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Generous Legislators? A Description of Vote Trading Agreements (Theory and Experiments)

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Abstract

Legislators trade influence to attain approval of their most preferred bills. The classical example is found in pork barrel politics with concentrated benefits and diffuse costs where logrolling agreements involving two (or more) legislators can load costs onto legislators excluded from the winning coalition. As Ferejohn *et al* (1987) show, the cheapest of these outcomes is the only agenda independent one. We model the bargaining game amongst legislators and show that this outcome can be supported by some specification of legislator preferences, but we shed light on a different outcome that has so far been overlooked in the literature: we may observe that legislators most affected by logrolling agreements (those who bear costs with no benefit) may break such coalitions. Specifically, in equilibrium some legislators 'generously' offer their support for some bills that are not to their benefit, and obtain NOTHING in exchange. We report evidence from experimental tests in which our theory predicts 2/3 of the subjects' behaviour.

JEL Classification: C72, C78, D72

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1 Introduction

Legislators usually need to reach decisions over multiple bills and try to have greater influence over ones that are more important to them. However, legislators can only cast a single vote in each decision, so the only way for an individual to exert more influence on a particular bill is by trading votes with fellow legislators. This trade of influence is often referred to as *logrolling* – if you'll help me roll my logs, I'll help you roll yours. The classical example is found in pork barrel politics with concentrated benefits and diffuse costs where agreements involving a majority of legislators allow the approval of a set of bills, each one only benefiting a minority. Despite the prevalence of such arrangements, we still lack a full theoretical characterisation of this phenomenon.

Stratmann (1992) provides empirical evidence of the existence of logrolling agreements in the Congressional amendments to the 1985 Farm Bill.[1] He shows the existence of a coalition of legislators favouring the dairy, sugar, peanut and tobacco bills. A detailed analysis of the data shows that the inclusion of the legislators in favour of the peanut bill was not necessary to form a winning coalition; indeed these legislators did not vote for the dairy bill but legislators with dairy interests still voted for the peanut bill. Why did this happen? Why did some legislators *generously* favour a bill that would only impose costs on them?

There is a large literature explaining the possibility of non-minimal winning coalitions but this will not be the focus of our analysis.[2] A detailed analysis of the 1985 Farm Bill shows that legislators representing dairy and wheat producers were the most numerous.[3] It is thus plausible to assume that these bills are the most costly. In a situation where dairy supporters have gathered enough support to approve their bill, they may try to prevent the approval of any other bill. In particular, they may devote most effort to preventing the approval of the most expensive one, i.e. the wheat bill. It then follows that their generous support for the peanut bill is a strategic move to avoid a logrolling coalition favouring the wheat and peanut bills. In the present paper we show that this is indeed a vote trading equilibrium, one that has been overlooked by the literature. It produces an outcome that appears to indicate the generosity of some legislators but in fact shows their strategic thinking: farsighted legislators break a logrolling coalition by offering their support to a subset of its bills.

The following payoff table illustrates the story above in a simplified Congress with only

[1] Irwin and Kroszner (1996) and Elvik (1995) also show the presence of logrolling agreements in the Smoot-Hawly Tariff Act and highway expenditures across Norway, respectively. Finally, Kardasheva (1008) also shows the existence of logrolling agreements between various EU institutions.

[2] For instance, Weingast (1979) shows that uncertainty about the composition of the winning coalition may lead to an ex-ante preference towards larger coalitions. Other works on non-minimal coalitions include Axelrod (1970), Groseclose and Snyder (1996) or Baron and Ferejohn (1989).

[3] We are indebted to Thomas Stratmann for giving us access to his dataset.

three legislators where bills need a majority of votes to be passed.

	peanut bill	**wheat bill**
peanut legislator	$10 - 3$	-6
wheat legislator	-3	$20 - 6$
dairy legislator	-3	-6

Benefits are concentrated and costs evenly distributed, thus a majority of the legislators wish the dismissal of each bill. However, the peanut and wheat legislators have incentives to logroll by jointly favouring the bill of the other. The dairy legislator loses the most from the logrolling outcome and may try to break a wheat-peanut coalition by appearing *generous* and supporting the peanut bill. This coalition may once again be broken when the minority voter (the wheat legislator) proposes to the dairy legislator that they implement the majoritarian outcome (once again). From a cooperative game theoretical perspective this game would be characterised by having an empty-core; that is, there are no stable coalitions. More precisely, Riker and Brams (1973) and Bernholz (1975) show that whenever a logrolling agreement can occur the voters preferences are such that they induce a cycle and no equilibrium ever exists. Their work is a contribution to the long-running debate between logrolling supporters and detractors, which began with the seminal work of Buchanan and Tullock (1962)[4]

In this paper, we take a novel approach by modelling the negotiation that leads to a vote trading agreement as a non-cooperative dynamic game. A negotiation is a *process of joint decision making. It is communication direct or tacit, between individuals who are trying to forge an agreement for mutual benefit.*[5] We model it as a repeated game with an endogenous status quo: in each period any alternative can challenge the status quo; the majoritarian winner becomes the status quo for next period; and the process only ends when no legislator wishes to continue the process any further. The status quo at the point when the process ends is the policy that is finally implemented. In our model, legislators are farsighted in the sense that they consider not only the benefits of voting in favour of a particular policy today, but also the benefits of alternatives that are likely to replace that policy in the future. Penn (2009) also considers farsighted legislators but looks at a situation where a policy is enacted every period and "decisions made today can greatly affect the types of decisions that are feasible tomorrow."

We show that the existence of a vote trading agreement depends on each bill's relative valuation (i.e. the comparison of costs and benefits across the different bills). Whenever the costs of the bills under consideration are too large relative to their concentrated benefits, no vote trading occurs and the majoritarian will is implemented. On the other hand, when the benefits of a set of bills are large enough, the negotiation process leads to

[4] See, for instance, Coleman (1966), Mueller (1967), Park (1967), Tullock (1970) and Ferejohn (1974).

[5] Young (1991), page 1.

a logrolling outcome where all bills that provide benefits to a majority of legislators are approved. Finally, and most interestingly, whenever the costs of the cheapest bill(s) included in a logrolling agreement are low enough, the legislators excluded from the coalition may form a new coalition in which they offer support for the cheapest bill(s) of the logrolling coalition. In this way, they can prevent the approval of many more bills.[6]

In the previous example, our model finds that there is a vote trading agreement where legislators with dairy interests may *generously* offer their support for the cheapest bill (i.e. the peanut bill). We should emphasise that this outcome arises because the dairy producers prefer an outcome where a single bill is approved rather than an outcome where two bills are approved. We should also note that although we are giving a cooperative interpretation to our results these equilibria are non-cooperative and only require legislators to be farsighted and anticipate the future consequences of voting in favour of any outcome.[7]

The literature on logrolling has focused solely on what we have called the logrolling outcome: a situation where various legislators support each other's bills.[8] For instance, Ferejohn, Fiorina and McKelvey (1987) show strategic voters can only implement agenda independent outcomes (these outcomes coincide with the Condorcet elements from the alternatives preferred to the status quo). In a pork barrel situation like the one in our previous example, approving both bills is the only agenda independent outcome. As a result, legislators with dairy interests offering their unilateral support for the peanut bill should never be observed. Other attempts at modelling vote trading have assumed a centralised market for votes (see Mueller (1967), Wilson (1969) and Phillipson and Snyder (1996). However, these models, by assuming that votes are perfectly tradeable and divisible goods, fail to take into account the institutional arrangements that determine the circumstances in which vote trading occurs.[9]

The difficulties of characterising vote trading agreements empirically are well known in the literature: consensus may be forged at the drafting stage, there may be unobserved compensations or payments, etc.[10] Our example above (extracted from Stratmann (1992)) may also be criticised on the grounds that the data does not capture all roll call votes so there is the possibility of unobserved vote trading agreements. In order to overcome these

[6] The threat of an outcome being the driving force of an *apparently implausible* equilibrium is not new. Within the political economy literature, see for instance Palfrey (1984) where the threat of entry of a third party can lead parties not to converge to the median policy.

[7] This rationale can be found in a variety of bargaining games. See for instance the work on legislative bargaining by Baron and Ferejohn (1989) where a minimum winning coalition is the equilibrium of a divide-the-dollar non-cooperative game

[8] The literature has always understood that *a logrolling situation exists if* xPy and vPw, but ywPxv; *where P stand for social preference as defined by the voting rule employed.* (Stratmann 1992, pg. 1163).

