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CDS Auctions [*]

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Abstract We analyze credit default swap settlement auctions theoretically and evaluate them empirically. In our theoretical analysis, we show that the current auction design may not result in the fair bond price and suggest modifications to the auction design to minimize mispricing. In our empirical study, we find support for our theoretical predictions. We show that an auction undervalues bonds by 10%, on average, on the day of the auction and link this undervaluation to the number of bonds that are exchanged during the auction. We also document a V-shaped pattern in underpricing during the days surrounding the auction: in the days leading up to the auction, the extent to which bonds are underpriced declines, while after the auction, the extent to which they are underpriced increases, with the smallest underpricing coming on the day of the auction.

JEL Classification Codes: G10, G13, D44

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Introduction

Credit Default Swaps (CDS) have been one of the most significant financial innovations in the last 20 years. They have become very popular among both investment and commercial banks, insurance companies, pension fund managers, and many other economic agents. As a result, the market has experienced enormous growth. According to the Bank of International Settlements (BIS), the notional amount of single-name CDS contracts grew from $5.1 trillion in December 2004 to $33.4 trillion in June 2008, and was still $18.4 trillion in June 2010 following a decline in the aftermath of the credit crisis.

The recent crisis put CDS in the spotlight, with policymakers now assigning a central role to CDS in many reforms. The success of these reforms depends on the efficient functioning of the CDS market and on a thorough understanding of how it operates. Recognizing this, a lot of research is dedicated to valuation of the contracts, econometric analysis of CDS premia, studying violations of the law of one price in the context of basis trades, impact of the search frictions, counterparty risk, private information and moral hazard problems associated with holding both bonds and CDS protection on the same entity.[1]

In this paper, we focus on another aspect of CDS. We study how the payoff on a CDS contract is determined upon a credit event. Our theoretical analysis of the unusual auction-based procedure reveals that this mechanism is vulnerable to mispricing relative to the fundamental value. The mispricing is attributable, in large part, to strategic bidding on the part of investors holding CDS. Empirically, we find that CDS auctions underprice the underlying securities by 10%, on average. Because this is an economically large magnitude, our findings may have implications for how CDS are valued, used and analyzed.

In a nutshell, a CDS is a contract that protects a buyer against the loss of a bond's principal in the case of a credit event (e.g., default, liquidation, debt restructuring, etc.). Initially, CDS were settled physically with the cheapest-to-deliver option. Under such settlement, the protection buyer has to deliver any bond of the reference

[1] This work includes, but is not limited to, Acharya and Johnson (2007), Arora, Gandhi, and Longstaff (2009), Bolton and Oehmke (2011), Duffie (1999), Duffie and Zhu (2011), Garleanu and Pedersen (2011), Longstaff, Mithal, and Neis (2005), Pan and Singleton (2008), and Parlour and Winton (2010).

entity to the protection seller in exchange for the bond's par value. As a result of the rapid development of the CDS market, the notional amount of outstanding CDS contracts came to exceed the notional amount of deliverable bonds by many times. This made physical settlement impractical and led the industry to develop a cash settlement mechanism. This mechanism is the object of our study.

While a myriad of derivatives are settled in cash, the settlement of CDS in cash is challenging for two reasons. First, the underlying bond market is opaque and illiquid, which makes establishing the benchmark bond price for cash settlement difficult. Second, the procedure for settlement of CDS provides market participants with an option to replicate an outcome of the physical settlement in which both the CDS and bond positions are closed simultaneously. Without this option, parties with positions in both CDS and the underlying bonds may be subject to recovery basis risk.[2]

The industry has developed a novel two-stage auction in response to these challenges. At the first stage of the auction, parties that wish to replicate the outcome of the physical settlement submit their requests for physical delivery via dealers. These requests for physical delivery are aggregated into the net open interest (NOI). Dealers also submit bid and offer prices with a commitment to transact in a predetermined minimal amount at the quoted prices. These quotations are used to construct the initial market midpoint price (IMM). The IMM is used to derive a limit on the final auction price that is imposed to avoid a potential price manipulation. The limit is referred to as the price cap. The NOI and the IMM are announced to all participants.

At the second stage, a uniform divisible good auction is implemented, in which the net open interest is cleared. Each participant can submit limit bids that are combined with the obligatory bids of dealers from the first stage. The price of a bid that clears the net open interest is declared to be the final auction price, which is then used to settle the CDS contracts in cash.

We analyze the auction outcomes from both theoretical and empirical perspectives.

[2] Imagine a party that hedges a long position in a bond by buying a CDS with the same notional amount. The final physically-settled position is known in advance: the protection buyer delivers a bond in exchange for the predetermined cash payment equal to the par value. However, the cash-settled position is uncertain before the auction: the protection buyer keeps the bond, pays the uncertain auction-determined bond value to the protection seller, and receives par value in exchange. The difference between the market value of the bond held by the protection buyer and the auction-determined value is the risky recovery basis.

Our analysis has to incorporate two unique features of the auction. The first one is the aforementioned two-stage process. The second one is that participants can have prior positions in the derivative contracts written on the asset being auctioned.

To study price formation, we formalize the auction using an idealized setup in which all auction participants are risk-neutral and have identical expected valuations of the bond, v. This case is not only tractable, but also provides a useful benchmark against which to test whether the auction leads to the fair-value price. In this case, the auction is supposed to result in a final auction price equal to v. We show, however, that the current auction design can result in the final price being either above or below v and propose several ways to minimize such deviations.

First, we analyze the second stage. Consider the case of positive NOI, that is, a second-stage auction in which the agents buy bonds. We show that agents with short CDS positions can bid above the true value of the bond, because it increases their payoff during settlement. This equilibrium behavior is more likely to occur when the net open interest is small compared to the total short CDS positions of participating agents. When this is the case, bidding above the fair value and realizing a loss from buying NOI units of bonds can be compensated by a reduction in the net pay on the existing CDS contracts. We also show that when the net open interest is large, there can exist "mispricing" equilibria in which the auction results in a price below v.

Second, we show that the presence of the cap from the first stage can result in either under- or overpricing in auction outcomes. A small absolute value of NOI makes it easy for agents to manipulate the action price to benefit from their CDS holdings. The cap restricts this behavior, which justifies including it in the auction design. However, the cap can also make fair-price equilibria infeasible by giving incentives to dealers to submit quotations below v in order to profit in the second stage.

In our analysis of empirical data, we find support for our theoretical predictions. We use TRACE bond data to construct the reference bond price. Using the reference bond price from the day before the auction, we show that that when the net open interest is to sell (which is a typical situation), the auction undervalues bonds by 10%, on average, and the degree of undervaluation increases with the NOI. We also document a V-shaped pattern in underpricing: during eight days before the auction, the extent to which bonds are underpriced declines from 35%, on average, to 10%,

on average, while during 12 days after the auction, the extent to which they are underpriced increases from 10%, on average to 35%, on average. Thus, we show that our conclusions about the effect of the auction on price are robust to our choice of the benchmark bond value. Finally, we find evidence that suggests that the price cap is binding primarily because total short CDS positions of participating agents is much larger than NOI.

Our results documenting underpricing during the auction prompt us to consider ways in which it might be possible to improve the outcomes. In a standard setting, in which agents have no prior positions in the derivative contracts written on the asset being auctioned, Kremer and Nyborg (2004) suggest a likely source of underpricing equilibria. They show that a simple change of the allocation rule from the pro-rata on the margin to the pro-rata destroys all underpricing equilibria. We show that the same change of the allocation rule would be beneficial in our setting as well. In addition, we suggest that imposing an auction price cap conditional on the outcomes of the first stage could further reduce mispricing in equilibrium outcomes.

To the best of our knowledge, there are three papers that examine CDS auctions. Two of them simply analyse empirical data from CDS auctions. Helwege, Maurer, Sarkar, and Wang (2009) evaluate an early sample of 10 auctions, of which only four used the current auction format, and find no evidence of mispricing. Similarly to us, Coudert and Gex (2010) document a large gap between a bond price on the auction date and the final auction price by studying a larger sample of auctions and using Bloomberg data for reference bond prices. However, they do not link the gap to the net open interest, nor do they provide any theoretical explanations for their findings. Finally, in an independent and contemporaneous study, Du and Zhu (2011) examine what types of outcome are possible in CDS auctions. They restrict their attention only to differentiable strategies, conclude that only "overpricing" equilibria can exist, and suggest changing the current auction format to a double auction. Unlike them, and similar to Wilson (1979) and Back and Zender (1993), we show that there can be a substantial underpricing in uniform divisible good auctions when participating agents have a small combined CDS position relative to the supply of bonds. We also suggest a number of modifications that are designed to deal with both over- and underpricing.

The remainder of the paper is organized as follows. Section 1 describes the CDS

auction methodology that is currently used. Section 2 describes the auction model. Section 3 provides the main theoretical analysis. Section 4 relates the predictions of our theoretical model to empirical data from CDS auctions. Section 5 discusses modifications that have the potential to improve the efficiency of the auction. Section 6 concludes. The appendix contains proofs that are are not provided in the main text.

1 The Auction Format

This discussion is based on a reading of the auction protocols, which are available from the ISDA website. The Dura auction, conducted on Novemeber 28, 2006, was the first auction that allowed single-name CDS to be settled in cash. All previous auctions were designed for cash-settling credit indexes and used different rules. The auction design used for Dura's bankruptcy and all the subsequent credit events consists of two stages.

In the first stage, participants in the auction submit their requests for physical settlement. Each request for physical settlement is an order to buy or sell bonds at par and must be, to the best of the relevant party's knowledge, in the same direction as, and not in excess of, its market position. This element of the design allows the participants to replicate the traditional physical settlement of the contracts. For example, if a party is long by one unit of protection (directionally equivalent to selling a bond) and submits a request to sell one bond at par, the resulting cash flow is identical to that of the physical settlement.

