simple lemma.

**Lemma 1.** *Greater severity of the firing rule is always associated with an increase in the arrival of resignations: for talent-pool sizes  $k$  and  $k'$  if  $q_k^\dagger > q_{k'}^\dagger$  then  $[\bar{\lambda} + \lambda(e_k)]q_k^\dagger > [\bar{\lambda} + \lambda(e_{k'})]q_{k'}^\dagger$ .*

Combining this lemma with our earlier results, an increase in the severity of the firing rule is associated with greater performance but more resignations; the last factor results in a reduction in the length of a ministerial career. However, we have not yet established the relationship between these co-moving variables and the size  $k$  of the talent pool.

To move further we consider the effect of a local increase in  $q_k^\dagger$ . When performance responds continuously to the firing probability such an increase is optimal if and only if

$$\underbrace{-\bar{s} \frac{d\lambda(e_k)}{dq_k^\dagger}}_{\text{fewer scandals}} > \underbrace{(W_k - W_{k-1}) \frac{d[(\bar{\lambda} + \lambda(e_k))q_k^\dagger]}{dq_k^\dagger}}_{\text{quicker talent depletion}}. \quad (6)$$

The left-hand side captures the effect of a stricter regime (higher  $q_k^\dagger$ ) on performance: a marginal increase in  $q_k^\dagger$  leads to fewer resignation calls and so the left-hand side is positive. The right-hand side captures the cost of this stricter regime: as the Prime Minister fires more often she runs down her talent pool. The size of the talent-depletion effect depends directly on the loss  $W_k - W_{k-1}$  experienced when a minister is fired. This implies that the Prime Minister is more willing to fire a member of her team whenever the marginal value of a talent pool member is small. Intuitively, this marginal value is small whenever the talent pool is large, as confirmed by the following lemma.

**Lemma 2.** *The value of the government is increasing in the size of the talent pool but exhibits decreasing returns to talent-pool size:  $W_k$  is increasing in  $k$ , while  $W_k - W_{k-1}$  is decreasing in  $k$ .*

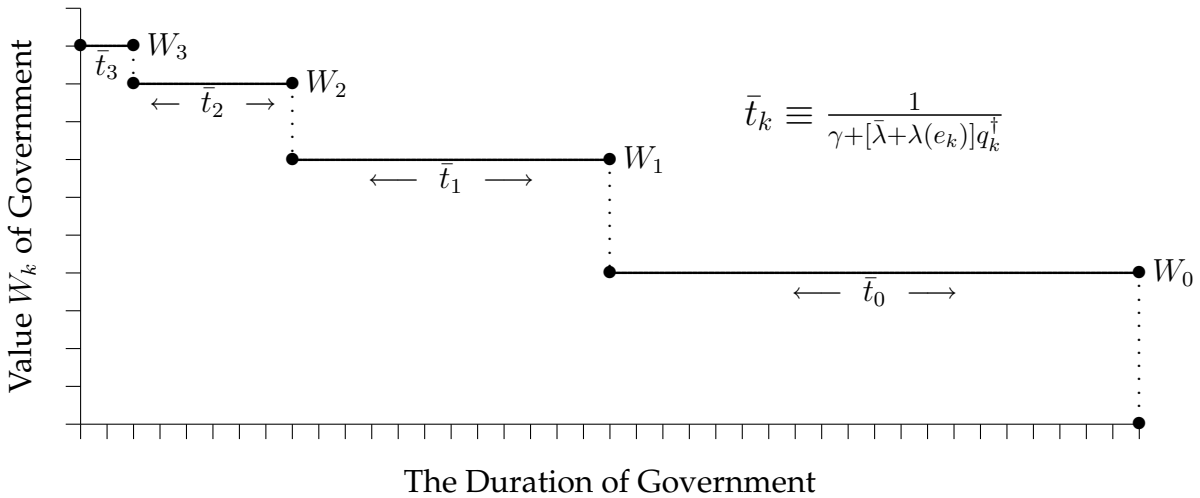
Combining Lemma 2 with the insight of Equation (6) we follow these logical steps: when the talent pool is large (high  $k$ ) the marginal value of a talent pool member is small; this reduces the severity of the talent-pool depletion effect; the Prime Minister is more willing to fire and hence  $q_k^\dagger$  is high; higher firing probabilities are associated with higher performance and the higher arrival rate of ministerial resignations. This logic results in our next formal proposition which is central to our paper.

**Proposition 4.** *Under an optimal firing rule, the firing probability  $q_k^\dagger$ , ministerial performance  $e_k$ , and the arrival rate  $q_k^\dagger \times [\bar{\lambda} + \lambda(e_k)]$  of resignations all increase with the size  $k$  of the talent pool. If  $\gamma$  is small enough, so that there is little exogenous risk of government failure, then the expected rate of decline  $q_k^\dagger [\bar{\lambda} + \lambda(e_k)] (W_k - W_{k-1})$  of the government's fortunes is increasing in  $k$ .*

Since  $k$  is inversely related to the number of ministers who have previously served, the result shows that a minister's hazard rate is affected not only by his own actions but also by the cumulative number of resignations from the government he serves. His expected time in post is longer when the talent pool is shallow. Of course, the talent pool evaporates over time, since the arrival rate of scandals and the firing probability are both strictly positive. We can, therefore, re-state Proposition 4 in terms of time-based expectations.

**Corollary to Proposition 4.** *Under an optimally chosen firing rule, the firing probability, the ministerial performance, the arrival rate of ministerial resignations, and (when  $\gamma$  is sufficiently small) the rate of decline of the government's fortunes are all expected to fall over time.*





*Notes.* Starting with a talent-pool of size  $k = 3$ , the figure illustrates the value  $W_k$  of the Prime Minister's career (or, equivalently, the value of her government) as her talent-pool declines. The height of each step reflects the decrease in value ( $W_k - W_{k-1}$ ) as she runs down her talent-pool. This loss increases as  $k$  declines and so  $q_k$  declines with  $k$ . The length  $\bar{t}_k$  of each step illustrates the expected tenure of a minister: this is inversely related to the risk he is exposed to in turn comprises the exogenous risk  $\gamma$  of government failure together with the endogenous risk  $[\bar{\lambda} + \lambda(e_k)]q_k^\dagger$  of his resignation. This tenure is increasing as the size of the talent-pool declines: the expected tenure of a minister increases although his performance decreases. The expected rate of decline of the government's fortunes is determined by  $(W_k - W_{k-1})/\bar{t}_k$  which is the steepness of each step. When  $\gamma$  is sufficiently small this expected rate of decline falls as the talent pool evaporates.

FIGURE 3. The Declining Talent Pool of Government

**A Decline in Government Performance.** The severity of the optimally chosen firing rule declines as the talent pool shrinks.<sup>9</sup> Hence, as time goes on, not only do ministers perform less well, they also stay in their positions for longer. This result has implications for the interpretation of published statistics on ministerial hazard rates. Berlinski, Dewan, and Dowding (2007) argued that “length of tenure must be some indicator of performance” and used fixed indicators of a minister's quality to explain variation in hazard rates. Our theoretical result provides a caveat: fixing his talent, a minister who enters the government late in its term (when the talent-pool is depleted) performs less well than one who served earlier (and was replaced) but survives for longer. Thus to understand the relationship between ministerial performance and tenure we must focus attention on the strategy deployed by the Prime Minister.

<sup>9</sup>This comparative-static prediction is consistent with results reported by Huber and Martinez-Gallardo (2003) who, when using the size of the legislature as their measure for talent-pool size, found a positive effect of an additional seat on ministerial hazard rates.

As a further illustration of Proposition 4 consider again the second United Kingdom government of Prime Minister John Major referred to in our introduction. Despite unexpectedly winning a majority in 1992, by the middle of 1993 the government had been rocked by successive scandals involving first David Mellor (Secretary of State for Heritage) and Michael Mates (Minister of State for Northern Ireland). Both were forced to resign and were replaced in the cabinet. As more ministers became tainted by scandal and forced to resign the performance of Major's government declined. Ministers became involved in public feuds over policy, most notably over the Maastricht Treaty, and began jockeying for position in the leadership contest that would follow Major's eventual downfall. By the middle of 1993 John Major had become so exasperated by the performance of three of his main cabinet colleagues that, in an astonishing outburst, he openly branded them as "bastards". And yet Major could not fire his cabinet rebels. One reason was that the reserve of talent had been so depleted as to make further turnover untenable. As recounted by Paxman (2003, p. 210), Tristan Garel-Jones observed that

"... things had got even worse. Not only had the overall number of Conservative MPs fallen, while the number of incompetents and has-beens had grown, there was also a much larger group who had already served in government and been worn out or found wanting by the process. Small wonder that it was so hard for John Major to give his administration an aura of either coherence or competence."