[9] The literature on non-cooperative legislative bargaining is closely related to our analysis but usually assumes a continuous space. See for instance Shepsle (1979), Baron and Ferejohn (1989) and Duggan and Kalandrakis (2007). The cooperative analysis of legislative bargaining is best analysed by Laver and Shepsle (1990).

[10] This sort of agreements is what Buchanan and Tullock (1962) called *implicit logrolling.*

difficulties we design a controlled laboratory experiment. We want to test whether our model of the negotiation captures the distinctive features of real vote trading agreements. For this reason, we embed no structure in our experiments and simply consider groups of three subjects that need to approve two bills and induce payments so that vote trading agreements are possible. We allow subjects to communicate freely (this communication can be private or public) and send binding proposals on the passage of the bills to the other subjects. The negotiation ends whenever a proposal is accepted.

Our experimental data provides supporting evidence for our theory. Most importantly, our theoretical predictions explain two thirds of the observations. Besides, the outcome that never constitutes an equilibrium (where only the most expensive bill is approved) is chosen in only 1% of the observations. Finally, and most reassuringly, the *generous* outcome where only the cheapest bill is approved is the most frequent outcome (more than half of our observations).

The paper is organised as follows. The model and equilibrium concept are introduced in Section 2. In Section 3 we solve the dynamic negotiation game and characterise its equilibria. A description of our experimental design and a detailed analysis of our data can be found in Section 4 below. Finally, Section 5 discussed the robustness of our results and concludes.

2 Model and equilibrium concept

We study situations where legislators can only obtain majority support for their bills if they form coalitions; legislators not included in such winning coalition incur the costs of the bills that are approved and reap no benefits. By regrouping legislators and bills we can think of a situation with three factions (none of which holds a majority of the seats) and two sets of bills benefiting two of the three factions. In order to avoid the question of whether the members of each faction have incentives to vote unanimously, we consider a simplified situation with three voters ($i = 1, 2, 3$) who need to decide whether to approve each of two bills.

The payoff table below summarises any situation where logrolling can occur.[11] The possibility of logrolling requires there to be a majority against each bill and that the individual

[11] The payoff table should be interpreted as the individual payoffs should each bill be approved; when a bill is dismissed the payoff is zero. All our results are robust to any affine transformation).

costs of each bill are smaller than the net benefits of the other bill (i.e. $m_1, m_2 \in (0, 1)$).

	first bill	second bill
voter 1	1	$-m_1$
voter 2	$-m_2$	1
voter 3	$-m3$	-1

Table 1: Payoffs when each of the bills is approved

It can be shown that under any other configuration of payoffs no vote trading agreements occur and the majoritarian will prevails. Without loss of generality we have normalised the voters' payoffs and assumed that $m_3 \in (0, 1)$. Considering $m_3 > 1$ simply requires relabelling the issues and renormalising the payoffs. Note that we are assuming that the costs of approving a bill may differ across voters.

Having two bills that can be approved or dismissed implies that there are four possible alternatives. We denote $\mathcal{A} := \{aa, dd, ad, da\}$ the set of alternatives where the coding of each alternative denotes the outcome on each bill. For instance ad is the alternative where the first bill is approved and the second bill dismissed, and aa is the alternative where both bills are approved (or passed). It is useful to rewrite the voters' preferences in terms of these alternatives where the payoffs have been linearly transformed and simplified using $\lambda_i := \frac{m_i}{1+m_i} \in \left(0, \frac{1}{2}\right)$:

voter 1	ad	$\succ$	aa	$\succ$	dd	$\succ$	da
voter 2	da	$\succ$	aa	$\succ$	dd	$\succ$	ad
voter 3	dd	$\succ$	ad	$\succ$	da	$\succ$	aa
obtained utility:	1		$1 - \lambda_i$		λ_i		0

Table 2: Preferences over the four possible alternatives

The cyclic relation we pointed out in the introduction is captured in Figure 1. The directed graph indicates which alternative is preferred by a majority of the voters (the direction of the arrow indicates the alternative that wins each pairwise vote).

We are now ready to describe our *negotiation game*, which captures the elementary aspects of any vote trading negotiation. There is an infinite horizon. At each period there is a status quo alternative; in the first period the status quo is the majoritarian outcome in both bills (dd). Voters decide by majority whether the status quo remains until the next period or is replaced by an alternative that has been selected with equal probability from the remaining alternatives. At the beginning of each period, any voter can pursue another round of negotiations. When no one wants to continue the negotiations, the status quo

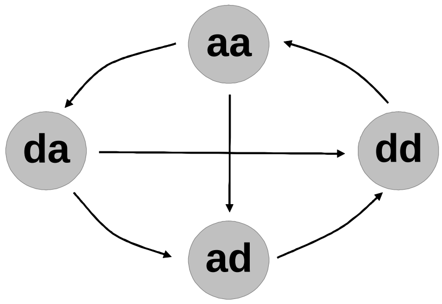

Figure 1: Majoritarian pairwise comparisons of instantaneous utilities.

alternative is implemented and payoffs realised. Finally, we assume that at any given period there is an exogenous probability $\beta \in (0, 1)$ that negotiations end and that the status quo is implemented. In most of our analysis we assume β to be arbitrarily close to 0.

Assuming an exogenous probability of termination allows us to capture the fact that the path through which an outcome is reached in a negotiation is not irrelevant (even if the probability is arbitrarily small). Dropping this modelling feature makes our negotiation game superfluous because the process through which voters reach an alternative is meaningless; voters only care about the final outcome and any alternative can be supported as an equilibrium of our negotiation game.

Two more features of our model deserve further attention. First of all, in our model alternatives that challenge the status quo are selected at random. This is intended to capture the lack of structure of any negotiation process where new alternatives may be proposed without restrictions. In contrast, most of the literature on legislative bargaining assumes that in each period a legislator is randomly assigned proposal power and decides on the alternative that challenges the status quo (e.g. Baron and Ferejohn (1989) or, more recently, Duggan and Kalandrakis (2007)). Secondly, in our model the process only ends when all the players want it to. Once again, with this distinctive feature we try to capture that parties involved in a negotiation may delay the final decision indefinitely if they believe that a better agreement is possible.

We assume that individual behaviour is Markovian: all that matters from the past is the current status quo. A Markovian pure strategy for voter i, σ_i, is defined as a pair (s_i, v_i). The first element of the pair denotes whether agent i wishes to stop the negotiations at any alternative, that is $s_i = \{s_i^x; x \in \mathcal{A}\}$, where $s_i^x = 1$ denotes his wish to stop negotiations at alternative x and $s_i^x = 0$ denotes his wish to continue negotiations at alternative x. The second element of the pair denotes the voting intentions of agent i, that is $v_i = \{v_i^{xy}; x, y \in \mathcal{A}, x \neq y\}$, where $v_i^{xy} \in \{x, y\}$ denotes his vote in a pairwise comparison between alternatives x and y.

The aggregate decisions at each node are denoted s^x and v^{xy}, for $x, y \in \mathcal{A}$. Any voter can delay the end of the negotiation process, so $s^x = s_1^x \cdot s_2^x \cdot s_3^x$. The majoritarian winner in the pairwise comparison between x and y is x if and only if $\#\{i : v_i^{xy} = x, i = 1, 2, 3\} \geq 2$. When $v^{xy} = x$ we may also write that $x \succ y$, i.e. the group preference ranks x above y.

The timing at each period of the negotiation game is as follows:

1. $x \in \mathcal{A}$ is the status quo
2. voters announce whether they wish to continue or stop the negotiation, s_i^x
3. if $s^x = 1$ the negotiation ends and the status quo is implemented
4. with probability β the negotiation ends and the status quo is implemented
5. any alternative $y \in \mathcal{A} \setminus x$ is equally likely to become the challenger
6. voters cast their vote v_i^{xy} and the majoritarian winner, v^{xy}, becomes the status quo for the next period

The continuation utility for a farsighted negotiator i is

$$U_i(x) = \begin{cases} u_i(x) & \text{if } s^x = 1 \\ \beta u_i(x) + \frac{1-\beta}{3}\left[U_i(v^{xy}) + U_i(v^{xz}) + U_i\left(v^{xt}\right)\right] & \text{if } s^x = 0 \end{cases}$$

where x, y, z and t are different alternatives from $\mathcal{A}$. The continuation utility at alternative x is the instantaneous utility at x ($u_i(x)$ as described in Table 2) should all voters decide to stop the negotiations at that node. On the other hand, if voters decide to continue negotiations for another period, there are two terms. The first term is the exogenous probability that negotiations end (β) times the instantaneous utility at x. The second term is the sum of the continuation utilities of the alternatives that win each pairwise vote (this last term is multiplied by the probability that negotiations do not end, $(1-\beta)$, and the probability each alternative becomes the challenger to the status quo, i.e. $\frac{1}{3}$).