In addition, a designated group of agents (dealers) makes a two-way market in the defaulted assets by submitting bids and offers with a predefined maximum spread and a predefined quotation size that is associated with it. The spread and quotation sizes are subject to specification prior to each auction and may vary for each auction depending on the liquidity of the defaulted assets.[3]

The inputs of the first stage are used to calculate a net open interest (NOI) and an 'initial market midpoint' (IMM), which are carried into the second part of the auction. The NOI is computed as a difference of the buy and sell requests for physical settlement. The IMM is set by discarding crossing/touching bids and

[3] The most common value of the spread is 2% of par. Quotation sizes range from \$1 to \$5 million; \$1 million is the most common amount.

offers, and taking the 'best half' of the bids and offers and calculating the average. The best half would be, respectively, the highest bids, and the lowest offers. A dealer makes a payment (an adjustment amount) to ISDA if her quotation is crossed and is on the wrong side of the IMM, that is, if a bid is higher than the IMM, and the NOI is to sell or if an offer is lower than the IMM, and the NOI is to buy. The adjustment amount is a product of the quotation amount and the difference between the quotation and IMM.

As an example, consider the Nortel Limited auction that took place on February 10, 2009. Table 1 lists the market quotes that were submitted. Next, all the bids are sorted in the descending offer and all the offers are sorted in the ascending offer. Then, the highest bid is matched with the lowest offer; the second highest bid is matched with the second lowest offer; and so on. Figure 1 displays the quotes from Table 1 that are organized this way. For example, we that the Citibank bid of 10.5 and the Barclays offer of 6.0 create a tradeable market.

IMM is computed based on the non-tradebale quotes. In our example, there are nine pairs of such quotes. First, the "best half", i.e. the first five pairs, of the non-tradebale quotes is selected. Second, IMM is computed as an average of both bid and offer quotes in the best half, rounded to the nearest one-eighth of one percentage point. In our example, the relevant bids are: three times 7.0 and two times 6.5; the relevant offers are: two times 8.0, two times 8.5, and 9. The average is 7.6 and the rounded average is 7.625.

Given the established IMM and the direction of open interest, the dealers whose quotes resulted in tradeable markets pay an adjustment amount to ISDA. In the case of Nortel, the open interest was to sell. Thus, the dealers whose bids crossed the markets has to pay an amount equal to (Bid-IMM) times the quotation amount, which was \$2 MM in the Nortel case. Thus, Citigroup had to pay $(10.5-7.625)/100 \times \$2MM = \$57500$ and Banc of America had to pay $(9.5-7.625)/100 \times \$2MM = \$37500$.

Finally, the direction of the open interest determines the cap on the final price that is determined in the second part of the auction. Because the open interest is to sell, the final price cannot exceed the IMM by 1.0. Thus, the cap price is 8.625 in the Nortel case. It is depicted in Figure 1.

After the IMM, the NOI, and the adjustment amounts are published, the second

stage of the auction begins. If the NOI is zero, the final price is set at the IMM. If the NOI is non-zero, the dealers submit the corresponding limit orders on behalf of their customers (even those without CDS positions) and for their own account to offset the NOI (if the $NOI > 0$, agents submit 'buy' limit orders and vice versa). In practice, it is unlikely that all agents who participate in the first stage, participate in the second stage as well. The participants in the CDS market are diverse in terms of their investment objectives and have various institutional constraints. For example, many mutual and pension funds may not be allowed to hold any of the defaulted bonds.

Upon submission of limit orders, the auction administrators match the open interest against all market bids from the first stage of the auction and limit bids from the second stage of the auction (if the NOI is to buy). They start with the highest bid and proceed to the remaining highest bid until either the entire net open interest has been matched, or all bids have been matched to the net open interest. In the former case, the final price is equal to the lowest bid that corresponds to the last matched limit order. However, if this lowest bid exceeds IMM by more than the cap amount (typically half of the bid-offer spread), the final price is IMM plus the cap amount. In the latter case, the final price will be zero and all bids will be filled on a pro rata basis. If the NOI is to sell, the entire procedure is similar. If there are not enough offers to match the entire net open interest to buy, the final price is set to par.

2 The Auction Model

The goal of this section is to formalize the auction process described in Section 1. There are two dates: $t = 0$ and $t = 1$. There is a set $\mathcal{N}$ of strategic players. The total number of agents is $|\mathcal{N}| = N$. A set of dealers $\mathcal{N}_d$ constitutes a subset of all players, $\mathcal{N}_d \subseteq \mathcal{N}$. Each agent $i \in \mathcal{N}$ is endowed with $n_i \in \mathbb{R}$ units of CDS contracts and $b_i \in \mathbb{R}$ units of bonds. Agents with positive (negative) n_i are called protection buyers (sellers). Because a CDS is a derivative contract, it is in zero net supply, $\sum_i n_i = 0$. One unit of bond pays $\tilde{v} \in [0, 100]$ at time $t = 1$. The auction takes place at time $t = 0$. It consists of two stages.

2.1 First Stage

At this stage, an agent i can submit a request to sell y_i (buy if $y_i < 0$) units of bonds at par (100). A protection buyer, $n_i > 0$, is only allowed to submit a request to sell $y_i \in [0, n_i]$ units of bonds. A protection seller, $n_i < 0$ can only submit a request to buy $y_i \in [n_i, 0]$ units of bonds. Given their requests, the NOI is determined as follows:

$$NOI = \sum_{i=1}^{N} y_i. \tag{1}$$

In addition, after observing the NOI, all dealers from the subset $\mathcal{N}_d$ are asked to quote a price π_i. Given π_i, a dealer i should be ready to sell and buy L units of bonds at bid and offer prices $\pi_i + s$ and $\pi_i - s$, $s > 0$. Given that L is small in practice, we consider the case of $L = 0$ in the sequel.[4] The first stage results in the auction initial market midpoint (IMM). The quotations of agents whose bids and offers cross are discarded. IMM is equal to the average of the remaining mid-quotations and denoted by p^M.

2.2 Second Stage

At this stage, there is a uniform divisible good auction. If $NOI = 0$ then $p^A = p^M$. If $NOI > 0$, the auction is to buy NOI units of bonds. In this case, each agent i can submit a left-continuous non-increasing demand schedule $x_i(p): \ [0, p^M + s] \to \mathbb{R}_+ \cup 0$. Let $\mathcal{N}_2, \mathcal{N}_2 \subseteq \mathcal{N}$ denote a subset of all agents who participate at the second stage. Let the total demand be $X(p) = \sum_{i \in \mathcal{N}_2} x_i(p)$. The final auction price p^A is the highest price, at which all NOI can be sold:

$$p^A = \max\{p | X(p) \geq NOI\}.$$

If $X(0) \leq NOI$, $p^A = 0$. Given p^A, the allocations $q_i(p^A)$ are given according to the pro-rata on the margin rule

$$q_i(p^A) = x_i^+(p^A) + \frac{x_i(p^A) - x_i^+(p^A)}{X(p^A) - X^+(p^A)}, \tag{2}$$

[4] The analysis of the case of nonzero L is available upon request.

where $x_i^+(p^A) = \lim_{p \downarrow p^A} x_i(p)$ and $X^+(p) = \lim_{p \downarrow p^A} X(p)$ are the individual and total demands above the auction clearing price.

If $NOI < 0$, the auction is to sell $|NOI|$ units of bonds. Each agent i can then submit a right-continuous non-decreasing supply schedule $x_i(p) : \ [100, p^M - s] \to \mathbb{R}_- \cup 0$. Again, the set of all agents participating at the second stage is denoted by $\mathcal{N}_2$, $\mathcal{N}_2 \subseteq \mathcal{N}$. The total supply is $X(p) = \sum_{i \in \mathcal{N}_2} x_i(p)$. The final auction price p^A is the lowest price, at which all NOI can be bought:

$$p^A = \min\{p | X(p) \leq NOI\}.$$

If $X(100) \geq NOI$, $p^A = 100$. Given p^A, the allocations $q_i(p^A)$ are given by

$$q_i(p^A) = x_i^-(p^A) + \frac{x_i(p^A) - x_i^-(p^A)}{X(p^A) - X^-(p^A)},$$

where $x_i^-(p^A) = \lim_{p \uparrow p^A} x_i(p)$ and $X^-(p) = \lim_{p \uparrow p^A} X(p)$ are the individual and total supplies below the auction clearing price.

2.3 Preferences

There are three groups of agents who participate in the auction. First, there are agents who participate in the second stage. We consider a setup in which we assume that these agents are risk-neutral and have identical expected valuations of the bond payoff equal to v. The agents objective is to maximize their wealth at date 1. This case is not only tractable, but also provides a useful benchmark against which to see whether the auction leads to the fair-value price. In this case, the auction is supposed to result in a final auction price equal to v.

Second, there are dealers who participate in both stages. We also assume that they are risk-neutral, have identical expected valuations of the bond payoff equal to v, and their objective is to maximize their wealth at date 1. Dealers are different from the rest of auction participants in that they submit quotes, π_i, in the first stage that become public after the auction. Thus, because of reputation concerns, dealers may be reluctant to quote prices very different from v unless this action results in a large gain. A simple way to model this is to assume that dealers' utility has an extra term $-\gamma(\pi - v)^2$, $\gamma \geq 0$.

Finally, there are agents who submit physical settlement requests, y_i, in the first stage (some of them, participate in the second stage as well). In practice, the choice of y_i is dictated by considerations outside of pure profit maximization associated with CDS settlement. For example, some agents may want to try getting a hold of as many bonds as possible to influence a company's management is bankruptcy. In contrast, other agents, such as pension funds or insurance companies, may not be allowed to hold bonds of defaulted companies. As another example, the logic of "arbitrage" trading strategies such as the CDS-bond basis trade necessitates physical settlement as a way to offset a bond position exactly. Therefore, in our subsequent analysis we do not model y_i explicitly and take them as given. Modeling physical settlement requests could be an interesting avenue for future research.

3 Analysis

We now provide the analysis of the auction described in the previous section. The auction can be solved using backward induction. First, we solve for the equilibrium outcome in the second stage of the auction for a given IMM and NOI. Second, we find optimal dealer quotations π_i in the first stage, given NOI and the equilibrium outcomes of the second stage.

