# Trade Dynamics in the Market for Federal Funds

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# Interbank markets

- ◇ Channel liquidity across the banking system
- ◇ Setting of the interest rate with shortest maturity
- ◇ Play key role in the implementation of monetary policy

# US Fed Funds Market

- Fed funds: Uncollateralized loans of reserve balances at Federal Reserve Banks
- Maturity: mostly overnight
- Participants: commercial banks, thrift institutions, agencies and branches of foreign banks in the United States, federal agencies, and government securities dealers
- Over-the-counter market

# OTC Frictions in the Fed Funds Market

- ◇ Price dispersion
- ◇ Intermediation
- ◇ Long & buy / Short & sell

# The Model

◇ Time is continuous $t \in [0, T]$, where $\tau = T - t$ is time remaining

◇ One asset: reserve balances $k(\tau) \in \mathbb{K} = \{0, 1, ..., K\}$

◇ Agents (*banks*):

- Unit measure of banks
- Hold reserve balances $k(\tau) \in \mathbb{K}$, and a common target $\overline{k} \in \mathbb{K}$
- Time preferences (constant discount rate $r$)
- Payoffs:

  $u_k \equiv$ Flow payoff from holding $k$ balances during the trading session

  $U_k \equiv$ Payoff from holding $k$ balances at end of the trading session

# The Model

- Payoff and flow payoff embed institutional features
  - Interest on reserves, deficiency charges, daylight and overnight overdrafts,...

- Market operates through search
  - Meetings are bilateral and random, at a Poisson rate $\alpha > 0$
  - Bargain over the size and rate of the loan of reserve balances

- Repayment $R$ occurs at $t = T + \Delta$

# Special Case: $\mathbb{K} = \{0, 1, 2\}$ and $\bar{k} = 1$

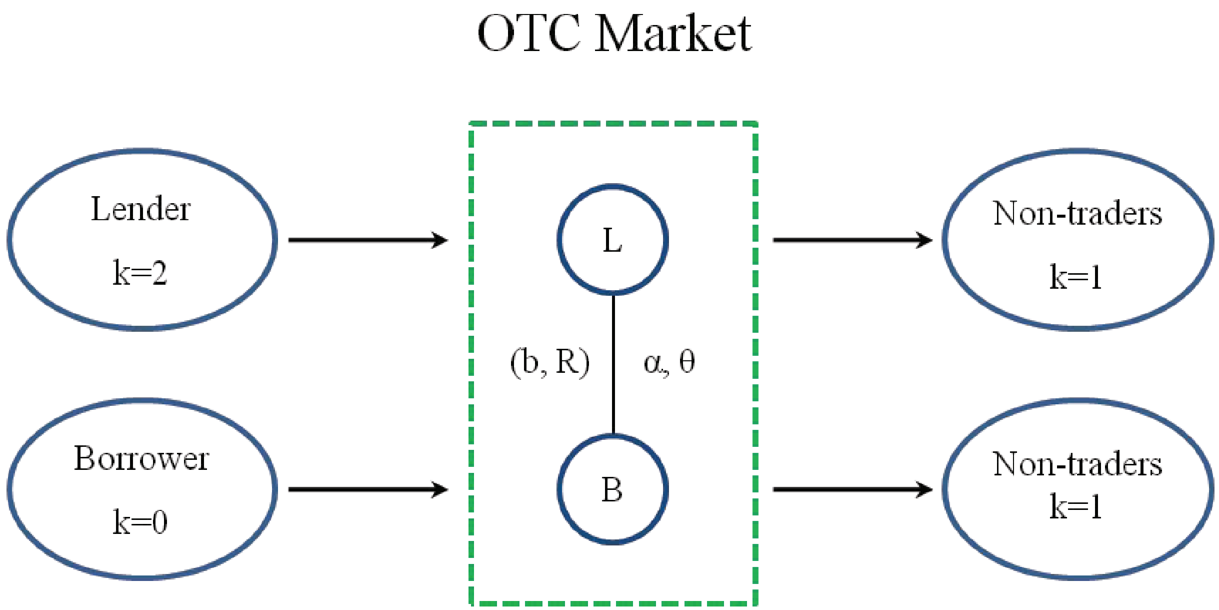



Figure: Market for Federal Funds

◇ The **size** of the loan is $b = 1$

(Only profitable trade is between $i = 0$ and $j = 2$)

