

# Knowing Who You are Matters: A Theory of Young Firms versus Mature Firms\*

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## Abstract

We develop a model of firm growth where the firm's capability must match with the project so that the firm can profitably commercialize the project. A young firm does not know its own capability and may learn it by attempting to commercialize a project. Only if this attempt is successful, the firm learns that its own capability matches with the commercialized project and becomes mature. Our model implies that a mature firm tends to abandon a project earlier than a young firm. This is because a mature firm discovers that the project may not fit its capability at an earlier stage of the project but a young firm cannot apply such an abandonment criterion not knowing its own capability. Further, we study stationary equilibria of the industry, where a firm may buy another firm's project by acquiring the selling firm. We find that a mature firm may acquire either a young firm or a mature firm, but a young firm will not acquire any firm, and that acquired projects perform better than internally commercialized projects. In an equilibrium in which a mature firm acquires another mature firm, the value of having a later-stage project becomes important relative to the value of being mature.

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## Abstract

We develop a model of firm growth where the firm's capability must match with the project so that the firm can profitably commercialize the project. A young firm does not know its own capability and may learn it by attempting to commercialize a project. Only if this attempt is successful, the firm learns that its own capability matches with the commercialized project and becomes mature. Our model implies that a mature firm tends to abandon a project earlier than a young firm. This is because a mature firm discovers that the project may not fit its capability at an earlier stage of the project but a young firm cannot apply such an abandonment criterion not knowing its own capability. Further, we study stationary equilibria of the industry, where a firm may buy another firm's project by acquiring the selling firm. We find that a mature firm may acquire either a young firm or a mature firm, but a young firm will not acquire any firm, and that acquired projects perform better than internally commercialized projects. In an equilibrium in which a mature firm acquires another mature firm, the value of having a later-stage project becomes important relative to the value of being mature.

# 1 Introduction

A substantial number of literatures in industrial organization reports that young firms and old firms are different. They sometimes appear to have different ordering as to which project to commercialize (Cassiman and Ueda, 2006). Young firms appear to be more innovative than established firms are (Scherer, 1980). Young firms are more likely to die than old firms.<sup>1</sup> Given small firms tend to be newer than large firms, evidence pointing the difference between small and large firms also join the league of the above literatures. Small firms tend to do more research than development (NSF, various years publication). Small firms appear to produce more innovations given the same number of employees. (Acs and Audretsch, 1988)

This paper proposes a new theory to explain the differences between young firms and old firms recognized in the literature above. Our basic idea is as follows. Each firm is born with a capability that allows the firm to successfully commercialize a certain type of projects. Nevertheless, the firm initially does not know which type of projects fits its own capability. The firm learns about its own capability only through the outcome of the commercialization it attempts. When the commercialization is successful, the firm learns that its capability is meant for the type of the project just successfully commercialized. Once the firm gets to know its capability, it becomes “mature” and optimally specializes in the same type of projects. Unlike the mature firm, the young firm does not distinguish one type of project from another *ex-ante*. Accordingly, the new firm undertakes whichever project comes along first. Therefore, this paper develops a model of firm growth where the firm’s preference as to which project to commercialize endogenously changes.

Each project goes through three stages: *incubation*, *development*, and *harvesting*. During the incubation stage, the firm writes up a detailed business plan and identifies what has to be done to bring the idea to the market. During the development stage, the business plan is executed, and only if it is

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<sup>1</sup>Studies using census data include Dunne, Roberts and Samuelson (1988, 1989). Evans (1987) and Hall (1987) report similar results using Dun and Bradstreet data and Compu-stat data respectively.

executed by the firm with the right capability, the project starts generating profits – harvesting. We study the stationary equilibrium of the economy where learning of own capability takes place in the manner described above. We study two types of industries. In the first type of industries, we do not allow any project transfer across firms abstracting from documented obstacles for technology transactions such as the appropriability problem advocated by Arrow (1962).<sup>2</sup> In the second type of industries, we allow project transfers but assume that the project is inalienable from the incubator company. Therefore, the buyer of the project has to acquire the incubator company to obtain the project. For instance, this case occurs when project-related knowledge is tacit and embedded into the whole organizational routine, when close cooperation of the incubator and the developer is necessary for developing the project, or both.<sup>3</sup>

For both types of industries just described, our model implies that a mature firm tends to abandon a project earlier than a young firm. This is because a mature firm chooses not to develop the incubated project if the project’s type does not match the firm’s capability, whereas a young firm cannot apply such an abandonment criterion not knowing its own capability. As a consequence, the young firm tends to insist on continuing the project it started. Our model predicts that the value of the mature firm increases more rapidly than that of the young firm when the firm progresses from an early stage of the project to a later stage of the project. This is because the mature firm chooses to move up the project cycle only if the project matches its capability and the commercialization will certainly generate a profit.

For the industry without project transfers across firms, our model implies that the ratio of commercialization spending to incubation spending

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<sup>2</sup>Related to this argument, von Hippel (1988) states that “users (of the innovations) can profit from such an innovation while keeping it hidden behind their factory walls as a trade secret.

<sup>3</sup>It is also possible to study the model in the context such that project transfer is possible and the incubator does not need to follow the project. This case occurs when project-related knowledge is codifiable and task-partitioning between the incubator and the developer is easy to implement (Arora, Fosfuri, and Gambardella, 2001). Chemical and petroleum industries where patent protection is effective (Levin et. al. 1987; Mansfield 1986) are supposedly in this third case.

is higher for a young firm than for a mature firm. It follows that an increase in the commercialization cost may hurt the young firm more than the mature firm. This is because the mature firm often abandons its projects before commercializing them, whereas the young firm does not. As a consequence, for a given project, the young firm incurs the cost of developing a project more often than the mature firm, and therefore an increase in the commercialization cost may hurt the young firm more than the mature firm.

For the industry with project transfers across firms, we focus on the case in which if a firm buys a project of another firm, then the buyer has to acquire the selling firm. If the cost of transferring a project is low enough, there exists a non-trivial stationary equilibrium in which a mature firm acquires either a young firm or another mature firm, but a young firm does not acquire a mature firm. This is because transferring an incubated project to a mature firm that possesses the capability suitable for such a project allows more profitable commercialization. As a consequence, our model also implies that the transferred projects are more successful than non-transferred projects on average, as the formers are developed by the firm with the right capability, whereas this may not be the case for the latter.

We find two types of non-trivial stationary equilibrium. In the first type of equilibrium, mature firms never incubate any projects by themselves, acquire incubated projects from young firms and develop them. In this equilibrium, we find a paradoxical result such that young firms attempt to commercialize their incubated projects more often if such an attempt is more likely to fail. This is because when such an attempt is likely to fail, only few young firms become mature firms. As a consequence, mature firms' demand for incubated projects is low in such a case and, due to this lack of sales opportunities, more young firms are forced to commercialize their incubated projects instead of selling them. In the second type of equilibrium, a mature firm does incubate its own project, chooses not to buy a project incubated by a young firm, and lets the young firm to develop it. This occurs because acquiring a project is costly due to the transfer cost. By incubating its own project, the mature firm bets on the possibility that it will incubate a project suitable for its capability, develop the project by itself, and save

on the transfer cost. Nevertheless, if the mature firm incubates the project that does not match with the firm's capability, the mature firm may sell the project and get acquired as there is no chance that the mature firm can successfully develop the incubated project.

An interesting question here is why a young firm still develops a project instead of selling it, while a mature firm ceases by being acquired. Intuitively, a mature firm should be more valuable than a young firm, as the mature firm knows its capability whereas the young firm does not. As a consequence, acquiring a young firm instead of acquiring a mature firm appears to cause lower losses of resources available in the economy. Nevertheless, if we compare the mature firm possessing the project unsuitable for the mature firm's capability with the young firm possessing the project that may fit the young firm's capability, it may be beneficial to keep the latter alive. As a consequence, mergers between two mature firms and the project development by young firms may coexist.