3.1 Second Stage

At this stage, there is a uniform divisible good auction, whose goal is to clear the net open interest generated in the first stage. A novel feature of our analysis is that we study auctions where participants have prior positions in the derivative contracts written on the asset being auctioned. We show that equilibrium outcomes in this case can be very different from those realized in "standard" auctions, that is, auctions in which $n_i = 0$ for all i.

We first consider the case in which all CDS positions are common knowledge. Later, we relax this assumption. Each agent i maximizes her profit at time 1 given

by

$$\begin{aligned}\Pi_i \quad = \quad & \underbrace{(v - p^A)q_i(p^A)}_{\text{auction-allocated bonds}} + \underbrace{(n_i - y_i) \times (100 - p^A)}_{\text{remaining CDS}} \\ + \quad & \underbrace{100y_i}_{\text{physical settlement}} + \underbrace{v(b_i - y_i)}_{\text{remaining bonds}} . \end{aligned} \tag{3}$$

Therefore, because each agent i takes NOI, a set of physical requests y_i, $i \in \mathcal{N}$, and a demand of other agents $x_{-i}(p)$ as given, her demand schedule $x_i(p)$ solves the following optimization problem:

$$\max_{x_i(p)} \; (v - p(x_i(p), x_{-i}(p)))\, q_i(x_i(p), x_{-i}(p)) + (n_i - y_i) \times (100 - p(x_i(p), x_{-i}(p)))\,. \tag{4}$$

The first term of this expression represents the payoff realized from participating in the auction, while the second term accounts for the payoff from the remaining CDS positions, $n_i - y_i$, which are settled in cash on the basis of the auction results.

To develop intuition about the forthcoming theoretical results, consider bidding incentives of the auction participants. The objective function (4) implies that, holding payoff from the auction constant, an agent who has a short (long) remaining CDS position is interested in the final price being as high (low) as possible. However, agents with opposing CDS positions do not have the same capacity to affect the auction price. The auction design restricts participants to submit one-sided limit orders, depending on the sign of the NOI. If the $NOI > 0$, the allowed limit orders are to buy, and, therefore, agents with short CDS positions are capable of bidding the price up. In contrast, the most agents with long CDS positions can do to bid the price down is not to bid at all. The situation is reversed when the $NOI < 0$.

Continuing with the case of the $NOI > 0$, consider an example of only one agent with a short CDS position. It is clear that she would be interested in bidding the price as high as possible if the NOI is smaller than the notional amount of her CDS. This is because the cost of purchasing bonds at a high auction price is offset by the benefit of cash-settling CDS at the same high price. In contrast, if the NOI is larger than the notional amount of her CDS position, she would not bid the price above the fair bond value v. This is because the cost of purchasing bonds at a price above v is not offset by the benefit of cash-settling CDS. We show in the sequel that this intuition can

be generalized to multiple agents as a long as we consider the size of their aggregate CDS positions relative to the NOI. For this reason we introduce Property 1 below.

Property 1 *When $NOI > 0$*

$$\sum_{i \in \mathcal{N}_2 : n_i < 0} |n_i - y_i| \geq NOI.$$

When $NOI < 0$

$$\sum_{i \in \mathcal{N}_2 : n_i > 0} n_i - y_i \geq |NOI|.$$

When Property 1 holds, the aggregate cash-settled CDS position of agents is larger than the absolute value of the NOI. We show next that this is the case when all agents participate in the second stage.

Lemma 1 *Suppose all agents participate in the second stage, $\mathcal{N}_2 = \mathcal{N}$. Then Property 1 holds.*

Proof. We prove the case of $NOI > 0$. The case of $NOI < 0$ is similar.

$$\sum_{i:n_i<0} n_i - y_i + NOI = \sum_{i:n_i<0} n_i - y_i + \sum_i y_i = \sum_{i:n_i<0} n_i + \sum_{i:n_i>0} y_i \leq \sum_{i:n_i<0} n_i + \sum_{i:n_i>0} n_i = 0.$$

QED.

The next two propositions show that whether Property 1 holds or not is critical for equilibrium outcomes. In particular, we demonstrate that when Property 1 holds the equilibrium auction price, p^A, is no less (no greater) than the bond payoff, v, when $NOI > 0$ ($NOI < 0$). This result is in contrast to that realized in "standard" auctions.[5] If Property 1 does not hold, outcomes are similar to those of the "standard" auctions.

Proposition 1 *Suppose that Property 1 holds. If $NOI > 0$ and $p^M + s > v$, then in any equilibrium, the final auction price $p^A \in [v, p^M + s]$. Furthermore, there always exists an equilibrium in which $p^A = p^M + s$. If $\max_{i:n_i<0} |n_i - y_i| \leq NOI$ then there also exists an equilibrium in which $p^A = v$. If $p^M + s < v$ then $p^A = p^M + s$. Likewise,*

[5] Contributions include, but are not limited to, Wilson, 1979; Back and Zender, 1993; Kremer and Nyborg, 2004.

if $NOI < 0$ and $p^M - s < v$, then in any equilibrium, $p^A \in [p^M - s, v]$. Furthermore, there always exists an equilibrium in which $p^A = p^M - s$. If $\max_{i:n_i>0}(n_i - y_i) \leq |NOI|$ then there also exists an equilibrium in which $p^A = v$. If $p^M - s > v$ then $p^A = p^M - s$.

Proof. See Appendix A.1.

We outline the intuition for the case of $NOI > 0$. If Property 1 holds, there is a subset of agents for whom a joint loss from acquiring the $|NOI|$ number of bonds at a price above v is dominated by a joint gain from having to pay less on a larger number of short CDS contracts that remain after the physical settlement. As a result, these agents bid aggressively and can push the auction price above v unless it is constrained by the IMM. In the latter case, $p^A = p^M + s$.

Proposition 2 *Suppose Property 1 does not hold. If $NOI > 0$, only equilibria with $p^A \leq \min\{p^M + s, v\}$ exist. Moreover, there exist infinitely many combinations of $n_i - y_i$ such that any $p \in [0, \min\{p^M + s, v\}]$ can be supported as a second-stage equilibrium. Likewise, if $NOI < 0$, only equilibria with $p^A \geq \max\{p^M - s, v\}$ exist. Furthermore, there exist infinitely many combinations of $n_i - y_i$ such that any $p \in [\max\{p^M - s, v\}, 100]$ can be supported as a second-stage equilibrium.*

Proof. See Appendix A.2.

The intuition for the proof is as follows. Suppose that $NOI > 0$ and $p^A > v$ is an equilibrium. First, note that only agents who hold a short position on CDS contracts are willing to bid above v. Because Property 1 does not hold, at least one of these agents ends up acquiring more bonds than her remaining short CDS contracts and therefore is better off reducing her demand for bonds. Therefore, just as in a "standard" auction without CDS positions, only equilibria with $p^A \leq v$ can exist. We construct such equilibria explicitly in the Appendix.

3.2 First stage

The role of the first stage is to determine the NOI and the IMM, which serves as a reference price. In our setting without uncertainty, the IMM does not carry any information. Nevertheless, it can still play an important role, because it provides a cap on the final price. We now show that the presence of the cap can result in either lower or larger mispricing in auction outcomes.

To be specific, we consider the case of $NOI > 0$. Proposition 1 shows that the final price could be anywhere between v and 100 if there were no cap. The cap restricts the final price to be no higher than $p^M + s$. Therefore, in the presence of the cap, the efficiency of the auction outcome depends on the bidding behavior of dealers in the first stage. If the quotations of all dealers coincide with the actual value, $\pi_i = v$, $p^M = v$. As a result, the final auction price does not deviate from v by more than s, which justifies the presence of the cap in the auction design. However, as we show in Lemma 2 below, the same presence of the cap gives incentives to dealers to submit quotations below v in order to profit in the second stage.

We consider a simple case, in which dealers have no reputation concerns ($\gamma = 0$ in Section 2.3) and no dealer has any CDS exposure, that is, $n_i = 0$ for all $i \in \mathcal{N}_d$.[6] Let $p^A(NOI)$ and $q(NOI)$ denote the auction final price and dealers' equilibrium allocation of bonds as a function of NOI (we restrict our attention to equilibria in which dealers play symmetric strategies). In the first stage, a dealer i then solves

$$\max_{\pi_i} \; \left(v - p^A(p^M(\pi_i), NOI)\right) \times q(p^M(\pi_i), NOI). \tag{5}$$

Lemma 2 *If $NOI > 0$, $\pi_i = 0$, $i \in \mathcal{N}_d$ is a first-stage equilibrium. Similarly, if $NOI < 0$, $\pi_i = 100$, $i \in \mathcal{N}_d$ is a first-stage equilibrium.*

Proof. As usual, we consider the case of $NOI > 0$. Because 0 is the lowest possible quotation, if a dealer deviates from it the only result can be a higher second-stage price, which will result in a lower profit for the dealer unless it leads to a larger bond allocation $q(p^M(\pi_i), NOI)$. The later, however, is independent from the dealer's quote because the final price is capped. Thus, no dealer will want to deviate. QED

Lemma 2 implies that the cap can lead to auction outcomes in which the final price is far from the true value. In general, the larger is the NOI, the larger is the profit that dealers get from playing this equilibrium strategy.

[6] This case is a simplification but is arguably realistic as dealers try to maintain zero CDS positions in their market-making capacity. The analysis of the case of nonzero dealers' CDS positions and $\gamma \neq 0$ is available upon request.

3.3 Private CDS positions

So far, we have restricted our attention to the simplest case in which agents' CDS positions are common knowledge. We now discuss how our results change if we relax this assumption. In practice, agents have private information about their CDS positions. To model this situation, we consider a variation of our base model. Specifically, in this section, we assume that CDS positions n_i are observed privately and are drawn from a joint distribution $F : \Omega \to \mathbb{R}^N$ that is common knowledge across agents. Each agent i knows only her own CDS position n_i. The next two propositions show that the results of Propositions 1 and 2 still generally hold. We consider the case of $NOI > 0$.

Proposition 3 *Suppose that F is such that NOI, n_i, and y_i satisfy Property 1 with probability 1. If $p^M + s \geq v$, in any equilibrium $p^A \geq v$.*

Proof. See Appendix A.3.

