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Abstract

This paper investigates the optimal size and focus of a VC's portfolio. We identify three main effects of portfolio size on the VC's and entrepreneurs' incentives. The first one is a rent extraction effect: the VC can extract higher rents in a larger portfolio by being able to reallocate his limited resources from one start-up to another. All else constant, this effect leads to stronger incentives for the VC and weaker incentives for the entrepreneurs. The second effect is a resource allocation effect: by diversifying investment over a larger number of start-ups, the VC can reallocate his resources among portfolio companies in case of project failure, allowing him to earn a greater return on his investment. This effect leads to stronger incentives for both the VCs and the entrepreneurs. The third effect is a value dilution effect: a large portfolio requires the VC to spread his limited resources across a large number of start-ups, diluting his value-adding role, which has a negative impact on both the VC's and entrepreneurs' incentives.

We find that the VC optimally chooses a small portfolio when the economy is endowed with start-ups with strong fundamentals which have a low level of relatedness. Under these conditions, the VC finds it optimal to promote strong entrepreneurial incentives by concentrating his limited resources on a small number of start-ups. The VC expands his portfolio size when the fundamentals of the start-ups get weaker and the relatedness of start-ups in the economy increases. Under these conditions, the VC trades off the stronger entrepreneurial incentives of a small portfolio for the greater rent extraction ability offered by a large portfolio. We also show that because of the complementarity between the VC's investment incentives and entrepreneurial effort, a large portfolio may lead to better incentives for both the VC and the entrepreneurs. Finally, we show that the VC benefits from managing his portfolio strategically by divesting some of the portfolio companies early, since this strategy allows the VC to extract more surplus from the remaining start-ups in his portfolio.

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1 Introduction

This paper takes a portfolio approach to analyze the investment strategy of a venture capitalist (VC), and investigates the optimal size and scope of a VC's portfolio. There is a large literature in financial economics analyzing various aspects of VC investment.[2] This literature has so far concentrated primarily on the bilateral relationship between an entrepreneur and a VC, where the VC provides monetary and non-monetary resources to turn the entrepreneur's project idea into a viable business.[3] However, VC funds typically invest in more than one start-up company at any given time, and engage in active portfolio management to maximize the return from their investment.

We address the following questions: What determines the size of a VC's portfolio? What are the benefits and costs of having a small versus a large portfolio? What are the strategic aspects of managing a portfolio of start-ups rather than a single start-up? Do VCs prefer having a diversified portfolio as opposed to a focused one? How do size and focus of a VC's portfolio affect performance? Are entrepreneurs better off obtaining financing from a VC with a large or a small portfolio?

Both entrepreneurs and VCs provide critical inputs for the success of a start-up company. Providing VCs with appropriate incentives to add value to their portfolio companies may often be as important as providing incentives to entrepreneurs to exert effort. VCs add value by acquiring knowledge, human capital and skills that are very often specific to their portfolio companies. These investments lead to specialization in the VC industry, where different VCs invest in different sets of skills. For example, some VCs specialize only in certain technologies and industries and keep their investment strategy focused, whereas others diversify into different industries.[4] Furthermore, younger venture capitalists are less experienced, and add less value than more experienced ones. A recent comprehensive study by Bottazzi, Da Rin and Hellmann (2004) documents that more experienced and better educated VCs (i.e. VCs with greater human capital) take a more active role in the management of their portfolio companies, suggesting that VCs' human capital is an important value driver in venture capital investments. In a recent paper, Kaplan and Schoar (2004) find substantial heterogeneity in performance across private equity funds, and suggest that VCs with superior skills and greater human capital can generate better results in their investments. Kaplan and Schoar (2004) also find that better performing VC funds grow proportionally slower, and argue that better

[2]See for example Sahlman (1990), Gompers (1995), Gompers and Lerner (1999), Hellmann (1998), Repullo and Suarez (2004), Hellmann and Puri (2000), Kaplan and Stromberg (2003) and (2004), Casamatta (2003), among others.

[3]Inderst and Muennich (2004), Kanniainen and Keuschnigg (2003) and Bernile and Lyandres (2003) provide a notable exception.

[4]In a recent study, Cummings (2004) documents systematic variations in both size and the geographical and industrial composition of VCs' portfolios.

VCs may choose to stay small (by deliberately limiting the amount of capital raised) to avoid dilution from allocating their limited amount of human capital over a large number of start-ups.[5] In another recent paper, Gompers, Kovner, Lerner and Scharfstein (2004) document that specialized VCs exhibit different investment behavior and performance than less specialized VCs, due to their industry-specific experience and human capital. Overall, these findings suggest that VCs' human capital plays a key role in managing and adding value to start-ups. In turn, entrepreneurs value VCs' human capital, since a skilled VC can help turn their start-up into a more valuable company.

The aim of our paper is to investigate the role of scarce human capital on a VC's investment strategy, such as portfolio size and composition, and on the performance of the VC's portfolio. In our model, we consider a VC facing a large number of entrepreneurs with project ideas. The VC must decide on the number of start-ups to invest in his portfolio. We simplify the VC's problem by limiting his choice to one or two start-ups (or none). Each start-up has two stages. Success of the first stage depends on the effort exerted by the entrepreneur. The payoff from the start-up is affected by a project-specific investment made by the VC. Entrepreneurial effort and VC's investment are not observable by each other, generating a double-sided moral hazard problem. After observing whether a project has been successful or not during its first stage, the VC chooses whether to continue the project into its second stage, or to divest (for example, to liquidate) the project. If the VC divests one of the projects, he can transfer (albeit imperfectly) his human capital and resources from the divested start-up to the surviving one. The VC's ability to transfer his resources from one start-up to the other depends on the degree of his portfolio's focus.

Continuation of a project into its second stage requires the participation and effort of both the VC and the entrepreneur, and is not contractible ex-ante. This implies that each entrepreneur must bargain (at the interim stage) with the VC on the continuation of her start-up. The outcome of bargaining will depend on the number of successful start-ups that the VC has in his portfolio. We show that the presence of a second start-up, by creating an outside option when the VC bargains with each entrepreneur, allows him to extract more surplus from the portfolio companies.

We study how the composition (size and focus) of the VC's portfolio affects the incentives of the VC and of the entrepreneurs in his portfolio, as well as portfolio performance. We identify three effects of

[5] For anecdotal evidence on the relevance of fund size see The Economist, which wrote on April 2, 2005 that "some venture capital funds say they have turned away money from investors in order to keep fund sizes down to an amount that can be managed responsibly". Note that a small fund size would imply a portfolio with a small number of start-ups, since many VC funds have requirements limiting the amount of capital that could be invested in a given start-up company.

portfolio size on incentives. The first one is a *rent extraction effect*: a large portfolio allows the VC to extract more surplus from his start-up companies by creating competition among them for his limited resources and human capital. The second advantage for the VC of a large portfolio is a *resource allocation effect*: the VC enjoys having a large portfolio because, by investing in a large number of different start-ups, he increases (all else equal) the probability that at least one of the start-ups will be successful. In this way, by reallocating his resources from the unsuccessful start-up to the successful one, the VC obtains a greater chance to earn a return on his initial investment. Thus, the VC benefits from diversifying his investment over a large number of portfolio companies. Note that this result does not depend on risk aversion - our agents are all risk-neutral - but it depends critically on the VC's ability to reallocate ex-post his resources and human capital from one start-up to another after observing whether they have been successful or not. The disadvantage of a large portfolio is a *value dilution effect*: when the VC has a large portfolio, he must spread his limited resources among a greater number of start-ups, reducing the value he can add to each individual start-up. This effect derives from the VC's limited resources and human capital.

The net effect of size on incentives derives from the interaction of the three effects we have identified. Typically, a small portfolio provides better incentives to the entrepreneur and worse incentives to the VC. This happens because, in a small portfolio, the VC extracts lower rents from each entrepreneur, and is able to devote more resources to each individual portfolio company. Thus, the rent extraction and the value dilution effects are weaker in a smaller portfolio, resulting in stronger entrepreneurial incentives. We show that the VC optimally chooses a small portfolio when the start-up fundamentals are strong, the relatedness of start-ups in the economy is low and the VC's bargaining power is low. The VC finds it optimal to have a large portfolio when start-up fundamentals are moderate, when the relatedness of start-ups in the economy is higher and the VC has high bargaining power.

An interesting implication of our model is that under certain conditions, entrepreneurs benefit from belonging to a large portfolio, rather than a small one, even if this means that the VC can extract more surplus from them. The complementary roles of the entrepreneurs and the VC for the start-up's success potential generate a strategic complementarity between entrepreneurial effort and the VC's investment incentives: entrepreneurs exert more effort if they expect the VC to make the start-up specific investment for their start-up, and the VC is more willing to make the start-up specific investments when he expects a high level of entrepreneurial effort. When the effect of complementarity dominates the rent extraction and the value dilution effects, the level of entrepreneurial effort will be higher in a large portfolio than in a small portfolio. Thus, both the VC and the entrepreneurs may be better-off with a large portfolio rather

than with a smaller one. We show that this possibility emerges when the entrepreneurs have start-ups of moderate value and the VC holding a small portfolio cannot extract sufficient surplus to compensate him for his initial investments.