Our paper is related to several strands of literature. Jovanovic (1982) is a seminal paper in modelling a new firm's learning process. Nevertheless, projects are homogeneous in the Jovanovic model, and therefore it does not address the issue of project selection that is the focus of our paper. Similar to our paper, Cassiman and Ueda (2004) model the difference between a new firm and a mature firm such that a new firm is indifferent between projects, whereas a mature firm is not. Nevertheless, different from this paper, Cassiman and Ueda do not study the process through which a new firm becomes a mature firm. Similar to our paper, Holmes and Schmitz (1990) model a multi-stage project and study individuals that possess the same commercialization skill but differ in the incubation skill. In their model, projects are transferred because some individuals are more talented in starting up new projects than others, whereas, in our model, projects are transferred because incubators may not possess the right capability to develop the projects. Holmes and Schmitz, and this paper share the same prediction such that transferred projects are more profitable than non-transferred projects. Different from Holmes and Schmitz, we are also able to predict that a business is transferred from a young firm to a mature firm. Similar to our paper,

Bernardo and Chowdhry (2000) study the project selection problem when a firm learns its capability by experimentation. They assume that a firm may possess either project-specific capability or general-management capability. In equilibrium, an established firm diversifies into a new area to see which type of capability the firm possesses. If the diversification is successful, the firm learns that it possesses the general-management capability. Otherwise, the firm learns that it possesses project-specific capability. Different from our paper, the focus of Bernardo and Chowdhry is the firms that have once successfully commercialized their past projects. As a result, their model is more appropriate for explaining the learning of mature firms rather than that of young firms, which is the focus of our paper. Finally, Mitchell (2000) studies the model in which a firm acquires a project specific capability through experience. In our model, the firm discovers instead of acquires such a capability.

The organization of this paper is as follows. Section 2 describes the basic model and the results. Section 3 introduces and characterizes the market for projects. Section 4 concludes.

## 2 The Model

We outline the lifecycle of the risk-neutral firm, which maximizes the expected present value of cash flows at the continuously compounded discount rate  $r$ . Firm's capability is denoted by  $s_f \in T_S$ , where  $T_S = \{1, 2, \dots, S\}$ . This capability defines which project the firm can commercialize profitably. The capability of the firm does not change over time. Project's type has the same space as  $T_S$ , and the firm can commercialize a project profitably if and only if the firm's capability and the project's type are the same. We call the firm "young" if it does not yet know its own capability and "mature" if it learned its own capability. We assume that the prior probability that a young firm possesses capability  $s_f$  is  $1/S$ ,  $\forall s_f \in T_S$ .

## 2.1 Project Arrival, Incubation, and Learning

The firm's lifecycle develops as follows. The firm emerges as a young firm and incubates a project incurring per period cost,  $c_0$ . The firm completes the incubation according to a negative exponential arrival process with rate of  $\lambda_0$ . The firm may also suddenly die according to a Poisson process with death rate,  $\mu_g > 0$ . This probability is invariant across firm's different stages. If we denote  $C_{g/0}$  be the expected present value of the total incubation cost,  $C_{g/0} = c_0 / (r + \lambda_0 + \mu_g)$ . Once the incubation is completed, the young firm learns the project type  $s_p \in T_S$ . We call the project young if it is not incubated, and mature if it is incubated. Once the project becomes mature, the young firm decides whether it attempts to commercialize the project or abandons it. If the project is developed, it costs  $c_1$  per period until the development is completed. The completion timing follows a negative exponential arrival process with rate  $\lambda_1$ . If we denote  $C_{g/1}$  as the present value of the expected development cost,  $C_{g/1} = c_1 / (r + \lambda_1 + \mu_g)$ . Once the development is completed, the project starts generating a per-period profit  $y > 0$  for an uncertain period if  $s_p = s_f$ , and otherwise "flops", that is, the project generates zero profit.<sup>4</sup> We interpret  $y$  as the net profit per unit time a firm enjoys from a successful project as long as it acts as a monopolist. However, after some time, the market becomes competitive driving profits to zero. If a young firm starts generating revenue from a project of type  $s_p$ , it learns that its capability is equal to  $s_p$  and therefore becomes mature. For simplicity, we assume that the young firm exits if the developed project flops. A rationale for this assumption is that the project failure may reveal not only the mismatch of the project and the firm's capability but also the firm's general ability to survive. This rationale is in the spirit of Jovanovic (1992), where there is no mismatch of project type and firm's capability but

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<sup>4</sup>A more general setting is to assume that  $y$  is a decreasing function of the distance between  $s_p$  and  $s_f$ . This generalization, however, adds the complex learning problem of a young firm. For instance, after generating a sufficiently high revenue, the young firm tries to commercialize if and only if the new project type is sufficiently close to that of the previous project. Studying this learning and experimentation process of the young firm is out of this paper's scope.



a new firm learns its cost.<sup>5</sup>

It is useful to discuss two issues on our basic setup at this moment. First, the order of the learning, project type first and capability later, is important in our model. If the firm would learn its capability first, we would not have the difference between a young firm and a mature firm that we derive later. Nevertheless, we can potentially generalize the learning process such that a firm may learn of its capability prior to learning of project type, as far as the firm may not learn of its capability precisely and therefore there is still value of learning from the commercialization outcome. Second, “firm” characterized by  $s_f$  may be a collection of resources such as entrepreneur, partners, knowledge, physical assets including location, or unique products or services. Our “firm” is a distinct entity from its project.

Similarly to a young firm, a mature firm may also die according to a Poisson process with death rate  $\mu_m > 0$ . A positive profit stream of the mature firm stops with the Poisson death rate  $\lambda_2$ . To maintain the model as simple as possible, we assume that  $\mu_m = \mu_g = \mu > 0$ . Therefore, if we denote  $Y$  be the expected present value of the revenue stream from the project,  $Y = y / (r + \lambda_2 + \mu)$ . Once the project “dies” in this way, the mature firm enters a project process similar to the one for a young firm. The mature firm first incubates a new project, and, once incubated, decides to develop it. If the mature firm decides not to commercialize the project, it waits for another project to arrive. The flow cost and the arrival and death rates are the same as described in the case of a young firm.

We denote the stage of firm development by  $\sigma \in \{g/0, g/1, m/0, m/1, m/2\}$ , where  $g$  means that a firm is young and  $m$  means that a firm is mature. The symbol  $/0$  means that the firm is incubating a project,  $/1$  means that the firm is developing a project, and  $/2$  means that the firm is generating revenues. These assumptions are summarized in the following table.

It is convenient to work with discount factors instead of discount rates, and

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<sup>5</sup>Without this assumption, the model needs to analyze the decision of more than one type of young firms. Some have not experimented at all and therefore do not have any idea about their types. Others have failed in one or more projects and therefore know which projects they are not good at.

	Young Firm		Mature Firm		
	Incubation	Development	Harvesting	Incubation	...
	0	1	2	0	...
Transition Rate	$\lambda_0$	$\lambda_1$	$\lambda_2$	$\lambda_0$	...
Death Rate	$\mu$	$\mu$	$\mu$	$\mu$	...
Cash Flow Rate	$-c_0$	$-c_1$	$y$	$-c_0$	...

Table 1: Summary of Notations

therefore we introduce the following notation:

$$\rho_j = \frac{\lambda_j}{\lambda_j + r + \mu}, \quad j = 0, 1, 2.$$

To make the model non-trivial, we make the following assumption.

**Assumption 1**

$$\Phi - \rho_0 \frac{(S-1)C_1}{S} + \rho_0 \rho_1 \rho_2 \frac{\alpha}{S} \Phi \geq 0, \quad (1)$$

where

$$\Phi \equiv \rho_0 \rho_1 \frac{Y}{S} - C_0 - \rho_0 \frac{C_1}{S}, \quad \text{and} \quad (2)$$

$$\alpha \equiv \left[ 1 - \frac{\rho_0 \rho_1 \rho_2}{S} - \frac{\rho_0 (S-1)}{S} \right]^{-1}. \quad (3)$$

Intuitively,  $\Phi$  is the present value of the expected cash flows from the entire cycle of one project for a *mature* firm. The value  $\Phi$  includes the incubation cost, the development cost, and the revenues if the project is matched with the firm's capability, but does not include the development cost if the project does not turn out to match the firm's capability. The left hand side of the equation (1) is the net present value of a young firm at its inception. The first two terms are the expected present value of the first project. Note that we subtract the extra term  $\rho_0 (S-1) C_1 / S$  because a young firm does not know its capability and always develops the incubated project. The third term is the option value of becoming a mature firm in the future. The variable  $\alpha$  is the multiplier to summarize the value of the projects that a mature firm will receive eternally. In sum, this assumption ensures that starting to incubate a project has a non-negative net present value for a young firm.

## 2.2 Development Policy and Value of the Firm

We now describe the development policy of the firm once the incubation of the project is finished and the project type is revealed. Here, the decision may depend on the firm's age. The mature firm knows its capability, therefore it knows with certainty if the project will or will not generate revenues once it is developed. The young firm, on the contrary, does not possess such prior knowledge and therefore the revenues that will be generated from the project are uncertain.

**Definition 1** *History is a function  $h_\tau : [0, \tau] \rightarrow \sigma$ .*

That is, the history tracks the firm's past stage of its lifecycle.

