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Building the city: sunk capital, sequencing, and institutional frictions

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Abstract

This paper models a growing city, and focuses on investment decisions and consequent patterns of land use and urban density and takes the model to data on Nairobi. We distinguish between formal and informal sector construction. The former can be built tall, but structures once built are durable and cannot be modified without complete demolition. In contrast, informal structures are malleable and do not involve sunk costs. As the city grows areas will initially be developed informally, and then formally; formal areas are redeveloped periodically. This process can be hindered by land right issues which raise the costs of converting informal to formal sector development. The size and shape of the city are sensitive to the expected returns to durable investments and to formalisation costs of converting informal to formal sector usage. In the empirics, we analyse Nairobi for 2003/4 and 2015, developing a novel set of facts about the evolution of the built environment. We study the evolution of building footprints and heights, churning, infill, and redevelopment of the formal sector. Volume of building space is growing at about 4.4% a year. We analyse the loss in revenues and land value from high formalisation costs, which inhibit conversion of slums near the centre to a higher and better use.

Keywords: city, urban, urban growth, slum development, urban structure, urban form, housing investment, capital durability.

JEL classification: O14, O18, R1, R3

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1. Introduction

This paper examines housing development and redevelopment in a growing city. Our focus is a city in a developing country, containing both formal and informal housing sectors. We look at formal sector development and redevelopment, at the allocation (and misallocation) of land between sectors, at the transition between the two, and at the role of property rights and expectations in altering paths of urban development. We develop a model of a growing city in which buildings are durable and investment decisions are taken on the basis of expectations about the future growth of the city. The work builds on the standard monocentric urban model and its dynamic extensions (e.g. Braid 2001). However, most of the urban literature is essentially static – and designed to analyse slowly changing developed country urban areas. The objective of this paper is to capture key features of developing country cities. The paper takes the model to the data for Nairobi, constructing an unusual data set on the built environment in 2003/4 and 2015, and using it to track the physical transformation of a city which shares common features with other cities in developing countries.

The features captured in the model are as follows. First, the city is growing in both population and area. This means that land rents and patterns of land use are changing through time. Second, the city contains 'formal' or modern structures. Formal buildings involve sunk capital costs, can be built tall, and are hard to modify once constructed. Since they are durable, investment decisions are based on expectations about future land rents, as driven by future incomes and populations. As the city grows there will be periodic demolition and redevelopment of formal areas. Third, the city may also contain informal structures, which we sometimes refer to as slums. Given the technology and materials used in construction, these building are not likely to be built tall, but they can be rebuilt and adjusted after their initial construction. We assume that capital used in such structures is not sunk, but remains perfectly malleable. Finally, and critical to some of our results, there is a cost of conversion of informal to formal land use, which we call a formalisation cost, that varies across properties in the city.

We show that as the city grows land will initially be developed with informal structures which are then replaced by formal structures, which will themselves be subject to intermittent redevelopment. The share of urban population in informal structures will generally decline through time. This decline is a consequence of rising land values (and hence a greater return to achieving density by building upwards) as the city expands. Formalisation costs means that informal structures may be very persistent, and spatial heterogeneity of these costs mean that they will continue to exist alongside formal structures, having long-lasting implications for the fabric of the city.

We take the model to the data for Nairobi, for which we have constructed a uniquely detailed data base of buildings. We know the counts and footprint of buildings throughout the urban

area for 2003/4 and 2015 based on tracings of all building polygons from aerial photos. For 2015 we also know heights of these buildings based on LiDAR data. The primary measure that we work with is the total volume of building space per unit area, by building type (formal or informal) and varying across the city and over time. We also use the data to analyse how Nairobi transforms from 2003/4 to 2015, tracing demolition, redevelopment, and in-fill at all locations. We have high resolution satellite data for 2003/4 and 2013, from which roads can be extracted. We also have on housing rents and land prices by location and for single points in time.

Nairobi conforms to predictions in our model and, in static monocentric models, that, in the formal sector: (1) house rents and land prices decline with distance to the centre and (2) building heights and total built cover to area decrease with distance to the centre. Beyond that, for our developing country context, we derive a novel set of facts. We start with the cross-section. (3) Consistent with the model, slums provide housing volume with high coverage to area ratio while the formal sector provides volume with height (implying much more land set aside for side streets and green space). (4) In comparing slum vs formal sector volume of space, in the core part of the city, slum and formal sectors actually provide a similar stock of built volume per unit area, albeit with slums at lower quality of building materials and amenities. However, (5) overall total volume of slum housing is a fraction of formal sector, given slums cover well under 20% of the built land.

For dynamics, the city changes dramatically from 2003/4 to 2015. For dynamics we have infill (a building in 2015 whose footprint overlaps with no 2004 building) and teardowns of 2004 buildings which are divided into 2 categories, demolition (no new building, yet) and redevelopment (a new building(s) overlapping torn down 2004 building(s)). All these then make up components of building count and volume changes. What do we see? (6) There is rapid growth, with total built volume just within the 2003/4 city effective boundary increasing by 50%, growth of about 4% a year, a substantial rate of capital accumulation. (7) Between 1-6 kms from the centre, redevelopment of formal sector buildings into higher new buildings alone accounts for large volume increases, with the total net increase in volume from redevelopment as a fraction of initial volume peaking at 35% at about 3 kms out. (8) Throughout the core of the city, within 1-8 kms of the city centre (CBD), there is enormous churning. About 35% of buildings from 2003/4 were torn down and about half of those were left vacant (‘demolitions’). Infill adds 40% to 2003/4 building counts at distance 1.5 to 4kms, rising to 80% and beyond as distance from the centre increases. However, churning nearer the centre involves relatively small areas and buildings, constrained by available plot sizes. Area of infill or demolition without replacement to total initial cover is about a third of the rates of corresponding building counts.

There is also slum churning and redevelopment, but our focus is on a relative lack of slum redevelopment into formal sector usage. (9) For slums, there is vast expansion towards the city fringe, while all remaining slum buildings very near the centre are demolished. However, it seems that in the mid-city development of slum into formal sector housing over the 11 years is slow. We explore the institutional context of Nairobi, to suggest there are high formalisation costs in traditional slums nearer the centre in Nairobi, meaning that a significant amount of land is not in highest and best use. Formalisation of slum areas in the distance band 2 to 6 kms from the CBD, which house about 350,000 people would result in an estimated $1.2 billion increase in inferred land values.

There are five novel aspects to the paper. First is the modelling. While Baird (2001) has a dynamic monocentric model with durable capital, no dynamic model deals with informality, formalisation costs, and expectations. Second are the data. While there is work on the USA using demographic census data to try to analyse redevelopment over of periods of time (Rosenthal and Brueckner, 2009), the work does not utilize data on buildings, with demolition, redevelopment, and intensification (infilling). As far as we know we are the first to utilize direct building information to detail the changes in the urban landscape. Third is a new set of facts about city development and redevelopment of the built environment. Fourth, we focus on a major developing country city, where population growth is much more rapid than in developed countries and where land market institutions are weak. Finally are the policy aspects where, in a general equilibrium context, we can think about the role of expectations and formalisation costs, and for the latter make inferences from the data about the impact on city development.

The analytical framework makes clear some of the risks faced by a growing city, and the role of policy in addressing these risks. Expectations are fundamentally important in investment decisions, and low expectations of the future development of the city have a major impact in distorting investment levels below their efficient levels. There are also major market failures that deter investment, such as inappropriate regulation and land titling or capital market imperfections. The consequences of such imperfections are long-lasting, given the durability of structures.

The paper is organised as follows. The basic model and core theoretical results are set out in section 2. Section 3 turns to data and analysis of Nairobi. Section 4 looks specifically at Nairobi slums, conversions costs, and misallocation. Section 5 concludes.

2. Theory

In this section we present the model of a growing city, focusing on investment decisions and consequent patterns of land use and urban density. The analysis is developed assuming that

prices of housing volume follow a given path over time; in the appendix we show how this price path can be endogenised in a complete open-city equilibrium. We start (section 2.1) by looking at building on a particular point in the city. We specify two alternative building technologies – formal and informal (or slum) – that can be used, and analyse the volume and form of buildings that each supply. We then (section 2.2) turn to choice of which technology is used in each place and how this choice evolves over time as the city grows. This can involve transition from informal to formal, and successive waves of formal sector demolition and reconstruction. Section 2.3 pulls in the spatial dimension, giving a complete description of both the cross-section of the city and its evolution through time. Section 2.4 adds some frictions to the model, in particular showing how barriers to conversion of informal development to formal can lead to a 'hotchpotch': co-existence of different building types and sizes throughout the city.

2.1 Building technology and housing supply

There are two distinct building technologies, formal and informal. Both deliver building volume but by different means, the formal sector (*F*) being able to build tall, and the informal sector (*I*) being able to 'crowd', increasing cover (the total building footprint) per unit of land. The volume of building delivered on a unit of land at a particular place, *x*, and time *t*, is the product of height and cover, $v_i(x,t) = h_i(x,t)c_i(x,t)$, $i = I, F$.

The informal sector is modelled as follows. First, it is unable to build high, so has height fixed at h_I; it can however increase building footprint, i.e. increase the proportion of each unit of land that is covered with buildings, $c_I(x,t)$. Informal sector construction materials are malleable and construction costs are a flow, occurring continuously through the life of the structure. This can be thought of as the rental on 'lego blocks' or 'meccano parts' used in construction, or as the cost of material whose life is one instant. We assume that these flow construction costs per unit volume (which, given h_I, is proportional to cover), are constant k_I, so construction costs per unit land are $k_I v_I(x,t)$. In reality in Nairobi, from the 10% sample of the 2009 Census, the majority (about 55%) of informal housing walls are corrugated iron sheets which can be reconfigured; most other slum housing involves mud construction (about 20%) and other material with short duration. Both sets of materials are not sufficiently load bearing to allow much in the way of height.

Crowding has the effect of reducing the quality of informal housing. We capture this by supposing that the price (and willingness to pay) for a unit of informal housing is the product of two elements; the price of informal housing of unit quality at place *x* at date *t*, $p_I(x,t)$, and

a quality factor, $q(v_I(x,t))$, diminishing and convex in crowding (as measured by volume per unit area). With this, land rent (i.e. revenue minus construction cost)[1], is

$$r_I(x,t) \equiv p_I(x,t)q(v_I(x,t))v_I(x,t) - k_I v_I(x,t). \tag{1}$$

The first order condition for choice of volume is (assuming that the effects of crowding are internalised in the actions of the developer receiving the rent),

$$\partial r_I(x,t)/\partial v_I(x,t) = p_I(x,t)\lfloor q(v_I(x,t)) + v_I(x,t)q'(v_I(x,t))\rfloor - k_I = 0. \tag{2}$$

If quality is isoelastic in cover, $q(v_I(x,t)) = v_I(x,t)^{(1-\lambda)/\lambda}$, $\lambda > 1$, then optimally chosen volume and maximised rent are respectively

$$v_I(x,t) = \left[\frac{p_I(x,t)}{k_I\lambda}\right]^{\frac{\lambda}{\lambda-1}}, \qquad r_I(x,t) = k_I(\lambda-1)\left[\frac{p_I(x,t)}{k_I\lambda}\right]^{\frac{\lambda}{\lambda-1}}. \tag{3}$$

It follows that land rent is fraction $(1-1/\lambda)$ of revenue earned by informal sector housing, i.e. $r_I(x,t) = [(\lambda-1)/\lambda]p_I(x,t)q(v_I(x,t))v_I(x,t)$.

The formal sector differs in a number of key respects. First, buildings are 'putty-clay', malleable at the date of construction but not thereafter. For simplicity we assume that formal sector land cover is not a choice variable but is set exogenously at c_F, and that volume is achieved by choice of height, $h_F(x,\tau_i)$. This is chosen at date of construction, denoted τ_i, and then fixed for the life of the structure, i.e. $v_F(x,\tau_i) = c_F h_F(x,\tau_i)$ is fixed at date τ_i until demolition at date τ_{i+1}. Subscript *i* is used to denote successive redevelopments of formal structures. Construction costs are one-off and sunk, and are an increasing and convex function of building volume, $k_F(v(x,\tau_i))$. This sunk cost of construction differs fundamentally from the flow cost in the slum sector. Demolition incurs neither costs nor benefits as materials cannot be recycled back to putty. In Nairobi over 90% of formal sector housing is made of stone or some type of brick/block, in contrast to slum materials.

While volume is fixed at date of construction, τ_i, the price of a unit of formal sector building volume, $p_F(x,t)$, may vary over the life of the building. The present value of rent (per unit land) that accrues over the life of a structure, $t \in [\tau_i, \tau_{i+1}]$, discounted to construction date τ_i at interest rate ρ is denoted $R_F(x,\tau_i)$,

[1] We reserve the word 'rent' for income accruing to land, and use the word 'price' (per unit volume) for housing services, although this is a per period flow, not a capital value.

$$R_F(x,\tau_i) \equiv \int_{\tau_i}^{\tau_{i+1}} p_F(x,t) v_F(x,\tau_i) e^{-\rho(t-\tau_i)} dt - k_F(v_F(x,\tau_i)). \quad (4)$$

The first order condition for choice of volume is,

$$\partial R_F(x,\tau_i)/\partial v_F(x,\tau_i) = \int_{\tau_i}^{\tau_{i+1}} p_F(x,t) e^{-\rho(t-\tau_i)} dt - k_F'(v_F(x,\tau_i)) = 0 \ . \quad (5)$$

We define the value-to-rent ratio on a newly constructed property as

$$\Phi(x,i) \equiv \int_{\tau_i}^{\tau_{i+1}} \left[p_F(x,t)/p_F(x,\tau_i) \right] e^{-\rho(t-\tau_i)} dt \ , \quad (6)$$

i.e. the present value of the price of a unit of formal housing space over its life relative to its price at date of construction.

We will often use an isoelastic form the cost function, $k_F(v_F) = k_F v_F{}^{\gamma}$, so the first order condition for choice of volume and maximised present value rent are,

$$v_F(x,\tau_i) = \left[\frac{p_F(x,\tau_i)\Phi(x,i)}{k_F \gamma} \right]^{\frac{1}{\gamma-1}}, \qquad R_F(x,\tau_i) = k_F(\gamma-1) \left[\frac{p_F(x,\tau_i)\Phi(x,i)}{k_F \gamma} \right]^{\frac{\gamma}{\gamma-1}}. \quad (7)$$

It is useful to have a continuous flow measure of rent, given by amortizing the one-off construction cost continuously over the life of the structure. If amortization is constant proportion *a* of revenue, then costs are fully covered by setting *a* to satisfy $k_F(v_F(x,\tau_i)) = \int_{\tau_i}^{\tau_{i+1}} a p_F(x,t) v_F(x,\tau_i) e^{-\rho(t-\tau_i)} dt = a p_F(x,\tau_i) v_F(x,\tau_i) \Phi(x,i)$. With $k_F(v_F) = k_F v_F{}^{\gamma}$ and (7), the amortization rate is $a = 1/\gamma$. Thus, flow land rent net of amortization is fraction $(1-1/\gamma)$ of revenue earned by land and structure together.

2.2 Land development and construction phases

Continuing to focus on a particular unit of land, *x*, we now look at the choice of when to develop (or redevelop) informal or formal structures. At some date (say time 0) the present value of earnings from a unit of land at *x* that has not yet been developed is

$$R(x) = \int_0^{\tau_0} r_0 e^{-\rho t} dt + \int_{\tau_0}^{\tau_1} r_I(x,t) e^{-\rho t} dt + \left[R_F(x,\tau_1) - D(x) \right] e^{-\rho \tau_1} + \sum_{i=2} R_F(x,\tau_i) e^{-\rho \tau_i} \ . \quad (8)$$

The first term is the present value of rent from undeveloped land (flow rent r_0 which we take to be constant), discounted at rate ρ and calculated up to the date of first development, denoted τ_0. The second term gives the present value of earnings from informally developed

land, during interval τ_0, τ_1. The first formal sector development, occurring at date τ_1 incurs a further one-time fixed cost $D(x)$ of converting to formality, the formalisation cost.[2] Formal sector development requires reasonably well defined property rights, such as land titling or formal leaseholds on land granted by the government. Obstacles to obtaining these rights may be substantial, particularly in African countries where much land is held traditionally under communal rights. $D(x)$ is the cost of overcoming these obstacles. The final term in (8) gives the discounted value of rents earned over the lives of consecutive formal sector buildings, constructed at dates $\tau_2, \tau_3...$

Dates of development and redevelopment are chosen to maximise $R(x)$. For the first development (which we assume for the moment to be informal), the optimal τ_0 simply equates flow land-rents on undeveloped and informal land, and is implicitly defined by

$$\frac{\partial R(x)}{\partial \tau_0} = e^{-\rho\tau_0}\left[r_0 - r_I(x,\tau_0)\right] = 0\,. \tag{9}$$

The first formal development takes place at date τ_1 satisfying

$$\frac{\partial R(x)}{\partial \tau_1} = e^{-\rho\tau_1}\left[r_I(x,\tau_1) - p_F(x,\tau_1)v_F(x,\tau_1) + \rho\left\{k_F(v_F(x,\tau_1)) + D(x)\right\}\right] = 0\,. \tag{10}$$

The first redevelopment of formal land is at date τ_2 satisfying

$$\frac{\partial R(x)}{\partial \tau_2} = e^{-\rho\tau_2}\left[p_F(x,\tau_2)v_F(x,\tau_1) - p_F(x,\tau_2)v_F(x,\tau_2) + \rho k_F(v_F(x,\tau_2))\right] = 0\,.$$

Generalising this for all redevelopments gives (see appendix for derivation):

$$p_F(x,\tau_{i+1})\left[v_F(x,\tau_{i+1}) - v_F(x,\tau_i)\right] = \rho k_F(v_F(x,\tau_{i+1}))\,, \quad \text{for } i \geq 1. \tag{11}$$

This condition says that demolition and reconstruction occurs at the date at which the revenue gain from the change in volume equals the interest cost of the construction expenditure incurred. Similar intuition applies to equation (10).