Relatedness of portfolio companies and thus the degree of portfolio focus (scope) affect the trade-off between having a large or a small portfolio. A high level of relatedness between portfolio companies allows the VC to reallocate resources more efficiently from one start-up to another. The VC benefits from a high level of focus in two different ways. First, when one of the start-ups fails, the VC transfers his resources and human capital to the successful start-up. The higher the level of focus, the higher the efficiency of the reallocation of resources. This implies that a greater level of focus reduces the inefficiency associated with spreading the VC's initial investment across several start-ups, increasing the benefits of the resource allocation effect. It also suggests that larger and more focused portfolios are optimal in the case of risky start-ups investing in related technologies with high uncertainty and failure rates.

The second benefit of focus is that it allows the VC to extract greater rents from his portfolio companies. This happens because a high degree of focus increases the value of the VC's outside option while he bargains with the entrepreneurs. Thus a higher degree of focus increases competition among portfolio companies and allows the VC to extract greater rents, increasing his investment incentives. Giving stronger incentives to the VC is important, for example, when the VC can add substantial value to his portfolio companies. This suggests that more experienced and better skilled VCs manage more focused portfolios.

Our model also implies that VCs actively manage their portfolios. We show that the VC with a large portfolio may find it optimal to divest one of his portfolio companies early, even if the company's early stage performance is positive. Early disposition of a start-up may result in the termination of an otherwise potentially viable venture, or in its sale to another VC fund, or in an early Initial Public Offer (IPO). This portfolio management strategy is desirable from the VC's point of view, since divesting a start-up allows the VC to add more value to the start-ups remaining in his portfolio and to extract more surplus from them. We find that the strategy of early divestiture may be optimal in cases where the portfolio companies have a high degree of relatedness and therefore, the VC has a focused portfolio. Thus, our paper offers a rationale for the IPO decision that is different from the traditional explanations, as the ones discussed in Subrahmanyam and Titman (1999) and Chemmanur and Fulghieri (1999), among others.

Our work is related to several papers in the recent literature. The closest paper to ours is Inderst and Muennich (2004). This paper shows that VCs may benefit from limiting the amount of capital they raise by having "shallow pockets," since competition for limited funds gives better incentives to

the entrepreneurs in their portfolios, even if it allows the VC to extract more surplus. Similarly, in our paper competition between entrepreneurs for the VC's resources (i.e. his human capital) allows the VC to extract more surplus from his portfolio companies. However, in our paper high rent extraction ability of the VC affects incentives through different dynamics. In Inderst and Muennich (2004) competition among portfolio companies is exploited by the VC to improve entrepreneurial incentives, but the VC's human capital and value-adding investment incentives are not considered. In our paper competition between start-ups worsens entrepreneurial incentives (all else equal), but it improves incentives for the VC, since it allows him to extract more surplus from the entrepreneurs. Competition may have a positive effect on entrepreneurial incentives only indirectly, through the complementarity of the VC's investment incentives and the entrepreneurs' effort. Another difference is that in Inderst and Muennich (2004), raising ex-ante limited funds commits the VC to divest one of the successful start-ups. In contrast, in our paper, the VC may be ex-post better off by divesting one of the start-ups, as early divestiture allows him to extract more surplus from the entrepreneur remaining in his portfolio. Finally, and most importantly, the main objective of our paper is to investigate the optimal size and focus of a VC's portfolio. Inderst and Muennich (2004) abstract from determining the optimal size of the VC's portfolio - the initial number of start-ups in the portfolio is taken as given - and do not consider the benefits and costs of having a focused portfolio, a key feature of our model.

Our paper is also related to the work by Kanniainen and Keuschnigg (2003), further extended by Bernile and Lyandres (2003). In these papers a VC has limited resources that he can devote to his portfolio companies, and adding an additional start-up to the portfolio always weakens both the VC's and the entrepreneurs' incentives. In our model, adding a new start-up induces competition among portfolio companies and allows the VC to extract more surplus at the bargaining stage. Due to the complementarity between entrepreneurial effort and VC's investment incentives, we show that a large portfolio may result in stronger incentives and be beneficial for both the entrepreneurs and the VC. In addition, in our model, the VC's ability to extract surplus depends on the degree of relatedness of the portfolio companies, and thus portfolio focus. Differently from Kanniainen and Keuschnigg (2003) and Bernile and Lyandres (2003) we derive the optimal size of a VC's portfolio by analyzing the combined impact of portfolio size and focus on incentives.

Our work also contributes to the literature stressing the active role of VCs in adding value to their portfolio companies, such as Casamatta (2003), Michelacci and Suarez (2004), and Repullo and Suarez (2004), among others. The main difference of our paper from this literature is that these papers consider

the incentive problems between a VC and an entrepreneur in isolation from the other start-ups in the VC's portfolio. In our paper, we consider the management of the VC's overall portfolio by focusing explicitly on the interactions between portfolio companies.

Finally, our paper relates to the literature on internal capital markets (see, for example, Gertner, Scharftein and Stein, 1994). This literature examines the role of internal capital markets in reallocating resources from one division to another and how resource reallocation affects incentives. In our paper, we show that a VC with a large portfolio, using his ability to reallocate resources from one start-up to another, creates competition among portfolio companies for his limited amount of resources. We show that due to the complementarity between entrepreneurial effort and VC's investment, competition can have positive or negative effects on incentives, depending on parameter values.

The paper is organized as follows. In Section 2, we describe our basic model. In Section 3, we examine the case where the VC has only one start-up. In Section 4, we study the case where the VC has multiple (two) start-ups. In Section 5, we determine the optimal portfolio size and derive the comparative static results of the model. In Section 6, we discuss the case in which the VC engages in active portfolio management, by selectively divesting one of his portfolio companies. In Section 7, we present the empirical implications of our model. Section 8 concludes. All proofs are in the Appendix.

2 The model

We consider an economy endowed with two types of risk neutral agents: venture capitalists (VCs) and entrepreneurs. Entrepreneurs are endowed with a project idea which can be turned, with the collaboration of a VC, into a final marketable product. VCs provide capital as well as other value adding activities for turning entrepreneurs' ideas into viable businesses.[6] We assume that the VCs are a scarce resource in the economy, and that they have access to an infinite supply of entrepreneurs with project ideas. This assumption reflects the notion that it takes time and experience to accumulate skills and human capital to become a VC.[7]

Entrepreneurs' project ideas can be turned into a final product in two stages.[8] The outcome of the first stage is either a success or a failure. If the first stage is successful, then the project is developed and

[6] The role of VCs as providers of capital and expertise to their portfolio companies is examined in several papers in the literature. See, for example, Kortum and Lerner (2000), Hellmann and Puri (2002), and Casamatta (2003).

[7] For example, on 27 November 2004, The Economist wrote: "perhaps there are simply just a few people in private equity who are very much better at it than their rivals" in explaining the substantial performance gap across private equity funds.

[8] Note that in the rest of the paper we will use the terms start-ups, projects and portfolio companies interchangeably.

commercialized during the second stage. If it is a failure, it has no value and is abandoned.

There are four dates in our economy, with no discounting between the dates. At $t = 0$, the VC chooses the number η of start-up companies to invest in his portfolio. He may invest in either one or two start-ups, or he may decide to make no investment; thus, $\eta \in \{0, 1, 2\}$. The development of each start-up requires the active involvement of both the VC and the entrepreneur. At $t = 1$, the VC makes a non-contractible start-up specific investment at a personal fixed cost of c, with $c > 0$. This investment can be interpreted as the cost of setting up the fund, developing the necessary business contacts useful in managing the portfolio companies, and acquiring all the project-specific skills and human capital that add value to a portfolio company.[9] For short, we will refer to these efforts as the VC's start-up specific investment.[10] If the VC makes no investment, the payoff of a given start-up is zero. Thus, the VC's investment and entrepreneurial effort are complementary inputs for the success of the start-up.

Entrepreneurs play a critical role during both the first and the second stage of a project. At $t = 1$, after observing the number of portfolio companies the VC has invested in his portfolio, each entrepreneur exerts effort p, at a cost of $\frac{k}{2}p^2$.[11] The parameter k measures the cost of exerting effort, with $k > 1$. Entrepreneurial effort determines the success probability of the first stage of the project, which becomes known at $t = 2$. Entrepreneurs' projects can be successful only if the VC makes the initial start-up specific investment at $t = 0$. Thus, entrepreneurs cannot start their projects at $t = 1$ without a VC, and then seek the involvement of a VC for the second stage, if the project is successful.

If the first stage of a given project is a failure, the project is terminated and both the VC and the entrepreneur obtain zero payoffs. If the first stage is a success, the second stage needs the active participation and effort of both the VC and the entrepreneur.[12] If neither the entrepreneur nor the VC participate to the second stage, the project is divested. We normalize the project's payoff to zero when divested. We assume contracts are incomplete in that it is not possible to contract ex-ante on the participation of either the entrepreneur or the VC to the second stage of the project. This assumption implies that both the VC and the entrepreneur can withdraw *at will* their involvement and human capital from the project at the second

[9] The importance of the VC's human capital in managing and adding value to portfolio companies is documented in Bottazzi, Da Rin and Hellmann (2004).

[10] Note that, for simplicity, we do not explicitly consider the VC's monetary investment into the start-ups. Such an expenditure can be thought of as being a part of the VC's initial fixed investment cost c. Alternatively, our analysis could easily be modified to incorporate explicitly an initial monetary outlay for each project.

[11] Note also that, while the entrepreneurial effort p is modelled as a continuous variable, with $p \in [0, 1]$, VC's input is a binary choice between making the initial investment, and thus paying the cost c, or not. We make these assumptions for analytical tractability, but the main results of our paper would also hold in a more general (but less tractable) model where the VC has a continuous choice variable as well.