#### THE LENGTH OF EFFECTIVE GOVERNMENT

An optimal firing rule yields declining ministerial performance. This decline might explain why governments push forward their programs early, and supports a common political intuition: commentators often see the "first hundred days of government" as being most important.<sup>10</sup> This raises questions: how long does a government remain effective? What determines the length of effective governance? Little attention has been paid to the duration of effective (rather than ineffective) governance. We take up this issue here.

**The Duration of Effective Government.** We have noted that performance depends on the talents and actions of office holders. As Figure 3 illustrates, this performance suffers as  $k$  declines: the lifetime of effective government is limited. When the hazard rate of

<sup>10</sup>The phrase originates, of course, with the "first hundred days" of the New Deal under Franklin D. Roosevelt, during which Congress passed an extraordinary amount of legislation. Whilst some administrations have sought to emulate FDR's achievement, most notably Ronald Reagan's first administration, others have fallen short of this benchmark or have set themselves more modest targets; John F. Kennedy famously remarked in his inaugural address that "it won't be done in the first hundred days."

scandals is linear in performance we can precisely describe the duration of government effectiveness. Recall that, for the linear specification, performance satisfies either  $e = \bar{e}$  or  $e = 0$ . This “all or nothing” nature of performance suggests a natural interpretation of effectiveness: the government is effective when ministers devote all of their available talent to the task of running government, and ineffective when they do not.

Whether government is effective depends on the standards operated by the Prime Minister: effective government requires the Prime Minister to operate a high standard ( $q_k^\dagger = q^*$ ) rather than a low standard ( $q_k^\dagger = q^\ddagger$ ). Proposition 4 establishes the relationship between  $q^\dagger$  and  $k$ . The Prime Minister operates a stricter threshold (higher  $q^\dagger$ ) early on and, as her talent pool shrinks, correspondingly relaxes her firing rule. Bringing these elements together leads directly to the following corollary to Proposition 4.

**Corollary to Proposition 4.** *When the arrival rate of scandals is linearly decreasing in performance, there is a critical size  $k^*$  of the talent pool such that the Prime Minister sets  $q_k^\dagger = q^*$  (generating high performance) when  $k \geq k^*$ , but sets  $q_k^\dagger = q^\ddagger$  (generating low performance) for  $k < k^*$ . As long as  $q_k^\dagger = q^*$  the government remains effective. Ministerial performance falls after  $n - k^*$  ministerial resignations, and from then on the government is ineffective.*

We have established that a government remains effective so long as the size of its talent pool exceeds a critical threshold. The duration of effective government stems from two elements: (i) the expected tenure of each minister in the high performance regime; and (ii) the number of ministers (that is,  $n - k^*$ ) involved in that regime.

The first factor may be calculated straightforwardly. In general, the expected tenure  $\bar{t}_k$  of a minister in the context of a  $k$ -strong talent pool satisfies  $\bar{t}_k = 1/(\gamma + [\bar{\lambda} + \lambda(e_k)]q_k^\dagger)$ . If the arrival rate of scandals is linearly decreasing in performance and  $k \geq k^*$  then  $q_k^\dagger = q^*$  (the firing probability emerging from Proposition 2) and  $\lambda(e_k) = \hat{e} - \bar{e}$ . Using Equation (4) the expected tenure of a minister in the effective phase of government satisfies

$$k \geq k^* \implies \bar{t}_k = \frac{1}{\gamma} \left[ \frac{\bar{v} - c(\bar{\lambda} + \hat{e})}{\bar{v} - c\bar{e}} \right].$$

An inspection of this expression confirms the predictions of Proposition 3: a minister’s tenure grows with his benefits of holding office and his talent, but decreases with the exogenous hazard rates of government termination and resignation calls.

The control of the second determinant of the duration of effective government (the number of ministers  $n - k^*$  affected by the high-performance regime or, equivalently, the talent-pool threshold  $k^*$ ) is the primary endogenous control variable of the Prime Minister and so we use this as our measure of the length of effective government.

**Definition.** *When the arrival rate of scandals is linearly decreasing in performance, the length of effective government is the number of ministers  $n - k^*$  who resign before the Prime Minister abandons her high-performance regime and switches to using the weakest feasible firing rule.*

We turn to analyze the determinants of this length, summarizing our results in the following proposition, before providing a more detailed discussion of our findings. For the purposes of this proposition we say that an endogenous variable is “U shaped” in a parameter if it is at first (at least weakly) decreasing and then (at least weakly) increasing, and so is maximized by the extreme (high and low) values of the parameter.<sup>11</sup>

**Proposition 5.** *When the arrival rate of scandals is linearly decreasing in performance, the length of effective government is decreasing in the Prime Minister’s value  $\bar{w}$  from holding office but increasing in the average severity  $\bar{s}$  of ministerial scandals. It is increasing in each minister’s value  $\bar{v}$  of holding office and in his maximum feasible performance  $\bar{e}$  but decreasing in the minister’s opportunity cost  $c$  of performance. Over the range of parameter values for which high performance is feasible (that is, when  $q^* \leq 1$ ) the length of effective government is a U-shaped function of the exogenous hazard rate  $\gamma$  of government termination and of the minimum firing probability  $q^\ddagger$ .*

The first pair of monotonic comparative-static predictions are unsurprising. When the Prime Minister cares deeply about holding on to office ( $\bar{w}$  is high) and worries little about scandals ( $\bar{s}$  is low) then she maximizes the duration of her tenure in office by keeping standards low and reducing the length of effective government.

The remaining monotonic predictions are also natural. When a minister values his office highly and performance cheap then he is influenced by a relatively low firing probability  $q^*$ . Furthermore, when his ability is high (that is, when  $\bar{e}$  is high) then a switch to the high-performance regime substantially reduces the arrival of scandals. These things extend the era of effective government, and also enhance the expected tenure of a high-performing minister; the duration of effective government rises unambiguously. These results provide further support for the hypothesis that high office benefits help government performance. As argued by Caselli and Morelli (2004), such benefits can attract talented politicians.<sup>12</sup> We have already established (Proposition 2) that such benefits induce greater effort. We now see that the Prime Minister responds by imposing higher standards for longer. Furthermore, when ministers are more talented (so that  $c$  is lower and  $\bar{e}$  is higher) the Prime Minister is more willing to exploit the talent available to her.

<sup>11</sup>Note that this definition of “U shaped” can also include monotonic functions. However, in the context of Proposition 5 it is always possible to choose values of  $\bar{w}$  and  $\bar{s}$  such that the length of effective government is a non-monotonic function of the parameters  $\gamma$  and  $q^\ddagger$ .

<sup>12</sup>This view was disputed by Mattozi and Merlo (2008). In their model a citizen entering political life signals her quality to the private sector. They found that higher salaries reduce the average quality of entrants.

**Stability and Effectiveness.** Political scientists have long studied government durability, the effects of exogenous shocks on government duration, and the stabilizing effects of institutions. However, as Laver (2003) has pointed out, there is very little theoretical literature on government duration. Moreover, neither the theoretical nor the empirical literature says much about the relationship between government survival and performance, although in a notable exception Warwick (1992) empirically revealed an indirect relationship via economic variables. As argued by Huber and Martinez-Gallardo (2004), it often implicitly assumed that government turnover is undesirable. Indeed Strøm (1985) explicitly used government duration as a performance indicator, arguing that since a government seeks tenure (amongst other things) it should be evaluated via this indicator.