For iso-elastic construction and quality costs, $q(v_I(x,t)) = v_I(x,t)^{(1-\lambda)/\lambda}$, $k_F(v_F) = k_F v_F{}^{\gamma}$, we can use the optimised values of v in equations (3) and (7) in equations (9) – (11) to generate

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[2] For simplicity, we do not let this depend on time. The dependence on location is drawn out in section 2.4

expressions for the dates at which sites are (re-)developed. The date at which site x becomes informally developed, τ_0, given by equation (9), is implicitly defined by

$$r_0 = \left[\frac{p_I(x, \tau_0)}{k_I \lambda} \right]^{\frac{\lambda}{\lambda - 1}} (\lambda - 1)\, k_I . \tag{9a}$$

The date at which informal settlement becomes formalised, τ_1, is given by equation (10) which using (3) and (7) becomes

$$\left[\frac{p_I(x, \tau_1)}{k_I \lambda} \right]^{\frac{\lambda}{\lambda - 1}} (\lambda - 1) k_I = \left[\frac{p_F(x, \tau_1)\Phi(x,1)}{k_F \gamma} \right]^{\frac{\gamma}{\gamma - 1}} k_F \left(\frac{\gamma}{\Phi(x,1)} - \rho \right) - \rho D(x) . \tag{10a}$$

The dates at which successive formal redevelopments of x take place, τ_i, $i > 1$, given by equation (11) can, using the iso-elastic functional forms, be expressed as

$$\left[\frac{p_F(x, \tau_i)\Phi(x,i)}{p_F(x, \tau_{i+1})\Phi(x, i+1)} \right]^{\frac{1}{\gamma - 1}} = \left(1 - \frac{\rho \Phi(x, i+1)}{\gamma} \right). \tag{11a}$$

2.3 Analysis

What do we learn from the characterisation of development stages given above? A benchmark case in which prices are growing at constant exponential rates $\hat{p}_I$, $\hat{p}_F > 0$ yields analytical results. The full general equilibrium model that supports constant exponential price growth is discussed in section 2.4 and detailed in the Theory Appendix; but for the present we simply assume these given price paths. We look at the time series development of a particular place, x, and then at the urban cross-section.

Urban dynamics: To draw out results we look first at successive redevelopments of formal areas of the city, and then turn to the city edge and informal development.

Proposition 1: If quality is iso-elastic in cover and constructions costs are iso-elastic in height, prices are growing at constant exponential rates $\hat{p}_I$, $\hat{p}_F > 0$, and agents have perfect foresight,

(i) The value-to-rent ratio takes constant value Φ, and the time interval between successive formal redevelopments is constant Δτ,

$$\Phi \equiv \int_0^{\Delta\tau} e^{(\hat{p}_F - \rho)t} dt = \frac{1 - e^{(\hat{p}_F - \rho)\Delta\tau}}{\rho - \hat{p}_F}, \qquad \Delta\tau = \frac{(\gamma - 1)}{\hat{p}_F} \ln\left[\frac{\gamma}{\gamma - \rho\Phi} \right]. \tag{12}$$

(ii) Successive rounds of formal sector building have greater volume (height) by a constant proportional factor.

$$\frac{v_F(x,\tau_{i+1})}{v_F(x,\tau_i)} = e^{\frac{\hat{p}_F \Delta\tau}{(\gamma-1)}} = \frac{\gamma}{\gamma-\rho\Phi}. \tag{13}$$

(iii) If the rate of growth of prices is the same in all locations, *x*, then so too are Φ, $\Delta\tau$, and volume growth.

The first part of this proposition comes from integrating equation (6) and using it in (11a), and noting that there is a unique solution solving the 2 parts of (12) with constant Φ and Δτ. The second part follows by using this in the first order condition for volume, (7). The third comes from noting that (12) and (13) do not depend on *x*. While value ratios and time intervals do not vary with *x*, the actual dates of redevelopment do, as discussed below.

What about the earlier stages of informal development? The first transition is (we have assumed) from agriculture to informal settlement. This occurs for land at *x* when the price of informal sector housing space reaches the point at which the right hand side of (9a) equals r_0. A period of informal settlement exists only if the return to informal settlement at τ_0 is greater than the return to commencing formal settlement, $r_I(x,\tau_0) > p_F(x,\tau_0)v_F(x,\tau_0) - \rho\{k_F(v_F(x,\tau_0)) + D(x,\tau_0)\}$ (see equation (10)). If not, then initial development will be formal, with date τ_1 implicitly defined by $r_0 = p_F(x,\tau_0)v_F(x,\tau_0) - \rho\{k_F(v_F(x,\tau_0)) + D(x,\tau_0)\}$.

The transition from informal to formal settlement is given by date τ_1 that solves (10a). There is a unique transition date satisfying the second order condition if the return to formal development is rising faster than the return to informal settlement (i.e. the right hand side of (10a) increasing faster than the left). If $D = 0$, a necessary and sufficient condition for this is that $\hat{p}_F\gamma/(\gamma-1) > \hat{p}_I\lambda/(\lambda-1)$. If $D > 0$, then this condition is sufficient but not necessary. We assume the condition to be satisfied, as it will be if prices (before being deflated for crowding) increase at the same rate and $\lambda > \gamma$. The interpretation of this inequality is that the elasticity of land rent with respect to price is greater in formal sector housing than informal (compare equations (3) and (7)). Essentially, there are sharper decreasing returns to increases in volume in the informal sector (where crowding reduces price) than in the formal sector (where building taller raises unit construction costs).

Figure 1 pulls these stages together and illustrates the development path.[3] Building volume is given on the vertical axis, and on the horizontal plane axes are time t and location x (distance from the CBD). The figure is constructed with house prices increasing exponentially with time and falling exponentially with distance from the CBD. We discuss the cross-section – variation across x at a given t – in the next sub-section, and now look just at the development of a particular location through time, i.e. fix x and look along a line sloping up and to the right. Initially (at low t) this land is rural. At date τ_0 (specific to location x), informal development takes place. The volume of informal development increases steadily as increasing p_I causes lego blocks to be rearranged and building cover to increase, although that is hard to see visually in Figure 1. Formal development takes place at τ_1 and, as illustrated, leads to an increase in volume, indicated by the second step. However, the sign of the change in volume depends on parameters, and it is possible that edge slums deliver more volume than does first stage formal development. Subsequent redevelopment occurs at fixed time interval $\Delta\tau$, and bring the same proportionate increase in volume, achieved by building taller. The timing and volume of each of these formal investments is based on perfect foresight about the growth of prices and the date of subsequent redevelopments.

The urban cross-section. We have so far concentrated on a single location, point x, and now place this in the context of a city where x measures distance from the CBD, and house prices decrease with distance. With prices decreasing in x, (9a), (10a) and (11a) can be interpreted as implicitly defining, for each date t, the city edge, $x_0(t)$, the location of formalisation, $x_1(t)$, and locations of successive redevelopments, $x_i(t)$, $i > 1$.[4] We illustrate and derive results for the urban cross-section assuming that the spatial price gradient is exponential with distance, at rates θ_I, θ_F, so together with exponential growth of prices through time,[5]

$$p_I(x,t) = \bar{p}_I e^{\hat{p}_I t} e^{-\theta_I x},\ p_F(x,t) = \bar{p}_F e^{\hat{p}_F t} e^{-\theta_F x}. \tag{14}$$

The urban cross-section at a point in time is indicated on Figure 1 by fixing a date and moving along a line sloping upwards to the left towards the CBD, with steps in volume occurring at locations $x_i(t)$, i = 0,1. 2... At the city edge land is informal and, moving towards the centre, locations that have been urban for longer have been through more stages of development and offer greater building volume per unit land. The increase in volume is achieved by increasing land cover in the informal area and by greater height in formal areas

[3] Parameters used in the simulation are given in the appendix.

[4] That is, instead of solving (10)-(12) for the date at which a particular location is developed, the equations give the location that undergoes development at a particular date.

[5] For generality we allow θ_I, θ_F to differ, as would be the case if e.g. occupants of informal housing travelled to the CBD less frequently than occupants of formal housing (see Appendix).

closer to the centre. We will see empirical data on these relationships in the following section.

Notice also that while the price of a unit volume of formal housing declines at rate θ_I with distance from the centre (by assumption in (14)), the observed price of a quality adjusted unit of informal housing is constant across space. The price is $p_I(x,t)q(v_I(x,t))$ and, with iso-elastic functional forms (using equation (3)), volume declines with price at rate λ/(λ - 1) and quality declines with volume at rate (1 - λ)/λ. Exact constancy is obviously a consequence of iso-elasticity, but the more general point is that crowding and quality reduction means that the price gradient for informal housing per unit volume is likely to be flatter across the city than that for formal housing, and could increase.

The evolving urban cross-section. Putting the parts together, we see how the urban cross-section evolves through time. Proposition 2 states results on how different stages of development (building types and heights) move across the city as it grows.

Proposition 2: If construction technologies are iso-elastic, prices are growing at constant exponential rates $\hat{p}_I, \hat{p}_F > 0$ and declining with distance at constant rates $\theta_I, \theta_F > 0$; and if conversion costs are the same at all locations and agents have perfect foresight, then:

(i) The distance between successive formal sector redevelopments, Δ*x*, is constant,

$$\Delta x = \Delta \tau \hat{p}_F / \theta_F = \frac{(\gamma - 1)}{\theta_F} \ln\left[\frac{\gamma}{\gamma - \rho\Phi}\right]. \qquad (15)$$

(ii) The distance from the CBD at which (re-) development occurs increases through time according to,

a. $dx_0 / dt = \hat{p}_I / \theta_I$,

b. If $D = 0$, $dx_1 / dt = \dfrac{\hat{p}_F \gamma(\lambda - 1) - \hat{p}_I \lambda(\gamma - 1)}{\theta_F \gamma(\lambda - 1) - \theta_I \lambda(\gamma - 1)}$,

c. $dx_i / dt = \hat{p}_F / \theta_F$, $i > 1$.

The proposition follows from using prices (14) in equations (9a)-(11a), noting Φ is a constant and recalling that, in the cross-section, equations are to be interpreted as giving places of transition at a given date. Thus, the price ratio in equation (11a) takes the form $p_F(x,\tau_i)/p_F(x,\tau_{i+1}) = e^{-\hat{p}_F \Delta\tau}$ in the time series (i.e. given *x*), and $p_F(x_i(t),t)/p_F(x_{i+1}(t),t) = e^{-\theta_F \Delta x}$ in the cross section (given *t*), this implying equation (15), analogous to (12).

This evolution is illustrated on Figure 1. Since the figure is constructed with $\hat{p}_I = \hat{p}_F$ and $\theta_I = \theta_F$, the lines along which development and redevelopment occur are parallel (see proposition 2.ii). It follows that the width of the informal area, $x_1 - x_0$, is constant through time. Hence one can show that, even in a circular city, the share of urban land area that is informal falls with time and as the city gets larger. Generally, the area of land occupied by the informal sector becomes narrower through time if price growth is faster in the formal sector than informal (quality unadjusted) or price gradient of the formal sector is flatter than that of the informal sector, $\hat{p}_F / \theta_F > \hat{p}_I / \theta_I$.[6]

While our analytical results are based on constant exponential price paths, we note that it is also possible to numerically compute the perfect foresight equilibrium for more general price paths, although we do not report those here.

2.4 Frictions and market imperfections

The analysis so far has concentrated on a benchmark case, and we now add two frictions to the model. The first is to add more heterogeneity across places, which we do by allowing formalisation costs, $D(x)$, to vary by place. Furthermore, we suppose that these costs may be due to institutional rather than real costs, so create inefficiency in the equilibrium outcome. Second, results so far have assumed perfect foresight; we relax this, and look at the implications of systematic deviations from perfect foresight.

Formalisation costs. Locations vary in their distance to the CBD and, potentially, in many other respects. One possibility is that that the cost of formalisation, $D(x)$, varies according to place. Figure 2a illustrates the implications of there being an interval of x within which $D(x)$ is particularly high. As expected, this extends the period during which the area is occupied by informal settlement.

Several other observations are noteworthy. First, a persistently informal area will see housing volume per unit area increase through time as informal buildings are reshaped and crowding increases; it is possible that it may come to have volume higher than the surrounding formal area, as illustrated in figure 2a and something we will see in the empirics. Second, a history of informality has a persistent legacy on the area. Formal development starts later, and so therefore does subsequent redevelopment; this impacts on building volume which depends on the price (and hence date) of redevelopment. Proposition 1 still holds for the time series evolution of each place, but looking across the urban cross-section there is now more variation in building volume and height, even in areas where there is no longer an

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[6] The general expression is $\frac{dx_1}{dt} - \frac{dx_0}{dt} = \frac{\gamma(\lambda-1)\{\hat{p}_I\theta_F - \hat{p}_F\theta_I\}}{\theta_I\{\theta_F\gamma(\lambda-1) - \theta_I\lambda(\gamma-1)\}}$.

informal sector presence. This is illustrated more vividly in figure 2b, in which *D*(*x*) was set randomly across space. All locations see volume increase with time, but initial and subsequent formal development takes place at different dates and build to different heights. This means that gradients of volume, density and land rents are not monotonically decreasing from the centre in such a city. Patterns are the hotchpotch we see in the data. Of course, a cross-section slice of Figure 2 just gives volumes along a single ray from the CBD. The heterogeneity exists along all such rays. In the following empirical section we will look at distance bands (concentric rings), in which formal and informal structures will coexist, as will buildings of different heights.

What are the welfare implications of this hotchpotch of different land use? Given exogeneity of wages and utility, welfare change is captured entirely in rents. Formalisation obstacles that are not real costs reduce welfare by distorting decisions, and in Section 4 we measure this by the loss of rent. Land rents are not generally observed, but housing prices are (i.e. gross revenue earned on each unit of volume). Following the structure of this model we know that – if construction costs are amortized as a constant fraction of revenues over the life of the structure – then land rents are fraction $(1-1/\gamma)$ of revenue earned in the formal sector and $(1-1/\lambda)$ of revenue earned in the informal sector.

Expectations: Analysis to this point has been based on optimisation with perfect foresight. What are the consequences of removing this assumption? Recall that $\Phi(x,i)$ is the value-to-rent ratio on a newly constructed property, and equations (12) give the perfect foresight values of this and of the expected length of life of the property, $\Delta\tau$. How do results change if construction decisions are based on a value-to-rent ratio that differs from the perfect foresight ratio?

The solid line on Figure 3 is a slice through Figure 1 at *t* = 180, maintaining $\hat{p}_F = \hat{p}_I$ and $\theta_I = \theta_F$. Given the parameters used, the perfect foresight value-to-rent ratio is Φ =26, and the interval between redevelopments is $\Delta\tau$ =70. The dashed line is constructed on the basis that developers have less positive expectations, and build on the basis of a value-to-rent ratio of 19.5 (imposed at 75% of the perfect foresight value). The transition from rural to informal settlement is unaffected by this, but formal development is based on these less optimistic expectations. As a consequence developers build less volume and hence buildings become obsolete more rapidly, so the interval between redevelopments drops to $\Delta\tau$ = 45.

The welfare cost of this imperfection is measured by its impact on land rents. Rather than looking at flow rents, we compute the present value of these rents, integrating over the locations and dates illustrated in figures 1and 2 (i.e. out to *t* =250 and to distance 60). Lower

expectations reduce the present value of land rents by 13.3%.[7] This is a substantial amount, particularly since the calculation does not take into account the fact that city population is smaller which would create further losses if there were urbanisation economies and/ or a wedge between urban and rural marginal products of labour.

2.5 Closing the model

To this point our analysis has focused on construction of the city, given time paths for the price of housing floor-space of each type at each location. The model can be completed by specifying household behaviour and hence the demand for space. This is constructed in a way consistent with the preceding analysis, merely offering a model of price growth in terms of growth in city incomes and productivity. The technical exposition is in the Theory Appendix and based on consumers with log linear preferences between commuting costs that decline exponentially with distance from the centre. The city is open with free migration from the outside where the outside option utility is given at any instant. The key driver of price (and population) growth is that in particular open city urban productivity relative to the outside utility level grows at a given rate.

3. Empirical work on Nairobi

The empirical work provides overall facts about the volume of built space in a city, examines key predictions of the model, and looks at a specific policy issue. So what are key predictions? In the cross section there are the usual items from the monocentric city model given above which we will review to do with land price, building heights, coverage and volume gradients. What is new in the cross section is the role of the slum versus the formal sector. On a city wide basis we can show how house prices, heights, coverage and volume differ between the two, as well as their overall contribution to a city's built stock.