◇ The **gains from trade**: $S(\tau) \equiv 2V_1(\tau) - V_2(\tau) - V_0(\tau)$ Conjecture: $S(\tau) > 0 \ \forall \tau \in [0, T]$ (to be verified later)

◇ Assumption: $2u_1 - u_2 - u_0 \geq 0$ and $2U_1 - U_2 - U_0 > 0$

◇ The bargaining problem simplifies to

$$\max_R \left[V_1(\tau) - V_0(\tau) - e^{-r(\tau+\Delta)}R\right]^\theta \left[V_1(\tau) - V_2(\tau) + e^{-r(\tau+\Delta)}R\right]^{1-\theta}$$

and the **repayment** amount $R$ (FOC)

$$e^{-r(\tau+\Delta)}R(\tau) = \theta\left[V_2(\tau) - V_1(\tau)\right] + (1-\theta)\left[V_1(\tau) - V_0(\tau)\right]$$

◇ Value functions:

$$
\begin{aligned}
rV_0(\tau) + \dot{V}_0(\tau) &= u_0 + \alpha n_2(\tau)\,\theta S(\tau) \\
rV_1(\tau) + \dot{V}_1(\tau) &= u_1 \\
rV_2(\tau) + \dot{V}_2(\tau) &= u_2 + \alpha n_0(\tau)\,(1-\theta)\,S(\tau)
\end{aligned}
$$

with $V_i(0) = U_i$ for $i = 0, 1, 2$.

◇ Given $\{n_k(T)\}$ the distribution of balances follows

$$
\begin{aligned}
\dot{n}_0(\tau) &= \alpha n_2(\tau)\, n_0(\tau) \\
\dot{n}_2(\tau) &= \alpha n_2(\tau)\, n_0(\tau)
\end{aligned}
$$

Then,

$$
\begin{aligned}
n_0(\tau) &= \frac{[n_2(T) - n_0(T)]\, n_0(T)}{n_2(T)\, e^{\alpha[n_2(T)-n_0(T)](T-\tau)} - n_0(T)} \\
n_2(\tau) &= n_0(\tau) + n_2(T) - n_0(T) \\
n_1(\tau) &= 1 - n_0(\tau) - n_2(\tau)
\end{aligned}
$$

## ◇ Surplus:

$$\dot{S}(\tau) + \delta(\tau) S(\tau) = 2u_1 - u_2 - u_0$$

where $\delta(\tau) \equiv \{r + \alpha[\theta n_2(\tau) + (1-\theta) n_0(\tau)]\}$. Then,

$$S(\tau) = \bar{u} \int_0^\tau e^{-\left[\bar{\delta}(\tau) - \bar{\delta}(z)\right]} dz + e^{-\bar{\delta}(\tau)} S(0)$$

where $\bar{u} \equiv 2u_1 - u_2 - u_0$ and $\bar{\delta}(\tau) \equiv \int_0^\tau \delta(x)\, dx$.

## ◇ Fed funds rate:

$$e^{-r(\tau+\Delta)} R(\tau) = V_2(\tau) - V_1(\tau) + (1-\theta) S(\tau)$$
$$R(\tau) = e^{\rho(\tau+\Delta)} \times 1$$

Then,

$$\rho(\tau) = r + \frac{\ln[V_2(\tau) - V_1(\tau) + (1-\theta) S(\tau)]}{\tau + \Delta}$$