**Definition 2** *A policy is a set-valued correspondence  $d : \{2^{T_S}, h_t\} \rightarrow \{0, 1\}$ , where  $T_S = \{1, 2, \dots, S\}$ ,  $2^{T_S}$  is the power set of  $T_S$ , that is, the set of all subsets of  $T_S$ . The decision  $d(\cdot) = 1$  if the firm decides to develop and  $d(\cdot) = 0$  otherwise.*

Note that for our model it would suffice to define decisions upon the singleton subsets of  $T_S$  since the project type is revealed with certainty after the incubation. However, our more general definition accounts for the possibility that there still exists some uncertainty regarding the project type after the incubation stage. For example, occurrence of  $T_S$  would imply that the incubation stage has been a total failure since no new information about the project type is revealed. For notational simplicity, we will write  $d(s)$  instead of  $d(\{s\})$  from now on.

We now define the value of the firm for each stage of the firm's lifecycle. Let  $i = \{g, m\}$  be the firm's "age." That is, if  $i = g$ , the firm is young and if  $i = m$ , the firm is mature. Let  $j = \{0, 1, 2\}$  be the project-related stage of the firm. That is, if  $j = 0$ , the firm is incubating a project, if  $j = 1$ , the firm is developing a project, and if  $j = 2$ , the firm is generating a positive revenue. Using this notation, let  $V_{i/j}(d)$  be the value of the firm that follows policy  $d$  when the firm's age is  $i$  and the firm's project-related stage is  $j$ . Then we define optimality as follows:

**Definition 3** A policy  $d^* \in 2^{Ts}$  is optimal for a firm if and only if  $V_{i/j}(d^*) \geq V_{i/j}(d), \forall d \in 2^{Ts}$ .

We introduce some terminology about firm's decisions that will be useful in the equilibrium analysis that follows.

**Definition 4** A policy  $d$  is called connected if for any  $s_1 < s_2$  with  $d(s_1, h_\tau) = d(s_2, h_\tau) = 1$  we have  $d(\tilde{s}, h_\tau) = 1, \forall \tilde{s} \in [s_1, s_2]$ . Policies that assign 1 for singletons will also be called connected by convention. A policy  $d$  is called monotonic if for any  $s_p \in T_S$  with  $d(s_p, h_\tau) = 1$  we have  $d(s, h_\tau) = 1$ , for all  $s$  such that  $s \geq s_p$ .

### 2.3 The Optimal Development Policy

We now characterize the optimal development policy of the firm. Recall that a mature firm knows its capability  $s_f$ . Intuitively, it would only develop projects that match its capability, since this guarantees the firm to generate revenues. If the firm's capability does not match with the project type, the firm will abandon the project in order to avoid incurring the development cost  $c_2$ . The next proposition formalizes this intuition.

**Proposition 1** The optimal policy for a mature firm of type  $s_f$ ,  $d^*$  is

$$d^*(s_p, h_\tau) = \begin{cases} 1, & \text{if } s_p = s_f \\ 0, & \text{otherwise} \end{cases}.$$

**Proof.** See Appendix.

The optimal policy of the mature firm is history independent. The firm maximizes its value by following the same decision rule for all the projects it encounters during its lifetime.

The young firm, in contrast with the mature firm, does not know its capability in advance. Therefore, there is uncertainty associated with the decision to develop an incubated project. The young firm will decide upon commercializing only if the expected benefit of a match exceeds the cost of developing the new project.

**Proposition 2** *The optimal policy for the young firm is to incubate any incoming project, independent of  $s_p$ .*

**Proof.** See Appendix.

The proof of this proposition formalizes the intuition that the policy of developing immediately brings  $V_{m/0}$  to the firm earlier than any other policy.

Proposition 1 and Proposition 2 make a sharp contrast between the mature firm and the young firm in terms of their development policy. When making the development decision, the mature firm is more selective than the young firm, which undertakes any kind of projects. In what follows, for notational simplicity, we drop the dependence of the firm value on the policy  $d$  with the understanding that we refer to the optimal policies by all firms.

## 2.4 The Value of the Firm

We now derive the closed form solutions for the firm value.

**Proposition 3** *The firm value for each product stage is given by the following set of the equations:*

$$V_{m/0} = \alpha\Phi, \quad (4)$$

where  $\alpha$  and  $\Phi$  are given in Assumption 1,

$$V_{m/2} = Y + \rho_2 V_{m/0}, \quad (5)$$

$$V_{m/1} = -C_1 + \rho_1 V_{m/2}, \quad (6)$$

$$V_{g/1} = -C_1 + \frac{\rho_1 V_{m/2}}{S}, \quad (7)$$

and

$$V_{g/0} = -C_0 + \rho_1 V_{g/1} \quad (8)$$

**Proof.** See Appendix.

Note that the value of the firm is linear on the project potential revenue  $Y$ , the incubation cost  $C_0$  and the development cost  $C_1$ . The value of the firm is decreasing in the number of project types  $S$ , because if  $S$  increases, then it becomes less likely that the young firm's capability matches with the

project the firm incubates. Observe that except  $V_0$ , which is nonnegative according to equation (1), all other values are strictly positive. Equation (1) implies  $\Phi > 0$  and  $\alpha > 0$ . Consequently  $V_{m/0} > 0$ , and it follows that  $V_{m/2} > 0$  by equation (5) and by  $Y > 0$ . To show that,  $V_{i/1} > 0$ ,  $i = \{g, m\}$ , it suffices to demonstrate that  $V_{g/1} > 0$ , because  $V_{m/1} > V_{g/1}$ . Sticking equation (5) for  $V_{m/2}$  into equation (7) gives

$$\begin{aligned} V_{g/1} &= -C_1 + \frac{\rho_1}{S}Y + \frac{\rho_1\rho_2}{S}V_{m/0} \\ &\geq -C_1 + \frac{\rho_1}{S}Y \\ &> 0. \end{aligned} \tag{9}$$

The last inequality follows equation (2) together with  $\Phi > 0$ . By using a similar logic, we can prove that  $V_{g/0} > 0$ .

#### 2.4.1 Value of Mature Firm versus Value of Young Firm

We now compare the value of a young firm and that of a mature firm for the same product stages. For the development stage, subtracting equation (7) from equation (6) and recursively using equation (5) and equation (6), we have

$$V_{m/1} - V_{g/1} = \rho_1 \frac{S-1}{S} (Y + \rho_2 \alpha \Phi). \tag{10}$$

This difference between the value of a mature firm and that of a young firm at the development stage comes from the payoff difference when the project developed by the young firm turns out not to match with the young firm's capability. If this is the case, the young firm does not enjoy both the value of positive revenue stream  $Y$  and the continuation value after the end of the revenue stream  $\alpha\Phi$ , whereas the mature firm enjoys both values.

Similarly for the incubation stage, we can compute

$$V_{m/0} - V_{g/0} = \rho_0 \frac{S-1}{S} (C_1 + \alpha\Phi). \tag{11}$$

The right hand side is the sum of the cost saving and the continuation value that the mature firm can potentially realize if the incubated project does not match with the firm's capability. For any product life stage, the value of

the mature firm is at least as much as that of a young firm. This is because the mature firm knows its capability and this information helps the firm to avoid incurring the development cost of the unmatched project.

How do exogenous variables affect the value of young and mature firms? The answers to this question is summarized in the following corollaries.

**Corollary 1** *The impact of the project revenue and of its duration on the firm values is as follows. For  $j = \{0, 1\}$ ,  $\partial V_{m/j}/\partial y > \partial V_{g/j}/\partial y > 0$  and  $\partial V_{m/j}/\partial \lambda_2 < \partial V_{g/j}/\partial \lambda_2 < 0$ . For  $j = 2$ ,  $\partial V_{m/j}/\partial y = \partial V_{g/j}/\partial y > 0$  and  $\partial V_{m/j}/\partial \lambda_2 = \partial V_{g/j}/\partial \lambda_2 < 0$ .*

**Proof.** To prove the positive impact of  $y$ , note that  $\Phi$  increases in  $y$ . To prove the negative impact of  $\lambda_2$ , note that differentiating  $\alpha\Phi$  by  $\lambda_2$  gives

$$\frac{\partial \alpha\Phi}{\partial \lambda_2} = \frac{\alpha\rho_0\rho_1 r}{S(r + \lambda_2)^2} \left( \alpha\Phi - \frac{y}{r} \right) < 0.$$

Together with these two facts, partial differentiation of equations (5), (6), (7), (4), (8), (10), and (11) gives the results. ■

This corollary states that the value of the firm is increasing in the project revenue  $y$  and the duration of this revenue  $1/\lambda_2$ . It also states that the sensitivity of the value to  $y$  and  $\lambda_2$  is higher for a mature firm than for a young firm. This asymmetric sensitivity is due to the following reason. The mature firm will stay in the market unless it involuntarily dies, whereas the young firm may exit the market if its project turns out not to match its own capability. Change in  $y$  and  $\lambda_2$  affects the firm value only for the state in which the firm stays in the market. A young firm is affected less because it, unlike the mature firm, has a lower probability to exit from the market.