Our equilibrium concepts looks at stationary Markovian (perfect Bayesian) equilibrium strategies under a weak dominance requirement (i.e. voters act as if their decision is pivotal):

Definition 1 *Given parameters $m_1, m_2, m_3, \beta \in (0,1)$, an equilibrium in pure strategies of the negotiation game described above is given by a set of strategies $\{\sigma_i = (s_i, v_i), i = 1, 2, 3\}$ such that for each voter $i = 1, 2$ and 3, the following conditions hold*

1. $\forall x, y \in \mathcal{A}$, $v_i^{xy} = x$ if $U_i(x) > U_i(y)$

2. $\forall x \in \mathcal{A}$*,* $s_i^x = 1$ *if* $u_i(x) \geq U_i(x \mid s^x = 0)$

and the voters' beliefs are confirmed in equilibrium.

In what follows we do not explicitly specify equilibrium beliefs and simply assume that they coincide with equilibrium behaviour. By definition 1, we know that once we find a set of strategies that satisfy conditions (1) and (2), equilibrium beliefs coincide with such strategies.

The difficulty of finding stationary Markovian (perfect Bayesian) equilibria is well known in the literature.[12] Rogers (1969) and Sobel (1971) prove their existence when the set of alternatives and actions is finite, and players are allowed to use mixed strategies. Penn (2009) and Duggan and Kalandrakis (2007) extend this result when the set of alternatives and actions is a multidimensional Euclidean space. One of the key differences between these two works is that the latter endogenises the agenda.

We depart from this branch of the literature by assuming pure strategies. Our objective is the characterisation of the equilibria of our specific negotiation game and our modelling assumptions make the problem solvable. A distinctive and elegant characteristic of our equilibria is that negotiations only terminate at alternatives that win all pairwise comparisons. The following definition ties down this idea.

Definition 2 *Alternative* $x \in \mathcal{A}$ *is a Farsighted Condorcet Winner (FCW), whenever there exist an equilibrium in which* x *wins all pairwise comparisons, i.e.* $v^{xy} = x, \forall y \in \mathcal{A} \setminus x$*.*

This definition is similar to the one used by Roberts (2007) where it is shown that Condorcet cycles may be avoided whenever voters implement one of three alternatives in each period (a very patient subject who is almost indifferent between his two top alternatives may prefer to always implement his second best alternative rather than cycling around the three available ones). In a similar vein, Bernheim and Nataraj (2004) look at a non game theoretic repeated majoritarian election and characterise the sequence of history dependent outcomes that win all pairwise comparisons.

3 The negotiation game

Our first key finding is that negotiations only stop when a FCW exists. Whenever there is (at least) an alternative that wins a pairwise comparison with the status quo, there is (at least) a player that prefers to continue the negotiation process.

[12] See the excellent review by Dutta and Sundaram (1998).

Proposition 1 *A negotiation game has an equilibrium that stops with alternative $x \in \mathcal{A}$ if and only if x is a Farsighted Condorcet Winner, i.e. $s^x = 0 \Leftrightarrow v^{xy} = x, \forall y \in \mathcal{A} \setminus x$.*

When there is a single alternative $y \in \mathcal{A}$ that wins the pairwise vote with the status quo $x \in \mathcal{A}$ we know that there are at least two voters for whom $U_i(y) > U_i(x)$. The continuation utility at x when $s^x = 0$ reads as follows:

$$U_i(x \mid s^x = 0) = \beta u_i(x) + \tfrac{1-\beta}{3}\left[U_i(x) + U_i(x) + U_i(y)\right].$$

Taking into account that $U_i(y) > U_i(x)$ we can rewrite the previous expression as $U_i(x \mid s^x = 0) > u_i(x)$. In other words, the voters that support the pairwise comparison between x and y prefer to continue the negotiation at node x. The rest of the proof can be found in the Appendix and shows that when the status quo loses two or three pairwise comparisons there is always a voter who prefers to continue the negotiations.

The fact that negotiations only stop at a FCW implies that equilibrium stopping conditions do not affect continuation utilities. It can be easily shown that the continuation utility at a FCW $x \in \mathcal{A}$ is the same whether or not the voters agree to stop:

$$U_i(x \mid s^x = 0) = \beta u_i(x) + \tfrac{1-\beta}{3}\left[U_i(x) + U_i(x) + U_i(x)\right] = u_i(x) = U_i(x \mid s^x = 1),$$

thus in all our subsequent analysis we can omit any reference to the stopping decisions. We simply need to specify the individual voting decisions, knowing that negotiation will only stop at a FCW.

Definition 2 implies that there can be at most one FCW. Moreover, when the exogenous probability of the negotiations ending (β) is arbitrarily small, only alternatives with $s^x = 1$ can be the outcome of our negotiations. It follows from Proposition 1 that:

Corollary 1 *As the exogenous probability of the negotiations ending tends to zero ($\beta \to 0$), the only alternative that can be implemented in a negotiation game is the FCW.*

In order to characterise all possible vote trading agreements we now find the equilibria with a FCW. When individual voting decisions lead to a FCW, group preferences among the remaining alternatives can contain a cycle or a loser. We define an alternative to a be a *loser* when it loses all pairwise comparisons. We analyse these two cases separately. All proofs of the Lemmas and Propositions can be found in the Appendix.

3.1 FCW and cycle among the remaining alternatives

Assume that *dd* is a FCW and that there is a cycle among the three remaining alternatives. For voters 1 and 3, the instantaneous utility from alternative *ad* is higher than their instantaneous utility for any other alternative in the cycle. It can be proved that these two voters always vote from ad in a pairwise vote with da or aa. It follows that:

Lemma 1 *The negotiation game has no equilibrium where alternative dd is the FCW and the remaining alternatives form a cycle.*

Alternatives ad and da play symmetric roles for voters 1 and 2. Alternative ad is the most preferred alternative for voter 1 and the least preferred for voter 2, whilst alternative da is the most preferred alternative for voter 2 and the least for voter 1. This implies that any pairwise vote involving ad (da) sees voter 1 (2) voting in favour and voter 2 (1) voting against. Thus, voter 3 is always pivotal in any vote involving any of these two alternatives. It follows that alternative da can never constitute a FCW –voter 3 cannot favour his second worst alternative over any other one.

Lemma 2 *The negotiation game has no equilibrium where alternative da is the FCW and the remaining alternatives form a cycle.*

When aa is a FCW there are two possible cycles among the remaining alternatives: $ad \succ dd \succ da \succ ad$ or $ad \succ da \succ dd \succ ad$. The first case is depicted in Figure 2.

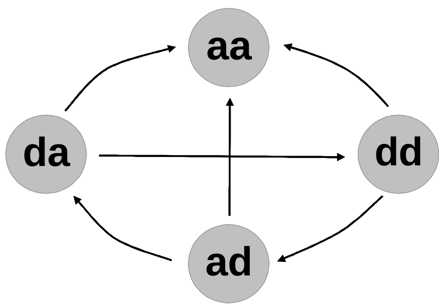

Figure 2: Equilibrium configuration when aa is a FCW and $ad \succ dd \succ da \succ ad$.

In any pairwise comparison, there will always be a majority in favour of one alternative. Hence, given any two pairwise votes there is always a voter that belongs to the majority on both votes. In other words, when the group preferences are such that $ad \succ dd \succ da \succ ad$, each player should satisfy one of the following chains of inequalities

$$\begin{cases} U_i\,(ad) > U_i\,(dd) > U_i\,(da) \\ U_j\,(da) > U_j\,(ad) > U_j\,(dd) \\ U_k\,(dd) > U_k\,(da) > U_k\,(ad) \end{cases}$$

In the Appendix we show that given an equilibrium configuration with a FCW and a cycle, each voter highest valued alternative should coincide with the one that yields highest instantaneous payoff (among the alternatives in the cycle). This implies that voters i, j and k correspond to voters 1, 2 and 3, respectively. In addition, we know that voter 3 never votes in favour of the implementation of his least preferred alternative (aa). As a result, aa is a FCW only when voters 1 and 2 favour it in all pairwise votes (i.e. $U_1(aa) > U_1(ad)$ and $U_2(aa) > U_2(da)$). In the Appendix we simplify all the above conditions using a non-recursive formulation of the continuation utilities and obtain the following result.