The intuition is similar to that in Proposition 1. If $NOI > 0$ and Property 1 holds almost surely, then for any realization of $\{n_i\}, i \in \mathcal{N}$, a subset of agents who hold a large number of short CDS contracts know that they receive sufficient support in pushing the price above v. As a result, they submit aggressive bidding schedules and the final price is above v in any state of the world.

Proposition 4 *Suppose that F is such that NOI, n_i, and y_i violates Property 1 with probability 1. In addition, suppose that there exists $\nu > 0$ such that $n_i - y_i > \nu$ whenever $n_i - y_i > 0$.*[7] *Then there exists an equilibrium in which $p^A < v$ with probability 1.*

Proof. See Appendix A.4.

Proposition 4 shows that when CDS positions, n_i, are private information, there exist a broad set of conditions that lead to underpricing (overpricing if $NOI < 0$) in any state of the world.

[7] This is a technical condition that allows us to obtain explicit closed-form solutions for equilibrium strategies. It ensures that the number of agents who submit non-zero demand schedules is constant across different realizations of uncertainty.

4 Empirical Evidence

We now describe our dataset and present the empirical analysis. First, we determine how the outcomes of the first stage affect the final outcomes. Next, we evaluate how the second stage of the auction affects the final price.

4.1 Data

Our data come from two primary sources. The details of the auction settlement are publicly available from the Creditfixings website (www.creditfixings.com). As of December 2010, there have been 86 CDS and Loan CDS auctions, which settle contracts on both US and international legal entities. To study the relationship between auction outcomes and bond values, we merge these data with the bond price data from the TRACE database. TRACE reports corporate bond trades for US companies only.Thus, our merged dataset contains 23 auctions.

Table 2 summarizes the results of the auctions for these firms. It reports a settlement date, a type of credit event, and auction outcomes. Most of the auctions took place in 2009 and were triggered by the Chapter 11 event. Of the 23 auctions, only two (Six Flags and General Motors) have the net open interest to but ($NOI < 0$). The full universe of CDS auctions contains 61 auctions that have the net open interest to sell, 19 auctions that have the net open interest to buy, and 6 auctions that have zero net open interest.

Table 3 provides summary statistics of deliverable bonds for each auction for which we have the bond data. Deliverable bonds are reported in auction protocols, which are available from the Creditfixings website. The notional amount of deliverable bonds outstanding exceeds \$10B for four companies (Lehman Brothers, Charter Communications, General Motors, and CIT Group). The table also reports the ratio of the net open interest to the notional amount of deliverable bonds (NOI/NAO). It shows how many bonds change hands during the auction as a percentage of the total amount of bonds. There is strong heterogeneity in NOI/NAO across different auctions. The absolute value ranges from 0.38% to 56.81%. In practice, NOI never exceeds NAO.

Following Bessembinder, Kahle, Maxwell, and Xuet (2009), we construct daily bond prices by weighing the price corresponding to each trade against the trade

size reported in TRACE. Bessembinder, Kahle, Maxwell, and Xuet (2009) advocate eliminating all trades under $100,000 because they are likely to be noninstitutional. The larger trades have lower execution costs; hence, they should reflect the underlying bond value with greater precision. For each company, we build a time-series of bond prices in the auction event window of -30 to +30 trading days. Because all credit events occurred within a calendar month of the CDS auction, our choice of the event window ensures that our sample contains all the data for the post-credit-event prices. The last column of Table 3, reports a weighted average bond price on the day before the auction.

One of the most important tests of the current auction format is whether the settlement price p^A equals the bond value v. Unfortunately, the true bond value is not observed. In light of this, we conduct our empirical analysis assuming that the weighted-average market price of bonds a day before an auction, p_{-1}, is a good proxy for the bond value v. Admittedly, this measure is not a perfect substitute for the true value of the bond, because many bond names are not very liquid. In addition, expectations as to the outcomes of CDS auctions may affect bond prices. Because of these reservations, we check whether our conclusions are robust to this assumption in a subsequent section.

4.2 The Impact of the First Stage

The theoretical results of section 3 imply that the first and the second stages of the auction are not independent. The first stage yields the mid-point price, p^M, which determines a cap on the final settlement price. Our model shows that when the final price, p^A, is capped, it can be both above and below the true value of the bond, v.

We focus on the case of $NOI > 0$. The first case can be realized only if the aggregate net CDS position of the agents who participate in the second stage is larger than the net open interest (Propositions 1 and 2). The second case is realized when the dealers set p^M so that $p^M + s$ is below v. Doing so prevents the agents from competing in prices above the cap at the second stage.

Our model suggests a way of differentiating between the two cases when the price cap is reached. In the first case, only agents who sold protection have an incentive to bid above the true value of the bond to minimize the amount paid to a CDS

counterparty. Simultaneously, they would like to minimize the amount of bonds acquired at the auction if the price is above v, for a given final auction price. Thus, they would never bid more than NOI at prices above v. In the second case, submitting a large demand at a cap price leads to a greater profit. Thus, in the presence of competition and sharing rules, agents have an incentive to bid substantially above NOI.

Of the 86 credit-event auctions, the final price is capped in 19.[8] Figure 2 shows the companies and the individual bids at the cap price . The bids are represented by different colors. The bid sizes are scaled by NOI to streamline the interpretation. For example, in total, there are seven bids at the cap price in the case of General Growth Properties. Six of them are equal to NOI and the seventh one is one-fourth of NOI.

We can see that in all but two auctions (Kaupthing Bank and Glitnir), the bids at the price cap do not exceed NOI. These results suggest that the cap is reached primarily because of the large effect of the CDS position. In this case, our model predicts that the final auction price will be above the true bond value. Of the 19 auctions, we have bond data for only five companies: Smurfit-Stone, Rouse, Charter Communications, Capmark, Bowater. Comparing the final action price from Table 2 with the bond price from Table 3 we can see that that, as anticipated, the bond price is below the final auction price in these cases.

We can compare the bond and auction prices for the rest of the companies with available TRACE data. Figure 3 shows the ratio of bond and final auction prices, p_{-1}/p^A. We see that in all but seven auctions the final auction price, p^A, is below the bond price, p_{-1}. The exceptions include the aforementioned five companies with capped auction prices. The remaining two exceptions, General Motors and Six Flags, did not reach the cap, but they have negative NOI, so their auction price is expected to have a reverse pattern.

4.3 Price Impact at the Second Stage

In the preceding section, we showed that when the price cap is not reached, the auction yields a price that is below the bond value. According to Propositions 1 and

[8] Of these 19 auctions, only one (Ecuador) has a negative NOI. So the above discussion for the case of positive NOI should be adjusted appropriately for Ecuador.

2, this can occur only if Property 1 does not hold. Recall that Property 1 relates the net open interest to the aggregate CDS positions of those agents who participate in the second stage of the auction. We do not have data on the latter. In consequence, we cannot test Propositions 1 and 2 directly.

Instead of testing these propositions, we provide empirical evidence that complements our theoretical analysis. Specifically, we study the effect of the NOI on the degree of price discrepancy that results from the auction. We scale the net open interest by the notional amount of deliverable bonds, NOI/NAO, to allow for a meaningful cross-sectional examination.

Table 3 and Figure 3 reveal that NOI/NAO has the largest values in the auctions where the largest discrepancy in prices occurs. At the same time, NOI/NAO has the lowest values in the auctions where the final price is capped, which is again consistent with Propositions 1. We quantify this relationship using a simple cross-sectional regression of p_{-1}/p^A on NOI/NAO :

$$p^{-1}/p^A = \alpha + \beta \times NOI/NAO + \varepsilon. \tag{6}$$

Figure 4 shows the results. The normalised NOI explains 65% of the variation in the ratio of the lag of the market price of bonds to the final price. The beta is significantly positive, which is consistent with our prediction that larger NOI increases the likelihood of equilibria in which mispricing occurs.

Our conclusions so far rest on the assumption that p_{-1} is a good proxy for the actual value v. One may argue that auctions exist precisely because it is difficult to establish a bond's fair value by observing the bond markets. Moreover, even if our representative bond price were to reflect the bond value accurately, it would be for the day before the auction. It is conceivable that the auction process establishes the correct value v that differs from p_{-1} simply because of the arrival of new information and/or the centralised clearing mechanism of the auction.

We expand the auction event window to check robustness of our results to these caveats. The shortest time between a credit event and an auction is 8 days in our sample. This prompts us to select an event window of -8 to +12 days. The choice of the right boundary is dictated by considerations of liquidity: liquidity generally declines after the auction. Figure 5 displays daily bond prices normalised by the

auction settlement price, equally weighted across all the 23 auctions for which we have the bond data. We see that the price generally declines, reaches its minimum on the auction day, then reverts back to its initial level. The figure shows that no matter which day we look at, the auction settlement price is, on average, at least 10% lower. In fact, p_{-1} turns out to be the most conservative proxy for v as the discrepancy between the bond and auction prices increases by up to 35% in both time directions.

The observed V shape of the discrepancy alleviates the concern that the correct value v differs from p_{-1} simply because the latter does not reflect the bond value correctly. If it were the case, one would expect bond prices to hover around the auction price after the auction. In practice, bond prices increase substantially after the auction.

5 Discussion

The empirical section documents that when NOI/NAO is large, the auction generally results in a price that is considerably below the bond value. We now discuss the likely cause of this price discrepancy and suggest several modifications to the auction design that can reduce mispricing in auction outcomes.

5.1 Allocation rule at the second stage

Proposition 2 shows that when Property 1 does not hold, the CDS auction is similar to a "standard" auction, so the price may be below v. Kremer and Nyborg (2004) show that in a setting without CDS positions, a simple change of the allocation rule from pro rata on the margin rule (2) to the pro-rata rule destroys all underpricing equilibria so that only $p^A = v$ remains. Under the pro-rata rule, the equilibrium allocations q_i are given by

$$q_i(p^A) = \frac{x_i(p^A)}{X(p^A)}.$$

The next proposition extends the result of Kremer and Nyborg (2004) to our setting. We demonstrate that if the conditions of Proposition 2 are satisfied and $p^M + s \geq v$, the only second-stage equilibrium price p^A under the pro-rata rule is equal to v. This is true even if the agents are allowed to hold non-zero quantities of CDS contracts.