**Corollary 2** *The impact of the incubation cost and its duration is as follows:  $\partial V_{m/0}/\partial c_0 < \partial V_{y/0}/\partial c_0 < 0$  and  $\partial V_{m/0}/\partial \lambda_0 < \partial V_{y/0}/\partial \lambda_0 < 0$ . For  $j = 3$ ,  $\partial V_{m/0}/\partial c_0 = \partial V_{g/0}/\partial c_0 < 0$  and  $\partial V_{m/0}/\partial \lambda_0 = \partial V_{y/0}/\partial \lambda_0 < 0$ .*

**Proof.** To prove the negative impact of  $c_0$ , note that  $\Phi$  decreases in  $c_0$ . To prove the positive impact of  $\lambda_0$ , note that both  $\alpha$  and  $\Phi$  increase in  $\lambda_0$ .

Partial differentiation of the equations (5), (6), (7), (4), (8), (10), and (11) gives the results. ■

This corollary states that the value of the firm decreases in the costs and durations of incubating projects. (Note that  $\lambda_0$  is negatively related with the duration of incubation.) And the sensitivity of the value to such costs and durations is higher for a mature firm than a young firm. The reason for this asymmetric sensitivity is the same as the one for Corollary 1.

Comparative statics with respect to the development cost is not as straightforward as the incubation cost, and therefore we divide the results in the two corollaries that follow. The next corollary is about comparative statics on the value of mature and young firms at their development and commercialization stages.

**Corollary 3** *The impact of the development cost and duration is as follows:  $\partial V_{m/1}/\partial c_1 < \partial V_{g/1}/\partial c_1 < 0$ ,  $\partial V_{m/2}/\partial c_1 = \partial V_{g/1}/\partial c_1 < 0$ ,  $\partial V_{m/1}/\partial \lambda_1 > \partial V_{g/1}/\partial \lambda_1 > 0$ , and  $\partial V_{m/2}/\partial \lambda_1 = \partial V_{g/2}/\partial \lambda_1 > 0$ .*

**Proof.** To prove the negative impact of  $c_1$ , note that  $\Phi$  decreases in  $c_1$ . To prove the positive impact of  $\lambda_1$ , note that  $\alpha$ ,  $\rho_1$ , and  $\Phi$  are all increasing in  $\lambda_1$ . Together with these facts, partial differentiation of the equations (5), (6), (7), and (10) combined with the equation (3) gives the results. ■

The value of the firm at the development and the commercialization stages respond to changes in the development cost similarly to changes in the incubation cost. This is because once development starts, both young and mature firms are at an equal footing.

The next corollary is about comparative statics on the value of mature and young firms at their incubation stage.

**Corollary 4** *The impact of the cost and the duration of developing on the value of firms at the incubation stage is as follows: For  $i = \{g, m\}$ ,*

1.  $\partial V_{i/0}/\partial c_1 < 0$  and  $\partial V_{i/0}/\partial \lambda_1 > 0$ .



2.

$$\partial V_{g/0}/\partial c_1 < \partial V_{m/0}/\partial c_1 < 0$$

*if and only if*

$$S - \rho_0 \rho_1 \rho_2 - \rho_0 (S - 1) - \rho_0 (r + \lambda_2) \leq 0; \quad (12)$$

3.

$$\partial V_{g/0}/\partial \lambda_1 > \partial V_{m/0}/\partial \lambda_1 > 0$$

*if and only if*

$$\frac{\alpha \rho_0}{S} (\alpha \rho_2 \Phi + Y) - \frac{c_1}{r} \leq 0. \quad (13)$$

**Proof.** The first part of the corollary can be obtained by differentiating the equations (4), and (8). To prove the second part of the corollary, note that differentiating  $C_1 + \alpha \Phi$  by  $c_1$  gives

$$\frac{\partial (C_1 + \alpha \Phi)}{\partial c_1} = \frac{S - \rho_0 \rho_1 \rho_2 - \rho_0 (S - 1) - \rho_0 (r + \lambda_2)}{(r + \lambda_1) (S - \rho_0 \rho_1 \rho_2 - \rho_0 (S - 1))}.$$

To prove the third part of the corollary, note that differentiating  $C_1 + \alpha \Phi$  by  $\lambda_1$  gives

$$\frac{\partial (C_1 + \alpha \Phi)}{\partial \lambda_1} = \left( \frac{\alpha \rho_0}{S} (\alpha \rho_2 \Phi + Y) - \frac{c_1}{r} \right) \frac{r}{(r + \lambda_1)^2}.$$

Together with equations (10) and (11), the second and the third parts of the corollary follow. ■

A change in the development cost  $c_1$  and the duration of development stage  $1/\lambda_1$  may or may not impact a mature firm more than a young firm. More costly development (high  $c_1$  or low  $\lambda_1$ ) impact the value of the firm in the search or incubation stage in two ways. First, it increases the cost of developing the current project in the future. This negative effect is more severe for a young firm than a mature firm, because the mature firm may not develop the current project at all if it will turn out not to match its capability. Second, more costly development increases the cost of developing future projects. This negative effect is more severe for a mature firm because

a mature firm will stay in the market in the future, whereas a young firm may not. These two effects impact on the difference between the values of a mature firm and a young firm in the opposite way. As a consequence, we do not have a clear result.

#### 2.4.2 Value of Progress for Mature Firms and Young Firms

We are now going to characterize how the firm value develops across the different firm stages.

**Corollary 5**  $V_{m/1} - V_{m/0} > V_{g/1} - V_{g/0}$ . *The difference in the firm value between development stage and incubation stage is larger for a mature firm than a young firm.*

**Proof.** Subtracting equation (10) from equation (11) gives

$$(V_{m/1} - V_{m/0}) - (V_{g/1} - V_{g/0}) = \frac{S-1}{S} [\rho_1 (Y + \rho_2 \alpha \Phi) - \rho_0 (C_1 + \alpha \Phi)].$$

Noting that  $\alpha \Phi = V_{m/0}$ ,  $Y + \rho_2 \alpha \Phi = V_{m/2}$  and rearranging equation (6) give  $\rho_1 V_{m/2} = V_{m/1} + C_1$ , we get

$$(V_{m/1} - V_{m/0}) - (V_{g/1} - V_{g/0}) = \frac{S-1}{S} [V_{m/1} + (1 - \rho_0) C_1 - \rho_0 V_{m/0}].$$

The right hand side is strictly positive since  $1 - \rho_0 > 0$ ,  $\rho_0 < 1$  and  $V_{m/1} > V_{m/0}$ , and the result follows. ■

Intuitively, if a mature firm is developing a project, that implies that the project type is matched with the firm's capability, and the development profit is guaranteed. Nevertheless, if a young firm is developing a project, it may not be matched with the firm's capability and therefore it may fail to generate revenues. Therefore, the progress from the incubation stage to the development stage conveys a stronger signal for a mature firm than a young firm.

**Corollary 6**  $V_{m/2} - V_{m/1} < V_{m/2} - V_{g/1}$ . *The difference in the firm value between the harvesting stage and the development stage is larger for a young firm than a mature firm.*

**Proof.** It immediately follows that  $V_{m/2} = V_{g/2}$  and  $V_{m/1} > V_{g/1}$ . ■

The young firm gains more from becoming a mature firm than a mature firm does from successfully commercializing a new project. This is because when a young firm progresses to the harvesting stage, this incident signals that the firm has a general ability to stay in the market. One knows that a mature firm possesses the ability to stay in the market when it is developing a project. Therefore, the value of a young firm increases more than the value of a mature firm, when the firm progresses from the development stage to the harvesting stage. This result is based on the same logic in Altı (2003). He argues that the information content of cash flow is bigger and so is the investment sensitivity to cash flow for a young firm than a mature firm.