[12] We can interpret the first stage of the project as the production of a prototype that can be turned into a final product, in the second stage, only with the active involvement of both the entrepreneur and the VC that have developed it.

stage. Note that it is quite plausible to assume, particularly in the context of VC investment, that neither the VC nor the entrepreneur(s) can commit ex-ante to the continuation of the project during its second stage. VCs very often finance entrepreneurs endowed with a project idea surrounded by great uncertainty. Not only it is very difficult to describe the final outcome of the project ex-ante, but also decisions about the level of resources to be allocated to the project, the level of VC's and entrepreneurs' involvement, and the contingencies (such as the state and progress of the project) under which such resources are made available to the entrepreneur are very often impossible to contract upon ex-ante.[13]

The entrepreneur's and VC's ability to withdraw their human capital from the project implies that their payoffs (the division of the surplus between them) from the implementation of the project cannot be contractually determined at the outset of the game, $t = 0$, but is determined at the interim stage, at $t = 2$, through bargaining. This also implies that contracts written ex-ante between the VC and the entrepreneur(s) on how to share the final surplus, such as equity contracts (or options, as in Noldeke and Schmidt, 1998), are ineffective since both the entrepreneur and the VC can (unilaterally) withdraw their participation and human capital from the implementation phase of the project.[14] Even if the VC and entrepreneur wrote, at the outset of the venture, a sharing rule on the final payoff of the start-up, the inability to contract ex-ante on the conditions specifying participation of the entrepreneur and the VC to the second stage of the project implies that any pre-existing sharing rule can be renegotiated away, and the division of the surplus is determined entirely by interim bargaining.[15] Thus, conditional on observing a successful outcome for the first stage of the project, at $t = 2$ the VC and the entrepreneur bargain over their compensation for the continuation of the start-up. The outcome of the bargaining process, which is described below, determines the allocation of the surplus between the VC and the entrepreneur, and thus affects their incentives.[16]

At $t = 3$, the payoff from the project is realized and distributed between the VC and the entrepreneur(s). The payoff realized at $t = 3$ depends on the number of start-ups the VC invested in his portfolio. If the VC invests in only one start-up (and he makes at $t = 1$ the start-up specific investment) the payoff from the start-up, if successful in the first stage and continued during its second stage, is 2Δ. If the VC invests

[13]Thus, contracts are incomplete in the sense of Hart and Moore (1990) and Grossman and Hart (1986). We recognize that contracts are a very important aspect of venture capital financing in real life. Kaplan and Stromberg (2003) document that VCs indeed use complex contracts designed to mitigate adverse selection, moral hazard, and hold-up problems. The main assumption of our paper is that, after all these contractual features are accounted for, VC contracts contain a significant degree of residual incompleteness and therefore are subject to renegotiation.

[14]For a further discussion on the role of employment *at will* and renegotiation on surplus allocation, see Stole and Zwiebel (1996a) and (1996b).

[15]Note also that such renegotiations occur very often in the real world (see, for example, Kaplan and Stromberg, 2003 and 2004).

[16]Note also that we rule out ex-ante transfers from one party to another at the beginning of the game.

in two start-ups (and he makes the start-up specific investment for each company in his portfolio), each start-up, if successful in its first stage and continued into its second stage, generates a payoff of Δ.[17] If one of the start-ups fails in its first stage, the VC can concentrate all his resources and human capital exclusively on the successful start-up, obtaining a payoff equal to $(1+\phi)\Delta$, with $0 \leq \phi \leq 1$. The value of the parameter ϕ depends on the ability of the VC to transfer the start-up-specific investment that he has made from one start up to the other. Thus, ϕ depends on the degree of relatedness of the two start-ups, and we will interpret it as representing the degree of "focus" or scope of the VC's portfolio.

In the following sections, we will first analyze the case where the VC invests in one start-up only and, next, we will examine the case where the VC invests in two start-ups. We will then characterize the VC's optimal choice of portfolio size.

3 The VC invests in one start-up

This section considers the case in which the VC invests in only one portfolio company, $\eta = 1$. We proceed backward, and first characterize the surplus allocation between the VC and the entrepreneur through interim bargaining. We next examine the ex-ante effort choice of the entrepreneur.

We model the bargaining game between the entrepreneur and the VC as a standard alternating-offers game, where the two parties make alternating offers under the threat that bargaining may break down with a certain exogenous probability.[18] If bargaining breaks down both the VC and the entrepreneur receive their outside options with a value that we have normalized to zero.[19] As the probability that bargaining breaks down tends to zero, one can show that the payoff of the subgame perfect equilibrium of the bargaining game between the VC and the entrepreneur is such that the VC and the entrepreneur receive a fraction $\frac{\lambda}{1+\lambda}$ and $\frac{1}{1+\lambda}$, respectively, of the surplus that they jointly generate, where we denote with $\lambda > 0$ the VC's bargaining power. This implies that the VC's payoff from the project, l^1_{VC}, is

[17] Note that we assume that the VC's human capital investment in the case of one or two start-ups have the same cost c. This assumption captures the notion that the VC has only limited time and resources at his disposal, and that he cannot expand his investment proportionally when he has two start-ups in his portfolio rather than only one. Thus, given limited resources, the VC faces two choices: he can either concentrate all his resources and human capital on only one start-up, or spread his resources and human capital over two start-ups, incurring the same cost c in each choice. This assumption also implies that, by incurring the fixed cost c, the VC can obtain a total potential payoff for his portfolio of 2Δ, which is then divided among the number of start-ups in the portfolio. This "linear" payoff structure implies that none of our results are driven by the presence of economies or diseconomies of scale in the VC's "production technology."

[18] See, for example, Binmore, Rubinstein and Wolinski (1986).

[19] Inderst and Muller (2004) and Michelacci and Suarez (2004) examine the effect of the presence of non-zero outside options for both entrepreneurs and VCs.

λ-multiple of the entrepreneur's payoff, $2\Delta - l_{VC}^1$, that is

$$\lambda(2\Delta - l_{VC}^1) = l_{VC}^1, \tag{1}$$

yielding

$$l_{VC}^1 = \frac{\lambda}{1+\lambda} 2\Delta. \tag{2}$$

Note that the share of the VC's surplus is an increasing function of his bargaining power λ.

Anticipating her expected share of the surplus from bargaining, the entrepreneur determines her level of effort p by maximizing her expected profit given by

$$\max_p \quad \pi_{EN}(1) \equiv p\frac{2\Delta}{1+\lambda} - \frac{k}{2}p^2. \tag{3}$$

Correspondingly, the VC's expected profit, is given by

$$\pi_{VC}(1) \equiv p\frac{\lambda 2\Delta}{1+\lambda} - c. \tag{4}$$

The following proposition characterizes the optimal effort level and the expected profits for this subgame.

Proposition 1 *If the VC makes the required start-up specific investment at $t = 1$, the optimal level of effort exerted by the entrepreneur is*

$$p^*(1) \equiv \frac{2\Delta}{(1+\lambda)\,k}. \tag{5}$$

The corresponding expected profits for the entrepreneur and the VC are

$$\pi_{EN}^*(1) = \frac{2\Delta^2}{(1+\lambda)^2\,k}, \tag{6}$$

$$\pi_{VC}^*(1) = \frac{4\Delta^2\lambda}{(1+\lambda)^2\,k} - c. \tag{7}$$

From (5) and (6), it can be immediately seen that both the level of entrepreneurial effort and the entrepreneur's profits are increasing functions of the project's payoff, Δ, and decreasing functions of the effort cost, k, and of the VC's bargaining power, λ. The impact of the VC's bargaining power λ on his profits, instead, is ambiguous and depends on its magnitude.

Lemma 1 *The VC's profits are increasing in his bargaining power λ for $\lambda \leq 1$ and decreasing in λ for*

$\lambda > 1$.

The ambiguity of the effect of the bargaining power on the VC's profits reflects the dual effect of λ on the VC's profits. The bargaining power λ affects both the VC's rent extraction ability and the level of entrepreneurial effort, two determinants of the VC's profits. A high level of bargaining power λ means a high rent extraction ability for the VC, but a low level of effort from the entrepreneur. When $0 < \lambda \leq 1$, the entrepreneur has a high bargaining power and she can extract large rents from the VC, leading to a high level of effort. In this case, an increase in the VC's bargaining power benefits the VC, since he can extract greater rents with only a minor impact on entrepreneurial effort. For $\lambda > 1$, the VC can extract large rents from the entrepreneur, leading to a low level of entrepreneurial effort. An increase in the level of VC's bargaining power deteriorates even further entrepreneur's incentives to exert effort, lowering the success probability of the start-up. Since the VC's profits depend directly on the success of the start-up in his portfolio, this is detrimental to the VC, and his expected profits decrease in λ.

Given (7), the VC has an incentive to make the start-up specific investment only if he expects positive expected profits. The following lemma characterizes the VC's decision.

Lemma 2 *The VC makes the start-up specific investment if and only if* $\Delta \geq \Delta_m(\lambda) \equiv \frac{1+\lambda}{2}\sqrt{\frac{ck}{\lambda}}$.

If the VC does not make the investment at $t = 1$, the payoff from the project is zero, and the entrepreneur does not exert any effort at all. In this case both parties obtain zero profits.

4 The VC invests in two start-ups

This section considers the case in which the VC invests in two start-ups, $\eta = 2$. We proceed again backward, and first characterize the outcome of the interim bargaining game between the VC and the two entrepreneurs. We next examine the effect of the surplus allocation resulting from interim bargaining on the ex-ante effort choice of the entrepreneurs.