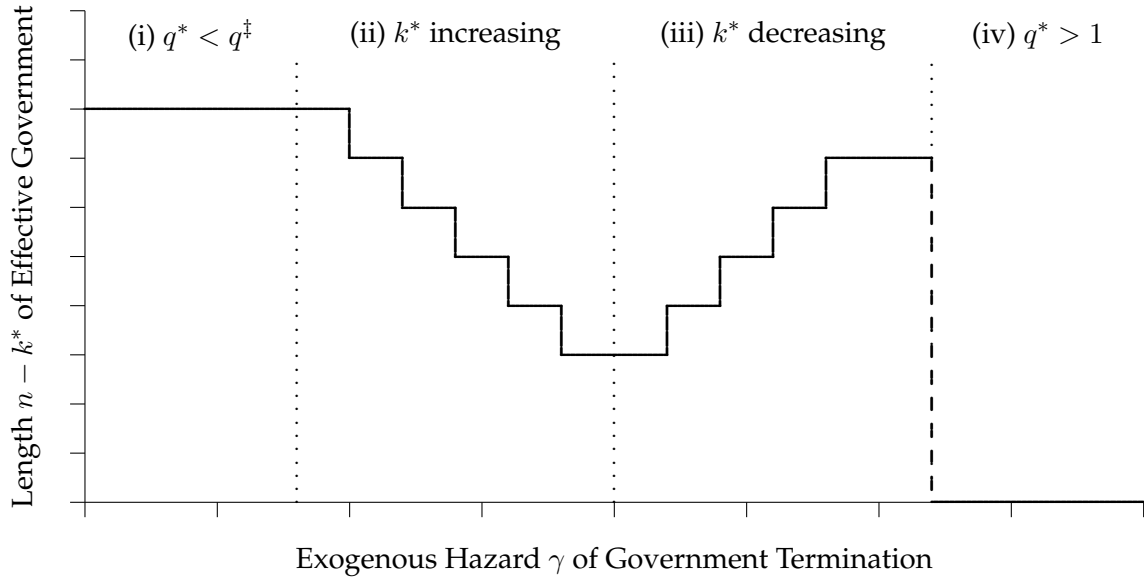
As we noted earlier in our paper, Sartori (1997) has questioned this view. In following Sartori's line of enquiry we have developed an agency model of ministerial turnover that incorporates Strøm's view, since our Prime Minister would (other things equal) wish to stay in office for longer. However, our Prime Minister also wishes to avoid costly scandals, and in our analysis we distinguish between government tenure and performance. This allows us to evaluate the effects of government stability (via the exogenous hazard rate  $\gamma$ ) on performance and the length of effective government.

The relationship between these variables turns out to be quite subtle: Proposition 5 reveals a non-monotonic relationship between the length of government and the arrival rate  $\gamma$  of exogenous government failure. The "U shaped" nature of this relationship ensures that the length of effective government is maximized when exogenous instability is largely absent ( $\gamma$  is low) or very important ( $\gamma$  is large). The non-monotonic impact of an increase in  $\gamma$  stems from a conflict between two conflicting forces.

Firstly, as  $\gamma$  increases it becomes harder to induce high performance from a minister since (from Proposition 2)  $q^*$  is increasing in  $\gamma$ . The increase in  $\gamma$  makes it more likely that a minister will exogenously lose his position, which lowers the value of his career. An income effect means that he now cares less about avoiding costly resignation calls.

Secondly, as  $\gamma$  increases the Prime Minister cares less about maintaining her talent pool. When  $\gamma$  is large her government is unlikely to survive long enough to run out of talent, and so the declining talent pool plays less on her fears. Under these circumstances the Prime Minister cares more about the current performance of her ministers and avoiding costly scandals than about the longer-term effect of talent-pool depletion: adopting a "live for today" attitude she endogenously increases the length of effective government.

Bring these two forces together, the first dominates for smaller  $\gamma$ , while the second can dominate for larger values of  $\gamma$ . Thus an increase in exogenous government instability



*Notes.* For region (i), the lower bound  $q^\ddagger$  on the firing probability is enough to induce high effort, whereas in region (iv) the value of a minister's career is so low that it is impossible to generate positive effort. The remaining two regions illustrate the "U shape" response described in Proposition 5.

FIGURE 4. The Duration of Effective Government

and reduction in the overall duration of government can result in an increase in the length of effective government. A caveat is that when  $\gamma$  is large the constraint  $q^* \leq 1$  can break. From Equation (4), if  $\gamma > (\bar{v}/c) - \bar{\lambda} - \hat{e}$  then it is impossible to induce high effort.

Hence, as  $\gamma$  increases, we can identify and illustrate (using Figure 4) four phases of response: (i) when  $\gamma$  is low enough to yield  $q^* < q^\ddagger$  then the government is always effective; (ii) for larger  $\gamma$ , the length of effective government falls as it becomes more costly to induce high performance; (iii) as the government becomes highly unstable the Prime Minister "lives for today" and so increases standards; and (iv) finally  $\gamma$  is so large that ministers no longer value their future careers and always supply low effort.

These results are significant for considering the effects of institutional changes designed to enhance government stability. Whilst the empirical evidence suggests that constitutions can be designed so as to enhance stability, by for example including a requirement for an investiture vote for an incoming government, our results suggest that such changes need not lead to more effective government. Any such constitutional engineering, to borrow Sartori's phrase, must be informed by an understanding of the underlying incentives and agency problems of governance, including the endogenous reaction to changes in institutional design by the executive leader.

**Imposing Ministerial Standards.** When faced with a scandal, the Prime Minister has discretion in some cases but not all: recall that if the severity  $s$  of a scandal exceeds  $s^\ddagger$  then the misdemeanor is so grave that the Prime Minister must let her minister go. What counts as a grave event may depend on the mood of the populace or on political circumstance.

The discretion enjoyed by a Prime Minister may be circumscribed by institutional procedures. For example, in the United Kingdom ministerial misdemeanors are referred to the Committee for Standards in Public Life, set up by Prime Minister John Major in response to growing concern over ministerial standards. The committee's judgement influences (although it need not determine) the Prime Minister's action. However, whenever the committee has reported on specific cases of alleged misconduct it has lowered the room for discretion enjoyed by the Prime Minister and in practice a rebuke from the committee terminates a ministerial career. For example, David Blunkett resigned as Home Secretary in December 2004 after the committee found that he had broken rules of conduct by fast-tracking a visa application for the nanny of his then lover Kimberly Quinn.

The imposition of standards in public life is seen as a means of enhancing accountability, leading to better performance and more effective government. However Proposition 5 states that the length of effective government is a "U shaped" function of the minimum feasible firing probability  $q^\ddagger$ . Of course, this minimum probability satisfies  $q^\ddagger = 1 - F(s^\ddagger)$  which grows with the any exogenous imposition of stricter standards (a reduction in  $s^\ddagger$ ). Thus, starting from a position of low standards (high  $s^\ddagger$ ) the gradual imposition of higher ministerial standards will first lead to a contraction in the length of effective government, and will only extend this length when such standards become sufficiently high (so that  $s^\ddagger$  is low). This non-monotonic response stems from the presence of two conflicting forces.

Firstly, an increase in exogenous standards, raising  $q^\ddagger$ , lowers the net cost of inducing high performance. This net cost is determined by the increase  $q^* - q^\ddagger$  in the firing probability needed for a minister to devote high effort to his tasks.

Secondly, the increase in  $q^\ddagger$  makes it difficult to maintain the longevity of the government in the low-performance regime. The Prime Minister responds by requiring a larger buffer of low-performing ministers. This effect is easiest to see by considering what happens when  $q^\ddagger = 0$ . In this case, the Prime Minister can insist on high performance from the first  $n - 1$  members of her talent pool, safe in the knowledge that she will never be forced to fire the last minister. Thus, when exogenous standards are eliminated the length of effective government is at least  $n - 1$ . As  $q^\ddagger$  increases this safety net disappears and the Prime Minister responds with lower standards by applying the  $q_k^\ddagger = q^*$  regime to fewer ministers. An observation of interest to us is that this second force can dominate the first.

**Corollary to Proposition 5.** *The exogenous imposition of higher ministerial standards can lead to an endogenous contraction in the length of effective government.*

The U-shaped response to  $q^\ddagger$  means that a contraction in the length of effective government happens when  $q^\ddagger$  is initially low. However, the clamor for stricter exogenous regulation of ministers is likely to be larger when existing standards are low. Under these circumstances our results suggests it pays to strengthen the powers of the Prime Minister (by allowing her to enjoy discretion via a lower  $q^\ddagger$ ) rather than to diminish them.

#### REPLENISHING THE TALENT POOL

The limited depth of the ministerial talent pool weakens the Prime Minister's willingness to fire and so dulls the incentives of her ministers. Here we turn our attention to the possibility that the talent pool may be periodically replenished. We consider the reactions of ministers to increased replenishment (Proposition 6), the impact of replenishment on the reaction of ministers to a stricter firing rule (Proposition 7), and the relationship between replenishment and the Prime Minister's credibility (Proposition 8).