# Policy issues: Impact of changes in IOR

- Assumptions: $r \approx 0$ and $u_i = 0$,

$$\rho_f(\tau) = \beta(\tau) i_f^e + [1 - \beta(\tau)] \left( i_f^r + P^r + i_f^{dc} \right)$$

where

$$\begin{aligned} \beta(\tau) &= \theta \quad \text{if } n_2(T) = n_0(T) \\ 0 \leq \beta(\tau) &\leq \theta \quad \text{if } n_2(T) < n_0(T) \\ \theta \leq \beta(\tau) &\leq 1 \quad \text{if } n_2(T) > n_0(T) \end{aligned}$$

- If $i_f^e = i_f^r = i_f$, then

$$\rho_f(\tau) = i_f + [1 - \beta(\tau)] \left( P^r + i_f^{dc} \right)$$

# General Case

- Banks hold reserve balances $k(\tau) \in \mathbb{K}$,

- Bargaining problem solves for loan terms: $b(\tau)$ and $R(\tau)$

- Under a restriction on flow payoffs and end-of-day payoffs,
  - An equilibrium exists
  - The paths $\boldsymbol{V}(\tau)$ and $\boldsymbol{n}(\tau)$ are unique
  - Equilibrium path for $\boldsymbol{\phi}(\tau)$: midpoint allocation

# Implications

◇ Positive implications - models delivers

– Time-varying distribution of fed fund rates

$$\rho_{ij}^{ks}(\tau) = \frac{\ln\left[\frac{R_{ij}^{ks}(\tau)}{k-i}\right]}{\tau+\Delta} = r + \frac{\ln\left[\frac{V_j(\tau)-V_s(\tau)}{j-s} + \frac{\frac{1}{2}S_{ij}^{ks}(\tau)}{j-s}\right]}{\tau+\Delta}$$

– Time-varying distribution of trade sizes and volume

$$\overline{v}(\tau) = \sum_{i\in\mathbb{K}}\sum_{j\in\mathbb{K}}\sum_{k\in\mathbb{K}}\sum_{s\in\mathbb{K}} \alpha n_i(\tau) n_j(\tau) \phi_{ij}^{ks}(\tau)|k-i|$$

– Endogenous intermediation

◇ Normative implications

Equilibrium supports an efficient allocation of reserve balances

# A numerical example with $\mathbb{K} = \{0, 1, \ldots, 49\}$

◇ Parameter values

| $i_f^r$ | $i_f^e$ | $i_f^{dc}$ | $i_+^d$ | $\Delta$ | $\Delta_f$ | $\Delta_f^r$ | $\Delta_f^{dc}$ | $\Delta_f^d$ | $T$ | $r$ | $\alpha$ | $\theta$ | $P^r$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\frac{0.0025}{360}$ | $\frac{0.0025}{360}$ | $\frac{0.0175}{360}$ | $\frac{0.00001}{360}$ | $\frac{22}{24}$ | $\frac{2.5}{24}$ | $\frac{2.5}{24}$ | $\frac{2.5}{24}$ | 0 | $\frac{2.5}{24}$ | $\frac{0.0001}{365}$ | 50 | 0.5 | $10^{-6}$ |

◇ Initial distribution:

$$n_k(T) = \frac{\lambda^k e^{-\lambda}}{k! \sum_{j=0}^{49} n_j(T)}, \text{with } \lambda = 10$$