4.1 Bargaining with two entrepreneurs

The bargaining process between the VC and the entrepreneurs, and therefore the allocation of the surplus among the parties, depends on whether only one or both projects have a successful outcome at their first stage at $t = 2$. There are three different possible cases (states of the world): (i) both projects are successful

in their first stage, state SS, (ii) one project is successful while the other one is a failure, state SF,[20] (iii) both projects are a failure, state FF.

We begin our analysis with the (simpler) case where only one start-up, say start-up i, is successful while the other, start-up j is a failure, state SF. In this case, the VC can concentrate his human capital exclusively to the successful start-up, increasing its payoff from Δ to $(1+\phi)\,\Delta$. Furthermore, since only one start-up in the VC's portfolio is successful, we assume that the entrepreneur and the VC engage in the same bargaining process with no outside options that we have described in the previous section. Thus, as before, the VC's and the entrepreneurs' payoffs, denoted by $l_{VC}(SF)$, $l_{EN_i}(SF)$ and $l_{EN_j}(SF)$, are now given, respectively, by

$$
\begin{aligned}
l_{VC}(SF) &= \frac{\lambda\,(1+\phi)\,\Delta}{1+\lambda} \equiv \bar{l}^{i}_{VC}, && (8)\\
l_{EN_i}(SF) &= (1+\phi)\Delta - l_{VC}(SF) = \frac{(1+\phi)\,\Delta}{1+\lambda}, && (9)\\
l_{EN_j}(SF) &= 0. && (10)
\end{aligned}
$$

Consider next the case where both projects have a successful outcome at the end of the first stage, state SS. In this case, the VC has two successful start-ups in his portfolio and he will bargain with both entrepreneurs. We model this process of "multilateral" bargaining between the VC and the two entrepreneurs in the same way as in Stole and Zwiebel (1996a). We assume that the VC leads individual bargaining sessions with one start-up at a time, starting, say, with entrepreneur i. Individual bargaining is modeled again as an alternating offer game, where bargaining may break down with a certain exogenous probability. If the VC and entrepreneur i reach an agreement, the VC starts a round of bargaining with entrepreneur j. If bargaining between the VC and entrepreneur i breaks down without an agreement, entrepreneur i drops from the bargaining process and the VC engages in bargaining with entrepreneur j, where both players have zero outside options. Equilibrium payoffs are subject to the (stability) condition that if the VC reaches an agreement with entrepreneur i, and bargaining with entrepreneur j breaks down, then entrepreneur j drops from the bargaining process and the VC and entrepreneur i renegotiate their original agreement through bargaining, where now both players have zero outside options. This condition ensures that the agreement reached between the VC and entrepreneur i (resp. j) anticipates the renegotiation that would take place if bargaining between entrepreneur j (resp. i) and the VC breaks

[20] Note that, given that the two entrepreneurs are identical, it is irrelevant which one of the two projects is successful. Thus, we will treat these two separate but symmetric cases effectively as a single case.

down.

We determine the payoffs from the overall multilateral bargaining process as follows. Let λ denote again the VC's bargaining power. Note that if bargaining with entrepreneur i (resp. j) breaks down, the VC has the option to bargain with entrepreneur j (resp. i), obtaining a payoff equal to $\bar{l}^j_{VC}$ (resp. $\bar{l}^i_{VC}$), defined in (8). Thus, $\bar{l}^j_{VC}$ (resp. $\bar{l}^i_{VC}$) represents the value of the VC's outside option while bargaining with entrepreneur i (resp. j). This implies that the surplus that the VC can extract from each entrepreneur in state SS, denoted by $l^i_{VC}(SS)$, $i = 1, 2$, must satisfy the following conditions (see Stole and Zwiebel, 1996a):

$$\lambda(\Delta - l^1_{VC}(SS)) = l_{VC}(SS) - \bar{l}^2_{VC}, \tag{11}$$

$$\lambda(\Delta - l^2_{VC}(SS)) = l_{VC}(SS) - \bar{l}^1_{VC}. \tag{12}$$

Condition (11) requires that the VC's incremental payoff from continuing start-up 1 in addition to start-up 2, given by the RHS of (11), is λ-multiple of entrepreneur 1's payoff from her start-up, given by $\Delta - l^1_{VC}(SS)$. The same condition holds for start-up 2 as well, giving (12). Solving the system of equations (11) and (12) for $l^1_{VC}(SS)$ and $l^2_{VC}(SS)$ leads to the following lemma.

Lemma 3 *The VC's and the entrepreneurs' payoffs in state SS, are equal to:*

$$l_{VC}(SS) \equiv l^1_{VC}(SS) + l^2_{VC}(SS) = \frac{(2+\lambda+\phi)\,\lambda}{(2+\lambda)\,(1+\lambda)} 2\Delta, \tag{13}$$

$$l_{EN,i}(SS) \equiv \Delta - l^i_{VC}(SS) = \frac{(2+(1-\phi)\lambda)\,\Delta}{(2+\lambda)\,(1+\lambda)},\ i = 1, 2. \tag{14}$$

Note that, by direct comparison of the VC's payoffs (13) with (2) and (8), it is easy to see that when the VC has a second (successful) start-up in his portfolio he obtains a fraction $\frac{(2+\lambda+\phi)\lambda}{(2+\lambda)(1+\lambda)}$ of the total surplus, which is always greater than the fraction $\frac{\lambda}{1+\lambda}$ that he obtains when he has only one (successful) start-up. Hence, the presence of a second (successful) start-up in the portfolio provides the VC with an outside option while bargaining with each start-up, and allows him to extract a greater fraction of the surplus from the entrepreneurs. Thus, the presence of a second start-up, and the ability to transfer resources from one start-up to the other, creates "competition" between entrepreneurs, allowing the VC to extract more surplus from his portfolio companies. The VC's ability to transfer ex-post resources from one start-up to another is critical, since when $\phi = 0$ the VC always extracts the same fraction of the surplus, given by

$\frac{\lambda}{1+\lambda}$, independent of the number of entrepreneurs he bargains with.[21]

The following lemma examines the effect of the VC's bargaining power, λ, and of the degree of portfolio focus, ϕ, on individual payoffs obtained in the SS state.

Lemma 4 *The VC's surplus from multilateral bargaining in state SS, $l_{VC}(SS)$, is an increasing function of project payoff, Δ, of his bargaining power, λ, and of the degree portfolio focus, ϕ. Each entrepreneur's surplus, $l_{EN,i}(SS), i = 1, 2$, is an increasing function of project payoff, Δ, and a decreasing function of the VC's bargaining power, λ, and the level of portfolio focus, ϕ.*

An increase in the level of project payoff, Δ, always increases the VC's and the entrepreneurs' payoffs. A greater level of the VC's bargaining power λ benefits the VC at the bargaining stage and, as expected, it hurts the entrepreneurs. Similarly, a greater level of portfolio focus ϕ benefits the VC but hurts the entrepreneurs. This property is due to the fact that a greater level of focus leads, in state SS, to a greater value of the outside option for the VC while he bargains with each entrepreneur. A greater outside option increases the VC's rent-extraction ability and allows him to extract a greater fraction of the total surplus from the entrepreneurs.

Note also that the VC's payoff is always (weakly) greater in the SS state than in the SF state, that is, $l_{VC}(SS) \geq l_{VC}(SF)$. Conversely, a successful entrepreneur's payoff is always greater in the SF state than in the SS state, that is, $l_{EN_i}(SF) \geq l_{EN_i}(SS)$. These properties reflect the fact that in the SF state the VC has only one successful start-up and thus no outside option, limiting his ability to extract surplus from the successful entrepreneur.

Finally, if both entrepreneurs fail in the first stage, state FF, both start-ups are terminated and all agents obtain zero payoffs.

4.2 The choice of effort by the entrepreneurs

If the VC makes the specific investment for each start-up, anticipating her payoffs from bargaining in the SS and SF states, given respectively by (14) and (9), entrepreneur i determines her effort level by maximizing her expected profits, given by

$$\max_{p_i} \ \pi_{ENi}(2) \equiv p_i p_j \frac{(2 + (1-\phi)\lambda)\,\Delta}{(2+\lambda)\,(1+\lambda)} + p_i(1-p_j)\frac{(1+\phi)\,\Delta}{1+\lambda} - \frac{k}{2}p_i^2, \quad i = 1, 2. \tag{15}$$

[21] Note also that when $\lambda = 1$ (that is, when the VC and the entrepreneurs have the same bargaining power) the payoffs (13) and (14) are the same as the players' Shapley value of the corresponding cooperative bargaining game.

Similarly, using the VC's payoffs from bargaining in the SS, SF (and FS) states given in (13) and (8), respectively, the VC's expected profit is given by:

$$\pi_{VC}(2) \equiv p_i p_j \frac{(2+\lambda+\phi)\,2\Delta\lambda}{(2+\lambda)\,(1+\lambda)} + p_i(1-p_j)\frac{(1+\phi)\,\lambda\Delta}{1+\lambda} + p_j(1-p_i)\frac{(1+\phi)\,\lambda\Delta}{1+\lambda} - c; i,j=1,2; i \neq j. \quad (16)$$

The first-order condition of (15) is given by

$$p_i(p_j) = \frac{(2+\lambda)\,(1+\phi) - 2\phi p_j\,(1+\lambda)}{(2+\lambda)\,(1+\lambda)\,k}\Delta. \quad (17)$$

Note that the first-order condition (17) implies that effort level exerted by entrepreneur i decreases in the effort level exerted by entrepreneur j and hence, the effort levels are strategic substitutes. This happens because, when the VC has two start-ups in his portfolio, he extracts higher surplus from entrepreneur i in the SS than in the SF state. Thus, competition between entrepreneurs, which is created by the VC's ability to reallocate his limited resources from one successful start-up to the other, makes entrepreneurial effort levels strategic substitutes, reducing entrepreneurial incentives to exert effort.