Proposition 5 *Suppose that the conditions of Proposition 2 are satisfied and the auction sharing rule is pro-rata. Then, if $NOI > 0$, all second-stage equilibria result in the unique final price $p^A = \min\{p^M + s, v\}$. If $NOI < 0$, all second-stage equilibria result in the unique final price $p^A = \max\{p^M - s, v\}$.*

Proof. See Appendix A.5.

Consider the case of the positive NOI to develop intuition for this result. According to Proposition 2, if Property 1 does not hold then the pro-rata on the margin allocation rule may inhibit competition and lead to underpricing equilibria. The presence of agents who are short CDS contracts does not help in this case. The pro-rata allocation rule (i) does not guarantee the agents their inframarginal demand above the clearing price and (ii) closely ties the proportion of allocated bonds to the ratio of individual to total demand at the clearing price. Therefore, a switch to such a rule increases competition for bonds among agents. As a result, even agents with long positions bid aggressively. If $p^A < v$ then demanding the NOI at a price only slightly higher than p^A allows an agent to capture at least half of the surplus. As a result, only fair-price equilibria survive.

5.2 The price cap

Our theoretical analysis in Section 3.2 shows that the presence of the cap can result in either lower or larger mispricing in auction outcomes. The cap is likely to help when $|NOI|$ is small and the temptation to manipulate the auction results is highest. At the same time, the cap allows dealers to limit competition in the second stage if they set the IMM away from the true bond price.

These results suggest that making the cap conditional on the outcome of the first stage of a CDS auction can lead to a better auction design. In our base model without uncertainty, the optimal conditional cap is trivial. Again, we consider the case of $NOI > 0$. If $p^M < v$, setting $s^* = v - p^M$ ensures that the set of second-stage equilibria includes v. If $p^M \geq v$, it is best to set $s^* = 0$.

In practice, v and n_i are unobservable. In this case, we suggest making the cap conditional on NOI and on the ratio p^M/p_{-1}. For example, if $p^M/p_{-1} \leq \alpha$ and the NOI is large, where $\alpha < 1$ is reasonably small, the auctioneer can set a larger cap; if $p^M/p_{-1} > \alpha$ and the NOI is small, a smaller cap can be set.

5.3 Risk-averse agents

So far, we have restricted our attention to the setting with risk-neutral agents. This allowed us to abstract from risk considerations. If agents are risk-averse, the reference entity's risk is generally priced. Even though a CDS is in a zero net supply, its settlement leads to a reallocation of risk among the participants in the auction; hence, it can lead to a different equilibrium bond price. In a particular scenario, when NOI/NOA is large and positive and there are only a few risk-averse agents willing to hold defaulted bonds, the auction results in a highly-concentrated ownership of the company's risk; hence, it can lead to a lower new equilibrium bond price.

Due to the fact that we do not have data on individual agents' bids and positions, we cannot determine whether the observed price discrepancy is due to the mispricing equilibria played or the risk-aversion channel. It is likely that both factors work together in the same direction. Data on individual agents' bids and positions could help to quantify the effect of the two factors on the observed relationship between the auction price and the size of the net open interest.

6 Conclusion

We have presented a theoretical and empirical analysis of how CDS contracts are settled when a credit event takes place. A two-stage auction-based procedure aims to establish a reference bond price for cash settlement and to provide market participants with an option to replicate an outcome of a physical settlement. The first stage determines the net open interest (NOI) in the physical settlement and the auction price cap (minimum or maximum price depending on whether the NOI is to sell or to buy). The second stage is a uniform divisible good auction with a marginal pro-rata allocation rule that establishes the final price by clearing the NOI.

In our theoretical analysis, we show that the auction may not result in the fair bond price. Whether the auction underprices or overprices the bond depends on the relative size of the NOI and the aggregate CDS positions of the agents who participate in the second stage of the auction.

In our analysis of empirical data, we find support for our theoretical predictions. The bonds are underpriced by 10%, on average, and the amount of underpricing

is related to the NOI. We propose introducing a pro-rata allocation rule and a conditional price cap to insure against both underpricing and overpricing equilibria in different auction settings.

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A Appendix

A.1 Proof of Proposition 1

We prove the case of the $NOI > 0$. The case of the $NOI < 0$ is similar. Suppose that $p^M + s > v$ and $p^A < v$. We show that this cannot be true in equilibrium. Let the equilibrium allocation of bonds for agent i be q_i. Consider a variation of a demand schedule of player i from x_i to x_i' that leads to an auction price $p \in [p^A, v]$. Denote the new bond allocation of agent i by q_i'. Since demand schedules are non-decreasing, $q_i' \geq q_i$. Agent $i's$ change in profit is thus

$$\delta_i = \left[(v - p^A)q_i - p^A(n_i - y_i)\right] - \left[(v - p)q_i' - p(n_i - y_i)\right] =$$

$$= (p - p^A)(n_i - y_i + q_i) - (v - p)(q_i' - q_i) \leq (p - p^A)(n_i - y_i + q_i). \qquad \text{(A1)}$$

Equilibrium conditions require that $\delta_i \geq 0$ for all i. Summing over all i such that $n_i < 0$, we have that it must be that

$$0 \leq \sum_{i:n_i<0} \delta_i \leq (v - p^A) \sum_{i:n_i<0} (n_i - y_i + q_i)\,.$$

Because all $q_i \geq 0$,

$$\sum_{i:n_i<0} (n_i - y_i + q_i) \leq \sum_{i:n_i<0} (n_i - y_i) + NOI \leq 0, \qquad \text{(A2)}$$

where we use Property 1. Thus in any equilibrium with $p^A < v$, it must be that $\delta_i = 0$ for all i with $n_i < 0$. (A1) and (A2) then imply that for any deviation x_i' that leads to $p \in [p^A, v]$ it must be that $q_i' = q_i$. Since this is true for any $p \in [p^A, v]$ it implies that the initial total demand $X(p)$ is constant over $[p^A, v]$, and therefore $p^A = v$. Thus we arrive at a contradiction.

Next, consider the following set of equilibrium strategies:

$$x_i(p): \quad \begin{cases} x_i = NOI \times (n_i - y_i)/(\sum_{i:n_i<0}(n_i - y_i)) & \text{if} \quad v < p \leq p^M + s, \\ x_i = NOI & \text{if} \quad p \leq v. \end{cases}$$

for agents with net negative CDS positions after physical request submission, and $x_i(p) \equiv 0$ for agents with positive CDS positions. It is not difficult to see that it supports $p^A = p^M + s$.

Consider now the following set of equilibrium strategies:

$$x_i(p): \quad \begin{cases} x_i = 0 & \text{if} \quad v < p \leq p^M + s, \\ x_i = NOI & \text{if} \quad p \leq v. \end{cases}$$

for agents with net negative CDS positions after physical request submission, and $x_i(p) \equiv 0$ for agents with positive CDS positions. It is not difficult to see that it supports $p^A = v$, provided that $\max_{i:n_i<0} |n_i - y_i| \leq NOI$. QED.

A.2 Proof of Proposition 2

We focus on the case of the $NOI > 0$. First, we prove that $p^A \leq v$. Suppose that it is not true and there exists an equilibrium with $p^A > v$. In such an equilibrium, since Property 1 does not hold, there exists i such that agent $i's$ equilibrium second stage allocation $q_i > |n_i - y_i|$. Consider a variation of this agent $i's$ demand schedule, in which she submits zero demand at $p^A > v$ and the NOI at $p^A = v$. Given this variation, the new auction price is higher than or equal to v. Thus, her profit increases at least by $(p^A - v)(q_i + n_i - y_i) > 0$. Thus $p^A > v$ cannot be an equilibrium outcome.

Next, we prove that equilibrium outcomes with p^A strictly lower that v can realize even if $p^M + s \geq v$. The proof is by construction. We construct an equilibrium in continuous strategies. Similar to Kremer and Nyborg (2004), one can show that any equilibrium which is obtained when restricting players to continuous strategies is still an equilibrium. In an equilibrium, the sum of the demand of agent i, $x_i(p^A)$ and the residual demand of other players, $x_{-i}(p^A)$ has to equal to the NOI. Therefore, F.O.C. for agent i at equilibrium price p^A is

$$\begin{aligned} (v - p^A)\frac{\partial x_{-i}(p^A)}{\partial p} + x_i(p^A) + n_i - y_i = 0 \qquad & \text{if} \qquad x_i(p^A) > 0, \\ (v - p^A)\frac{\partial x_{-i}(p^A)}{\partial p} + x_i(p^A) + n_i - y_i \geq 0 \qquad & \text{if} \qquad x_i(p^A) = 0. \end{aligned} \tag{A3}$$

Consider the following set of strategies:[9]

$$x_i = [c(v-p)^\gamma - n_i + y_i]_+ \tag{A4}$$

Suppose one wants to support $p^A = 0$. For $p^A = 0$ to be an equilibrium it must be that, in addition to optimality conditions (A3), the total demand at $p = 0$ equals to NOI:

$$\sum_{i \in \mathcal{N}_\in} [cv^\gamma - n_i + y_i]_+ = NOI. \tag{A5}$$

Equation (A5) defines c for any γ, which we denote as $c(\gamma)$. Suppose that the set of $n_i - y_i$ is such that there exists a solution to the following equation:

$$\gamma^{-1} = \sum_{i \in \mathcal{N}_\in} \mathbf{1}_{\{c(\gamma)v^\gamma - n_i + y_i \geq 0\}} - 1. \tag{A6}$$

Let us find conditions for the existence. For convenience, let us assume w.l.o.g. that all $n_i - y_i$ are sorted in the increasing order. Consider $\gamma = 1/(N_2 - 1)$, $N_2 = |\mathcal{N}_2|$. If for all i, $c(\gamma)v^\gamma - n_i + y_i \geq 0$ then γ is the sought solution. If there exists i such that $c(\gamma)v^\gamma - n_i + y_i < 0$ then discard the most positive $n_i - y_i$, consider $\gamma = 1/(N_2 - 2)$, and repeat the whole process until we get such γ that all $c(\gamma)v^\gamma - n_i + y_i \geq 0$. Let M be the corresponding number of agents who survived the elimination procedure, so that $\gamma = 1/(M-1)$. Suppose that

$$\left(NOI + \sum_{i=1}^{M}(n_i - y_i)\right)/M - (n_i - y_i) \;\geq\; 0, \quad i = 1..M, \tag{A7}$$

$$\left(NOI + \sum_{i=1}^{M}(n_i - y_i)\right)/(M-1) - (n_{M+1} - y_{M+1}) \;<\; 0. \tag{A8}$$

Condition (A7) ensures that $x_i(0) > 0$, for $i = 1..M$. Condition (A8) ensures that (A3) holds, so that $M+1$ agent does not want to submit a positive demand at zero price. Substituting demand schedules (A4) with such a γ into F.O.C. (A3) one can see that they support $p^A = 0$ as the equilibrium price. Also, it can be shown that if $p^A = 0$ can be supported as an equilibrium then any other price $p^A \in [0, v]$ can be

[9] A similar result can be shown using other strategies, e.g., linear strategies.

supported as well. QED.