## 2.5 Related Evidence

Proposition 1 and Proposition 2 together imply that the mature firm may abandon its project right after the incubation, whereas the young firm does not. Related to this result, Kaplan, Sensoys, and Stromberg (2005) find that venture capital funded firms rarely change their business plans. If we interpret the search and incubation stages as research and the development stage as literally development, our model predicts that a mature firm spends relatively more on research and less on development than a young firm does. This is not supported by data. For instance NSF (2002) reports that the ratio of development to research is 3.1 for firms with not less than 5,000 of employment and this ratio drops to 2.5 for firms with less than 5,000 of employment.<sup>6</sup>

Cockburn and Henderson (2001) examine the performance of drug development for clinical research projects of 10 multinational pharmaceutical firms. They find that among 585 projects with their outcomes known 269

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<sup>6</sup>Similar to Jovanovic (1982), our model predicts that a young firm is less likely to progress to a profitable stage of the project than a mature firm. Consistent with this prediction, Macher and Boerner, (2005) based on the data on drug development projects from CROs (Contract Research Organizations), find that more experienced CROs are less likely to fail in clinical trials than less experienced CROs. We can interpret experience as firm's maturity.

projects were abandoned even before any regulatory filings were commenced (Table 2). Unfortunately, the authors do not have data for young pharmaceutical firms, so we cannot compare whether young firms would be slower on relinquishing projects that our model predicts. Finally, Danzon, Nicholson and Perreira (2005), using the data on 900 pharmaceutical firms, find that experience do not change the probability that the firm completes the phase 1 clinical trial. Jovanovic (1982) suggests that this probability should be higher for a more experienced firm, because such a firm, on average, should have a better ability than a young firm. Our model may explain this contradiction between the theory and the data, as we demonstrate the opposite force such that a young firm may be more persistent than a mature firm. This is because a young firm cannot screen out the project early based on the mismatch between the project type and its own type.

Finally, Corollary 6 suggests that the value of young firms is more sensitive to cash flow announcements than that of mature firms. Supporting this argument, Zhang (2006) finds that stock return of younger firms are more sensitive to revisions of analysts' earning forecasts (Table III).

### 3 Project Transfer

So far we have prohibited technology transfer between firms. In this section, we introduce an opportunity for a firm to sell the project it incubated to another firm.

To motivate our model without technology transfer in the last section and also the model with technology transfer in this section, it is useful to discuss what makes technology transfer easier in one industry than in another. Gans and Stern (2000) argue that severity of the appropriation problem associated with knowledge sales influences the commercialization strategy of start-up firms. The start-up firms may develop a project internally or sell it to an established firm that already possesses the commercialization capability. In order to sell its project, the start-up firm needs to disclose its content to a potential buyer. As a result, if intellectual property rights to the project are not well protected, the potential buyer may commercialize

it without paying to the start-up firm. If this misappropriation is a serious problem, the start-up firm chooses to commercialize its project internally and to sell it to the established firm otherwise. Confirming this hypothesis, Gans, Hsu, and Stern (2002) find that start-up firms with patents, which are less likely subject to the appropriation problem, chooses to commercialize through cooperation from established firms. In particular, they find that such cooperations are more often in biotechnology than other industries. According to this finding, our model in the previous section without technology transfer fits better for non-biotechnology firms and our model in this section with technology transfer fits better for bio-technology firms.

### 3.1 Exploration versus Exploitation

Note that there are two regimes. In the first regime, the mass of mature firms losing revenue generating projects is smaller than the mass of young firms completing the incubation of projects. In this regime, such mature firms instantaneously purchase projects from young firms. Unsold projects are developed by young firms. We call this regime *exploitation* because the supply of projects are abundant and the mature firms focus on exploiting such opportunities. In the second regime, the mass of mature firms losing revenue generating projects is greater than the mass of young and mature firms completing the incubation of projects. In this regime, some of mature firms losing revenue generating projects instantaneously purchase projects from young firms and other mature firms have to start from incubating projects. Therefore, no young firm develops a project. We call this regime *exploration* because the supply of projects are low and the mature firms also incubate projects and explore opportunities.

### 3.2 Stationary Equilibrium

The basic structure of our model with technology transfer is as follows. The entire economy is described by the distribution of firms at different life stages. Let  $G_1(t)$  be the mass of young firms incubating at time  $t$ ,  $G_2(t, s)$  be the mass of young firms developing the project  $s$  at time  $t$ , and  $M_i(t, s)$ ,

$i \in \{0, 1, 2\}$  be the mass of mature firms with capability  $s$  in stage  $i$  at time  $t$ . Let  $N(t)$  be the mass of new firms entering in the market at time  $t$ . Let  $p(t, s)$  be the price of a project with type  $s$  at time  $t$ .

We focus on stationary equilibrium such that the distribution of firm types and the price of each project stays the same over time. Therefore, we drop  $t$  from notation. More formally, we define a stationary equilibrium as follows:

**Definition 5** *A stationary equilibrium with project transfers is a collection of firm masses in the economy  $N, G_0, G_1, M_0(s), M_1(s), M_2(s)$ , a project transfer price  $p(s)$  and a collection of firm development policies such that*

- *the mass of firms at any given life stage and the price of each project are time invariant, and*
- *given the project price, all firms follow the optimal development policies.*

Before starting to characterize the stationary equilibria, it is useful to introduce the following notation. Let

$$G_1 = \sum_{s=1}^S G_1(s)$$

$$M_1 = \sum_{s=1}^S M_1(s)$$

By the law of large numbers,  $G_1(s) = \frac{1}{S}G_1$ , and  $M_1(s) = \frac{1}{S}M_1$ , for all  $s$ . By symmetry,  $p(s) = p$  for all  $s$ .

### 3.3 Project Transfer by Acquisition

In what follows, we study stationary equilibria when a firm immediately dies if it sells its project. This case is relevant when tacit knowledge embedded in the whole organization of the incubator is important for developing and commercializing the project. In this case, not only the project but also its

incubator herself will be sold to the buyer of the project. Therefore, the incubator is not able to recycle her ability to incubate another project.

This assumption is relevant in many industries. Arora, Fosfuri, and Gambardella (2001) summarizes the intellectual history as to the rarity of research specialized firms as follows. “Stigler (1951) argued that division of labor could also embrace the innovation process and industry evolution would lead to the rise of stand-alone R&D labs selling their research outcomes to other parties. Thus, far, this prediction had not come true. Mowery (1983) showed that employment of scientific personnel in independent research organizations dropped between the two wars. More generally, the historical evidence suggests that since the nineteenth century, manufacturing companies have increasingly internalized R&D operations. (Chandler 1990), Nelson and Winter (1982) and Teece (1988), along with others, have explained this by emphasizing the tacit and idiosyncratic nature of knowledge and technologies.”

In addition to assuming that project transfer occurs as a project buyer’s acquisition of a seller firm, we make a few assumptions concerning project transfer. First, we assume that the seller of the project incurs  $\psi \geq 0$  of deadweight cost at the moment of the sales. Second, we assume that a firm has to make the decision to sell its project at the moment it has incubated the project. Once it starts to develop the project, it can no longer sell the project to another firm. Third, we assume that a mature firm has to make the decision to buy an incubated project at the moment the firm’s harvesting period has ended. Once it starts to incubate a project, it can no longer purchase an incubated project from other firms. Note that a young firm never buys an incubated project from a mature firm, whereas the other around is possible. This is because a mature firm with the right capability for a project is willing to pay a higher price for the project than a young firm. And therefore, in what follow, we ignore the possibility of project transfer from a mature firm to a young firm. Under our assumptions, firms make their decisions concerning whether to enter the project transfer market as follows.

Once a young firm incubates a project, the firm may either (1) develop it and obtain  $V_{g/1}$ , (2) sell it for price  $p$ , incur  $\psi$ , and cease operating, or (3) discard the incubated project, begin incubating another project and get  $V_{g/0}$ . The third option is clearly dominated by one of the other two options in the steady state, where  $V_{g/0}$ ,  $V_{g/1}$ , and  $p$  stay the same over time. As a result, the young firm should develop its own project if  $V_{g/1} > p - \psi$ , should sell it if  $p - \psi > V_{g/1}$ , and should be indifferent between the two options if  $V_{g/1} = p - \psi$ .

When a harvesting period of a mature firm ends, the firm has two options. First, it may buy a mature project that fits its own capability from either a young firm or a mature firm, and proceed to develop it. This gives the firm  $V_{m/1} - p$ . Second, the firm may begin incubating and this gives the firm  $V_{m/0}$ . As a result, the mature firm should buy a project if  $V_{m/1} - p > V_{m/0}$ , should start incubating if  $V_{m/0} > V_{m/1} - p$ , and should be indifferent between the two options if  $V_{m/0} = V_{m/1} - p$ .

When a mature firm incubates a project which fits the firm's capability, the firm should develop it. If the mature firm incubates a project which does not fit the firm's capability, the mature firm may either sell it for price  $p$ , incur  $\psi$  and cease operating or discard the incubated project and incubate another project. As a result, the mature firm should sell the project if  $p - \psi > V_{m/0}$ , should not sell it if  $p - \psi < V_{m/0}$ , and should be indifferent between the two options if  $p - \psi = V_{m/0}$ .