The dynamics uses building footprint polygons from high resolution data for 2003/4 and 2015 and building heights for 2015. For the first time that we know of, the evolution of the built environment of a city can be tracked and the predictions of a dynamic model examined. We note the changes in height, cover, and volume overall and within the slum and formal sector and we note the degree of churning of individual buildings. The churning and volume changes indicate a city in rapid evolution in both the slum and formal sectors. Building to a higher height as in the model plays a critical role in creating new volume, as does effective extension of the city.

[7] As a percentage of the excess of urban land rent over the rent earned by land in non-urban use, r_0.

For slums we ask if they seem to move away from the centre and spring up on the edge and whether their role shrinking or rising? Key to the last question is a policy issue.

What is the role of formalisation costs? For Nairobi, based on “accidents” of history, we have an empirical counterpart: slum settlements where formalisation costs are high. We explore the role of these costs on the building of Nairobi, in particular to quantify the lost revenue from high formalisation costs near the city centre and an estimate of the lost land values (as a welfare measure), because of inability to convert slum lands to their highest and best use.

In this section we first describe the data in more detail. Then we present cross-section results, and analyse the dynamics of how Nairobi’s built environment changed from 2003/4 to 2015. Finally, we focus directly on the issue of slums and potential inefficiencies driven by artificially high formalisation costs.

3.1 Data and mapping

We develop a data set for Nairobi which defines characteristics of the built environment at a very fine spatial resolution for 2 points in time. Characteristics are defined at no more than 40 cm resolution and, based on that, then mapped for 3mx3m cells and aggregated preserving details to a grid of 150m x 150m. There are thousands of these in the region; and for the sample we focus on, the 2003/4 built area of the city, we start with 6470 grid cells, from which we remove 726 grid squares in major ‘permanent’ public use described below and listed in the Appendix.

Our main data, which we refer to as footprint data, is building footprint based on tracing of buildings from aerial photo images for 2003/4 and 2015. We received the 2003/4 footprint data from the Nairobi City Council with digitized polygons for every building in the administrative boundary of Nairobi. As far as we can tell, this data was created by the Japan International Cooperation Agency (JICA) and the Government of the Republic of Kenya under the Japanese Government Technical Cooperation Program, and mostly based on aerial images taken in February 2003 at a scale of 1:15,000 (Williams, et al. 2014).[8] In January 2015, imagery at (10-20cm resolution) was recorded and digitized into building footprints by Ramani Geosystems. The key methodological imagery work has been to overlay the 2003/4 and 2015 images to determine which building footprints are unchanged since 2003/4, which buildings were demolished with no replacement on the prior site, which buildings were redeveloped and finally where and to what extent infill occurred with new building on sites where no 2004 buildings existed. Overlay is complicated by variations in the way buildings were traced and specifically aligned in 2004 versus 2015, as well as by tracing error. The

[8] We base this off documentation from the Center for Sustainable Urban Development (CSUD) at Columbia University, who use a highly detailed landuse map from the JICA.

Data Methodology Appendix describes the issue and the algorithm used to overlay and identify types of changes.

These data are supplemented by building height data for 2015 from LiDAR (0.3-1m resolution) which was used to create a Digital Elevation Model. To get heights for 2003/4 we assign to buildings in a grid square in a sector (slum or formal) the average height of unchanged buildings in that sector in queen neighbouring grid squares. This will tend to overstate average 2003/4 heights and understate volume changes since it is likely 2003/4 shorter buildings were the ones demolished. Certainly demolished buildings have relatively small footprints. We also use high resolution SPOT satellite data for the years (circa) 2003/4 and 2013 to measure road coverage.

For Nairobi we have two classifications of slums which we utilize. For 2003/4, a land use map was prepared by the CSUD at Columbia University based on a more detailed, copyrighted, landuse map created by the JICA and the Government of the Republic of Kenya under the Japanese Government Technical Cooperation Program which was published and printed by the survey of KENYA 1000 in March 2005. Columbia categorized polygons as slums if they contained small mostly temporary buildings that are randomly distributed in high density clusters, with a statement: "It should also be noted that in some cases the JICA maps labelled these areas as slums on the map and that is the reason we included it here. It was hard to categorize slums so this label was only used when it was clear that this was the type of land use" (See Williams, et al. 2014 for their full methodology). Second, in 2011, slums were mapped by IPE Global under the Kenya Informal Settlements program, and we digitized these maps. IPE mapping of settlements was done using satellite imagery and topographic maps with imprecisely defined criteria. The general idea is that slums are "unplanned settlements" which have some aspects of low house quality, poor infrastructure, or insecure tenure. The 2011 designation has many more slums than in 2003/4. Some 2011 areas had housing in 2003/4 not then defined as slums; in most cases these areas subsequently experienced enormous infill of small densely packed buildings. It is clear however that the effective definitions differ across years and cannot be used to distinguish new slums or even to some extent slums which no longer exist. We rely on the 2011 mapping despite some misclassification issues we will see. But we do look within 2004 slums especially those near the centre to see what happened within those slums.

In Figure 4 we show these two mappings of slums and we also define the area of the city we will work with. We adopt a fairly conservative definition of the boundary: that for a (150mx150m) grid cell to be in the city on the outer edge a smoothed (by 900 meter squares) building cover must be 10% or more of the area. Figures 4a and 4b show the city respectively in dark outline in 2003/4 and in 2015. For each year we mark the slums as recorded at that time: 2003/4 in 4a and 2011in Figure 4b. We also mark the radius in red near the CBD in

which there are no slums as defined in each time period and the city centre with a yellow star. The city centre is the brightest lit pixel in night lights data in the early 1990's.

In either Figure 4a or 4b, we see the intensive margin which is the 2003/4 city. We focus on this margin for examining key aspects of dynamics. City shape is not a nice regular circle. It is bounded to the south by an airport and then a large national park and to the immediate north of the centre by a preserved state forest. We can see also that there is a big extensive margin to the city. Apart from spread, what we take from the figures concerns slums. As the model predicts, slums are not prevalent near the centre; and the area with no slums near the centre expands considerably between the two years, from a 0.775 km to a 2.0 km radius around the centre by 2011, although some of this change could involve differential classification in 2004 versus 2011. The maps suggest considerable slum expansion at the 2004 fringe of the city, as predicted in the model. Finally we note the large slum of Kibera directly south-west of the centre (ranging from 3-5 kms of the centre). In Section 4, we will focus in part on Kibera.

In Figure 5 we show a 3-D map of the city for the 2003/4 boundary, which gives average height of all buildings in public or private use in each grid square (assigned to slum or formal sector by where the centroid of the gird square lies). Calculations are discussed below. Blank areas are those which have missing data in 2004 (the Moi airbase, the State House, and the Ministry of State for Defence) and large areas that have no cover (in particular the Kibera golf course). The city does look monocentric with high heights but variable spikiness at the centre and then diminishing. Slum areas in red are generally low. In the north-east they also reveal misclassification problems; satellite images indicate that those tall areas are not slums!

For the empirical analysis we adjust the areas of analysis in Figures 4b and 5 in two ways, related to two major issues in analysing the data to compare slums and the formal sector. Sectoral classification focuses on slums, and does so with very tight boundaries cutting off vacant land adjacent to the slum (or a river dividing a slum) and even edge slum housing. The formal sector is a residual of everything else in the city. To do a proper comparison, we first remove all grid squares entirely in permanent public use (or not traced in 2004), which serve both slum and formal sector residents. This includes airports, the President's palace, a railyard, a garbage dump, a golf course, major stadiums and parks, colleges and universities, and the like. The Appendix gives a full list of public uses. Overall we remove 11% of land in the 2004 city boundary; but, at the centre from 0-1 km with parks and the President's palace, it is 25%. Note neighbourhood schools are left in and appear in both slum and non-slum areas; side streets and all-purpose streets are left in. Roads as we will see later are much more prevalent in the formal compared to the slum sector. However the greater number of side streets helps to give the formal sector the high amenity levels reflected in house rents. The second issue is the tight mapping. To offset this, we adjust the IPE boundaries by; first,

classifying buildings as slum if their centre lies within the original slum boundary, and then assigning each 3mx3m pixel of non-built land to slum if the nearest building is classified as slum, and formal otherwise.

The analysis also makes use of two other data sets. First is a cross section of georeferenced household level data from the 2012 'Kenya: State of the Cities' survey by the National Opinion Research Center (NORC). This is the first data set to record *household* rent (with detailed house and some neighbourhood characteristics) in Nairobi for a sample that is stratified between slum and formal areas (based on the 2009 Census). Although there have been previous studies of household rents in Nairobi's slums (Guylani and Talukdar, 2008), they rely on data restricted to slum areas, and so offer no analysis of the relationship between the slum and formal housing markets. In addition to rent data, for 2015, we have property values that have been scraped from property24.co.ke. We focus on the vacant land listings with information on asking price and plot area, for which we have information for 80% of the listings. These listings are only found in the formal sector.

3.2 Defining the built features of a city in the cross-section

To analyse the built environment and the dynamics of change, we must define some key concepts and a basic decomposition of the sources of building volume in a city.

Each cell (3x3m) is classified as either informal/slum (I) or formal (F) by the adjusted IPE map. These cells are then aggregated up to 150m x 150m grid squares. As noted earlier we remove grid squares that are entirely in public use, so what is left is just slum and formal. We have the following definitions.

$a_i(\chi)$ is defined as the area (m^2) of grid square χ that is occupied by type *i*, $i = I, F$ (as defined by the binary classification at the 3x3m level). The total area is $a_I(\chi) + a_F(\chi) = 22500$.

$c_i(\chi)$ is the building footprint in area $a_i(\chi)$ of grid square χ (in m^2).

$\bar{h}_i(\chi)$, is the average height of covered area $c_i(\chi)$.

$v_i(\chi) = \bar{h}_i(\chi)\, c_i(\chi)$ is the total volume of built space of type i in the grid square (in m^3).

Aggregation: We will relate outcomes with respect to their distance *x* to the city centre. We define the area at *x* as all grid squares within a ring at *x*. Unless otherwise noted, all figures are done with smoothed moving ring widths of 300m. We will also show heterogeneity within *x* in certain dimensions. It is here that we have our key concepts in defining aspects of the built space. First is the 'cover area ratio' (CAR) by type of use

$$CAR_i(x) = c_i(x) / a_i(x) \qquad (16)$$

where $a_i(x)=\sum_{\chi\in x}a_i(\chi)$ is the total area of type *i* at x and $c_i(x)=\sum_{\chi\in x}c_i(\chi)$ is the total building type *i* footprint at *x*. Then there is average height of built space and volume of built space, where

$$\bar{h}_i(x)=\sum_{\chi\in x}\bar{h}_i(\chi)c_i(\chi)/c_i(x). \qquad (17)$$

This leads to a new concept the 'built volume to area ratio' (BVAR). This is like a floor to area ratio (FAR) except it is in cubic meters of space (related to floor space by dividing by average height 3-3.1 m) and the area is not lot size but all unbuilt land which includes side streets, vacant lots, and small (but not large) public uses.

$$BVAR_i(x)=v_i(x)/a_i(x), \qquad (18)$$

where $v_i(x)=\sum_{\chi\in x}v_i(\chi)=\bar{h}_i(x)c_i(x)$ is total volume supplied by type *i* at *x*. We note that at each *x*, the share of area in slums is $\rho_I=a_I(x)/\sum_i a_i(x)$ and the share in formal is $(1-\rho_I)$.

For total volume $v(x)=\sum_i v_i(x)$, we can now do a fundamental decomposition:

$$\begin{aligned} v(x) &= \underset{\text{total area}}{a(x)}\ \{\ \underset{\text{share in slum}}{\rho_I}\ \underbrace{BVAR_I(x)}_{\text{built vol to area}}+\underbrace{(1-\rho_I)\cdot BVAR_F(x)}_{formal}\} \\ &= \underset{\text{total area}}{a(x)}\ \{\ \underset{\text{share in slum}}{\rho_I}\ \underset{\text{avg. height}}{\bar{h}_I(x)}\ \underbrace{CAR_I(x)}_{\text{cover/area}}+\underbrace{(1-\rho_I)\cdot\bar{h}_F(x)\cdot CAR_F(x)}_{formal}\} \end{aligned} \qquad (19)$$

Graphs will show the components of these in the cross section and dynamics, so show both overall determinants of volume and the role of height and CAR in driving BVAR is each sectors.

3.3 Nairobi in the 2015 cross-section

Prices and heights. In Figure 6, we show the land sales price gradient for sales price per square meter of vacant land, which on principle corresponds in the theory section of the present value of future land rents. This rises sharply as we approach the centre, rising about fivefold from 10 kms out, and much more from the edge of the city (not shown). Corresponding to this, in Figure 7a average building height ($\bar{h}_i(\mathrm{X})$) in meters in the formal sector declines sharply from almost 30m at the centre until levelling out at about 7-8m. These are smoothed curves for grid squares whose centroid is in a 300m moving window going out from the centre.[9] In the slums, height is flat at under 5m throughout, as the technology

[9] This is STATA local mean smoothing with an Epanechnikov kernel, with default settings

modelling in Section 2 would suggest. The building materials of slum housing do not permit building high. Figure 7b reports heights in just the residential sector by floors from the NORC survey. Again heights in the formal sector decline sharply as we move away from the city centre while those in slums are flat or even rise modestly. Figure 7c shows the variability of height in meters within sector. Especially near the centre in the formal sector there is enormous spikiness or variability as we combine office towers, historical buildings, all-purpose buildings like parking garages and shops wedged between tall buildings. In slums, especially the older ones from 3-6 kms out there is little variability; and the variability further out reflects some misclassification issues noted in the discussion of Figure 5. One comment is that Africa experts gave the impression that African cities were built without height. Nairobi clearly does not fit this description. Overall, buildings from 0-1 kms of the centre average (at the 3m x 3m pixel) 10 stories (at 3.1m a storey) and in Figure 7c, 5% of these pixels are over 16 stories.

Volume. We now turn to equation (19) and the decomposition of the components of volume. First in Figure 8 is the share of land in slum and formal use. The share of formal sector (with its roads) is very high: 100% near the centre. Slums occupy no more than 20% of non-public land at any distance up to 10kms from the centre. Figure 9 shows that across the two sectors, there are enormous differences in how housing is produced. In slums the cover to area ratio is very high, over 50% between 3 and 7 kms out; and, in the older slums, nearer the city centre building CAR averages near 60%. In the formal sector CAR never averages above 30% and in the core of the city bumps along at about 25%. This means that slums have little green/open space around houses and little in the way of side streets. We also give coverage adding in roads within each sector by dashed lines in the figure. Much more coverage by way of roads is added to the formal than the slum sector, where in the formal sector it looks like roads are about 15% of coverage near the centre.[10] The slum versus formal sector coverage is a fundamental quality of life issue: access to green space and connections to the rest of the city is much greater in formal compared to slum areas. Finally, we note that, slum CAR with or without roads declines sharply with distance (and opportunity cost of land), as predicted in the model. Surprisingly, the pattern in the formal sector shows little decline, but that more mimics the model where we assumed constant CAR in the formal sector, in order to focus on height.

Combining Figures 7 and 9, slums produce housing with intense ground cover but little height while in the formal sector the opposite is the case. Figure 10 gives this net: built volume to area ratio (BVAR). In the formal sector up to almost 2 kms (where there are also non slums) BVAR is very high, averaging around 7 metres of vertical space per metre of

[10] We know overall roads are about 22% of total area of the city centre, implying that roads in public sector use grid squares we have removed is high near the centre.

ground area, or 7 cubic metres of space per metre of ground area.[11] At 2km and beyond, slums and the formal sector deliver essentially the same BVAR, so height and CAR differences cancel out, and both decline with distance from the centre. This is consistent with the theory where we saw that, dependent on parameters, cover in slums and height in formal areas could deliver the same housing volume. At 6.5 kms, the BVAR in slums does bump up, but as we saw earlier in Figure 5 and will revisit below, this is due to misclassification of tall formal sector buildings as slums. Figure 10b shows again the high heterogeneity in BVAR in the formal sector as different grid cells have more or less roads and other non-building use, and as building heights differ between newer and more historical or utilitarian uses (parking garage). In the core older slum areas from 3-5 kms, there is only modest variation in BVAR.

There is a new fact here for the opposing views of whether formal sector height trumps slum coverage in providing volume of built space. In Nairobi they do equally well on average, albeit at very different quality levels. However later we will argue that Figure 10 for the formal sector reflects an average of locked historical BVAR and redeveloped BVAR based on current demanded height for new buildings. We will see later, that at least from 2-5 kms out, redeveloped BVAR in the formal sector does to some degree dominate what is provided in slums.

Figure 11 pulls the whole decomposition together and shows total volume and then the share in formal throughout the city. There are two key takeaways. First slum volume is never a big part of the picture, given slum share in land is never over 20%. Second total volume rises sharply to peak at almost 13.5 million cubic meters at 3.5 kms from the centre as the amount of potentially available land in any circumference increases; but then it falls to average around 7-8 million. As we noted in Figures 4-5, Nairobi has little available land beyond 4-5 kms to the direct north and south.