Proposition 2 *The negotiation game has an equilibrium where alternative aa is the FCW and the remaining alternatives form a cycle. This equilibrium holds only when:*

1. $\lambda_1 < \frac{3(1-\beta)}{2\beta^2-\beta+8}, \lambda_2 < \frac{1-\beta}{3}, \lambda_3 > \frac{\beta+2}{\beta+5}$ *and* $ad \succ dd \succ da \succ ad$

   or,

2. $\lambda_1 < \frac{1-\beta}{3}, \lambda_2 < \frac{3(1-\beta)}{2\beta^2-\beta+8}, \lambda_3 > \frac{1-\beta}{4-\beta}$ *and* $ad \succ da \succ dd \succ ad$.

A decrease in the exogenous probability of stopping the negotiation (β) relaxes all conditions (the bounds for the first case are illustrated in Figure 3). When β is close to 1 the possibility of sustaining a FCW vanishes. This can be easily explained because when β is close to 1 we come close to our initial static problem where no Condorcet Winner exists.

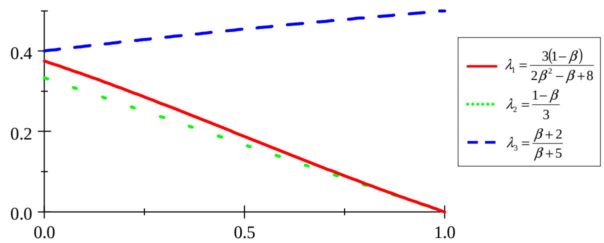

Figure 3: Bounds on $\lambda_i, i = 1, 2, 3$ that ensure aa is the FCW and $ad \succ dd \succ da \succ ad$.

The FCW, alternative aa, is the least preferred alternative by voter 3. This implies that voter 3 never votes in favour of this alternative as its continuation utility is 0, $U_3(aa) = 0$. This implies that aa can only be a FCW when both voters 1 and 2 prefer it above any other alternative. In particular, their valuation of aa needs to be above the continuation utilities on other nodes; $(1-\lambda_1)$ and $(1-\lambda_2)$ need to be large enough. Or, as specified in Proposition 2, λ_1 and λ_2 are bounded above. In the first case (when group preferences are $ad \succ dd \succ da \succ ad$) the bound for voter 1 is slightly above the one for voter 2. This occurs because voter 1's most preferred alternative is followed by his least preferred alternative; instead, voter 2's most preferred alternative is followed by his third ranked alternative. This implies that voter 1's expected utility at his most preferred alternative of the cycle is lower than the expected utility of voter 2 at his preferred alternative. It follows that voter 1 is (slightly) more inclined to vote in favour of the FCW than voter 2.

The cycle requires voter 3 to favour *da* over *ad*. This vote allows him to *move* towards his most preferred alternative (*dd*) but is a vote *against* his instantaneous utilities. This is why in equilibrium we need $u_3(da) = \lambda_3$ and $u_3(ad) = (1 - \lambda_3)$ to be close enough, i.e. λ_3 bounded below.

The equilibrium when group preferences are $ad \succ da \succ dd \succ ad$, implies the reasoning above is reversed and so are the bounds for voters 1 and 2. Voter 3's bound is now relaxed because the chain of inequalities required for his pairwise votes ($U_3(dd) > U_3(ad) > U_3(da)$) coincides with his instantaneous valuations ($u_3(dd) > u_3(ad) > u_3(da)$). However, λ_3 cannot be too small, otherwise the gains from moving towards his most preferred alternative would not outweigh the costs of being closer to his least preferred one.

The case where *ad* is a FCW and group preferences cycle among the remaining alternatives can be characterised analogously.

Proposition 3 *The negotiation game has an equilibrium where alternative ad is the FCW and the remaining alternatives form a cycle. This equilibrium holds only when:*

1. $\lambda_1 < \frac{\beta+2}{\beta+5}, \lambda_2 > \frac{1-\beta}{4-\beta}$, *and* $\lambda_3 < \frac{3(1-\beta)}{2\beta^2-\beta+8}$ *when* $aa \succ dd \succ da \succ aa$

   or,

2. $\lambda_1 < \frac{1-\beta}{4-\beta}, \lambda_2 > \frac{\beta+2}{\beta+5}$, *and* $\lambda_3 < \frac{1-\beta}{3}$ *when* $aa \succ da \succ dd \succ aa$.

The interpretation of this equilibrium is analogous to the previous one while taking into account that the FCW is now the least preferred alternative of voter 2 –i.e. λ_2 is bounded below, and λ_1 and λ_3 are bounded above.

3.2 FCW and a loser among the remaining alternatives

Consider the four different alternatives x, y, z, and $t \in \mathcal{A}$. Assume that x is a FCW and t is an alternative that loses all pairwise comparisons; without loss of generality, further assume that the group prefers y to z, i.e. $v^{yz} = y$. With these assumptions, we can write the continuation utilities:

$$\begin{aligned} U_i(x) &= \beta u_i(x) + \tfrac{1-\beta}{3}\left[U_i(x) + U_i(x) + U_i(x)\right] \\ U_i(y) &= \beta u_i(y) + \tfrac{1-\beta}{3}\left[U_i(x) + U_i(y) + U_i(y)\right] \\ U_i(z) &= \beta u_i(z) + \tfrac{1-\beta}{3}\left[U_i(x) + U_i(y) + U_i(z)\right] \\ U_i(t) &= \beta u_i(t) + \tfrac{1-\beta}{3}\left[U_i(x) + U_i(y) + U_i(z)\right]. \end{aligned} \tag{1}$$

A first exploration of these continuation utilities tells us that the described configuration is an equilibrium only when the following Lemma holds.

Lemma 3 *The negotiation game has an equilibrium where alternative $x \in \mathcal{A}$ is the FCW, there is a loser $t \in \mathcal{A}$, and the other remaining alternatives are such that the group prefers y to z, only when each inequality in $u_i(x) > u_i(y) > u_i(z) > u_i(t)$ is satisfied by a majority of voters.*

The Lemma's proof just requires comparing the continuation utilities. For instance, we know that there is a majority that votes for z in the pairwise vote between z and t, i.e. a majority of voters satisfy $U_i(z) > U_i(t)$. The second term of the continuation utilities at z and t coincide (see (1)) thus there is a majority of voters for which $u_i(z) > u_i(t)$.

Lemma 3 allows us to characterise all pairwise comparisons when da is the FCW: ad loses all pairwise comparisons and $aa \succ dd$. It was noted earlier that when da is the FCW, voter 3's decision on any pairwise vote with da is pivotal (voter 1 always votes against da and voter 2 always favours it). In particular, voter 3 should favour da over his most preferred alternative dd but this can never happen. It follows that:

Lemma 4 *The negotiation game has no equilibrium where alternative da is the FCW and there is a loser among the remaining alternatives.*

Lemma 3 tells us necessary conditions for an equilibrium configuration with a FCW and a loser. These conditions are not sufficient as they only ensure that three pairwise comparisons are met. Three further comparisons need to be met: when x is the FCW, t is the loser and $y \succ z$ we also need a majority of voters satisfying each of the following inequalities:

$$\begin{cases} U_i(x) > U_i(z) \\ U_i(x) > U_i(t) \\ U_i(y) > U_i(t) \end{cases} \tag{2}$$

When aa is the FCW, Lemma 3 implies that da is the loser. The first two conditions in 2 can now be read as: the FCW needs to obtain a majority of votes when compared with ad and when compared with da. It can be shown that the pivotal voter in those decisions is voter 1 but he cannot simultaneously favour the necessary outcome in both of these pairwise votes.

Lemma 5 *The negotiation game has no equilibrium where alternative aa is the FCW and there is a loser among the remaining alternatives.*

Finally, the proof of the following Proposition requires imposing Lemma 3's conditions when dd or ad are FCW and the conditions in (2).

Proposition 4 *The negotiation game has an equilibrium with a FCW and a loser only when*

1. *ad is the FCW, da is the loser, $aa \succ dd$, $\lambda_2 < \frac{1-\beta}{4-\beta}$ and $\lambda_3 < \frac{1-\beta}{\beta+2}$*

   or

2. *dd is the FCW, aa is the loser, $ad \succ da$, $\lambda_1 > \frac{1-\beta}{4-\beta}$ and $\lambda_2 > \frac{3(2\beta+1)}{2(\beta+2)^2}$.*

In both cases we have a situation where there is no constraint on the voter whose ideal alternative is the FCW. When ad is the FCW we need voter 3 to always support such alternative (as voter 2 always opposes such alternative) but this can only happen when voter 3's instantaneous valuation at this alternative $(1 - \lambda_3)$ is large enough, i.e. λ_3 is bounded above; the bound on λ_2 is required to ensure da loses the pairwise vote with aa. When dd is the FCW we need voters 1 and 2 to support such alternative in different pairwise comparisons but this only happens when their valuation of alternative dd is large enough, i.e. λ_1 and λ_2 are bounded below.