It is convenient to point a necessary condition for an equilibrium to have a positive mass of mature firms. Without young firm's growth, the mass of mature firms declines over time because of a positive death rate  $\mu$ . Therefore,

$$p - \psi \leq V_{g/1}. \quad (14)$$

### 3.4 Characteristics of Stationary Equilibrium

The stationary equilibrium is characterized by the following system of equations.

$$(\mu + \lambda_0) G_0 = N, \quad (15)$$



$$(\mu + \lambda_1) G_1 = \lambda_0 G_0 (1 - H_g), \quad (16)$$

$$(\mu + \lambda_1) M_1 = \lambda_0 \left( G_0 H_g + \frac{S-1}{S} M_0 H_m + \frac{1}{S} M_0 \right), \quad (17)$$

$$(\mu + \lambda_2) M_2 = \frac{\lambda_1 G_1}{S} + \lambda_1 M_1, \quad (18)$$

and

$$\left( \mu + \lambda_0 \left( \frac{1}{S} + \frac{S-1}{S} H_m \right) \right) M_0 = \lambda_2 M_2 - \lambda_0 \left( G_0 H_g + \frac{S-1}{S} M_0 H_m \right), \quad (19)$$

where  $H_g$  is the proportion of projects incubated by young firms and sold to mature firms and  $H_m$  is the ratio of sold projects to all projects incubated by mature firms with unmatched capabilities. Equation (15) ensures the outflow and the inflow of incubating new firms are the same. Note that among the young firms that have just finished their incubations, the fraction of  $H_g$  is sold to mature firms and the rest will develop their projects by their own. Then, equation (16) ensures that the outflow and the inflow of developing young firms are the same. Note that the right hand side of equation (17) is the inflow of developing mature firms. It consists of the mass of projects transferred from young firms, the mass of projects the same mature firms have incubated and are going to develop, and the mass of projects transferred from mature firms to other mature firms. Then, equation (17) ensures that the outflow and the inflow of developing mature firms are the same. Equation (18) ensures that the outflow and the inflow of harvesting firms are the same. The left hand side of equation (19) is the outflow of incubating mature firms and it consists of those who have just died, those who have just incubated projects and are going to develop them by their own, and those who have just incubated projects and sold their projects to other mature firms. The right hand side of equation (19) is the inflow of incubating mature firms and it is equal to the mass of mature firms that finished harvesting minus those who have just bought projects from either young or other mature firms.

### 3.4.1 Exploitation Equilibrium

We begin with examining if there exists an equilibrium in which mature firms specialize in developing projects and incubate no projects. In such an equilibrium,  $M_0 = 0$ . We call such an equilibrium *exploitation equilibrium*. The following proposition summarizes the results.

**Proposition 4** *If*

$$\psi \leq \frac{S-1}{S} \rho_1 Y - \alpha \Phi \left( 1 - \rho_1 \rho_2 + \frac{\rho_1 \rho_2}{S} \right),$$

*then there exists exploitation equilibrium such that  $M_1, M_2 > 0$ . In this equilibrium, the project price is given by*

$$p = \frac{S(1 - \rho_1 \rho_2) \psi + \rho_1 Y}{S(1 - \rho_1 \rho_2) + \rho_1 \rho_2} - C_1. \quad (20)$$

**Proof.** See Appendix.

For the exploitation equilibrium to exist, the transfer cost  $\psi$  needs to be sufficiently low, so that a young firm does not require too high a price to sell its incubated project to a mature firm. Note that  $\Phi$  is decreasing in the incubation cost  $C_0$ . Then, this proposition claims that  $C_0$  needs to be sufficiently high for the exploitation equilibrium to exist. This is intuitive. If incubating a new project would not be too costly, a mature firm would find it more profitable to incubate a project internally, instead of acquiring from a young firm. The project price is decreasing in the development cost, as a young firm is willing to sell its project at a lower price if not selling results in a higher development cost. The project price is increasing in the revenues, as a young firm is less willing to sell its project if not selling can potentially bring in higher revenues and therefore the price needs to increase in order to induce the young firm to sell its incubated project to a mature firm. The project price is decreasing in  $S$  because an increase in  $S$  makes it less likely that the young firm's attempt to develop a project turns out successful.<sup>7</sup>

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<sup>7</sup>Differentiating  $p$  with respect to  $S$  gives

$$\frac{dp}{dS} = \frac{\rho_1 (1 - \rho_1 \rho_2) (\rho_2 \psi - Y)}{(S(1 - \rho_1 \rho_2) + \rho_1 \rho_2)^2}.$$

Given the death rate  $\mu$ , the entry mass  $N$ , and the rates to move to the next stage,  $\lambda_0$ ,  $\lambda_1$  and  $\lambda_2$ , one can solve for the complete distribution  $G_0$ ,  $G_1$ ,  $M_2$  and  $M_1$ . In what follows, we characterize the stationary equilibrium.

**Proposition 5** *In the exploitation equilibrium, the rate of acquisition defined by  $\lambda_2 M_2 / \lambda_0 G_0$  is decreasing in  $\mu$  and  $S$  and increasing in  $\lambda_1$  and  $\lambda_2$ .*

**Proof.** See Appendix.

Intuitions behind this proposition are as follows. When  $\mu$  and  $S$  are high, less firms become mature and, as a result, mature firms' demand for projects is low and less acquisitions occur. The positive relation between  $S$  and the acquisition rate is paradoxical. If  $S$  is high, commercialization attempts by new firms are more likely to fail. Nevertheless, the rate of such an attempt is higher when  $S$  is high. When  $\lambda_1$  and  $\lambda_2$  are high, mature firms come back to the project market quickly and, as a result, demand for projects increase. As a consequence, the acquisition rate increases.

**Proposition 6** *In the exploitation equilibrium, the values of mature firms  $V_2$  and  $V_{m/1}$  are both increasing in  $S$ . The values of young firms  $V_{g/0}$  and  $V_{g/1}$  are both decreasing in  $S$ .*

**Proof.** See Appendix.

Intuitions behind this proposition are similar to how entry deterrence asymmetrically influences incumbents and new entrants. When  $S$  is high, it is unlikely that a young firm can successfully develop the project it has just incubated. Developing by itself is an outside option of the young firm when it negotiates the price of selling the incubated project to a mature firm. If this outside option is not attractive, the bargaining power of the young firm falls and so does the price of the project. Opposite to young firms, a mature firm can buy a project at a cheaper price if  $S$  is high, and therefore the value of the mature firm rises.

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This is negative because  $Y > \rho_2 \psi$  by the existence condition for the exploitation equilibrium described in Proposition 4.

### 3.4.2 Equilibrium with Mergers between Two Mature Firms

We are now going to examine if there exists a stationary equilibrium in which some mature firms incubate and two mature firms merge. Note that there always exists a stationary equilibrium in which mature firms incubate and they do not merge, if the transfer cost  $\psi$  is sufficiently high. We have already analyzed this case in Section 2.

We begin our analysis by describing the necessary conditions for a stationary equilibrium with mergers between mature firms to exist. The first condition is that a mature firm needs to be willing to incubate instead of buying an already incubated project, that is,  $V_{m/0} \geq V_{m/1} - p$ . The second condition is that a mature firm is also willing to buy an incubated project, that is,  $V_{m/1} - p \geq V_{m/0}$ . Therefore,

$$p = V_{m/1} - V_{m/0}. \quad (21)$$

In addition, a mature firm should be willing to sell its incubated project instead of incubating another project, that is,  $V_{m/0} \leq p - \psi$ . Combining with equation (14) gives

$$V_{m/0} \leq p - \psi \leq V_{g/1}. \quad (22)$$

The following proposition characterizes the stationary equilibrium with mergers between mature firms.

