LSE RICAFE 2

Working Paper NO. 049

Leverage and Firm Scope

Elisa Luciano
Università di Torino and Collegio Carlo Alberto

Giovanna Nicodano
Università di Torino and Collegio Carlo Alberto

January 2008

RICAFE2 - Regional Comparative Advantage and Knowledge Based Entrepreneurship

A project financed by the European Commission, DG Research
under the 'Citizens and governance in a knowledge-based society' (FP6) programme
Contract No : grant CIT5-CT-2006-028942.

Financial Markets Group, London School of Economics and Political Science, LSE, UK
Dipartimento di Scienze Economiche e Finanziarie Prato, Università di Torino, TORINO, Italy
Centre for Financial Studies, CFS, Germany
Haute Etudes Commerciales, HEC, France
Baltic International Center for Economic Policy Studies, BICEPS, Latvia
Amsterdam University, UVA, Netherlands
Neaman Institute for Advanced Studies in Science and Technology at Technion, TECHNION, Israel
Indian School of Business, ISB, India
Tilburg University, UTIL.CER, Netherlands
University of Southern Switzerland, USI, Switzerland
The Institute of Physics, IBP, Serbia

Leverage and Firm Scope*

Elisa Luciano, Giovanna Nicodano†
Università di Torino and Collegio Carlo Alberto

This draft: January 20th, 2008

Abstract

This paper proposes a financial theory of firm scope explaining why groups emerge despite shareholders' value destruction, in a framework without agency costs. We allow stand alone firms, conglomerate mergers and group affiliates to optimally issue debt against their cash flows, when there are bankruptcy costs and non-neutral taxation. We show that groups destroy equity value even if they maximize total firm value, by levering up the subsidiary which gets support from its parent.This optimal capital structure implied by our model is commonly observed in the private equity industry. Such asymmetric leverage allows to preserve diversification benefits even when cash flow correlation is perfect across affiliates. Finally, the risk and size characteristics of holdings and subsidiaries also emerge endogenously.

Keywords: business groups, internal capital market, leverage, bankruptcy costs, limited liability, private equity, LBOs

JEL classification numbers: G32, G34, L22

*We thank Marc Deloof, Julia Hirsch, Rafael Repullo, Jean Tirole and Yishay Yafeh for helpful suggestions, as well as participants to the Internal Capital Market Workshop (Antwerp), to seminars at the Norwegian School of Management and Collegio Carlo Alberto and to the RICAFE2 Conference (BICEPS Riga) for comments. Alessandro Dovis provided us with excellent research assistance. Financial support from MIUR and European Commission (grant CIT5-CT-2006-028942 RICAFE2) is gratefully acknowledged by Giovanna Nicodano.

†Corresponding Author. Address: University of Turin, Corso Unione Sovietica, 218bis - 10134 Turin, ITALY. giovanna.nicodano@unito.it, Ph: +39-011.670 5006, Fax: +39-011.6706062

Non technical abstract

An entrepreneur with two firms can organize them either as stand alone units, or conglomerates or parent-subsidiary structures. Several papers analyze the relative advantages of the first two organizations, but we know little about the third despite its diffusion the world. What we know is paradoxical, in that the equity prices of group-affiliated firms are often lower than those of comparable stand alone units: why should an entrepreneur incorporate his/her firms as a group, if it can organize them as stand alone units? Moreover, there is no model that helps pricing claims issued by group-affiliated firms and forecast the price effects of (reverse) carve-outs. This paper builds such a pricing model, and shows when parent-subsidiaries structures maximize firm value even if they may destroy shareholders' value. Our pricing model determines the value of debt and equity in a parent and its subsidiary as a function of: (a) the characteristics of the two firms, that are summarized in average cash flow (size), cash flow volatility (risk) and cash -flow correlation (diversification) across the two units; (b) the corporate income tax rate; (c) the cost that lenders bear in the event of default. The face value of debt is set equal to the level that maximizes total firm value, taking into account the possibility to deduct interest from taxes. The holding may transfer funds to its subsidiary through the internal capital market, while enjoying limited liability vis-à-vis the subsidiary's external debt obligations. Thus, the holding supports its insolvent subsidiary only when both can survive.

We find that the optimal face value of debt is equal to zero in the holding while it is very large in the subsidiary. This allows the holding to save on bankruptcy costs, while permitting the subsidiary to maximize the tax shield. As a consequence, equity value in the subsidiary is very low, as its shareholders very seldom earn positive profits. Similarly, the value of equity in the holding is reduced by the transfers to subsidiary lenders. However, total group value exceeds that of two stand alone companies. This is the outcome of higher debt capacity due to support from the holding, which leads to lower tax burden in the subsidiary and lower bankruptcy costs in the holding.

Leland (2007) studies stand alones and conglomerates in this type of set up. Divisions of a conglomerate always support each other, while stand alone firms never do. He finds that conglomerates have higher values than stand alones when they are able to raise more debt, which occurs if the two production units are sufficiently diversified but not too different in terms of risk and size. Gains deriving from conglomeration

disappear when cash flows are perfectly correlated. In groups, instead, such gains remain and reach the highest level, because zero leverage preserves the holding ability to rescue its subsidiary. Thus we show that, in the absence of agency costs, the total value of group affiliates exceeds that of both stand-alone companies and conglomerates irrespective of diversification opportunities offered by the internal capital market. The reason is that they are able to maximize the tax shield and minimize bankruptcy costs by optimizing joint capital structure.

Tax elusion lies behind value creation in a parent-subsidiary structure. In several countries, regulation prevents firms from deducting interest payments above a certain threshold. We therefore analyze a situation when the parent is allowed to optimize leverage, while the subsidiary may only raise as much debt as a stand alone company. Groups still create more value than stand alone firms. Yet, it is no longer the case that perfect correlation between production results in the highest value for the group.

When we allow for different characteristics of units, we determine which unit should be the holding company. Consistent with empirical evidence we find that subsidiaries are riskier than their holding companies - because safer units more easily support riskier ones. Secondly, shielding income from taxes by levering up has higher value with a riskier cash flow. The value benefits of groups - relative to both the stand alone and conglomerate cases - increase in the risk asymmetry because the option of abandoning a subsidiary in bankruptcy is more valuable the riskier is the subsidiary.

The relevance of our model goes beyond the case of business groups found in continental European countries and in emerging markets. It applies more generally to the case of a parent and subsidiary firms that are separately incorporated, and can issue debt and equity against their own cash flows. Similar organizational structures are present in innovative industries in the US and, more generally, in the private equity industry. Our assumption of zero agency costs applies well to this industry, because of some features of the governance structure of private equity deals that provide incentives for monitors and managers. Moreover, our characterization of highly leveraged subsidiaries that exploit tax elusion appears to be consistent with some stylized facts concerning private equity subsidiaries, that appear to seldom go bankrupt despite a 95% - or higher - leverage.

1 Introduction

Companies are organized as groups in several developing and developed countries (Khanna and Yafeh, 2006; Barca and Becht, 2001). The pervasiveness of groups goes along with evidence that this organizational mode destroys equity value with respect to a stand alone benchmark. The expropriation of unsophisticated minority shareholders by controlling shareholders may be able to reconcile these paradoxical regularities when affiliated firms are listed on public exchanges, as suggested by Almeida and Wolfenzon (2006). It can however explain neither the reason why unlisted groups are common, nor why they thrive in countries - such as Scandinavian ones - with strict enforcement of securities regulation and social stigma for diversion. This paper proposes a theory of firm scope explaining why groups emerge despite shareholders' value destruction, without any reference to agency costs.

For this purpose, we extend the set-up used to study stand-alone firms and conglomerates by Leland (2007). Thus, we focus on purely financial synergies arising from firm combinations and assume exogenous operating cash flows from two production units. We depart from the Modigliani Miller theorem (Modigliani and Miller 1958, Stiglitz 1969) by postulating bankruptcy costs and the possibility to deduct interests from taxes, and allow each firm type to optimally issue debt against its cash flows. In the group case, we allow the holding to transfer funds to its subsidiary through the internal capital market [1], while enjoying limited liability [2] vis-à-vis the subsidiary's external debt obligations. More precisely, we assume that the holding supports its insolvent subsidiary when both can survive. On the contrary, divisions of a conglomerate always support each other, while stand alone firms never do.

In the absence of agency costs, we show that the total value of group affiliates exceeds that of both stand-alone companies and conglomerates irrespective of diversification opportunities offered by the internal capital market. The reason is that they are able to maximize the tax shield and minimize bankruptcy costs by optimizing joint capital structure. Specifically, the subsidiary levers up more than if it were stand alone, while the holding company raises no debt. Thus the holding never goes bankrupt while contributing to minimize the subsidiary insolvency. In turn, the interest burden on the subsidiary dramatically reduces taxation. Our model thus fits into the literature that highlights the "bright side" of

[1] Khanna and Palepu (2000) observe that Indian group firms assist each other in times of financial distress, while Bertrand et al. (2002) document cash transfers in several forms - from asset sales to internal loans at subsidized rates.

[2] This is a common characteristic across major jurisdictions. See Blumberg (1989) for the US, and Hadden (1996) on Britain, France, Germany and the US.

internal capital markets. Our emphasis is however on the possibility for groups to optimize the trade off between bankruptcy risk and the tax burden, whereas previous research focuses on their ability to exploit diversification and circumvent imperfections arising from asymmetric information (Stein, 1997; Gertner, Scharfstein and Stein, 1994). In prior models (Leland 2007) gains deriving from internal capital markets disappear when cash flow correlation is perfect across symmetric subsidiaries. In groups such gains remain, because lower leverage preserves the holding ability to rescue its subsidiary. Clearly, unmodelled agency costs may counterbalance in practice the advantages we highlight - in which case stand alone organizations or conglomerates will dominate.

Value destruction in groups appears by now as an empirical regularity. In several countries and over different time spans, shareholders' value is often lower in groups than in competing organizations even when this cannot be ascribed to worse operating performance (Bennedsen and Nielsen, 2006; Claessens et al., 2002; Khanna and Yafeh, 2006). One view holds that controlling shareholders divert resources to the expense of minority shareholders. According to this view, groups emerge when entrepreneurs prefer to fund activities indirectly, through another company, rather than directly. This is the case when the present value of the project, net of diversion, is negative, because part of the burden is shifted onto the minority shareholders of the company (Almeida and Wolfenzon, 2006). Our explanation is able to account for equity discounts without postulating the diversion of resources from minority shareholders. In order to minimize its own bankruptcy probability and to rescue the subsidiary as much as it can afford to, it is optimal for the holding to reduce its leverage. The cash transfer from the holding benefits the subsidiary's debtholders, whose claims increase in value. The equity value of the subsidiary however falls to almost zero because its leverage is much larger than the stand alone one, allowing to exploit the tax shelter but increasing the likelihood of default. Thus, the equity value of two stand alone firms exceeds that of a holding and its subsidiary. However, the opposite holds for debt value. Overall, the increase in debt value brought about by groups more than offsets equity value destruction.

Tax elusion is partly the cause of value creation in a parent-subsidiary structure. In several countries, regulation prevents firms from deducting interest payments above a certain threshold. We therefore analyze a situation when only the parent is allowed to optimize leverage, while the subsidiary may only raise as much debt as a stand alone company. Groups still create more value than stand alone firms. Yet, it is no longer the case that perfect correlation between production results in the highest value for the group, because of the higher bankruptcy probability.

Now the holding, being almost as levered as its subsidiary, is often unable to rescue its subsidiary and may actually default.