The following proposition and lemmas characterize the Nash-equilibrium level of effort and equilibrium payoffs in this subgame.

Proposition 2 *If the VC has made at $t = 1$ start-up specific investment for each start-up, the Nash-equilibrium level of efforts is given by*

$$p_1^* = p_2^* \equiv p^*(2) \equiv \frac{(2+\lambda)\,(1+\phi)\Delta}{(1+\lambda)\,((2+\lambda)k + 2\Delta\phi)}. \quad (18)$$

The corresponding level of expected profits for the VC and the entrepreneurs are given by

$$\pi_{VC}^*(2) = \frac{2\lambda\,(2+\lambda)\,((2+\lambda)k+\Delta\phi)\,(1+\phi)^2\,\Delta^2}{(1+\lambda)^2\,((2+\lambda)k+2\Delta\phi)^2} - c, \quad (19)$$

$$\pi_{EN1}^*(2) = \pi_{EN2}^*(2) = \frac{k}{2}\left(\frac{(2+\lambda)\,(1+\phi)\,\Delta}{(1+\lambda)\,((2+\lambda)k+2\Delta\phi)}\right)^2. \quad (20)$$

Lemma 5 *The Nash-equilibrium level of entrepreneurial effort $p^*(2)$ is increasing in the level of project payoff, Δ, and the level of portfolio focus, ϕ, and decreasing in the cost of effort, k, and the VC's bargaining power, λ.*

An increase in the project's payoff, Δ, increases overall surplus and always leads to a greater level of

entrepreneurial effort. The focus parameter ϕ has two opposing effects on the level of effort chosen by the entrepreneur. On the one hand, a higher degree of focus between the start-up companies allows the VC to extract more surplus from the entrepreneurs in the SS state, with a negative effect on entrepreneurial effort. On the other hand, a more focused portfolio allows the VC to reallocate more efficiently his resources to the successful start-up in the SF state, where only one of the start-ups has been successful in the first stage. Thus, the greater ability of the VC to add more value to the successful venture in this state has a positive effect on entrepreneurial effort. As it turns out, the second, value enhancing effect, dominates the first, rent extraction effect, and the overall impact of an increase in focus on the level of effort is always positive. Finally, as expected, entrepreneurial effort is a decreasing function of both the cost of providing effort, k, and the VC's bargaining power, λ.

Lemma 6 *The entrepreneurs' and the VC's expected profits are increasing in project payoff, Δ, and the degree of portfolio focus ϕ, and are decreasing in the entrepreneur's cost of exerting effort, k. Moreover, the entrepreneurs' expected profits are decreasing in the VC's bargaining power λ. The VC's payoff is increasing in his bargaining power, λ, if and only if $\lambda < \frac{\sqrt{9k^2+8k\phi\Delta}-k}{2k}$.*

Note that, similarly to the single start-up case discussed in Lemma 1, the VC benefits from having a greater bargaining power only when his bargaining power is not too high. When the VC has a relatively low bargaining power, $\lambda < \frac{\sqrt{9k^2+8k\phi\Delta}-k}{2k}$, he can extract a low surplus from the entrepreneurs, providing them with powerful incentives to exert effort. In this case, an increase in the bargaining power λ benefits the VC from a greater surplus extraction ability with a relatively small adverse effect on entrepreneurial effort and probability of success. When, instead, the VC has a high bargaining power, $\lambda > \frac{\sqrt{9k^2+8k\phi\Delta}-k}{2k}$, the negative effect on incentives of a further increase in λ overshadows the positive effect on the VC's rent extraction ability, making the VC worse off.

4.2.1 The impact of portfolio size on entrepreneurial effort

Entrepreneurial incentives to exert effort and, in turn, the VC's investment incentives depend on the number of start-ups in the VC's portfolio. An important question in the analysis that follows is whether the entrepreneurs have better incentives to exert effort when the VC has one or two start-ups in his portfolio.

Lemma 7 *If the VC is induced to make the necessary start-up specific investments, an entrepreneur*

always has greater incentives to exert effort when his start-up is the only one in the VC's portfolio:

$$p^*(1) > p^*(2).$$

Furthermore, the difference between the levels of effort, $p^*(1) - p^*(2)$, increases in project payoff, Δ, and decreases in the degree of focus, ϕ, in the VC's bargaining power, λ, and in the entrepreneur's cost of effort, k.

Note that each entrepreneur has an incentive to exert effort only if she expects the VC to make the start-up specific investment. In turn, the VC is willing to make the start-up specific investments only if he expects a positive profit, net of his total investment cost c. If the VC has sufficient incentives to make the start-up specific investments with either one or two start-ups, entrepreneurial incentives to exert effort are always worse when the VC has two start-ups rather than only one. This is due to the fact that, when the VC has two start-ups in his portfolio, he adds less value to each start-up and is able to extract more surplus from the entrepreneurs. Thus, conditional on the VC making the start-up specific investments, entrepreneurial incentives are always worse when the start-ups belong to a large portfolio, leading to lower effort. Furthermore, the difference between the level of effort in the two cases is increasing in the project payoff, Δ. This property is due to the fact that each entrepreneur benefits less from an increase in the project payoff when the VC has two portfolio companies, since the VC can extract a greater fraction of the surplus from the entrepreneurs. Also, the difference between the level of effort in the two cases is decreasing in the level of focus, ϕ. This property can be seen by noting that, from Lemma 5, an increase in the degree of focus, ϕ, increases $p^*(2)$, while it has no effect on $p^*(1)$, which reduces the difference between the two effort levels. An increase in the VC's bargaining power λ always reduces entrepreneurial effort, but relatively less when the VC has two portfolio companies. This happens because an increase in the VC's bargaining power has a greater impact on the VC's rent extraction ability in a small portfolio than in a large portfolio, since the VC is already extracting greater rents from the entrepreneurs in a large portfolio. Finally, an increase in the cost of effort, k, always reduces entrepreneurial effort, but relatively more when the VC has only one portfolio company.

5 The optimal size of the VC's portfolio

The VC chooses the size of his portfolio, that is, whether to invest in one or two start-ups, as a result of the interaction of three distinct effects. The first one is the *rent extraction* effect: the VC can extract more surplus when he has two start-ups in his portfolio than only one. Thus, this effect always induces the VC to prefer (all else equal) a larger portfolio, rather than a smaller one. The second one is the *resource allocation effect*: by investing in two, rather than only one portfolio company, the VC may increase the expected value of his portfolio. This effect depends critically on the VC's ability to reallocate ex-post his resources and human capital from one start-up to the other. Therefore, the strength of this effect depends on the degree of focus of the portfolio, ϕ. Note that, if the success probability of each start-up is fixed and the same whether the VC has one or two start-ups, this effect always leads the VC to prefer a large portfolio to a small one.[22] The third effect is the *value dilution* effect: a larger portfolio requires the VC to spread his fixed amount of resources and human capital over a larger number of companies. The result is that the VC adds lower value to each start-up in a large portfolio: he will add only Δ to each portfolio company in the SS state, and $(1+\phi)\Delta$ in the SF state. Thus, the value dilution effect favors a smaller portfolio.

The optimal portfolio size depends on the interaction of the three effects we have identified, and on their impact on the VC's and the entrepreneurs' incentives, as follows. First, in a large portfolio, the rent extraction effect impacts entrepreneurial incentives negatively and the VC's investment incentives positively. A focused portfolio strengthens this effect, since focus allows the VC to extract more surplus from the start-ups. Second, in a large portfolio, dilution from spreading the VC's resources and human capital over two start-ups lowers the payoff from exerting effort, and thus reduces both entrepreneurial effort and the VC's investment incentives. Third, a large portfolio allows the VC to reallocate his human capital from one start-up to another, in case one of the start-ups fails, affecting both the VC's and entrepreneurs' incentives positively. The resource reallocation effect is stronger the higher the level of the portfolio focus.

Finally, complementarity between entrepreneurial effort and the VC's investment incentives creates a positive externality. The VC is more willing to make the initial start-up specific investment when he

[22] This property can be seen as follows. If the VC has only one start-up, and the entrepreneur exerts effort p, total expected value of the VC's portfolio is $p2\Delta$. If the VC has two start-ups in his portfolio, and each entrepreneur exerts effort q, the total expected value of the VC's portfolio is $2q^2\Delta + 2q(1-q)(1+\phi)\Delta = 2q\Delta + 2q(1-q)\phi\Delta$. It is easy to see that the expected value of the VC's portfolio is larger with two start-ups than with only one when $\phi > \frac{p-q}{q(1-q)}$, which is always the case when $p = q$ and $\phi > 0$. Note, however, that in our analysis, p and q are determined endogenously as a function of portfolio size and focus.

expects a higher level of entrepreneurial effort. Similarly, each entrepreneur responds positively to the VC's willingness to make the start-up specific investment by exerting a higher level of effort. Hence, both the VC and the entrepreneurs may be better off with a large portfolio than with a small portfolio.

The following proposition characterizes the optimal portfolio size (see Figure 1).