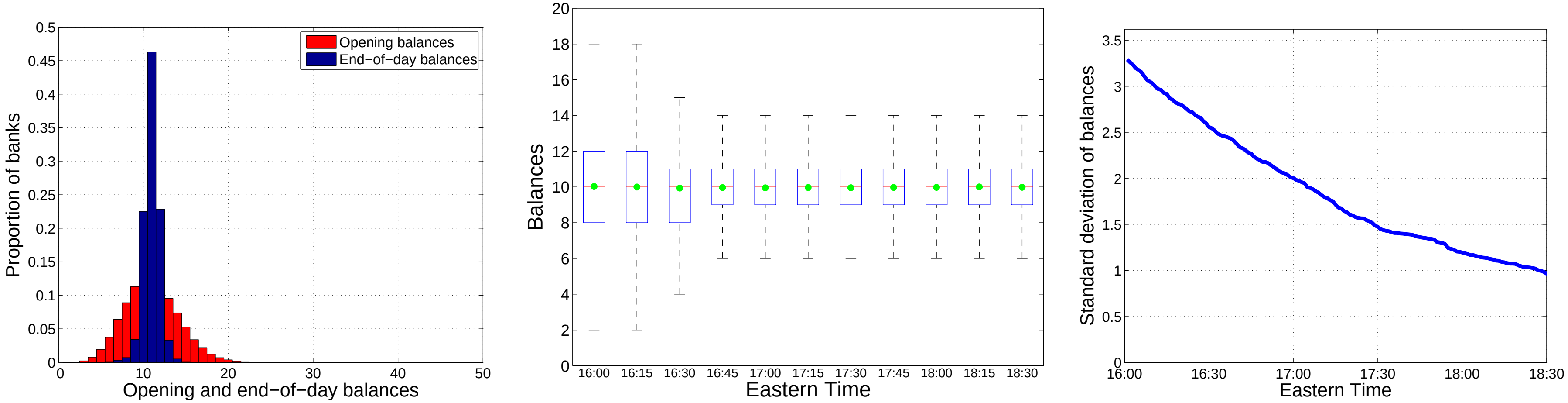



Figure: Balances

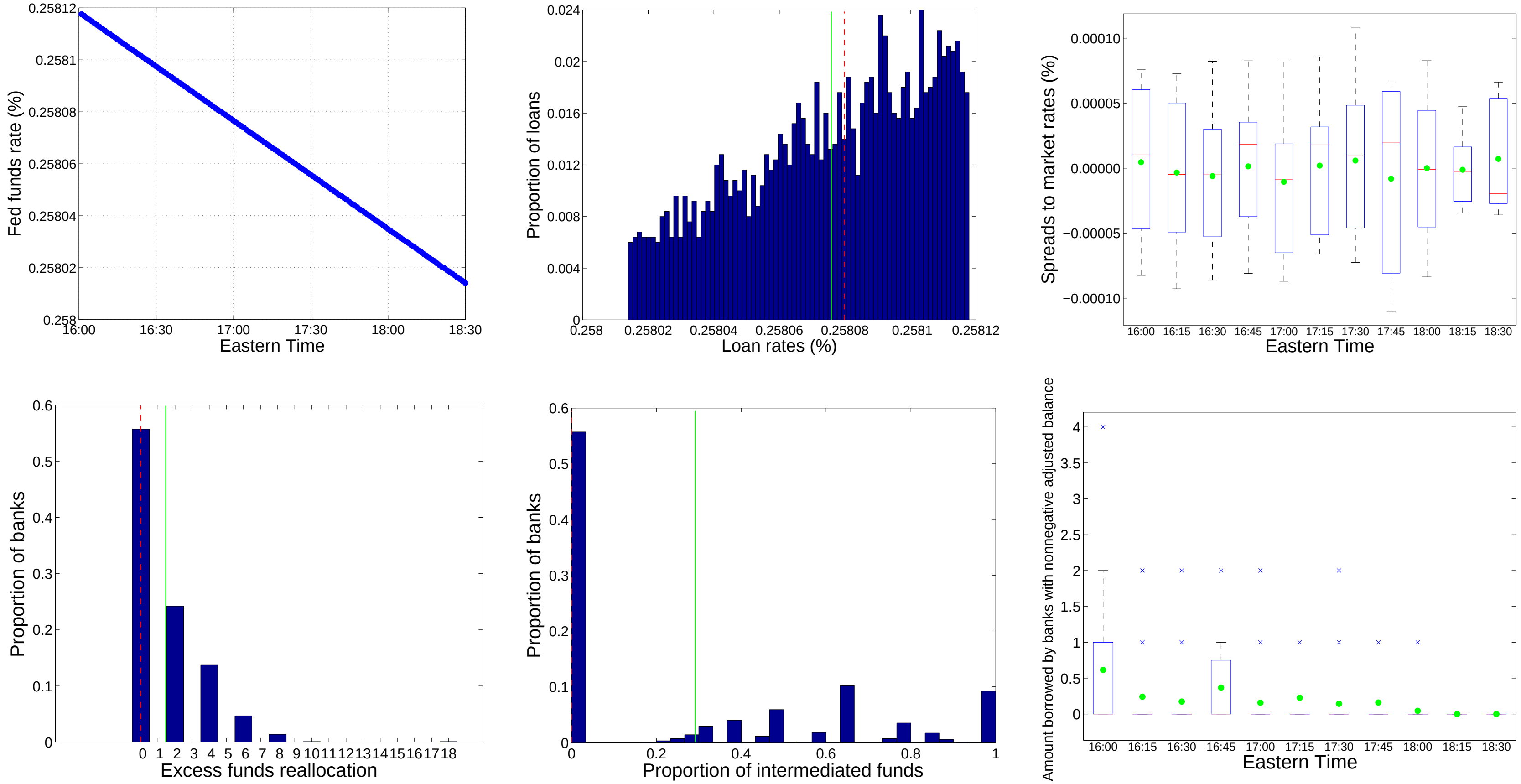



Figure: Price dispersion and intermediation

# Impact of changes in IORR

(a) Required/Total=10% (b) Required/Total=90%

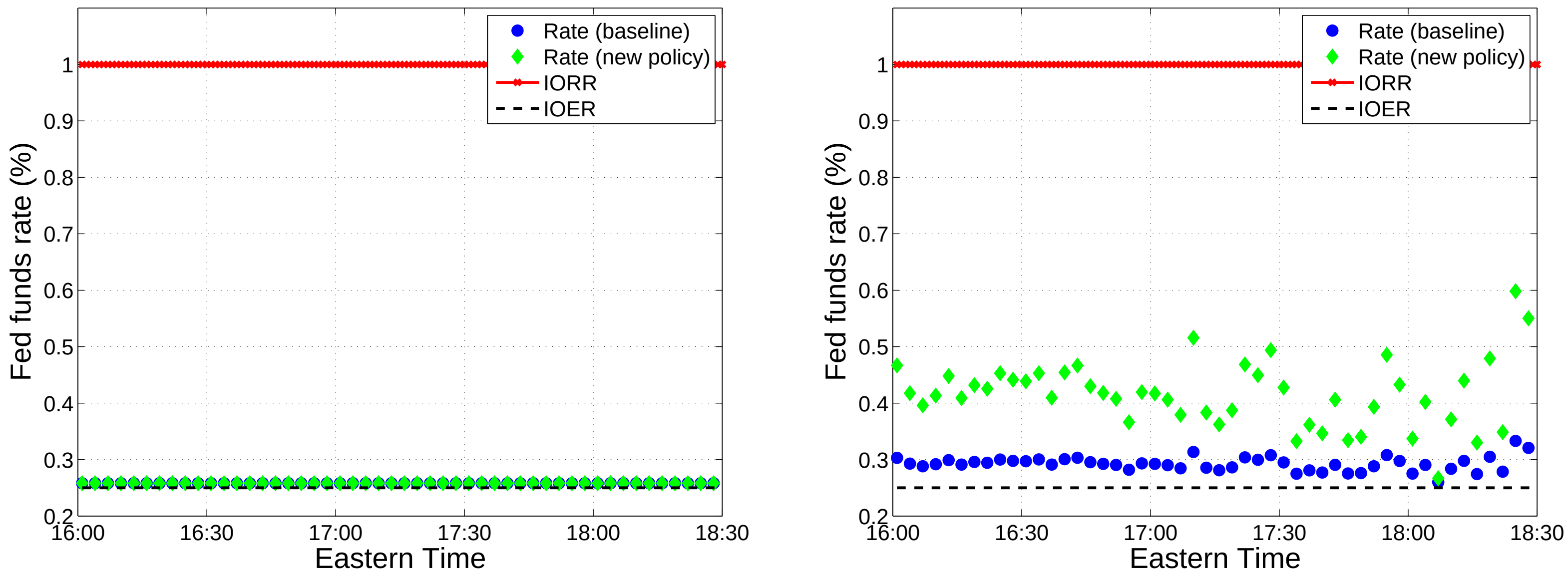



Figure: Increase in IORR to 1%

# Extensions

1. Heterogeneity in bargaining power

   Large banks typically exhibit stronger bargaining positions

2. Heterogeneity in contact rates

   Some participants trade more frequently than others

3. Heterogeneity in payoffs

   Some institutions hold higher excess reserves

   Some participants do not earn interest on reserves (GSEs)

# Concluding Remarks

◇ A dynamic equilibrium model of trade in the fed funds market which incorporates two distinctive features of an OTC market:

1. Search for counterparties

2. Bargaining over the terms of the loan (size and rate)

◇ Study the determinants of fed funds rate, trading volume and intermediation... as well as the efficient reallocation of funds

# Future Research

◇ Fed funds brokers

◇ Random payment shocks

◇ Quantitative analysis with ex-ante heterogeneity

◇ Policy questions:

- Changes to interest on reserves (level, who earns interest)
- Changes to the level of reserves (draining reserves)