When we allow for different characteristics of units, we are able to endogenously characterize group optimal organizational form. Consistent with empirical evidence (Robinson, 2006; Bianco and Nicodano, 2006), we find that subsidiaries are riskier than their holding companies. Robinson relates this finding to the inability of guaranteeing funding to highly risky project inside a multidivisional corporation. This prompts the signing of a legally binding contract between two separately incorporated units. We offer a different perspective as to why the riskier unit is the subsidiary, i.e. the firm that receives support. First, a riskier holding would often be unable to rescue its subsidiary. Secondly, shielding income from taxes by levering up has higher value with a riskier cash flow, and leverage must be optimally located in the subsidiary in order to control default costs. Interestingly, we find that the value benefits of groups - relative to both the stand alone and conglomerate cases - increase in the risk asymmetry. Indeed, the option of abandoning a subsidiary in bankruptcy is more valuable the riskier is the subsidiary relative to the holding.

Some key assumptions concerning the modelling of tax rates and bankruptcy costs are borrowed from Leland (2007) with no ad hoc modification. In particular, there are no tax refunds being paid to the firm when its income is negative. In the real world, some losses may be carried forward by companies, in order to reduce the asymmetric nature of taxation - which however remains. Bankruptcy costs, which the firm pays only when it does not meet its debt obligations, are proportional to cash-flows. This is a well accepted assumption in structural models of credit risk.

In our full information model, the holding company guarantees the rescue of its subsidiary when this is possible without compromising its own solvency. Such guarantee is ex-ante optimal as lenders, who save on the deadweight cost of bankruptcy, provide better credit conditions.[3] In reality, holding companies write comfort letters assuring subsidiaries' lenders that they would assist them in distress. These do not undermine their limited liability, because they are legally unenforceable and discretionary contracts that allow the holding companies not to honor them when they would not be able to preserve their own integrity. In a

[3] Ex post, also a stand alone firm may find it optimal to acquire a distressed one - as in Fluck and Llynch (1999). However, this would not affect as much credit conditions offered by lenders ex ante, because they cannot attribute to ex post acquisitions the same probability. Fo instance, acquisitions are not observed - and recovery rates are especially low - in the case of distressed industries (Acharya et al., 2005).

reputation game, such letters create value (Boot et al, 1993).

We use a structural model of credit risk, assuming away agency and strategic problems, in order to analyze how the internal capital market affects optimal leverage and value. On the contrary several papers study how agency problems in internal capital markets affect product market competition and investment choice[4]. Cestone and Fumagalli (2005) analyze the specificities of group internal capital market by both assuming limited liability of the holding and allowing subsidiaries to raise their own debt, as we do. They study how transfers from the holding impact on the conditions obtained by subsidiaries from its outside financiers, when managerial effort cannot be observed. Better credit conditions are the outcome of increased managerial effort, rather than that of reduced tax burden.

Another related paper, Huizinga et al (2007), studies tax arbitrage in multinational groups that is engineered by raising more debt in high-tax countries. In our model, groups minimize the tax burden through debt even if there is no tax rate differential between the holding and its subsidiary.

The relevance of our model goes beyond the case of business groups found in continental European countries and in emerging markets. It applies more generally to the case of a parent and subsidiary firm that are separately incorporated, and can issue debt and equity against their own cash flows. Similar organizational structures are present in innovative industries in the US (Allen, 1998; Sahlman, 1990; Mathews and Robinson, 2006) and, more generally, in the private equity industry. Our assumption of zero agency costs applies well to this industry, because of some features of the governance structure of private equity deals that provide incentives for monitors and managers (Jensen, 2007). Moreover, our characterization of highly leveraged subsidiaries appears to be consistent with some stylized facts concerning private equity subsidiaries, that appear to seldom go bankrupt despite a 95% - or higher - leverage. While Jensen (2007) emphasizes the incentive function of high debt, we stress its tax advantages as in Kaplan (1989).

The paper is organized as follows. Section 2 presents the set up, and three organizational modes for two activities - stand alone, group and conglomerate. Section 3 analyzes these three cases through a numerical example, so as to understand their respective properties. It then compares optimal leverage, value of debt and equity across the three modes, as the correlation between cash flows varies. Section 4 allows units to

[4]Most focus on aspects, such as cash-flow pooling, that are typical of both conglomerates and groups. See, among others, Rajan Servaes Zingales (2000) and Faure Grimaud and Inderst (2005).

differ in mean cash flow, volatility and bankruptcy costs, characterizing optimal group structure. Section 5 examines the case of a constrained subsidiary and that of a holding company receiving dividends.

2 The common set up

In this section we review Leland set up and his analysis of stand alone firms. We then extend it to the group case. We consider a no arbitrage environment with two dates $t = \{0, T\}$, where T is the length of time spanned by the dates. There are two activities, and each activity i generates a random future operational (net) cash flow value X_i at time $t = T$. X_i is a continuous random variable that may take on negative values. The riskfree interest rate over the time period T is r_T. No arbitrage implies that the value of the operational cash flow at $t = 0$ is its discounted expected value:

$$X_{0i} = (1 + r_T)^{-1}\mathrm{E}X_i \tag{1}$$

where $\mathrm{E}X_i$ is evaluated under the risk neutral measure. The owners can "walk away" from negative cash flows thanks to limited liability. Thus the (pre-tax) value of each activity with limited liability is

$$H_{0i} = (1 + r_T)^{-1}\mathrm{E}X_i^+ \tag{2}$$

where $X_i^+ = \max(X_i, 0)$, and the pre-tax value of limited liability is

$$L_{0i} = H_{0i} - X_{0i} \geq 0 \tag{3}$$

Now consider a tax rate on future cash flows equal to τ_i. The aftertax value of the unlevered firm, which corresponds to its equity value, is

$$V_{0i} = (1 - \tau_i)H_{0i} \tag{4}$$

The present value of taxes it pays is

$$T_{0i}(0) = \tau_i H_{0i} \tag{5}$$

At time $t = 0$ the entrepreneur can lever the firm by issuing zero-coupon debt. Let its principal value be P_i, and assume it is due, with absolute priority, at $t = T$. Let $D_{0i}(P_i)$ denote the value, at $t = 0$, of such debt, which is cashed-in by the entrepreneur at issuance. We assume that there is an incentive to issue debt, as interest is a deductible expense. The promised interest payment is equal to:

$$P_i - D_{0i}(P_i) \tag{6}$$

In turn, taxable income is the operational one net of interest payment:

$$X_i - (P_i - D_{0i}(P_i)) \tag{7}$$

and the zero-tax level of cash flow with positive leverage, X_i^Z, is

$$X_i^Z(P_i) = P_i - D_{0i}(P_i) \tag{8}$$

Hereafter the argument P_i of D_{0i} and X_i^Z is often suppressed.

We assume that no tax refunds are paid by the tax authority to the owners of the activity if $X_i < X_i^Z$. It follows that operational cash flows, net of tax payments, are[5]

$$X_i^n = X_i^+ - \tau_i(X_i - X_i^Z)^+ = \begin{cases} 0 & X_i < 0 \\ X_i & 0 < X_i < X_i^Z \\ X_i(1-\tau) + \tau X_i^Z & X_i > X_i^Z \end{cases} \tag{9}$$

The present value of future tax payments of the levered firm is equal to:

$$T_{0i}(P_i) = \tau_i(1 + r_T)^{-1}\mathrm{E}(X_i - X_i^Z)^+ \tag{10}$$

Clearly, some value gains obtain when (10) is lower than (5). However, issuing debt has costs as well. Similarly to Merton (1974), default occurs when net operational cash flow is smaller than the face value of the debt:

$$X_i^n < P_i \tag{11}$$

Having defined the default threshold X_i^d as

$$X_i^d(P_i) = P_i + \frac{\tau_i}{1-\tau_i} D_{0i}(P_i) = \frac{P_i - \tau_i X_i^Z}{1-\tau_i} \tag{12}$$

the default triggering condition (11) can be written in terms of the pre tax cash flows as $X_i < X_i^d$. In the event of default, we assume that bondholders will receive a fraction $(1-\alpha_i)$ of operational cash flow, X_i, when this is positive; a fraction α_i of or cash flows is instead lost upon liquidation. There is then a trade-off between the dissipative default costs, $\alpha_i X_i$, and the tax savings possibly generated by debt. Given such trade-off, the entrepreneur chooses the face value of debt so as to maximize the time zero combined value of equity and debt.

The time zero value of equity and debt is the expected present value of cash flows accruing to shareholders and lenders respectively, evaluated under the risk neutral measure. Such cash flows vary with organizational structure, which we analyze below.

[5] Having assumed X_i continuous, we omit the boundary values in this and the following inequalities on payoffs.

2.1 The stand alone case

We now assume that the two activities, $i = 1, 2$, are separately incorporated and independently managed. Based on the arguments in the previous section, the face value of debt in firm i, P_i, maximizes the sum of cashed-in debt, $D_{0i}(P_i)$, and the value of levered equity, $E_{0i}(P_i)$.[6] This sum coincides with levered firm value at time 0, $\nu_{0i}(P_i)$:

$$\nu_{0i}(P_i) = E_{0i}(P_i) + D_{0i}(P_i) \tag{13}$$

We now determine the two elements on the right-hand side of equation 14 as a function of the payoff to financiers at time T. The cash flow to shareholders at t=T, E_i, is operational cash flow less taxes and the repayment of principal, when the difference is positive:

$$E_i(P_i) = (X_i^n - P_i)^+ \tag{14}$$

Indeed, limited liability ensures that shareholders bear no responsibility when the difference is negative.

By no arbitrage the value of equity is simply[7]

$$E_{0i}(P) = (1 + r_T)^{-1}\mathrm{E}(X_i^n - P_i)^+ \tag{15}$$

The cash flows D_i to lenders at time $t = T$ will equal P_i when $X_i > X_i^d$ and the firm is solvent. When the firm is insolvent, i.e. $X_i < X_i^d$, debtholders become the residual claimants. They receive cash flows net of bankruptcy costs, $(1 - \alpha_i)X_i$, if cash flows net of interests are lower or equal to zero ($X_i \leq X_i^Z$). Recalling that the government has priority for tax payments before lenders, debtholders will also bear a tax liability $\tau_i(X_i - X_i^Z)$ in default when $X_i^Z < X_i < X_i^d$. The payoff to lenders is therefore

$$D_i(P_i) = \begin{cases} (1-\alpha_i)X_i & 0 < X_i < X_i^Z \\ (1-\alpha_i)X_i - \tau_i(X_i - X_i^Z) & X_i^Z < X_i < X_i^d \\ P_i & X_i > X_i^d \end{cases} \tag{16}$$

and it can be represented as follows:

Insert here Figure 1

[6] This is the objective if the entrepreneur sells her activity, as the cash inflow from the sale is precisely the sum of debt and equity. It is also the objective when she keeps a fraction of equity capital, in which case her cash inflow is reduced by the amount she does not sell.

[7] Notice that E_{0i} is a call option with underlying X_i^n and exercise price P_i. It depends on debt principal both directly and indirectly, through the tax shield X_i^Z that enters the underlying.

The present value of lenders' payoff 16, $D_{0i}(P_i)$, is the value of zero-coupon debt given the principal P_i:

$$D_{0i}(P_i) = (1+r_T)^{-1}\mathrm{E}\left[\begin{array}{c}(1-\alpha_i)X_i\ \mathbf{1}_{\{0<X_i<X_i^Z\}}+\\ \left[(1-\alpha_i)X_i-\tau_i(X_i-X_i^Z)\right]\mathbf{1}_{\{X_i^Z<X_i<X_i^d\}}+\\ +P_i\ \mathbf{1}_{\{X_i>X_i^d\}}\end{array}\right] \tag{17}$$

where $\mathbf{1}_{\{\bullet\}}$ is the usual indicator function.[8] Note that (17) is an implicit equation, since X_i^Z and X_i^d are themselves function of D_{0i} through (8) and (12). Numerical methods are required for its solution. Since D_{0i} determines the thresholds and the latter enter the equity value, the solution approach consists in finding a fixed point for D_{0i} and then determine X_i^Z, X_i^d and E_{0i}.