3.4 The dynamics of the built environment in Nairobi

We continue the decomposition analysis and develop the framework for the empirical analysis of dynamics. Then we proceed to the data.

3.4.1 Defining the dynamics of the built environment in a city

For changes, we start with the basic decomposition $v(\mathrm{x}) = \underbrace{a(x)}_{\text{total area}} \{ \underbrace{\rho_I}_{\text{share in slum}} \underbrace{BVAR_I(x)}_{\text{built vol to area}} + \underbrace{(1-\rho_I)\cdot BVAR_F(x)}_{formal} \}$. Here we cannot distinguish and thus

[11] If one wants to compare this to the usual floor to area measure (FAR) after dividing by 3-3.1 m per floor, we note that the base is not lot size but all land not in public use including all land in transport.

treat as constant the classification of cells by I or F, therefore holding constant ρ_I and $a_i(x)$. What can change at any χ (and hence x) are $\bar{h}_i(x)$ and the $c_i(x)$ in $CAR_i(x)$ and hence $BVAR_i(x) = CAR_i(x) \cdot \bar{h}_i(x)$. These then give the percent changes in $v(x)$ and $v_i(x)$.

Of particular interest is to decompose in each sector the changes in $c_i(x)\bar{h}_i(x)$ into changes due to infill (a new building with a footprint that did not overlap with any building in 2003/4), demolition (a building in 2003/4 that has been demolished is now all open space) and redevelopment (a 2003/4 building which has been replaced by a new building with a different (usually larger) footprint. Notation is a little tricky. Note for redevelopment, we have both a net change in footprint ($\Delta c_i^R(x)$) and a new footprint ($c_i^{R,0}(x) + \Delta c_i^R(x)$). We obtain these by overlaying images polygons from 2003/4 with 2015; details on methodology are in the Appendix. Specific infill, demolition and redevelopment cover and volume change definitions are given as:

Infill (N): $\Delta c_i^N(x)$, $\Delta v_i^N(x) = \bar{h}_i^{N,1}(x)\Delta c_i^N(x)$, where 1 is 2015. (a)

Demolition (D): $-\Delta c_i^D(x)$, $-\Delta v_i^D(x) = \bar{h}_i^{D,0}(x)\Delta c_i^D(x)$ where 0 is 2003/4 (b) (20)

Redevelopment (R): $\Delta c_i^R(x)$, $\Delta v_i^R(x) = h_i^{R,1}(x)[c_i^{R,0}(x) + \Delta c_i^R(x)] - h_i^{R,0}(x)c_i^{R,0}(x)$ (c)

Besides height, volume and coverage differences by these categories we will look at differences in building count patterns.

3.4.2. Results on Dynamics

Between 2003/4 and 2015 there is dramatic change in the city. There is substantial infill especially farther from the centre and substantial increase in heights nearer the centre achieved through redevelopment. In the first kilometre from the centre however there is less change. Use in the centre is locked in historical buildings and roads, and sky-scrapers built over the last 35 years.

To see the drivers of change we first focus on the formal sector. Figure 12 shows average height of unchanged buildings and redeveloped ones. From 1.5 to 5 kms, redeveloped buildings generally average twice the height of unchanged buildings. This is building higher with redevelopment which the model predicts for the formal sector, as a city grows and land prices rise. Infill is at a lower height than either redeveloped or existing buildings at least out to 6 kms, a detail we discuss below.

Figures 13a and 13b give the present change in cover and in volume by infill, redevelopment and demolition. Since total areas in formal usage are fixed, these relate also to the percent changes in CAR and BVAR by each source and in total. Since what 13a shows is net

redevelopment change in cover (2015 minus 2004 footprint sizes), throughout beyond 1 km infill dominates in contribution to total coverage change. As we move further from the centre where there is more available land, infill in coverage is enormous. However, because of the high heights of redeveloped buildings, from 1-4.5 kms, volume changes due to development dominate infill, but not further out. The net increase in volume (new volume minus old) due to redevelopment alone accounts for 35% of original total volume at 3kms out. Formal sector total volume changes peak at near 70% at about 4.5 kms out, where infill starts to take off.

Figure 14 shows the overall percent change in volume in the formal sector (already given in 13b), the slum sector, and then the area weighted average of the two to get total change in volume. Again given fixed areas in usage, volume changes and BVAR changes mimic each other. Overall there are 45-70% increases in total volume from 2-8 kms out. Until 9 kms out *within* sector percent increases in the formal sector generally dominate those in the slums, showing the increasing relative role of the formal sector in the main part of the city. Total volume change as a weighted average by area where slums never have more than 20% of land at any distance are completely dominated by (and hence mimic) formal sector volume changes.

As a basic fact, overall in the 2004 city (the intensive margin), total built volume for non-public use increased by 53% from 2003/4 to 2015, about a 4% annual increase in this major form of wealth. Including (unchanging) public in the base, total change is about 50%. At the extensive margin, the ring in Figure 4b between the 2004 and 2015 boundaries accounts for 19% of total volume in all uses in 2004 for the city out to the 2015 boundary. The increase in volume in all uses at the extensive margin is 96%. Overall within the 2015 boundary, volume in all uses increases by 62%, about a 4.4% annual rate of increase. This compares with an annual population growth rate which approaches 4%. Slum changes overall are modestly smaller, at 55%, so the slum share falls.

Slums

Having just reintroduced slums, we note their changes. The key problem is that we don't directly capture slum conversion and what Figure 14 shows is changes in areas defined as slums in 2011. In the Appendix we show the same graphs (12- 13) as we did for the formal sector (as well as another one to match 16 below). Basically heights of all buildings in slums are low and thus coverage and volume percent changes mimic each other. Infill and redevelopment follow similar patterns to the formal sector. Note we expect slum redevelopment per se given the impermanence of the basic structures. However there is also slum redevelopment into the formal sector. Figure 15 tries to look at an aspect of slum transition to the formal sector. It shows the height of redeveloped buildings compared to unchanged buildings for slums defined as of 2003/4. Most (65%) buildings at 3m high are cut

from the figure. The rest shows the increase in density at high heights for redeveloped versus unchanged buildings. Redevelopment is to a higher height; and, based on the increased density at very high heights, some of that must be slum redevelopment into formal sector usage.

Churning of small lots

In Figure 15, we focus on some details, which provide novel facts and relate to an unusual feature: churning. In solid lines we show the changes in building counts as a percent of 2003/4 counts and in dashed lines the same for area covered. There is enormous churning in counts. Demolition and redevelopment each are about 15-20% of 2003/4 building counts from 2-6 kms out, so that about 35% of buildings there are torn down and 50% of those are redeveloped. Infill by counts is enormous everywhere, adding about 40% to 2003/4 counts from 1.5 to 4kms out, with that percent then escalating as x increases (which is why the graph is cut at 8kms rather than 10). However as a percent of 2003/4 cover, these changes are quite modest near the centre. Infill adds 10-18% to 2003/04 cover from 1.5 to 4kms. Around 5% of 2003/04 cover is demolished (less than 1/3 of the count) and not redeveloped by 2015. The 2003/4 footprint size ($c_i^{R,0}(x)$) involved in redevelopment from 2-6 kms bounces along at 6-10% of 2003/4 cover, under half the rate of count of buildings redeveloped. However, redeveloped buildings have a distinct and very large increase ($\Delta c_i^R(x)$) in average footprint size. The increase in footprint size of redeveloped building averages 100% at 3 kms which rises to 200% by 6kms.

What is going on, for the high churning in counts versus the much small areas affected? First, in-fill very near the centre is constrained by prior development on small lots. For example, in a sampling of 50 in-fill buildings from 0-1.5kms, 32% involve building on top of small parking lots near the centre (which is only 10-12% further out). In another sampling of 50 demolitions from 0-1.5kms, these former small buildings without redevelopment to date are currently parking areas (27%), roads (15%), gardens for others (10%), and small sandwiched spaces (19%); only 29% are more open spaces, mostly with vegetation. Further out at (1.5 – 3kms and at 5-6kms which are similar), a sample of 100 has more garden (19%) and road usage (40%) with less open space (18%) and parking (14%). Churning due to infill and demolition without redevelopment typically is constrained by small lots demarcated under prior planning and land right histories. Redeveloped lots are typically different. These often involve situations where coverage can be extended and/or land assembled to increase footprint size, so as to build to a higher height.

4. Slum redevelopment and lack thereof

We know that slums very near the centre seem to have disappeared up to 2kms out. But beyond that, although we lack a consistent definition of slums over time, there does not seem to be any massive slum redevelopment. Why is that?

We argue that, due to land market 'institutions' and lack of reform, formalisation costs nearer the centre are very high. This comes from two sources. First IPE (2012) produced a map of which slum lands are under government control versus under private ownership. As Figure 17 shows, government ownership is 100% near the centre and then declines as we move out, while private rises and there is a residual (Nairobi City Council, mixed private and government, temporary occupation licenses, and road and riparian reserves). If private is truly private, formalisation in response to market forces should be more forthcoming. However that will not be the case for the government owned slums near the centre. Why is that the case?

The literature on Nairobi slums, some focused on Kibera, suggests government owned slums are intractable problems. Research studies and government reports discuss corruption, the array of actors involved in slums, and 'outright plunder' (Marx, Stoker and Suri 2013 and Southall 2005). Studies suggest slum housing is almost all rental and the housing is operated by slum lords who make high profits. Guylani and Talukdar (2008) estimate payback periods on an investment in a single room of 20.4 months. In Kibera, of 120 slum lords surveyed, 41% were government officials, 16% (often the biggest holders) were politicians, and 42% were other absentee owners (Syagga, Mitullah, and Karirah-Gitau 2002 as cited in Gulyani and Talukdar 2008). The political economy issue is that if the government were to take the land and auction it for formal use, the slumlords would have no claim to the revenue since they don't own the land and their presence is at best quasi-legal. They would simply lose profitable businesses. Having well connected bureaucrats and political figures opposed to conversion presents a problem.

For Kibera, the problem is accentuated by Kibera's history, and we suspect the history of many government owned slums. The 1000 acres in Kibera was awarded to Nubians soldiers in 1912, albeit without formal title. They immediately occupied a portion of the land but at independence their claims (but not tenancy) were revoked, and land reverted in theory to the government. The large portion of Kibera not occupied by Nubians was settled on by others and had titles illegally allocated by local chiefs and bureaucrats. The moral claim of the Nubian descendants to at least the land they occupy is well recognized but the unwillingness to grant them title is yet another road block to redevelopment (Joireman and Vanderpoel, 2011).[12]

[12] Further documentation on the Nubian settlers in Kibera can be found online at Kenya's Nubians, who also argue that the Nubians have a valid claim to the land in Kibera. (http://www.nubiansinkenya.com/)

We now turn to an estimate of some of the benefits of redevelopment of slum lands from 2-6kms out (of which about 75% is in Kibera). The 2-6km area houses about 300,000-400,000 people, roughly 15-20% of Nairobi's slum population. Figure 18a plots log price per square meter of floor space from the NORC data for 2012 in slums versus the formal sector by distance from the centre. We use a regression where the only covariates are distance, slum and their interaction. We infer that the gap in price between the formal and slum sector reflects quality differences: quality in floor space provided in iron sheet or mud dwellings including facilities compared to permanent structures and quality of amenities offered by green space and side roads, as well as disamenities from crowding per se. One might argue that socio-economic status also differs and that is not fundamental to the built environment of slums in explaining these price differentials. Based on results in the Appendix, it would seem little of the price gap would be explained by such differences.[13]

In Figure 18a, the formal sector unit volume (floor space) rent gradient declines with distance from the city centre. But the slum one does not; if anything, it rises. The key explanation comes from the model in Section 2 and Figure 9. In Figure 9, slums nearer the centre have much less green and road space, and more crowding compared to those further out. In the benchmark case in the section 2 theory, quality adjusted price of slum housing across the city is constant and, as we saw there, for some functional forms this price could increase with distance. A second explanation has to do with possible job access. Slum residents there may be more likely to have jobs involving a commute to the outskirts and industrial activities than to the professional and tradeable business service sector in the city centre. Finally, in government slums near the centre, there may be less incentive for individual slumlords to invest in slum amenities because they do not own the land. In private slums, owners have a longer view and can potentially reap the benefits of investments since they own the land, especially for collective decisions to improve the slum.

If we converted slums from 2-6kms out what would be the gains? First we calculate the BVAR for redeveloped buildings in the slum vs formal sector, which is shown in Figure 18b. This graph also removes from consideration all slum grid squares where average height exceeds 9 meters and thus are misclassified (this is about 4% of all slum grid squares). The

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[13] In the Appendix we present a hedonic regression of rent per square meter on all sorts of house characteristics, a slum dummy, distance and slum x distance slum, as well as percent of the population which has some college (the key socio-economic variable after much experimentation). From that, we can take the coefficient on percent college (0.615) and multiply by 0.22, the difference in average percent between slums and formal sector. That 13% is an estimate of the portion of the price gap in Figure 18a explained by socio-economic spillover differences between sectors. At 2kms, that is a modest part of the 250% by which formal rents exceed slum ones. We use raw rents rather than predicting rents for typical slum versus formal sector houses in the hedonic for two reasons. The Rsq is modest (0.39); and related we do not think the slum and slum x distance interaction terms capture how amenities vary within slums. The latter statement requires explanation.

BVAR takes 2015 CAR in slums versus the formal sector at each x[14] but applies heights of redeveloped buildings to get BVAR for new slum vs formal sector developments. Here now at 2-5 kms the formal sector does provide more BVAR with its high heights. We then calculate revenues for slums (slum BVAR times slum area times slum volume price) and for formal sector redevelopment (formal BVAR times slum area times formal sector volume price). Relevant numbers are given in Table 1. We convert these monthly revenues to 2015 annual revenues in dollars (where house price appreciation from 2012 in Nairobi is 8% a year) and then obtain the present value of an indefinite stream discounting at 4%. This is done for each distance ring from 2-3 kms up to 5-6 kms. The bottom row of Table 1 shows the present value of revenues in each ring in millions of dollars. If we sum up, we get \$3.49 billion for conversion. We have no cost side to this conversion per se. If we think land rent revenues are, say, 35% of house price revenues at this distance from the centre (Duranton and Puga, 2015), the implication would be that formalisation raises land values by about \$1.25 billion, a measure of welfare cost of indefinite non-conversion at today's price conditions and heights. It is of course possible that land rents are a higher share of house revenues in slums than in formal areas but then the calculation depends on the actual share numbers. In the theory section we set slum land shares at 70% ($\lambda = 3.3$) and formal at 50 ($\gamma = 2$). These numbers generate about 10-15% larger land value gains than the 35%.

Whatever the exact magnitude, there is a vast surplus in land values which could be used to buy-out vested interests of slum lords hindering formalization of lands, as well as helping with relocation. One solution might be to give longer term residents ownership of their units and land, allowing redevelopers to buy them out in a timely (and voluntary) fashion; but that solution would require settling with slum lords.

5. Conclusions

The model and data both suggest that in the formal sector house rents and land prices decline with distance to the centre; consequently building heights decrease with distance to the centre. Heights in slum areas are much lower than in the formal sector near the centre and lower throughout the city. However intensity of land cover within slums is very high. Slums account for a small fraction of total housing space overall at any distance from the centre. Between 1-6 kms from the centre from 2004 to 2015 there is major redevelopment of 2004 formal sector buildings into higher height new buildings. Expansion of the informal sector is towards the city fringe, with intensive demolition very near the centre. We find that there is high intensification of land use with infill of new buildings through much of the city

[14] Note CAR is not well defined for infill vs redevelopment since it is an overall area concept bringing in side streets and green spaces.

especially on the fringes. We find that development of the informal into formal sector housing mid-city over the 11 years is slow.

In the model we explore the role of expectations in altering (re)development paths. Under-estimating future demand growth leads to stunted city heights and spatial size. In the model we explore the cost of converting slum to formal use; and in the data for the common institutional context of Nairobi, we explore misallocation of land between slums and formal sector usage, based on formalisation costs arising from poor institutions. We argue that slum ‘ownership’ by government means unresolved land right issues and corruption with vested slum interests of political figures, with a significant welfare loss.

Theory Appendix

Derivation of equation (11): Derivation of (11) uses

$$\partial R_F(x,\tau_i)/\partial\tau_i = -p_F(x,t)v_F(x,\tau_i) + \rho\int_{\tau_i}^{\tau_{i+1}} p_F(x,t)v_F(x,\tau_i)e^{-\rho(t-\tau_i)}dt$$

$$= -p_F(x,t)v_F(x,\tau_i) + \rho\left[R_F(x,\tau_i) + k_F(v_F(x,\tau_i))\right].$$

And the fact that volume is optimised.

Parameters for figures

Parameter values in figure 1 are: $c_F = 1$, $c_I = 0.1$, $\gamma = 2$, $\lambda = 3.33$, $\rho = 0.05$, $\hat{p}_I = \hat{p}_F = 0.015$, $\theta_F = \theta_I = 0.05$, $r_0 = 4$, $\bar{p}_F = 5, \bar{p}_I = 4$. Simulation is done with time running to t = 800, and reported up to t = 250. Distance running to x = 60. In figure 1 formalisation cost D=1000, in figure 2a D = 1000 or D=5000 (10> x <20) (and, for comparison, the construction cost of the first formal sector structure built is 4500).