**Expanding the Talent Pool.** When the talent-pool is finite, then so is the lifespan of effective government. This problem might be mitigated when a larger talent pool is available or when, once talent has been lost, new talent can emerge. This suggests, in turn, that restrictive selection methods might damage government performance. For example, in parliamentary systems, since the executive must maintain the confidence of the legislature (and in some cases faces a formal vote of investiture) selection into government is usually restricted to members of the legislative body. As one observer recently put it:

“The notion that any government could staff its entire ministerial team from the small talent pool of MPs in parliament is absurd. No private sector company would operate like this.”<sup>13</sup>

Lifting this restriction may be seen as raising the size of the ministerial talent pool. Of course, the value of the Prime Minister's career (and of her government) is increasing in the state variable  $k$  and hence in the initial size  $n$  of her talent pool. This suggests that more talent is always a good thing, at least from her perspective.

Of course, things are not so straightforward if the expansion of the talent pool changes its composition. For instance, an expansion may bring in members with objectives that are not well aligned with those of the government. In the context of the model, this might correspond to an increase in the opportunity cost of effort  $c$ : there may well be a trade-off

<sup>13</sup>Vernon Bogdanor, quoted in “Bevin Offers Hope to Outsiders,” *Financial Times*, London, June 29, 2007.



between the size of the talent pool and its composition. Rather than pursue this formally, we instead develop the idea that a finite talent pool may be periodically refreshed.

**Introducing New Talent.** Whereas the pool of ministerial talent may be fixed at a particular time, and subject to an upper bound (perhaps the size of the legislature), we can envisage a world in which talent flows into the pool. For instance, a new backbencher who was initially unsuitable for office might gain enough experience to become a candidate for office. We adapt our model so that as talent evaporates it might be replenished. Formally, we suppose that new talent arrives with Poisson arrival rate  $\beta$  so long as  $k < n$ . We think of a situation where  $\beta$  is low as corresponding to the adoption of restrictive selection methods: for example, the government might be selected from amongst the talents in the majority party; or a strict minimal winning coalition of parties; or, even more restrictively, from certain factions or families within those parties. A higher  $\beta$  might correspond, on the other hand, with a situation where the Prime Minister can develop the candidacy of other citizens, whether elected or not. When  $\beta = 0$  then, as we have seen, there is a risk that the government terminates prematurely due to endogenous talent-pool depletion. Conversely, as  $\beta \rightarrow \infty$  the talent-pool is replenished as quickly as it is depleted, thus ensuring that the only source of government termination is exogenous.

**The Effect of Replenishment.** We imagine that the Prime Minister continues to employ a firing rule contingent only on the size of the talent pool. In this situation, the problem faced by a minister in office changes: this minister recognizes that the talent pool may expand, which will change the firing probability he faces and hence the value of his career. Subscripting the value of a ministerial career by the current talent-pool size, with a  $k$ -strong talent pool the arrival of a new member of the pool changes the value of the minister's career by  $V_{k+1} - V_k$ . Modifying Equation (1) appropriately, we obtain

$$\underbrace{\bar{v} - ce_k}_{\text{flow benefit and cost}} = \underbrace{[\bar{\lambda} + \lambda(e_k)]q_k^\dagger V_k}_{\text{risk of resignation}} - \underbrace{\beta[V_{k+1} - V_k]}_{\text{effect of replenishment}} + \underbrace{\gamma V_k}_{\text{exogenous risk}}. \quad (7)$$

As  $\beta$  is exogenous, the first-order condition of Equation (2) is unchanged and given by

$$c = -\lambda(e)q_k^\dagger V_k. \quad (8)$$

It might seem reasonable to suggest that an increase in the replenishment rate will lead a minister to fear for his position and hence focus on his tasks. However, any change in the replenishment rate influences the behavior of a minister only via the income effect on the value of his career  $V_k$ . If it lowers the value of his career then his fear of resignation will fall it with it, leading to a reduction in performance. The value of his career does indeed

fall with an increase in  $\beta$  whenever the firing probability  $q_k$  increases with the size of the talent pool, leading to the following simple proposition.

**Proposition 6.** *If the firing probability is increasing in the size of the talent pool then, fixing the firing strategy, an increase in the replenishment rate reduces ministerial performance.*

A higher firing probability reduces the value of a minister's career. The presence of replenishment means that the minister faces the possibility that he will be exposed to a harsher regime, which reduces  $V_k$ . We have already seen that the firing probability increases with the size of the talent pool in a world without replenishment (Proposition 4) and so, beginning from this world, we obtain the following corollary.

**Corollary to Proposition 6.** *Beginning from zero replenishment, so that the firing probability increases with the size of the talent pool, a local increase in replenishment reduces performance.*

So far we have changed the replenishment rate, whilst fixing the firing probabilities. Of course we might also consider the effect of a marginal increase in the severity of the firing rule when fixing the replenishment rate. This situation is more complicated than in a world without replenishment. In the context of a  $k$ -strong talent pool, a minister considers not only the immediate effect of  $q_k^\dagger$  on his career, but also the effect of  $q_j^\dagger$  for  $j > k$ .

**Proposition 7.** *Fixing a positive replenishment rate  $\beta > 0$  an increase in the firing probability  $q_k^\dagger$  for a  $k$ -strong talent pool increases the performance of a minister facing such a talent pool ( $e_k$  rises) but reduces the performance of ministers when the talent pool size is smaller ( $e_j$  falls for  $j < k$ ).*

As we noted in our earlier discussion, an increase in the severity of the firing rule can have both an income and a substitution effect. Following a rise in  $q_k^\dagger$ , both effects are present when we consider the choice of  $e_k$ ; the substitution effect dominates and so performance rises. However, for all talent-pool sizes  $j \neq k$  only the income effect is present. For  $j > k$ , the firing probability doesn't matter for the minister (the talent pool can only shrink if he is fired) whereas for  $j < k$  the income effect works against the minister's performance.

Drawing the results of Proposition 6 and 7, we suggest that replenishment of the talent pool can sometimes diminish performance, suggesting that the relaxation of restrictions on the selection of ministers is not necessarily an unalloyed blessing.

**Replenishment and Reputation.** Ministers respond to the threat of resignation by performing well, but they are subject to shocks: there will be occasions when, despite their efforts, they fail. To provide incentives the Prime Minister must implement her firing rule. But will she carry out her threats? Here we show how the presence of a replenishing talent pool can help support the credibility of the Prime Minister's firing rule.

The credibility issue arises due to the finite depth of the talent pool. Eventually the pool is drained of talent. When this is so, the “last man standing” is pivotal to the government, and so the Prime Minister will never willingly fire him: any firing rule specifying  $q_0^\dagger > q^\ddagger$  lacks credibility. Insisting on dynamic consistency, the firing rule must satisfy  $q_0^\dagger = q^\ddagger$ . (Equivalently, the threshold on scandals must satisfy  $s_0^\dagger = s^\ddagger$ .) Of course, we can now apply the same logic for the  $k = 1$  case: even if the Prime Minister sacrifices her reputation by failing to implement a firing rule  $q_1^\dagger > q^\ddagger$ , this will have no repercussions in the future. Continuing this argument iteratively, we conclude that  $q_k^\dagger = q^\ddagger$  for all  $k$ .

Of course, the argument deployed here relies upon a talent pool that dries up over a government’s lifespan. But what if new talent may emerge? To explore this issue we consider our model with an arrival rate of scandals that is linear in performance and, to simplify notation, we set  $\hat{e} = 0$ .<sup>14</sup> We also restrict our attention to a two-state world with  $k \in \{0, 1\}$ , so that the talent-pool is either “full” or “empty”. When the talent-pool is empty, it is replenished (it switches from  $k = 0$  to  $k = 1$ ) at the Poisson arrival rate  $\beta > 0$ .

In this simple setting, the last-man-standing argument continues to apply when the talent pool is empty, and hence  $q_0^\dagger = q^\ddagger$ . Setting  $q_1^\dagger > q^\ddagger$  only makes sense if it induces high effort, and it does so if and only if  $q_1^\dagger \geq q^*$ . Hence, when choosing the firing probability for the “full” talent pool the Prime Minister will choose from two options:  $q_1^\dagger \in \{q^\ddagger, q^*\}$ .