The value increase due to leverage, $\nu_{0i}(P_i) - V_{0i}$, reflects tax savings from interest deduction less default costs. Thus the value of the levered firm can also be expressed as

$$\nu_{0i}(P_i) = V_{0i} + TS_i(P_i) - DC_i(P_i) \tag{18}$$

where $TS_i(P_i)$ is the present value of tax savings, equal to the difference in taxes between the levered and unlevered firm

$$TS_i(P_i) = T_i(0) - T_i(P_i) = \tau_i\,(1+r_T)^{-1}\left[\mathrm{E}X_i^+ - \mathrm{E}(X_i - X_i^Z)^+\right] \tag{19}$$

and $DCi(P_i)$ is the present value of the default costs incurred in because of leverage:

$$DC_i(P_i) = \alpha_i(1+r_T)^{-1}\mathrm{E}\left[X_i\ \mathbf{1}_{\{0<X_i<X_i^d\}}\right] \tag{20}$$

As V_{0i} in equation (18) is independent of P_i, the optimal leverage problem can also be posed as choosing the debt level P_i to maximize tax savings less default costs.

2.2 Groups

We now analyze the case where the two activities belong to the same entrepreneur, who is interested in the sum of their values. They are separately incorporated but jointly managed, and are organized as subsidiary and parent (or holding). For the time being, we posit that the holding has an infinitesimal equity share in the subsidiary. We later

[8] Due to default costs and tax savings, debt $D_{0i}(P_i)$ is a portfolio of plain vanilla puts and the present value of the principal.

remove this simplifying assumption, so as to study what happens when the holding receives dividends from its subsidiary.

When unlevered, the value of the holding and of the subsidiary is equal to V_{0i} $i = 1, 2$, and coincides with (4). The levered value of equity is denoted as E_{0h} and E_{0s} for the holding ($i = h$) and the subsidiary ($i = s$) respectively. Let D_{0h} and D_{0s} be the corresponding time-0 values of debt that is cashed in. The entrepreneur then chooses the face value of debt in the holding and in the subsidiary, P_h and P_s, so as to maximize the sum $E_{0h} + D_{0h} + E_{0s} + D_{0s}$, which is the levered group ($i = g$) value:

$$\nu_{0g} = v_0(P_h, P_s) = E_{0h} + D_{0h} + E_{0s} + D_{0s} \tag{21}$$

subject to the state contingent cash flows to shareholders and lenders, that we now characterize.

In particular, we assume that the holding company may transfer cash to its subsidiary so that it can honour debt obligations, when this allows the survival of both. Following the legal literature, we assume that the holding company otherwise enjoys limited liability, being not responsible for the subsidiary's debt obligations if the subsidiary defaults.[9] Let us denote with X_s and X_h the pretax operational cash flows of the two units. A necessary condition for the transfer is that the subsidiary is unable to meet its debt obligations, whereas the holding is not in a default state. Moreover, limited liability ensures that there is no rescue if the operational cash flows of the subsidiary are negative, as the holding would otherwise bear an operational loss that it could have avoided. Thus, the holding rescues its subsidiary only if:

$$\begin{cases} 0 < X_s < X_s^d, \\ X_h > X_h^d \end{cases} \tag{22}$$

where the default thresholds X_i^d, $i = h, s$ and the tax shield X_i^Z correspond to (12) and (8) respectively.

The transfer takes place only if the transfer allows the subsidiary to meets its debt obligations. This is the case only if the after-tax holding cash flow, net of debt repayment, exceeds the corresponding difference for the subsidiary:

$$X_h^n - P_h > P_s - X_s^n \tag{23}$$

Overall, a transfer will occur if and only if both (22) and (23) hold:

[9] We know that some jurisdictions require minority shareholders to be present in the subsidiary for this to be the case. We however abstract from the identity of shareholders at present.

$$\begin{cases} 0 < X_s < X_s^d, \\ X_h > X_h^d \\ X_h^n - P_h > P_s - X_s^n \end{cases} \tag{24}$$

In what follows, we denote the occurrence of these conditions as event A and the amount of the transfer as $(P_s - X_s^n)\mathbf{1}_{\{A\}}$.

We can now proceed to determine the payoffs to shareholders and lenders at t=T. The cash-flow accruing to shareholders of the holding company is equal to the stand alone one, $(X_h^n - P_h)^+$, less the transfer amount:

$$E_h(P_h,P_s) = (X_h^n - P_h)^+ - (P_s - X_s^n)\mathbf{1}_{\{A\}} \tag{25}$$

It follows that the equity value is

$$E_{0h}(P_h,P_s) = (1+r_T)^{-1}\mathrm{E}\left[(X_h^n - P_h)^+ - (P_s - X_s^n)\mathbf{1}_{\{A\}}\right] \tag{26}$$

The payoffs to subsidiary lenders are the same as in the stand alone case, outside the states when a transfer takes place. It must instead be augmented by the transfer in the transfer area, as shown in Figure 2.

Insert here Figure 2

The value of debt is the present expected value of these final payoffs:

$$D_{0s}(P_s, P_h) =$$

$$(1+r_T)^{-1}\,\mathrm{E}\left[\begin{array}{c} X_s(1-\alpha)\mathbf{1}_{\{B\}}+ \\ +\left[X_s(1-\alpha) - \tau(X_s - X_s^Z)\right]\mathbf{1}_{\{C\}}+ \\ +P_s\left[\mathbf{1}_{\{A\}} + \mathbf{1}_{\left\{X_s > X_s^d\right\}}\right] \end{array}\right] \tag{27}$$

where the subsidiary defaults in both states B and C, however lenders have to pay taxes in state C only ($X_s^Z < X_s < X_s^d$) as the tax shield protects them in state B ($0 < X_s < X_s^Z$). In both cases the holding does not supports its subsidiary because its own cash flow is insufficient and there is no transfer. The reader can notice that the value of debt depends on debt face values of both subsidiary and holding companies, since the transfers, tax shields and default thresholds do as well. Thus both E_{0h} and D_{0s} depend on both principals P_h and P_s, which are simultaneously determined.

The payoffs to lenders of the holding do not change with respect to the stand alone case, as the transfer to the subsidiary occurs only after the service of the holding debt. Similarly, equity holders of the subsidiary are unaffected, as the transfer occurs for the sake of servicing

debt. As a consequence equations (17) and (14) still hold for $i = h$ and $i = s$ respectively.

The tax shield enjoyed by the two companies - that together form a group - and their total default costs amount to:

$$TS_g = TS_s + TS_h = \tau(1+r_T)^{-1} \begin{bmatrix} \mathrm{E}X_s^+ - \mathrm{E}(X_s - X_s^Z)^+ + \\ +\mathrm{E}X_h^+ - \mathrm{E}(X_h - X_h^Z)^+ \end{bmatrix} \tag{28}$$

$$DC_g = \alpha(1+r_T)^{-1}\mathrm{E}\left[X_s \mathbf{1}_{\{B\}} + X_s \mathbf{1}_{\{C\}} + X_h \mathbf{1}_{\{0<X_h<X_h^d\}}\right] \tag{29}$$

2.3 Conglomerate

A conglomerate obtains when the two activities are incorporated as one company, and jointly managed by the entrepreneur. In this case claims are issued against the sum of the cash flows of the original activities:

$$X_m = X_1 + X_2 \tag{30}$$

One activity thus subsidizes the other in case of default. There is a unique choice variable, the face value of merger debt, P_m, which maximizes the entrepreneurs' wealth, i.e. merger value:

$$\nu_{0m} = \nu_0(P_m) = E_0(P_m) + D_0(P_m) \tag{31}$$

where $E_0(P_m)$ and $D_0(P_m)$ are computed according to (14) and (16) with $i = m$.

The equivalent problem is to maximize tax savings

$$TS_m = \tau(1+r_T)^{-1}\left[\mathrm{E}X_m{}^+ - \mathrm{E}(X_m - X_m^Z)^+\right] \tag{32}$$

less default costs:

$$DC_m = \alpha(1+r_T)^{-1}\mathrm{E}\left[X_m \mathbf{1}_{\{0<X_m<X_m^d\}}\right] \tag{33}$$

where X_m^Z and X_m^d are defined as in (8) and (12).

We now turn to the study of optimal firm scope with endogenous leverage. Appendix A derives the equations for value differentials between groups - on one side - and conglomerates or stand alones on the other. Appendix B discusses a simple infinite horizon extension of the model.

3 Numerical analysis

This section analyzes the properties of different organizational modes through numerical methods, assuming that the annual cash flow distribution is Normal. The parameters are borrowed from the base case in Leland (2007), which is consistent with a typical firm that issues BBB-rated unsecured debt. Table 1 reports parameter values. Expected operational cash flow for each activity, $Mu = 127.6$, is chosen such that its present value is $X_0 = 100$. Operational cash flow at the end of 5 years has standard deviation (Std) of 49.2, consistent with an annual standard deviation of cash flows equal to 22.0 ($= 49.2/\sqrt{5}$) if annual cash flows are independently distributed in time. Henceforth we express volatility σ as an annual percent of initial activity value X_0, e.g. $\sigma = 22\%$. The tax rate $\tau = 20\%$ and the default cost parameter $\alpha = 23\%$ are chosen so as to generate optimal leverage and recovery rates consistent with the BBB choice.

Insert here Table 1

Table 2 shows the optimal capital structure for a firm with base-case parameters. The first column reports values for a stand alone. The second and the third refer to holding and subsidiary respectively, while the last column to half of a conglomerate - when the correlation coefficient between the units cash flows is equal to 0.2. Also this correlation is chosen as in Leland.

Insert here Table 2

3.1 Holding and subsidiary versus stand alone

The optimal face value of debt, equal to 57.1 for a stand alone company, reaches 219 in the subsidiary while it is equal to zero in the holding company. The yield spread paid by the subsidiary is unsurprisingly very large (8.4 versus 1.2%), because of such a debt. However, the spread in the stand alone would be higher if it raised as much debt as the subsidiary, as it would not receive transfers in case of default.

Large debt in the subsidiary maximizes the tax advantage of debt[10] while, at the same time, zero debt in the holding drives its expected default costs to zero. The tax savings brought about by leverage in the subsidiary reach 14.53, as opposed to 2.33 in a stand alone, while its expected bankruptcy costs are 7.98, versus 0.90 for a stand alone. The

[10] We conjecture that loss offset provisions in the fiscal code would reduce the optimal level of debt in the subsidiary.

value increase due to leveraging jumps from 2.36 for two stand alone firms to 6.31 for the group ($\nu_{0g}^{*} - V_{0h} - V_{0s} = 6.31$). This jump is the effect of differential tax savings and differential default costs, and captures the value of the internal capital market in groups.

These intercorporate links destroy equity value - namely, $E_h^* + E_s^* < E_1^* + E_2^*$. Specifically, the value of equity in the stand alone is larger than in the subsidiary (39.01 instead of 0.037), because of its much lower default probability. The value of equity in the holding is however larger than the stand alone one (49.2 versus 39.01). Indeed, while part of its cash flow is being transferred to the subsidiary lenders, the holding has no debt burden.

The beneficiaries of such a transfer are the debtholders in the subsidiary, who therefore charge a lower spread than if it were stand alone with the same face value of debt. Ultimately, though, the beneficiaries of the internal capital market are the initial owners of the two activities, who could sell them for more. Indeed, the total value of two independent activities is equal to 162.46, while the total value of the group is 165.914.

This result implies that an entrepreneur would sell its two production units as a group rather than two stand alone companies. This is true despite lower equity value.

