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Mortgage Hedging in Fixed Income Markets*

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Abstract

We study the feedback from hedging mortgage portfolios on the level and volatility of interest rates. We incorporate the supply shocks resulting from hedging into an otherwise standard dynamic term structure model, and derive two sets of predictions which are strongly supported by the data: First, the duration of mortgage-backed securities (MBS) positively predicts excess bond returns, especially for longer maturities. Second, MBS convexity increases yield and swaption implied volatilities, and this effect has a hump-shaped term structure. Empirically, neither duration, nor convexity are spanned by yield factors. A calibrated version of our model replicates salient features of first and second moments of bond yields.

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The effect that the hedging of large mortgage portfolios has on the term structure of interest rates is a concern for policy makers and practitioners alike. For example, the recent decision by the Federal Reserve to initiate a third round of quantitative easing by purchasing mortgage debt has created widespread expectations that lower hedging volume will lead to lower bond volatility.¹ Indeed, right after Chairman Bernanke’s announcement, yields on agency mortgage bonds and Treasury bond implied volatility fell to a record low.

Mortgage loans and mortgage-backed securities (MBS) expose investors to interest rate risk. Unlike with regular bonds, this exposure can change significantly with interest rate conditions.² Mortgages typically feature an embedded prepayment option that makes the convexity of mortgage-backed securities negative: Lower interest rates increase the prepayment risk of outstanding mortgages and thereby considerably decrease their duration. This is equivalent to a negative shock to the supply of long-term bonds and can have an effect on their prices. Mortgage investors who aim to keep the duration of their portfolios constant (either for hedging or portfolio re-balancing reasons) take a long position in bonds, which pushes interest rates further down. Thus, negative convexity creates an amplification channel for interest rate shocks.

In this paper, we formally model the term structure of interest rates as resulting from the interaction between (i) exogenous shocks to the short rate and (ii) changes in the net supply of long-term bonds that are endogenously driven by the interest rate risk exposure of mortgages. The latter effect is in line with recent literature where the net supply of bonds is held by risk-averse financial intermediaries in exchange for a premium (e.g., Vayanos and Vila (2009)).

The term structure model we derive takes the form of a standard Vasicek (1977) short rate model augmented by an additional factor that represents the duration of outstanding MBS. This factor drives the market price of interest rate risk and affects

¹See [Bloomberg \(2012\)](#).

²The upper panel of Figure 1 plots the 10-year yield together with the duration of a MBS portfolio and the duration of a generic portfolio consisting of Treasury bonds with maturities ranging between 5 and 7 years. The figure clearly reveals the tight relationship between the interest rate environment and MBS duration, as well as the apparent lack of significant variation in the Treasury duration.

the risk premia of long-term bonds but not the dynamics of the risk-free rate itself. Its contribution tends to zero if hedging activity is small or market liquidity is high.

The model has two sets of testable predictions pertaining to both levels and volatilities of yields and bond returns. First, the duration of outstanding MBS predicts bond excess returns. Moreover, this effect is stronger for longer maturity bonds. This happens because the market price of interest rate risk is proportional to the quantity of duration risk that investors have to hold. Longer maturity bonds with a higher exposure to interest rate risk are more strongly affected through this channel.

Second, the average volatility of all yields is increasing in the convexity of outstanding MBS through the negative convexity amplification channel discussed above. This effect has a hump-shaped pattern across maturities with intermediate maturities being most strongly affected. In the model, supply shocks create transitory variations in the market price of interest rate risk. Short-maturity bonds are a close substitute to the short rate and are not very sensitive to variations in the market price of risk, while for very long maturities, the market price of risk is expected to mean revert. This explains why the effect is strongest on the volatility of bonds with intermediate maturities. A similar intuition applies to the implied volatility of interest rate derivatives: Our model is able to accommodate a hump-shaped term structure of implied volatility of swaptions.

Finally, our model has implications regarding the spanning of MBS duration by yield factors. The model captures the fact that mortgage refinancing depends on the current level of interest rates relative to past rates at which outstanding mortgages were originated. Because of this history dependence, duration cannot be expressed as a function of the current short rate. This motivates us to investigate the way in which the cross-section of yields empirically spans the duration factor.

As a first test we calibrate our model. We find that MBS duration is well described by a process buffeted by the same innovations as long rates but with higher mean-reversion. This behavior would, for instance, result from a model where MBS duration is driven by a mortgage refinancing incentive, i.e. the difference between the current level of long rates and the average coupon paid on outstanding mortgages, which itself is akin to a weighted average of past interest rates. Higher mean-reversion captures

the refinancing burnout effect: Aggregate MBS duration tends to revert back up when refinancing activity lowers the average coupon. We observe that the half-life of duration is about six months, suggesting that it varies at a much higher frequency than the business cycle or the supply of government debt. Calibrated to MBS duration dynamics, our model matches the upward sloping term structure of predictability of bond excess returns by MBS duration together with the hump-shaped term structure of unconditional bond yield volatilities and implied volatilities on swaptions.

We then take the model to the data and test the aforementioned theoretical predictions using linear regressions. We first regress bond yields and bond excess returns onto measures of MBS duration and find a strong positive link between duration and levels of bond yields and returns. Moreover, the link becomes stronger as maturity increases. The relationship is also economically significant: A one standard deviation change in MBS duration implies a change of 47 basis points in bond yields at longer maturities and expected 10-year bond returns over one year change by 210 basis points. These numbers are comparable to results reported for the first round of quantitative easing by the Fed (QE1). For example, Gagnon, Raskin, Remache, and Sack (2010) estimate that QE1 operations, which mainly involved the purchase of mortgages in 2008 and 2009, lowered the ten year term premium by between 30 and 100 basis points.

For second moments of bond yields, we find that higher MBS convexity (in absolute terms) increases bond yield volatility significantly. For example, a one standard deviation change in MBS convexity implies an up to 40 basis point change in bond yield volatilities. As implied by the model, the effect is strongest for intermediate maturities between two and three years. Moreover, this strong link remains if we add other determinants of bond yield volatility. Hedging of prepayment risk is mostly done through swaption contracts. We therefore test the implications of convexity onto the implied volatility of swaptions with different maturity. We find that the effect is hump shaped and highly significant, in line with the theory. We also test how MBS convexity affects bond return volatility—both implied and realized—and find that the estimated slope coefficients are highly significant. To test how MBS convexity affects compensation for volatility risk in fixed income markets, we use measures of bond variance risk premia and straddle returns

from at-the-money options on Treasury futures. In line with intuition, we find that MBS convexity loads positively and highly significantly on these proxies of volatility risk.

Since both MBS duration and convexity depend on interest rate conditions, it is natural to ask whether duration and convexity contain any information above the one encoded in yields. We find that neither duration nor convexity are spanned by bond yield factors. Running regressions from bond excess returns and bond volatility on duration (or convexity) and the bond yield factors still produces statistically and economically significant estimates. Moreover, we find that MBS duration partially drives out the level factor of bond yields as a predictor of future excess returns.

Our work is related to several strands in the asset pricing literature. We make use of the preferred habitat model of the term structure developed by Vayanos and Vila (2009). In their model, the term structure of interest rates is determined by the interaction of preferred habitat investors and risk-averse arbitrageurs. The latter incorporate expected short rates into bond prices and bring yields in line with each other by smoothing demand and supply pressure. These arbitrageurs trade the slope of the term structure by buying long-term bonds and selling short-term securities and vice versa. Since arbitrageurs are risk-averse, they demand higher risk premia as their relative exposure to long-term bonds increases. Thus, excess supply matters. Greenwood and Vayanos (2012) use this theoretical framework to study the implications of a change in the maturity structure of government debt supply, similar to the one undertaken in 2011 by the Federal Reserve during “Operation Twist”. Our paper is different in at least three respects. First, in our model the variation in the net supply of bonds is driven endogenously by hedging activity and not exogenously by the government. Second, the supply factor in Greenwood and Vayanos (2012) explains low frequency variation in risk premia. As the authors point out, movements in maturity-weighted government debt to GDP occur at a lower frequency than movements in the short rate. Our hedging demand factor, on the other hand, explains variations in risk premia at a higher frequency than movements in the level of interest rates. Finally, Greenwood and Vayanos (2012) posit that the government adjusts the maturity structure of its debt in a way that stabilizes bond markets. For instance, when interest rates are high, the government will finance itself with shorter

maturity debt and thereby reduce the quantity of interest rate risk held by agents. Our effect goes exactly in the opposite direction: Because of the negative convexity in MBS, hedging activity amplifies interest rates shocks.

This paper contributes to the literature on equilibrium term structure models, e.g., Le, Singleton, and Dai (2010), Xiong and Yan (2010), Ehling, Gallmeyer, Heyerdahl-Larsen, and Illeditsch (2013), Bansal and Shaliastovich (2013), and Le and Singleton (2013), among others. Contrary to these papers, in which bond risk premia are determined by macroeconomic fundamentals or differences in beliefs about these fundamentals, we focus on the aggregate demand and supply of bonds as the main driver of risk premia. Recent empirical work by Joslin, Pribsch, and Singleton (2012) and Le and Singleton (2013) document the existence of unspanned factors that are unrelated to the cross-section of yields but have a significant impact on risk premia. Moreover, Le and Singleton (2013) show that time-varying market prices of risk, in addition to time-varying quantities of risk, are required to explain bond risk premia. In our model, duration drives the variation in the market price of risk and hence, bond risk premia. Moreover, duration is unspanned by the yield factors. Finally, Joslin, Le, and Singleton (2013) and Fenou and Fontaine (2013) develop term structure models that include additional lags in the dynamics of yield factors. Similarly, in our paper, MBS duration depends on both current and past yields.

There is an important literature that studies the optimal prepayment in the MBS market; see, e.g., Schwartz and Torous (1989) for an example of a prepayment model, and Stanton (1995), Stanton and Wallace (1998), Longstaff (2005) and, more recently, Agarwal, Driscoll, and Laibson (2013) for examples of optimal prepayment. Closest to us are Gabaix, Krishnamurthy, and Vigneron (2007), who study the effect of limits to arbitrage in the MBS market and show that mortgage prepayment risk carries a positive risk premium. The authors do not study the term structure of interest rates and are mainly interested in the risk premium due to mortgage prepayment risk.

Our paper is also related in spirit to the recent literature that studies the role of hedgers and arbitrageurs in the determination of market prices of risk in commodities

futures (Acharya, Lochstoer, and Ramadorai (2013) and Hamilton and Wu (2012b)) or equity options markets (Gârleanu, Pedersen, and Poteshman (2009)).

This is not the first paper that studies how hedging in fixed income markets affects bond volatilities. For example, Duarte (2008) and Perli and Sack (2003) test the presence of a linkage between various proxies for hedging activity and either implied or realized volatilities. Duarte (2008) assumes that hedging in bond and interest rate options markets have separate effects on implied and realized volatilities. In contemporaneous empirical work, Hanson (2012) studies the impact of MBS duration onto bond excess returns. Li and Wei (2013) study a no arbitrage model of the term structure that includes an un-spanned MBS supply factor. Hamilton and Wu (2012a) estimate a no arbitrage term structure model with a Treasury supply factor to study the implications of “Operation Twist”. In our paper, we build a micro-founded model that allows us to inspect the mechanism through which hedging affects long-term interest rates. The model is embedded in a standard dynamic term structure framework and has implications for bond risk premia, and both implied and realized volatilities.

Finally, our paper is related to the literature that studies the effect of the recent Fed’s purchase of long-term assets on interest rates (see e.g., Gagnon, Raskin, Remache, and Sack (2010), Krishnamurthy and Vissing-Jorgensen (2011), D’Amico, English, Lopez-Salido, and Nelson (2012), D’Amico and King (2013)). While our model mainly focusses on the relationship between long-term yields and volatilities and hedging in MBS markets, other papers present evidence for alternative channels. For example, Krishnamurthy and Vissing-Jorgensen (2011) report that QE1, which involved large purchases of agency MBS, led not only to large reductions in mortgage rates, but also helped drive down Treasury yields and caused a drop in the default risk premium of corporate bonds. They interpret their findings as evidence for a long-term safety channel. D’Amico and King (2013) emphasize a scarcity channel, i.e. a localized effect of supply shocks on yields of nearby maturities.

The remainder of the paper is organized as follows. Section 1 sets up a model of the term structure of interest rates and hedging demand, and presents a calibration of this

model. Section 2 describes the data used and Section 3 empirically tests our hypotheses. Finally, Section 4 concludes. Proofs are deferred to the Appendix.

1 Model

Our model builds on the premise that bond prices are determined by the actions of different market participants: The supply of Treasuries by the government (Greenwood and Vayanos (2012)), demand by pension funds and other preferred-habitat investors (Vayanos and Vila (2009)), foreign investors (Beltran, Kretchmer, Marquez, and Thomas (2012)), and institutional investors like hedge funds and mutual funds (Lou, Yan, and Zhang (2013)).

In this paper, we propose a new channel founded in the interaction between institutional investors in mortgage-related securities, and risk-averse financial intermediaries. We proceed as follows. First, we describe the variation in the net supply of bonds resulting from hedging and portfolio re-balancing by mortgage investors. Then, we explicitly model intermediaries who behave as the marginal investors in bond markets and determine asset prices as a function of the net supply provided by all other market participants. We derive equilibrium prices in closed form and study how mortgage duration and convexity affect the first and second moments of bond yields and returns.

1.1 Bond market

Time is continuous and goes from zero to infinity. We denote the time- t price of a zero coupon bond paying one dollar at maturity $t + \tau$ by Λ_t^τ , and its yield by $y_t^\tau = -\frac{1}{\tau} \log \Lambda_t^\tau$. The short rate r_t is the limit of y_t^τ when $\tau \rightarrow 0$. We take r_t as exogenous and assume that its dynamics under the physical probability measure are given by

$$dr_t = \kappa(\theta - r_t)dt + \sigma dB_t, \tag{1}$$

where θ is the long run mean of r_t , κ is the speed of mean reversion, and σ is the volatility of the short rate.

At each date t , there exists a continuum of zero-coupon bonds with time to maturity $\tau \in (0, T]$ in total net supply of s_t^τ . We think of the net supply as the difference between the maturity structures of government debt and holdings of institutional investors. We focus exclusively on the transitory changes in the supply driven by the variation in the interest rate risk exposure of mortgage-related positions. Motivated by the data, we implicitly take the view that the interest rate risk exposure of mortgages with an embedded prepayment option varies more and at a higher frequency than other participants in the bond market adjust their issues and holdings. Specifically, we assume that the dollar duration of the net supply of all bonds depends exclusively on the MBS duration factor, D_t , that has the following dynamics

$$dD_t = -\kappa_D D_t dt + \eta_y (dy_t^{\bar{\tau}} - E_t[dy_t^{\bar{\tau}}]), \quad (2)$$

where $D_t \equiv -dMBS_t/dy_t^{\bar{\tau}}$ is the observable sensitivity of the aggregate MBS portfolio to the changes in a reference long-maturity rate $y_t^{\bar{\tau}}$. Note that D_t is itself buffeted by innovations to long rates and hence its dynamics are to be determined endogenously.³

Financial intermediaries invest in the bond market. Intermediaries are competitive and have mean-variance preference over the instantaneous change in the value of their bond portfolio. If x_t^τ denotes their holdings in maturity- τ bonds at time t , the intermediaries' budget constraint becomes

$$dW_t = \left(W_t - \int_0^T x_t^\tau \Lambda_t^\tau d\tau \right) r_t dt + \int_0^T x_t^\tau \Lambda_t^\tau \frac{d\Lambda_t^\tau}{\Lambda_t^\tau} d\tau, \quad (3)$$

and their optimization problem is given by

$$\max_{\{x_t^\tau\}_{\tau \in (0, T]}} E_t[dW_t] - \frac{\alpha}{2} \text{Var}_t[dW_t], \quad (4)$$

³Intuitively, we think of the reference maturity $\bar{\tau}$ as ten years. Our model could accommodate a more elaborate form for the dependence of the duration factor on yields. Yet, the parsimonious specification we propose captures the key property that the duration factor, which in our model determines bond risk premia, itself depends on long-term yields. Moreover, it provides an excellent empirical description of MBS duration dynamics. Figure 1 illustrates the close relationship between MBS duration and the ten-year yield (the plot looks virtually the same if we use the five-year yield instead, whose duration is closer to the average MBS duration).

where α is their absolute risk aversion. Since intermediaries have to take the other side of the trade in the bond market, the market clearing condition is given by

$$x_t^\tau = s_t^\tau, \quad (5)$$

for all t and τ .

1.2 Equilibrium term structure

Before solving for equilibrium yields, we determine the market price of interest rate risk.

Lemma 1. *Given (1), (3), (4), and (5), the unique market price of interest rate risk is proportional to the dollar sensitivity of the total net supply of bonds to the changes in the short rate:*

$$\lambda_t = \alpha \sigma \frac{d \left(\int_0^T s_t^\tau \Lambda_t^\tau d\tau \right)}{dr_t}. \quad (6)$$

Lemma 1 follows from the absence of arbitrage and implies that, regardless of the specific maturity composition of the net supply of bonds, the market price of interest rate risk is determined by its dollar duration. In particular, this means that in order to derive the equilibrium term structure, it is not necessary to explicitly model the full dynamics of the net supply of bonds, but it is sufficient to capture the sensitivity of the net supply of bonds to movements in interest rates, or the dollar duration of the net supply of bonds. Omitting constants, we rewrite (6) as

$$\lambda_t = -\alpha \sigma_y^{\bar{r}} D_t, \quad (7)$$

where $\sigma_y^{\bar{r}}$, the volatility of the reference yield $y_t^{\bar{r}}$, is a constant to be determined endogenously.

Equations (1), (2) and (7) together imply that the dynamics of the short rate and the MBS duration factor under the risk-neutral measure become

$$dr_t = [\kappa(\theta - r_t) + \alpha \sigma \sigma_y^{\bar{r}} D_t] dt + \sigma dB_t^{\mathbb{Q}} \text{ and} \quad (8)$$

$$dD_t = -\kappa_D^{\mathbb{Q}} D_t dt + \eta_y \sigma_y^{\bar{r}} dB_t^{\mathbb{Q}}, \quad (9)$$

where $\kappa_D^{\mathbb{Q}} = \kappa_D - \alpha\eta_y (\sigma_y^{\bar{\tau}})^2$ is the speed of mean reversion of MBS duration under the risk-neutral measure.

We now have all the ingredients to solve for the equilibrium term structure.