**Proposition 7** *Let*

$$\begin{aligned} \alpha_{m/0} &= \left(1 - \frac{\rho_0 \rho_1 \rho_2}{S}\right)^{-1}, \\ \alpha_p &= (1 - \rho_1 \rho_2) \alpha_{m/0}, \end{aligned}$$

$$\sigma_1 = \frac{1}{S - (S - 1) \alpha_{m/0} \rho_0},$$

$$\sigma_2 = \frac{1}{S + (S - 1) \alpha_p \rho_0} \text{ and}$$

$$\sigma_3 = \frac{1}{S^2 - \rho_0 \rho_1 \rho_2 \alpha_{m/0} (S - 1)}.$$

If

$$\begin{aligned} & (\alpha_{m/0}\rho_0\sigma_1 - (S - \alpha_p\rho_0)\sigma_2)(\rho_1Y - C_1) + S\psi\sigma_2 \\ \leq & (\alpha_p\sigma_2 + \sigma_1)SC_0, \end{aligned} \quad (23)$$

and

$$\begin{aligned} & ((S - \alpha_p\rho_0)\sigma_2 - \rho_0\rho_1\rho_2\alpha_{m/0}\sigma_3)(\rho_1Y - C_1) + (\rho_1\rho_2\alpha_{m/0}\sigma_3 + \alpha_p\sigma_2)SC_0 \\ \leq & S^2\left(\frac{\rho_1}{S}Y - C_1\right)\sigma_3 + S\psi\sigma_2, \end{aligned} \quad (24)$$

then there exists a stationary equilibrium in which mature firms merge. The equilibrium price of the project is

$$p = \sigma_2((\rho_1Y - C_1)(S - \alpha_p\rho_0) + \alpha_pSC_0 + (S - 1)\alpha_p\rho_0\psi). \quad (25)$$

**Proof.** See Appendix.

Similar to the results in the exploitation equilibrium, the project price is increasing in both the revenue and the transfer cost and decreasing in the development cost. Different from the results in the exploitation equilibrium, the project price does depend on the incubation cost and their relation is positive. In an equilibrium with mergers between two mature firms, a mature firm needs to be indifferent between the two options. One is to incubate a project and the other is to buy an incubated project. If the incubation cost is high, the project price must be also high so that the two options continue to be equally attractive.

### 3.5 Related Evidence

Our model implies that transferred projects had a greater life expectancy and also a higher profitability than non-transferred project. This is because transferred projects are developed by the firms with the matching capabilities, whereas non-transferred projects may not be. Consistent with our model, Ravenscraft and Scherer (1987) find that acquired lines of business are of higher quality (usually measured by profitability) than those not acquired. Jain and Kini (1994) find that IPO firms operating performance

is worse than matching firm performance. Given that many IPO firms are new firms, their evidence suggest that development by new firms perform worse than that by mature firms as consistent with the theory. Finally, Churchill (1955) finds that transferred businesses are more profitable than non-transferred businesses.

Our model characterizes patterns of acquisitions. First, our model predicts that a mature firm acquires a young firm, but not the other way around. There is little evidence on this, whereas there is a plenty of evidence documenting that an acquirer is bigger than its target. Given that firm size and age are positively related, it is suggestive to cite the evidence on size. Based on data on all U.S. publicly traded firms that announced M&A between 1994 and 2000, Mitchell, Pulvino, and Stafford (2004) find that the ratio of the median target equity value to acquirer equity value varies between 0.17 (in 1994) and 0.10 (in 2000). That is, the size of acquirer is usually 6-10 times as much as that of its target, supporting our theoretical prediction. Second, our model predicts that a mature firm who has lost the cash cow project will engage in acquisitions. Consistent with this prediction, Higgins and Rodriguez (2006) find that pharmaceutical firms that are more desperate for blockbuster drugs are more likely to engage in an outsourcing-type acquisition.

Our model also explains what determines the rate of acquisition. Brau, Francis, and Kohers (2003) find that takeovers are more often than IPOs in manufacturing industries than others.

## 4 Conclusion

This paper models the difference of a mature firm and a young firm by the knowledge of own capability; a mature firm knows its own capability but a young firm does not know. This simple difference is enough to generate rich predictions. Our model implies that a mature firm tends to abandon a project earlier than a young firm. This is because a mature firm discovers that the project may not fit its capability at an earlier stage of the project but a young firm cannot apply such an abandonment criterion not knowing

its own capability. Our model also implies that a new firm may sell its project to a mature firms, but not the other way around, and sold projects perform better than unsold projects. Finally, we also find a paradoxical result that when new firms' attempts to commercialize are more likely to fail, they are more likely to make such attempts.

Numerous interesting extensions are not explored in this paper. First, we focus on stationary equilibria and did not study transitional dynamics of the economy. Introducing dynamics and characterizing the lifecycle of industries may generate interesting results. Second, introducing search friction regarding project transfers may enrich the structure of the model. Third, the model neglects the possibility that a new firm may make more than one commercialization attempt to discover its own capability. Lastly, we assume that project types remain the same over time. Time-changing project type space appears to be more relevant for explaining firm dynamics in an innovative environment.

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## Appendix

This appendix gathers the proofs of the propositions.

### Proof of Proposition 1

It suffices to show that

$$V_{m/1}(d^*) \geq V_{m/1}(d), \forall d \in 2^{Ts}.$$

Suppose that the type of the incubated project is  $s_p = s_f$ . Let  $\hat{d}, d$  be such that  $\hat{d}(s_p) = 1, d(s_p) = 0$ . We will show that  $V_{m/1}(\hat{d}) \geq V_{m/1}(d)$ . Note that we do not constraint the firm policy in subsequent decisions. If the firm does not develop this project, the value of the firm is equal to  $V_{m/0}(d)$ . If the firm does develop this project, the value of the firm is equal to

$$\begin{aligned} V_{m/1}(\hat{d}) &= \rho_1 Y - C_1 + \rho_2 \rho_1 V_{m/0}(\hat{d}) \implies \\ V_{m/1}(\hat{d}) &\geq \rho_1 Y - C_1 + \rho_2 \rho_1 V_{m/0}(d) \end{aligned}$$

The second line follows from the fact that the continuation decisions of the firm are unconstrained, so given  $d$  the firm can do at least as well in policy  $\hat{d}$  by replicating  $d$  in all subsequent histories. We want to show

$$\rho_1 Y - C_1 > (1 - \rho_2 \rho_1) V_{m/0}(d)$$

Let

$$\int_0^{\lambda_1^{-1} + \lambda_2^{-1}} z e^{-rt} dt = \rho_1 Y - C_1.$$

That is,  $z$  is per period cash flow for the expected duration of development plus commercialization periods that is equivalent to the payoff of developing the matched project. Solving this equation gives

$$z = \frac{r(\rho_1 Y - C_1)}{1 - e^{-r(\lambda_1^{-1} + \lambda_2^{-1})}}.$$

Since [the perpetuity of the cream skimming period should be larger than the value of the firm of non-skimming period,]

$$z > r V_{m/0}(d). \tag{26}$$

Integrating both sides of the equation (26) from 0 to  $\lambda_0^{-1} + \lambda_0^{-2}$  give

$$\rho_1 Y - C_1 > \left(1 - e^{-r(\lambda_1^{-1} + \lambda_2^{-1})}\right) V_{m/0}(d).$$

Now we show that

$$e^{-r(\lambda_1^{-1} + \lambda_2^{-1})} < \rho_1 \rho_2$$

which is alternatively

$$e^{r/\lambda_2} e^{r/\lambda_3} > \rho_2^{-1} \rho_3^{-1},$$

which is true, since  $e^x > 1 + x$ , for  $x > 0$ .

Suppose now that the type of the incubated project is  $s_p \neq s_f$ . Let  $\hat{d}, d$  be such that  $\hat{d}(s_p) = 0, d(s_p) = 1$ . We will show that  $V_{m/1}(\hat{d}) \geq V_{m/1}(d)$ . If the firm does not develop this project, the value of the firm is equal to  $V_{m/0}(\hat{d})$ . If the firm does develop this project, the firm's value is less than  $V_{m/0}(\hat{d})$  because it is equal to the present value of making loss plus the discounted value of  $V_{m/0}(d)$ , which is not more than  $V_{m/0}(\hat{d})$ . Since  $\hat{d} = d^*$ , the proof is complete. Q.E.D.

## Proof of Proposition 2

Suppose that the young firm has just incubated a project. The value from developing the project given the policy  $d$  is

$$V_{g/1}(d) = -C_1 + \rho_1 \left( \frac{1}{S} V_{g/2}(d) + \frac{S-1}{S} 0 \right) = -C_1 + \frac{1}{S} \rho_1 V_{m/2}, \quad (27)$$

where the second equality follows by noting that a young firm becomes mature when it starts generating revenue, since it infers its type from the project type that it incubated. This expression is positive by Assumption 1, so it remains to prove that the value from developing immediately is greater than the value from not developing the project  $V_{g/0}(d)$ .

$$\begin{aligned} V_{g/0} &= -C_0 + \rho_0(-C_1 + \rho_1 V_{g/2}) \implies \\ V_{g/2} - V_{g/0} &= V_{g/2}(1 - \rho_0) + C_0 \end{aligned}$$

The last expression is positive, so the proof is complete. Q.E.D.