A.3 Proof of Proposition 3

As usual, we provide a proof for the case of the $NOI > 0$. Suppose that there exists a set of equilibrium strategies, which lead to $p^A < v$ on a set of positive measure. Denote this set by $\mathcal{S}$. We show that this cannot be true. Let the equilibrium allocation of bonds for agent i be q_i, which now depends on the realization of a state $\omega \in \Omega$. Consider a variation of a demand schedule of player i in which it demands a higher amount of bonds x_i' for $p \leq v$. This variation leads to an auction price $p \in [p^A, v]$ and a new bond allocation $q_i' \geq q_i$ whenever $p^A \in \mathcal{S}$ and to the same equilibrium price and bond allocation whenever $p^A \in \Omega/\mathcal{S}$. Agent $i's$ expected change in profit from this variation is

$$\begin{aligned} \delta_i &= \int_{\Omega} \left(\left[(v - p^A) q_i - p^A (n_i - y_i) \right] - \left[(v - p) q_i' - p(n_i - y_i) \right] \right) dF(n_i, \cdot) && \text{(A9)} \\ &\leq (n_i - y_i + q_i) \int_{\Omega/\mathcal{S}} (p - p^A) dF(n_i, \cdot) \leq (n_i - y_i + q_i) \int_{\Omega/\mathcal{S}} (v - p^A) dF(n_i, \cdot). \end{aligned}$$

Equilibrium conditions require that $\delta_i \geq 0$ for all i. Summing over all i with $n_i < 0$, we have that it must be that

$$\max_i \left\{ \int_{\Omega/\mathcal{S}} (v - p^A) dF(n_i, \cdot) \right\} \sum_{i:n_i<0} (n_i - y_i + q_i) \geq \sum_{i:n_i<0} \delta_i \geq 0. \tag{A10}$$

Since all $q_i \geq 0$

$$\sum_{i:n_i<0} (n_i - y_i + q_i) \leq \sum_{i:n_i<0} (n_i - y_i) + NOI = \sum_{i:n_i<0} n_i + \sum_{i:n_i>0} y_i \leq \sum_{i:\widehat{n}_i<0} n_i + \sum_{i:n_i>0} n_i = 0.$$

Thus it must be that $\delta_i = 0$ for all i with $n_i < 0$. As in Proposition 1, this implies that for any deviation x_i' that leads to $p \in [p^A, v]$, $q_i' = q_i$. Since this is true for any $p \in [p^A, v]$ it implies that the initial total demand $X(p)$ is constant over $[p^A, v]$ for $\omega \in \mathcal{S}$, and therefore $p^A = v$ $\omega \in \mathcal{S}$. Thus we arrive at a contradiction. QED.

A.4 Proof of Proposition 4

The proof is by construction. We consider the case when F is such that there are $M \geq 2$ agents with negative CDS positions who participate in the second stage. Cases with $M = 1$ and $M = 0$ are simpler and available upon request. As in Proposition 2, we construct equilibria in continuous strategies and consider the case when $p^M + s \geq v$. To simplify notation, define $\widehat{n}_i = n_i - y_i$. Following Wilson (1979), define a function $H(p, x_i(p))$ as

$$H(p, x_i(p)) = Prob\{p^A \leq p\} = Prob\{x_{-i}(p, \widehat{n}_{-i}) \leq NOI - x_i(p)\}. \tag{11}$$

Then agent $i's$ expected profit as a function of its demand $x_i(p)$ can be written as

$$\int_0^{100} [(v - p)x_i(p) - \widehat{n}_i p] \, dH(p, x_i(p)). \tag{12}$$

Integrating (13) by parts we arrive at

$$\int_0^{100} \left[-(v - p)x_i'(p) + \widehat{n}_i + x_i(p)\right] H(p, x_i(p))dp + (v-100)x_i(100) - 100\widehat{n}_i - vx_i(0)H(0, x_i(0)). \tag{13}$$

Euler equation takes form of

$$\frac{d}{dp}\left[(v - p)H(p, x_i(p))\right] + H(p, x_i(p)) + [-(v - p)x_i'(p) + \widehat{n}_i + x_i(p)]H_x'(p, x_i(p)) = 0,$$

which simplifies to

$$(v - p)H_p'(p, x_i(p)) + [\widehat{n}_i + x_i(p)]H_x'(p, x_i(p)) = 0, \quad \forall p \in [0, 100]. \tag{14}$$

Since each agent can submit only non-negative demand schedule, optimality condition (14) is in fact,

$$\begin{aligned} (v - p)H_p'(p, x_i(p)) + [\widehat{n}_i + x_i(p)]H_x'(p, x_i(p)) = 0, \quad \forall p : x_i(p) > 0, \\ (v - p)H_p'(p, x_i(p)) + [\widehat{n}_i + x_i(p)]H_x'(p, x_i(p)) \leq 0, \quad \forall p : x_i(p) = 0. \end{aligned} \tag{15}$$

Suppose that each agent i with $\widehat{n}_i < 0$ submits a demand schedule

$$x_i(p) = c(v - p)^{1/(M-1)} - \widehat{n}_i, \tag{16}$$

and agents with positive CDS positions submit zero demand. Consider first optimality conditions (15) for agents with negative CDS positions. From the the definition of $H(p, x_i(p))$, for such an agent i,

$$H_p'(p, x_i(p)) = -c(v - p)^{1/(M-1)-1} H_x'(p, x_i(p)).$$

Therefore, optimality conditions (15) become

$$\left[-c(v-p)^{1/(M-1)}+\widehat{n}_i+x_i(p)\right]H'_x(p,x_i(p))=0, \tag{17}$$

which, given (16), holds true for every p. Consider now optimality conditions (15) for agents with positive CDS positions. From the the definition of $H(p,x_i(p))$, for such an agent i,

$$H'_p(p,x_i(p))=-c(v-p)^{1/(M-1)-1}\frac{M}{M-1}H'_x(p,x_i(p)).$$

Therefore, optimality conditions (15) become

$$-c(v-p)^{1/(M-1)}\frac{M}{M-1}+\widehat{n}_i\geq 0, \quad \forall p: H'_x(p,0)<0, \tag{18}$$

where we used the fact that $H'_x(p,0)\leq 0$. Consider

$$c=\frac{c'NOI}{Mv^{1/(M-1)}}. \tag{19}$$

In equilibrium,

$$X(p)=NOI \quad \Leftrightarrow \quad p^A=v\left(1-\frac{NOI+\sum_{i\in\mathcal{N}_2:n_i<0}\widehat{n}_i}{c'NOI}\right). \tag{20}$$

Since Property 1 does not hold, $v(1-1/c')\leq p^A<v$ with probability one. Finally, it is always possible to choose c (and therefore c') such that optimality conditions (18) for agents with positive CDS positions hold true. QED.

A.5 Proof of Proposition 5

As usual, we focus on the case of the $NOI>0$. Note that the pro-rata sharing rule satisfies the *majority property* (Kremer, Nyborg (2004)): an agent whose demand at the clearing price is above 50% of the total demand at this price is guaranteed to get at least $(50\%+\eta)\times NOI$, where $\eta>0$.
First, suppose that $v\leq p^M+s$. The proof that p^A cannot be above v is the same as in Proposition 2. We prove that p^A cannot be below v. Suppose $p^A<v$. The part of each agent i's utility that depends on its equilibrium allocation and the final price is:

$$(v-p^A)\times q_i-p^A\times\widehat{n}_i.$$

Suppose first that there is at least one agent for which $q_i<0.5$. Suppose this agent changes its demand schedule to:

$$x'_i(p)=\begin{cases}NOI, & p\leq p^A+\varepsilon\\ 0, & \text{otherwise,}\end{cases} \tag{21}$$

where $0 < \varepsilon < v - p^A$. Under such a deviation, the new clearing price is $p^A + \varepsilon$. Because $X_{-i}(p^A + \varepsilon) < NOI$ (otherwise, $p^A + \varepsilon$ would have been the clearing price), agent i demands more than 50% at $p^A + \varepsilon$ and under the pro-rata sharing rule gets $q'_i > 0.5$. The lower bound on agent i's relevant part of utility is now:

$$(v - p^A - \varepsilon) \times 0.5 - (p^A + \varepsilon) \times \widehat{n}_i.$$

We can write the difference between agent i's utility under deviation and under the assumed equilibrium:

$$(0.5 - q_i) \times (v - p^A) - \varepsilon(\widehat{n}_i + 0.5). \tag{22}$$

For ε small enough, and under assumption that $p^A < v$, (22) is greater than zero, hence equilibria with $p^A < v$ cannot exist.