Theorem 1. *In the term structure model described by (8) and (9), equilibrium yields are*

$$y_t^{\tau} = \mathcal{A}(\tau) + \mathcal{B}(\tau) r_t + \mathcal{C}(\tau) D_t, \quad (10)$$

where

$$\mathcal{B}(\tau) = \frac{1 - e^{-\kappa\tau}}{\kappa\tau}, \quad (11)$$

$$\mathcal{C}(\tau) = -\frac{\alpha\sigma\sigma_y^{\bar{\tau}}}{\kappa - \kappa_D^{\mathbb{Q}}} \left(\frac{1 - e^{-\kappa\tau}}{\kappa\tau} - \frac{1 - e^{-\kappa_D^{\mathbb{Q}}\tau}}{\kappa_D^{\mathbb{Q}}\tau} \right), \quad (12)$$

$\mathcal{A}(\tau)$ is given by (A-8), and $\sigma_y^{\bar{\tau}}$ solves

$$\frac{\sigma_y^{\bar{\tau}}}{\sigma} = \frac{1 - e^{-\kappa\bar{\tau}}}{\kappa\bar{\tau}} - \frac{\kappa_D - \kappa_D^{\mathbb{Q}}}{\kappa - \kappa_D^{\mathbb{Q}}} \left(\frac{1 - e^{-\kappa\bar{\tau}}}{\kappa\bar{\tau}} - \frac{1 - e^{-\kappa_D^{\mathbb{Q}}\bar{\tau}}}{\kappa_D^{\mathbb{Q}}\bar{\tau}} \right). \quad (13)$$

Equation (13) has a solution whenever α is below a threshold $\bar{\alpha} > 0$.

Two comments are in order regarding our assumptions. First, as can be seen from equation (2), the duration of MBS is itself driven by innovations to interest rates. In reality, MBS duration depends primarily on the likelihood that borrowers re-finance their mortgages, which itself depends on the level of interest rates.⁴ Lower rates increase the probability of borrowers pre-paying their mortgages, leading to a lower duration. In

⁴The likelihood of refinancing depends also on other factors and not just the level of interest rates (see Boudoukh, Richardson, Stanton, and Whitelaw (1997)). For example, Chen, Michaux, and Roussanov (2012) show that mortgage holders are more likely to refinance when macroeconomic uncertainty is high even conditioning on the level of interest rates. Also, the choice between fixed and adjustable-rate mortgages, that influences the aggregate duration of outstanding mortgage portfolio, could depend on the entire term structure rather than a specific maturity. Koijen, van Hemert, and van Nieuwerburgh (2009) study the link between the term structure of yields and households' mortgage choice. The authors find high predictive power from the long-term bond risk premium onto the share of adjustable-rate mortgages.

other terms, the short American option embedded in mortgage loans is the source of negative (dollar) convexity, which is formally equal to:

$$\frac{d^2 MBS_t}{d(y_t^\tau)^2} = -\frac{dD_t}{dy_t^\tau} = -\eta_y < 0.$$

Second, while a unique shock is buffeting both the short rate and the MBS duration factor, the latter has its own frequency ($\kappa_D \neq \kappa$) and cannot, in general, be expressed as a function of the former. The resulting additional factor allows us to interpret MBS duration as depending on the history of interest rates.⁵ This is important because the borrowers' incentive to re-finance their mortgages is a function of the current level of interest rates relative to the average coupon paid by them on outstanding loans, which is akin to a weighted average of past interest rates.

1.3 Model implications

Our model has a series of implications regarding the effect of hedging demand on yields, expected returns, bond and yield volatility. We summarize them in two propositions from which we derive our empirical hypotheses.

Proposition 1. *The dollar duration of MBS positively predicts excess bond returns for all maturities and the effect is stronger for longer maturities. The effect of the dollar duration of MBS on yields is positive and, for a bounded set of maturities, is either increasing or hump-shaped in maturity.*

Proposition 1 is equivalent to the main result in Greenwood and Vayanos (2012) on the duration of government debt. The market price of interest-rate risk depends on the quantity of this risk that intermediaries hold to satisfy the hedging demand. In turn, bonds with higher exposure to interest-rate risk are more affected. As a result, MBS duration should predict excess bond returns and the effect is stronger for longer maturity bonds.

⁵Formally, when $\kappa_D \neq \kappa$, interest rates in our model are non-Markovian with respect to the short rate r_t alone. However, their history-dependence can be summarized by an additional Markovian factor, namely the duration D_t .

More formally, for the slope coefficient $\beta^{\tau,h}$ of the regression of excess returns on bonds with maturity τ over horizon h on the MBS duration factor D_t , we verify that $\lim_{\tau \rightarrow h} \beta^{\tau,h} = 0$ and $d\beta^{\tau,h}/d\tau > 0$ and hence $\beta^{\tau,h}$ is positive and increasing across maturities.

Note that the effect of MBS duration on the level of yields is not necessarily monotonic in the maturity. A yield depends on the average of risk premia over the life of the bond. Higher risk premia increase yields. However, because of mean reversion in interest rates and hedging activity, we expect risk premia at longer horizons to be lower.

Proposition 2. *The volatility of all yields is increasing in the negative convexity of MBS and the effect is strongest for intermediary maturities, i.e. is hump-shaped. The effect of negative convexity on the volatility of bond returns is positive and increasing in maturity.*

Higher MBS convexity implies that the quantity of duration risk and therefore the market price of risk are more sensitive to changes in interest rates. Moreover, because MBS convexity is negative, hedging activity amplifies, rather than offsets, the effect that the initial shock to the short rate has on long-term interest rates. As a result, interest rate volatility is higher.

The link between convexity and volatility has a term-structure dimension. Short-maturity yields are close to the short rate and therefore are not significantly affected by the variations in the market price of risk. For long maturities, we expect the duration of MBS to revert to its long-term mean. At the limit, yields at infinite horizon should not be affected by current changes in the short rate and MBS duration at all.⁶ As a result, the effect of MBS convexity on yield volatilities has a hump-shaped term structure.⁷

⁶This is just an application of a more general argument by Dybvig, Ingersoll, and Ross (1996) on why the yield volatility curve for long maturities should be downward sloping.

⁷When the negative convexity effect is strong enough, our model generates a hump-shaped term structure of unconditional yield volatilities. Note that under the risk-neutral measure, our model can be represented as an additive two-factor short rate model in which the factors are perfectly negatively correlated when $\kappa_D > \kappa$. Additive two-factor models with negatively correlated factors are known for being able to generate hump-shaped term structures of interest rate volatilities; see Brigo and Mercurio (2006).

Finally, even for low levels of intermediary risk aversion α , hedging activity can cause potentially significant volatility in yields and risk premia. This is because in addition to the direct amplification effect, where the fluctuations in the market price of risk amplify the effect of short rate shocks on long rates, negative convexity also creates a feedback loop effect. Through the latter, movements in long rates caused by fluctuations in MBS duration themselves affect MBS duration.

More formally, the effect of MBS convexity on interest rate volatility can be understood in two steps. First, for the instantaneous volatility of the maturity- τ yield, σ_y^τ , and $\gamma \equiv -\frac{d^2 MBS_t}{dr_t^2}$, we verify that $d\sigma_y^\tau/d\gamma > 0$. Moreover, $\lim_{\tau \rightarrow 0} \sigma_y^\tau = \sigma$ and $\lim_{\tau \rightarrow \infty} \sigma_y^\tau = 0$ are both independent of γ . Hence, yield volatility is increasing in γ , and since it does not change for extreme values, the effect is hump-shaped across maturities.

Next, we can relate the (negative of the) observed convexity η_y to γ . We show that around $\alpha = 0$,

$$\gamma \approx \frac{h_0 \eta_y}{1 - \alpha h_1 \eta_y},$$

where both linearization coefficients are positive: $h_0, h_1 > 0$. Compared to the $\alpha = 0$ case where MBS duration has no effect on interest rates, MBS duration sensitivity to short-rate shocks increases by a factor $\frac{1}{1 - \alpha h_1 \eta_y}$. In other words, because of the feedback between long rates and duration, a smaller shock to the short rate is required to produce an equilibrium change in duration of a given magnitude.

1.4 Calibration

With the model solutions at hand, we can now calibrate our model to the data. The goal is to check whether our model can jointly replicate the first and second moment features in the data, namely the predictive power of duration for bond excess returns and the hump shape of the term structure of bond yield and swaption implied volatility. To estimate the parameters of the duration and short yield, we discretize the processes and estimate them using weekly data from 1990 to 2012. Appendix B outlines all the calibration steps in detail.

We first estimate a simple duration model that captures the intuition that MBS duration depends on the current level of interest rates relative to a weighted average of past rates. Using the estimates, we set the parameters of the duration factor to $\kappa_D = 1.48$ and $\eta_y = 0.95$. In addition, we calibrate the short rate process (1) to historical data and obtain that its mean reversion and volatility are given by $\kappa = 0.13$ and $\sigma = 0.0133$, respectively. Finally, we match the volatility of the ten year yield by choosing α . Table 1 summarizes all parameters.⁸

Figure 2 illustrates the model fit for the duration time series. As we can see, the model explains almost all of the variation in MBS duration: The autoregressive part of the model produces an R^2 of 68%, adding the yield innovation increases the R^2 to 87%. This becomes even more evident if we compare our model to a model where we include all yield principal components and other potential factors that predict MBS duration. For example, following Chen, Michaux, and Roussanov (2012) who argue that macroeconomic uncertainty affects the refinancing activity of mortgage holders, we add as measure of macroeconomic uncertainty the level of VIX. As we can see in Figure 2, our model captures better the time series behavior of duration than a “PCA+” model, which includes all the yield principal components together with the VIX.

[Insert Table 1 and Figure 2 here.]

From Table 1, we note that the MBS duration factor has a higher frequency than the short rate, i.e., $\kappa_D > \kappa$. This implies that the short rate has a half-life of about five years whereas the half-life of duration is much shorter at six months. The higher mean-reversion captures the refinancing burnout effect. Low interest rates increase the probability of prepayment and lower the duration of MBS. However, aggregate MBS duration tends to reset when mortgages are eventually prepaid or simply mature and new MBS are issued based on new pools of mortgages with lower coupons. It is also interesting to compare the half-life of MBS duration to the numbers reported in Greenwood and Vayanos (2012). The calibrated parameters in their paper imply that shocks to government supply work at a much lower frequency than our duration factor. In their

⁸Calibrating the model to the five-year yield instead gives virtually identical results.

model, the half-life of government supply is around 33 years. This suggests that their factor moves at an even lower frequency than the business cycle, whereas aggregate MBS duration has a higher frequency.

Using these parameters, we can now check whether the model is able to reproduce several stylized facts in the data. We are first interested in whether the model can reproduce the hump shape in the term structure of yield volatility. The upper left panel of Figure 3 plots the term structure of bond yield volatility in the model and in the data. Overall, the fit is good and more importantly, consistent with the data, the model produces a hump at a maturity of two years.

Recall that in addition to the direct effect of MBS duration on long-term yields, negative convexity creates a feedback from yields to duration. To assess the importance of the feedback loop on yield volatilities, we compare our calibration results to the case where we allow only for a direct channel from duration onto yields.⁹ In the upper right panel of Figure 3, we plot three different term structures of bond yield volatility: (i) Based on a Vasicek model, (ii) based on a model that allows for the feedback and finally (iii) based on a model without feedback effects. We note that allowing for the additional duration factor but shutting down the feedback channel leads to an overall lower term structure of volatility, however, the hump shape is still present.

An additional way to illustrate the effect of convexity on volatility is to consider volatilities implied by swaptions. Because our model falls into the two-factor additive Gaussian class, swaption pricing is straightforward (see Appendix B). The lower left panel of Figure 3 reports implied volatilities on swaptions written on the 10-year swap rate. We note two things: First, the overall fit is very good, second, as volatilities of bond yields, the term structure of volatilities implied by swaptions displays a pronounced hump.

⁹Formally, this is done by replacing the endogenously determined duration factor volatility $\eta_y \sigma_y^{\bar{\tau}}$ by $\eta_y \mathcal{B}(\bar{\tau}) \sigma$, which still gives (13) for $\sigma_y^{\bar{\tau}}$ to solve in Theorem 1, but this time with $\kappa_D^{\mathbb{Q}} = \kappa_D - \alpha \eta_y \mathcal{B}(\bar{\tau}) \sigma \sigma_y^{\bar{\tau}}$.

The other main prediction from our model, related to Proposition 1, is that MBS duration predicts bond excess returns. To test this, we run regressions from model-implied bond excess returns onto duration:

$$rx_{t+1}^\tau = \alpha^\tau + \beta^\tau D_t + \epsilon_{t+1}^\tau,$$

where rx_{t+1}^τ are annual bond excess returns and D_t is the duration factor. Model-implied coefficients β^τ are available in closed form. The lower right panel of Figure 3 depicts the model-implied (solid line) and the empirical (dashed line) slope coefficients, $\hat{\beta}^\tau$, for each maturity τ . Consistent with the model prediction, we find that the effect of duration is increasing with maturity. Model-implied slope coefficients compare reasonably at short maturities, but underestimate the effect at longer maturities compared to the data.

[Insert Figure 3 here.]

An important question is how the duration factor and yield factors are linked. In our model, duration is not directly determined by the current level of interest rates, the past levels matter as well. However, because risk premia depend on MBS duration, the cross-section of yields should span the duration factor. In the calibrated model, the first principal component explains 99.94% of the variation in yields, with the second component fully accounting for the remaining 0.06%. At the same time, regressing the duration factor on the first principal component yields an R^2 of 36%.¹⁰ In other words, the yield factor that accounts for virtually all of the variation in the shape of the yield curve does not span D_t , which is the variable relevant for forecasting excess returns.

We summarize that our model is able to accommodate salient features of first and second moments of bond yields. Moreover, the model is also able to replicate the hump shape found in the term structure of volatilities of interest rate derivatives and the predictability of duration for bond risk premia.

¹⁰Note that the structure of our model allows us to compute the unconditional covariance matrix of yields in closed form and obtain the numbers above theoretically, rather than via simulations. See Appendix B.

2 Data

We use data from several sources. From Bloomberg, we collect data on mortgage securities and mortgage issuance. Furthermore, we use Treasury data from the Federal Reserve Board and agency bond index data from Datastream. Data are weekly and span the time period from January 1997 through December 2012 for a total of 835 observations.

2.1 Mortgage data

We use estimates of MBS duration and convexity from Barclays. The Barclays US MBS index covers mortgage-backed pass-through securities guaranteed by Ginnie Mae, Fannie Mae, and Freddie Mac. The index is comprised of pass-throughs backed by conventional fixed rate mortgages and is formed by grouping the universe of over one million agency MBS pools into generic pools based on agency, program (i.e., 30-year, 15-year, etc.), coupon (e.g., 6.0%, 6.5%, etc.), and vintage year (e.g., 2011, 2012, etc.). A generic pool is included in the index if it has a weighted-average contractual maturity greater than one-year and more than USD 250 million outstanding. The average duration in our sample is 4.5 years. The low average duration is mainly driven by the prepayment option of the underlying mortgages. There are two major drops in the duration time series: The first one in May 2003 and the second one in January 2009. We note that the first drop corresponds to the record high refinancing rate in the second half of 2002 and first half of 2003 due to exceptionally low levels of yields. The second fall corresponds to the Federal Reserve’s MBS purchase program which was announced in November 2008 but did not actually commence before January 2009 (Hancock and Passmore (2010)). MBS display negative convexity and the average convexity in our sample is around -1.5 .

2.2 Interest rates and excess returns

We use the Gürkaynak, Sack, and Wright (2007, GSW henceforth), zero coupon yield data available from the Federal Reserve Board. Unlike the Fama and Bliss discount bond database from CRSP, the GSW data is available at the weekly frequency. We use the raw data to calculate annual Treasury bond excess returns for two to ten year bonds.

We denote the return on a τ -year bond with log price Λ_t^τ by $r_{t+1}^\tau = \Lambda_{t+1}^{\tau-1} - \Lambda_t^\tau$. The annual excess bond return is defined as $rx_{t+1}^\tau \equiv r_{t+1}^\tau - y_t^1$, where y_t^1 is the one year yield. As we have weekly data, the annual excess returns are overlapping by 51 weeks. From the same data, we also construct a tent-shaped factor from forward rates, the Cochrane and Piazzesi (2005) factor (CP factor, labeled cp_t).

We also calculate the slope of the term structure as the difference between the ten year and the one year zero coupon yield (labeled $slope_t$). For short term interest rates, we take one-month and three-month USD LIBOR rates from Bloomberg.

2.3 Bond volatility

Using the GSW yields ranging from six months to ten years, we estimate a time-varying term structure of yield volatility. We sample the data at the weekly frequency and take log yield changes. We then construct rolling window measures of realized volatility which represent the conditional bond yield volatility. The resulting term structure of unconditional volatility exhibits a hump shape consistent with the stylized facts reported in Dai and Singleton (2010), with the volatility peak being at the two year maturity.

Mueller, Vedolin, and Yen (2012) calculate measures of model-free implied and realized bond market volatilities for the one-month horizon using Treasury futures and options data from the Chicago Mercantile Exchange (CME). We use their data for the 30-year Treasury bond and henceforth label the realized volatility trv_t and the implied volatility tiv_t . Moreover, the difference between the expected variance under the risk-neutral (tiv^2) and physical probability measure (trv^2) is defined as the variance risk premium ($vrp_t \equiv tiv_t^2 - trv_t^2$). We also construct a measure of volatility of variance for both the realized and implied proxies. Rolling volatility is calculated using a 52 week window and we label the two time-series $tivov_t$ and $trvov_t$, for implied and realized volatility of variance, respectively.

From Bloomberg, we also get implied volatility for at-the-money swaptions for different maturities ranging from 1 to 10 years and we fix the tenor to 10 years. We label these swaption implied volatilities by $iv^{\tau y 10y}$, where $\tau = 1, \dots, 10$.

Another way to capture bond price volatility risk is through returns on at-the-money straddles, which are portfolios mainly exposed to volatility risk (see Collin-Dufresne and Goldstein (2002)). We construct monthly straddle returns using at-the-money options written on the 30-year Treasury bond. We label this time-series straddle_t.

2.4 Other variables

Motivated by Hu, Pan, and Wang (2012) who report a link between swaption implied volatility and measures of liquidity in bond markets, we use their proxy of noise illiquidity, which measures an average yield pricing error from the Nelson, Siegel and Svensson model (see Svensson (2004)). We also use the funding liquidity proxy of Fontaine and Garcia (2012) which measures the on-/off-the-run premium across bonds with different maturities.

3 Empirical analysis

Based on our model, we deduce a number of empirical predictions that we now proceed to test in the data by running linear regressions. All regressions are standardized, meaning that we first de-mean and then divide each variable by its standard deviation to make slope coefficients comparable across different regressors.

3.1 Bond risk premia and bond yields

Hypothesis 1. *A regression of bond excess returns on the dollar duration of MBS yields a positive slope coefficient for all maturities, moreover the effect is stronger for longer maturities. The same applies to yields.*

This hypothesis is equivalent to Proposition 1. To test Hypothesis 1, we run linear regressions of excess returns and yields on MBS duration. The regression for excess returns is as follows:

$$rx_{t+1}^\tau = \beta_1^\tau \text{duration}_t + \beta_2^\tau \text{slope}_t + \beta_3^\tau \text{cp}_t + \epsilon_{t+1},$$

where slope_t is the slope at time t and cp_t is the CP factor at time t . The univariate results are depicted in Figure 4 (middle two panels) and multivariate results are presented in Table 2.