# Appendix

# OTC frictions: Price dispersion

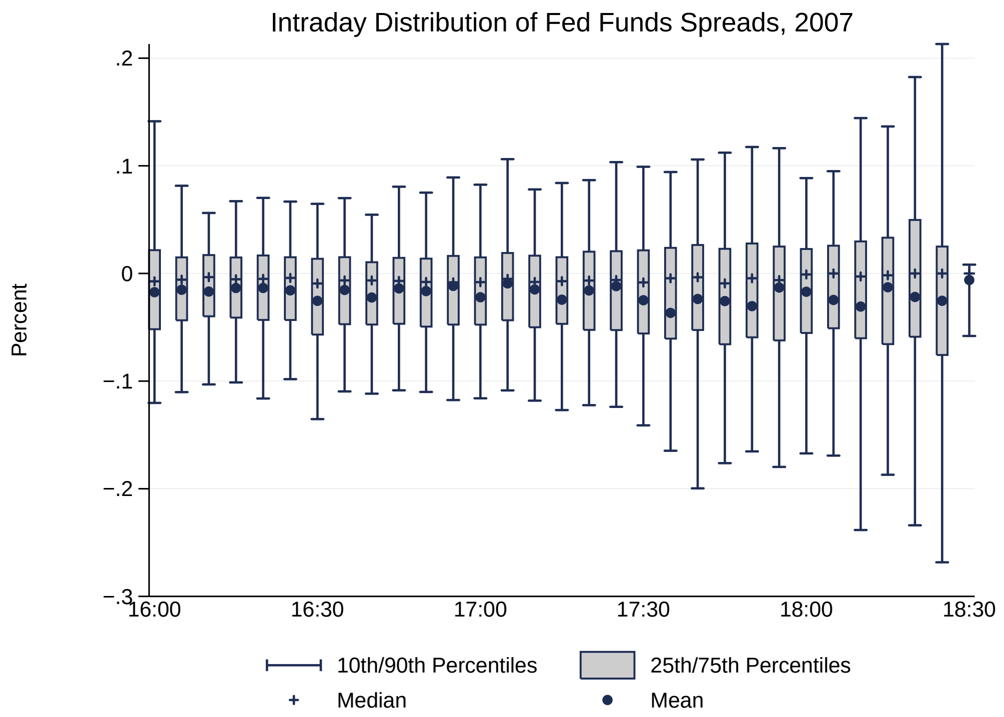



Figure: Spread to daily value-weighted rate

# OTC frictions: Intermediation

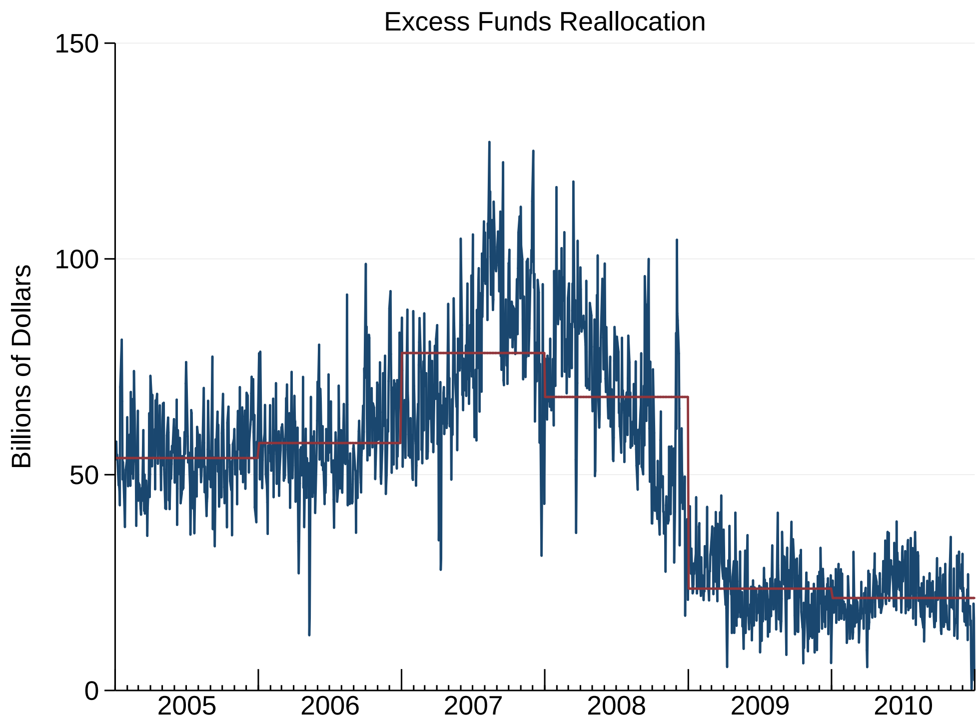