Figure 2 helps understanding the optimal leverage strategy of the group. By setting debt to zero in the holding, its default threshold coincides with the horizontal axis. This maximizes the transfer area A, given X_s^d. Raising more debt in the subsidiary ensures that its tax threshold moves to the right - making it less likely that taxes will be paid both when the firm is bankrupt and when it is not. Clearly also X_s^d moves to the right, but the transfers will often make sure that the subsidiary does not default - while enjoying tax privileges.

Modigliani and Miller (1963) argued that, in a world without bankruptcy costs, optimal leverage may reach 99% because of the tax advantage of debt. Subsidiaries display a similar leverage because state contingent transfers are able to reduce the likelihood of bankruptcy.

3.2 Group versus conglomerate

Merging two activities allows the conglomerate to raise higher debt than the two stand alone because of cash flow pooling between the two units, that reduces the probability of default (Leland, 2007). This brings enhanced tax advantages because of interest deductions, as predicted by Lewellen (1971). It also allows to use the losses from one unit to offset taxable income from the other unit, thus reducing the negative impact of tax asymmetries (Majd and Myers, 1987). However it makes possible for one unprofitable activity to absorb the cash flows of a profitable one,

reducing the value of limited liability for the unlevered company - the "Sarig effect". The value of conglomerates exceeds that of two stand alone activities for a correlation coefficient between cash flows equal to 0.2. These results can be visualized by comparing the first with the last column of Table 2.

We now turn to the comparison between a conglomerate and a group. Our model shows that optimal group debt is far greater than that of conglomerates (219 versus 117.4). The novel reason is that the group is able to better exploit the asymmetry of taxation, thanks to the possibility of raising different debt levels in two companies. Indeed, raising more debt from the subsidiary increases its no tax threshold, which reaches 102.32, against a mean cash flow of 127.63. As a consequence, the tax burden in a group falls to 25.37 as opposed to 35.62 in the conglomerate - and the one in the subsidiary is as low as 5.42 compared to 17.81 in a merged unit.

However, the default threshold of the subsidiary is pushed up to 248.169. Such a burdensome level of the debt service can be sustained because of transfers from the holding company. At the optimum, there are both larger tax savings (14.53 versus 4.38) and larger default costs (7.98 versus 1.24) in the group than in the conglomerate. However, the value increase thanks to leveraging is twice as large in groups (6.31) than in conglomerates (3.05). As a consequence, the value of a groups, 165.914, exceeds that of a conglomerate, 163.14. Since both have an internal capital market, the value increase is due to the specificities of the group - namely limited liability associated to separate incorporation.

In the following section we assess whether these patterns hold when diversification opportunities change.

3.3 Capital structure and value with changing correlation

Stand alone firms do not exploit diversification. Conglomerates exploit it but are subject to the Sarig effect. Total debt capacity is largest in groups[11], because they exploit the diversification potential without incurring into the Sarig effect, thanks to the possibility to abandon a defaulting subsidiary. One may however expect that, as correlation among cash flows increases, the transfers from the holding to the subsidiary will become less likely and hence the optimal face values of debt will converge to the stand alone level because of vanishing diversification opportunities. The first part of this reasoning is correct: as the correlation

[11] Deloof and Vershueren (2006) find tha intragroup guarantees increase total debt raised from third parties by Belgian firms.

coefficient ρ increases, the probability of a transfer from the holding to the subsidiary halves as the correlation coefficient goes from -0.8 to 0.8. The second part of the argument is however incorrect: debt in the holding continues to be zero, because this allows to eliminate the holding bankruptcy costs.

The other surprising result is that the optimal face value of debt in the subsidiary increases in ρ, as evident in the upper left panel of Figure 3. In conglomerates the opposite holds: as diversification opportunities vanish, debt gets as risky as the one of stand alone and converges to the same value - as the probability of paying twice bankruptcy costs increases. On the contrary, in groups the holding company never incurs into bankruptcy costs. The reason is that they can have different levels of debt in the two units. This ensures that transfers will indeed take place even in the limiting case of $\rho = 1$, because the holding is unlevered while the subsidiary has high leverage. Clearly, the market value of subsidiary debt dramatically falls as correlation increases (see upper right panel of Figure 3), because lenders required spread grows from 0.017 for $\rho =$ -0.8 to 0.104 for $\rho = 0.8$.

Group equity value $(E_1^* + E_2^*)$ is lower than in the case of both independent companies and conglomerates, confirming shareholders' value destruction in business groups for all correlation coefficients (lower left panel of Figure 3). This is due to the transfer of cash flows to the benefit of debt-holders.

Finally, and most importantly, the total value of the group exceeds that of stand alone and conglomerates (Bottom right panel of Figure 3) whether or not diversification benefits exist. Moreover, the value differential increases as correlation between cash flows grows above -0.5. The tax burden on groups falls due to the increased face value of debt, while the one on stand alone firms is constant and the one on conglomerates increases. At the same time, default costs in the subsidiary increase more rapidly than in conglomerates, however not as quickly to compensate the increase in tax savings. Indeed, the difference between the default threshold and the no tax threshold in the subsidiary shrinks as correlation increases. In Figure 2, the area A' gets larger relative to A''. Thus the states of nature when the subsidiary defaults and pays taxes decrease.

We can summarize the results obtained so far in the following proposition.

Proposition 1 *: assume no agency costs, positive proportional bankruptcy costs, fiscal deductibility of interest for BBB calibrated companies and the provision of state-contingent support by the holding company to*

its subsidiary (H1). Then the value of a group exceeds that of both conglomerates and stand alone companies. The value of its equity claims is lower, while that of debt claims is larger than in other organizations.

The higher value of groups, which is implied by our model, may explain one empirical regularity - namely the diffusion of groups around the world. The model also indicates that groups destroy equity value, which is another stylized fact.

Last but not least, the tax burden of debt drops from 17.62% to 5.42% of operating cash flow in Table 2. In firms taken private through MBOs, the tax burden dropped from 20% to 1% in the first two years and to 4.8% in the third year (Kaplan, 1989). In Table 2, the default threshold for a subsidiary is 248, way above the mean operating income. In a sample of distressed highly leverage transactions, all sample firms had operating margins in excess of the industry median (Andrade and Kaplan, 1998). The model implies leverage in excess of 95% in subsidiaries. This is also observed in the private equity industry, where our assumption of no agency costs applies well. Private equity partners often need to raise new funds in the market because of the limited temporal commitments of financiers, and this is possible only if their reputation is good. Moreover, subsidiary managers receive bonuses only when they repay their debt obligations (Jensen, 2007).

4 Characterizing holding and subsidiaries

So far numerical results refer to two symmetric activities, that differ only because one is assumed to support the other. The analysis below allows to endogenously derive the relative characteristics of holding and subsidiaries that maximize group value, which is a further contribution of this paper relative to prior literature. Our simulations refer to the cases when affiliated units differ in cash flow volatility (σ), size(μ) and proportional bankruptcy costs (α). The following proposition summarizes our main findings.

Proposition 2 *: assume H1. Then the default costs and the size of the holding are at least as large as those of its subsidiary. The subsidiary, in turn, is at least as risky as the holding. The value benefits of a group structure relative to competing organizations increase with risk and bankruptcy costs asymmetries between the holding company and its subsidiary.*

Activities with relatively larger bankruptcy costs should be holding companies. This is because, under the optimal capital structure, they never pay them. Table 3 displays numerical results. When correlation

between cash-flows is equal to 0.2, the optimal capital structure and group value do not change as default costs in the subsidiary increase from 23% to 75%. In the case of stand alone firms, the optimal value drops instead from 81.23 (see Table 2) to 80.83 and the face value of debt from 57.2 to 33. It follows that value gains from group structure increase with asymmetric default costs. This holds true also with respect to the conglomerate merger case, whose value falls from 163.14 to 162.47 with asymmetric default costs.

Unreported optimizations assess the cost of a suboptimal group structure, associated with the subsidiary bearing higher proportional default costs than its holding. Its face value of debt falls to 107 (as opposed to 219). Due to a reduced tax shield, the value of the group is now lower (162.37) than the one of the merger(162.47), and this holds for all correlations between -0.8 and +0.8[12]. However, even a suboptimal group dominates the stand alone organizations.

Table 4 concerns the case of asymmetric volatilities, when one unit has an annualized cash flow volatility of 44% as opposed to 22% of the other and correlation is still 0.2. We know that the tax shield has higher value with a riskier cash flow; it is therefore unsurprising to find higher optimal debt in all organizations. The higher volatility unit, when incorporated as a stand alone, has face value of debt equal to 83 instead of 57 in the base case, and is accordingly charged a much higher spread (6.2% as opposed to 1.2%) by lenders. This implies increased tax savings (4.66 versus 2.31), and a higher value of the riskier stand alone (84.84 versus 81.23) - even though its equity value drops from 39.32 to 36.1. The total value of these two stand alone is equal to 166.07, while the conglomerate value is 163.26[13]. Merging the two units thus destroys value as the riskier division is more likely to drag the safer one in default.

This is not a problem for the group, as the holding can use its limited liability. Consistent with empirical evidence (Bianco and Nicodano, 2006), we find that subsidiaries are riskier than their holding companies in the optimal group structure. A riskier holding incurs into larger losses more often than a safer one. Hence it would not be able to rescue its subsidiary as often as a safer one. Moreover, a riskier holding would suffer more than a riskier subsidiary from the asymmetric nature of taxation, as it uses less the tax shield of debt. Going back to Table 4, subsidiary debt reaches 223 (up from 219 in the base case), ensuring that interests shield the larger profits - which are now more likely - from taxes. Total group value is now equal to 170.13, which not only exceeds

[12] This result highlights, in passing, that allowing for conditional support and differential debt levels does not - *per se* - guarantee higher total value.

[13] It is smaller than 166.07 for all correlations levels

value of competing organizational forms but also the value of the group in the base case (163.14), when the subsidiary is less risky. These results may rationalize the reason why strategic alliances (Robinson, 2006), venture capital funds (Sahlman, 1990) and innovative firms (Allen, 1998) often adopt a keiretsu structure, with riskier ventures incorporated as subsidiaries.

Up to now we have seen that debt is always optimally located in the subsidiary. The last case we examine is the one of differing size. Simulation results in Table 5 refer to a situation where the expected operational cash flow of one activity is five times the one of the other and $\rho = 0.2$. When the two units are incorporated as stand alone, their joint value - obtained by summing figures in the first two columns - is equal to 162.46 and their joint principal is equal to 114. A conglomerate has a value which is only slightly above (162.98), suggesting that asymmetric size is detrimental to conglomerates because the larger company is more likely to drag the smaller, healthy one into insolvency. [14]

Group value, in the fifth column, also falls relative to the symmetric case, but marginally so (165.65 instead of 165.91) - provided that the smaller activity is downstream. One implication of our model is that smaller activities should be the ones that receive conditional support. The smaller subsidiary - see the fourth column - raises less debt than the one of the base case (121.33 as opposed to 219), with two consequences. On the one hand the holding company wants to raise debt as well so as to reduce taxation, on the other it can do so without compromising the provision of support to its subsidiary- thanks to its size. Debt appears in the holding company (63.33)as well.[15]

A group with a large subsidiary would be suboptimal, as the large subsidiary could hardly be rescued by a small holding. Consequently, it would raise less debt with a corresponding reduction in both the tax shield and group value (163.25). This is however still higher than in the competing organizations.

All the previous qualitative results under asymmetry hold true when correlation varies: we conclude that the ability of groups to create value by optimally trading off taxes and bankruptcy costs is a strikingly robust

[14] Rajan and Zingales (2000) find that observed conglomerate discounts are positively related to the diversity of investment opportunities across divisions.