The univariate regression results indicate that MBS duration is a highly significant predictor of bond excess returns at all maturities. The link has the expected positive sign and is increasing with maturity: For any one standard deviation increase in duration there is a 210 basis point increase in expected bond returns at the ten year maturity. Adjusted R^2 range from 2% for the shortest maturity to 14% for the longest maturity.

Adding regressors does not deteriorate the significance of duration, moreover, estimated slope coefficients on duration remain remarkably stable. When we include the slope of the term structure and the CP factor to the regressions, the estimated coefficients on duration remain highly significant for five year maturities and longer. All three regressors combined explain between 21% and 46% of the time variation of annual bond excess returns.

The second part of Hypothesis 1 implies that higher duration increases yields and that the effect should be more pronounced for longer maturity bonds. We run the following regression:

$$y_t^\tau = \beta_1^\tau \text{duration}_t + \beta_2^\tau \text{slope}_t + \beta_3^\tau \text{cp}_t + \epsilon_t,$$

where slope_t is the slope at time t and cp_t is the CP factor at time t . The univariate regression results are presented in Figure 4 (upper two panels) and multivariate regression results are reported in Table 3. The slope coefficient has the expected sign and is highly significant as indicated by the 95% confidence intervals. Adjusted R^2 range from 18% for the shortest maturity to almost 30% for the ten year yield. The relationship between duration and bond yields is also economically significant: For a one standard deviation change in duration, there is a more than 50 basis point change in ten year yields. To relate our results with other work in the literature, we can compare these numbers to the findings in Gagnon, Raskin, Remache, and Sack (2011) and D’Amico, English, Lopez-Salido, and Nelson (2012), who report that the Fed’s QE1 and QE2 com-

binned lowered the ten year term premium by between 30 and 100 basis points. Since only QE1 involved the purchase of MBS, our numbers align nicely with these estimates. As for the multivariate regressions, we find that duration remains highly significant if we add the slope of the term structure or the CP factor to the regression. Furthermore, the effect of MBS duration on yields is still increasing with maturity.

[Insert Figure 4 and Tables 2 and 3 here.]

We conclude that there is a strong link between bond risk premia and levels of bond yields and MBS duration. The effect is stronger for longer maturity bonds and remains significant when we add other predictors to the regressions.

3.2 Bond yield volatility

Proposition 2 states that bond yield volatility is endogenously stochastic and driven by convexity, moreover, the effect should be the strongest at intermediate maturities (hump). We therefore test the following hypothesis in the data.

Hypothesis 2. *The volatility of all yields is increasing in the negative convexity of MBS. Moreover, the effect is the largest for intermediate maturities, i.e. is hump-shaped.*

We run the following regression from conditional bond yield volatility onto convexity and a set of other predictors:

$$\text{vol}_t^\tau = \beta_1^\tau \text{convexity}_t + \beta_2^\tau \text{tiv}_t + \beta_3^\tau \text{illiq}_t + \epsilon_t,$$

where vol_t^τ is the conditional bond yield volatility at time t of a bond with maturity $\tau = 1, \dots, 10$ years, tiv_t is the Treasury implied volatility at time t and illiq_t the illiquidity factor at time t . The univariate results are presented in the lower two panels of Figure 4, which plot the estimated slope coefficients of convexity, i.e. $\hat{\beta}_1$ (left panel), and the associated adjusted R^2 (right panel). In line with our model predictions, we find a significant effect from convexity onto bond yield volatility and the effect is the

largest for the intermediate maturities. The estimated slope coefficients produce the hump shaped feature similar to the one observed in the unconditional averages of yield volatility. Adjusted R^2 range from 6% for the shortest maturities, increase to 7% for the two and three year maturities and decrease again to 3% for longer maturities. Estimated coefficients are not only statistically but also economically significant: For any one standard deviation change in the MBS convexity, there is a 0.3 standard deviation change in the bond yield volatility, which corresponds to approximately 120 basis points. Multivariate regressions are presented in Table 4.

[Insert Table 4 here.]

When we add implied volatility or illiquidity into the regression, convexity still remains highly statistically significant. The estimated coefficients reveal that the effect is the largest for the intermediate maturities of five years as indicated by the size of the coefficient. This also makes sense economically because the average duration in our sample is around 4.5 years and hedging is therefore expected to focus on this maturity. Unsurprisingly, implied volatility loads positively on bond yield volatility. Higher bond yield volatility goes hand in hand with higher implied volatility in bond option markets. Illiquidity has the expected positive sign as bond volatility tends to be high when markets are illiquid. However, the effect becomes insignificant as we add the Treasury implied volatility into the regression. All three factors together explain between 50% and 62% of the time variation in bond yield volatility across different maturities.

3.3 Other volatility regressions

As outlined in Proposition 2, our model implies that the volatility of bond returns is positively related to the hedging demand. We formulate a testable empirical prediction in the following hypothesis:

Hypothesis 3. *Bond return volatility, both implied and realized, are increasing as MBS convexity is increasing. Moreover, the associated volatility of volatility and variance risk premia are increasing as well (in absolute terms).*

In the following, we use both implied volatility from options on Treasuries and swaptions. The reason is that hedging activity could potentially affect both. For example, Wooldridge (2001) notes that non-government securities were routinely hedged in the Treasury market until the financial crisis of 1998 when investors started hedging their interest rate exposure in the swaption market. This point is also made in Perli and Sack (2003), Duarte (2008), or Feldhütter and Lando (2008).

To test the above hypothesis, we run regressions of the following type:

$$\text{tiv}_t/\text{trv}_t/\text{iv}^{\tau y 10y}/\text{vrp}_t/\text{straddle}_t/\text{tivov}_t/\text{trvov}_t = \beta_1 \text{convexity}_t + \beta_2 \text{illiq}_t + \beta_3 \text{lagged}_{t-1} + \epsilon_t,$$

where tiv_t (trv_t) is the implied (realized) volatility of the 30 year Treasury bond at time t , $\text{iv}^{\tau y 10y}$ is the τ year maturity implied volatility from swaptions on the 10-year swap rate, vrp_t is the associated variance risk premium defined as the difference between the implied and realized measure, tivov_t (trvov_t) is the volatility of implied (realized) variance, illiq_t is the illiquidity measure and lagged_{t-1} are the LHS variables lagged for one period given the high persistence in the time-series itself. The volatility of variance regressions are motivated by the results in Perli and Sack (2003), who find that mortgage hedging has not only a direct effect on the level of volatility but also on the changes in expected volatility, which is larger whenever hedging is intense. The results of these regressions are found in Tables 5 and 6.

[Insert Tables 5 and 6 here.]

The data confirms our model predictions. The estimated coefficients are significant and also carry the expected sign: Higher MBS convexity implies a larger variance risk premium, larger implied and realized volatility, and a larger volatility of variance. In economic terms, this implies that for any one standard deviation increase in MBS convexity there is a 60 (40) basis point increase in implied (realized) volatility. MBS convexity also helps to explain returns of straddle strategies. The estimated slope coefficient of convexity is significant, albeit only at the 10% confidence level.

When we add other regressors, $\hat{\beta}_1$ remains significant. Illiquidity is significant and carries the expected positive sign: Higher illiquidity implies higher volatility, a higher associated variance risk premium, and higher returns on a straddle strategy.

The same picture emerges for implied volatilities from swaptions: Higher convexity induces higher volatility on swaptions. Estimated slope coefficients are significant and robust to the inclusion of other regressands. Interestingly, in the univariate regressions, we also find a hump shaped pattern similar to the one observed in the bond yield volatility regressions.

Overall, we conclude that both MBS duration and convexity are significantly linked to first and second moments of bond yields and bond returns. The empirical results are in line with the theoretical predictions and also hold when adding other control variables to the regressions.

3.4 What is MBS duration and convexity capturing beyond information in yields?

Households refinance mortgages when interest rates drop and it is therefore tempting to assume that MBS duration is a mere reflection of information already contained in yields. Most models in fixed income decompose movements in yields into principal components and argue that the first three factors explain most of the variation in yields. In the following, we ask whether MBS duration encodes any information that helps us to predict bond returns which is not already contained in bond yields. As a first test, we look at unconditional correlations between MBS duration and the first five principal components (PCs) given in Table 7 (Panel A). Unsurprisingly, we find that MBS duration is highly correlated with the first PC, the level factor, but has almost no correlation with higher order PCs. To test more formally whether duration is spanned by the yield factors we run the following regression similar to Joslin, Priebsch, and Singleton (2012):

$$\text{duration}_t = \alpha + \beta_1 \text{level}_t + \beta_2 \text{slope}_t + \beta_3 \text{curvature}_t + \epsilon_t,$$

where level_t , slope_t and curvature_t are the first three PCs from the cross-section of yields. The residual ϵ_t represent the risks which cannot be replicated using the yield PCs,

henceforth, we label this residual $\overline{\text{duration}}_t$. For our sample period, the projection of duration onto the first three PCs of yields gives us an adjusted R^2 of 25%. This means that 75% of the variation in MBS duration arises from risks which are distinct from these PCs. Moreover, a regression from the duration time series on bond yields should produce fitted residuals which are close to uncorrelated. In our regression, we find that the AR(1) of residuals is 0.94 and the associated Durbin and Watson statistic is 0.1, which clearly rejects the null of zero autocorrelation.

This now begs the question of whether duration contains any information beyond these principal components to predict bond returns. To test this, we regress bond excess returns on MBS duration and the first three PCs. Panel B of Table 7 shows the results. Once we add MBS duration into the regression, the coefficient for the level factor becomes insignificant for maturities equal and larger than five years. The significance of the duration factor, however, remains qualitatively very similar to the results reported in Table 2. Finally, Panel C reports the slope coefficients for the orthogonalized duration time series, $\overline{\text{duration}}_t$. Comparing significance of the coefficients for duration between Panel B and C, we virtually do not find any difference whether we use the orthogonalized or non-orthogonalized time series.

[Insert Table 7 here.]

Finally, it is also natural to investigate how MBS convexity is related to level, slope, and curvature of yields. Running a regression from MBS convexity onto the first three PCs, we find that the regressors explain around 40% of the variation and the AR(1) coefficient of residuals is 0.85. We therefore conclude that—just as duration—convexity is not spanned by the yield factors. Since in our model, convexity is mainly related to bond yield volatility, we repeat the same exercise using the PCs calculated from the cross-section of bond yield volatilities. Similar to the cross-section of yields, three factors are essentially enough to fully capture the dynamics of bond yield volatilities: The first three factors explain 91.8%, 6.5% and 1.15% of the overall variation, respectively. Running a regression from convexity onto the first three volatility PCs, we find that they explain little of the variation in convexity as the adjusted R^2 is only about 10%.

3.5 MBS duration and other funding factors

Le and Singleton (2013) demonstrate excellent forecasting power of proxies of funding risk for bond excess returns beyond the information contained in yields. In particular, the authors show that the realized jump mean constructed by Wright and Zhou (2009) has substantial forecasting power for bond excess returns relative to yield PCs. Interestingly, this jump amplitude factor is highly correlated with the funding liquidity proxy of Fontaine and Garcia (2012). In the following, we ask whether the MBS duration factor is subsumed by these proxies of funding risk. Our motivation is twofold: First, we note the rather high (negative) correlation between MBS duration and realized jump mean which is depicted in Figure 5. Secondly, similar to MBS duration, the mean jump size exhibits low correlation with the yield PCs: The highest correlation is with the 4th PC which is 25%. Running a regression from the mean jump size onto the yield PCs leads to an R^2 of only 14%.

If the mean jump size and MBS duration are both unspanned factors and both are possibly related to friction-like phenomena, then it is natural to ask how each of these factors would perform in multivariate predictive regressions of bond excess returns. To answer this question, we run monthly regressions from bond excess returns onto duration and measures of funding risk like the jump amplitude factor, J_t and the funding liquidity proxy of Fontaine and Garcia (2012), F_t . The results are reported in Panel A of Table 8.

[Insert Figure 5 and Table 8 here.]

The economic and statistical significance of MBS duration is not affected if we add either the funding liquidity or jump amplitude factor into the regression. Estimated coefficients for duration are significant for all maturities and remain quantitatively similar to the ones reported before.¹¹ In Panel B, we report adjusted R^2 from regressing bond excess returns onto the same factors but including the yield PCs. In line with the weekly

¹¹One could suspect that multicollinearity affects the results. To check this, we run the same regressions, but use the residual from a contemporaneous regression from the jump amplitude onto duration. The results do not change qualitatively with respect to those reported in Table 8.

results in Table 7, the first two lines reveal that including duration into the regression almost doubles the R^2 for long maturities. While adding the funding liquidity proxy does not alter the R^2 much, including J_t does increase the R^2 by another 10%.

3.6 MBS duration and macro factors

MBS duration depends on interest rate conditions, and hence, we expect some variation of duration to be driven by the macroeconomy. However, from our previous analysis, we know that duration moves at a much higher frequency than the business cycle since the half-life is only 6 months. In the following, we study whether duration explains anything of bond risk premia beyond macroeconomic factors.

Ludvigson and Ng (2009) exploit information in 132 different realized macroeconomic and financial series and explore the predictive content of these series for bond risk premia. The authors find that the main principal components extracted from this panel are statistically significant even in the presence of the CP factor and substantially improve predictability. To test whether our MBS duration factor is robust to the inclusion of these macroeconomic factors, we compute the eight static macroeconomic factors, F^j , $j = 1, \dots, 8$, for an updated data set through December 2012. In the following, we report regressions from bond risk premia onto duration and the macroeconomic factors at the monthly frequency. To save space, we defer the bond yield regressions to the Online Appendix.

[Insert Table 9 and Figure 6 here.]

Table 9 presents the results. While the effect of duration at the shortest maturities is negligible, at longer maturities, the impact remains almost unaltered compared to the results for weekly returns reported in Table 2. At longer maturities, almost none of the macroeconomic factors is significant and more importantly, the significance of the duration factor is virtually unchanged when moving from the univariate regression results to the regressions which include the macro factors. This is intuitive, as the macro

variables mainly affect the short-term rates because of monetary policy, whereas MBS duration mainly influences longer maturity assets.

To further improve our understanding of how duration affects bond risk premia, we plot in Figure 6 (upper panel) fitted values from an univariate regression from the 10-year bond risk premium onto duration. We can compare this with predicted values from a regression using the CP factor, which is said to be business cycle related. Two observations are noteworthy: First, the fitted values from the duration regression pick up some of the cyclicalities of the bond risk premium, similar to the CP factor. Second, duration does exceptionally well in predicting the bond risk premium in two distinct periods: 2003 and 2008. Reverting back to Figure 1, these are exactly the periods when refinancing activity was the highest.

Lastly, we can compare the predictive power of the jump amplitude factor which is plotted in Figure 6 (lower panel). Interestingly, the jump factor also tracks nicely the downturns in the bond risk premium in 2003 and 2008, similar to MBS duration.

We summarize our findings as follows: MBS duration is a strong predictor of bond risk premia at longer horizons and its predictive power is not subsumed by macroeconomic factors. Moreover, MBS duration seems to be related to friction-like phenomena.

3.7 The impact of MBS duration and convexity over time

Figure 7 plots the ratio of mortgages outstanding and GDP from 1990 to 2012 together with the amount outstanding in mortgages and Treasuries. As one can see, the importance of the mortgage market vis-à-vis both GDP and Treasuries has increased over the past 20 years. Both ratios peak in 2010 and since then have declined. If mortgage markets have become more important over time, one would expect the effect of MBS duration and convexity to increase over time. To test this hypothesis, we run similar regressions as above but interact the duration and convexity measures with the mortgage and GDP ratio. The results are reported in Table 10.

[Insert Figure 7 and Table 10 here.]

The results indicate that MBS duration and convexity are still highly significant predictors of bond excess returns and bond yield volatility. For the bond yield volatility regressions, we find that the interaction term is able to explain 24% of the time variation in long term bond yield volatility. Overall, we conclude that MBS duration and convexity are robust predictors for levels and second moments of bond yields.

3.8 Robustness

One issue with using annual bond excess returns is the low number of independent variables. In our data sample of 16 years, we have a maximum of 16 independent observations. To address this issue, we use actual bond returns for different maturity bins available from CRSP. Data is monthly and represents an equally-weighted average of holding period returns for each bond in the portfolio. We calculate excess returns by subtracting the T-bill. Because of the large impact of monetary policy on T-bills in the past couple of years, we also use the 1-month Eurodollar deposit rate. Results are reported in Table 11.

[Insert Table 11 here.]

We note that while the adjusted R^2 almost halves compared to the previous results, the estimated coefficient of duration is still significant. Moreover, these results hold whether we use the T-bill or the Eurodollar rate to construct the excess returns.

4 Conclusion

This paper studies the effect of dynamic hedging of large positions in mortgage backed securities on the term structure of bond yields, expected returns, and bond volatilities. We first build a micro-founded model of these demand pressures in bond markets and build it into an otherwise standard one factor term structure model. Despite its simple structure, our model has interesting implications for first and second moments of bond

yields: Our model is able to replicate the predictive power of MBS duration for bond excess returns and also features a hump shaped term structure of bond yield and swaption implied volatilities.

We then empirically test our model predictions using 16 years of data and find our hypotheses confirmed. For example, we find a strong positive link between measures of MBS duration, bond yields, and bond excess returns. The relationship is not only statistically significant but also economically relevant. This relationship remains strong when we add other standard regressors. We then proceed to study the relationship between MBS hedging and bond yield volatilities and bond variance risk premia. In line, with our theoretical predictions, we find that MBS convexity significantly affects bond yield volatilities and it produces the predicted hump shaped feature. A higher MBS convexity not only significantly increases bond yield volatility but also measures of bond return volatility and bond variance risk premia.

While MBS duration and convexity are naturally related to information in the term structure of bond yields, we provide novel evidence that duration and convexity are not spanned by the usual bond yield factors (principal components). For example, we show that MBS duration contains information which is beyond the one captured in the level factor. A more formal investigation of unspanning and its implications for second moments of bond returns is an exciting avenue which we leave to future research.