Suppose, then, that the Prime Minister would strictly prefer to set  $q_1^\dagger = q^* > q^\ddagger$  if only this were credible. This generates performances of  $e_1 = \bar{e}$  and  $e_0 = 0$ .<sup>15</sup> Using Equation (5) for the case  $k = 1$  and modifying the corresponding equation for  $k = 0$  yields

$$\begin{aligned} \bar{w} - (\bar{\lambda} - \bar{e})\bar{s} &= q^*(\bar{\lambda} - \bar{e})[W_1 - W_0] + \gamma W_1. \\ \text{and } \bar{w} - \bar{\lambda}\bar{s} &= q^\ddagger\bar{\lambda}W_0 + \gamma W_0 - \underbrace{\beta(W_1 - W_0)}_{\text{effect of replenishment}}. \end{aligned}$$

The second equation differs from earlier analysis via the inclusion of the final term: this captures the replenishment effect. These equations solve to yield  $W_1$  and  $W_0$ . If instead the Prime Minister were to set  $q_1^\dagger = q^\ddagger$  then the corresponding values  $\tilde{W}_1$  and  $\tilde{W}_0$  satisfy

$$\begin{aligned} \bar{w} - \bar{\lambda}\bar{s} &= q^\ddagger\bar{\lambda}[\tilde{W}_1 - \tilde{W}_0] + \gamma\tilde{W}_1. \\ \text{and } \bar{w} - \bar{\lambda}\bar{s} &= q^\ddagger\bar{\lambda}\tilde{W}_0 + \gamma\tilde{W}_0 - \beta[\tilde{W}_1 - \tilde{W}_0]. \end{aligned}$$

Once again, we can solve this pair of equations to obtain  $\tilde{W}_0$  and  $\tilde{W}_1$ . We are considering a case where the Prime Minister would wish to implement the high-performance regime

<sup>14</sup>Setting  $\hat{e} = 0$  is without loss of generality, since  $\hat{e}$  can be absorbed into the exogenous scandal hazard  $\bar{\lambda}$ . Doing so, the assumption that  $\hat{e} > \bar{e}$  translates to  $\bar{\lambda} > \bar{e}$ .

<sup>15</sup>Setting  $q_1^\dagger = q_0^\dagger = q^\ddagger$  yields  $e_1 = e_0 = 0$ . Raising  $q_1^\dagger$  can only reduce  $e_0$ , via the negative income effect.

when  $k = 1$ , so that  $q_1^\dagger = q^*$ . Here, this desire is captured by the inequality  $W_1 \geq \tilde{W}_1$ . This inequality may determine what is desirable; but is it credible?

Defining  $s^*$  to satisfy  $q^* = 1 - F(s^*)$ , credibility requires that, if the minister is hit by a scandal of with severity satisfying  $s^* < s \leq s^\ddagger$ , the Prime Minister will demand his resignation. Of course, the Prime Minister is tempted to renege by keeping her minister in post after such a scandal. In order to prevent her from doing so, let us suppose that if she deviates in this way then future ministers refuse to believe her threats and hence, facing effective firing probabilities of  $q_1^\dagger = q_0^\dagger = q^\ddagger$ , they then supply zero effort.

The trade-off is this: if the Prime Minister fires her scandal-hit minister then the value of her career falls from  $W_1$  to  $W_0$ ; she maintains her reputation but loses a member of her talent pool. On the other hand, if she retains him then her value shifts from  $W_1$  to  $\tilde{W}_1$ ; her talent pool remains intact but her threats no longer carry weight. Summarizing, for the high-performance regime to be desirable we require  $W_1 \geq \tilde{W}_1$ , but for it to be credible we require  $W_0 \geq \tilde{W}_1$ . Now,  $W_1 > W_0$  and hence the credibility constraint is harder to satisfy than the the desirability constraint. This means that there are parameter values for which the Prime Minister will wish to impose high performance but is unable to maintain credibility. Whether these inequalities are satisfied depends on the replenishment rate.

**Proposition 8.** *Suppose that (i) the arrival rate of scandals is linearly decreasing in performance; (ii)  $k \in \{0, 1\}$  so that the talent pool is either full or empty; and (iii) an empty talent pool is replenished with arrival rate  $\beta$ . There are two critical replenishment rates satisfying  $\beta_H > \beta_L \geq 0$  such that if  $\beta \geq \beta_H$  then the Prime Minister wishes to set  $q_1^\dagger = q^*$  and is able to do so credibly; but if  $\beta_H > \beta > \beta_L$  then she wishes to set  $q_1^\dagger = q^*$  but is unable to do so credibly.*

As the replenishment rate rises the Prime Minister is less concerned with running out of talent (after all, this talent is replaced quickly) and so the desire for higher standards is natural. Furthermore, this also makes her more willing to sacrifice her minister when  $k = 1$ , for precisely the same reasons. However, when  $\beta_L < \beta < \beta_H$  replenishment is not quite rapid enough for her to resist the temptation to renege on her promised threat.

#### CONCLUDING REMARKS

We have built a model in which a principal (a Prime Minister) provides incentives by firing those seen to fail. This feature is common to other agency models (Huber and Gordon, 2002; Gailmard and Patty, 2007; Shotts and Wiseman, 2008) but here we have developed this framework to analyze situations where only some potential replacements are talented. Our model sheds light on an inherent tension between the need to provide incentives through turnover and the need to maintain talented agents in post.

The optimal firing rule weakens incentives over the lifespan of a government. Our model thus explains variations (over time and in the cross-section) in ministerial turnover and in the related performance of governments. For example, we find that anything that increases the value of a ministerial career increases performance whilst reducing ministerial hazard rates. Consistent with recent studies of political careers (Besley, 2004; Caselli and Morelli, 2004; Messner and Polborn, 2004) we find that an increase in ministerial salaries and office seeking perks is related to lower levels of turnover, although here, and in contrast to those papers, this effect is due to the better performance of talented ministers.

An increase in the exogenous risk of government termination reduces the value of ministers' careers, increases turnover, and can reduce performance. This is consistent with the Huber and Martinez-Gallardo (2004) analysis of the French Fourth Republic, and offers some micro-foundations for their findings. But our findings are at odds with other claims made in the empirical literature: Berlinski, Dewan, and Dowding (2007) suggested tenure as a measure of ministerial performance, while Huber and Martinez-Gallardo (2004) used it to measure experience, implicitly assuming that longer tenure is desirable. We show that variation in tenure can reflect differences in the way the Prime Minister manages her ministers. Furthermore, ministers who perform less well can survive for longer.

An important implication of our analysis is that the length of effective government is finite. This leads naturally to the question: how long does a government remain effective and what helps to sustain effective governance? In We obtained some surprising findings. Whilst some parliamentary democracies deploy committees to oversee public standards, our results do not chime with a conventional wisdom that an exogenous increase in the standards imposed on ministers leads to better performance: limiting the discretion of the Prime Minister can lead her to relax her own standards, thus curtailing the length of time her government remains effective. We have also argued that introducing measures to enhance government durability need not increase the length of effective government.

In our view, a limit to the pool of available talent lies at the root of the decline in government performance. When the talent pool has evaporated an incumbent minister is essentially a lame duck: lacking incentives he does relatively little to enhance the government's performance. A (currently popular) resolution for perceived poor performance is for the Prime Minister to look beyond her own back-benches in filling vacant posts: choosing from "all the talents" ensures that her talent pool is always replenished. We find that the introduction of new blood is, however, no panacea: replenishment can dilute the incentives of incumbent ministers further. Such replenishment does however enhance a Prime Minister's concern for her reputation, allowing her to commit credibly to her firing rule in situations where, absent replenishment, she would be unable to do so.