Figure: Excess funds reallocation

# OTC frictions: Intermediation

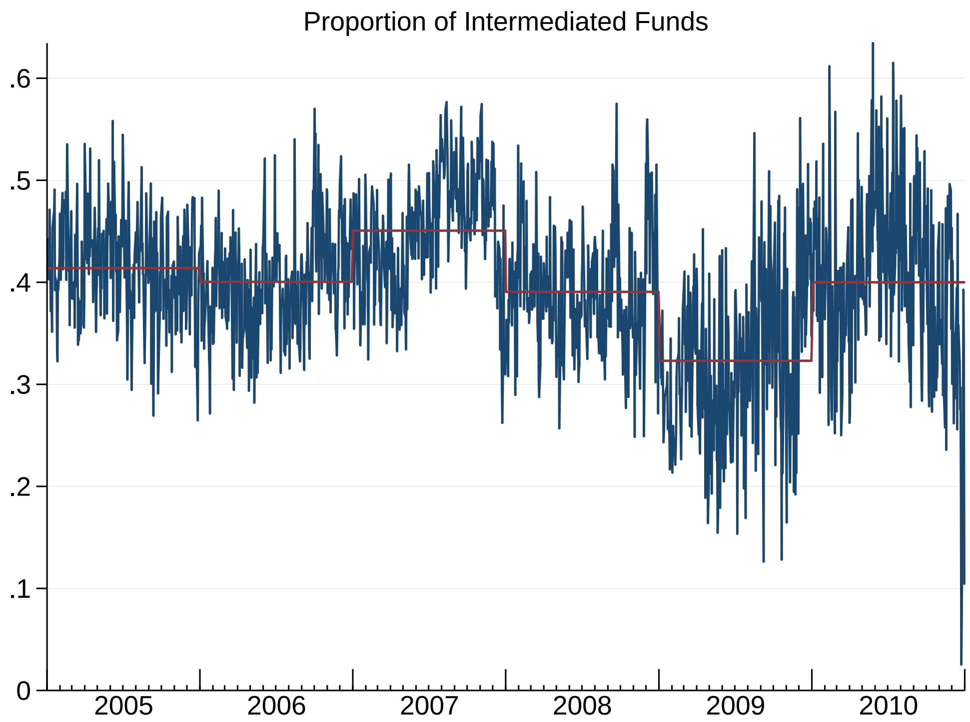
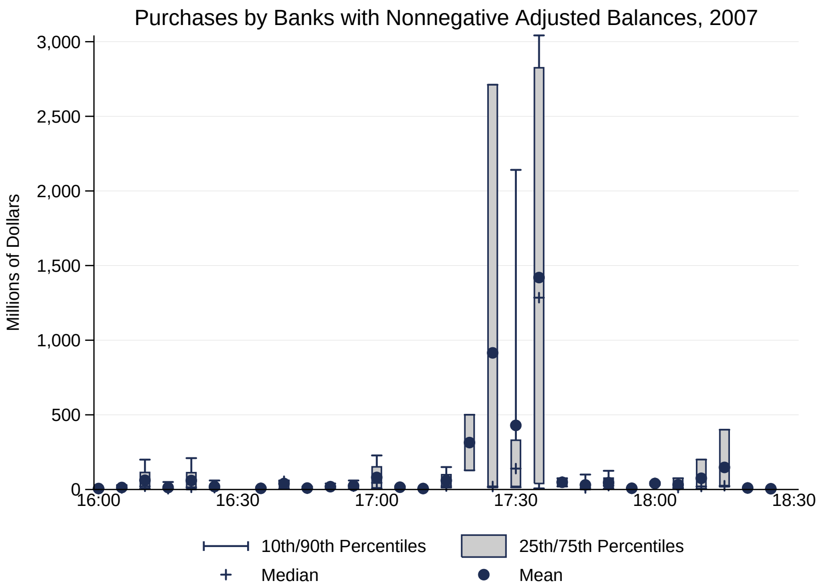