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Appendix A Proofs and derivations

Proof of Lemma 1. For notational simplicity let us suppose that bond prices are in the form

$$\frac{d\Lambda_t^\tau}{\Lambda_t^\tau} = \mu_t^\tau dt - \sigma_t^\tau dB_t. \quad (\text{A-1})$$

Substituting (A-1) into intermediaries' budget constraint, (3), we get

$$dW_t = \left[r_t W_t + \int_0^T x_t^\tau \Lambda_t^\tau (\mu_t^\tau - r_t) d\tau \right] dt - \left[\int_0^T x_t^\tau \Lambda_t^\tau \sigma_t^\tau d\tau \right] dB_t,$$

therefore (4) simplifies to

$$\max_{\{x_t^\tau\}_{\tau \in (0, T]}} \int_0^T x_t^\tau \Lambda_t^\tau (\mu_t^\tau - r_t) d\tau - \frac{\alpha}{2} \left[\int_0^T x_t^\tau \Lambda_t^\tau \sigma_t^\tau d\tau \right]^2. \quad (\text{A-2})$$

Because markets are complete, by no-arbitrage, there exists a unique market price of interest rate risk across all bonds that satisfies

$$\lambda_t = \frac{E_t \left(\frac{d\Lambda_t^\tau}{\Lambda_t^\tau} \right) / dt - r_t}{\frac{1}{\Lambda_t^\tau} \frac{d\Lambda_t^\tau}{dr_t} \sigma} = \frac{\mu_t^\tau - r_t}{-\sigma_t^\tau}, \quad (\text{A-3})$$

and introducing

$$x_t = \frac{d \left(\int_0^T x_t^\tau \Lambda_t^\tau d\tau \right)}{dr_t} = \int_0^T x_t^\tau \frac{d\Lambda_t^\tau}{dr_t} d\tau = -\frac{1}{\sigma} \int_0^T x_t^\tau \Lambda_t^\tau \sigma_t^\tau d\tau, \quad (\text{A-4})$$

for the total exposure of interest rate risk borne by intermediaries, their maximization problem (A-2) reduces to

$$\max_{x_t} \lambda_t x_t - \frac{\alpha \sigma}{2} x_t^2. \quad (\text{A-5})$$

The first order condition of (A-5) together with the market clearing condition (5) determine the equilibrium market price of risk and provides (6). \square

Proof of Theorem 1. We conjecture equilibrium yields in the model defined by (8) and (9) to be in the form (10), i.e. bond prices can be written as

$$\Lambda_t^\tau = e^{-[\tau \mathcal{A}(\tau) + \tau \mathcal{B}(\tau) r_t + \tau \mathcal{C}(\tau) D_t]}.$$

Writing down the standard pricing PDE, we obtain an equation that is affine in r_t and D_t . Collecting the r_t , D_t , and constant terms, respectively, we get a set of ODEs:

$$1 = \tau \mathcal{B}'(\tau) + \mathcal{B}(\tau) + \kappa \tau \mathcal{B}(\tau), \quad (\text{A-6})$$

$$0 = \tau \mathcal{C}'(\tau) + \mathcal{C}(\tau) + \kappa_D^{\mathbb{Q}} \tau \mathcal{C}(\tau) - \alpha \sigma \sigma_y^\tau \tau \mathcal{B}(\tau), \text{ and} \quad (\text{A-7})$$

$$0 = \tau \mathcal{A}'(\tau) + \mathcal{A}(\tau) - \kappa \theta \tau \mathcal{B}(\tau) + \frac{1}{2} \sigma^2 \tau^2 \mathcal{B}^2(\tau) + \frac{1}{2} (\sigma_y^\tau)^2 \tau^2 \mathcal{C}^2(\tau) + \sigma \sigma_y^\tau \tau^2 \mathcal{B}(\tau) \mathcal{C}(\tau),$$

with terminal conditions $\mathcal{A}(0) = \mathcal{B}(0) = \mathcal{C}(0) = 0$. The solution to (A-6) is then given by (11), and the solution to (A-6) is (12). Moreover, the function $\mathcal{A}(\tau)$ is given by

$$\begin{aligned} \mathcal{A}(\tau) = & \theta\omega(\kappa\tau) + \frac{\sigma^2}{\kappa^2} \left(\frac{\kappa - \kappa_D}{\kappa - \kappa_D^{\mathbb{Q}}} \right)^2 \left[\frac{1}{2}\omega(2\kappa\tau) - \omega(\kappa\tau) \right] \\ & + \frac{\sigma^2}{(\kappa_D^{\mathbb{Q}})^2} \left(\frac{\kappa_D - \kappa_D^{\mathbb{Q}}}{\kappa - \kappa_D^{\mathbb{Q}}} \right)^2 \left[\frac{1}{2}\omega(2\kappa_D^{\mathbb{Q}}\tau) - \omega(\kappa_D^{\mathbb{Q}}\tau) \right] \\ & + \frac{\sigma^2}{\kappa\kappa_D^{\mathbb{Q}}} \frac{\kappa_D - \kappa_D^{\mathbb{Q}}}{\kappa - \kappa_D^{\mathbb{Q}}} \frac{\kappa - \kappa_D}{\kappa - \kappa_D^{\mathbb{Q}}} \left[\omega\left(\left(\kappa + \kappa_D^{\mathbb{Q}}\right)\tau\right) - \omega(\kappa\tau) - \omega(\kappa_D^{\mathbb{Q}}\tau) \right], \end{aligned} \quad (\text{A-8})$$

where the function $\omega(\cdot)$ is defined as $\omega(x) = 1 - \frac{1-e^{-x}}{x}$ for all $x \neq 0$.

Finally, from (10), the volatility of the reference yield has to solve

$$\sigma_y^{\bar{\tau}} = \mathcal{B}(\bar{\tau})\sigma + \mathcal{C}(\bar{\tau})\eta_y\sigma_y^{\bar{\tau}}. \quad (\text{A-9})$$

Substituting (11) and (12) into this equation, we obtain (13).

To complete the proof of the Theorem, we show that there exists $\bar{\alpha} > 0$ such that (13) has a meaningful solution for all $0 \leq \alpha < \bar{\alpha}$. First, we show that all $\sigma_y^{\bar{\tau}}$ solutions of (13) are non-negative. It is straightforward from (11) that $\mathcal{B}(\tau)$ is positive and decreasing. Also, as the function $x \mapsto \frac{1-e^{-x}}{x}$ is decreasing, from (12) it must be that $\mathcal{C}(\tau)$ and $\sigma_y^{\bar{\tau}}$ have the same sign. However, as $\eta_y > 0$, the right-hand side of (A-9) is then positive, implying both $\sigma_y^{\bar{\tau}} \geq 0$ and $\mathcal{C}(\tau) \geq 0$.

Second, following Greenwood and Vayanos (2012), a sufficient condition for the existence of a solution to (13) is that the left-hand side is greater than the right-hand side when $\kappa_D^{\mathbb{Q}} = 0$ - this means that under the risk-neutral measure the model does not explode. After some algebra, this is equivalent to

$$\alpha < \frac{\kappa_D}{\eta_y \left(\frac{\kappa - \kappa_D}{\kappa} \frac{1 - e^{-\kappa\bar{\tau}}}{\kappa\bar{\tau}} + \frac{\kappa_D}{\kappa} \right)^2 \sigma^2}. \quad (\text{A-10})$$

Defining $\bar{\alpha}$ as the right-hand side of (A-10), which is certainly positive, (13) has at least one solution whenever $\alpha < \bar{\alpha}$. \square

Proof of Proposition 1. The theoretical slope coefficient of the regression of excess returns over horizon h on maturity- τ bonds on the duration factor D_t is equal to

$$\begin{aligned} \beta^{\tau,h} &= \tau\mathcal{C}(\tau) - h\mathcal{C}(h) - (\tau - h)\mathcal{C}(\tau - h)e^{-\kappa_D h}, \\ &= -\frac{\alpha\sigma\sigma_y^{\bar{\tau}}e^{-\kappa_D h}}{\kappa - \kappa_D^{\mathbb{Q}}} \left[\frac{1 - e^{-\kappa_D^{\mathbb{Q}}(\tau-h)}}{\kappa_D^{\mathbb{Q}}} \left(1 - e^{-(\kappa_D^{\mathbb{Q}} - \kappa_D)h} \right) - \frac{1 - e^{-\kappa(\tau-h)}}{\kappa} \left(1 - e^{-(\kappa - \kappa_D)h} \right) \right]. \end{aligned}$$

Regarding the sign of $\beta^{\tau,h}$ and its behavior across maturities, first we have $\lim_{\tau \rightarrow h} \beta^{\tau,h} = 0$. Moreover,

$$\frac{d\beta^{\tau,h}}{d\tau} = -\frac{\alpha\sigma\sigma_y^{\bar{\tau}}e^{-\kappa_D h}}{\kappa - \kappa_D^{\mathbb{Q}}} \left[e^{-\kappa_D^{\mathbb{Q}}(\tau-h)} \left(1 - e^{-(\kappa_D^{\mathbb{Q}} - \kappa_D)h} \right) - e^{-\kappa(\tau-h)} \left(1 - e^{-(\kappa - \kappa_D)h} \right) \right].$$

We focus only on the case when $\kappa_D > \kappa$, because it holds for the parameters we obtain from calibrating the model, see Table 1. If also $\kappa_D^{\mathbb{Q}} > \kappa$, the term inside the bracket is positive, and $d\beta^{\tau,h}/d\tau > 0$. If, on the other hand, $\kappa_D^{\mathbb{Q}} < \kappa < \kappa_D$, the term in the bracket is negative and $d\beta^{\tau,h}/d\tau > 0$ again. Therefore, it is sufficient that $\kappa < \kappa_D$ to have $d\beta^{\tau,h}/d\tau > 0$, and hence $\beta^{\tau,h}$ being positive and increasing in maturities.

The effect of duration on yields, from (10), is given by $\mathcal{C}(\tau)$. From the Proof of Theorem 1, $\mathcal{C}(\tau) \geq 0$. Moreover, it is easy to show that

$$\lim_{\tau \rightarrow 0} \mathcal{C}(\tau) = \lim_{\tau \rightarrow \infty} \mathcal{C}(\tau) = 0,$$

which implies that the effect is either increasing across maturities if τ is small, or first increasing then decreasing if τ is sufficiently large. This completes the proof. \square

Proof of Proposition 2. From (10), bond yield volatility is given by

$$\sigma_y^\tau = \mathcal{B}(\tau) \sigma + \mathcal{C}(\tau) \eta_y \sigma_y^{\bar{\tau}}. \quad (\text{A-11})$$

We determine the effect of convexity $-\eta_y$ on yield volatilities in two steps.

First, we focus on $\frac{d\sigma_y^\tau}{d\gamma}$, where $\gamma \equiv -\frac{d^2 MBS_t}{dr_t^2}$. Straightforward from its definition, we have

$$\gamma \sigma^2 = \eta_y (\sigma_y^{\bar{\tau}})^2, \quad (\text{A-12})$$

and hence $\kappa_D^{\mathbb{Q}} = \kappa_D - \alpha \sigma^2 \gamma$. Using (12) and (A-12) we rewrite $\mathcal{C}(\tau) \eta_y \sigma_y^{\bar{\tau}}$ as

$$\mathcal{C}(\tau) \eta_y \sigma_y^{\bar{\tau}} = -\frac{\alpha \sigma^3 \gamma}{\kappa - \kappa_D^{\mathbb{Q}}} \left(\frac{1 - e^{-\kappa \tau}}{\kappa \tau} - \frac{1 - e^{-\kappa_D^{\mathbb{Q}} \tau}}{\kappa_D^{\mathbb{Q}} \tau} \right),$$

and since $\mathcal{B}(\tau)$ is only a function of κ , we obtain

$$\begin{aligned} \frac{d\sigma_y^\tau}{d\gamma} &= -\frac{\alpha \sigma^3}{(\kappa - \kappa_D^{\mathbb{Q}})} \left(\frac{1 - e^{-\kappa \tau}}{\kappa \tau} - \frac{1 - e^{-\kappa_D^{\mathbb{Q}} \tau}}{\kappa_D^{\mathbb{Q}} \tau} \right) \\ &\quad + \frac{\alpha \sigma^3 (\kappa_D - \kappa_D^{\mathbb{Q}})}{(\kappa - \kappa_D^{\mathbb{Q}})^2} \left(\frac{1 - e^{-\kappa \tau}}{\kappa \tau} - \frac{1 - e^{-\kappa_D^{\mathbb{Q}} \tau}}{\kappa_D^{\mathbb{Q}} \tau} + \left(\kappa - \kappa_D^{\mathbb{Q}} \right) \frac{1 - e^{-\kappa_D^{\mathbb{Q}} \tau} - \kappa_D^{\mathbb{Q}} \tau e^{-\kappa_D^{\mathbb{Q}} \tau}}{(\kappa_D^{\mathbb{Q}})^2 \tau} \right). \end{aligned} \quad (\text{A-13})$$

As the function $f(x) = \frac{1 - e^{-x\tau}}{x\tau}$ is decreasing for any $\tau > 0$, the first term on the RHS is always positive. Moreover, it is also a convex function, hence

$$\frac{1 - e^{-\kappa \tau}}{\kappa \tau} - \frac{1 - e^{-\kappa_D^{\mathbb{Q}} \tau}}{\kappa_D^{\mathbb{Q}} \tau} + \left(\kappa - \kappa_D^{\mathbb{Q}} \right) \frac{1 - e^{-\kappa_D^{\mathbb{Q}} \tau} - \kappa_D^{\mathbb{Q}} \tau e^{-\kappa_D^{\mathbb{Q}} \tau}}{(\kappa_D^{\mathbb{Q}})^2 \tau} \geq 0,$$

and the second term on the RHS of (A-13) is also positive. Therefore, $\frac{d\sigma_y^\tau}{d\gamma} > 0$.

Second, we relate γ to η_y by considering (A-12) around $\alpha = 0$. From (A-11) we have

$$\left(\frac{\sigma_y^{\bar{\tau}}}{\sigma} \right)^2 = \left(\mathcal{B}(\bar{\tau}) + \mathcal{C}(\bar{\tau}) \frac{\sigma_D}{\sigma} \right)^2 \approx h_0 + \alpha h_1 \gamma, \quad (\text{A-14})$$

where

$$h_0 \equiv \left(\mathcal{B}(\bar{\tau}) + \mathcal{C}(\bar{\tau}) \frac{\sigma_D}{\sigma} \right)^2 \Big|_{\alpha=0} = \mathcal{B}^2(\bar{\tau}), \text{ and}$$

$$h_1 \equiv \frac{1}{\gamma} \frac{d \left(\mathcal{B}(\bar{\tau}) + \mathcal{C}(\bar{\tau}) \frac{\sigma_D}{\sigma} \right)^2}{d\alpha} \Big|_{\alpha=0} = -2\mathcal{B}(\bar{\tau}) \frac{\sigma^2}{\kappa - \kappa_D} \left(\frac{1 - e^{-\kappa\bar{\tau}}}{\kappa\bar{\tau}} - \frac{1 - e^{-\kappa_D\bar{\tau}}}{\kappa_D\bar{\tau}} \right).$$

Combining (A-12) and (A-14), we obtain that around $\alpha = 0$,

$$\gamma = \frac{h_0 \eta_y}{1 - \alpha h_1 \eta_y}.$$

Since $h_0, h_1 > 0$, γ is increasing in η_y , and together with the first part of the proof we get

$$\frac{d\sigma_y^\tau}{d\eta_y} > 0. \quad (\text{A-15})$$

Third, we trivially verify that

$$\lim_{\tau \rightarrow 0} \sigma_y^\tau = \sigma \text{ and } \lim_{\tau \rightarrow \infty} \sigma_y^\tau = 0.$$

Therefore, the effect of negative convexity on yield volatilities tends to zero at very short and very long maturities, and hence it must be hump-shaped.

Finally, since bond return volatility satisfies $\sigma_t^\tau = \tau \sigma_y^\tau$, (A-15) also implies that bond return volatility increases when convexity becomes more negative. \square

Appendix B Calibration

Duration dynamics. The discrete-time dynamics of the duration factor and the reference 10-year yield in our model can be approximated (omitting constant terms) by¹²

$$\begin{bmatrix} y_{t+\Delta} \\ D_{t+\Delta} \end{bmatrix} = \begin{bmatrix} 1 - \kappa\Delta & (\kappa - \kappa_D)\Delta\mathcal{C} \\ 0 & 1 - \kappa_D\Delta \end{bmatrix} \begin{bmatrix} y_t \\ D_t \end{bmatrix} + \begin{bmatrix} \mathcal{B}\sigma + \mathcal{C}\sigma_D \\ \sigma_D \end{bmatrix} \sqrt{\Delta}\epsilon_t.$$

We specify the following model for duration that has two components. First, duration is affine in the reference 10-year yield, y^τ , and a “coupon variable”, w_t , that captures the path-dependence of duration

$$D_t = \eta_y y_t^\tau + \eta_w w_t.$$

Second, average mortgage coupons change with actual yields, we assume that

$$dw_t = \kappa_w (y_t^\tau - w_t) dt, \quad (\text{A-16})$$

where (A-16) can be approximated in discrete time by

$$w_{t+\Delta} = (\kappa_w \Delta) y_t^\tau + (1 - \kappa_w \Delta) w_t.$$

¹²The factor dynamics are approximated by

$$\begin{bmatrix} r_{t+\Delta} \\ D_{t+\Delta} \end{bmatrix} = \begin{bmatrix} 1 - \kappa\Delta & 0 \\ 0 & 1 - \kappa_D\Delta \end{bmatrix} \begin{bmatrix} r_t \\ D_t \end{bmatrix} + \begin{bmatrix} \sigma \\ \sigma_D \end{bmatrix} \sqrt{\Delta}\epsilon_t.$$

We can then substitute out the coupon variable w_t . Estimating this equation using monthly data ($\Delta = 1/12$) on MBS duration and 10-year yields, we find the following point estimates

$$D_{t+\Delta} = \underbrace{(1 - \kappa_w \Delta)}_{0.89 \text{ (32.37)}} D_t + \underbrace{\eta_y}_{0.94 \text{ (17.47)}} y_{t+\Delta}^{\bar{}} + \underbrace{[\eta_w \kappa_w \Delta - \eta_y (1 - \kappa_w \Delta)]}_{-0.92 \text{ (-16.54)}} y_t^{\bar{}}, \quad (\text{A-17})$$

where the numbers in parentheses are Newey and West (1987) corrected t-statistics. The regression yields an adjusted R^2 of 87%.

Figure 3 illustrates the model fit. Using the above estimates of η_y and $\eta_w \kappa_w \Delta - \eta_y (1 - \kappa_w \Delta)$ note that if interest rates are persistent enough we have that

$$\eta_y (1 - \kappa \Delta) + [\eta_w \kappa_w \Delta - \eta_y (1 - \kappa_w \Delta)] \approx 0$$

and

$$-\eta_y \kappa + (\eta_y + \eta_w) \kappa_w \approx 0.$$

In other words, duration is well described by a process buffeted by innovations to the 10-year yield but with a mean reversion different from that of interest rates

$$dD_t = [-\kappa_w + \eta_y \mathcal{C} (\kappa - \kappa_D)] D_t dt + \eta_y (dy_t^{\bar{}} - E_t [dy_t^{\bar{}}]),$$

thus justifying the form of the process (2).