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#### OMITTED PROOFS

*Proof of Proposition 1.* Differentiating Equation (2) with respect to  $q^\dagger$  we obtain

$$\underbrace{\lambda''(e)q^\dagger V}_{+ve} \times \frac{de}{dq^\dagger} = \underbrace{-\lambda'(e)}_{+ve} \times \left[ V + q^\dagger \frac{dV}{dq^\dagger} \right] \implies \frac{de}{dq^\dagger} > 0 \Leftrightarrow V + q^\dagger \frac{dV}{dq^\dagger} > 0.$$

Next, differentiating Equation (1) with respect to  $q^\dagger$  we obtain

$$\begin{aligned} \gamma \frac{dV}{dq^\dagger} &= -\underbrace{[c + \lambda'(e)q^\dagger V]}_{\text{zero from Eq. (2)}} \frac{de}{dq^\dagger} - [\bar{\lambda} + \lambda(e)] \left[ V + q^\dagger \frac{dV}{dq^\dagger} \right] \\ &\implies V + q^\dagger \frac{dV}{dq^\dagger} = V \left[ 1 - \frac{\bar{\lambda} + \lambda(e)}{(\gamma/q^\dagger) + \bar{\lambda} + \lambda(e)} \right] > 0 \implies \frac{de}{dq^\dagger} > 0. \end{aligned}$$

This argument extends straightforwardly to the boundary cases of  $e = 0$  and  $e = \bar{e}$ . For the remaining claims, an increase in  $\bar{v}$  or a reduction in either  $\gamma$  or  $\bar{\lambda}$  serve to increase  $V$ . Using Equation (2), when  $V$  increases  $e$  must rise. An increase in  $c$  leads to a reduction in  $V$  and a consequent fall in  $e$  via the same logic;  $c$  also enters Equation (2) directly, and this direct effect also reduces  $e$ . Lifting  $\bar{e}$  can only increase effort, since it loosens the constraint  $e \leq \bar{e}$ .

Turning to the specific example, when the hazard rate of scandals is inversely proportional to performance then  $\lambda(e) = 1/e$  and  $\lambda'(e) = -1/e^2$ . Equations (1) and (2) become

$$\bar{v} - ce = \left[ \bar{\lambda} + \frac{1}{e} \right] q^\dagger V + \gamma V \quad \text{and} \quad c = \frac{q^\dagger V}{e^2}.$$

Using these two equations to eliminate  $V$  generates the quadratic  $[\gamma c + \bar{\lambda} c q^\dagger] e^2 + 2c q^\dagger e - q^\dagger \bar{v} = 0$ . Solving for the unique positive root yields the stated solution for  $e$ .  $\square$

*Proof of Proposition 2.* If the arrival rate of scandals is linearly decreasing in performance then  $\lambda(e) = \hat{e} - e$  and so  $\lambda'(e) = -1$ . The linearity of  $\lambda(e)$  ensures that the optimally chosen effort must satisfy  $e = \bar{e}$  or  $e = 0$ . Using Equation (1), the corresponding career values satisfy

$$V|_{e=\bar{e}} = \frac{\bar{v} - c\bar{e}}{\gamma + q^\dagger(\bar{\lambda} + \hat{e} - \bar{e})} \quad \text{and} \quad V|_{e=0} = \frac{\bar{v}}{\gamma + q^\dagger(\bar{\lambda} + \hat{e})}.$$

The minister exerts high effort if and only if the first expression exceeds the second. (We assume that he chooses  $e = \bar{e}$  whenever indifferent.) Straightforward algebraic manipulations confirm that this is so if and only if  $q^\dagger \geq q^*$  where  $q^*$  is given in Equation (4). Of course, this latter inequality can only be satisfied if  $q^* \leq 1$ , which holds if and only if  $\bar{v} \geq c[\gamma + \bar{\lambda} + \hat{e}]$ .  $\square$

*Proof of Proposition 3.* Follows from Propositions 1 and 2, together with arguments in the text.  $\square$

*Proof of Lemma 1.* Follows from the argument given in the text.  $\square$

*Proof of Lemma 2.* We first show the (obvious) result that  $W_k$  is increasing in  $k$ , so that  $W_k > W_{k-1}$ . Defining  $W_{-1} \equiv 0$  (corresponding to a failed government following complete depletion of the talent pool) this is certainly true for  $k = 0$ . We will show inductively that is true for all larger  $k$ . To do this, first re-arrange Equation (5) to obtain an explicit solution for  $W_k$ :

$$W_k = \frac{\bar{w} + [\bar{\lambda} + \lambda(e_k)][q_k^\dagger W_{k-1} - \bar{s}]}{\gamma + q_k^\dagger[\bar{\lambda} + \lambda(e_k)]},$$

Now consider  $W_{k+1}$ . This value is optimized by choosing a firing probability  $q_{k+1}^\dagger$ . Suppose that the Prime Minister instead chooses a firing probability  $q_k^\dagger$  when there are  $k + 1$  members of her talent pool, and write  $\tilde{W}_{k+1}$  for the associated present value of her government. This satisfies

$$\tilde{W}_{k+1} = \frac{\bar{w} + [\bar{\lambda} + \lambda(e_k)][q_k^\dagger W_k - \bar{s}]}{\gamma + q_k^\dagger[\bar{\lambda} + \lambda(e_k)]}.$$

Since  $q_{k+1}^\dagger$  is the optimal choice, it must be that  $W_{k+1} \geq \tilde{W}_{k+1}$ . Now,

$$W_{k+1} - W_k \geq \tilde{W}_{k+1} - W_k = \frac{q_k^\dagger[\bar{\lambda} + \lambda(e_k)][W_k - W_{k-1}]}{\gamma + q_k^\dagger[\bar{\lambda} + \lambda(e_k)]}.$$

By inspection if  $W_k > W_{k-1}$  then  $W_{k+1} > W_k$ . We have already noted that  $W_k > W_{k-1}$  for  $k = 1$ . Hence, by the principle of induction,  $W_k$  is strictly increasing in  $k$ .



We now show that there are decreasing returns to the talent pool size:  $W_{k+1} - W_k \leq W_k - W_{k-1}$  for each  $k$ . To do this, suppose that the Prime Minister chooses a firing probability  $q_{k+1}$  when there are  $k$  members of her talent pool, and write  $\hat{W}_k$  for the associated present value of her government:

$$\hat{W}_k = \frac{\bar{w} + [\bar{\lambda} + \lambda(e_{k+1})][q_{k+1}^\dagger W_{k-1} - \bar{s}]}{\gamma + q_{k+1}^\dagger [\bar{\lambda} + \lambda(e_{k+1})]}.$$

Since  $q_k^\dagger$  is the optimal choice, it must be that  $W_k \geq \hat{W}_k$ . Hence

$$W_{k+1} - W_k \leq W_{k+1} - \hat{W}_k = \frac{q_{k+1}^\dagger [\bar{\lambda} + \lambda(e_{k+1})][W_k - W_{k-1}]}{\gamma + q_{k+1}^\dagger [\bar{\lambda} + \lambda(e_{k+1})]} \leq W_k - W_{k-1}.$$

Note that the final inequality is strict whenever  $\gamma > 0$ . □

*Proof of Proposition 4.* When there are  $k$  members of the talent pool, the Prime Minister finds it optimal to use the firing probability  $q_k^\dagger$ , and hence she must be (at least weakly) worse off using the firing probability  $q_{k-1}^\dagger$ . Using Equation (5), this means that

$$[\lambda(e_{k-1}) - \lambda(e_k)]\bar{s} \geq \left( q_k^\dagger [\bar{\lambda} + \lambda(e_k)] - q_{k-1}^\dagger [\bar{\lambda} + \lambda(e_{k-1})] \right) [W_k - W_{k-1}].$$

When there are  $k-1$  members of the talent pool, the Prime Minister prefers to use the firing probability  $q_{k-1}^\dagger$  rather than  $q_k^\dagger$ . Using Equation (5) once more, we obtain

$$[\lambda(e_{k-1}) - \lambda(e_k)]\bar{s} \leq \left( q_k^\dagger [\bar{\lambda} + \lambda(e_k)] - q_{k-1}^\dagger [\bar{\lambda} + \lambda(e_{k-1})] \right) [W_{k-1} - W_{k-2}].$$

These two inequalities combine straightforwardly to yield

$$0 \geq \left( q_k^\dagger [\bar{\lambda} + \lambda(e_k)] - q_{k-1}^\dagger [\bar{\lambda} + \lambda(e_{k-1})] \right) \underbrace{([W_k - W_{k-1}] - [W_{k-1} - W_{k-2}])}_{\text{strictly negative}}.$$

The second term on the right-hand side is strictly negative, since there are decreasing returns to the size of the talent pool (Lemma 2). For the inequality to be satisfied, we must have  $q_k^\dagger [\bar{\lambda} + \lambda(e_k)] \geq q_{k-1}^\dagger [\bar{\lambda} + \lambda(e_{k-1})]$ , so that (as claimed) the arrival rate of resignations increases with  $k$ . Using Lemma 1, this implies the firing probability is increasing in  $k$ , so that  $q_k^\dagger \geq q_{k-1}^\dagger$ . Finally, applying Proposition 1, we conclude that the minister's performance is increasing in  $k$ .