3.3 Interpretation of our results

Legislators trade influence to attain approval of their most preferred bills. In our model there are two bills whose benefits are concentrated on voters 1 and 2's constituencies. The fact that each bill benefits only one legislator but imposes costs on the remaining two implies that there is no majoritarian support for either bill.

The literature on logrolling has focused on the fact that voters 1 and 2 may trade their votes and support each other's bills. In this paper we have constructed a precise model of the negotiations leading to this trading agreements and shown all possible vote trading agreements in terms of the bills' relative payoffs for each voter. We have shown that:

- When two voters may gain from favouring each other bills but the gains from such an agreement aren't large enough, negotiations lead to the majoritarian outcome where no bill is passed. In such a case, a farsighted voter prefers implementing the majoritarian outcome, given the risk of implementing a much worse alternative where only the bill that does not benefit his own constituency is passed.
- The logrolling outcome where both bills are passed is observed when the benefits from the logrolling agreement are large enough. In other words, when the targeted

benefits of the bills are much larger than their costs. There is an additional insight from our analysis: this equilibrium can only be supported when the costs to the third voter from both bills are sufficiently similar.

- If, on the other hand, when voter 3 bears much lower costs from one bill than from the other one, he votes in favour of (only) approving the cheapest bill. Heuristically, we can interpret this equilibrium as one where voter 3, in the light of the risk of seeing both bills passed, and taking into account that one of the bills is very cheap, breaks the logrolling coalition. There is a further requirement for this equilibrium to be sustained: the gains from a logrolling agreement need to be large enough for at least one of the voters.

Once we allow legislators to negotiate their way to a better outcome than the majoritarian one, they may end up not approving the bills that benefit a minimal winning coalition. The presence of the logrolling outcome gives incentives to the voter(s) not included in the logrolling coalition to accept the implementation of his least costly bill(s). In our framework, this implies that the third voter implements his second best alternative (ad) and avoids his worst alternative (aa). This outcome is only observed when voter 3's costs of implementing the first bill are low enough and when the threat of a logrolling outcome is significant, i.e. when the gains from a logrolling agreement are high for (at least) one voter.

Finally, we need to emphasize that even though we provide an almost cooperative heuristic of the equilibria, these arise from a non-cooperative behaviour among voters. We require voters to consider not only the instantaneous benefits from choosing an alternative but also its future consequences in the negotiation.

4 Experimental evidence

Vote trading agreements are difficult to observe: roll call votes may only be the tip of the iceberg. Strategic agents foresee the behaviour of other agents and modify the drafting of the bills they want to see approved by accommodating the opinions of a majority of legislators (Tsebelis 2002). Mattila and Lane (2001) analyse voting in the European Union Council of Ministers and argue that the adoption of legislation in the Council is preceded by an extensive preparatory phase in various EU bodies. During the period from 1994 to 1998 analysis of the Council's roll call data shows that decisions were unanimously adopted 78% of the time.[13] This last aspect reinforces the idea that votes are probably just one of the many tools that legislators use to reach their goals. It is for this reason

[13] On the practice of *universalism* in US politics see for instance Shepsle and Weingast (1981) and references therein.

that we do not attempt to apply our model to real world data. Instead, we test our theory with controlled laboratory experiments.

We run 2 sessions with 18 subjects per session. Students are recruited through the online recruitment system ORSEE (Greiner (2004)) and the experiment takes place on networked personal computers in the LEEX at Universitat Pompeu Fabra. The experiment is programmed and conducted with the software z-Tree (Fischbacher (2007)).[14] The same procedure is used in all sessions. Instructions (see Appendix) are read aloud and questions answered in private. Students are asked to answer a questionnaire to check their full understanding of the experimental design (if any of their answers are wrong the experimenter refers privately to the section of the instructions where the correct answer is provided). Students are isolated and are not allowed to communicate.

Each session consists of 20 periods. At the start of each period subjects are randomly matched into groups of three. Subjects' payoffs (expressed in euro cents) depend on the outcome for each bill. The table below captures a possible set of payments (a screenshot image of the program can be found in the Appendix):

	first bill	**second bill**
Participant A	passed 700	not passed 200
Participant B	not passed 300	passed 600
Participant C	not passed 100	not passed 800

Table 3: One possible configuration of subjects' payoffs

For example, Participant B's payoff from the first bill is 0 if the bill is passed and 300 if it is not passed. We avoid *framing effects* by normalising payoffs so that the maximum any subject can obtain in a given period is 900.[15] Additionally, payoffs are never negative and are multiples of 100 (strictly higher than 0) and are drawn from a uniform distribution.[16] Each subject's role in each period is randomly selected and may change in every period – a subject can play as Participant A, B or C.

At the beginning of each period, payoffs are announced and subjects can communicate freely with the other members of the group through three chat boxes: the first chat box allows public communication with all members of the group; the other two boxes allow private (confidential) communication with each member of the group.[17] After three minutes of communication subjects can propose binding agreements on both bills. When a

[14] The data and programme code for the experiment are available upon request.

[15] Framing effects imply that voters may behave differently when they are assigned payments (300,600) or (5,10), see the seminal reference Kahneman and Tversky (1983). Note that the normalisation is done without loss of generality from the perspective of our theoretical model.

[16] That is, (800,100), (700,200), (600,300), etc. are all equally likely.

[17] There was a unique restriction in the subjects' communication: they could not identify themselves. Failure to satisfy this rule implied the subject being excluded from the laboratory participant list (no subject breached that rule).

subject accepts a proposal, negotiations end and that proposal becomes the final outcome of the period. If no proposal is accepted within five minutes of the start of the period, both issues are dismissed. At the end of each period the computer announces the final outcome and payoff to each member of the group.

The final payment of the session is computed by adding the payoff obtained in three (randomly selected) periods and the show up fee of 3€. At the end of each session participants are asked to fill in a questionnaire on the computer and are given their final payment in private. The average final payment is 17.50€ (minimum payment is 6€ and maximum payment is 27€). Session length, including waiting time and payment, is around an hour and a half.

Our data contains 240 observations where a group of three subjects negotiated and obtained one of four possible final outcomes. Looking at the frequency of each outcome (Table 4) we can highlight some interesting features.

outcome	**frequency**
aa (approve both bills)	24 %
ad (approve /dismiss)	57 %
da (dismiss / approve)	1 %
dd (dismiss both bills)	18 %

Table 4: Frequencies of final outcomes

First, and most importantly, the generous outcome (*ad*) where only one bill is passed is the most frequent one. This highlights the relevance of the situation described in the introduction where the legislators favouring the dairy bill supported the peanut bill obtaining nothing in exchange. Second, the outcome *da* (never supported by any parameter configuration in our model) is the least likely in our data: there are only three observations where the first bill is dismissed and the second one approved (two in the first period and one in period 11).

Subjects never exhaust the total time of five minutes per period and given that the communication among them is unrestricted we consider the predictions of our model when the exogenous factor of ending the negotiation is arbitrarily small. In other words, we consider the bounds of our equilibria when $\beta \to 0$. We can now determine whether the final outcome of each negotiation coincides with the one predicted by our theory. Table 5 below shows the frequency counts of each outcome depending on what the theory predicted. Note that there are situations where the theory remains silent ($\oslash$) because the payoffs do not satisfy the set of constraints in Propositions 2, 3 or 4. Note also that there are situations where there are multiplicity of equilibria.

realised	**equilibria**					
outcome	$\oslash$	*aa*	*aa* and *ad*	*ad*	*ad* and *dd*	*dd*
aa	8	2	10	33	3	1
ad	7	3	21	93	4	8
da	1	0	2	0	0	0
dd	8	1	8	19	2	6

Table 5: Frequency counts of realised outcomes vs. theory prediction

When there is no equilibrium, it is equally likely that the negotiations end in *aa*, *ad*, or *dd*. Table 5 shows that the generous outcome is not only the most frequent outcome but it is the most frequent in each and every situation where an equilibrium of our negotiation game is predicted. When the generous outcome can be sustained as an equilibrium of our negotiation game, subjects attain it 64% of the time.