We re-estimate (A-17) explicitly imposing the restriction (2) and obtain the following point estimates

$$D_{t+\Delta} = \underbrace{(1 - \kappa_w \Delta)}_{0.91 \text{ (40.49)}} D_t + \underbrace{\eta_y}_{0.95 \text{ (17.74)}} y_{t+\Delta}^{\bar{}} + \underbrace{[\eta_w \kappa_w \Delta - \eta_y (1 - \kappa_w \Delta)]}_{-0.94 \text{ (-17.74)}} y_t^{\bar{}}, \quad (\text{A-18})$$

with an adjusted R^2 of 87%. Finally, we calibrate the parameters of the duration factor such that

$$-\kappa_w + \eta_y \mathcal{C} (\kappa - \kappa_D) = \kappa_D$$

and

$$\sigma_D = \eta_y (\mathcal{B} \sigma + \mathcal{C} \sigma_D).$$

Swaption implied volatility. At time t the k -year forward swap rate that starts at $t + \tau$ and pays every h years is given by

$$S_t^{\tau, \tau+k} = \frac{\Lambda_t^{\tau} - \Lambda_t^{\tau+k}}{h \sum_{i=1}^{k/h} \Lambda_t^{\tau+ih}},$$

which is also the strike rate of the corresponding ATM swaption.

Since our model falls into the two-factor additive Gaussian class, exact swaption prices can be numerically computed based on Theorem 4.2.3 of Brigo and Mercurio (2006). Alternatively, we can use an approximation based on Schrager and Pelsner (2006). After some algebra, the price at time t of an ATM payer or receiver swaption with maturity τ and tenor k (i.e., the underlying swap starts at $t + \tau$ and ends at $t + \tau + k$) that pays every h years, is approximately

$$PS_t^{ATM} = \frac{\sigma}{\sqrt{2\pi}} \Sigma_{\tau, \tau+k},$$

where

$$\begin{aligned}\Sigma_{\tau, \tau+k}^2 &= \left(\frac{\kappa - \kappa_D}{\kappa - \kappa_D^{\mathbb{Q}}} \right)^2 \frac{e^{2\kappa\tau} - 1}{2\kappa} \xi_1^2 + \left(\frac{\alpha \eta_y (\sigma_y^{\bar{\tau}})^2}{\kappa - \kappa_D^{\mathbb{Q}}} \right)^2 \frac{e^{2\kappa_D^{\mathbb{Q}}\tau} - 1}{2\kappa_D^{\mathbb{Q}}} \xi_2^2 \\ &\quad + 2 \frac{\kappa - \kappa_D}{\kappa - \kappa_D^{\mathbb{Q}}} \frac{\alpha \eta_y (\sigma_y^{\bar{\tau}})^2}{\kappa - \kappa_D^{\mathbb{Q}}} \frac{e^{(\kappa + \kappa_D^{\mathbb{Q}})\tau} - 1}{\kappa + \kappa_D^{\mathbb{Q}}} \xi_1 \xi_2,\end{aligned}$$

with

$$\xi_1 = e^{-\kappa\tau} \Lambda_t^\tau - e^{-\kappa(\tau+k)} \Lambda_t^{\tau+k} - \left(\Lambda_t^{\tau+k} - \Lambda_t^\tau \right) \frac{\sum_{i=1}^{k/h} e^{-\kappa(\tau+ih)} \Lambda_t^{\tau+ih}}{\sum_{i=1}^{k/h} \Lambda_t^{\tau+ih}},$$

and

$$\xi_2 = e^{-\kappa_D^{\mathbb{Q}}\tau} \Lambda_t^\tau - e^{-\kappa_D^{\mathbb{Q}}(\tau+k)} \Lambda_t^{\tau+k} - \left(\Lambda_t^{\tau+k} - \Lambda_t^\tau \right) \frac{\sum_{i=1}^{k/h} e^{-\kappa_D^{\mathbb{Q}}(\tau+ih)} \Lambda_t^{\tau+ih}}{\sum_{i=1}^{k/h} \Lambda_t^{\tau+ih}}.$$

From here, the swaption implied volatility, v_τ , solves

$$PS_t^{ATM} = S_t^{\tau, \tau+k} \left[2\Phi \left(\frac{v_\tau \sqrt{\tau}}{2} \right) - 1 \right].$$

Setting the year fraction to be $h = 0.25$ and the tenor $k = 10$ years, the lower left panel of Figure 3 plots the swaption implied volatility for different maturities τ in our model, a one-factor Vasicek model (i.e., when setting $\alpha = 0$), and contrasts them with the data.

Principal components. The unconditional covariance matrix of $\mathcal{F}_t = [r_t \ D_t]^\top$ is given by

$$\Sigma_{\mathcal{F}} = \begin{pmatrix} \frac{\sigma^2}{2\kappa} & \frac{\sigma\sigma_D}{\kappa + \kappa_D} \\ \frac{\sigma\sigma_D}{\kappa + \kappa_D} & \frac{\sigma_D^2}{2\kappa_D} \end{pmatrix}.$$

Equation (10) implies that yields are affine in factors and the vector of yields for maturities of interest can be written $y_t = A + B\mathcal{F}_t$. The unconditional covariance matrix of yields is readily obtained as

$$\Sigma_y = B\Sigma_{\mathcal{F}}B^\top.$$

Principal components of yields are

$$pc_t = \mathcal{Q}^\top y_t,$$

where \mathcal{Q} is the matrix of ordered eigenvectors of Σ_y . Given the overall affine structure, it is possible to calculate the theoretical covariance matrix of principal components and factors, theoretical regression coefficients and R^2 s.

Appendix C Tables

Table 1
Calibrated parameters

This table reports parameters used for the calibration exercise. Appendix [B](#) outlines all the calibration steps in detail. Data is weekly and runs from 1990 through 2012.

κ	0.1300
σ	0.0133
κ_D	1.4800
η_y	0.9500
$\alpha\sigma_y^{\bar{\tau}}$	45

Table 2
Bond risk premia regressions

This table reports estimated coefficients from regressing annual bond excess returns, rx_{t+1}^τ , onto a set of variables:

$$rx_{t+1}^\tau = \beta_1^\tau \text{duration}_t + \beta_2^\tau \text{slope}_t + \beta_3^\tau \text{cp}_t + \epsilon_{t+1}^\tau,$$

where slope_t is the slope of the term structure at time t and cp_t is the CP factor at time t . t-Statistics presented in parentheses are calculated using Newey and West (1987). All variables are standardized to have mean zero and a standard deviation of one. Data is weekly and runs from 1997 through 2011.

	2y	3y	4y	5y	6y	7y	8y	9y	10y
duration	0.116 (1.63)	0.167 (2.28)	0.212 (2.88)	0.251 (3.38)	0.285 (3.80)	0.314 (4.16)	0.340 (4.46)	0.362 (4.72)	0.381 (4.96)
Adj. R^2	1.34%	2.78%	4.48%	6.29%	8.11%	9.87%	11.53%	13.07%	14.51%
duration	0.121 (1.60)	0.197 (2.51)	0.263 (3.37)	0.319 (4.17)	0.367 (4.91)	0.408 (5.58)	0.442 (6.18)	0.471 (6.73)	0.495 (7.24)
slope	0.022 (0.21)	0.137 (1.32)	0.229 (2.25)	0.306 (3.10)	0.370 (3.87)	0.421 (4.55)	0.461 (5.14)	0.491 (5.64)	0.513 (6.06)
Adj. R^2	1.26%	4.43%	9.35%	15.09%	21.02%	26.64%	31.66%	35.95%	39.48%
duration	-0.006 (-0.07)	0.080 (0.94)	0.152 (1.81)	0.215 (2.63)	0.269 (3.39)	0.316 (4.11)	0.357 (4.77)	0.392 (5.39)	0.423 (5.95)
slope	-0.186 (-1.86)	-0.056 (-0.57)	0.047 (0.49)	0.134 (1.44)	0.208 (2.27)	0.270 (2.97)	0.321 (3.56)	0.362 (4.03)	0.394 (4.40)
cp factor	0.502 (6.49)	0.464 (6.23)	0.437 (5.88)	0.413 (5.49)	0.389 (5.09)	0.364 (4.68)	0.338 (4.28)	0.312 (3.89)	0.287 (3.52)
Adj. R^2	21.57%	21.83%	24.76%	28.87%	33.23%	37.32%	40.87%	43.79%	46.09%

Table 3
Bond yield regressions

This table reports estimated coefficients from regressing bond yields, y_t^T , onto a set of variables:

$$y_t^T = \beta_1^T \text{duration}_t + \beta_2^T \text{slope}_t + \beta_3^T \text{cp}_t + \epsilon_t^T,$$

where slope_t is the slope of the term structure at time t and cp_t is the CP factor at time t . t-Statistics presented in parentheses are calculated using Newey and West (1987). All variables are standardized to have mean zero and a standard deviation of one. Data is weekly and runs from 1997 through 2012.

	1y	2y	3y	4y	5y	6y	7y	8y	9y	10y
duration	0.400 (5.31)	0.442 (5.83)	0.470 (6.11)	0.491 (6.24)	0.507 (6.27)	0.519 (6.24)	0.528 (6.16)	0.534 (6.06)	0.537 (5.96)	0.539 (5.87)
Adj. R^2	16.04%	19.52%	22.07%	24.08%	25.69%	26.95%	27.87%	28.49%	28.85%	29.01%
duration	0.215 (4.42)	0.268 (4.88)	0.308 (5.02)	0.341 (5.05)	0.368 (5.04)	0.391 (5.02)	0.410 (4.98)	0.425 (4.95)	0.437 (4.91)	0.446 (4.88)
slope	-0.854 (-27.40)	-0.801 (-21.12)	-0.745 (-17.00)	-0.691 (-14.28)	-0.639 (-12.28)	-0.589 (-10.71)	-0.542 (-9.42)	-0.499 (-8.36)	-0.460 (-7.48)	-0.425 (-6.74)
Adj. R^2	85.57%	80.58%	74.93%	69.55%	64.56%	59.99%	55.86%	52.18%	48.96%	46.18%
duration	0.039 (2.32)	0.079 (3.24)	0.101 (3.39)	0.115 (3.43)	0.125 (3.43)	0.133 (3.41)	0.139 (3.38)	0.144 (3.34)	0.148 (3.29)	0.152 (3.24)
slope	-1.031 (-44.82)	-0.998 (-29.29)	-0.970 (-22.93)	-0.945 (-19.72)	-0.921 (-17.69)	-0.896 (-16.18)	-0.869 (-14.94)	-0.841 (-13.84)	-0.813 (-12.86)	-0.784 (-11.97)
cp factor	0.261 (12.69)	0.283 (10.00)	0.320 (9.21)	0.362 (9.22)	0.403 (9.48)	0.442 (9.77)	0.476 (9.98)	0.505 (10.09)	0.528 (10.09)	0.545 (10.00)
Adj. R^2	95.94%	92.05%	88.31%	85.22%	82.59%	80.18%	77.83%	75.46%	73.05%	70.62%

Table 4
Bond yield volatility regressions

This table reports estimated coefficients from regressing bond yield volatility, vol_t^τ , onto a set of variables.

$$\text{vol}_t^\tau = \beta_1^\tau \text{convexity}_t + \beta_2^\tau \text{illiq}_t + \beta_3^\tau \text{tiv}_t + \epsilon_t^\tau,$$

where tiv_t is the implied volatility of options on the 30-year Treasury at time t and illiq_t the illiquidity factor at time t . t-Statistics presented in parentheses are calculated using Newey and West (1987). All variables are standardized to have mean zero and a standard deviation of one. Data is weekly and runs from 1997 through 2011.

	1y	2y	3y	4y	5y	6y	7y	8y	9y	10y
convexity	0.248 (3.24)	0.264 (3.38)	0.258 (3.32)	0.247 (3.19)	0.231 (3.01)	0.213 (2.81)	0.195 (2.61)	0.180 (2.44)	0.166 (2.28)	0.154 (2.15)
Adj. R^2	6.17%	6.96%	6.66%	6.10%	5.34%	4.54%	3.82%	3.22%	2.75%	2.38%
convexity	0.301 (4.51)	0.312 (4.50)	0.307 (4.49)	0.299 (4.46)	0.286 (4.38)	0.271 (4.27)	0.257 (4.15)	0.243 (4.01)	0.231 (3.87)	0.220 (3.73)
illiq	0.478 (6.52)	0.438 (5.37)	0.450 (5.20)	0.474 (5.17)	0.505 (5.19)	0.535 (5.18)	0.560 (5.10)	0.579 (4.95)	0.592 (4.78)	0.601 (4.61)
Adj. R^2	28.64%	25.82%	26.54%	28.21%	30.40%	32.68%	34.69%	36.25%	37.34%	38.02%
convexity	0.183 (3.51)	0.167 (3.29)	0.159 (3.18)	0.152 (3.08)	0.142 (2.94)	0.130 (2.79)	0.118 (2.63)	0.108 (2.48)	0.099 (2.34)	0.092 (2.21)
illiq	-0.035 (-0.43)	-0.190 (-2.16)	-0.193 (-2.04)	-0.165 (-1.67)	-0.124 (-1.24)	-0.081 (-0.79)	-0.041 (-0.39)	-0.006 (-0.06)	0.021 (0.20)	0.042 (0.38)
tiv	0.685 (8.69)	0.840 (10.41)	0.858 (10.30)	0.854 (9.92)	0.840 (9.44)	0.822 (8.98)	0.802 (8.56)	0.782 (8.21)	0.763 (7.91)	0.746 (7.67)
Adj. R^2	49.16%	56.68%	58.80%	60.12%	61.32%	62.27%	62.83%	62.99%	62.82%	62.40%

Table 5
Swaption implied volatility regression

This table reports estimated coefficients from regressing τ -year maturity implied volatility of swaptions written on the 10-year swap rate, $iv_t^{\tau y 10y}$, onto a set of variables.

$$iv_t^{\tau y 10y} = \beta_1^\tau \text{convexity}_t + \beta_2^\tau \text{illiq}_t + \beta_3^\tau iv_{t-1}^{\tau y 10y} + \epsilon_t^\tau,$$

where illiq_t is the illiquidity factor at time t . t-Statistics presented in parentheses are calculated using Newey and West (1987). All variables are standardized to have mean zero and a standard deviation of one. Data is weekly and runs from 2002 through 2011.

	1y	2y	3y	4y	5y	7y	10y
convexity	0.272 (2.33)	0.281 (2.39)	0.270 (2.29)	0.270 (2.26)	0.262 (2.14)	0.245 (1.87)	0.215 (1.54)
Adj. R^2	7.42%	7.89%	7.31%	7.28%	6.89%	6.02%	4.64%
convexity	0.309 (3.09)	0.316 (3.06)	0.306 (2.91)	0.306 (2.83)	0.298 (2.61)	0.280 (2.23)	0.248 (1.82)
illiq	0.622 (9.08)	0.598 (8.59)	0.608 (8.96)	0.610 (9.12)	0.602 (9.00)	0.581 (8.75)	0.553 (7.92)
Adj. R^2	45.82%	43.37%	44.05%	44.21%	42.91%	39.54%	34.99%
convexity	0.038 (2.95)	0.040 (3.00)	0.044 (2.87)	0.047 (2.77)	0.048 (2.64)	0.051 (2.49)	0.053 (2.49)
illiq	0.042 (1.61)	0.042 (1.51)	0.046 (1.54)	0.048 (1.51)	0.049 (1.51)	0.048 (1.40)	0.047 (1.28)
lagged	0.945 (53.08)	0.942 (51.14)	0.934 (44.61)	0.928 (39.93)	0.923 (37.33)	0.918 (34.12)	0.916 (30.95)
Adj. R^2	97.29%	96.73%	95.93%	95.18%	94.48%	93.38%	92.76%

Table 6
Bond return volatility and bond variance risk

This table reports estimated coefficients from regressing treasury implied volatility (tiv), realized volatility (trv), the bond variance risk premium (vrp), returns on a monthly straddle strategy (straddle), implied and realized measures of volatility of volatility (tivov/trvov) on a set of variables:

$$\text{tiv}_t/\text{trv}_t/\text{vrp}_t/\text{straddle}_t/\text{tivov}_t/\text{trvov}_t = \beta_1\text{convexity}_t + \beta_2\text{illiq}_t + \beta_3\text{lagged}_{t-1} + \epsilon_t,$$

where illiq_t is the illiquidity factor at time *t* and lagged_{*t*-1} is the LHS variable lagged for one period. t-Statistics presented in parentheses are calculated using Newey and West (1987). All variables are standardized to have mean zero and a standard deviation of one. Data for tiv, trv, vrp, tivov, and trvov are weekly and straddle returns are monthly. All data runs from 1997 to 2011.

	tiv	trv	vrp	straddle	tivov	trvov
convexity	0.186 (2.47)	0.146 (1.75)	0.187 (3.29)	0.090 (1.93)	0.152 (2.69)	0.159 (2.59)
Adj. <i>R</i> ²	3.45%	2.12%	3.48%	1.10%	2.32%	2.52%
convexity	0.267 (3.65)	0.232 (2.87)	0.262 (4.41)	0.096 (1.95)	0.213 (4.11)	0.212 (3.86)
illiq	0.684 (14.06)	0.700 (15.57)	0.628 (6.11)	0.106 (2.52)	0.545 (4.86)	0.473 (6.14)
Adj. <i>R</i> ²	49.28%	50.18%	42.06%	1.30%	31.55%	24.55%
convexity	0.027 (2.61)	0.017 (1.81)	0.064 (2.96)	0.096 (2.18)	0.004 (1.31)	0.005 (1.76)
illiq	0.071 (3.33)	0.053 (3.14)	0.159 (3.20)	0.105 (2.53)	0.032 (3.74)	0.022 (3.05)
lagged	0.900 (44.82)	0.928 (60.50)	0.749 (18.80)	0.005 (0.07)	0.982 (164.54)	0.987 (103.67)
Adj. <i>R</i> ²	90.68%	93.57%	74.87%	1.00%	99.74%	99.57%

Table 7
Duration and yield factors

Panel A reports the unconditional correlation between MBS duration and the first five principal components (PCA) of bond yields. Panel B reports estimated coefficients from regressing bond excess returns onto duration and the first three PCAs.

$$rx_{t+1}^{\tau} = \beta_1^{\tau} \text{duration}_t + \beta_2^{\tau} \text{level}_t + \beta_3^{\tau} \text{slope}_t + \beta_4^{\tau} \text{curvature}_t + \epsilon_{t+1}^{\tau},$$

where level_t , slope_t and curvature_t represent the first three principal components of bond yields. Panel C reports the estimated coefficients from a regression of bond excess returns onto the orthogonalized duration time series, $\bar{\text{duration}}_t$. We orthogonalize duration with respect to the first five PCs by running a contemporaneous regression of duration on the PCs and take the residual. t-Statistics presented in parentheses are calculated using Newey and West (1987). All variables are standardized to have mean zero and a standard deviation of one. Data is weekly and runs from 1997 through 2011.