[15] In Belgian groups, both holding and subsidiaries raise debt, and subsidiaries account for 8% the size of their group (Dewaelheyns and Van Hulle, 2007). Also Italian holding groups raise debt from holding companies (Bianco and Nicodano, 2006). Some ambiguity however emerges in interpreting empirical results, because the supporting companies of our model could indifferently be a holding or a financial subsidiary, depending on the group organizion.

result - absent incentive problems.

5 Adding some real-world features

In the next section we will allow for the case where the subsidiary cannot raise more debt than a stand alone, for either regulatory constraints or incentive reasons. Next, we will investigate the case when the holding owns part of its subsidiary equity.

5.1 The case of a constrained subsidiary

Several countries set upper limits to interest deductions in order to prevent tax elusion or thin capitalization. This tax provision clearly reduces the incentive to lever up the subsidiary. Specific regulation applies also to group affiliated firms that are listed on public exchanges. Several jurisdictions impose to subsidiary managers to act in the interest of the subsidiary shareholders, rather than implementing what is optimal for the group (Hadden, 1996). In this case, a high level of debt and a value of equity close to zero in the subsidiary could be considered as a violation of the rule.

We therefore investigate the effects of these policies by imposing a subsidiary debt level equal to the stand alone one, and optimize leverage in the holding.[16]

Table 6 describes value and equilibrium capital structure for this case, as correlation changes. Let us focus on 0.2 correlation, so that results directly compare to Table 2. The holding company has a positive face value of debt in this case (54), which implies that tax savings are almost equally shared by the subsidiary (2.11) and by the holding (2.15). Spreads are respectively equal to 0.58% and 1.1%: this differential is due to the higher default probability of the holding company. The two companies are indeed almost equal in everything but the fact that the holding supports its subsidiary, whereas the opposite is not true. Thus expected default costs are down to 23% for the subsidiary, but up to 73% for its parent.

Equity value in groups is now marginally higher (79.08) than in stand alone companies (79.02). Group value (162.79) still exceeds that of two stand-alone companies (162.46), but the differential - obviously - shrinks. Indeed total tax savings fall to 4.27 as opposed to 14.53 in the unconstrained case. Such regulation achieves a reduction in dissipative bankruptcy costs together with a fall in tax elusion. However tax elusion falls more than expected default costs, which are down to 0.96 from an

[16] Low levels of debt in subsidiaries may also be the outcome of agency problems. Non-related lenders may otherwise charge very high interest rates anticipating risk shifting problems (Bianco and Nicodano, 2006)

unconstrained level of 7.98. The value of a parent-subsidiary structure is now decreasing in the correlation coefficient between cash flows. When cash flows are positively correlated, the holding is often unable to rescue its subsidiary - and this increases expected bankruptcy costs.

Comparison with the conglomerate indicates that a group - such as the one we are describing- could not emerge endogenously, as its value is lower (162.79 versus 163.15). The reason is simple: the face value of debt raised by the optimizing conglomerate (117.4) exceeds the one in groups (111.2), implying a lower tax burden (35.62 versus 35.63). In spite of the Sarig effect, merger is the optimal arrangement, because it leverages more. It is however possible to show that for a face value of debt in the subsidiary equal to 80 - hence still much lower than the unconstrained optimal level (219) - the value of groups (163.31) again exceeds the conglomerate merger highest price (163.15). Thus the group is able to conservatively use debt in the subsidiary for tax shielding purposes, keeping the default probability of the holding company low enough so as to dominate on competing organizations.

Previous results qualitatively obtain also with changing correlations between activities cash flows.

Proposition 3 *Assume H1. Consider a subsidiary with face value of debt equal to that of a BBB stand alone company. Then the overall group value exceeds that of the corresponding stand alone companies (but not that of the conglomerate merger). Overall equity (debt) value is higher (lower) than the sum of the stand alone ones, for positive cash flow correlation. Viceversa for negative correlation.*

5.2 Capital structure and intercorporate dividends

We normally observe that the holding keeps a fraction ω of the subsidiary shares. When this is the case, the face values of debt in the two companies maximize the proceeds from the group sale, which must be reduced by the sum kept by the holding, ωE_{0s}:

$$\nu_{0g} = \nu_0(P_h, P_s) = E_{0h} + D_{0h} + (1-\omega)E_{0s} + D_{0s} \tag{34}$$

Thus $\omega = 1$ corresponds to a wholly owned subsidiary, while $\omega = 0$ is the case we analyzed above, namely extreme separation between ownership and control.

The holding is entitled to a share ω of the total payoff to subsidiary equityholders, which may allow it to avoid default in some states. Appendix C characterizes the state contingent flow of dividends to the holding, as well as its new default thresholds.

Unreported results for $\omega = 0.5$ and $\omega = 1$ show that - in our unconstrained model - ownership and the associated dividends do not affect at all optimal group capital structure, and the highly levered subsidiary pays dividends very rarely.[17] Thus, state contingent transfers like dividends - that may support a defaulting company but are not targeted to that purpose - do not alter equilibrium tax savings and default costs. Results on comparative value of groups, which are summarized in Proposition 1, carry over to this generalized case.

Ownership levels matter for firm value when incentive problems plague relationships between majority and minority shareholders (see Almeida and Wolfenzon, 2006, among others). In our set up without agency problems, ownership levels are irrelevant for group leverage and value, as long as there are no external constraints.

6 Concluding comments

This paper contributes to our understanding of firm scope, by showing that parent- subsidiary structures are value maximizing arrangements in environments where bankruptcy costs and non- neutral taxation make firm financial policy relevant. Our results may thus explain why groups are such a resilient organizational structure worldwide. They may also shed some light on the reason why they are emerging again in innovative industries in the U.S., where the risk of subsidiary activities is large. Importantly, we provide a pricing model for group affiliated companies - albeit in a simplified setting without agency problems vis à vis lenders and minority shareholders. This feature allows to understand if and why shares of affiliated firms trade at a discount, pointing to a solution for this pricing puzzle. Furthermore, it paves the way for predicting the value effects of corporate restructurings, such as carve-outs.[18] Clearly, we are still far from a fully realistic picture of such complex firm structures, as we ignore the possibility of intragroup lending - which is quite common even though many affiliated companies do not exploit it. We leave this extension for further work.

In contrast with previous literature on groups, we do not assume the extraction of private benefits by controlling shareholders. However our framework lends itself to a theoretical examination of the nature of private benefits, once one allows for differences between shareholders.

[17]It follows that intercorporate taxation of dividends - advocated by Morck (2004) in order to eliminate groups - would not be very effective in this unconstrained case.

[18]This paper does not consider the comparative credit risk properties of stand alones, conglomerates and groups for reasons of space. We analyze these in a companion work, which focuses on selective versus joint defaults of group affiliated firms. (Luciano and Nicodano,2007)

We conjecture that there would be no difference in the price of majority and minority shares as long as minority shareholders invest the same stake in both the subsidiary and the holding company. The benefits of transfers from one company to the other would in fact be equally shared by both minority and controlling equityholders. A wedge would instead appear when stakes differ, if transfers do not benefit the minority shareholders in the subsidiary. Such a wedge would then become an endogenous private benefit from control.

An empirical investigation of the explanatory power of this new hypothesis would be useful, also in view of the policy stance against groups that is prevailing in Europe. Dismantling groups would cause a welfare loss for the corporate sector, with redistributions from lenders to equityholders - if our hypotheses hold and agency costs do not compensate the reduction in default costs brought about by groups. However, a full-blown picture of group value and capital structure requires data on both holdings and subsidiaries - that are often non listed. Moreover, the standard comparison between group-affiliated and stand-alone companies appears not to be viable in the light of this model. Indeed, Tobin's q is not a substitute for firm value. Furthermore, leverage cannot be considered as a control variable, being endogenously determined. Last but not least, the inference on average capital structure, value of claims and risk of group affiliated companies will depend on the relative number of holdings and subsidiaries sampled by the econometrician.

We however envisage two simpler applications of our structural model, once relevant features of the tax and regulatory environment are accommodated. Indeed, a calibration of cash-flow parameters should be able to assess whether it is able to replicate the observed holding company discount (Cornell and Liu, 2001) and the price effects of carve-out announcements (Lamont and Thaler, 2003; Schipper and Smith, 1986).

This model may finally turn out to be useful to the understanding of several other institutions which are characterized by separate incorporation and yet linked by an internal capital market, such as financial conglomerates, project financing and closed-end funds.

Appendix A

A.1. Groups versus stand alone

When a group forms out of two stand alone companies, the following value change obtains:

$$\Delta\nu_{0g} = \nu_0(P_h, P_s) - \nu_0(P_1) - \nu_0(P_2) = \Delta TS_g - \Delta DC_g \qquad (35)$$

where the differences in tax savings and default costs are respectively:

$$\Delta TS_g = TS_s + TS_h - TS_1 - TS_2 \tag{36}$$
$$= \tau(1+r_T)^{-1}\begin{bmatrix} \mathrm{E}(X_1 - X_1^Z)^+ + \mathrm{E}(X_2 - X_2^Z)^+ + \\ -\mathrm{E}(X_s - X_s^Z)^+ - \mathrm{E}(X_h - X_h^Z)^+ \end{bmatrix}$$
$$\Delta DC_g = DC_g - DC_1 - DC_2 = \alpha(1+r_T)^{-1} \times \tag{37}$$
$$\begin{cases} \mathrm{E}\left[X_s\mathbf{1}_{\{B\}} + X_s\mathbf{1}_{\{C\}}\right] + \mathrm{E}\left[X_h\mathbf{1}_{\{0<X_h<X_h^d\}}\right] + \\ -\mathrm{E}\left[X_1\mathbf{1}_{\{0<X_1<X_1^d\}} + X_2\mathbf{1}_{\{0<X_2<X_2^d\}}\right] \end{cases}$$

A.2. Value comparison: conglomerates versus stand alone and groups

Leland (2007) decomposes the total value differential relative to the stand alone case, $\Delta\nu_{0m}$, into a differential value for the unlevered company and a leverage effect:

$$\Delta\nu_{0m} = \nu_0(P_m) - \nu_0(P_1) - \nu_0(P_1) = \underbrace{\Delta V_{0m}}_{\text{Sarig effect}} + \underbrace{\Delta TS_m - \Delta DC_m}_{\text{Leverage effect}} \tag{38}$$

where

$$\Delta V_{0m} = (1-\tau)(1+r_T)^{-1}\left[\mathrm{E}(X_1+X_2)^+ - \mathrm{E}X_1^+ - \mathrm{E}X_2^+\right] \tag{39}$$

This term - the "Sarig effect" - can be negative as limited liability for stand alone companies provides a valuable option to walk away from future losses, which is partially given up by joint incorporation. Observe that there is no Sarig effect in Equation 35 as group affiliated companies - like stand alone ones - retain their limited liability.