Overall, subjects reach an outcome that coincides with the one(s) predicted by the theory 64% of the time. When we eliminate the cases where there is multiplicity of equilibria, subjects match the theoretical prediction 61% of the time. Finally, it is worth noting that there is no time trend so we cannot claim that subjects learn to play the equilibrium.

A critical aspect of this experiment is the fact that most payoff configurations replicate a situation where the generous outcome (where only one bill is passed) is the predicted equilibrium. This is a consequence of our uniform prior assumption on each individuals's payoffs and the bounds where each equilibrium is supported when β tends to 0. Whilst we realised this fact when we designed the experiment, we decided to preserve this design for two reasons. First, it allows us to show the high likelihood of being in parameter configurations where the generous outcome is an equilibrium. Second, it allows us to test the main contribution of this paper: whether the generous outcome (that has been overlooked by the literature) is supported when subjects negotiate vote trading agreements.

Finally, we check the robustness of our experimental design to the fact that subjects cannot send proposals (and so are forced to negotiate) in the first three minutes of each period. For this purpose we run a pilot session in which binding proposals could be sent from the onset of each period. We observe more variance in the data but qualitatively nothing changes: the generous outcome is the most frequent, *da* is only observed 4 times and subjects match the theoretical prediction 50% of the time.

5 Concluding remarks

All our analysis builds on Proposition 1 where we show that voters only stop negotiations at a FCW. When we assume that the exogenous factors that end negotiations are negligible ($\beta \to 0$) we find that negotiations can only implement FCW. However, there are situations where no FCW exists.[18] In such circumstances, our theory predicts that parties will never agree to a negotiated outcome. Following the work by Eliaz, Ray and Razin (2004), in which the aversion towards disagreement is explicitly modelled in a model with incomplete information where agents need to decide over two alternatives, we could think of an extended model where agents only engage in negotiations when they anticipate that they will reach an agreed outcome (i.e. they enter the negotiation game only when it has a FCW). A detailed analysis of the further strategic incentives that this extension may introduce is left for future work.

Another aspect that hasn't been mentioned is the welfare consequences of vote trading agreements. When we look at the majoritarian pairwise comparisons of instantaneous utilities (see Figure 1) we can see that there is no pairwise comparison supported by all voters. It then follows that all four agreements are Pareto optimal and, from this perspective, vote trading agreements can never be harmful. Any further optimality analysis requires inter-personal comparisons of utility and will always depend on the particular normalisation of payoffs used.

Designing more elaborate experiments is certainly an area of fruitful research. We need to further embed the institutional features that may restrict or enhance particular vote trading agreements (complex setting with more voters and issues, different amendment rules, repeated interaction...). It is only then that we will be able to see whether our model still captures the distinctive features of vote trading agreements.

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6 Appendix

Proof of Proposition 1. When the status quo is a FCW, there are no gains from continuing the negotiations, thus $s_i^x = 1$ for $i = 1, 2$, and 3.

When the status quo $x \in \mathcal{A}$ only loses one pairwise comparison against $y \in \mathcal{A}$, the continuation utility for any voter when $s^x = 0$ is,

$$U_i(x \mid s^x = 0) = \beta u_i(x) + \tfrac{1-\beta}{3}\left[U_i(x) + U_i(x) + U_i(y)\right].$$

Note that the term inside the square brackets captures the fact that x wins two pairwise comparisons and loses the comparison with y. Using the fact that there is a majority of voters such that $U_i(y) > U_i(x)$ we have that $U_i(x \mid s^x = 0) > u_i(x)$. In other words, voters that prefer y to x prefer continuing the negotiations at node x.

When the status quo $x \in \mathcal{A}$ loses two pairwise comparisons (against $y, z \in \mathcal{A}$) there is at least one voter such that $U_i(y), U_i(z) > U_i(x)$. We can follow the same steps as above and show that:

$$U_i(x \mid s^x = 0) = \beta u_i(x) + \tfrac{1-\beta}{3}\left[U_i(x) + U_i(y) + U_i(z)\right] > u_i(x).$$

Once again, there is a voter that prefers continuing the negotiation at node x.

When the status quo loses the three pairwise comparisons we need to show that there is still an individual that prefers continuing the negotiations. When a voter's preferences with respect to alternative x coincide with the group preferences we can replicate the above analysis. However, when each voter's preferences coincide with the group preferences on two issues but disagree on one issue, we need to show that the benefits of continuing the negotiations outweigh the costs. Without loss of generality, we can assume that the following inequalities hold:

$$\begin{aligned} U_1(y), U_1(z) &> U_1(x) > U_1(t) \\ U_2(z), U_2(t) &> U_2(x) > U_2(y) \\ U_3(t), U_3(y) &> U_3(x) > U_3(z) \end{aligned}$$

A necessary condition for these inequalities to hold is that x is not the least preferred alternative for any player, i.e. $u_i(x) \neq 0, \forall i$. This implies that x can only be *dd*. However, *dd* is voter 3's preferred alternative thus there can not be two alternatives t and y whose continuation utilities are higher than $U_3(x)$. ■

Proof of Lemma 1. Assume there is an equilibrium with a FCW at $x \in \mathcal{A}$ and a cycle among alternatives $y, z, t \in \mathcal{A}$. Without loss of generality, group preferences over the alternatives of the cycle can be assumed to be: $y \succ z \succ t \succ y$. Expected utilities for player i read as follows

$$\begin{aligned}
&U_i(x) = \beta u_i(x) + \tfrac{1-\beta}{3}\left[U_i(x) + U_i(x) + U_i(x)\right] = u_i(x)\\
&U_i(y) = \beta u_i(y) + \tfrac{1-\beta}{3}\left[U_i(x) + U_i(y) + U_i(t)\right] = \tfrac{3\beta}{2+\beta}u_i(y) + \tfrac{1-\beta}{2+\beta}\left[U_i(x) + U_i(t)\right]\\
&U_i(z) = \beta u_i(z) + \tfrac{1-\beta}{3}\left[U_i(x) + U_i(z) + U_i(y)\right] = \tfrac{3\beta}{2+\beta}u_i(z) + \tfrac{1-\beta}{2+\beta}\left[U_i(x) + U_i(y)\right]\\
&U_i(t) = \beta u_i(t) + \tfrac{1-\beta}{3}\left[U_i(x) + U_i(t) + U_i(z)\right] = \tfrac{3\beta}{2+\beta}u_i(t) + \tfrac{1-\beta}{2+\beta}\left[U_i(x) + U_i(z)\right].
\end{aligned}$$

In any pairwise comparison, there must be a majority in favour of one alternative. Hence, given any two pairwise votes there always exists a voter that belongs to the majority on both votes, i.e. $\forall x, y, z, t \in \mathcal{A}$ such that $x \neq y, z \neq t$, there exists $i \in \{1, 2, 3\}$ such that $v_i^{xy} = v^{xy}$ and $v_i^{zt} = v^{zt}$. In other words, there always exists a voter whose preferences over two pairwise comparisons coincide with the group ones.

Therefore, when group preferences are such that $y \succ z \succ t \succ y$., there should exist a voter $i \in \{1, 2, 3\}$ for which

$$U_i(y) > U_i(z) > U_i(t). \tag{3}$$

We can rewrite the first inequality in (3) using the above expression for the continuation utilities: $3\beta u_i(y) + (1-\beta)U_i(t) > 3\beta u_i(z) + (1-\beta)U_i(y)$. This last inequality together with (3) implies that $u_i(y) > u_i(z)$. Similarly, $U_i(y) > U_i(t)$ can be rewritten as: $3\beta u_i(y) + (1-\beta)U_i(t) > 3\beta u_i(t) + (1-\beta)U_i(z)$ which implies that $u_i(y) > u_i(t)$.