### Proof of Proposition 3

According to the asset pricing formula, the value of the firm depending on its project cycle is

$$(r + \lambda_0) V_{i/0} = -c_0 + \frac{\lambda_0}{S} V_{i/1} + \frac{\lambda_0 (S-1)}{S} V_{i/0} \quad i \in \{g, m\}, \quad (28)$$

$$(r + \lambda_1) V_{g/1} = -c_1 + \frac{\lambda_1 V_{g/2}}{S}, \quad (29)$$

$$(r + \lambda_1) V_{m/1} = -c_1 + \lambda_1 V_{m/2}, \quad (30)$$

and

$$(r + \lambda_2) V_{i/2} = y + \lambda_2 V_{m/0} \quad i \in \{y, m\}.$$

As there are eight independent equations and eight unknowns, we can obtain the explicit formula of each firm value. Solving this system yields the results. Q.E.D.

### Proof of Proposition 4

As previously argued, equation (14) needs to hold in a stationary equilibrium such that  $M_1, M_2 > 0$ . It also must hold that  $p \geq \psi + V_{g/0}$  and a young firm is willing to sell its incubated project in a stationary equilibrium with  $M_0 = 0$ . Because otherwise a mature firm that finished harvesting would have an incentive to incubate. As a consequence,

$$p = \psi + V_{g/1}. \quad (31)$$

In a stationary equilibrium of interest, the system of the following Bellman equations holds:

$$\begin{aligned} V_2 &= Y + \rho_2 (V_{m/1} - p), \\ V_{m/1} &= -C_1 + \rho_1 V_2, \\ V_{g/1} &= -C_1 + \frac{\rho_1}{S} V_2, \text{ and} \\ V_{g/0} &= -C_0 + \rho_0 V_{g/1}. \end{aligned} \quad (32)$$

Solving for  $V_2$ ,  $V_{m/1}$  and then  $V_{g/1}$  gives

$$V_2 = \frac{Y - \rho_2(p + C_1)}{1 - \rho_1\rho_2}, \quad (33)$$

$$V_{m/1} = -C_1 + \rho_1 \frac{Y - \rho_2(p + C_1)}{1 - \rho_1\rho_2} \text{ and} \quad (34)$$

$$V_{g/1} = -C_1 + \frac{\rho_1}{S} \frac{Y - \rho_2(p + C_1)}{1 - \rho_1\rho_2}. \quad (35)$$

Sticking  $V_{g/1}$  into equation (31) and solving for  $p$  give equation (20).

It remains to show that a mature firm is willing to pay this price to start developing a project immediately, instead of incubating a project by itself. The value of the strategy not to buy an incubated project is the same as the value function of the incubating mature firm given in the case without project transfers. Let  $\hat{V}_{m/0}$  be this value, then due to equation (4)

$$\hat{V}_{m/0} = \alpha\Phi.$$

If the mature firm buys an incubated project instead, due to equations (34) and (20), the firm gets

$$V_{m/1} - p = \frac{(S - 1)\rho_1 Y - S\psi}{S(1 - \rho_1\rho_2) + \rho_1\rho_2}.$$

Therefore, if

$$\frac{S - 1}{S}\rho_1 Y - \frac{\alpha\Phi}{S}(S(1 - \rho_1\rho_2) + \rho_1\rho_2) \geq \psi,$$

then it is optimal for the mature firm to buy an incubated project.

### Proof of Proposition 5

Sticking  $M_0 = 0$  into the system of equations (15)-(19) gives the stationary distribution of firms expressed by equations (36).

$$\begin{aligned}
G_0 &= \frac{1}{\mu + \lambda_0} N, \\
G_1 &= \frac{\lambda_0 S \mu (\mu + \lambda_1 + \lambda_2)}{(\mu + \lambda_0) (\mu + \lambda_1) \{\lambda_1 \lambda_2 + S \mu (\mu + \lambda_1 + \lambda_2)\}} N, \\
M_0 &= 0, \\
M_1 &= \frac{\lambda_0 \lambda_1 \lambda_2}{(\mu + \lambda_0) (\mu + \lambda_1) \{\lambda_1 \lambda_2 + S \mu (\mu + \lambda_1 + \lambda_2)\}} N, \\
M_2 &= \frac{\lambda_0 \lambda_1}{(\mu + \lambda_0) \{\lambda_1 \lambda_2 + S \mu (\mu + \lambda_1 + \lambda_2)\}} N,
\end{aligned} \tag{36}$$

Therefore, the rate of acquisition is

$$R = \frac{\lambda_2 M_2}{\lambda_0 G_0} = \left( 1 + \frac{\mu S (\mu + \lambda_1 + \lambda_2)}{\lambda_1 \lambda_2} \right)^{-1},$$

which is decreasing in  $\mu$  and  $S$ , and increasing in  $\lambda_1$  and  $\lambda_2$ . Q.E.D.

### Proof of Proposition 6

Sticking equation (20) into equation (34) for  $p$  gives the closed form solution of  $V_{m/1}$

$$V_{m/1} = \frac{Y - \rho_2 \psi}{S (1 - \rho_1 \rho_2) + \rho_1 \rho_2} \rho_1 S - C_1,$$

Sticking equation (20) into equation (33) for  $p$  gives the closed form solution of  $V_2$

$$V_2 = \frac{S (Y - \rho_2 \psi)}{S (1 - \rho_1 \rho_2) + \rho_1 \rho_2}.$$

Sticking equation (20) into equation (35) for  $p$  gives the closed form solution of  $V_{g/1}$

$$V_{g/1} = \frac{(Y - \rho_2 \psi) \rho_1}{S (1 - \rho_1 \rho_2) + \rho_1 \rho_2} - C_1.$$

Sticking equation (20) into equation (32) for  $p$  gives the closed form solution of  $V_{g/0}$

$$V_{g/0} = \frac{(Y - \rho_2 \psi) \rho_0 \rho_1}{S (1 - \rho_1 \rho_2) + \rho_1 \rho_2} - \rho_0 C_1 - C_0.$$



It is obvious that both  $V_{g/0}$  and  $V_{g/1}$  are decreasing in  $S$ . Differentiating  $V_{m/1}$  and  $V_2$  by  $S$  gives

$$\frac{dV_{m/1}}{dS} = \frac{\rho_1^2 \rho_2 (Y - \rho_2 \psi)}{(S(1 - \rho_1 \rho_2) + \rho_1 \rho_2)^2} > 0$$

and

$$\frac{dV_2}{dS} = \frac{\rho_1 \rho_2 (Y - \rho_2 \psi)}{(S(1 - \rho_1 \rho_2) + \rho_1 \rho_2)^2} > 0.$$

Therefore, the proposition follows. Q.E.D.

### Proof of Proposition 7

As previously argued, equation (21) needs to hold in a stationary equilibrium such that  $M_1, M_2 > 0$ . In a stationary equilibrium of interest, the system of the following Bellman equations holds:

$$\begin{aligned} V_2 &= Y + \rho_2 V_{m/0}, \\ V_{m/0} &= -C_0 + \rho_0 \left( \frac{V_{m/1}}{S} + \frac{S-1}{S} (p - \psi) \right), \\ V_{m/1} &= -C_1 + \rho_1 V_2, \\ V_{g/1} &= -C_1 + \frac{\rho_1}{S} V_2, \text{ and} \\ V_{g/0} &= -C_0 + \rho_0 V_{g/1}. \end{aligned}$$

Solving for  $V_2$ ,  $V_{m/0}$ ,  $V_{m/1}$ ,  $V_{g/1}$  and  $V_{g/0}$  gives

$$V_2 = Y + \rho_2 \alpha_{m/0} \Phi_{m/0}, \tag{37}$$

$$V_{m/0} = \alpha_{m/0} \Phi_{m/0}, \tag{38}$$

$$V_{m/1} = -C_1 + \rho_1 (Y + \rho_2 \alpha_{m/0} \Phi_{m/0}), \tag{39}$$

$$V_{g/1} = -C_1 + \frac{\rho_1}{S} (Y + \rho_2 \alpha_{m/0} \Phi_{m/0}) \text{ and} \tag{39}$$

$$V_{g/0} = -C_0 + \rho_0 \left( \frac{\rho_1}{S} Y - C_1 + \frac{\rho_1 \rho_2 \alpha_{m/0} \Phi_{m/0}}{S} \right), \tag{40}$$

where

$$\alpha_{m/0} = \left( 1 - \frac{\rho_0 \rho_1 \rho_2}{S} \right)^{-1}$$

and

$$\Phi_{m/0} = \frac{\rho_0}{S} (\rho_1 Y - C_1 + (S-1)(p - \psi)) - C_0.$$

Substituting equations (37) and (38) for  $V_{m/0}$  and  $V_{m/1}$  into equation (21) and solving for  $p$  give equation (25).

It remains to show that equation (22) is satisfied. Substituting into equation (22), equation (37) for  $V_{m/0}$ , equation (39) for  $V_{g/1}$ , and equation (25) for  $p$  give the necessary conditions (23) and (24).