Section 2.5: Closing the model

Households: At date t a representative urban household living at distance x from the CBD receives income net of commuting costs $w(t)T(x)$, where $w(t)$ is the wage at date t (the same for all households), and $T(x)$ is the fraction remaining after commuting costs. Each household makes a discrete choice between formal and informal sector housing. For the chosen sector, the household chooses $s_i(x,t)$ units of housing (i.e. volume), at price $p_F(x,t)$ per unit in the formal sector, and $p_I(x,t)q(x,t)$ in the informal sector. Utility is derived from the volume consumed, its quality and formal/informal status, and consumption of a numeraire good (equal to wage income net of commuting and housing costs). For each type of housing,

$$u_F(x,t) = u\left(s_F(x,t),\quad w(t)T_F(x) - p_F(x,t)s_F(x,t) : F\right).$$

$$u_I(x,t) = u\left(s_I(x,t)q(x,t),\quad w(t)T_I(x) - p_I(x,t)q(x,t)s_I(x,t) : I\right).$$

If preferences are Cobb-Douglas then

$$u_F(x,t) = s_F(x,t)^{\alpha_F}\left\{w(t)T_F(x) - s_F(x,t)p_F(x,t)\right\}^{1-\alpha_F}$$

$$u_I(x,t) = q_i(x,t)^{\alpha_I}\, s_I(x,t)^{\alpha_I}\left\{w(t)T_I(x) - s_I(x,t)p_i(x,t)q(x,t)\right\}^{1-\alpha_I},$$

Consumers take price and quality of housing at each place as given, and the quantity of housing space, $s_i(x,t)$, is chosen to maximise utility. Optimal choice gives,

$$s_F(x,t) = \alpha_F w(t)T_F(x)/\,p_F(x,t), \qquad s_I(x,t) = \alpha_I w(t)T_I(x)/\,p_I(x,t)q(x,t).$$

Maximised utility for each type of house is

$$U_F(x,t) = A_F\, p_F(x,t)^{-\alpha_F}\, w(t)T_F(x), \qquad U_I(x,t) = A_I\, p_I(x,t)^{-\alpha_F}\, w(t)T_I(x),$$

$$A_i \equiv \alpha_i^{-\alpha_i}(1-\alpha_i)^{1-\alpha_i}$$

Free choice of location and housing type means that, at any occupied location and housing type, utility equals a common city wide utility level, $\overline{U}(t)$. Prices of formal and informal (quality one) housing must therefore satisfy

$$p_I(x,t) = \left(\frac{w(t)T_I(x)}{A_I\overline{U}(t)}\right)^{1/\alpha_I}, \qquad p_F(x,t) = \left(\frac{w(t)T_F(x)}{A_F\overline{U}(t)}\right)^{1/\alpha_F}, \tag{A3a}$$

Constant exponential growth of the price of space is achieved by assuming that urban wages relative to outside utility grow at constant rate g. Similarly, constant exponential decline with respect to distance is achieved by the share of income net of commuting declining with distance at rates $\hat{T}_I, \hat{T}_F$, so $p_i(x,t) = \left(\overline{w}e^{gt-\hat{T}_i x}/\overline{U}(t)\right)^{1/\alpha_I}$, $i = I, F$. This gives prices rising through time at constant rates $\hat{p}_I = g/\alpha_I$, $\hat{p}_F = g/\alpha_F$, and declining with distance, $\theta_I = -\hat{T}_I/\alpha_I$, $\theta_F = -\hat{T}_F/\alpha_F$.

Labour and population: To complete the model, we note that population at a point is v/s, total volume supplied divided by consumption of floor space per household. Total city population at date t is therefore

$$L(t) = \sum_{i=1}^{i\max(t)} \int_{x_{i+1}(t)}^{x_i(t)} v_F(x,\tau_i)/s_F(x,t)dx + \int_{x_1(t)}^{x_0(t)} v_I(x,t)/s_I(x,t)dx. \tag{A5}$$

The oldest formal development has been redeveloped the most times (which, at date t, we denote $i$max(t)). Notice that this expression assumes that the city is linear (or a set of rays), not a disc; adjustment to (A5) to capture the latter is straightforward.

The final element is to close the model, either by setting $\overline{U}(t)$ exogenously with $L(t)$ endogenous (open city), or with $L(t)$ exogenous and determining the equilibrium city wide level of utility (closed city). The analysis in the body of the paper follows the open city route, with exogenous growth of urban wages relative to outside utility driving housing price growth.

Data Methodology Appendix

This Appendix has two components. The first deals with measures on cover/footprint and volume we use to analysis. The second gives the algorithm used to extract unchanged buildings, redeveloped buildings and infill from the overlay of 2004 and 2015 depiction of building polygons.

Measures of cover and volume

Our unit of analysis is 150x150m grid squares. For calculating cover within the grid square in a usage, each of these is broken into 50 3m by 3m cells and use type classified by what is at the centroid of the 3m square. There are three uses: vacant land, slum area and formal. Each 3x3 square is given the type of cover there in whichever time period. For each 150x150 square we sum across the 50 cells to get for example total building cover in each type. If for example a 150m by 150m gird has only formal sector buildings the square meter coverage can take values of 9, 18, 27, etc. up to 450. And the same

for areas that are always slums. Most 150x150 squares are either all slum or all formal sector. However there are about 12% which are mixed grid squares, for which we record the cover or volume of slum and formal separately.

For average coverage in a grid square in the formal sector, before smoothing in a year in a given distance ring, the total area of all cover in 3x3 squares is summed up for all 150x150 meter squares whose centroid falls in a narrow distance ring. That sum is then divided by the **total** number of 150x150 grid squares in that distance band. The same procedure follows for slums. For Volume for 2015, for each 3x3m square which is formal sector, we have the height of the building whose cover is over the centroid of that square. So volume for that 3x3 square is 9 times the height in meters of the building from LiDAR data. We then sum across the grid squares occupied with formal usage for 150x150m grid squares in each distance ring and then average by the total number of 150x150 meter grid squares in the ring. For 2004 we have no height data. To infer 2004 heights, we use what we think is an upper bound on height: the height of unchanged buildings, where we presume demolished buildings between 2004 and 2015 are likely to be of lower height than those which survive. To assign a height to a 3mx3m square in 2004 in formal sector usage, we take the average height in 2015 of all buildings that were there in 2004 for all 3x3m formal sector unchanged buildings in the own 150x150m grids square and its 8 queen neighbours. Height is the height assigned to each 3x3m square in usage in a distance ring from the centre averaged over all such cells, to effectively get a coverage weighted average of individual building heights.

How do we measure change between 2004 and 2015? For demolition, at the 3x3m level the square is defined as demolition if its centroid is covered by a 2004 building which has been replaced by open space. Demolished coverage is lost 2004 cover; demolished volume is assessed as before using the average height of unchanged buildings in the neighbourhood. Infill is new buildings which do now overlap with any 2004 buildings; a 3x3m square is infill if its centroid is covered by such a building on 2015 where there was no building in 2004. Infill cover and volume are assessed from 2015 data. Net redevelopment in coverage takes coverage in the new 2015 buildings and subtracts the coverage of old 2004 buildings. So for each 150m150m meter square we have for redeveloped buildings, we have total coverage in 2004 measured at the 3x3m level (centroid covered by the old 2004 building(s)) and we have total coverage in 2015 measured at the 3x3m squares (centroid covered by the new replacement 2015 building(s)). Net redevelopment at the 150x150sqaure is the difference. In general, the same buildings are drawn in 2015 to have modestly more coverage than in 2004 so coverage change is likely to be an upper bound. Net volume change again assigns heights in 2004 to the 3x3m coverage based on neighbourhood averages for unchanged buildings and uses 2015 height information on the new buildings.

Overlaying Buildings

We match buildings across time by overlaying 2015 and 2004 building polygon data in order to track the persistency, demolition, construction and reconstruction of buildings over time. Since buildings are not identified across time our links rely on a shape matching algorithm. For each building, the algorithm determines whether it was there in the other period, or not, by comparing it with the buildings that overlap in the other time period.

This task is not straightforward, since the same building can be recorded in different ways depending on the aerial imagery used, whether building height was available, and the idiosyncrasies of the human digitizer.

Data and definitions

For 2004 we use a building dataset received from the Nairobi City Council with digitized polygons for every building, roughly 340,000 in the administrative boundary of Nairobi. For 2015 we use a similar dataset that was created by Ramani Geosystems using imagery (10-20cm resolution) and LiDAR (0.3-

1m resolution). We have 2015 data for a wider extent, and consequently many more buildings, about 1.14 million. The LiDAR data in 2015 were used to measure heights of objects. With use of the aerial imagery and heights in 2015, a 3D model was created by hand, and rooftops extracted from this model.

Here we define the nomenclature that we use. First, a *trace* is the collection of polygon vertices that make up its outline. A *shape* is the area enclosed by the trace, and it can be thought of as a representation of the rooftop of a building. A *cavity* is an empty hole completely enclosed in a shape. A *candidate pair* is the set of any two shapes in different time periods which spatially intersect. A *link* is the relationship between a set of candidates in one period to a set of candidates in the opposite time period.

Pre-processing

Before running our shape matching algorithm we clean up the data sets. First we take care of no data areas. There are some areas that were not delineated in 2004, including the Moi Air Base, and the Nairobi State House. We drop all buildings in these areas for both 2004 and 2015. We drop roughly 1,500 buildings from the 2015 data, and 100 buildings from the 2004 data. Next we deal with overlapping shapes. While the 2004 data has no overlapping shapes, in the 2015 data there are some shapes that overlap. This is most often the same building traced multiple times. We identify all such overlapping polygons and discard the smaller version, until no overlaps remain. We drop about 1,400 buildings from the 2015 data this way. We also decide to drop small shapes, in part because the 2015 data has many very small shapes, while the 2004 data does not. In order to avoid complications of censoring in the 2004 data, we simply drop all shapes that have an area of less than $1m^2$. We drop 2 small buildings in 2004, and 462 small buildings in 2015.

Another issue is that buildings are often defined as contiguous shapes in 2004, but broken up in 2015. For the majority of buildings we cannot aggregate the broken up pieces in 2015 since it is hard to identify such cases in general. To match these cases across time we rely on our one to many, and many to many matching algorithms defined below. However, in the specific case where a building is completely enclosed in another the task is much easier. First, we find all cavities present in each period, then we take all building shapes that overlap with the cavities in the same time period. After identifying all shapes that intersect a cavity, we redefine both shapes, the original shape containing the cavity and the shape intersecting it, as a single new shape.

Shape Matching Algorithm

After the pre-processing of each cross-section is complete, we run our shape matching algorithm to establish links between buildings across time periods. For any given building we consider 5 possible scenarios; that it has a link to no building, that it has a link to one building (one to one match), that it has a link to multiple buildings (one to many), that it is part of a group of buildings that match to one building (many to one), or that it is a part of a group of buildings that matches to a group of buildings (many to many). We follow and approach similar to Yeom et al (2015) however, due to the inherent difficulty of inconsistent tracings we contribute to their method by introducing the one to many and many to many approaches. We assign each link a measure of fit that we call the overlay ratio. We then choose optimal links based on the overlay ratio. Finally, we categorize links as matched or not using a strict cut-off on the overlay ratio of 0.5. Other cu-offs such as 0.4, 0.6 and 0.7 produced more errors in categorization.

Candidates

For all buildings A in the first time period, and B in the second time period we identify the set of candidates:

$$CP = \{(A, B);\ Area(A \cap B) \neq 0\}$$

For each candidate pair we find the ratio of the intersection area over the area of each shape, so if shapes A and B intersect, we find $r_{AB} = \frac{Area(A \cup B)}{Area(A)}$ and $r_{BA} = \frac{Area(A \cap B)}{Area(B)}$
We link all shapes which do not belong to a candidate pair to the empty set.

One to One Matching
First we consider candidate pairs to be links on their own. For each pair, we calculate the overlay ratio as the intersection area over union area, so if A and B are candidate pair, we find:

$$R_{AB} = \frac{Area(A \cap B)}{Area(A \cup B)} = \frac{Area(A \cap B)}{Area(A) + Area(B) - Area(A \cap B)}$$

One to Many Matching
For each time period separately, we identify all candidate pair links for which their intersection to area ratio is above threshold θ. For shape A we define a group $= \{B; r_{BA} \geq \theta\}$. Now we calculate the overlay ratio of one to many links as the intersection area over union area ratio:

$$R_{AG} = \frac{Area(A \cap \bigcup_{B \in G} B)}{Area(A \cup \bigcup_{B \in G} B)} = \frac{\sum_{B \in G} Area(A \cap B)}{\sum_{B \in G} Area(A \cup B)}$$

Many to Many Matching
Here we have two cases, one when the shapes are fairly similar, which we capture in previous sections (one to one, or many to one). The other is inconsistent shapes that form the same structure. To capture these we consider both time periods at the once, we clean the candidate pair list, keeping links for which either ratio is above a threshold θ_1:

$$LC = \{(A, B);\ r_{AB} \geq \theta_1\ or\ r_{BA} \geq \theta_1\}$$

Then we condition to only keep shape for which the total ratio intersection is above threshold θ_2, so shape A will be included if $\sum_{B \in \{x|(A,x) \in LC\}} r_{AB} \geq \theta_2$. Now we are left with a new candidate list, which we convert to sets $LC = \{(\{A\}, \{B\})\}$ and start merging them:

$$if\ G_i \cap G_j \neq \emptyset\ or\ H_i \cap H_j \neq \emptyset : LC = \{(G_i \cup G_j, H_i \cup H_j)\} \cup LC/\{(G_i, H_i), (G_j, H_j)\}, \mathrm{i} \neq \mathrm{j}$$

We keep doing this until we can no longer merge any two rows. At this point we calculate the overlay ratio of many to many links as the intersection area over union section ratio:

$$R_{GH} = \frac{Area(\bigcup_{A \in G} A \cap \bigcup_{B \in H} B)}{Area(\bigcup_{A \in G} A \cup \bigcup_{B \in H} B)}$$

ICP Translation
We encounter a problem when the two shapes or groups of shapes are similar but do not overlap well, this usually stems from the angle at which the images were taken, and is especially prevalent with tall buildings. To address this issue, we translate one trace towards the other, and then recalculate the overlay ratio. As in Besl and McKay (1992), we use the iterative closest point (ICP) method to estimate this translation. To perform the ICP we ignore any cavity points as we found they often cause less suitable translation. We found that for similar shapes this will optimize the intersection area.

Optimal Linking
In the end, we rank all links by their overlay ratio. We iteratively keep the link with the highest overlay ratio, or discard it if at least one of the buildings in the link has already been confirmed in a separate link. From the list of optimal links, we define a link to be a match if its overlay ratio, or the overlay ratio after ICP translation is above 0.5. We then define all matched candidates as unchanged, and the

remaining candidates as redeveloped. All buildings that were not considered as candidates are defined as infill, if from 2015, and demolished, if from 2004.

Accuracy Assessment

In order to assess the performance of the polygon matching algorithm we manually classified links between 2004 and 2015 for a random sample of buildings. We sampled 48 150x150m gridcells, stratifying over slum, non-slum within 3km, non-slum within 6km, and non-slum further than 6km to the CBD. The sample consists of over 2,250 buildings in 2004 and 3,500 buildings in 2015.

Results

We first break down matches by their mapping type. There are five types of manual link: redeveloped/infill/demolished (0), one to one match (1), one to many match (2), many to one match (3), and many to many match (4). For the algorithm we further split (0) into infill/demolished (-1) and redeveloped (0). Appendix table 1 shows the correspondence between the two mappings by building (a) and roof area (b). We can see that most errors come from the one to one matches, however, the many to many matches have the worst performance. Overall the diagonal values are quite high, which means not only are we matching buildings well, but also the algorithm is recognising the clumping of buildings as a human does (bear in mind that, for example, the one to one matches which we 'misclassify' as many to many will still be classified as match in the final data). Finally we have perfect correspondence for demolition and in 2015 nearly perfect for infill.

Next we compare buildings that were matched by the algorithm and those matched manually. For now we use a cut-off of the overlay ratio of 0.5, later we explore the effect of different cut-offs on performance. As seen in appendix table 1 infill and demolition are classified with almost perfect correspondence. For this reason we ignore buildings with these mappings and focus on accuracy of redevelopment and unchanged. In appendix table 2 we condense mappings 1, 2, 3, and 4 into category 1, while redevelopment, or category 0, remains the same.

We define precision P (negative predictive value *NPV*) as the fraction of buildings classified as unchanged (redeveloped) by the algorithm that are correct, recall R (true negative rate *TNR*) as the fraction of buildings classified as unchanged (redeveloped) by hand that the algorithm gets correct, and the F1 score (F) as the weighted average of the two.

$$P = \frac{True\ Positive}{Positive\ Predictions}, \qquad NPV = \frac{True\ Negative}{Negative\ Predictions}, \qquad R = \frac{True\ Positive}{Positive\ Condition},$$

$$TNR = \frac{True\ Negative}{Negative\ Condition}, \qquad F = \frac{2 * P * R}{P + R}$$

The confusion matrix in table 2 is done across all sampled buildings in 2004 and weights observations by buildings (1) and roof area (2). The F1 score is high in both cases, but in part this is due to relative success classifying unchanged buildings: precision for buildings that were classified as redeveloped by the algorithm is 76% of buildings and 72% of roof area, while recall of true redeveloped buildings is 83% of buildings and 74% of roof area

In our first attempt we arbitrarily picked 50% as a cut off of the overlay ratio. Here we take a closer look at this choice. Using our manually classified links we can maximize the F1 score with respect to the cut off. In appendix figure 1 we plot the F1 score weighted by roof area against cut-offs of the overlay ratio for the 2004 data. We find that the highest F1 score comes just below 50% suggesting our first estimate was not far off.