Proposition 3 *There are critical values* $\{\phi^M, \Delta_1^M, \Delta_2^M\}$ *(defined in the appendix) such that the VC's optimal portfolio composition is as follows:*

i) the VC does not invest in any start-up if $0 \leq \Delta < \Delta_1^M(\phi, \lambda, k)$;

ii) the VC invests in one start-up if $0 \leq \phi < \phi^M$ *and* $\Delta \geq \Delta_m(\lambda)$;

iii) the VC invests in two start-ups if $\phi^M \leq \phi \leq 1$ *and* $\Delta_1^M(\phi, \lambda, k) \leq \Delta < \Delta_2^M(\phi, \lambda, k)$, *and in one start-up if* $\phi^M \leq \phi \leq 1$ *and* $\Delta \geq \Delta_2^M(\phi, \lambda, k)$.

Furthermore, $\frac{\partial \Delta_1^M(\phi,\lambda,k)}{\partial \phi} \leq 0$ *and* $\frac{\partial \Delta_2^M(\phi,\lambda,k)}{\partial \phi} > 0$.

There are four relevant cases in our model, which are generated by the VC's incentives to exert effort in a small and in a large portfolio (identified in Figure 1 as Case 1 through Case 4). In Case 1, we have that $\pi_{VC}^*(1) < 0$ and $\pi_{VC}^*(2) < 0$: the VC does not have sufficient incentives to make the start-up specific investments with either one or two start-ups in his portfolio. Thus, entrepreneurs as well will not exert any effort and the project opportunities will not be exploited, even in those cases where undertaking the projects is socially optimal. This case occurs when the potential payoff from the project, Δ, and the relatedness of the start-ups in the economy, ϕ, are so low that the VC cannot extract sufficient rents to be induced to make the initial start-up specific investments. Formally, this case arises when $\Delta < \Delta_1^M$.

The second and third cases are symmetric. In Case 2, $\pi_{VC}^*(1) \geq 0$ but $\pi_{VC}^*(2) < 0$: the VC has sufficient incentives to make the start-up specific investment if he undertakes only one start-up, but he will not make these investments if he undertakes two start-ups. In Case 3, $\pi_{VC}^*(1) < 0$ but $\pi_{VC}^*(2) \geq 0$: the VC will be willing to make the start-up specific investments if he undertakes two start-ups, but not if he undertakes one start-up only.

Case 2 emerges when the payoff from the project is sufficiently large, $\Delta \geq \Delta_m$, but the relatedness of the two portfolio companies is low, $0 \leq \phi < \phi^M$. The intuition for this result is as follows. From Lemma 7, we know that the larger the value of Δ, the greater the differential between the levels of entrepreneurial effort in a small portfolio and in a large one, which makes a small portfolio more attractive. Furthermore, a lower

value of ϕ decreases the VC's rent extraction ability, reduces the efficiency of reallocating resources among portfolio companies, and thus reduces the benefits of a large portfolio. Therefore, in Case 2, expected profits with two start-ups are not sufficient to induce the VC to make the start-up specific investments, whereas investing in only one start-up gives sufficient incentives to the VC to induce his initial investment. Note also that a high value of Δ, by providing stronger entrepreneurial incentives, increases VC's expected profits even though in a smaller portfolio the VC has a lower rent extraction ability.

Case 3 is the mirror image of Case 2. In this case the VC is motivated to make start-up specific investment only if he has two start-ups in his portfolio, but not when he has only one. This case occurs when the payoff from the project is moderate, $\Delta_1 \leq \Delta < \Delta_m$, and the relatedness of the potential portfolio companies is relatively large, $\phi \geq \phi^M$. As discussed in Case 1 above, when $\Delta < \Delta_m$, with only one start-up the VC cannot obtain sufficient profits to compensate him for the cost of making the initial investment. Anticipating that the VC will not be willing to make any investment, the entrepreneur does not exert any effort either. Non-contractibility of VC's effort and the inability of the two parties to reallocate ex-ante the surplus from the project make it impossible for the entrepreneur to compensate the VC for his valuable start-up specific investment. Thus, the project is not undertaken in a small portfolio, even if it is potentially profitable. A large portfolio, instead, allows the VC to extract more rents from each entrepreneur and to benefit from ex-post resource reallocation allowed by a large portfolio, inducing him to make the initial start-up specific investment. Note that, for a given Δ, this case occurs when the VC can invest in start-ups which are sufficiently related, so that the degree of focus of the resulting portfolio is sufficiently large, $\phi \geq \phi^M$. As discussed above, greater portfolio focus enhances the positive rent extraction effect, reduces the negative dilution effect, and increases the strength of the resource reallocation effect. Hence, now the VC is willing to make the initial start-up specific investments. Anticipating the improved incentives of the VC, the entrepreneurs exert effort. Thus, they too are better off than if each entrepreneur were the only start-up in the VC's portfolio.

In the last case, Case 4, we have that $\pi^*_{VC}(1) \geq 0$ and $\pi^*_{VC}(2) \geq 0$: the VC has sufficient incentives to make the initial investment with either one or two start-ups. Now, the VC selects the number of portfolio companies that maximizes his overall profit. When the degree of relatedness of the start-ups is low, i.e. when $0 \leq \phi < \phi^M$, the VC invests in one portfolio company only (provided that the project payoff is sufficiently large). In this case, adding a second start-up to the VC's portfolio increases only modestly the VC's rent extraction ability and generates a limited value of reallocation effect (which are both increasing in ϕ) while, from Lemma 7, it results in a significant reduction in entrepreneurial effort. Thus, the VC

prefers a smaller portfolio, and he optimally invests in only one start-up.

When the degree of relatedness of the start-ups is sufficiently large (and therefore the resulting portfolio is sufficiently focused), that is, when $\phi^M < \phi \leq 1$, the VC invests in two start-ups when project payoff is moderate, that is, when $\Delta_m \leq \Delta < \Delta_2^M$, and in one start-up when project payoff is sufficiently high, that is, when $\Delta \geq \Delta_2^M$. Remember again that, from Lemma 7, the difference in effort levels in a small and large portfolio, $p^*(1) - p^*(2)$, increases in the project payoff, Δ and decreases in portfolio focus, ϕ. This implies that for moderate values of Δ and high values of ϕ, the VC benefits from the greater rent extraction ability and the reallocation effect provided by a large portfolio, with only a limited negative impact on entrepreneurial incentives. Thus, the advantages of a large portfolio dominate those of a small portfolio and the VC optimally invests in two start-ups. Note also that the threshold level $\Delta_2^M(\phi, \lambda, k)$ is increasing in the level focus ϕ, which implies that the VC is more likely to form a large portfolio when he can invest in start-ups with a greater degree of focus.

When the payoff from the project, Δ, is sufficiently large, that is, when $\Delta \geq \Delta_2^M$, the difference between the level of entrepreneurial effort in large and small portfolio, $p^*(1) - p^*(2)$, becomes so large that the VC prefers to forego the higher rent extraction ability and the resource reallocation benefits of a large portfolio and, rather, to invest only in one start-up to obtain strong entrepreneurial incentives. Another way to look at this result is that the VC's greater rent extraction ability in a large portfolio affects entrepreneurial effort and the quality of each start-up (that is, their first-stage success probability) so negatively that the VC prefers to give up the higher rents and the resource reallocation benefits of a large portfolio in order to provide the entrepreneur with stronger incentives (and thus to benefit from a greater success probability of the start-up) by holding a small portfolio.

The following proposition examines the effect of the VC's bargaining power, λ, and the cost of effort, k, on optimal portfolio size.

Proposition 4 *We have the following comparative statics results:*

i) $\frac{\partial \Delta_2^M(\phi,\lambda,k)}{\partial \lambda} > 0.$

ii) $\frac{\partial \Delta_2^M(\phi,\lambda,k)}{\partial k} > 0.$

Proposition 4, part (i), shows that an increase in the level of the VC's bargaining power leads the VC to form a larger portfolio. From Lemma 7 we know that the difference in effort levels, $p^*(1)-p^*(2)$, decreases in λ. This implies that an increase in the VC's bargaining power decreases (all else equal) the advantage of

a small portfolio in terms of better entrepreneurial incentives. Thus, the VC finds the large portfolio more attractive. Part (ii) of Proposition 4 reveals that an increase in the entrepreneurs' cost of exerting effort, k, makes a large portfolio more desirable. An increase in the value of k reduces entrepreneurial effort, leading to a lower success probability for each start-up and to a riskier portfolio (higher failure probability). Moreover, the benefits of a small portfolio in terms of strong entrepreneurial incentives become weaker, because the effort advantage of a small portfolio over a large portfolio, $p^*(1) - p^*(2)$, decreases in k as established in Lemma 7. Thus, the VC will expand the size of his portfolio in order to maximize the probability that at least one of the start-ups will be successful.

6 Portfolio Management

In this section we show that when the VC invests in two start-ups, he may find it optimal to divest one of the start-ups early, even if it has been successful in its first stage. We will show that this strategy can be optimal from the VC's point of view, since the possibility of divesting one of the start-ups allows the VC to extract more surplus from the one remaining in his portfolio. Thus, the VC can increase the return on his portfolio by engaging in active portfolio management and divesting one of the start-ups.[23]

We modify our basic model as follows. Consider the case in which the VC invests in two start-ups and both entrepreneurs have a successful first stage, state SS. The VC now faces two choices: either to continue both start-ups, or to divest one of them and to dedicate himself entirely to the remaining one. If the VC chooses not to continue a successful start-up, he can divest it, for example through a sale to another VC (or some private buyer), or even take it public in an IPO. We assume that the proceeds from divesting the start-up are lower than the payoff that can be obtained if the original VC continues the start-up (given by Δ) and, for simplicity, are normalized to zero.[24]

The VC now has the option to bargain with only one entrepreneur for the continuation of only her start-up and the termination of the other, which we will denote as *bilateral* bargaining, or to engage, as before, in multilateral bargaining with both entrepreneurs for the continuation of both start-ups. This choice is important because the VC may be able to extract a different surplus depending on whether he continues one or both start-ups.[25]

[23] For an interesting example of VCs' active engagement in portfolio management see The Economist, November 27, 2004, which wrote that "Google's founders would have preferred to wait longer to do their IPO, but had to rush it because venture capitalists, including Kleiner Perkins, wanted to cash in."