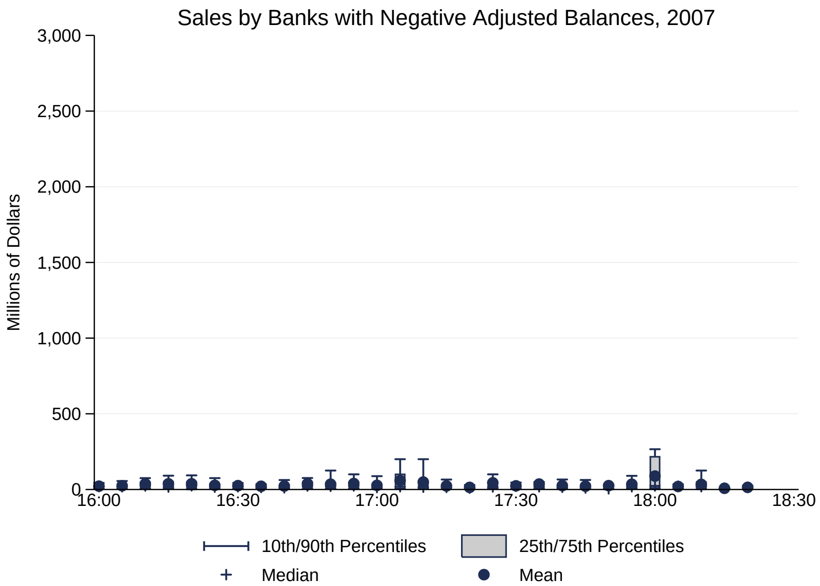



Figure: Proportion of intermediated funds

# OTC frictions: Intermediation




Purchases by banks with ‘high’ balances & Sales by banks with ‘low’ balances