Turning to the final claim of the proposition, the left-hand side of Equation (5) is increasing in  $k$ , and hence so is the right-hand side. For  $\gamma$  small enough, the right-hand side is the expected rate of decline as defined in the statement of the proposition. □

*Proof of Proposition 5.* An optimal firing rule satisfies  $q_k^\dagger \in \{q^\dagger, q^*\}$  for each  $k$  since intermediate values of  $q^\dagger$  generate more resignations than  $q^\dagger$  while failing to elicit positive performance (Proposition 2). Applying Proposition 4, the optimal firing probability is increasing in  $k$ . Thus there must be a critical talent-pool size  $k^*$  such that  $q_k^\dagger = q^\dagger$  for  $k < k^*$  and  $q_k^\dagger = q^*$  for  $k \geq k^*$ .

We now calculate  $k^*$ . We write  $W_k^{\hat{k}}$  for the present value obtained by setting  $q^\dagger = q^*$  if and only if  $k \geq \hat{k}$  and  $W_k^\infty$  for the present value of setting  $q_k^\dagger = q^\dagger$  for every feasible talent-pool size. If the

critical threshold  $k^*$  is chosen optimally then it must be the case that  $W_{k^*}^{k^*} \geq W_{k^*}^\infty$  since otherwise the Prime Minister could do better by increasing the threshold to  $k^* + 1$ . It must also be the case that  $W_{k^*-1}^{k^*-1} \leq W_{k^*-1}^\infty$  since otherwise she would gain by switching to  $k^* - 1$ .

To establish when these inequalities are satisfied we need to calculate  $W_k^{\hat{k}}$  and  $W_k^\infty$ . Using Equation (5) and setting  $\hat{e} = 0$  to simplify notation (this is without loss of generality):

$$W_k^{\hat{k}} = \frac{\bar{w} + [\bar{\lambda} - \bar{e}][q^* W_{k-1}^\infty - \bar{s}]}{\gamma + q^*[\bar{\lambda} - \bar{e}]} \quad \text{and} \quad W_k^\infty = \frac{\bar{w} + \bar{\lambda}[q^\dagger W_{k-1}^\infty - \bar{s}]}{\gamma + \bar{\lambda}q^\dagger}.$$

Using these two expressions straightforward but tedious algebra reveals that

$$W_k^{\hat{k}} \geq W_k^\infty \iff W_{k-1}^\infty \geq \frac{1}{\gamma} \left[ \bar{w} - \bar{\lambda}\bar{s} - \frac{\bar{e}\bar{s}(\gamma + \bar{\lambda}q^\dagger)}{\bar{\lambda}(q^* - q^\dagger) - \bar{e}q^*} \right].$$

Next, we solve for  $W_k^\infty$  by repeated substitution to obtain

$$W_k^\infty = \frac{\bar{w} - \bar{\lambda}\bar{s}}{\gamma} \left[ 1 - \left( \frac{\bar{\lambda}q^\dagger}{\gamma + \bar{\lambda}q^\dagger} \right)^{k+1} \right]$$

and so

$$W_k^{\hat{k}} \geq W_k^\infty \iff (\bar{\lambda}(q^* - q^\dagger) - \bar{e}q^*) \left( \frac{\bar{\lambda}q^\dagger}{\gamma + \bar{\lambda}q^\dagger} \right)^{\hat{k}} \leq \frac{\bar{e}\bar{s}(\gamma + \bar{\lambda}q^\dagger)}{\bar{w} - \bar{\lambda}\bar{s}}.$$

A first case to consider is when  $\bar{\lambda}(q^* - q^\dagger) \leq \bar{e}q^*$ . If this holds then the left-hand side of the inequality is negative, and so  $W_k^{\hat{k}} \geq W_k^\infty$  for all  $\hat{k}$ . This implies that the optimal talent-pool threshold must satisfy  $k^* = 0$ . The alternative case is when  $\bar{\lambda}(q^* - q^\dagger) > \bar{e}q^*$ . In that case the inequality holds so long as  $\hat{k}$  is sufficiently large. Recalling that an optimal threshold  $k^*$  satisfies  $W_{k^*}^{k^*} \geq W_{k^*}^\infty$  and  $W_{k^*-1}^{k^*-1} \leq W_{k^*-1}^\infty$ , we bring this pair of equalities together to obtain

$$\left( \frac{\bar{\lambda}q^\dagger}{\gamma + \bar{\lambda}q^\dagger} \right)^{k^*} \leq \frac{\bar{e}\bar{s}(\gamma + \bar{\lambda}q^\dagger)}{(\bar{w} - \bar{\lambda}\bar{s})(\bar{\lambda}(q^* - q^\dagger) - \bar{e}q^*)} \leq \left( \frac{\bar{\lambda}q^\dagger}{\gamma + \bar{\lambda}q^\dagger} \right)^{k^*-1}.$$

Generically there is a unique integer  $k^*$  satisfying these inequalities. This is the smallest integer greater than  $\tilde{k}$ , where  $\tilde{k}$  satisfies

$$\begin{aligned} \left( \frac{\bar{\lambda}q^\dagger}{\gamma + \bar{\lambda}q^\dagger} \right)^{\tilde{k}} &= \frac{\bar{e}\bar{s}(\gamma + \bar{\lambda}q^\dagger)}{(\bar{w} - \bar{\lambda}\bar{s})(\bar{\lambda}(q^* - q^\dagger) - \bar{e}q^*)} \\ \iff \tilde{k} &= \frac{\log[(\bar{w} - \bar{\lambda}\bar{s})(\bar{\lambda}(q^* - q^\dagger) - \bar{e}q^*)] - \log[\bar{e}\bar{s}(\gamma + \bar{\lambda}q^\dagger)]}{\log[\gamma + \bar{\lambda}q^\dagger] - \log[\bar{\lambda}q^\dagger]}. \end{aligned}$$

Recall that we are dealing with the case  $(\bar{\lambda} - \bar{e})q^* > \bar{\lambda}q^\dagger$ , so that the  $\tilde{k}$  is well defined. By inspection,  $\tilde{k}$  is increasing in  $\bar{w}$  and decreasing in  $\bar{s}$ , which implies that the length of effective government  $n - k^*$  is decreasing in  $\bar{w}$  and increasing in  $\bar{s}$ , as claimed in the proposition.

The parameters  $\bar{v}$  and  $c$  enter only via the firing probability  $q^*$ . By inspection,  $\tilde{k}$  is increasing in  $q^*$ . (This is because  $\bar{\lambda} > \bar{e}$ , which is a necessary consequence of the fact we absorbed  $\hat{e}$  into  $\bar{\lambda}$  when we set  $\hat{e} = 0$  in order to simplify notation.)  $q^*$  is increasing in  $c$  and decreasing in  $\bar{v}$  (Proposition 2), and so  $\tilde{k}$  responds in the same way. Hence  $n - k^*$  is decreasing in  $c$  and increasing in  $\bar{v}$ .

Next we turn attention to the effects of changes in  $\gamma$  and  $q^\dagger$ . We will show that  $\tilde{k}$  is quasi-concave in  $\gamma$  and quasi-concave in  $q^\dagger$  when evaluated at parameter values which ensure that the expression for  $\tilde{k}$  is well defined (requiring  $(\bar{\lambda} - \bar{e})q^* > \bar{\lambda}q^\dagger$ ) and high performance is feasible (requiring  $q^* < 1$ ).

It proves convenient to use the notation  $A \equiv \log[(\bar{w} - \bar{\lambda}\bar{s})(\bar{\lambda}(q^* - q^\dagger) - \bar{e}q^*)] - \log[\bar{e}\bar{s}(\gamma + \bar{\lambda}q^\dagger)]$  for the numerator of  $\tilde{k}$  and  $B \equiv \log[\gamma + \bar{\lambda}q^\dagger] - \log[\bar{\lambda}q^\dagger]$  for its denominator.