Panel A: Unconditional Correlations									
PC1	PC2	PC3	PC4	PC5					
0.55	0.12	-0.14	-0.01	-0.11					

Panel B: Bond Excess Return Regression									
	2y	3y	4y	5y	6y	7y	8y	9y	10y
duration	0.064 (0.87)	0.144 (1.88)	0.216 (2.84)	0.278 (3.70)	0.331 (4.45)	0.374 (5.05)	0.409 (5.54)	0.438 (5.93)	0.462 (6.24)
level	0.420 (3.78)	0.318 (3.01)	0.233 (2.26)	0.164 (1.59)	0.108 (1.03)	0.063 (0.58)	0.028 (0.25)	0.001 (0.00)	-0.020 (-0.17)
slope	0.334 (2.68)	0.374 (2.94)	0.403 (3.20)	0.428 (3.50)	0.449 (3.81)	0.466 (4.13)	0.480 (4.43)	0.491 (4.70)	0.497 (4.93)
curvature	0.318 (2.75)	0.284 (2.46)	0.270 (2.37)	0.262 (2.38)	0.254 (2.42)	0.246 (2.47)	0.237 (2.51)	0.228 (2.52)	0.218 (2.51)
Adj. R^2	14.54%	14.36%	17.19%	21.49%	26.33%	31.07%	35.36%	39.04%	42.08%

Panel C: Bond Excess Return Regression with Duration only									
duration	0.020 (0.30)	0.084 (1.22)	0.144 (2.05)	0.196 (2.75)	0.242 (3.31)	0.281 (3.74)	0.314 (4.07)	0.342 (4.33)	0.366 (4.56)
Adj. R^2	0.04%	0.71%	2.06%	3.86%	5.85%	7.89%	9.84%	11.68%	13.39%

Table 8
Duration and other proxies of frictions

Panel A reports estimated coefficients from regressing bond excess returns onto duration, funding liquidity, and a mean jump factor.

$$rx_{t+1}^{\tau} = \beta_1^{\tau} \text{duration}_t + \beta_2^{\tau} F_t + \beta_3^{\tau} \text{JM}_t + \epsilon_{t+1}^{\tau},$$

where F_t and JM_t are funding liquidity and realized jump mean, respectively. Panel B reports adjusted R^2 from regression bond excess returns onto the first three PCs, duration, funding liquidity, and the realized jump mean. All variables are standardized to have mean zero and a standard deviation of one. Data is monthly and runs from 1990 through 2011.

Panel A: Bond Excess Returns									
	2y	3y	4y	5y	6y	7y	8y	9y	10y
duration	0.191 (2.20)	0.210 (2.48)	0.230 (2.78)	0.249 (3.04)	0.267 (3.26)	0.284 (3.43)	0.299 (3.57)	0.314 (3.68)	0.328 (3.79)
funding liquidity	-0.121 (-1.32)	-0.119 (-1.34)	-0.111 (-1.30)	-0.102 (-1.22)	-0.093 (-1.11)	-0.084 (-0.98)	-0.075 (-0.85)	-0.067 (-0.73)	-0.059 (-0.63)
jump amplitude	-0.485 (-3.33)	-0.513 (-4.05)	-0.524 (-4.78)	-0.527 (-5.45)	-0.523 (-5.93)	-0.514 (-6.12)	-0.500 (-6.05)	-0.483 (-5.78)	-0.464 (-5.42)
Adj. R^2	33.57%	37.74%	39.99%	41.26%	41.71%	41.49%	40.77%	39.73%	38.55%

Panel B: Adj. R^2									
PC1-3	18.30%	13.09%	11.44%	11.43%	12.24%	13.43%	14.70%	15.86%	16.81%
PC1-3+D	20.92%	18.34%	19.52%	22.30%	25.73%	29.24%	32.49%	35.29%	37.60%
PC1-3+D+F	22.92%	19.73%	20.26%	22.56%	25.66%	29.01%	32.23%	35.13%	37.61%
PC1-3+D+F+J	48.02%	44.73%	43.89%	44.42%	45.53%	46.78%	47.92%	48.83%	49.50%

Table 9
Bond Risk Premia Regressions Monthly with Macro

This table reports estimated coefficients from regressing bond excess returns, rx_{t+1}^τ , onto a set of variables:

$$rx_{t+1}^\tau = \beta_1^\tau \text{duration}_t + \beta_2^\tau \text{slope}_t + \beta_3^\tau \text{cp}_t + \sum_{i=1}^8 F_t^i + \epsilon_t,$$

where slope_t is the slope at time t , cp_t is the CP factor at time t , and F_t^i are the Ludvigson and Ng (2009) macro factors. t-Statistics presented in parentheses are calculated using Newey and West (1987). All variables are standardized to have mean zero and a standard deviation of one. Data is monthly and runs from 1997 through 2011.

	2y	3y	4y	5y	6y	7y	8y	9y	10y
duration	0.133 (1.25)	0.168 (1.60)	0.201 (1.95)	0.231 (2.27)	0.259 (2.56)	0.285 (2.84)	0.310 (3.09)	0.333 (3.32)	0.354 (3.53)
Adj. R^2	1.77%	2.83%	4.03%	5.34%	6.72%	8.15%	9.61%	11.07%	12.50%
duration	0.081 (0.63)	0.142 (1.04)	0.189 (1.38)	0.229 (1.67)	0.264 (1.96)	0.297 (2.23)	0.327 (2.51)	0.356 (2.78)	0.382 (3.04)
slope	-0.612 (-3.48)	-0.507 (-2.91)	-0.410 (-2.42)	-0.317 (-1.94)	-0.229 (-1.45)	-0.145 (-0.95)	-0.067 (-0.45)	0.004 (0.03)	0.066 (0.48)
cp factor	0.584 (5.86)	0.548 (5.88)	0.519 (5.73)	0.496 (5.56)	0.474 (5.37)	0.451 (5.15)	0.428 (4.90)	0.404 (4.61)	0.381 (4.28)
F^1	-0.588 (-5.21)	-0.535 (-4.88)	-0.476 (-4.52)	-0.418 (-4.15)	-0.361 (-3.76)	-0.307 (-3.34)	-0.257 (-2.91)	-0.211 (-2.45)	-0.168 (-1.98)
F^2	-0.111 (-2.75)	-0.111 (-2.60)	-0.110 (-2.57)	-0.108 (-2.57)	-0.105 (-2.56)	-0.101 (-2.52)	-0.096 (-2.44)	-0.089 (-2.31)	-0.082 (-2.13)
F^3	0.322 (3.58)	0.386 (4.13)	0.427 (4.56)	0.449 (4.80)	0.456 (4.88)	0.452 (4.82)	0.442 (4.66)	0.427 (4.42)	0.410 (4.14)
F^4	0.016 (0.17)	0.015 (0.15)	0.024 (0.23)	0.033 (0.32)	0.041 (0.40)	0.047 (0.47)	0.050 (0.53)	0.053 (0.57)	0.055 (0.62)
F^5	0.025 (0.49)	0.018 (0.37)	0.012 (0.25)	0.007 (0.16)	0.004 (0.09)	0.002 (0.05)	0.001 (0.01)	-0.001 (-0.01)	-0.002 (-0.04)
F^6	-0.067 (-0.83)	-0.091 (-1.12)	-0.095 (-1.20)	-0.090 (-1.16)	-0.080 (-1.07)	-0.068 (-0.93)	-0.056 (-0.78)	-0.045 (-0.63)	-0.034 (-0.49)
F^7	-0.116 (-1.44)	-0.121 (-1.48)	-0.123 (-1.52)	-0.125 (-1.56)	-0.125 (-1.58)	-0.124 (-1.58)	-0.121 (-1.56)	-0.117 (-1.53)	-0.111 (-1.48)
F^8	-0.014 (-0.22)	-0.017 (-0.25)	-0.020 (-0.28)	-0.021 (-0.29)	-0.020 (-0.29)	-0.018 (-0.26)	-0.015 (-0.22)	-0.011 (-0.17)	-0.007 (-0.11)
Adj. R^2	39.04%	37.12%	37.41%	38.76%	40.58%	42.52%	44.37%	46.01%	47.36%

Table 10
Regressions with interaction terms

This table reports estimated coefficients from regressing annual bond excess returns (Panel A) and bond yield volatility (Panel B) onto MBS duration and convexity interacted with the ratio between total mortgages outstanding and GDP. t-Statistics presented in parentheses are calculated using Newey and West (1987). All variables are standardized to have mean zero and a standard deviation of one. Data is quarterly and runs from 1997 through 2011.

Panel A: Bond Excess Return Regression									
	2y	3y	4y	5y	6y	7y	8y	9y	10y
duration × ratio	0.131 (0.83)	0.216 (1.39)	0.270 (1.77)	0.305 (2.01)	0.326 (2.15)	0.339 (2.22)	0.346 (2.25)	0.350 (2.25)	0.351 (2.24)
Adj. R ²	1.70%	4.68%	7.31%	9.29%	10.65%	11.50%	11.99%	12.23%	12.31%
duration × ratio	0.134 (0.82)	0.213 (1.36)	0.263 (1.73)	0.293 (2.00)	0.312 (2.18)	0.322 (2.31)	0.328 (2.38)	0.330 (2.43)	0.330 (2.45)
slope	-0.042 (-0.27)	0.037 (0.25)	0.096 (0.65)	0.142 (0.99)	0.180 (1.27)	0.210 (1.50)	0.233 (1.68)	0.251 (1.82)	0.264 (1.92)
Adj. R ²	0.74%	3.71%	7.16%	10.27%	12.85%	14.90%	16.45%	17.56%	18.32%
duration × ratio	0.329 (2.39)	0.396 (2.93)	0.435 (3.32)	0.458 (3.59)	0.470 (3.77)	0.476 (3.89)	0.478 (3.95)	0.477 (3.97)	0.474 (3.97)
slope	-0.197 (-1.18)	-0.124 (-0.76)	-0.075 (-0.48)	-0.038 (-0.26)	-0.008 (-0.06)	0.016 (0.12)	0.035 (0.27)	0.050 (0.39)	0.062 (0.48)
cp factor	0.564 (3.94)	0.543 (4.12)	0.533 (4.43)	0.526 (4.75)	0.522 (5.03)	0.518 (5.20)	0.514 (5.28)	0.510 (5.26)	0.506 (5.18)
Adj. R ²	25.25%	26.89%	29.13%	31.04%	32.51%	33.57%	34.26%	34.64%	34.77%

Panel B: Bond Yield Volatility Regression										
	1y	2y	3y	4y	5y	6y	7y	8y	9y	10y
convexity × ratio	0.404 (2.48)	0.422 (2.77)	0.439 (2.89)	0.453 (2.99)	0.461 (3.08)	0.467 (3.16)	0.472 (3.23)	0.477 (3.30)	0.484 (3.37)	0.492 (3.43)
Adj. R ²	16.35%	17.82%	19.30%	20.49%	21.23%	21.76%	22.25%	22.80%	23.44%	24.17%
convexity × ratio	0.347 (3.25)	0.364 (3.83)	0.382 (4.10)	0.396 (4.30)	0.406 (4.47)	0.413 (4.62)	0.419 (4.76)	0.425 (4.87)	0.432 (4.97)	0.440 (5.06)
illiq	0.635 (10.02)	0.647 (10.44)	0.639 (10.08)	0.623 (8.91)	0.608 (7.82)	0.596 (7.14)	0.587 (6.74)	0.579 (6.52)	0.572 (6.40)	0.566 (6.36)
Adj. R ²	55.58%	58.52%	59.04%	58.25%	57.17%	56.25%	55.62%	55.26%	55.12%	55.14%
convexity × ratio	0.211 (2.76)	0.218 (3.31)	0.240 (3.74)	0.258 (4.02)	0.271 (4.25)	0.280 (4.46)	0.287 (4.66)	0.293 (4.86)	0.300 (5.04)	0.308 (5.23)
illiq	0.229 (2.06)	0.209 (1.86)	0.214 (1.85)	0.209 (1.75)	0.203 (1.65)	0.197 (1.57)	0.191 (1.50)	0.184 (1.44)	0.176 (1.37)	0.168 (1.30)
tiv	0.596 (5.04)	0.642 (4.88)	0.623 (4.76)	0.607 (4.77)	0.594 (4.74)	0.585 (4.70)	0.581 (4.65)	0.580 (4.59)	0.581 (4.52)	0.583 (4.44)
Adj. R ²	71.55%	77.22%	76.58%	74.87%	73.03%	71.62%	70.73%	70.31%	70.23%	70.36%

Table 11
Robustness Checks

This table reports estimated coefficients from regressing bond portfolio excess returns onto duration.

$$\text{rxpf}_{t+1}^{\tau} = \beta_1^{\tau} \text{duration}_t + \epsilon_{t+j}^{\tau},$$

where rxpf_{t+1} are monthly excess returns on the CRSP bond portfolios with maturities between 5 and 10 years and larger than 10 years. Returns are in excess of either the 1-month T-bill or Eurodollar deposit rate. All variables are standardized to have mean zero and a standard deviation of one. Data is monthly and runs from 1990 through 2011.

	T-bill		ED rate	
	$\geq 5y < 10y$	$> 10y$	$\geq 5y < 10y$	$\geq 5y < 10y$
duration	0.159 (2.88)	0.163 (3.17)	0.156 (2.79)	0.161 (3.11)
Adj. R^2	2.53%	2.67%	2.42%	2.60%
duration	0.267 (3.73)	0.268 (3.84)	0.266 (3.80)	0.268 (3.89)
slope	0.226 (2.69)	0.220 (2.69)	0.230 (2.85)	0.222 (2.78)
Adj. R^2	6.05%	5.99%	6.09%	6.01%
duration	0.231 (2.72)	0.244 (3.11)	0.229 (2.78)	0.243 (3.15)
slope	0.174 (1.58)	0.185 (1.85)	0.177 (1.67)	0.187 (1.91)
cp factor	0.087 (1.24)	0.058 (0.86)	0.089 (1.28)	0.059 (0.88)
Adj. R^2	6.18%	5.82%	6.25%	5.85%

Appendix D Figures

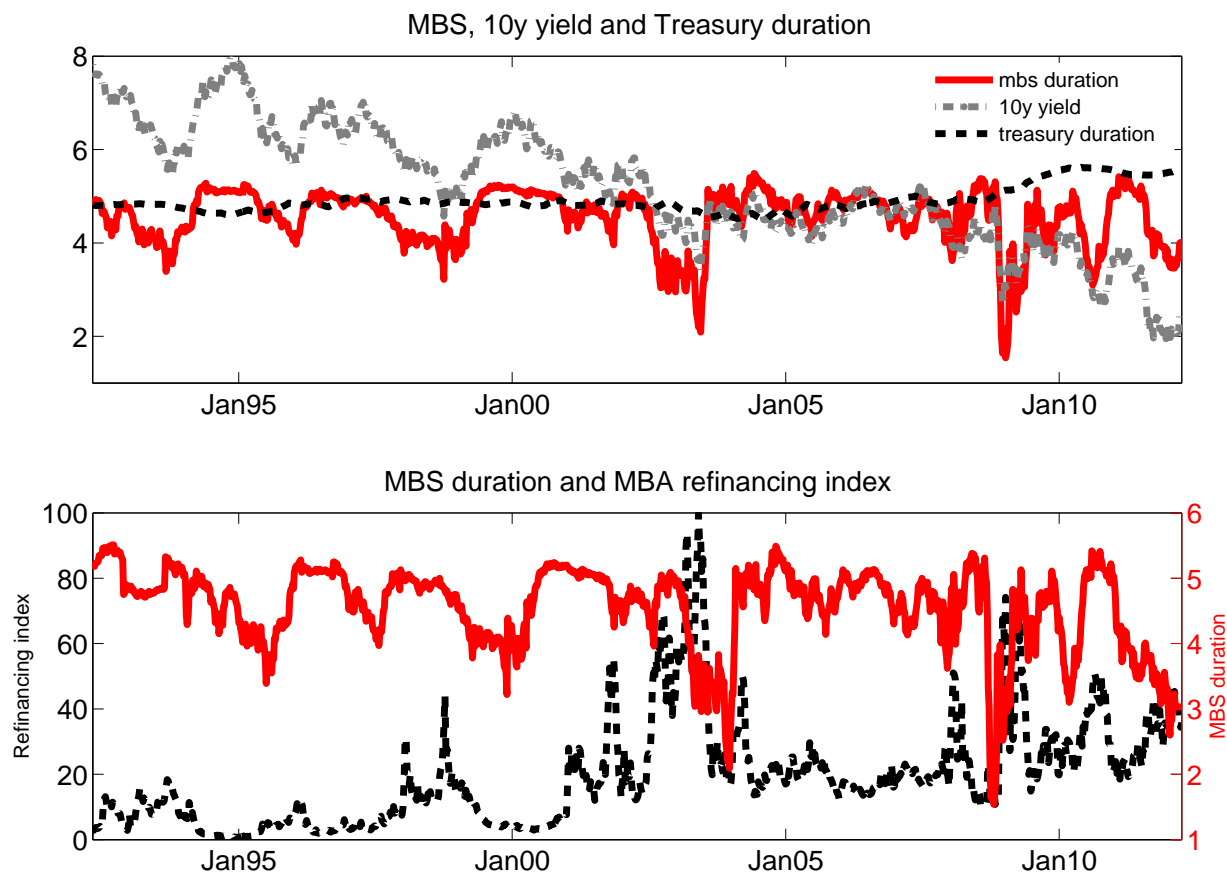


Figure 1. MBS duration

The upper panel plots MBS duration together with the 10y yield and the duration of a generic bond portfolio consisting of bonds with maturities ranging between five and seven years. The lower panel plots MBS duration together with the Mortgage Bankers Association (MBA) refinancing index (in hundreds). The refinancing index covers all mortgage applications to refinance an existing mortgage. Data is weekly and runs from January 1990 to December 2012.