If there are no agents with $q_i < 0.5$, we are in an auction with two bidders only, and each of them gets exactly $0.5 \times NOI$. At a price $p^A + \varepsilon$, $0 < \varepsilon < p^M + s - v$, there is at least one player (player i), for which $x_i(p^A + \varepsilon) < 0.5$. Then, if the opposite agent uses (21), the new clearing price is $p^A + \varepsilon$ and this agent gets at least $(0.5 + \eta) \times NOI$. For ε small enough, the difference between agent i's utility under deviation and under the assumed equilibrium is:

$$\eta \times (v - p^A) - \varepsilon(\widehat{n}_i + 0.5 + \eta) > 0. \tag{23}$$

Therefore, equilibria with $p^A < v$ cannot exist. We conclude that if $v \leq p^M + s$, then $p^A = v$ is the only clearing price in any equilibrium under the pro-rata sharing rule.

Finally, suppose that $p^M + s < v$. The proof for this case is the same, except there is no feasible deviation to a higher price if $p^A = p^M + s$. Hence, $p^A = p^M + s < v$ is the only clearing price in any equilibrium under the pro-rata sharing rule. QED.

Table 1
Nortel Limited market quotes

Dealer	Bid	Offer
Banc of America Securities LLC	9.5	11.5
Barclays Bank PLC	4.0	6.0
BNP Paribas	7.0	9.0
Citigroup Global Markets Inc.	10.5	12.5
Credit Suisse International	6.5	8.5
Deutsche Bank AG	6.0	8.0
Goldman Sachs & Co.	6.0	8.0
J.P. Morgan Securities Inc.	7.0	9.0
Morgan Stanley & Co. Incorporated	5.0	7.0
The Royal Bank of Scotland PLC	6.5	8.5
UBS Securities LLC	7.0	9.0

Table 1 shows two-way quotes submitted by dealers at the first stage of Nortel Ltd. auction.

Table 2
Auction Summaries

Name	Date	Credit Event	Inside Market Quote	Net Open Interest	Final Price
Dura	28 Nov 2006	Chapter 11	24.875	20.000	24.125
Dura Subordinated	28 Nov 2006	Chapter 11	4.250	77.000	3.500
Quebecor	19 Feb 2008	Chapter 11	42.125	66.000	41.250
Lehman Brothers	10 Oct 2008	Chapter 11	9.750	4920.000	8.625
Washington Mutual	23 Oct 2008	Chapter 11	63.625	988.000	57.000
Tribune	6 Jan 2009	Chapter 11	3.500	765.000	1.500
Lyondell	3 Feb 2009	Chapter 11	23.250	143.238	15.500
Nortel Corp.	10 Feb 2009	Chapter 11	12.125	290.470	12.000
Smurfit-Stone	19 Feb 2009	Chapter 11	7.875	128.675	8.875
Chemtura	14 Apr 2009	Chapter 11	20.875	98.738	15.000
Great Lakes	14 Apr 2009	Ch 11 of Chemtura	22.875	130.672	18.250
Rouse	15 Apr 2009	Downgrade to D	28.250	8.585	29.250
Abitibi	17 Apr 2009	Acceleration of Debt	3.750	234.247	3.250
Charter Communications	21 Apr 2009	Chapter 11	1.375	49.2	2.375
Capmark	22 Apr 2009	Default	22.375	115.050	23.375
Idearc	23 Apr 2009	Chapter 11	1.375	889.557	1.750
Bowater	12 May 2009	Chapter 11	14.000	117.583	15.000
R.H.Donnelly Corp.	11 Jun 2009	Chapter 11	4.875	143.900	4.875
General Motors	12 Jun 2009	Chapter 11	11.000	-529.098	12.500
Visteon	23 Jun 2009	Chapter 11	4.750	179.677	3.000
Six Flags	9 Jul 2009	Chapter 11	13.000	-62.000	14.000
Lear	21 Jul 2009	Chapter 11	40.125	172.528	38.500
CIT	1 Nov 2009	Chapter 11	70.250	728.980	68.125

Table 2 summarizes auction results for 23 US firms for which the TRACE data is available. It reports a settlement date, a type of credit event, inside market quote (per 100 of paer), net open interest (in millions of USD), and final auction settlement price (per 100 of par).

Table 3
Tradable Deliverable Bond Summary Statictics

Name	Number of deliverable bonds	Notional amount outstanding (NAO)	NOI/NOA (%)	Average price on the day before the auction
Dura	1	350,000	5.71	25.16
Dura Subordinated	1	458,500	16.79	5.34
Quebecor	2	600,000	11.00	42.00
Lehman Brothers	157	42,873,290	11.47	12.98
Washington Mutual	9	4,750,000	20.80	64.79
Tribune	6	1,346,515	56.81	4.31
Lyondell	3	475,000	30.15	26.57
Nortel Corp.	5	3,149,800	9.22	14.19
Smurfit-Stone	5	2,275,000	5.65	7.77
Chemtura	3	1,050,000	9.40	26.5
Great Lakes	1	400,000	32.65	26.71
Rouse	4	1,350,000	0.63	29.00
Abitibi	10	3,000,000	7.81	4.61
Charter Communications	17	12,769,495	0.38	2.00
Capmark	2	1,700,000	6.79	22.75
Idearc	1	2,849,875	31.21	2.15
Bowater	6	1,875,000	6.27	14.12
R.H.Donnelly Corp.	7	3,770,255	3.81	5.12
General Motors	16	18,180,552	-2.91	11.17
Visteon	2	1,150,000	15.62	74.87
Six Flags	4	1,495,000	-4.14	13.26
Lear	3	1,298,750	13.28	39.27
CIT	281	22,584,893	3.29	69.35

Table 3 provides summary statistics of deliverable bonds for 23 US firms for which the TRACE data is available. The third column reports the ratio of the net open interest (NOI) from Table 2 to notional amount outstanding of deliverable bonds. The last column shows a weighted average bond price on the day before the auction, constructed as described in Section 4.1.

Figure 1: IMM determination: the case of Nortel

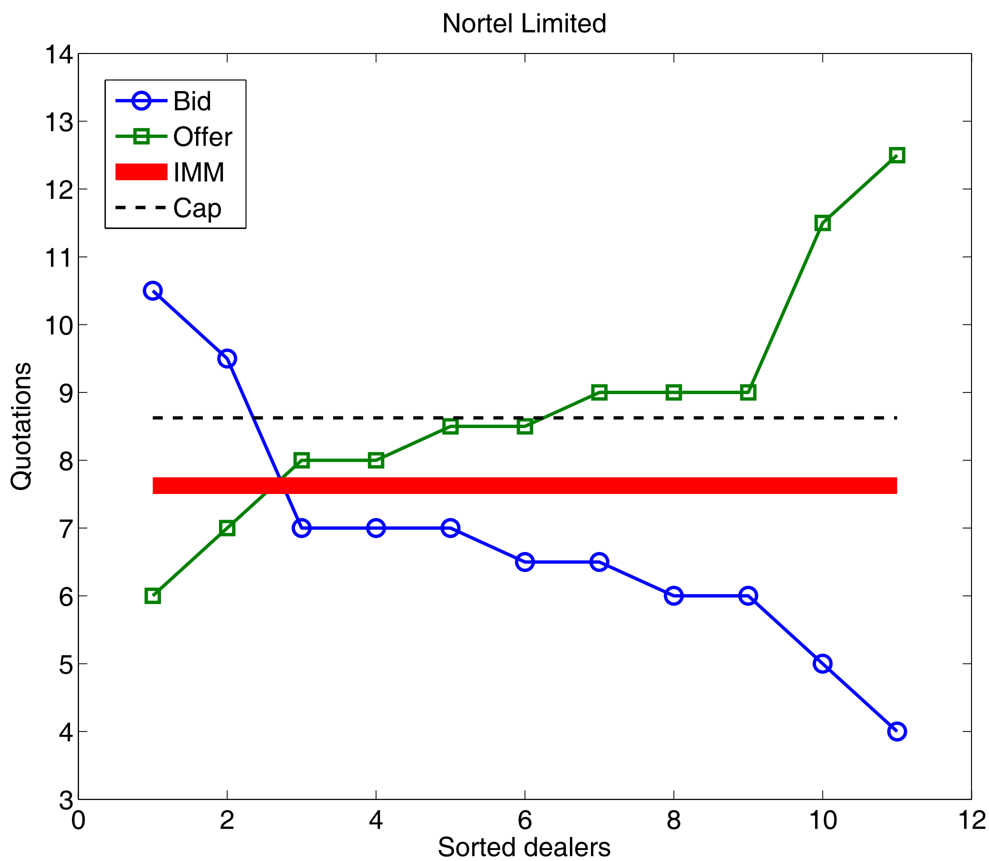

Figure 1 displays all the bids sorted in the descending order and all the offers sorted in the ascending order. The tradeable quotes (bid greater than offer) are discarded for the purposes of computing IMM. The dealers that quote tradeable markets has to pay a penalty (adjustment amount) to ISDA. The cap price is higher than the IMM by 1% of par and is used in the course of determination of the final price. (If the open interest is to buy the cap price is below the IMM).