[24] This assumption reflects that the incumbent VC, because of the initial specific investment he made, can generate a greater payoff than the one that can be obtained upon divestiture.

[25] Note that the results of this section depend on the difference in the surplus that the VC can obtain by continuing one or

6.1 Bilateral bargaining

We model the process of bilateral bargaining between the VC and the entrepreneur in state SS as follows. The VC selects, with equal probability, one of the two successful start-ups, say start-up i, and negotiates with entrepreneur i the payoff that he will receive for his exclusive participation to the continuation of start-up i only.[26] Bilateral bargaining is modeled again as the one in which the two parties make alternating offers under the threat that bargaining may break down with a certain exogenous probability. If bargaining with start-up i breaks down, the VC has the option to go to the start-up j and start a new round of bargaining with entrepreneur j. Thus, the payoff from bargaining with start-up j, which is again equal to $\bar{l}_{VC}^{j}$ (defined in (8)), represents the VC's outside option while bargaining with start-up i. Both entrepreneurs, instead, have again zero outside options. As the probability that the bargaining process breaks down tends to zero, the payoff of the subgame perfect equilibrium of the bargaining game between the VC and entrepreneur i is such that the VC and the entrepreneur receive the value of their outside options (the value that they can obtain in case of a breakdown in the bargaining game) plus a fraction $\frac{\lambda}{1+\lambda}$ and $\frac{1}{1+\lambda}$, respectively, of the surplus that they jointly generate, net of the sum of their outside options.[27] Thus, the surplus that the VC can extract through bilateral bargaining with start-up i in state SS, defined as $l_{VC}^{B}(SS)$, satisfies

$$\lambda\left[(1+\phi)\Delta - l_{VC}^{B}(SS)\right] = l_{VC}^{B}(SS) - \bar{l}_{VC}^{j}. \tag{21}$$

Solving (21) for $l_{VC}^{B}(SS)$, we obtain the VC's and entrepreneur i's payoffs as follows:

$$\begin{aligned} l_{VC}^{B}(SS) &\equiv \bar{l}_{VC}^{j} + \frac{\lambda}{1+\lambda}\left[(1+\phi)\Delta - l_{j}^{B}\right] = \frac{(2+\lambda)\,\lambda\,(1+\phi)\,\Delta}{(1+\lambda)^{2}}, && (22)\\ l_{EN_i}^{B}(SS) &\equiv (1+\phi)\Delta - l_{VC}^{B}(SS) = \frac{(1+\phi)\,\Delta}{(1+\lambda)^{2}}, && (23)\\ l_{EN_j}^{B}(SS) &= 0. && (24) \end{aligned}$$

two start-ups, and, therefore, they depend on the specific bargaining games that the VC can play with the entrepreneurs in these two cases.

[26] This may be achieved, for example, by negotiating an agreement, at the bargaining stage, between the VC and the entrepreneur that limits the VC's ability to participate to other projects. This implies that the VC can *commit not to* participate to the continuation of the other project (and thus, effectively, to divest it). The VC's ability to make such a commitment at this stage of the game (after the realization of the state of the world) is a weaker requirement than the assumption that the VC can, at the beginning of the game, *commit to* continue a project under predetermined circumstances, a possibility that we have ruled out. Note also that the use of such provisions is very common in stock purchase agreements. For a discussion of covenants in shareholders agreements, see Gompers and Lerner (1996) and Chemla, Habib and Ljungqvist (2002).

[27] See, again, Binmore, Rubinstein and Wolinski (1986).

Note that the total surplus that the VC and entrepreneur i can generate from start-up i is now $(1+\ \phi)\Delta$, which depends on the degree of focus ϕ of the VC's portfolio. This is due to the fact if the VC divests start-up j, he can reallocate his resources and human capital to start-up i, increasing its value potential by $\phi\Delta$.

An important question that follows is whether the VC can obtain a greater payoff by continuing both start-ups or by divesting one of them and continuing only the remaining one. This choice depends on whether the VC can extract more surplus by engaging in multilateral or bilateral bargaining with the entrepreneurs. The surplus that the VC can extract by engaging in either multilateral or bilateral bargaining is determined by the structure of the respective bargaining (sub)games. As a result, the VC may (under the conditions detailed below) prefer to engage in bilateral bargaining and to divest one the successful start-ups, if he will be able to extract more surplus in this way than he would obtain by continuing both start-ups.[28] The next proposition examines the VC's choice.

Proposition 5 *The VC prefers continuation of both projects (and thus engages in multilateral bargaining) if* $\phi \leq\ \bar{\phi} \equiv \frac{\lambda(2+\lambda)}{\lambda^2+2\lambda+2}$, *and he prefers continuation of only one project (and thus engages in bilateral bargaining) if* $\phi >\ \bar{\phi}$. *Correspondingly, the VC prefers continuation of both start-ups if* $\lambda \geq\ \bar{\lambda} \equiv \frac{\sqrt{1-\phi^2}-(1-\phi)}{(1-\phi)}$, *and he prefers continuation of only one start-up if* $\lambda <\ \bar{\lambda}$.

The factors affecting the choice between continuing only one start-up (and thus engaging in bilateral bargaining), or continuing both start-ups (and thus engaging in multilateral bargaining) can be summarized as follows.

Note first that when the VC divests one of the start-ups, from (22), he will receive a fraction $\frac{(2+\lambda)\lambda}{(1+\lambda)^2}$ of the total surplus given by $(1+\phi)\,\Delta$. When the VC continues both start-ups, and thus engages in multilateral bargaining with both entrepreneurs, from (13), he will receive a fraction $\frac{(2+\lambda+\phi)\lambda}{(2+\lambda)(1+\lambda)}$ of the total surplus given by 2Δ. By direct comparison, it is easy to see that $\frac{(2+\lambda)\lambda}{(1+\lambda)^2} > \frac{(2+\lambda+\phi)\lambda}{(2+\lambda)(1+\lambda)}$, for all $0 \leq \phi \leq 1$. This implies that, by divesting one of the start-ups in his portfolio, the VC maximizes his rent extraction ability by increasing competition between entrepreneurs for his resources. Thus, the VC can increase his rents by actively managing his portfolio and divesting one of the start-ups.

[28] In other words, the VC may be willing to inefficiently terminate one of the two start-ups if, given the structure of the respective bargaining games, he cannot internalize a sufficiently large part of the efficiency gains that can be obtained by continuing both start-ups. The question of whether the VC can internalize, through multilateral bargaining, a sufficient portion of the efficiency gains to induce him to always make the socially efficient decision is essentially an empirical one. Anecdotal evidence that VCs engage in socially inefficient practices while actively managing their portfolios in order to maximize the return from their investment is provided, for example, in the case of Google mentioned earlier (see footnote 23). Thus, our model may shed light on such instances where VCs make inefficient decisions to maximize their own welfare.

The VC's decision on whether or not to divest one of the two successful start-ups is based on the following trade off. If the VC chooses to continue only one start-up, he will be able to extract a greater fraction of the surplus from the surviving one, but at the cost of reducing the total value of his portfolio from 2Δ to $(1+\phi)\Delta$. If the VC continues both start-ups, he must now reach an agreement with both entrepreneurs, which will reduce the fraction of the combined surplus that he can obtain from the two entrepreneurs. The advantage of continuing both projects, is that the VC can realize the full value of its portfolio, given by 2Δ. Note that even if divesting one of the start-ups is socially inefficient, it may still be optimal for the VC to do so, since in this way he can extract a higher portion of the value of the start-up that is continued.[29]

Proposition 5 states that when the loss from divesting one of the projects is not too large, that is, when $\phi > \bar{\phi}$, the VC finds it optimal to exploit the better bargaining position afforded by bilateral bargaining, and thus prefers to continue one start-up only. When there is no value loss in reallocating resources from one start-up to the other, that is, when $\phi = 1$, the VC always prefers to divest one of the two start-ups. When, instead, the loss from divesting one project is sufficiently large, $\phi \leq \bar{\phi}$, the VC prefers to give up the better bargaining position provided by bilateral bargaining in order to realize the full potential of his portfolio by continuing both start-ups.

Proposition 5 also reveals that the VC always continues both start-ups when his bargaining power is sufficiently high, that is, when $\lambda \geq \bar{\lambda}$. The reason is that, when the VC already has a high level of bargaining power, the incremental rents that he obtains from one start-up by terminating the other is limited. Hence, in this case, the VC is better off by keeping both start-ups in his portfolio and not incurring any efficiency losses from early termination. In contrast, when the VC has low bargaining power, he prefers to inefficiently divest one of the two start-ups to be able to extract more surplus from the remaining one.