In figure 1 we plot lines for each method of calculating the overlay ratio: without ICP, with ICP, and the maximum of the two. Around 50% we can see that the maximum performs best, but with only a very slight improvement over the ICP alone, which is in turn marginally better than without the ICP.

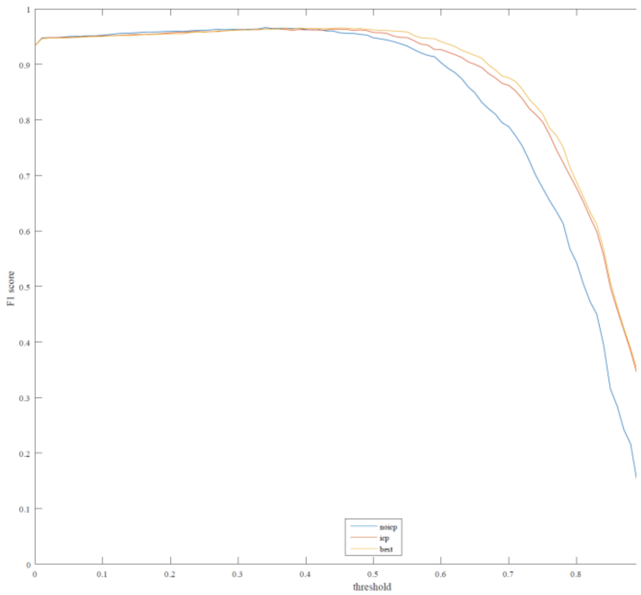
Appendix Table 1 – Mapping Correspondence 2004

a) Weighted by Building						
	Algo=-1	Algo=0	Algo=1	Algo=2	Algo=3	Algo=4
Manual=0	280	433	41	16	11	20
Manual=1	0	25	712	10	1	25
Manual=2	0	29	21	266	0	20
Manual=3	0	18	6	0	137	1
Manual=4	0	65	52	24	63	135
b) Weighted by Area (sq-m)						
	Algo=-1	Algo=0	Algo=1	Algo=2	Algo=3	Algo=4
Manual=0	12708	28187	4913	2780	943	1043
Manual=1	0	908	112762	4180	279	1775
Manual=2	0	3575	2328	89472	0	2819
Manual=3	0	910	1053	0	14148	23
Manual=4	0	5317	5528	4795	4464	14262
Mapping definitions: -1 demolition or infill; 0 redevelopment; 1 one to one match; 2 one to many match; 3 many to one match; 4 many to many match						

Appendix Table 2 – Matching all areas 2004

a) Weighted by Building			
	Algo=0	Algo=1	Recall
Manual=0	433	88	0.83
Manual=1	137	1473	0.91
Precision	0.76	0.94	F=0.93
b) Weighted by Area (sq-m)			
	Algo=0	Algo=1	Recall
Manual=0	28187	9679	0.74
Manual=1	10710	257888	0.96
Precision	0.72	0.96	F=0.96

Appendix Figure 1



Appendix: List of public uses

Recreational
a) Impala club, Kenya Harlequins, and Rugby Union of East Africa (0.14kmsq)
b) Golf Course (0.9kmsq)
c) Arboretum (0.25kmsq)
d) Central park, Uhuru park, railway club, railway golf course (0.5kmsq)
e) Nyayo stadium (0.1kmsq)
f) City park, Simba Union, Premier Club (1.1kmsq)
g) Barclays, Stima, KCB, Ruaraka, Utali clubs, and FOX drive in cinema (0.3kmsq)

Undeveloped
a) Makdara Railway Yard (1kmsq)
b) John Michuki Memorial Park (0.1kmsq)

Special use -- Includes poorly traced areas
a) State House
b) Ministry of State for Defence
c) Forces Memorial Hospital and Administration Police Camp
d) Langata Army Barracks
e) Armed Forces
f) Moi Airbase
g) Kahawa Garrison

Public utility
a) Dandora dump (0.5kmsq)
b) Sewage works (0.25kmsq)

Public use
a) Communications Commission of Kenya (0.1kmsq)

b) Langata Womens prison (0.2kmsq)
c) Nairobi and Kenyatta hospitals, Milimani Police Station, Civil Service club
d) Mbagathi hospital, Kenya Medical Research Institute, Monalisa funeral home
e) National museums of Kenya
f) Kenya convention centre and railway museum
g) Industrial area prison
h) Mathari mental hospital, Mathare police station, traffic police, Kenya police, Ruaraka complex, and National youth service
i) Jamahuri show ground

Educational (not primary and secondary schools)
a) University of Nairobi and other colleges
b) Kenya Institute of Highways & Built Technology
c) Railway Training Institute
d) Kenya Veterinary Vaccines Production Institute
e) Moi Forces Academy
f) NYS engineering, Kenya Institute of Monetary Studies, KCA university, KPLC training, Utali college

Appendix: Hedonic regression based on NORC data for 2012

	(1)	(2)
	Ln Rent per m-sq	Ln Rent per m-sq
Distance to Centre	-0.0748***	-0.0275*
	(0.0171)	(0.0142)
Slum=1 X Distance to Centre	0.122***	0.0522**
	(0.0315)	(0.0253)
Slum=1	-1.422***	-0.425*
	(0.246)	(0.221)
Tenancy Agreement=No Written Agreement		-0.163
		(0.115)
Piped Water in Compound=no		-0.218**
		(0.0846)
# Bathrooms=One		-0.193**
		(0.0851)
# Bathrooms=Two+		0.0334
		(0.0997)
Type of Structure=Shared House		-0.655
		(0.437)
Type of Structure=Single-storey with shared facilities		0.222**
		(0.0896)
Type of Structure=Room in house		-0.423***
		(0.149)
Type of Structure=Shack		-0.364
		(0.256)
Type of Structure=Multi-storey private bath		0.169
		(0.165)
Type of Structure=Multi-storey shared bath		0.408***
		(0.0922)

Type of Walls=Brick/Block		0.338***
		(0.119)
Type of Walls=Mud/Wood		-0.00749
		(0.176)
Type of Walls=Mud/Cement		-0.160
		(0.254)
Type of Walls=Wood only		0.450**
		(0.220)
Type of Walls=Corrugated iron sheet		0.230*
		(0.131)
Type of Walls=Tin		0.356**
		(0.166)
Type of Floor=Tiles		0.839***
		(0.204)
Type of Floor=Cement		0.143
		(0.0956)
Times Flooded Last Rainy Season=Once		-0.338**
		(0.152)
Times Flooded Last Rainy Season=2-3 times		-0.338*
		(0.171)
Times Flooded Last Rainy Season=More than 3 times		-0.303***
		(0.110)
Ln # Floors		0.147**
		(0.0657)
% EA Building Cover 2015		0.0549
		(0.254)
Ln EA Building Density 2015		-14.31*
		(8.200)
EA frac household heads with some post-secondary		0.615***
		(0.187)
Constant	6.223***	5.262***
	(0.166)	(0.303)
Observations	1008	927
R-squared	0.121	0.386
Standard errors in parentheses		
="* p<0.10	** p<0.05	*** p<0.01"

Slum Dynamics. Slum figures corresponding to Figures 12, 13 and 16

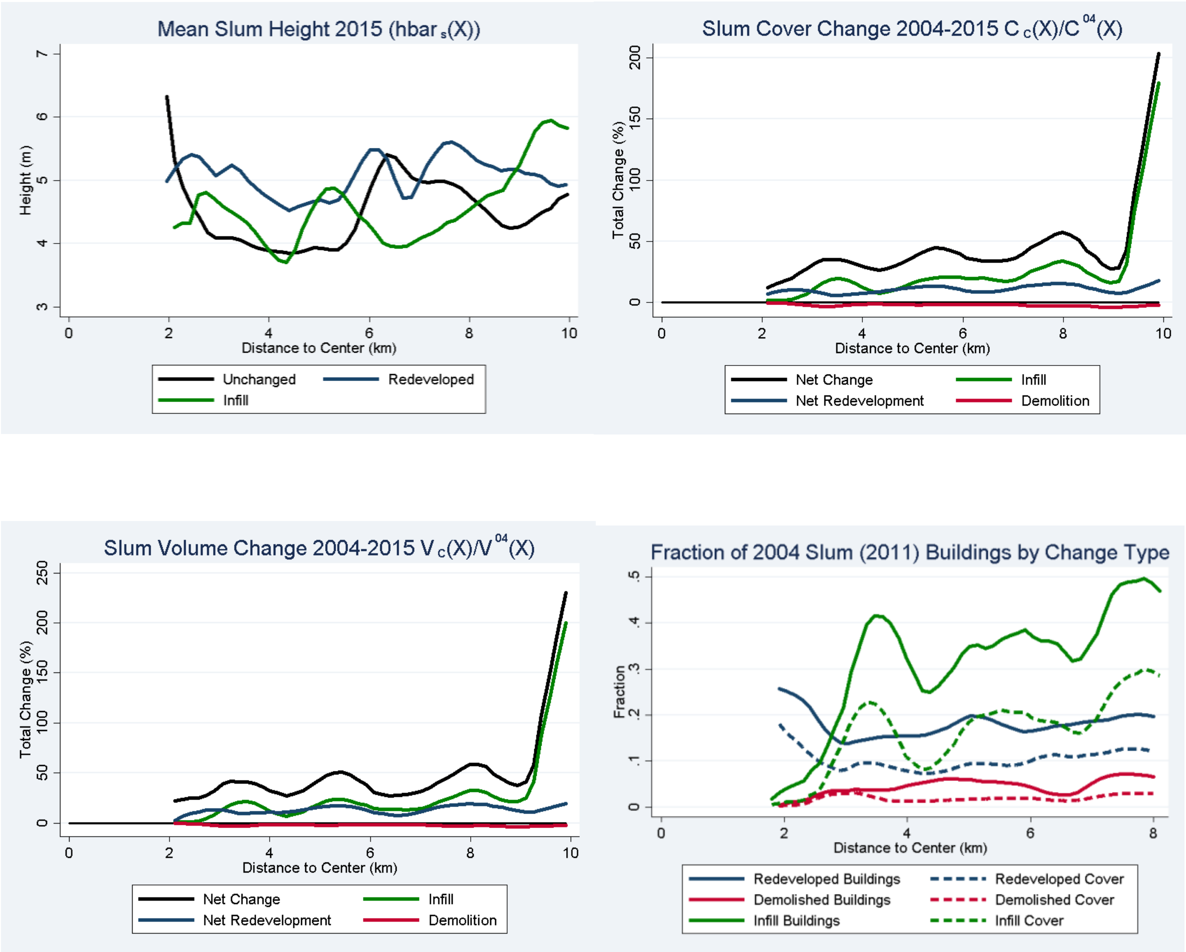


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Figure 1: Urban development with perfect foresight

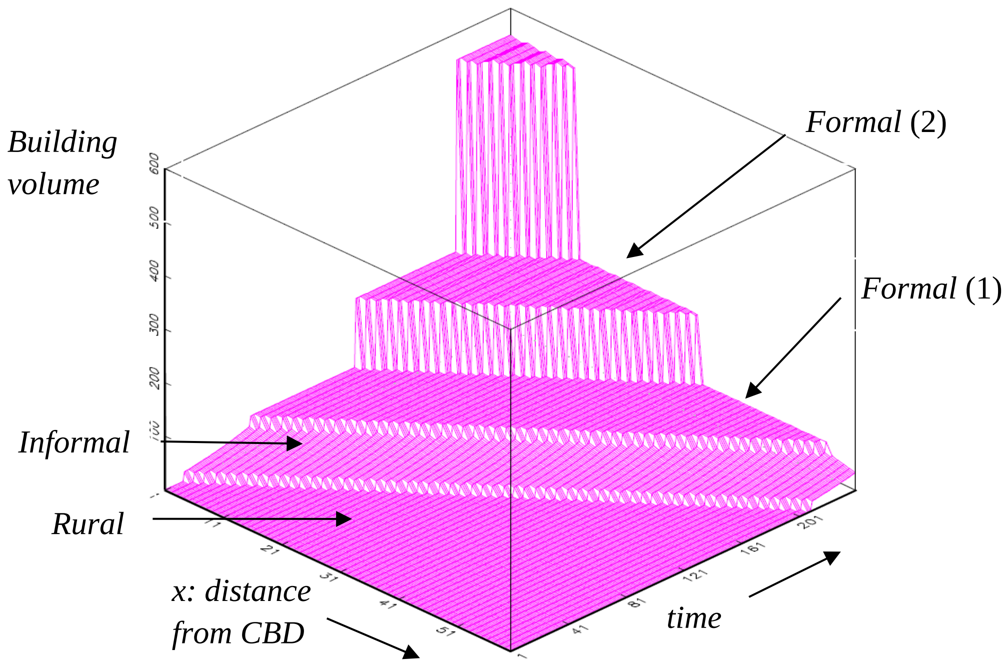


Figure 2a: Heterogeneous formalisation costs

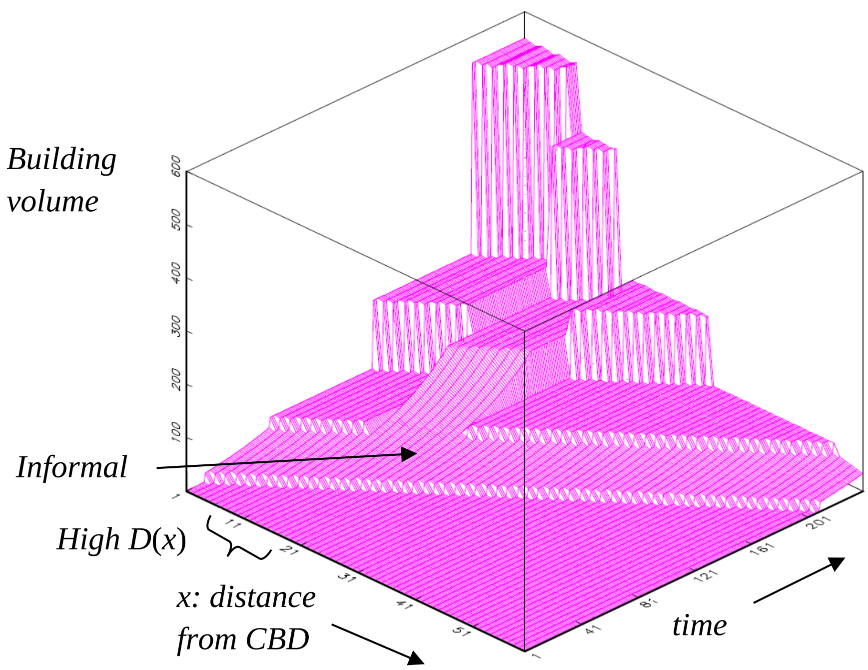


Figure 2b: Random variation in formalisation costs

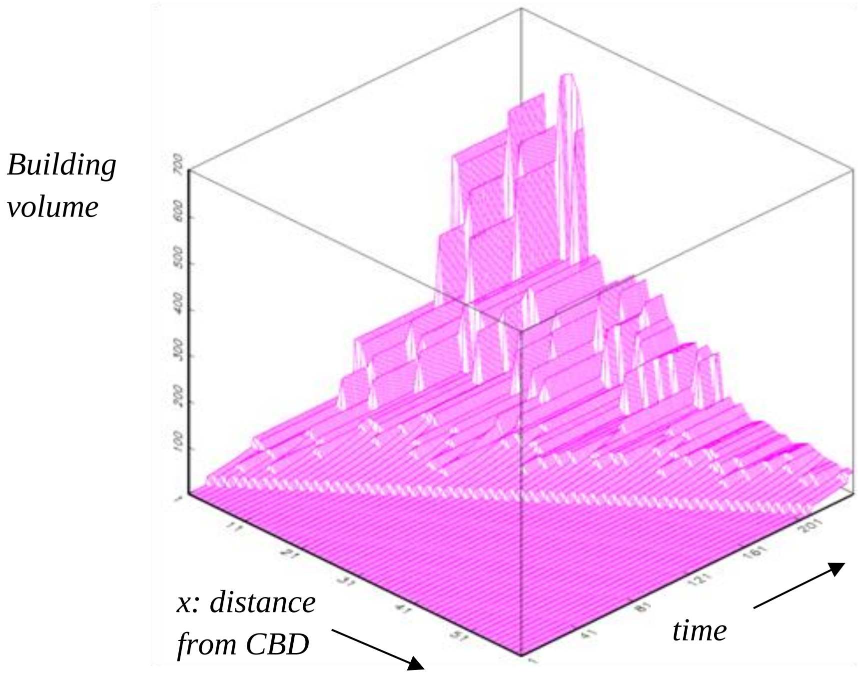


Figure 3: Expectations: volume profile of city at t = 180.