In turn, the leverage effect is composed of the tax shield ΔTS_m and the change in bankruptcy cost ΔDC_m.

$$\Delta TS_m = TS_m - TS_1 - TS_2 = \tag{40}$$
$$= \tau(1+r_T)^{-1}\begin{bmatrix} -\mathrm{E}(X_m - X_m^Z)^+ + \mathrm{E}(X_1 - X_1^Z)^+ + \\ +\mathrm{E}(X_2 - X_2^Z)^+ + \mathrm{E}X_m{}^+ - \mathrm{E}X_1^+ - \mathrm{E}X_2 \end{bmatrix}$$
$$\Delta DC_m = DC_m - DC_1 - DC_2 = \tag{41}$$
$$= \alpha(1+r_T)^{-1}\begin{bmatrix} \mathrm{E}\left[X_m\mathbf{1}_{\{0<X_m<X_m^d\}}\right] + \\ -\mathrm{E}\left[X_1\mathbf{1}_{\{0<X_1<X_1^d\}} + X_2\mathbf{1}_{\{0<X_2<X_2^d\}}\right] \end{bmatrix}$$

We now compare the conglomerate to a group. Tax savings, default costs and value differentials are respectively equal to:

$$\Delta TS_g - \Delta TS_m =$$

$$= \tau(1+r_T)^{-1}\begin{bmatrix} -\mathrm{E}(X_s - X_s^Z)^+ - \mathrm{E}(X_h - X_h^Z)^+ + \\ +\mathrm{E}(X_m - X_m^Z)^+ - \mathrm{E}X_m{}^+ + \mathrm{E}X_1^+ + \mathrm{E}X_2^+ \end{bmatrix} \tag{42}$$

$$\Delta DC_g - \Delta DC_m =$$

$$= \alpha(1+r_T)^{-1}\left\{ \begin{array}{c} \mathrm{E}\left[X_s \mathbf{1}_{\{B\}} + X_s \mathbf{1}_{\{C\}}\right] + \\ +\mathrm{E}\left[X_h \mathbf{1}_{\{0<X_h<X_h^d\}}\right] - \mathrm{E}\left[X_m \mathbf{1}_{\{0<X_m<X_m^d\}}\right] \end{array} \right. \tag{43}$$

$$\Delta\nu_0 = \nu_0(P_h, P_s) - \nu_0(P_m) = \Delta\nu_{0g} - \Delta\nu_{0m} = \tag{44}$$
$$\Delta TS_g - \Delta DC_g - \Delta V_{0m} - \Delta TS_m + \Delta DC_m$$

The Sarig effect is present in the last equation, as each division of a conglomerate gives up limited liability whereas parent and subsidiary do not.

Appendix B - Infinite horizon

Leland extends his stand alone and merger model to an infinite horizon, by assuming that the problem is stationary and that cash flows are independently distributed in time. At each length T cycle, the leverage selection is repeated. Optimal policies do not change, because of stationarity. In addition, Leland posits that the entrepreneur is endowed with a cash flow X independently of whether default occurred in the previous run or not. In his case, the overall value accruing to equity holders in the stand alone and merger case is respectively

$$\sum_{t=0}^{+\infty}(1+r_T)^{-t}\left[\nu_{01}^* + \nu_{02}^*\right] = \frac{\nu_{01}^* + \nu_{02}^*}{d_T}$$

and

$$\sum_{t=0}^{+\infty}(1+r_T)^{-t}\,\nu_{0m}^* = \frac{\nu_{0m}^*}{d_T}$$

where $d_T = r_T/(1+r_T)$. By a similar reasoning, group value is:

$$\sum_{t=0}^{+\infty}(1+r_T)^{-t}\,\nu_{0g}^* = \frac{\nu_{0g}^*}{d_T} \tag{45}$$

This value is derived under the assumption that rescue indeed occurs when expected, i.e. that the stated guarantee is enforceable - which is consistent with our full information setting[19].

[19] An extension could allow for limited enforceability, which applies to comfort letters, in a game where credit conditions from a given round onwards depend on whether rescue took place. Then shareholders would tradeoff future gains from improved credit conditions with current gains from not supporting the subsidiary even when the parent has sufficient cash flows (as in Boot et al., 1993).

Appendix C - Groups with finite ownership

Let ω be the ownership share of the holding in the subsidiary ($\omega \in (0,1]$). The cash flows of the holding must be augmented for dividend, net of intercorporate taxation. If there is no such taxation, these cash flows are

$$X_h^n + \omega(X_s^n - P_s)^+ \tag{46}$$

Observe that dividends are zero in default states, that is when $X_s < X_s^d$, or $X_s^n < P_s$. Below we distinguish between four specifications for them, depending on whether they are smaller or greater than the tax shield and on whether dividends are being paid out or not:

$$\begin{cases} X_h & X_s^n < P_s, X_h < X_h^Z \\ X_h - \tau(X_h - X_h^Z) & X_s^n < P_s, X_h > X_h^Z \\ X_h + \omega\left[X_s - \tau(X_s - X_s^Z) - P_s\right] & X_s^n > P_s, X_h < X_h^Z \\ X_h - \tau(X_h - X_h^Z) + \omega\left[X_s - \tau(X_s - X_s^Z) - P_s\right] & X_s^n > P_s, \\ & X_h > X_h^Z \end{cases} \tag{47}$$

Within these cash flows, the payoff to lenders is equal to:

$$\begin{cases} 0 & X_h^n + \omega(X_s^n - P_s)^+ < 0 \\ (1-\alpha)\left[X_h^n + w(X_s^n - P_s)^+\right] & 0 < X_h^n + \omega(X_s^n - P_s)^+ < P_h \\ P_h & X_h^n + \omega(X_s^n - P_s)^+ > P_h \end{cases}$$

In the first case cash flows, gross of any dividends, are negative. Therefore lenders lose all their capital. In the second case, the holding defaults and lenders receive all cash flows by absolute priority. In the last case, the holding is solvent.

When dividends are paid out, the holding default threshold, $\bar{X}_h^d$, depends on the subsidiary cash flow X_s. It is the level of operational cash flows, net of taxes but gross of dividends, that equals P_h:

$$\bar{X}_h^d - \tau(\bar{X}_h^d - X_h^Z)^+ + \omega(X_s - \tau(X_s - X_s^Z) - P_s) = P_h \tag{48}$$

As a result, the holding default occurrence can be written in terms of the subsidiary cash flows as follows:

$$\begin{cases} X_h < X_h^d,\ X_s < X_s^d \\ X_h^Z < X_h < X_h^d,\ X_s^d < X_s < X_s^d - (X_h - X_h^d)/\omega \\ 0 < X_h < X_h^Z,\ X_s^d < X_s < X_s^d - \frac{X_h - P_h}{\omega(1-\tau)} \end{cases} \tag{49}$$

We visualize the default threshold and the lenders' payoff in Figure 4.

Insert here Figure 4

The holding equity holders receive nothing below the default threshold. Above it, they receive cash flows gross of any dividends.

The new value of the holding equity and debt value obtain by discounting the expectation of these new cash flows to shareholders and lenders, respectively. The problem is complicated by the fact that they depend on the face value of the subsidiary debt (this is not the case without dividends). The values of subsidiary debt and equity are instead unaffected by the payment of dividends to the holding: therefore, they can be represented as in (16) and (14).

References

Acharya, V.V., S.T. Bharath and A. Srinivasan, (2005), Does Industry-wide Distress Affect Defaulted Firms? Evidence from Creditor Recoveries, mimeo

Allen, J.W.,(1998), Capital Markets and Corporate Structure: The Equity Carve-Outs of Thermo Electron, *Journal of Financial Economics*, 48, 99-124

Almeida, H. and D. Wolfenzon, (2006), A Theory of Pyramidal Ownership and Family Business Groups, *Journal of Finance*, 79,2637-2680

Andrade, G. and S.N. Kaplan, (1998), How Costly is Financial (not Economic) Distress? Evidence from Highly Leveraged Transactions that Became Distressed, *Journal of Finance,* 53, 1443-1493

Barca, F. and M. Becht (eds.), (2001), The Control of Corporate Europe, (Oxford, Oxford University Press).

Bennedsen, M. and K.M. Nielsen, (2006), The Principle of Proportional Ownership, Investor Protection and Firm Value in Western Europe, ECGI WP. 134/2006

Bertrand M., Metha P. and S. Mullainathan, (2002), Ferreting Out Tunneling: An Application to Indian Business Groups, *Quarterly Journal of Economics*, 117, 121-148

Bianco, M. and G. Nicodano, (2006), Business Groups and Debt, *European Economic Review*

Blumberg Phillip I., (1989), The Law of Corporate Groups, Boston: Little, Brown and Company

Boot, A.W.A., S.I. Greenbaum and A.V. Thakor, (1993), Reputation and Discretion in Financial Contracting, *The American Economic Review*, 83(5), 1165-1183

Cestone, G., and C. Fumagalli, (2005), The Strategic Impact of Resource Flexibility in Business Groups, *Rand Journal of Economics*, 36, 193-214

Claessens, S., Djankov S., Fan J.P.H. and L.H.P. Lang, (2002) Disentangling the Incentive and Entrenchment Effect of Large Shareholders, *Journal of Finance*, 57,2741-71

Cornell, B. and Q. Liu, (2001), The Holding Company Puzzle: When Is the Whole Worth Less Than One of the Parts?, *Journal-of-Corporate Finance*, 7(4), 341-66

Deloof M.and I. Vershueren, (2006) How Does Intragroup Financing Affect Leverage? Belgian Evidence, *Journal of Accounting Auditing and Finance,* 21, 83-108

Dewaelheyns N. and C. Van Hulle, (2007), Internal Capital Markets and Capital Structure: Bank versus Internal Debt, Katholieke Universiteit Leuven, wp 0703

Faure-Grimaud, A. and R. Inderst, (2005), Conglomerate Entrenchment under Optimal Financial Contracting, *American Economic Review*,

Fluck, Z. and A., Lynch, (1999), Why Do Firms Merge and Then Divest? A Theory of Financial Synergy, *The Journal of Business*, (72) 3

Gertner R.H., Scharfstein D.S., and J. Stein, (1994), Internal versus External Capital Market, *Quarterly Journal of Economics*, 109, 1211-1230

Hadden Thomas, Liabilities in Corporate Groups, in Rossi Guido et al. eds., (1996), I Gruppi di Società, Proceedings of the International Conference on Business Groups, Venice, Milano: Giuffrè, 1361-1388

Huizinga H., Laeven L. and G. Nicodème, Capital Structure and International Debt Shifting, IMF February 2007

Jensen, M., (2007), The Economic Case For Private Equity, Harvard NOM Research Paper No. 07-02

Kaplan S., (1989), Management Buyouts: Evidence on Taxes as a Source of Value, *The Journal of Finance*, 44(3), 611-632

Khanna, T, and K. Palepu, (2000), Is Group Membership Profitable in Emerging Markets? An Analysis of Diversified Indian Business Groups, *Journal of Finance*, 55, 867-891

Khanna, T., and Y. Yafeh, (2006), Business Groups in Emerging Markets: Paragons or Parasites?, ECGI Working Paper.

Lamont, O.A. and R. H. Thaler, (2003), Can the market add and subtract? Mispricing in tech stock carve-outs, *Journal of Political Economy*, 111, 227-268

Leland, H., (2007), Purely financial synergies and the optimal scope of the firm: implications for mergers, spin offs, and structured finance, *Journal of Finance*, 62, 765-807

Lewellen, W., (1971), A Pure Financial Rationale for the Conglomerate Mergers, *Journal of Finance*, 26, 521-537

Luciano, E. and G. Nicodano, (2007), Credit risk and parent-subsidiary links, wp

Majd, S. and S.C. Myers, (1987), Tax Asymmetries and corporate income tax reform, in M. Feldstein, ed., Effects of taxation on capital accumulation (University of Chicago Press, Chicago, IL)

Mansi, S.A. and D.M.Reeb, (2002), Corporate Diversification: What Gets Discounted?, *Journal of Finance*, 57(5), 2167-83

Mathews, Robinson (2006), Market Structure, Internal Capital Markets, and the Boundaries of the Firm, wp

Merton, R.C., (1974), On the pricing of corporate debt: The risk structure of interest rates, *Journal of Finance*, 449-70

Modigliani, F., and M.H. Miller, (1958), The Cost of Capital, Cor-

poration Finance and the Theory of Investment, *American Economic Review*

Modigliani, F., and M.H. Miller, (1963), The Cost of Capital, Corporation Finance and the Theory of Investment: A Correction, *American Economic Review, 53,*433-443

Morck, R., (2004), How to Eliminate Pyramidal Business Groups – The Double Taxation of Inter-Corporate Dividends and Other Incisive Uses of Tax Policy, NBER Working Paper No. 10944.