By repeating this process with the other two chains of inequalities among the alternatives in the cycle, we obtain that for each alternative in the cycle, there exists a voter whose instantaneous utility for that alternative is higher than his instantaneous utility for any other alternative in the cycle, i.e.

$$\begin{aligned}
&\exists i \in \{1, 2, 3\} : u_i(y) > u_i(z), u_i(t)\\
&\exists i \in \{1, 2, 3\} : u_i(z) > u_i(t), u_i(y)\\
&\exists i \in \{1, 2, 3\} : u_i(t) > u_i(y), u_i(z).
\end{aligned}$$

It follows, that dd cannot be a FCW when the three remaining alternatives form a cycle because voters 1 and 3's instantaneous utility at ad is higher than their instantaneous utilities at da and aa. ■

Proof of Lemma 2. Whenever there is a FCW in $x \in \mathcal{A}$ and the group preference among the remaining alternatives is such that $y \succ z \succ t \succ y$ we can rewrite the continuation utilities in such a way as to avoid a its recursive formulation. That is,

$$\begin{cases} U_i\left(x\right) = u_i\left(x\right) \\ U_i\left(y\right) = \frac{1-\beta}{2\beta+1} u_i\left(x\right) + 3\beta \frac{(\beta+2)^2 u_i(y) + (\beta-1)^2 u_i(z) + (1-\beta)(\beta+2) u_i(t)}{(2\beta+1)\left(\beta+\beta^2+7\right)} \\ U_i\left(z\right) = \frac{1-\beta}{2\beta+1} u_i\left(x\right) + 3\beta \frac{(1-\beta)(\beta+2) u_i(y) + (\beta+2)^2 u_i(z) + (1-\beta)^2 u_i(t)}{(2\beta+1)\left(\beta+\beta^2+7\right)} \\ U_i\left(t\right) = \frac{1-\beta}{2\beta+1} u_i\left(x\right) + 3\beta \frac{(1-\beta)^2 u_i(y) + (1-\beta)(\beta+2) u_i(z) + (\beta+2)^2 u_i(t)}{(2\beta+1)\left(\beta+\beta^2+7\right)} \end{cases}$$

There are two possibilities where da is a FCW and the remaining alternatives form a cycle due to the fact that the cycle can go two ways $ad \succ dd \succ aa \succ ad$ or $ad \succ aa \succ dd \succ ad$. In either case, voter 1 always votes in favour of moving towards alternative da (it is his most preferred one) and voter 2 always votes against it (it is his worst alternative). Therefore, da can only be a FCW when voter 3 favours it in all pairwise comparison. In particular: $U_3(da) > U_3(dd)$. We just need to rewrite this inequality using the above expression of the continuation utilities. When $ad \succ dd \succ aa \succ ad$ these read as follows:

$$\begin{cases} U_3\left(da\right) = u_3\left(da\right) \\ U_3\left(dd\right) = \frac{1-\beta}{2\beta+1} u_3\left(da\right) + 3\beta \frac{(1-\beta)(\beta+2) u_3(ad) + (\beta+2)^2 u_3(dd) + (1-\beta)^2 u_3(aa)}{(2\beta+1)\left(\beta+\beta^2+7\right)} \end{cases}.$$

By substituting the values of voter 3's instantaneous utilities it can be checked that the inequality $U_3(da) > U_3(dd)$ can never be satisfied. The same is true in the second case when the cycle is $ad \succ aa \succ dd \succ ad$. ■

Proof of Proposition 2. There always exists a voter whose preferences over any two pairwise comparisons coincide with the group ones. For instance, there exists $i \in \{1, 2, 3\}$ such that $U_i\left(y\right) > Ui\left(z\right) > U_i\left(t\right)$. Using the reformulation of the continuation utilities in Lemma 2, these inequalities can be rewritten as:

$$u_i\left(y\right) > \frac{3u_i\left(z\right) - (1-\beta)\, u_i\left(t\right)}{\beta+2} \quad \text{and} \quad u_i\left(y\right) > \frac{3u_i\left(t\right) - (\beta+2)\, u_i\left(z\right)}{1-\beta}. \tag{4}$$

When aa is a FCW and group preference are such that $ad \succ dd \succ da \succ ad$, the following inequalities should hold:

$$\begin{cases} U_1\left(ad\right) > U_1\left(dd\right) > U_1\left(da\right) \\ U_2\left(da\right) > U_2\left(ad\right) > U_2\left(dd\right) \\ U_3\left(dd\right) > U_3\left(da\right) > U_3\left(ad\right) \end{cases}$$

Note that we are taking into account that the alternative that yields the highest continuation payoff should also be the one that yields the highest instantaneous payoff (see proof of Lemma 1). Using conditions (4) together with the fact that the FCW needs to win all pairwise comparisons (i.e. $U_1\left(aa\right) > U_1\left(ad\right)$ and $U_2\left(aa\right) > U_2\left(da\right)$) we obtain the conditions $\lambda_1 < \frac{3(1-\beta)}{2\beta^2-\beta+8}$, $\lambda_2 < \frac{1-\beta}{3}$, and $\lambda_3 > \frac{\beta+2}{\beta+5}$. The second case where group preference are such that $ad \succ da \succ dd \succ ad$ is proved analogously. ■

Proof of Lemma 3. It has already been shown in the main text that a majority of voters need to satisfy: $u_i(z) > u_i(t)$. We know that a majority of voters satisfy $U_i(x) > U_i(y)$. These continuation utilities can be rewritten as follows:

$$U_i(x) = \tfrac{3\beta}{2\beta+1} u_i(x) + \tfrac{1-\beta}{2\beta+1} U_i(x) \quad \text{and} \quad U_i(y) = \tfrac{3\beta}{2\beta+1} u_i(y) + \tfrac{1-\beta}{2\beta+1} U_i(x).$$

It follows that a majority of voters satisfy $U_i(x) > U_i(y)$ if and only if a majority of voters satisfy $u_i(x) > u_i(y)$. Finally, by rewriting the continuation utilities at y and z,

$$U_i(y) = \tfrac{3\beta}{\beta+2} u_i(y) + \tfrac{1-\beta}{\beta+2}\left[U_i(x) + U_i(y)\right] \quad \text{and} \quad U_i(z) = \tfrac{3\beta}{\beta+2} u_i(z) + \tfrac{1-\beta}{\beta+2}\left[U_i(x) + U_i(y)\right].$$

we see that a majority of voters satisfy $U_i(y) > U_i(z)$ if and only if a majority of voters satisfy $u_i(y) > u_i(z)$. ■

Proof of Lemma 4. Whenever x is the FCW, t a loser and $y \succ z$, the non-recursive formulation of continuation utilities reads as follows:

$$\begin{aligned}
U_i(x) &= u_i(x) \\
U_i(y) &= \tfrac{1-\beta}{2\beta+1} u_i(x) + \tfrac{3\beta}{2\beta+1} u_i(y) \\
U_i(z) &= \tfrac{1-\beta}{2\beta+1} u_i(x) + \tfrac{3\beta(1-\beta)}{(\beta+2)(2\beta+1)} u_i(y) + \tfrac{3\beta}{\beta+2} u_i(z) \\
U_i(t) &= \tfrac{1-\beta}{2\beta+1} u_i(x) + \tfrac{3\beta(1-\beta)}{(\beta+2)(2\beta+1)} u_i(y) + \tfrac{\beta(1-\beta)}{\beta+2} u_i(z) + \beta u_i(t)
\end{aligned}$$

Lemma 3 implies that when the FCW is da, ad is the loser and $aa \succ dd$. Following Lemma 3's notation we have that $x = da$, $y = aa$, $z = dd$, and $t = ad$. The FCW being at da implies that voter 3 is pivotal in all decisions involving da, in particular: $U_3(da) > U_3(dd)$. By using the expression of the continuation utilities above we have that $U_3(da) > U_3(dd)$ is satisfied if and only if $\lambda_3 > \frac{2\beta+1}{\beta+2}$. The fact that the latter inequality can never be satisfied (recall that $\lambda_3 \in \left(0, \frac{1}{2}\right)$) implies that da cannot be a FCW. ■

Proof of Lemma 5. There is an equilibrium where x is a FCW, t a loser and $y \succ z$ when each of following inequalities is satisfied by a majority of voters: $U_i(x) > U_i(z)$, $U_i(x) > U_i(t)$, and $U_i(y) > U_i(t)$. It is convenient to rewrite these inequalities avoiding the recursive formulation of the continuation utilities:

$$\left\{\begin{array}{l}
u_i(x) > \frac{1-\beta}{\beta+2} u_i(y) + \frac{2\beta+1}{\beta+2} u_i(z) \\
u_i(y) > \frac{1-\beta}{3} u_i(z) + \frac{\beta+2}{3} u_i(t) \\
u_i(x) > \frac{1-\beta}{\beta+2} u_i(y) + \frac{(2\beta+1)(1-\beta)}{3(\beta+2)} u_i(z) + \frac{(2\beta+1)}{3} u_i(t)
\end{array}\right. \tag{5}$$

Lemma 3 implies that when the FCW is aa, da is the loser and $dd \succ ad$. Following Lemma 3's notation we have that $x = aa$, $y = dd$, $z = ad$, and $t = da$. The first two inequalities in (5) need to be satisfied by (at least) 2 voters each. It can be shown that voter 3 always satisfies the first one but never the second one, while voter 2 always satisfies the second one but never the first one. Therefore, voter 1 is pivotal in both decisions. When we substitute the instantaneous utility values for voter 1 we see that there does not exist a $\lambda_1 \in \left(0, \frac{1}{2}\right)$ satisfying both inequalities simultaneously. ■

Proof of Proposition 4. When ad is the FCW, Lemma 3 tells us that two cases are possible:

1. $x = ad$, $y = da$, $z = aa$, and $t = dd$
2. $x = ad$, $y = aa$, $z = dd$, and $t = da$

In the first case we can show that the three conditions in (5).cannot be satisfied. The first inequality is never met by voter 1 and always met by voter 2. The second inequality is never met by voter 2 and always met by voter 1. Therefore, voter 3 is pivotal in both comparisons but there is no parameter $\lambda_3 \in \left(0, \frac{1}{2}\right)$ that satisfies both conditions simultaneously.