We begin by considering the effect of an increase in the hazard rate  $\gamma$  of exogenous government termination. By inspection,  $B$  is increasing in  $\gamma$  and hence

$$\frac{\partial \tilde{k}}{\partial \gamma} > 0 \iff \frac{\partial A / \partial \gamma}{\partial B / \partial \gamma} > \tilde{k}.$$

To establish quasi-concavity we investigate the sign of the second derivative of  $\tilde{k}$  when evaluated at a stationary point. Straightforwardly,

$$\frac{\partial \tilde{k}}{\partial \gamma} = 0 \implies \frac{\partial^2 \tilde{k}}{\partial \gamma^2} < 0 \iff \frac{\partial}{\partial \gamma} \left[ \frac{\partial A / \partial \gamma}{\partial B / \partial \gamma} \right] < 0.$$

Differentiating and re-arranging we obtain

$$\frac{\partial A / \partial \gamma}{\partial B / \partial \gamma} = \frac{(\bar{\lambda} - \bar{e})q^* + \frac{(\bar{\lambda} - \bar{e})c\bar{\lambda}q^\dagger}{v - c\bar{\lambda}}}{(\bar{\lambda} - \bar{e})q^* - \bar{\lambda}q^\dagger} - 1.$$

By inspection this is strictly decreasing in  $q^*$  and so in turn  $q^*$  is strictly increasing in  $\gamma$ . Hence the expression is decreasing in  $\gamma$ , which establishes the quasi-concavity of  $\tilde{k}$  in  $\gamma$ .

We now turn attention to  $q^\dagger$ . We note that the denominator  $B$  is decreasing in  $q^\dagger$ . This implies that

$$\frac{\partial \tilde{k}}{\partial q^\dagger} = 0 \implies \frac{\partial^2 \tilde{k}}{\partial (q^\dagger)^2} < 0 \iff \frac{\partial}{\partial q^\dagger} \left[ \frac{\partial A / \partial q^\dagger}{\partial B / \partial q^\dagger} \right] > 0.$$

Straightforward differentiation of  $A$  and  $B$  yields

$$\frac{\partial A}{\partial q^\dagger} = - \left[ \frac{\bar{\lambda}}{(\bar{\lambda} - \bar{e})q^* - \bar{\lambda}q^\dagger} + \frac{\bar{\lambda}}{\gamma + \bar{\lambda}q^\dagger} \right] \quad \text{and} \quad \frac{\partial B}{\partial q^\dagger} = - \left[ \frac{1}{q^\dagger} - \frac{\bar{\lambda}}{\gamma + \bar{\lambda}q^\dagger} \right].$$

Taking the ratio of these two derivatives and re-arranging we obtain

$$\frac{\partial A / \partial q^\dagger}{\partial B / \partial q^\dagger} = \frac{\bar{\lambda}q^\dagger}{\gamma} \left[ \frac{\gamma + (\bar{\lambda} - \bar{e})q^*}{(\bar{\lambda} - \bar{e})q^* - \bar{\lambda}q^\dagger} \right].$$

By inspection this is strictly increasing in  $q^\dagger$  which establishes the quasi-concavity of  $\tilde{k}$  in  $q^\dagger$ .  $\square$

*Proof of Proposition 6.* The proposition follows from the argument in the text: increased replenishment speeds ministers toward harsher regimes (given that  $q_k^\dagger$  is increasing in  $k$ ) and hence lowers the value of each ministerial career. The income effect drives performance down.  $\square$

*Proof of Proposition 7.* The increase in  $q_k^\dagger$  reduces  $V_j$  for all  $j < k$ . This income effect lowers  $e_j$  for all  $j < k$ . Using Equation (8),  $e_k$  is increasing in  $q_k^\dagger$  if and only if  $q_k^\dagger V_k$  is increasing in  $q_k^\dagger$ . This is so,

following the argument used to prove Proposition 1. To see this, rewrite Equation (7) in the form

$$(\bar{v} + \beta V_{k+1}) - ce_k = [\bar{\lambda} + \lambda(e_k)]q_k^\dagger V_k + (\beta + \gamma)V_k.$$

This takes the same form as Equation (1) but where  $\bar{v}$  has been replaced by  $\bar{v} + \beta V_{k+1}$  (noting that  $V_{k+1}$  is independent of  $q_k^\dagger$ ) and  $\gamma$  has been replaced by  $\gamma + \beta$ .  $\square$

*Proof of Proposition 8.* Solving explicitly for  $W_0$ ,  $W_1$ , and  $\tilde{W}_1$  yields

$$\begin{aligned} W_0 &= \frac{[\beta + \gamma + (\bar{\lambda} - \bar{e})q^*](\bar{w} - \bar{\lambda}\bar{s}) + \beta\bar{e}\bar{s}}{\beta\gamma + (\gamma + \bar{\lambda}q^\dagger)[\gamma + (\bar{\lambda} - \bar{e})q^*]}, \\ W_1 &= \frac{[\beta + \gamma + \bar{\lambda}q^\dagger + (\bar{\lambda} - \bar{e})q^*](\bar{w} - \bar{\lambda}\bar{s}) + (\beta + \gamma + \bar{\lambda}q^\dagger)\bar{e}\bar{s}}{\beta\gamma + (\gamma + \bar{\lambda}q^\dagger)[\gamma + (\bar{\lambda} - \bar{e})q^*]} \\ \text{and } \tilde{W}_1 &= \frac{[\beta + \gamma + 2\bar{\lambda}q^\dagger](\bar{w} - \bar{\lambda}\bar{s})}{\beta\gamma + (\gamma + \bar{\lambda}q^\dagger)^2}. \end{aligned}$$

By inspection  $W_1 > W_0$  so that (naturally) a full talent pool is preferred. There are three possibilities: (a)  $\tilde{W}_1 \geq W_1 > W_0$  so that high performance (when the talent pool is full) is neither feasible nor strictly desirable; (b)  $W_1 > W_0 \geq \tilde{W}_1$  so that high performance is both feasible and strictly desirable; and (c)  $W_1 > \tilde{W}_1 > W_0$  high performance is strictly desired but, alas, is not feasible.

We turn our attention to which of these cases holds as  $\beta$  varies. For large  $\beta$ :

$$\lim_{\beta \rightarrow \infty} W_0 = \lim_{\beta \rightarrow \infty} W_1 = \frac{\bar{w} - \bar{\lambda}\bar{s} + \bar{e}\bar{s}}{\gamma} \quad \text{and} \quad \lim_{\beta \rightarrow \infty} \tilde{W}_1 = \frac{\bar{w} - \bar{\lambda}\bar{s}}{\gamma}.$$

By inspection  $W_1 > W_0 \geq \tilde{W}_1$  must hold for  $\beta$  sufficiently large. Evaluating at  $\beta = 0$ ,

$$\begin{aligned} W_0 &= \frac{[\gamma + (\bar{\lambda} - \bar{e})q^*](\bar{w} - \bar{\lambda}\bar{s})}{(\gamma + \bar{\lambda}q^\dagger)[\gamma + (\bar{\lambda} - \bar{e})q^*]}, \\ W_1 &= \frac{[\gamma + \bar{\lambda}q^\dagger + (\bar{\lambda} - \bar{e})q^*](\bar{w} - \bar{\lambda}\bar{s}) + (\gamma + \bar{\lambda}q^\dagger)\bar{e}\bar{s}}{(\gamma + \bar{\lambda}q^\dagger)[\gamma + (\bar{\lambda} - \bar{e})q^*]} \\ \text{and } \tilde{W}_1 &= \frac{[\gamma + 2\bar{\lambda}q^\dagger](\bar{w} - \bar{\lambda}\bar{s})}{(\gamma + \bar{\lambda}q^\dagger)^2}. \end{aligned}$$

From these expressions we conclude that  $\tilde{W}_1 > W_0$  for small  $\beta$ . Drawing our observations together, case (b) obtains for  $\beta$  large enough, and set  $\beta_H \equiv \inf\{\hat{\beta} : \beta \geq \hat{\beta} \Rightarrow W_1 > W_0 \geq \tilde{W}_1\} > 0$ . ( $\beta_H > 0$  holds since case (b) cannot hold for  $\beta$  small enough.) By continuity, for small  $\varepsilon > 0$  we can ensure that  $W_1 > \tilde{W}_1 > W_0$  for  $\beta \in (\beta_H - \varepsilon, \beta_H)$  so that case (c) obtains. Hence there is a  $\beta_L < \beta_H$  such that high performance is strictly desirable but not feasible when  $\beta_H > \beta > \beta_L$ .  $\square$