Rajan, R., H. Servaes, and L. Zingales, (2000), The Cost of Diversity: The Diversification Discount and Inefficient Investment, Journal of Finance, 55, 35-80.

Robinson D.T., (2006), Strategic Alliances and the Boundaries of the Firm, Duke University,

Sahlman, W.A., (1990) The Structure and Governance of Venture-Capital Organizations, *Journal of Financial Economics*, 473-521

Schipper, K. and A. Smith, (1986), A comparison of equity carve-outs and seasoned equity offerings: share price effects of corporate restructuring, *Journal of Financial Economics,* 15, 153-186.

Stein, J.C., (1997), Internal Capital Markets and the Competition for Corporate Resources, *The Journal of Finance*, 111-113

Stiglitz, J.E., (1969), A Re-Examination of the Modigliani-Miller Theorem, *American Economic Review*, 784-793

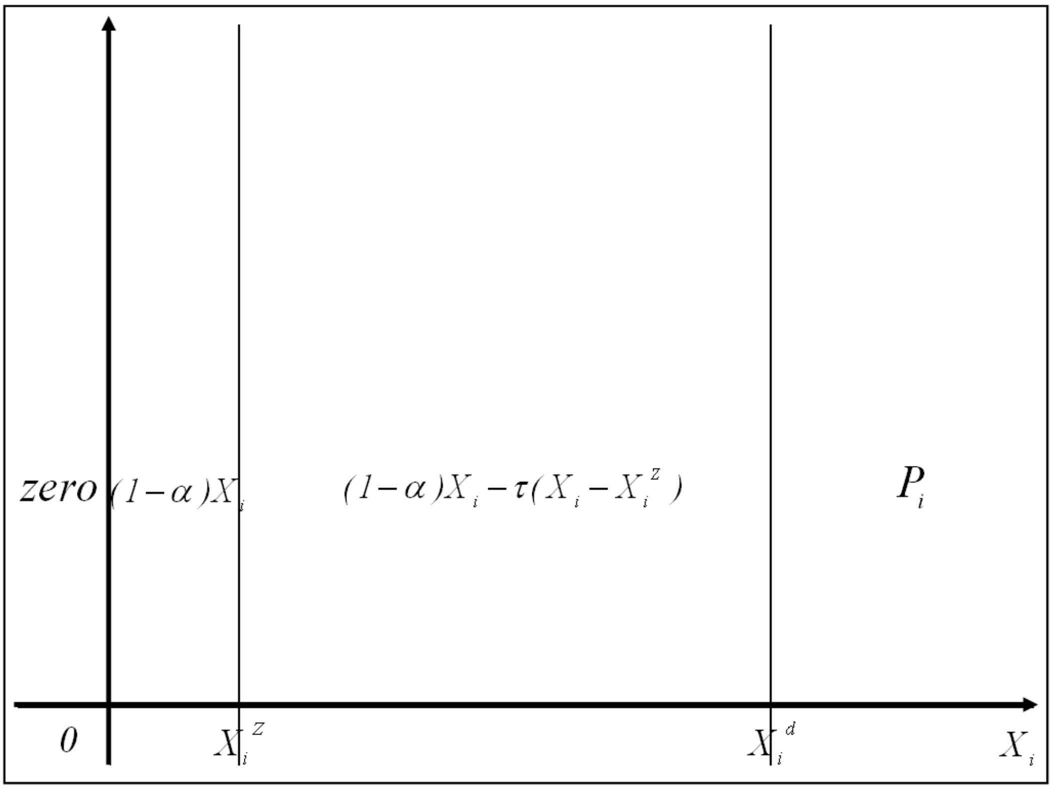

Figure 1: This figure represents the payoff to the lenders of a stand alone company. Lenders are fully reimbursed (P_i) when cash flow X_i exceeds the default threshold X_i^d. Otherwise, they receive a fraction $(1-\alpha)$ of positive cash flows. However, when the stand alone defaults and its cash flow exceeds the no tax level X_i^z, debtholders pay taxes on top of bankruptcy costs.

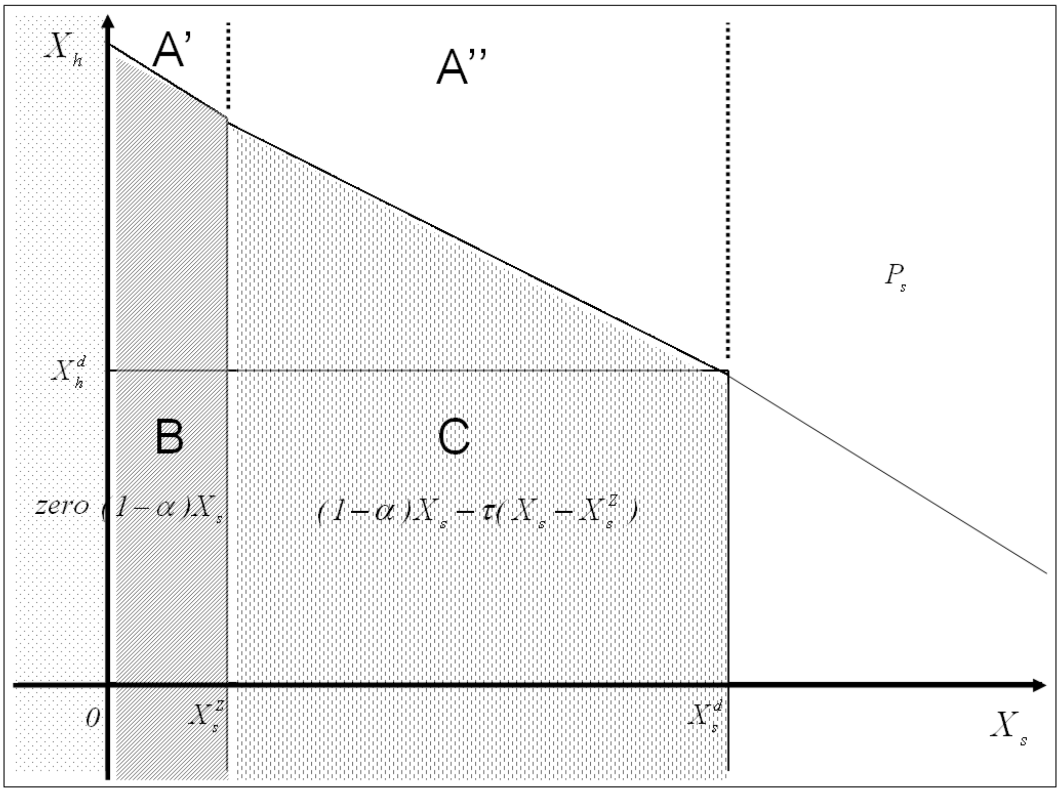

Figure 2: This figure represents the payoffs to subsidiary lenders as a function of the subsidiary cash flows (on the horizontal axis) and of the holding cash flows (on the vertical axis) for the case of infinitesimal ownership share. The figure reproduces the one for stand alone firms when X_h is lower than the holding default threshold X_h^d, as in this case the holding is unable to help its subisdiary. The area of the transifer is $A = A' \cup A''$. In A'', the subsidiary does not default thanks to the transfer, but it pays taxes. In A', the subsidiary saves on both default costs and taxes thanks to the transfer. The pale grey zone is event B, the dark grey is C in equation (27).

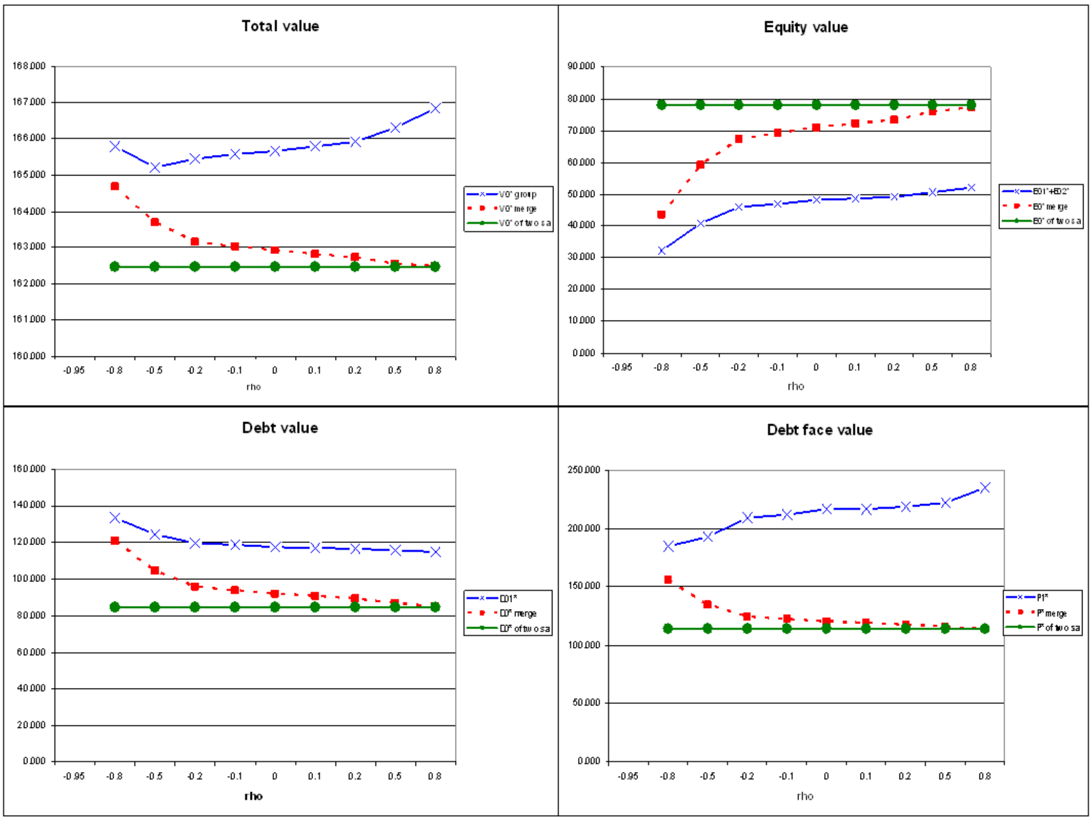

Figure 3: The upper left panel displays the value of a group (stars), a conglomerate (big and small dots) and two stand alone firms (dotted) as the correlation coefficient between the activities cash glows varies between -0.8 and +0.8. Similarly, the upper right panel displays the value of equity, the lower left panel the market value of debt and the last one the face value of debt.