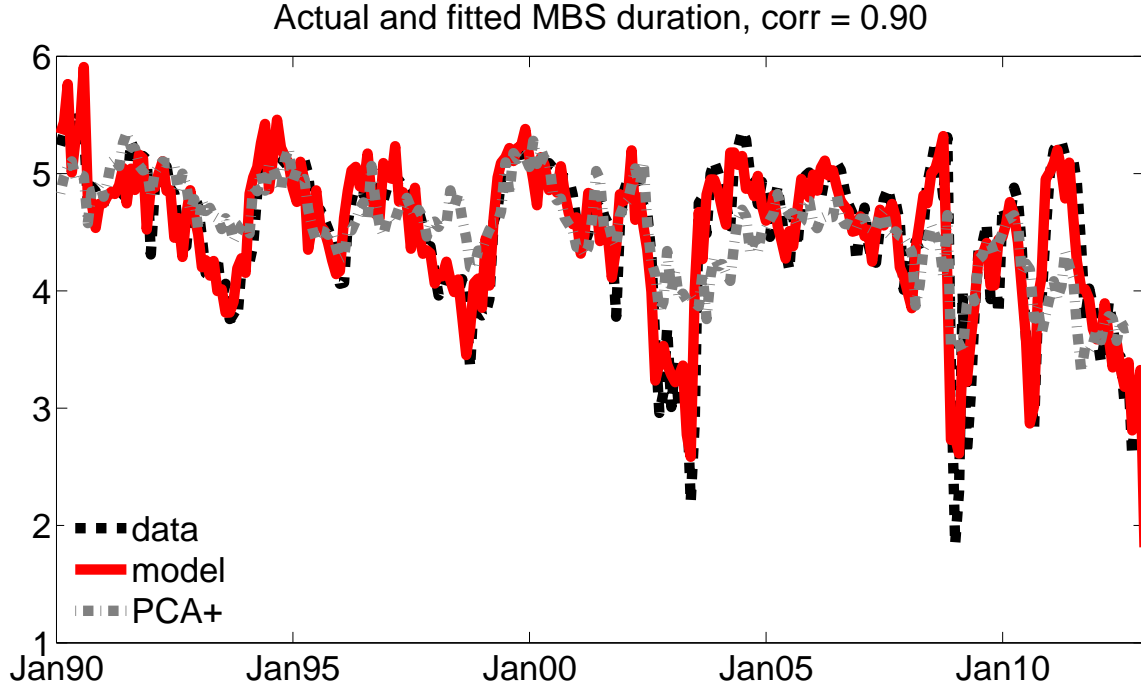


Figure 2. Actual and fitted MBS duration

This figure plots actual MBS duration (data) together with the fitted time-series as implied by our model (model) and a model which includes all yield PCs together with a measure of macroeconomic uncertainty (VIX). Our model is described in equation [A-17](#). The “PCA+” model looks as follows:

$$\text{duration}_t = \alpha + \beta_i \sum_{i=1}^{10} \text{PC}_i + \beta_{11} \text{VIX}_t + \epsilon_t,$$

where PC_i are the yield principal components and VIX_t the model-free implied volatility on 1m options on the S&P500. Data runs from January 1990 to December 2012.

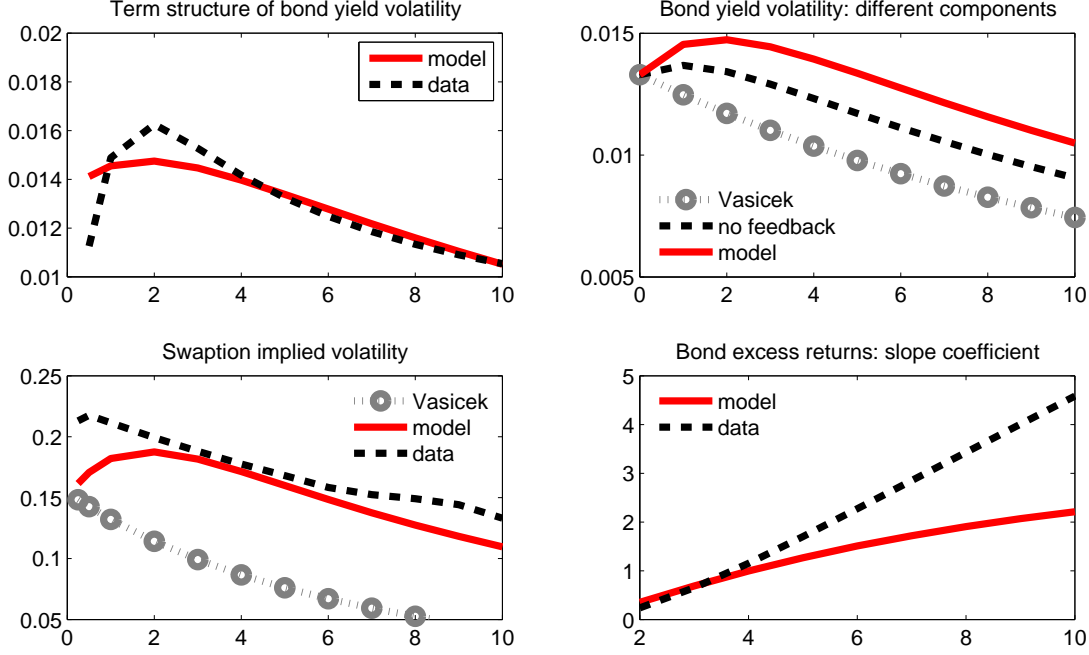


Figure 3. Calibrated term structure of volatility and predictability of bond excess returns

The left upper panel plots the term structure of bond yield volatility as implied by our model using the calibrated parameters in Table 1 (solid line) together with the data (dashed line). The upper right panel depicts the term structure of bond yield volatility from a simple one-factor Vasicek model (dotted line with markers), a model that only allows for the direct effect of duration on yields while shutting down the volatility feedback loop (dashed line), and our benchmark model (solid line). The lower left panel plots swaption implied volatility with different maturity on the 10y swap rate for our model (solid line), a one-factor Vasicek model (dotted line with markers) and the data (dashed line). The lower right panel plots the estimated slope coefficients from the following regression:

$$rx_{t+1}^{\tau} = \alpha^{\tau} + \beta^{\tau} D_t + \epsilon_{t+1}^{\tau},$$

where rx_{t+1}^{τ} is the excess return of a bond with maturity $\tau = 2, \dots, 10$ and D_t is MBS duration as implied by our model. The dashed line depicts the empirical coefficients while the solid line depicts the model implied coefficients that can be calculated in closed form.

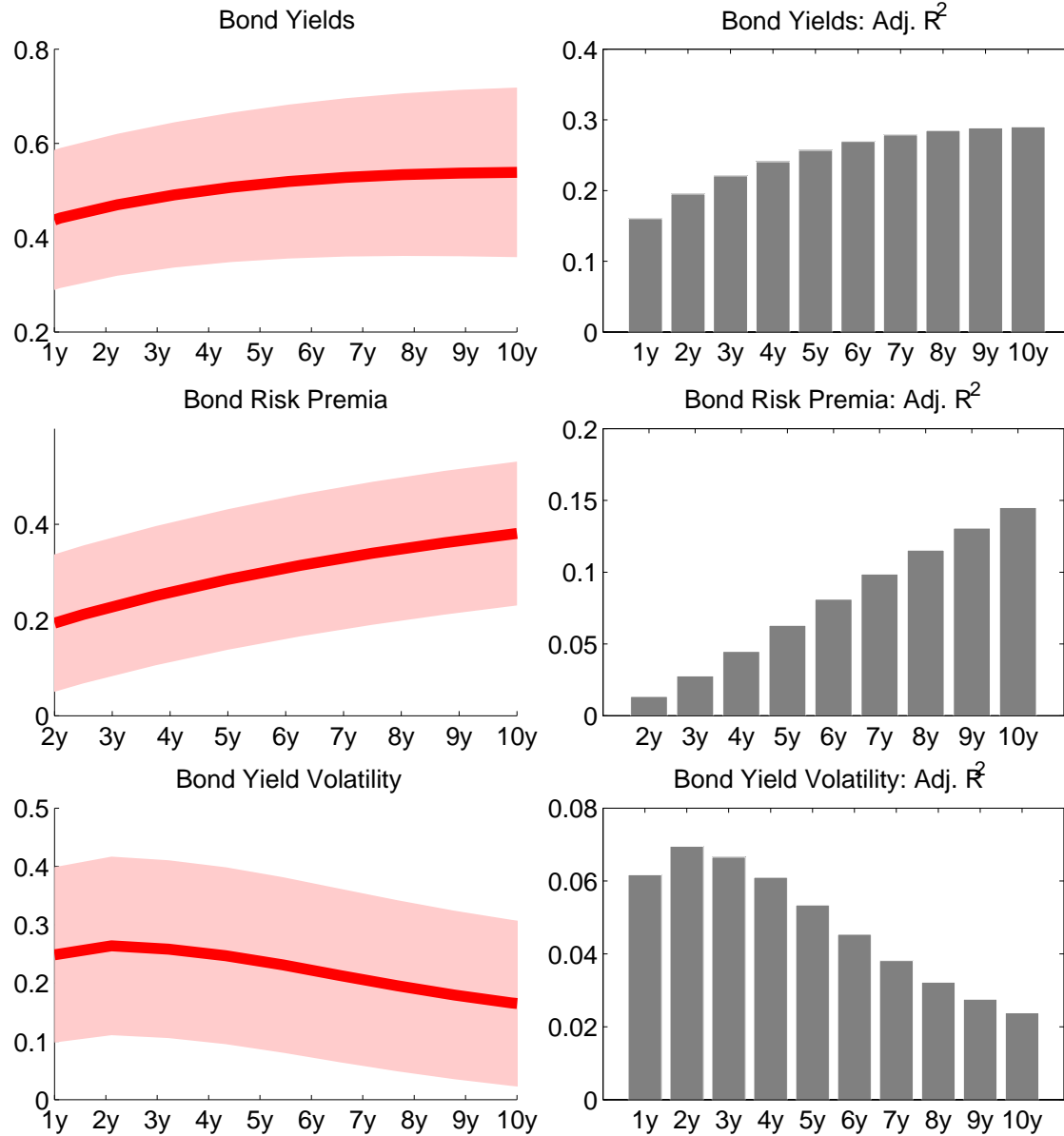


Figure 4. Univariate regression coefficients

This figure plots estimated coefficients and adjusted R^2 from univariate regressions of bond yields (upper panel), bond excess returns (middle panels) and bond yield volatilities (lower panels) onto MBS duration (bond yields and bond excess returns) and MBS convexity (bond yield volatilities). All variables are standardized, i.e. mean of zero and standard deviation of one. Data is weekly and runs from 1997 through 2011. Shaded areas represent confidence levels on the 95% level.

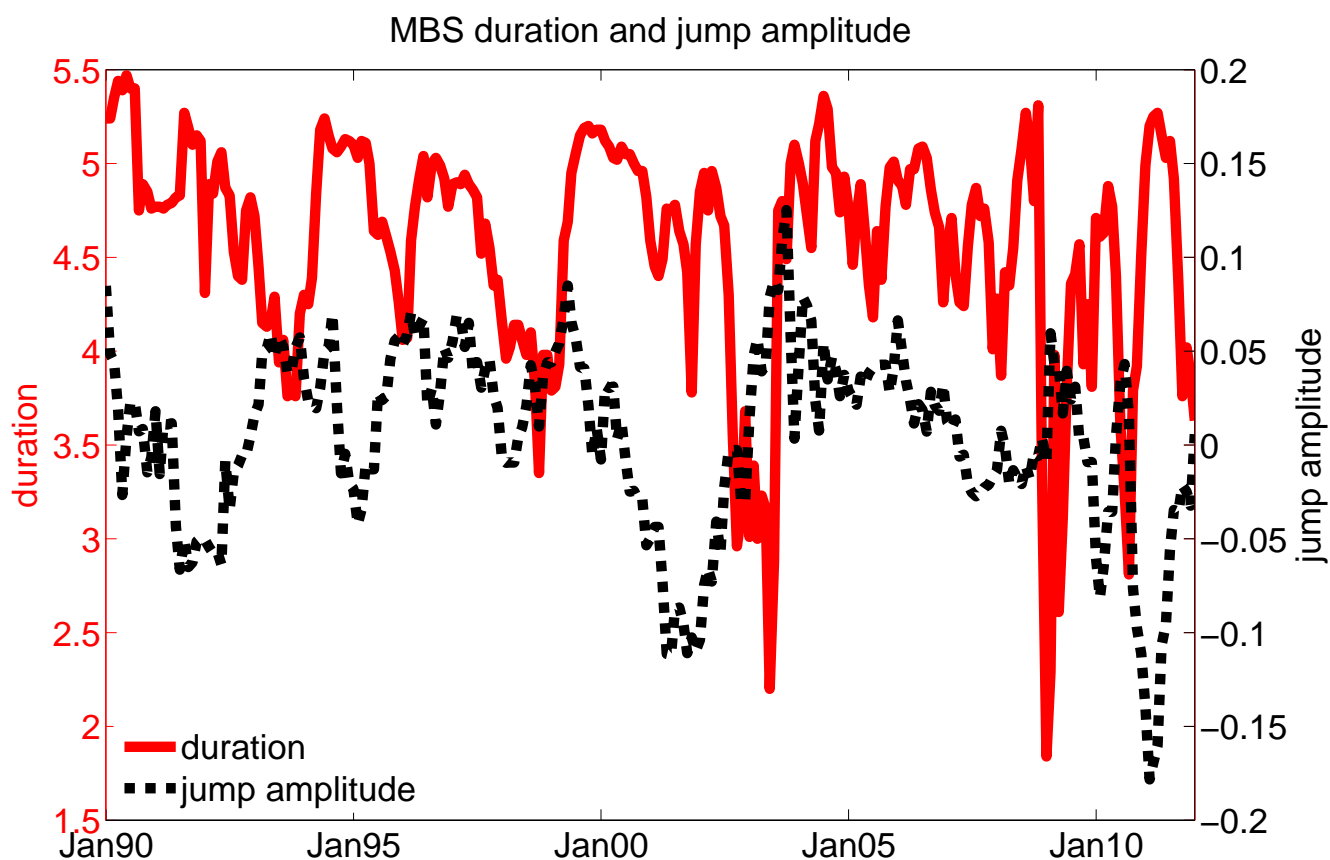


Figure 5. MBS duration and jump amplitude

This figure plots MBS duration together with the jump amplitude measure of Wright and Zhou (2009). Data is monthly from 1990 through 2012.

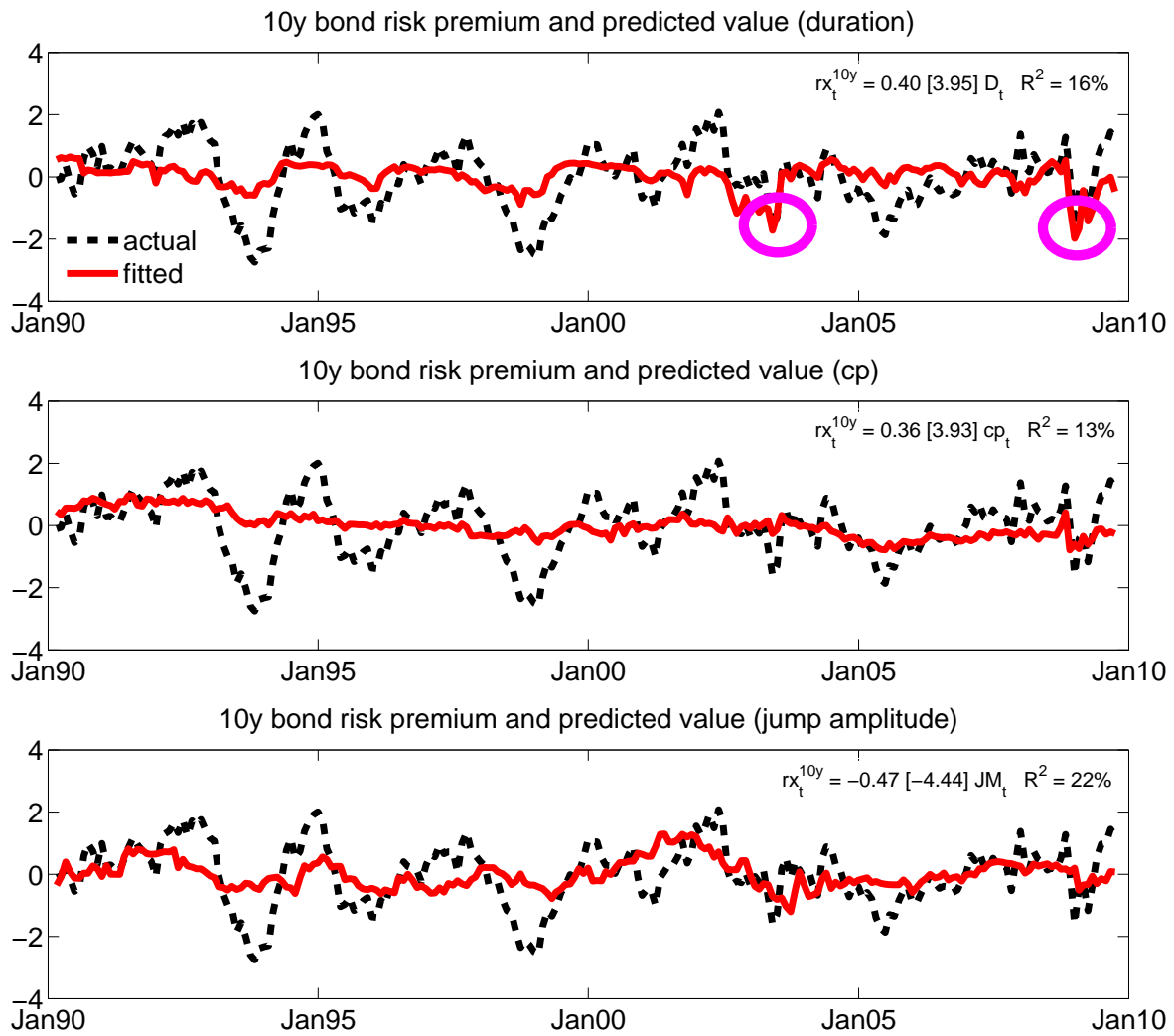


Figure 6. Actual and fitted bond risk premia

These figures plot actual and fitted 10y bond risk premia. The upper panel plots the fitted time-series from a predictive univariate regression using MBS duration, the middle (lower) panel plots the same for the CP factor (jump amplitude). Data is monthly from 1990 through 2012.

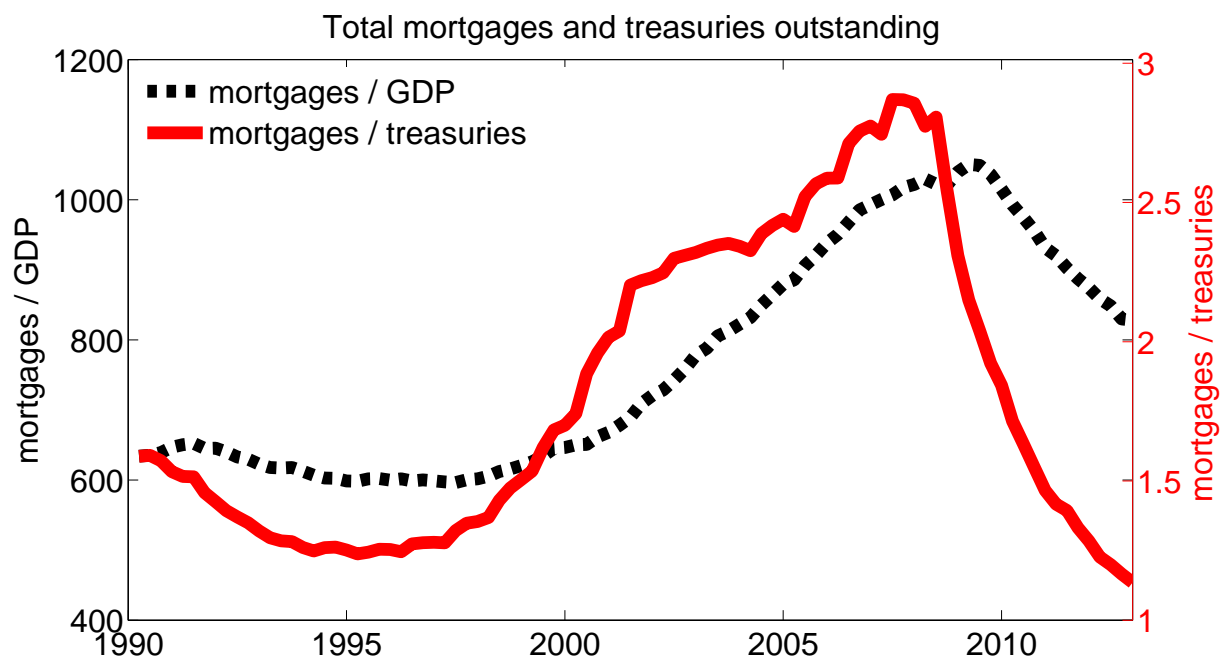


Figure 7. Total mortgages and treasuries outstanding

This figure plots total nominal value outstanding of all mortgages divided by GDP (left axis) and the total amount outstanding of mortgages divided by amount outstanding in Treasuries (right axis). Data is quarterly from 1990 to 2012.