Figure 2: Bids at the cap price

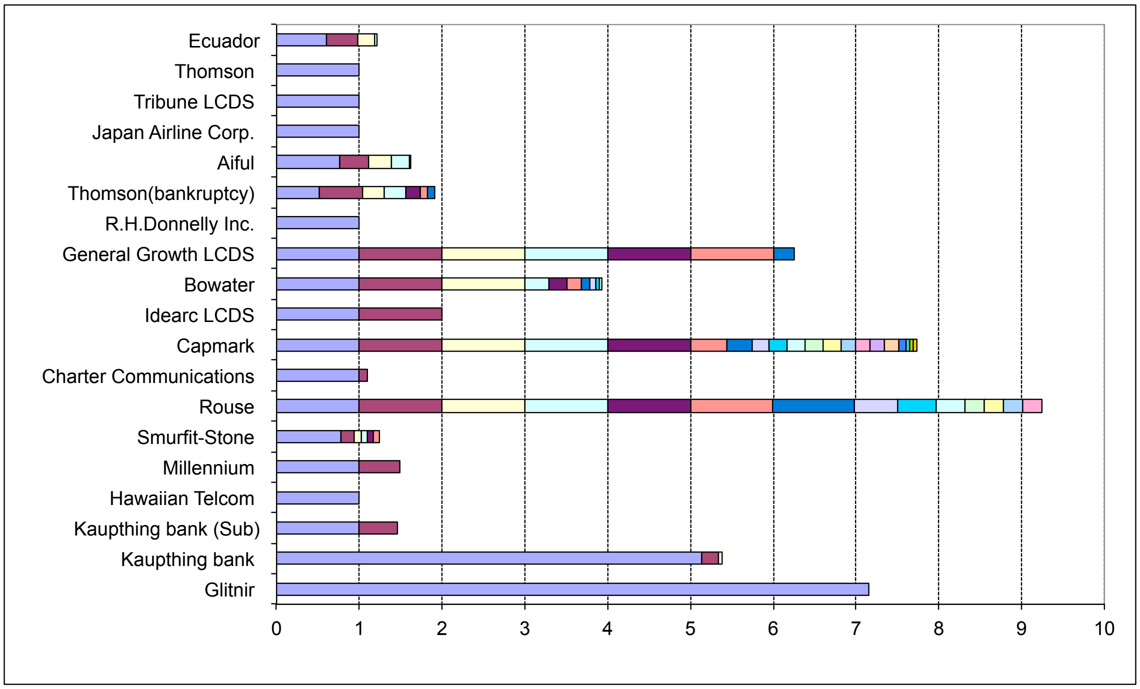

Figure 2 shows the individual bids scaled by NOI at the cap price represented by different colors in auctions, in which the final price is capped.

Figure 3: Bond and Auction Prices

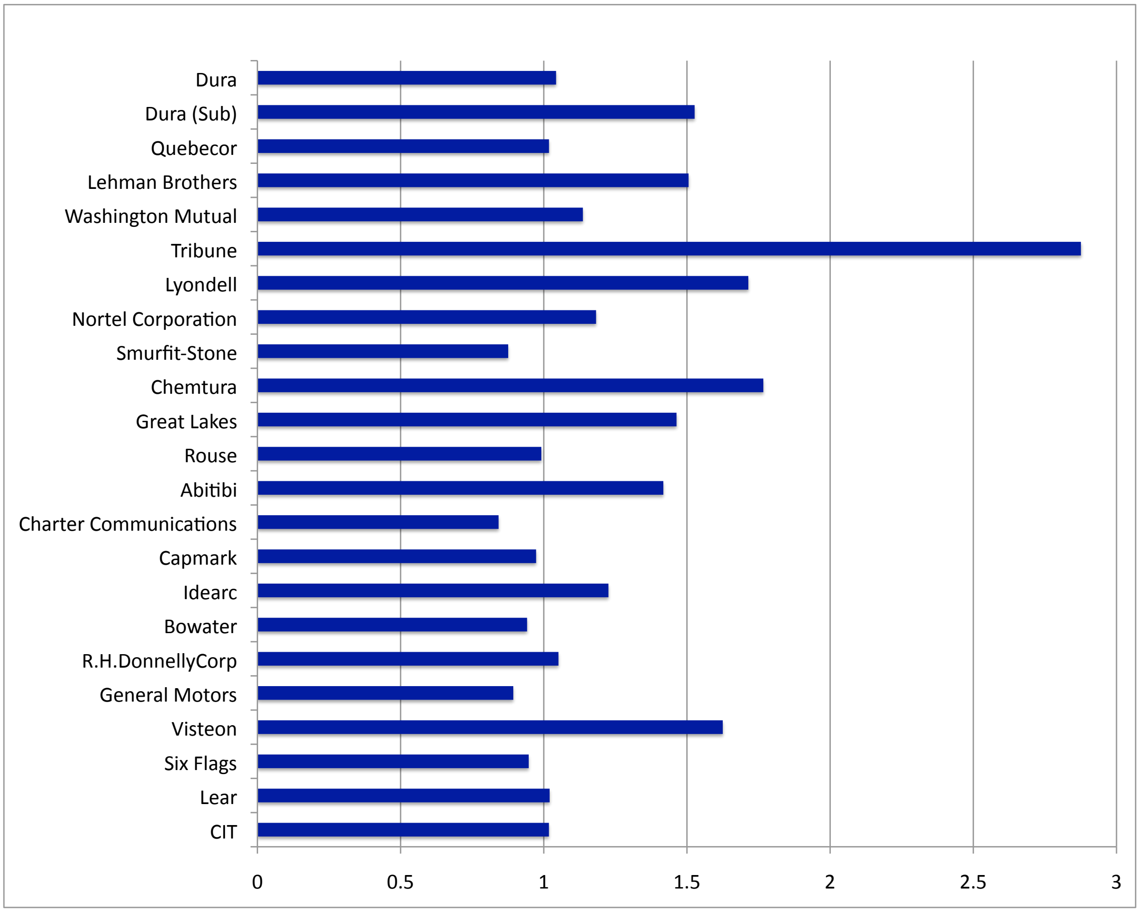

Figure 3 shows the representative bond prices from the day before the auction scaled by the auction price.

Figure 4: Price Discount

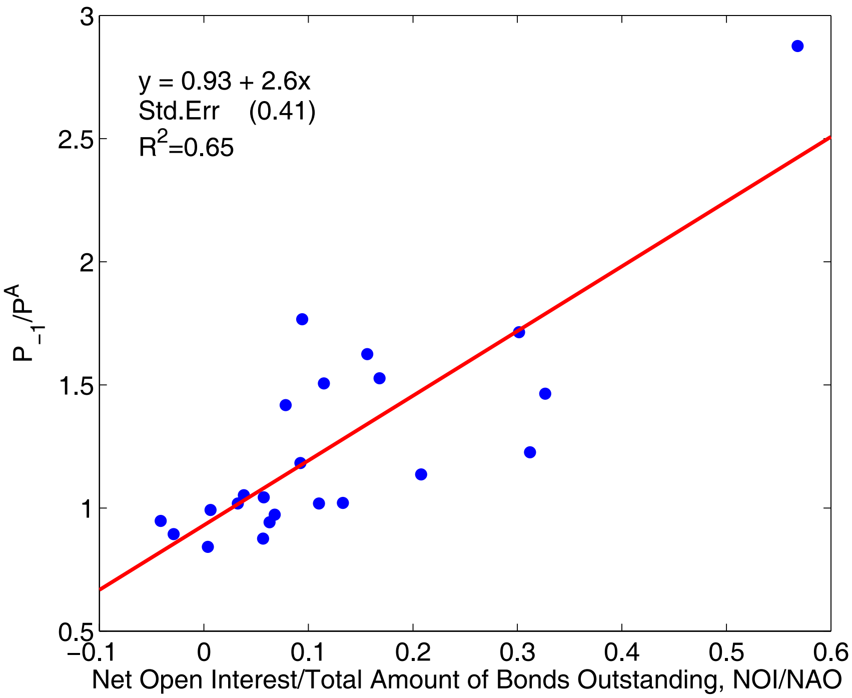

Figure 4 shows results of an OLS regression of the ratio of the weighted-average market price of bonds a day before an auction to the final auction price on the scaled NOI:

$$y_i = \alpha + \beta \times NOI_i/NAO_i + \varepsilon_i.$$

Figure 5: Price Impact

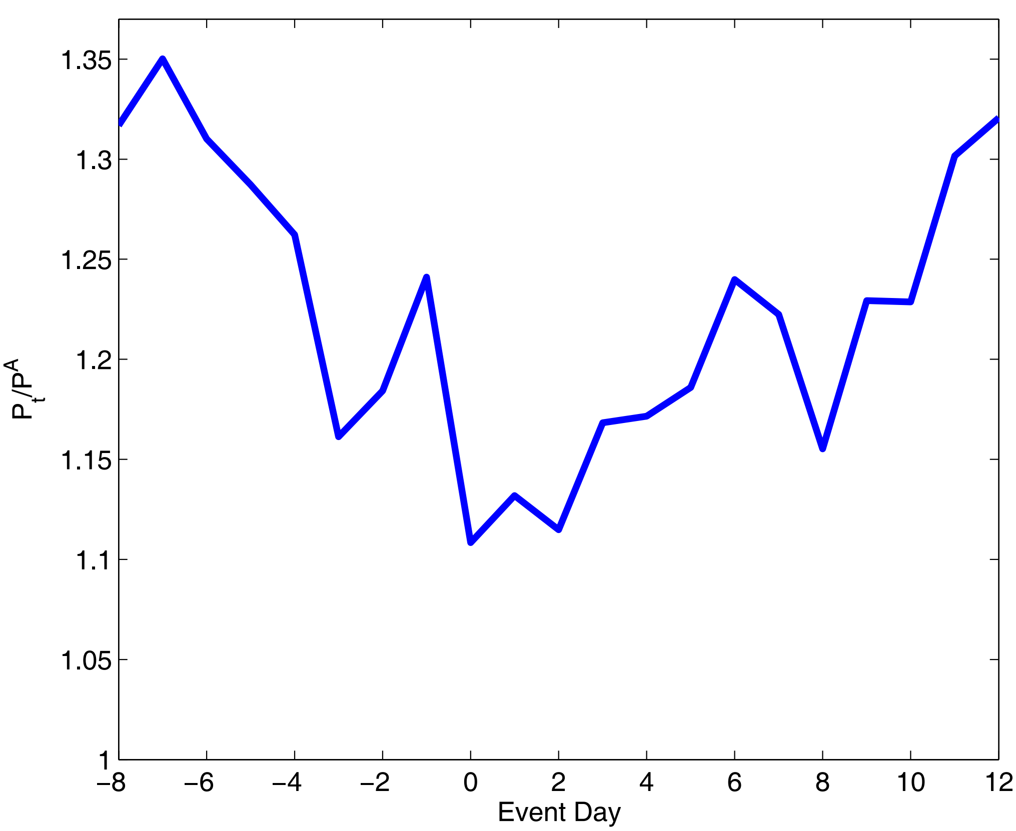

Figure 5 shows daily volume-weighted bond prices, constructed as described in Section 4.1, normalized by the auction settlement price, p^A, and averaged across 23 auctions reported in Table 2.