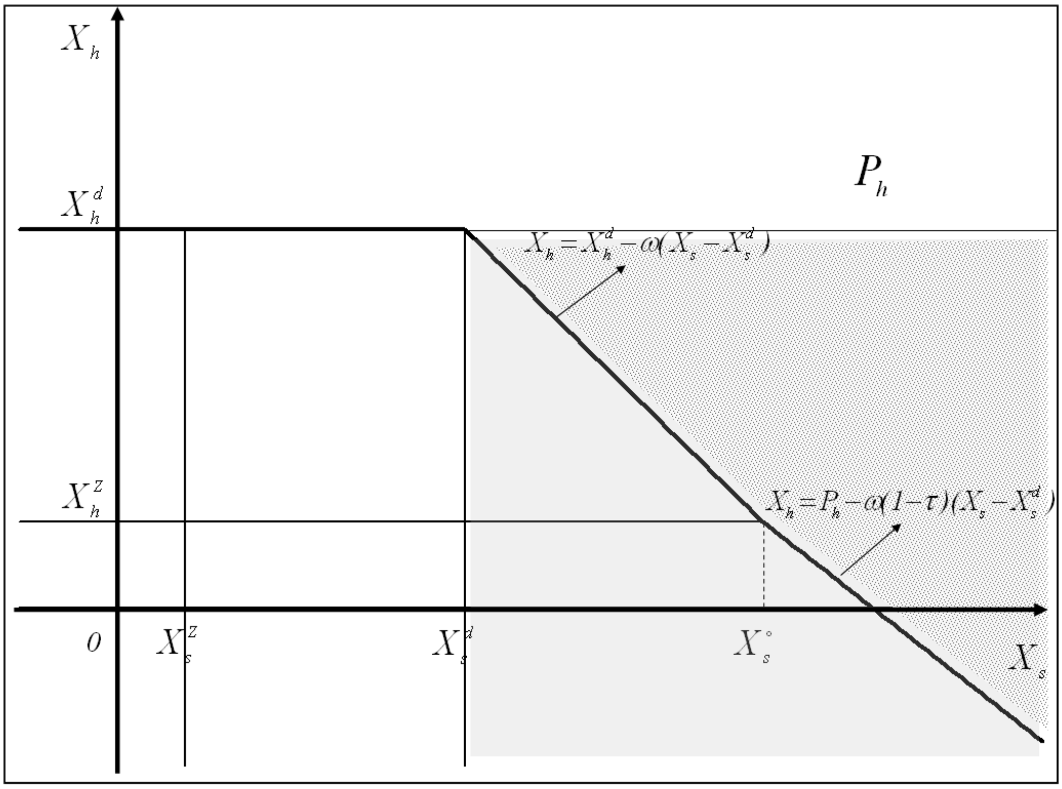

Figure 4: This figure represents the payoffs to holding lenders as a function of the subsidiary cash flows (on the horizontal axis) and of the holding cash flows (on the vertical axis) for the case of positive ownership share. Consider the shaded area where the holding would default ($X_h < X_h^d$) and the subsidiary does not default ($X_s < X_s^d$). The holding now receives dividends proportional to its ownership share, ω, that allow its survival in the shaded area with dots.

Table 1: Base Case Parameters

Variables	Symbols	Values
Annual Riskfree Rate	r	5.00%
Time Period/Debt Maturity (yrs)	T	5.00
T-period Riskfree Rate	$r_T = (1+r)^T - 1$	27.63%
Capitalization Factor	$Z = (1+r_T)/r_T$	4.62
Unlevered Firm Variables		
Expected Future Operational Cash Flow at T	Mu	127.63
Expected Operational Cash Flow Value (PV)	$X_0 = Mu/(1+r)^T$	100.00
Cash Flow Volatility at T	Std	49.19
Annualized Operational Cash Flow Volatility	$\sigma = Std/T^{0.5}$	22.00
Tax Rate	τ	20%
Value of Unlevered Firm w/Limited Liability	V_0	80.05
Value of Limited Liability	L_0	0.057

Table 2: Optimal Capital Structure and Value

	Symbols	Values				
		Stand Alone	Holding	Subsidiary	1/2 Group	1/2 Conglomerate
Default Costs	α	23%	23%	23%	23%	23%
Optimal Face Value of Debt	P^*	57.20	0	219	109.5	58.70
Default Threshold	X^{d*}	67.75	0	248.169	-	69.87
No Tax Profit Level	X^{Z*}	14.98	0	102.32	-	14.00
Value of Optimal Debt	D_0^*	42.22	0	116.68	58.34	44.70
Optimal Leverage Ratio	D_0^*/ν_0^*	52%	0	99.9%	70.3%	55%
Annual Yield Spread of Debt (%)	$(P^*/D_0^*)^{1/T} - 1 - r$	1.26%	//	8.4%	-	0.6%
Value of Optimal Equity	E_0^*	39.01	49.2	0.037	24.62	36.88
Optimal Levered Firm Value	$\nu_0^* = D_0^* + E_0^*$	81.23	49.2	116.71	82. 95	81.57
Tax Burden	T_0^*	17.62	19.95	5.42	12. 69	17.81
Tax Savings of Leverage	TS_0^*	2.33	0	14.53	7.27	2.19
Expected Default Costs	DC_0^*	0.90	0	7.98	3.99	0.62
Value of Optimal Leveraging	$\nu_0^* - V_0$	1.18	-30.60	36.91	3.15	1.53
Capitalized Value of Optimal Leverage	$Z(\nu_0^* - V_0)/V_0$	6. 81%	-1.77	2.14	18.2 4%	10.3%

Table 3: Asymmetric alfas: capital structure and value across organizational forms, $\rho = 0.2$

Variables	Symbols	Values					
		S. Alone	S. Alone	Holding	Subsidiary	1/2 Group	1/2 Conglomerate
Default Costs	α	75%	23%	75%	23%	23%	75%
Optimal Face Value of Debt	P^*	33	57.20	0	219	109.5	46.5
Default Threshold	X^{d*}	39.247	67.75	0	248.17	-	55. 43
No Tax Profit Level	X^{Z*}	8.01	14.98	0	102.32	-	10. 79
Value of Optimal Debt	D_0^*	24.99	42.22	0	116.68	58.34	35. 71
Value of Optimal Equity	E_0^*	55.84	39.01	49.2	0.037	24.62	45. 52
Optimal Levered Firm Value	$\nu_0^* = D_0^* + E_0^*$	80. 83	81.23	49.2	116.71	82. 95	81. 23
Optimal Leverage Ratio	D_0^*/ν_0^*	30.92%	52%	0	99.9%	70.3%	44%
Annual Yield Spread of Debt (%)	y	0.7%	1.26%	//	8.4%	-	0.4%
Tax Burden	T_0^*	18.76	17.62	19.95	5.42	12. 69	18. 31
Tax Savings of Leverage	TS_0^*	1.25	2.33	0	14.53	7.27	1. 69
Expected Default Costs	DC_0^*	0.46	0.90	0	7.98	3.99	0.455
Value of Optimal Leveraging	$\nu_0^* - V_0$	0.78	1.18	-	-	3.15	1.18
Capitalized Optimal Value of Leveraging	$Z(\nu_0^* - V_0)/V_0$	4. 50%	6. 81%			18.2 4%	6. 81%

Table 4: Asymmetric volatilities: capital structure and value across organizational forms, $\rho = 0.2, \sigma_s = 44\%, \sigma_h = 22\%$

Variables	Symbols	Values					
		S.A.($\sigma = 44\%$)	S. A.($\sigma = 22\%$)	Holding	Subsidiary	1/2 Group	1/2 Conglomerate
Default Costs	α	23%	23%	23%	23%	23%	23%
Optimal Face Value of Debt	P^*	83	57.20	0	223	111.5	59
Default Threshold	X^{d*}	95.19	67.75	0	248.169	-	34.75
No Tax Profit Level	X^{Z*}	34.25	14.98	0	102.32	-	8.50
Value of Optimal Debt	D_0^*	48.75	42.22	0	106.83	53.41	41.98
Value of Optimal Equity	E_0^*	36.10	39.01	60.29	3.01	31.65	39.64
Optimal Levered Firm Value	$\nu_0^* = D_0^* + E_0^*$	84.84	81.23	60.29	109.84	85.06	81.62
Optimal Leverage Ratio	D_0^*/ν_0^*	57.46%	52%	0	97.3%	62.8%	51.4%
Tax Burden	T_0^*	16.05	1.26%	19.95	7.01	13.48	17.45
Tax Savings of Leverage	TS_0^*	4.66	17.62	0	13.59	6.80	2.60
Expected Default Costs	DC_0^*	2.64	2.33	0	5.53	2.765	1.18
Annual Yield Spread of Debt (%)	y	6.2%	0.90	//	10.9%	-	2%
Value of Optimal Leveraging	$\nu_0^* - V_0$	4.79	1.18	-	-	5.26	1.58
Capitalized Value of Optimal Leveraging	$Z(\nu_0^* - V_0)/V_0$	27.64%	6.81%			30.45%	9.12%

Table 5: Asymmetric size: capital structure and value across organizational forms, $\rho = 0.2,\ V_{h0} = 167, V_{s0} = 33.$

Variables	Symbols	Values					
		S. Alone(1/3)	S. Alone(5/3)	Holding	Subsidiary	1/2 Group	1/2 Conglomerate
Default Costs	α	23%	23%	23%	23%	23%	23%
Optimal Face Value of Debt	P^*	19	95	63.33	121.33	92.33	57
Default Threshold	X^{d*}	22. 50	112. 54			-	67. 81
No Tax Profit Level	X^{Z*}	4.98	24.85			-	13. 765
Value of Optimal Debt	D_0^*	14.02	70.15	48.25	70.26	59.25	43.24
Value of Optimal Equity	E_0^*	13.04	65.24	47.15	0	23. 58	38. 255
Optimal Levered Firm Value	$\nu_0^* = D_0^* + E_0^*$	27.06	135.38	95.39	70.26	82. 83	81.49
Optimal Leverage Ratio	D_0^*/ν_0^*	51.8 1%	51.8 1%	50.57%	100%	71.54%	53.06%
Annual Yield Spread of Debt (%)	y	1.2%	1.2%	0.6%	6.5%	-	1.1%
Tax Burden	T_0^*	5.90	29.49	30.90	0.48	15. 69	17. 78
Tax Savings of Leverage	TS_0^*	0.77	3.87	2.35	6.17	8. 52	2.23
Expected Default Costs	DC_0^*	0.30	1.48	0.37	2.10	1.23	0.75
Value of Optimal Leveraging	$\nu_0^* - V_0$	0.38	1. 96	-	-	3.03	1.44
Capitalized Value of Optimal Leveraging	$Z(\nu_0^* - V_0)/V_0$	6. 58%	6. 79%			17.5 4%	8. 31%

Note: group principal is calculated as the sum of holding and subsidiary principals.

Table 6: Capital structure and value with constrained subsidiary, different values of rho

Variables	Symbols	ρ			
		-0.8	0	0.2	0.8
Face Value of Subsidiary Debt	P_{0s}^*	57.2	57.2	57.2	57.2
Face Value of Parent Debt	P_{0h}^*	57	56	54	51
Value of Subsidiary Debt	D_{0s}^*	42.09	41.44	40.12	38.11
Value of Parent Debt	D_{0h}^*	44.47	43.82	43.59	42.78
Levered Group Value	ν_{0g}^*	162.95	162.83	162.79	162.68
Group Leverage Ratio	D_{0g}^*/ν_{0g}^*	0.531	0.524	0.514	0.497
Subsidiary Optimal Leverage Ratio	D_{0s}^*/ν_{0s}^*	0.521	0.517	0.508	0.495
Equity Value of Parent	E_h^*	37.69	38.78	40.26	42.85
Equity Value of Subsidiary	E_s^*	38.70	38.79	38.82	38.93
Equity Value of Group	E_g^*	76.39	77.5680	79.0810	81.7880
Default Threshold	X_g^d	135.84	134.51	132.13	128.42
No Tax Profit Level	X_g^Z	28. 04	27. 94	27. 49	27. 31
Tax Burden	T_0	35.606	35.560	35.630	35.657
Parent Tax Savings of Leverage	TS_{0h}^*	2.3162	2.2619	2.1559	2.0028
Subsidiary Tax Savings of Leverage	TS_{0s}^*	1.9781	2.0784	2.1145	2.2400
Parent Expected Default Costs	DC_{0h}^*	0.8870	0.8338	0.7347	0.6026
Subsidiary Expected Default Costs	DC_{0s}^*	0.0002	0.1670	0.2303	0.5159
Parent Yield Spread	y_h	1.25%	1.21%	1.12%	1.00%
Subsidiary Yield Spread	y_s	0.16%	0.47%	0.58%	0.98%
Value of Optimal Leverage	$\upsilon_0^* - V_0$	3.35	3.23	3.19	3.08
Capitalized Value of Optimal Leverage	$Z(\upsilon_0^* - V_0)/V_0$	9. 70%	9.3 5%	9. 23%	8. 92%

Note: group figures obtain by summing up the holding and the subsidiary figures.