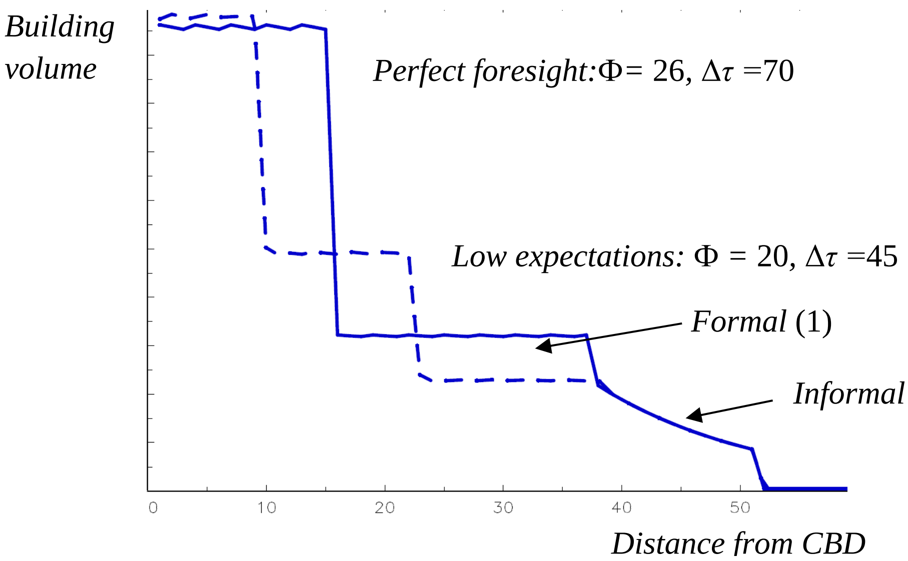
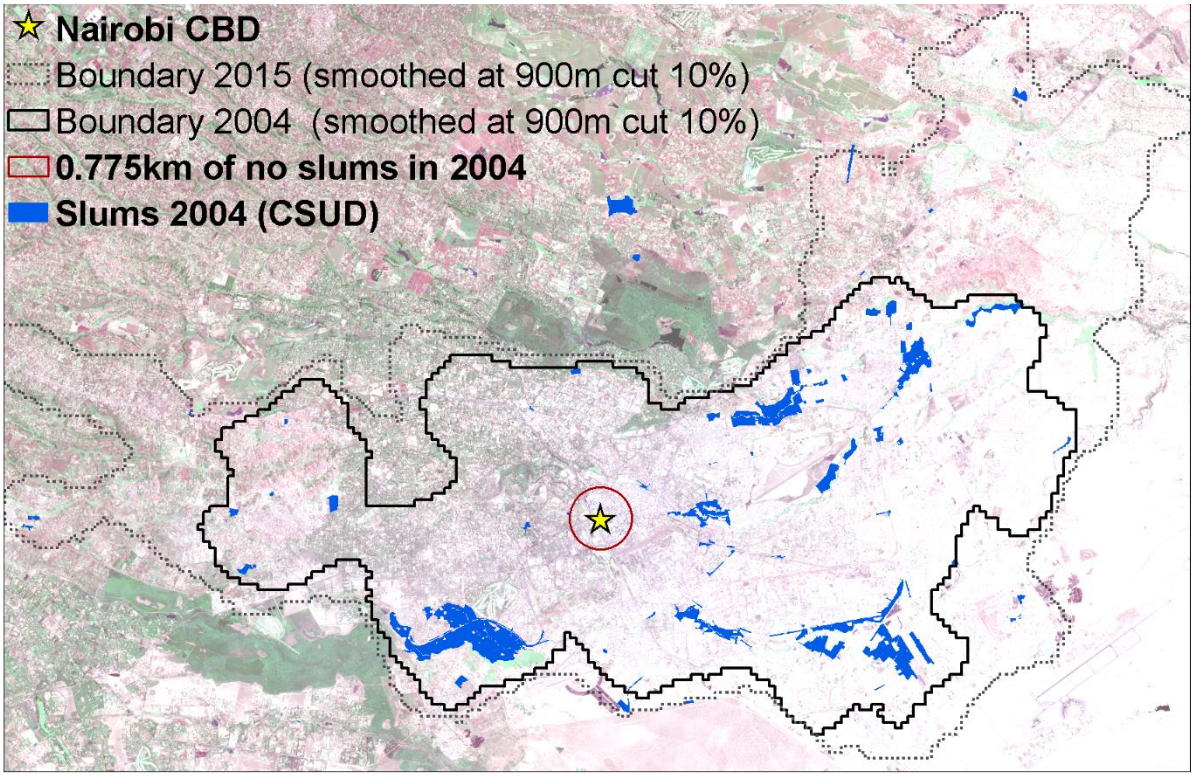


Figure 4 City shape **a. City in 2004**



b. City in 2015

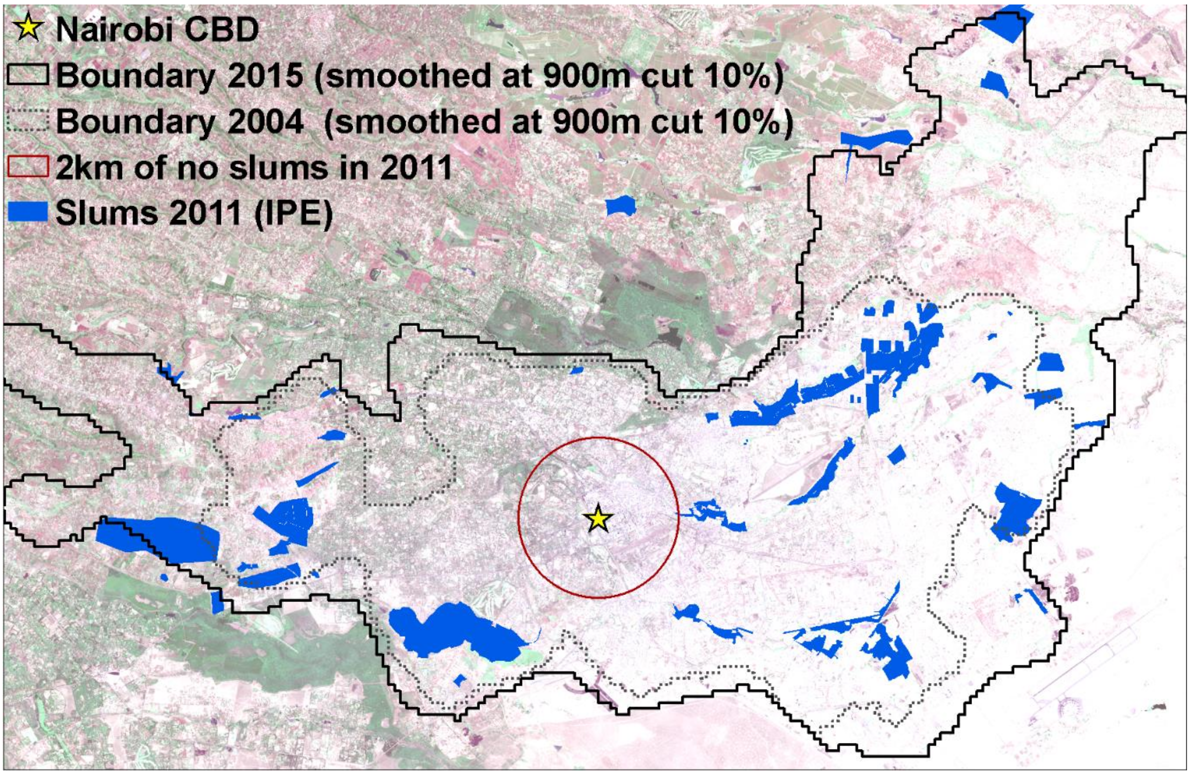


Figure 5. 3-D average height of buildings by grid square in the formal and slum sectors

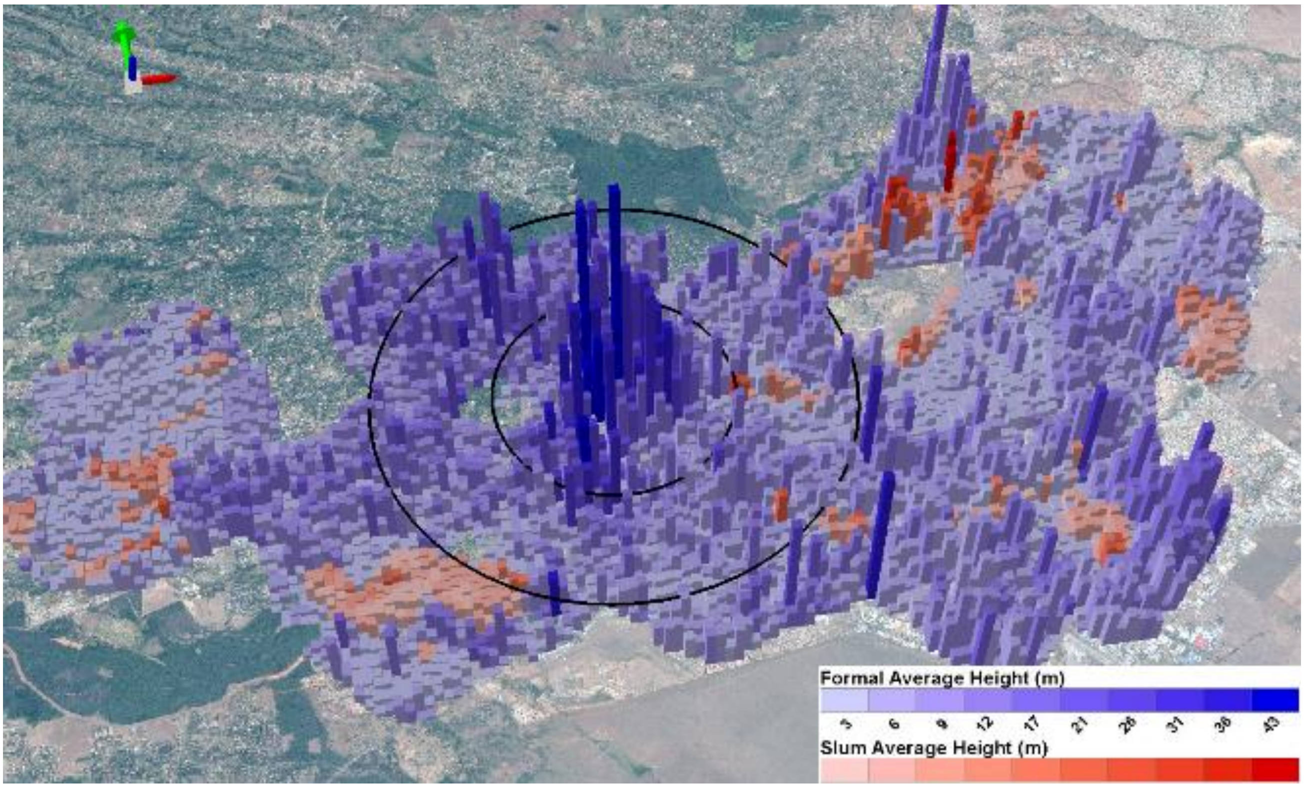
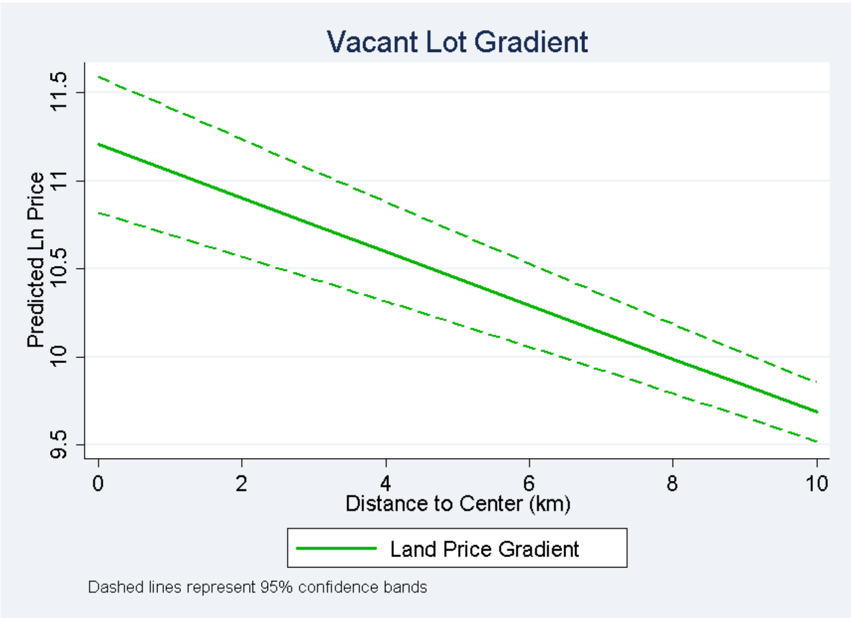


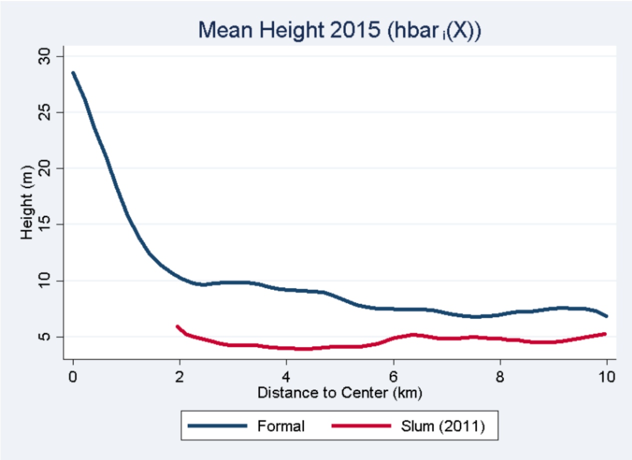
Figure 6. Land prices per square meter land



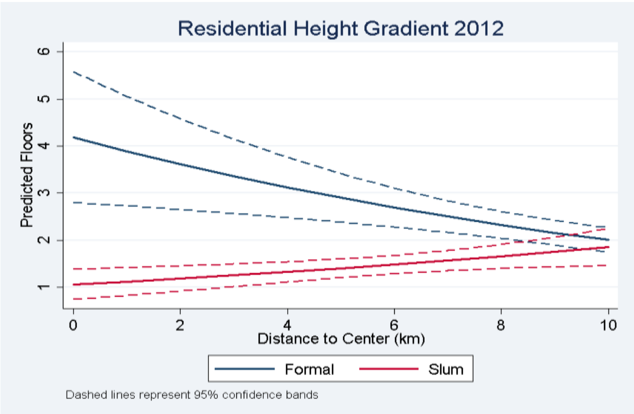
This is a regression relationship for price per square meter as a function of distance from the centre, controlling for whether the address was imprecise (usually a lot in an inferior area) and for lot size. Overall R^2 is 0.57.

Figure 7

a. Mean height in meters

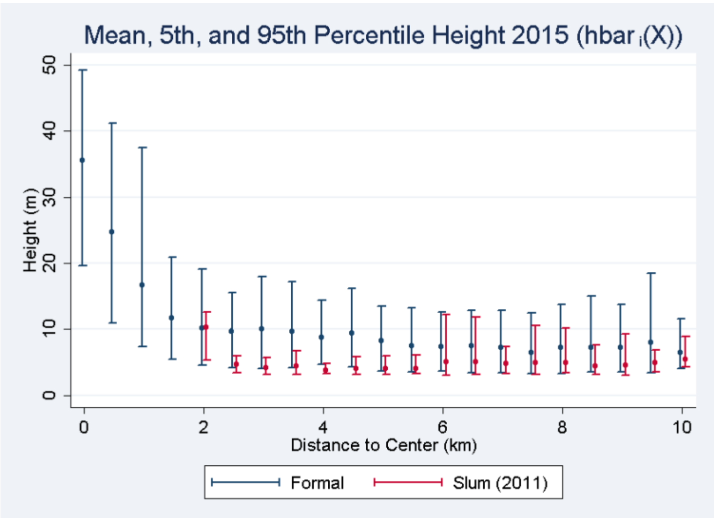


b. Residential height in floors from survey data



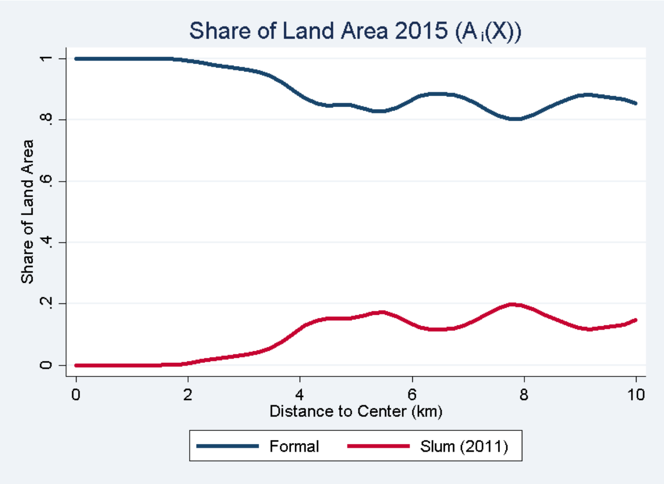
*Estimated from a Poisson regression of individual height floors on distance by type of use.

c. Variability in height



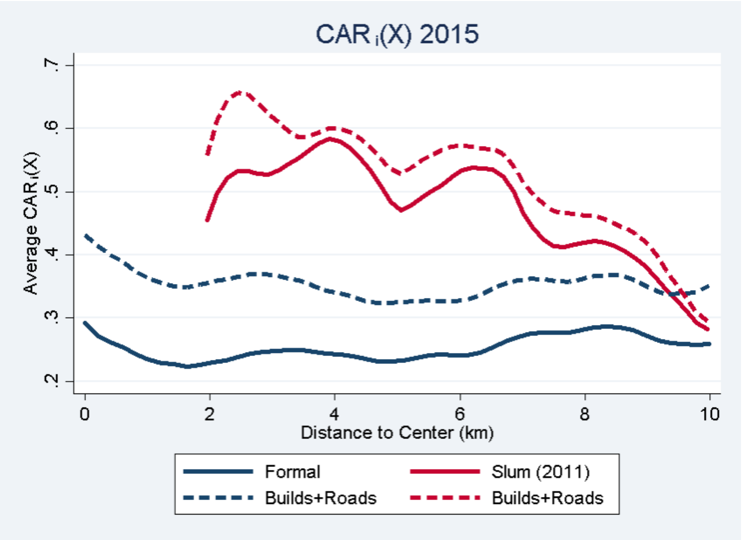
These are mean and percentiles based on pixel (3mx3m) heights.