Online Appendix to “Hedging in Fixed Income Markets”

This online appendix consists of two sections. We first give some institutional background on mortgage backed securities (MBS). The reason for this is that we often get asked whether MBS really constitute an important asset class. Our answer is: They are. A second question which repeatedly pops up is whether we have any evidence of significant hedging activity in the MBS market. One standard approach usually taken in the literature (see e.g., Acharya, Lochstoer, and Ramadorai (2013)) is to look at net positions of hedgers in the futures market through data provided by the U.S. Commodity Futures Trading Commission. While there is data available for Treasury options and futures, it would be farfetched to assume that this data really represents the hedging incentives of MBS holders alone. Another way to pin down hedging activity is to directly look into the annual filings of large players in the MBS market. According to the Financial Accounting Standard (FAS) 133, any firm is required to publish the fair value of derivatives designated as hedging instrument. One of the largest holders of MBS are the government sponsored enterprises such as Fannie Mae and Freddie Mac. In the following, we hand collect hedging demand of Fannie Mae from their 10K filings and relate this to our proxy of MBS duration.¹

The second part of this appendix looks into the significance of duration for bond yields in the presence of macroeconomic variables. Since macroeconomic variables are not available on a weekly frequency, we run similar regressions as in the main part of the paper but instead of using weekly data, we run regressions at the monthly frequency.

Mortgage Backed Securities

Mortgages, together with U.S. Treasuries and corporate bonds, constitute almost 70% of the total outstanding U.S. bond market debt. Figure OA-1 plots the time series of average daily trading volume in the U.S. debt market. While in the beginning of the 1990ies mortgage backed securities (MBS) only made a small fraction of overall trading, nowadays MBS are the second largest asset class in the U.S. fixed income market.

Mortgage-related issuance increased from 500 USD billions in 1996 to 2000 USD billions in 2012. The peak has been in 2003 when issuance has been at 3172 USD billions. Since 2008 almost 100% of mortgage issuance has been through the three major agencies, Fannie Mae, Freddie Mac, and Ginnie Mae. Up until autumn 2008, the major players in the MBS market were the two GSEs (Fannie Mae and Freddie Mac), financial institutions and foreign investors like central banks and sovereign wealth funds. Since 2008, foreign buyers and sovereign wealth funds have greatly curtailed or ceased their purchases of MBS due to the large mortgage purchases of the Federal Reserve. Figure OA-2 summarizes the average holdings from 1996 to 2012.

As the U.S. residential mortgage market has grown tremendously over the past couple of years, the Federal National Mortgage Association’s (Fannie Mae) portfolio has grown in tandem. As of December 2012, Fannie Mae’s portfolio of mortgage assets is worth approximately

¹Unfortunately, Freddie Mac’s 10K filings are not as detailed as the ones by Fannie Mae, therefore, we cannot carry out a similar exercise for Freddie Mac.

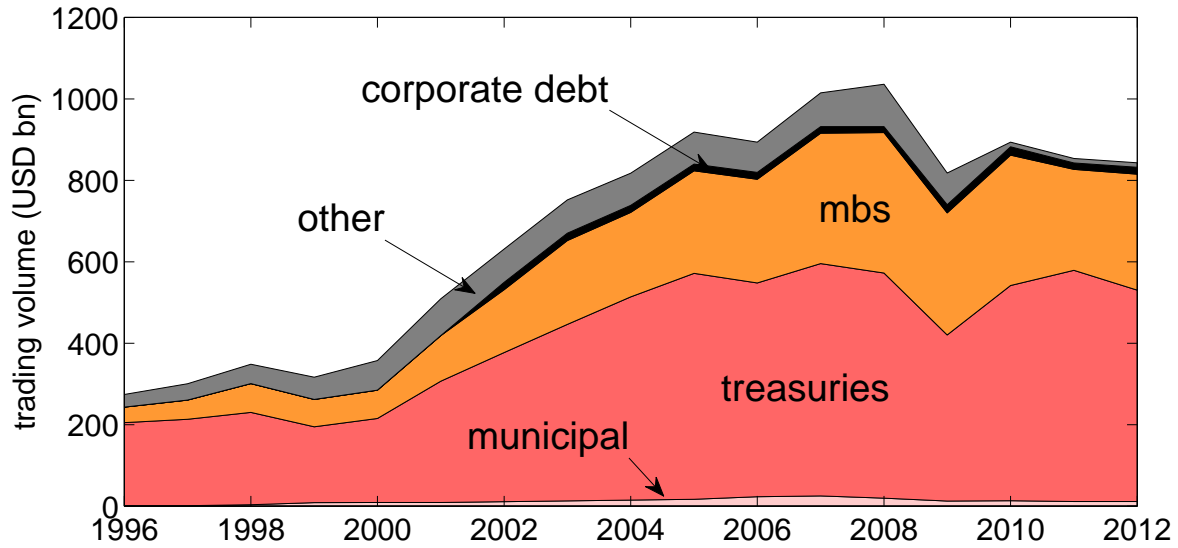


Figure OA-1. U.S. Bond Market Daily Average Trading Volume (USD bn)

This figure plots daily average trading volume in USD billions. Source: Securities Industry and Financial Markets Association (SIFMA).

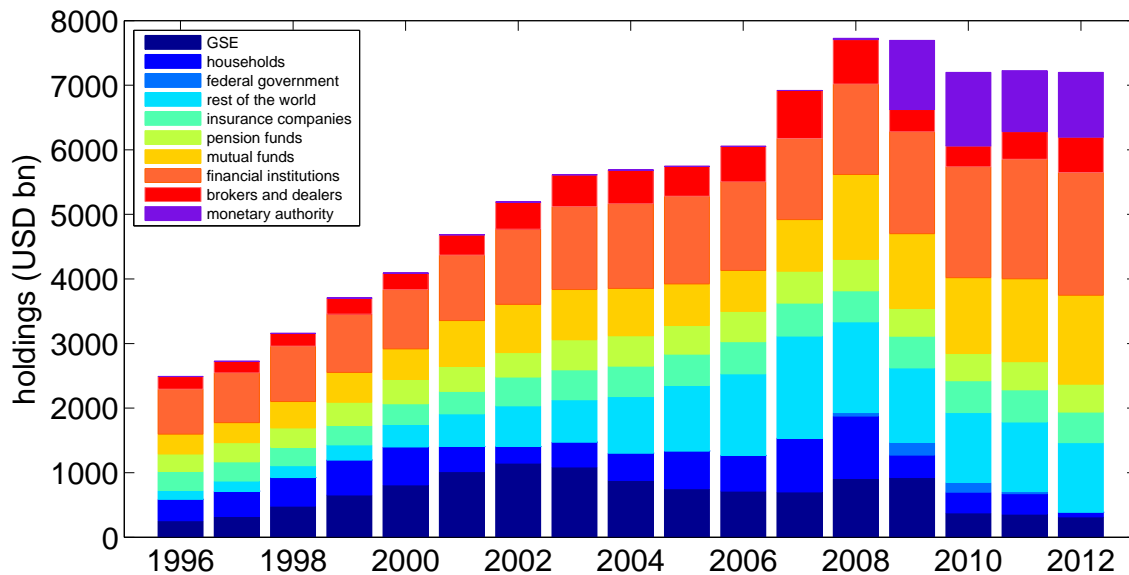


Figure OA-2. MBS Holders (USD mn)

This figure plots holdings of Agency- and GSE-Backed Securities in USD billions. Source: Flow of Funds Tables Federal Reserve.

USD 633 billions of which USD 184 billions are MBS. In addition, Fannie Mae issued USD 865 billions in MBS in all of 2012. Over the past couple of years, critics of GSEs have argued that the GSEs' retained portfolios of mortgages and mortgage securities are so large and contain

such high levels of risk from changes in interest rates that they threaten the stability of the financial system. The large market concentration and the associated hedging activity makes the market more vulnerable and is hence a concern for policymakers (see e.g., Parkinson, Gibson, Mosser, Walter, and LaTorre (2005)). A natural question is therefore, whether this hedging activity of a small group of investors really has an impact on say duration and convexity of MBS. Fannie Mae and Freddie Mac together rank among the world’s largest buyers of interest rate derivatives. They use these assets to either transform short-term debt into synthetic long-term debt and thereby reduce a potential duration mismatch or to hedge the prepayment options on their mortgage assets. The latter is implemented through a so called imperfect dynamic hedging strategy in the interest rate derivatives market (see Jaffee (2003)).

In the following, we study the relation between Fannie Mae’s hedging demand and average MBS duration. To this end, we construct a proxy of hedging demand from the annual reports of Fannie Mae. To this end, we use data on hedging activity from Fannie Mae’s 10-K SEC filings available through EDGAR. The EDGAR database contains quarterly and annual reports in electronic form from 2000 onwards. We are particularly interested in the Financial Accounting Standards Board’s (FAS) 133 regulation which requires firms to report the amount of hedging using derivatives at fair value. Both GSE’s actively manage their interest rate risk through the use of derivatives. The 10-K filings mention three sources of risk: (i) interest rate risk, (ii) liquidity risk, and (iii) spread risk. The exposure to interest rate risk relates to the cash flow and/or market price variability of their assets and liabilities attributable to movements in market interest rates. Liquidity risk is the potential inability to meet funding obligations. And the exposure to spread risk relates to the possibility that interest rates in different market sectors, such as the mortgage and debt markets, will not move in tandem. While Fannie Mae historically has actively managed the former two, they do not risk manage spread risk. In their 2012 10-K filings, Fannie Mae stresses the role of duration and prepayment risk:

Risk management derivative instruments are an integral part of our management of interest rate risk. We supplement our issuance of debt securities with derivative instruments to further reduce duration risk, which includes prepayment risk. We purchase option-based risk management derivatives to economically hedge prepayment risk.

To get a grasp of how much Fannie Mae hedges, we manually collect data on their hedging activity from the 10-K filings in the section “Interest Rate Risk Management and Other Market Risks”. We define as hedging activity the total notional amount of derivatives used for risk management purposes, labeled as “Total risk management derivatives”. The time series is plotted in Figure OA-3 together with the MBS duration time series. Two things are noteworthy. First, there is a large negative correlation between the two time series which is more than -50%.² Also, both time-series experience two peaks namely during 2003 and 2008 when refinancing was large.

Robustness Checks

Regression with Macro Variables

In Section 3.6 of the main paper, we study the predictive power of MBS duration beyond macro variables for bond excess returns. In the following, we run the same regressions using

²The small number of data points is obviously too small to draw any statistical conclusions.

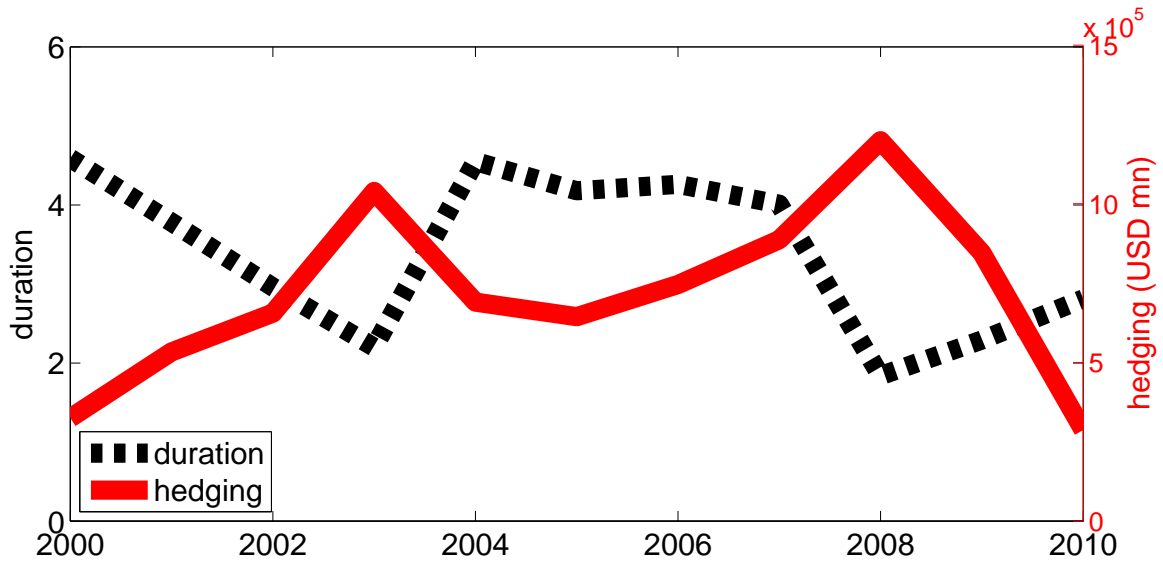


Figure OA-3. Fannie Mae hedging volume and MBS duration

This figure plots derivatives hedging positions (in USD mn) from the 10-K SEC filings of Fannie Mae together with MBS duration. Data is annual and runs from 2000 to 2010.

bond yields as the LHS variables. Table OA-1 reports the results. We note two things. First of all, the statistical significance of the duration factor is not reduced when we switch from the weekly to the monthly frequency. Secondly, adding the macro factors into the regressions does not have an effect on the economic or statistical significance for duration. In terms of economic significance, none of the Ludvigson and Ng (2009) factors is nearly as large as the duration factor. We conclude that similar to the bond excess return regressions presented in the paper, macro variables do not deter the predictive power of duration for bond yields.

Table OA-1
Bond Yield Regressions Monthly

This table reports estimated coefficients from regressing bond yields, y_t^τ , onto a set of variables:

$$y_t^\tau = \beta_1^\tau \text{duration}_t + \beta_2^\tau \text{slope}_t + \beta_3^\tau \text{cp}_t + \sum_{i=1}^8 F_t^i + \epsilon_t,$$

where slope_t is the slope at time t , cp_t is the CP factor at time t , and F_t^i are the Ludvigson and Ng (2009) macro factors. t-Statistics presented in parentheses are calculated using Newey and West (1987). All variables are standardized to have mean zero and a standard deviation of one. Data are monthly and run from 1997 through 2012.

	1y	2y	3y	4y	5y	6y	7y	8y	9y	10y
duration	0.423 (3.86)	0.464 (4.21)	0.492 (4.38)	0.513 (4.45)	0.528 (4.45)	0.540 (4.40)	0.549 (4.33)	0.554 (4.25)	0.557 (4.17)	0.558 (4.09)
Adj. R^2	17.92%	21.54%	24.19%	26.27%	27.93%	29.21%	30.13%	30.73%	31.07%	31.19%
duration	0.055 (2.60)	0.118 (4.32)	0.152 (4.57)	0.175 (4.59)	0.191 (4.57)	0.205 (4.54)	0.216 (4.50)	0.225 (4.46)	0.230 (4.40)	0.233 (4.32)
slope	-0.985 (-28.65)	-0.912 (-19.66)	-0.859 (-15.48)	-0.817 (-13.05)	-0.782 (-11.35)	-0.748 (-10.03)	-0.713 (-8.94)	-0.677 (-8.02)	-0.639 (-7.22)	-0.600 (-6.54)
cp factor	0.248 (7.22)	0.259 (6.42)	0.291 (6.32)	0.329 (6.42)	0.368 (6.54)	0.404 (6.62)	0.438 (6.63)	0.467 (6.59)	0.492 (6.51)	0.513 (6.42)
F^1	0.002 (0.07)	0.025 (0.84)	0.040 (1.16)	0.046 (1.21)	0.047 (1.13)	0.046 (1.00)	0.044 (0.88)	0.043 (0.81)	0.045 (0.79)	0.050 (0.84)
F^2	-0.003 (-0.21)	0.018 (1.35)	0.030 (1.81)	0.035 (1.86)	0.037 (1.79)	0.037 (1.66)	0.036 (1.53)	0.035 (1.43)	0.035 (1.36)	0.036 (1.35)
F^3	-0.003 (-0.12)	-0.041 (-1.40)	-0.064 (-1.87)	-0.073 (-1.92)	-0.075 (-1.81)	-0.075 (-1.66)	-0.074 (-1.52)	-0.074 (-1.43)	-0.077 (-1.39)	-0.081 (-1.40)
F^4	-0.014 (-0.91)	-0.049 (-2.50)	-0.068 (-2.88)	-0.079 (-2.93)	-0.086 (-2.87)	-0.090 (-2.79)	-0.093 (-2.71)	-0.095 (-2.64)	-0.096 (-2.57)	-0.096 (-2.50)
F^5	-0.006 (-0.33)	-0.002 (-0.09)	0.000 (-0.02)	0.000 (0.00)	0.000 (-0.01)	-0.001 (-0.02)	-0.001 (-0.03)	-0.002 (-0.05)	-0.003 (-0.07)	-0.004 (-0.09)
F^6	0.071 (2.96)	0.074 (2.62)	0.081 (2.47)	0.089 (2.44)	0.100 (2.45)	0.110 (2.49)	0.121 (2.53)	0.130 (2.56)	0.139 (2.59)	0.146 (2.62)
F^7	0.030 (2.02)	0.052 (2.60)	0.064 (2.71)	0.073 (2.70)	0.079 (2.67)	0.084 (2.64)	0.089 (2.62)	0.094 (2.60)	0.098 (2.60)	0.102 (2.60)
F^8	0.048 (3.73)	0.074 (4.17)	0.091 (4.22)	0.104 (4.23)	0.114 (4.23)	0.122 (4.22)	0.129 (4.21)	0.135 (4.20)	0.140 (4.18)	0.145 (4.17)
Adj. R^2	96.78%	94.54%	92.34%	90.28%	88.28%	86.27%	84.26%	82.28%	80.39%	78.64%