In the second case we require that a majority of voters satisfy each inequality in (5). First note that voter 1 satisfies all inequalities. The first inequality is never met by voter 2, and voter 3 only satisfies it when $\lambda_3 < \frac{1-\beta}{\beta+2}$. The second inequality is never met by voter 3, and always met by voter 2. Finally, the third inequality is never met by voter 3, and voter 2 only satisfies it when $\lambda_2 < \frac{1-\beta}{\beta+2}$.

When dd is the FCW, Lemma 3 tells us that $x = dd$, $y = ad$, $z = da$, and $t = aa$. Once again, we require that a majority of voters satisfy each inequality in (5). First note that voter 3 satisfies all inequalities. The first inequality is never met by voter 2, and voter 1 only satisfies it when $\lambda_1 > \frac{1-\beta}{4-\beta}$. The second inequality is never met by voter 1, and voter 2 only satisfies it when $\lambda_2 > \frac{3(2\beta+1)}{2(\beta+2)^2}$. Finally, the third inequality is always met by voter 1. ■

INSTRUCTIONS

(Translated from the original Spanish instructions)

We are grateful for your participation. The sum of money you will earn during the session will be given privately to you at the end of the experiment. From now on (and until the end of the experiment) you cannot talk to any other participant. If you have a question, please raise your hand and one of the instructors will answer your questions privately. Please do not ask anything aloud!

These experiments consist of 20 periods. The rules are the same for all participants and for all periods. At the beginning of each period participants are arranged in groups of three. Participants in your group are randomly selected at each period. None of you knows who the other participants are.

You and the other participants decide over the approval or dismissal of two bills. There are four possible outcomes: 1) approve both bills; 2) approve the first bill and dismiss the second one; 3) dismiss the first bill and approve the second one; and 4) dismiss both bills. These results determine the profits that you and the other participants make in each period. Remember that the participants with whom you are interacting are selected randomly in each period.

1. Your valuations

At the beginning of each period the ‘valuations’ for your group are announced. The valuations indicate how much each participant can earn if each of the bills is approved or dismissed. These valuations are expressed in terms of Euro cents.

The following table shows a possible combination of valuations for a group of three participants:

	Bill 1	**Bill 2**
Participant A	Approve 700	Dismiss 200
Participant B	Dismiss 300	Approve 600
Participant C	Dismiss 100	Dismiss 800

In each period you are randomly assigned a role (A, B, or C). Your earnings depend on whether the final decision for each bill coincides with the one shown in the table. That is, if both bills are dismissed, *Participant A* earns his/her valuation for bill 2 (200), *Participant B* earns his/her valuation for bill 1 (300), and *Participant C* earns his/her valuations for both bills (100+800).

2. Negotiation

In each period you can communicate with the members in your group through chat windows. Communication is unrestricted, except that you cannot identify yourself. Anyone who breaks this rule will be excluded from the Leex participant list.

There are three Chat windows. The messages written in the first one are public and are received by all three members of the group. The ones in the other two windows, however, are private, and allow confidential communication with each of the other two members of your group.

After 3 minutes of negotiations, you can send proposals that the other members of the group can accept or reject. Similarly, you are able to accept or reject proposals that other members of the group may be sending. When any member of the group accepts a proposal, this becomes the period's final decision. If no one accepts any proposal, both issues are dismissed.

3. Negotiation screen

As you can see in the following page, in each period the screen is divided into three parts:

- **Valuations**. On the left of the screen are the valuations of each member of your group. The ones that correspond to you are labelled as such. Below this table there is a reminder that both issues are dismissed when no agreement is reached.

- **Send / Cancel proposal**. On the top right of the screen there is a window through which you can send proposal to the remaining members of your group. You have to tick whether you wish to approve both issues (aprobar / aprobar), approve the first and dismiss the second (aprobar / denegar), dismiss the first and approve the second (denegar / aprobar) or dismiss both issues. You can always cancel any of your proposals. Remember that your proposal becomes the period's final decision when another member of the group accepts it.

- **Accept / Reject proposal.** On the top right of the screen you can also accept or reject the proposals that other members of the group send. An accepted proposal becomes the period's final decision

- ***Chat***. On the bottom right of the screen, you have three Chat Windows. The first one allows you to send public messages to the other two members of your group. The remaining two allow you to send private (confidential) messages to each of them individually. At the bottom of these windows you can write your messages and send them by pressing *enter*.

Periodo

1

Tiempo restante: 127

	Cuestión 1		Cuestión 2	
Tus Valoraciones	Aprobar	700	Denegar	200
Participante B	Denegar	300	Aprobar	600
Participante C	Denegar	100	Denegar	800

Si no llegáis a un acuerdo en el tiempo concedido, ambas cuestiones serán denegadas.

Tu Propuesta

- Aprobar / Aprobar
- Aprobar / Denegar
- Denegar / Aprobar
- Denegar / Denegar

Enviar Propuesta

El participante C propone:

Cuestión 1: aprobar

Cuestión 2: aprobar

Aceptar Rechazar

Mensajes Públicos

Aquí puedes escribir tus mensajes públicos

Mensajes privados (entre el participante B y tú)

Aquí puedes enviar mensajes privados al participante B

Mensajes privados (entre el participante C y tú)

Aquí puedes enviar mensajes privados al participante C

4. Timing of each period

Each period ends when one member of your group has accepted a proposal or, if no one accepts a any proposals, when you reach the time limit of 5 minutes. Recall that during the first 3 minutes of each period you can only send messages. It is after these first 3 minutes have elapsed that you can send proposals and accept or reject the proposals the other members of your group send. It is only necessary for one participant to accept a proposal for it to become the final decision for that period. If no member of the group accepts a proposal within the prescribed time of 5 minutes, the period ends and both issues are dismissed.

On the top right of the screen you can see the time remaining in the present period.

5. Earnings per period

In each period your earnings depend on whether each issue is approved or dismissed. For instance, when your valuations are *Participant A*'s in the next table,

	Bill 1	**Bill 2**
Participant A (Your valuations)	Approve 700	Dismiss 200
Participant B	Dismiss 300	Approve 600
Participant C	Dismiss 100	Dismiss 800

you earn 700 when the first bill is approved or 0 when it is dismissed; and you earn 0 when the second bill is approved or 200 when it is dismissed. The next list shows your possible gains given the four possible outcomes:

Approve / Approve	700 +	0
Approve / Dismiss	700 +	200
Dismiss / Approve	0 +	0
Dismiss / Dismiss	0 +	200

6. Information at the end of each period and final payments

At the end of each period, the computer announces the results of the negotiation and your earnings in that period.

After the last period, 3 periods are randomly selected and you are paid the sum of your earnings in those periods. Additionally, you receive 3 euros for participating in this experiment.

7. Questionnaire

1. **Circle the correct answer. Before negotiating…**

   - You know your valuations? YES NO
   - You know the valuations of the participants you are matched with? YES NO
   - Your own valuations and theirs can be different? YES NO
   - You know who the other participants you are matched with are? YES NO

2. **Imagine you have the following valuation table:**

	Bill 1	Bill 2
Participant A	Approve 700	Dismiss 200
Your valuations	Dismiss 300	Approve 600
Participant C	Dismiss 100	Dismiss 800

   - How much do you earn when issue 1 is dismissed? ___________
   - How much do you earn when issue 1 is approved? ___________
   - How much do you earn when issue 2 is dismissed? ___________
   - How much do you earn when both issue are dismissed? ___________
   - How much does *Participant A* earn when both issues are dismissed? ___________
   - How much does *Participant C* earn when both issues are dismissed? ___________

3. **¿How many periods are randomly selected to determine your final payment ?** _________

4. **¿What happens when no one accepts a proposal within the first five minutes?** _________