Figure 8 Share of land in slums versus formal sector



The lines add up to 1, but we show both for ease of reading.

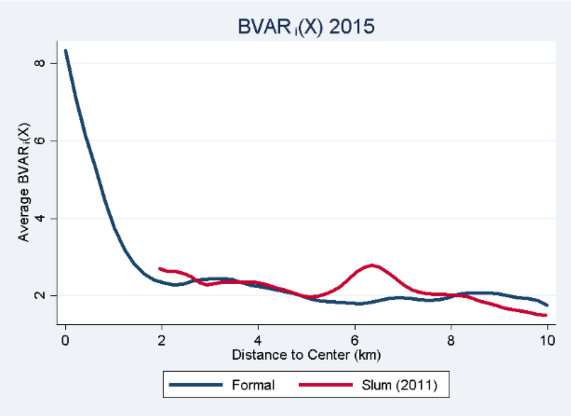
Figure 9. Cover to area ratio, without (building cover, CAR) and with roads



Solid lines are building cover to area. Dashed lines add road cover to building cover in the numerator. Note road cover in the formal sector far exceeds that in slums. Roads include any paved roads, which can accommodate at least two cars passing.

Figure 10. Built volume per unit area (BVAR)

a. Smoothed means



b. Heterogeneity

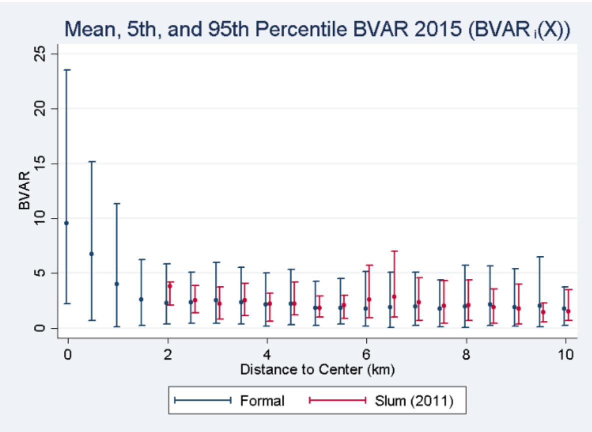
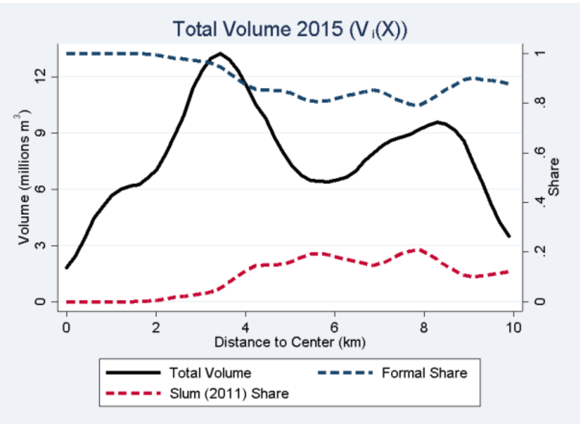


Figure 11. Total volume by distance and sector



The dashed lines add up to 1, but we show both for ease of reading.

Figure 12. Building higher in the formal sector

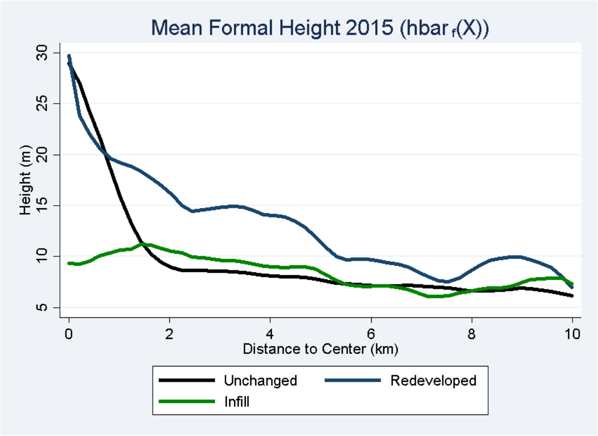
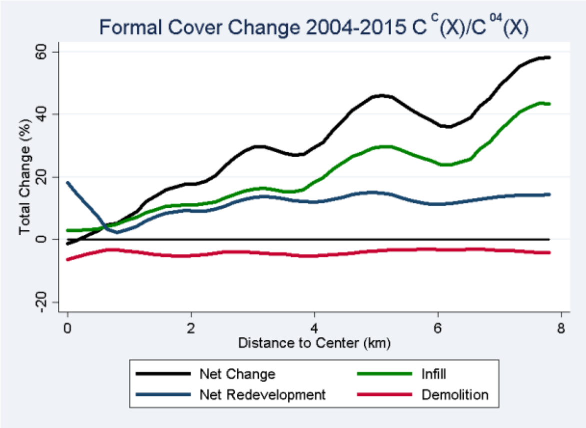
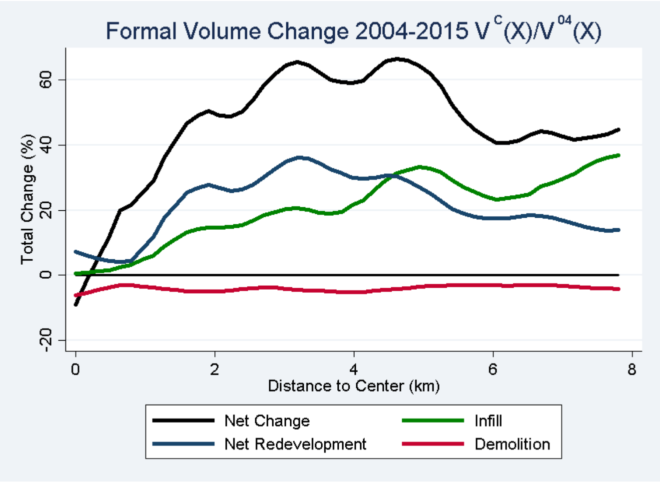


Figure 13a Changes in cover (and CAR) and volume (and BVAR)

a. Cover



b. Volume



This takes the components (and total) of net volume increase and divides by total initial volume at each radius.

Figure 14. Growth in total volume and by sector

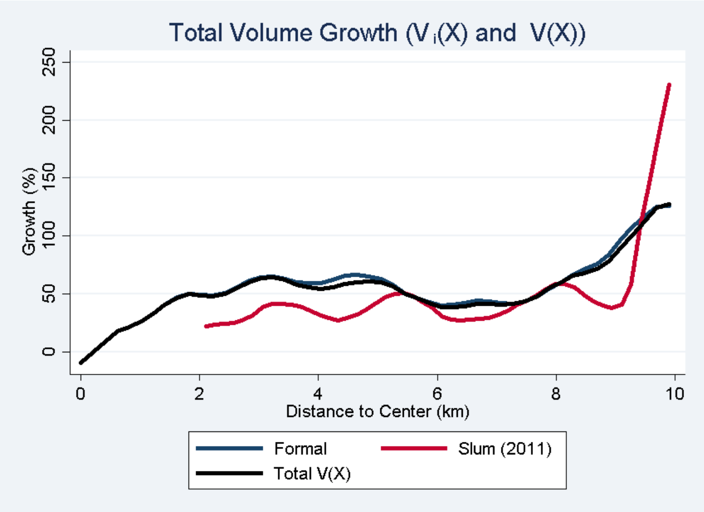


Figure 15 Height changes in 2004 Slums (Not showing 65+% at 3 meters)

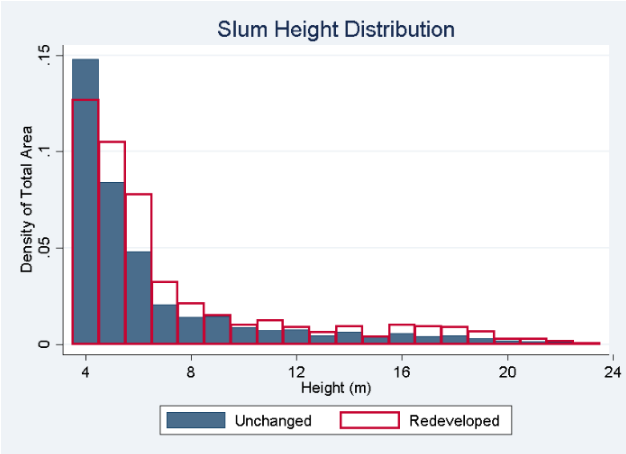
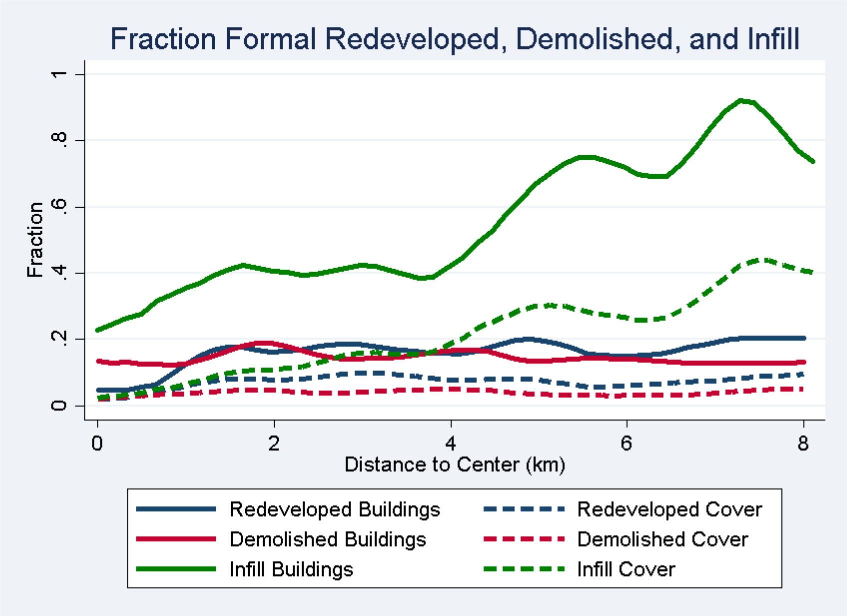


Figure 16 Churning: Changes in counts and cover due to infill, redevelopment and demolition as a fraction of 2003/4 counts and cover (in the formal sector)



In the formal sector, this takes counts of 2015 infill, 2004 demolished and 2004 buildings that are redeveloped in a radius and divides by original counts in that radius. The cover ratios are calculated on the same basis. E.g., for redeveloped, it is the 2004 footprint of 2004 buildings redeveloped divided by 2004 building cover.

Figure 17. Slum ownership

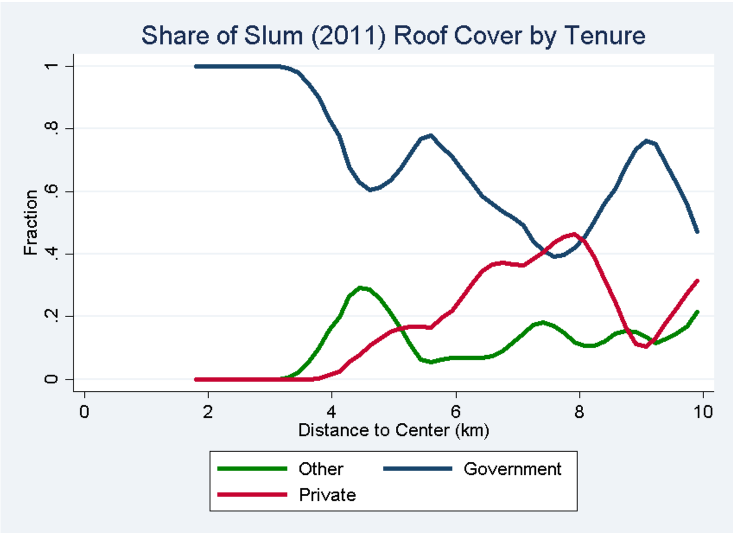
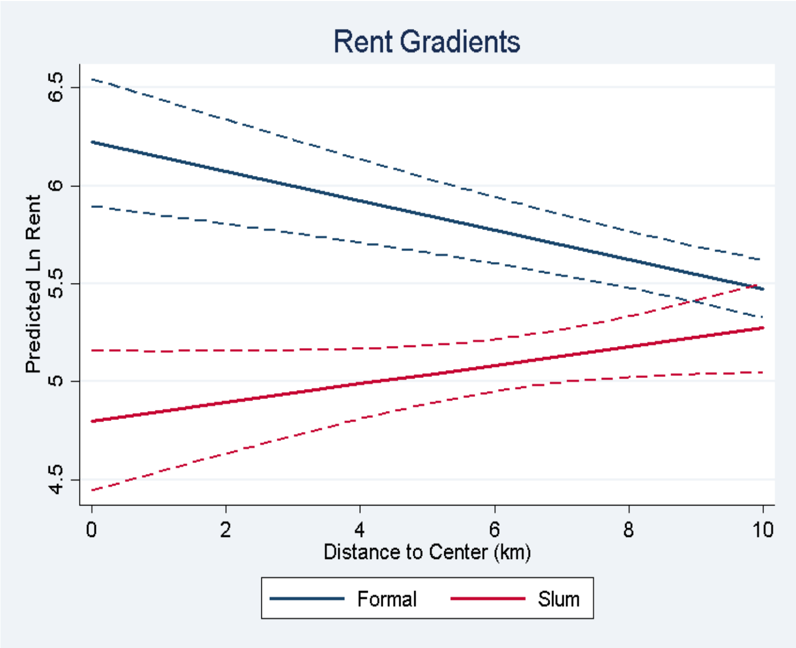


Figure 18. The formal sector-slum rent gap per sq m of standardized housing and in BVAR under redevelopment

a. Rent gap per sq meter of floor space



* This is a regression where the only covariates are slum, distance to centre and slum*distance to the centre.

b. Built volume to area ratio for redeveloped lands

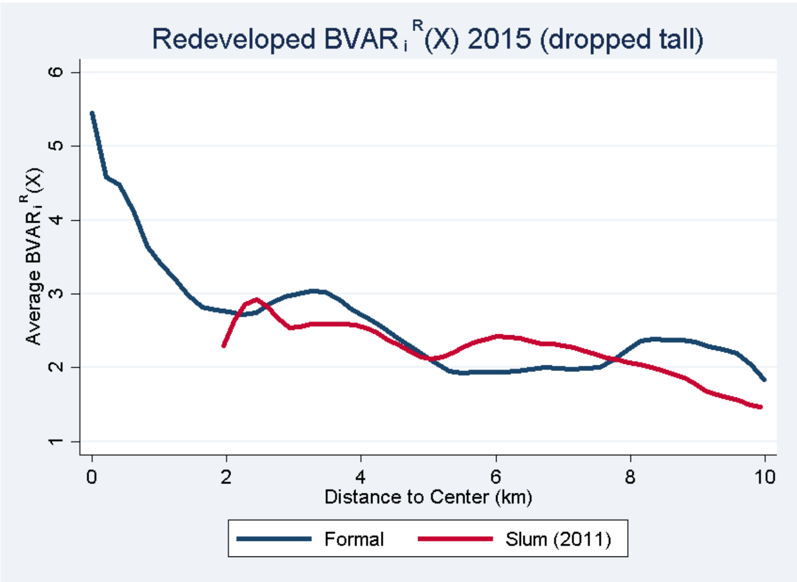


Table 1. IPE 2011 Slums: ignoring cells with hbar slum > 9m

	1km	2km	3km	4km	5km
Formal redevelopment BVAR (Intensity) 2015	2.86	2.73	3.07	2.422	1.93
Slum BVAR 2015	2.23	2.98	2.56	2.31	2.30
Slum Land Area 2015	2718	263430	1129311	2263428	1946034
Raw rents:					
Formal rent	450.55	418.10	387.99	360.04	334.11
Slum rent	130.49	136.81	143.44	150.39	157.67
Rent gains from conversion (Million USD)	**3.31**	**234.86**	**1132.71**	**1448.78**	**670.52**