6.2 The optimal size of the VC's portfolio

We now determine the VC's ex-ante choice of the number of companies η to include in his portfolio. If $\phi \leq \bar{\phi}$, the VC will choose in the SS state to continue both start-ups, and Proposition 3 will remain valid. Thus, in the remainder of the analysis, we will assume that $\phi > \bar{\phi}$, and we will characterize the VC's optimal choice of portfolio size when he divests one of the successful start-ups in the SS state. We

[29] Note that this feature of our model is reminiscent of the overemployment result in Stole and Zwiebel (1996b), which shows that a given firm may find it optimal to overemploy (hire a higher number of employees than the first best) to increase its rent extraction ability in the intrafirm wage negotiations with its employees. In our model, the VC may invest in two start-ups ex-ante and terminate one of the start-ups inefficiently ex-post in order to extract higher rents from the remaining start-up.

assume now that, in the SS state, the VC chooses with equal probability one of the successful start-ups, and engages in bilateral bargaining with the corresponding entrepreneur. In the SF and FF states, the game unfolds as before. Proceeding backward, and conditional on the VC making the start-up specific investment for each start-up at $t = 1$, anticipating her payoffs from bargaining in the SS and SF states, given respectively by (23) and (9) entrepreneur i, $i = 1, 2$, determines her level of effort p_i by maximizing her expected profit, given by:

$$\max_{p_i} \ \pi^B_{ENi}(2) \equiv \ p_i p_j \frac{1}{2} \frac{(1+\phi)\,\Delta}{(1+\lambda)^2} + p_i(1-p_j)\frac{(1+\phi)\,\Delta}{1+\lambda} - \frac{k}{2} p_i^2. \tag{25}$$

Correspondingly, using the VC's payoffs from bargaining in the SS, SF (and FS) states, given respectively in (22) and (8), the VC's expected profit is obtained by:

$$\pi^B_{VC}(2) \equiv p_i p_j \frac{(2+\lambda)(1+\phi)\,\lambda\Delta}{(1+\lambda)^2} + p_i(1-p_j)\frac{(1+\phi)\,\lambda\Delta}{1+\lambda} + p_j(1-p_i)\frac{(1+\phi)\,\lambda\Delta}{1+\lambda} - c; i,j = 1,2; i \neq j. \tag{26}$$

The first-order condition of (25) is given by

$$p^B_i(p_j) = \frac{1}{2}\,(1+\phi)\,\frac{2(1+\lambda) - p_j\,(1+2\lambda)}{(1+\lambda)^2\,k}\Delta\ . \tag{27}$$

Inspection of (27) reveals that the effort level exerted by entrepreneur i decreases in the effort level exerted by entrepreneur j, and hence, the effort levels are again strategic substitutes.

The following proposition and lemma characterize the Nash-equilibrium level of effort and equilibrium payoffs in this subgame.

Proposition 6 *The Nash-equilibrium level of efforts is given by*

$$p^{B*}_i = p^{B*}_j = p^{B*}(2) \equiv \frac{(1+\lambda)(1+\phi)\,2\Delta}{2k(1+\lambda)^2 + (1+2\lambda)(1+\phi)\Delta}. \tag{28}$$

The corresponding level of expected profits for the VC and the entrepreneurs are given by

$$\pi^{B*}_{VC}(2) = \frac{4\lambda\,(1+\lambda)\,(2k(1+\lambda) + (1+\phi)\Delta)\,(1+\phi)^2\,\Delta^2}{(2k(1+\lambda)^2 + (1+2\lambda)(1+\phi)\Delta)^2} - c, \tag{29}$$

$$\pi^{B*}_{ENi}(2) = \frac{2k\,((1+\lambda)\,(1+\phi)\,\Delta)^2}{(2k(1+\lambda)^2 + (1+2\lambda)(1+\phi)\Delta)^2}; \quad i = 1,2. \tag{30}$$

Lemma 8 *The entrepreneurs' and the VC's expected profits are increasing functions of both project payoff*

Δ and the degree of portfolio focus ϕ, and are decreasing functions of the entrepreneur's cost of exerting effort k. Moreover, the entrepreneurs' expected profits are a decreasing function of the VC's bargaining power λ. The VC's payoff is an increasing function of his bargaining power if and only if $\lambda < \sqrt{1 + \frac{(1+\phi)\Delta}{2k}}$.

Lemma 8 mirrors Lemma 6. Both the VC and the entrepreneurs benefit from a greater payoff from the project, Δ, and a greater portfolio focus, ϕ, and are hurt by an increase in the cost of effort, k. Note that, again, the VC benefits from an increase in his bargaining power only when his bargaining power λ is not too high, that is, when $\lambda < \sqrt{1 + \frac{(1+\phi)\Delta}{2k}}$. When the VC has low bargaining power, entrepreneurial incentives are sufficiently strong; thus, an increase in the bargaining power of the VC will increase the VC's rent extraction ability with a relatively smaller negative impact on entrepreneurial incentives. Conversely, when the VC has a high bargaining power, entrepreneurial incentives are already low, and an increase in the VC's bargaining power will hurt the VC.

We can now characterize the VC's ex-ante choice of portfolio size, as follows.

Proposition 7 *Let $\bar{\phi} \leq \phi \leq 1$ and $c < \bar{c}(\lambda, k)$ (defined in the appendix). There are critical values $\{\Delta_1^B, \Delta_2^B\}$ (defined in the appendix) such that the VC's optimal portfolio size is as follows:*

i) the VC does not invest in any start-up if $0 \leq \Delta < \Delta_1^B(\phi, \lambda, k)$;

ii) the VC invests in two start-ups if $\Delta_1^B(\phi, \lambda, k) \leq \Delta < \Delta_2^B(\phi, \lambda, k)$, and in one start-up if $\Delta \geq \Delta_2^B(\phi, \lambda, k)$.

Furthermore, $\frac{\partial \Delta_1^B(\phi,\lambda,k)}{\partial \phi} \leq 0$ and $\frac{\partial \Delta_2^B(\phi,\lambda,k)}{\partial \phi} > 0$.

Proposition 7 mirrors Proposition 3. When project payoff is very low, that is, when $\Delta < \Delta_1^B(\phi, \lambda, k)$, the VC does not have sufficient incentives to invest in start–up specific human capital, and the projects are not undertaken. When project payoff is moderate, that is, when $\Delta_1^B(\phi, \lambda, k) \leq \Delta < \Delta_2^B(\phi, \lambda, k)$, the VC chooses a larger portfolio by investing in two portfolio companies. Note that now, differently from the case discussed in Proposition 3, when both start-ups are successful (state SS) the VC will divest one of the two portfolio companies. When the project's payoff is sufficiently large, $\Delta \geq \Delta_2^B(\phi, \lambda, k)$, the VC chooses a smaller portfolio by investing in only one start-up.

It is interesting to compare Proposition 7 with Proposition 3 to understand how the optimal portfolio size changes when the VC engages in portfolio management. Our last proposition examines the impact of VC's portfolio management activity on the optimal portfolio size. We address this question in the

(analytically tractable) case where entrepreneurs and the VC have the same bargaining power, that is, when $\lambda = 1$.

Proposition 8 *Let $\bar{\phi} \leq \phi \leq 1$, $\lambda = 1$, and $c < \bar{c}(1, k)$. There is a $\hat{\phi}$ such that:*

$$i) \text{ if } \hat{\phi} < \phi \leq 1 : \Delta_1^B(\phi, 1, k) < \Delta_1^M(\phi, 1, k) \text{ and } \Delta_2^B(\phi, 1, k) > \Delta_2^M(\phi, 1, k);$$

$$ii) \text{ if } \bar{\phi} \leq \phi \leq \hat{\phi} : \Delta_1^B(\phi, 1, k) \geq \Delta_1^M(\phi, 1, k) \text{ and } \Delta_2^B(\phi, 1, k) \leq \Delta_2^M(\phi, 1, k) \ .$$

Proposition 8 reveals that the impact of portfolio management on the ex-ante choice of portfolio size depends on the relatedness of the start-ups. When start-ups are highly related, that is, when $\hat{\phi} < \phi \leq 1$ (and therefore the VC can form a portfolio with a high degree of focus), we obtain that $\Delta_2^B(\phi, 1, k) > \Delta_2^M(\phi, 1, k)$. Since the VC invests in one start-up only when project payoff is sufficiently large (i.e. when $\Delta > \Delta_2^\tau$, $\tau = B, M$), the inequality $\Delta_2^B > \Delta_2^M$ implies that the region in which the VC invests in two start-ups rather than only one enlarges under active portfolio management. When the VC can form a highly focused portfolio, he can divest one of the successful start-ups with little loss in value, reducing the inefficiency of the divestiture. Thus, active portfolio management, by enhancing the VC's rent-extraction ability, induces the VC to choose a larger portfolio. Note that the greater rent extraction ability given by active portfolio management also means that the VC has stronger incentives to make the initial project specific investments at $t = 1$. Thus, active portfolio management enlarges the set of feasible start-ups, which gives that $\Delta_1^B(\phi, 1, k) < \Delta_1^M(\phi, 1, k)$. This also implies that there are start-ups with a low payoff, i.e. those with $\Delta \in [\Delta_1^B(\phi, 1, k), \Delta_1^M(\phi, 1, k))$, that would be financed by the VC only if he can increase his rent extraction ability by engaging in portfolio management. Note that, in this case, the entrepreneurs as well are ex-ante better-off, since the VC will be willing to finance their start-up only if he can engage in portfolio management, even if this implies that one of the start-ups is divested in the SS state, and that the VC extract more rents from the one remaining in his portfolio.

These results are reversed when the portfolio companies are only moderately related, that is, when $\bar{\phi} \leq \phi \leq \hat{\phi}$. In this case, the VC's portfolio has a moderate degree of focus, and the possibility of active portfolio management shrinks the region where the VC invests in two start-ups, which formally translates into $\Delta_2^B(\phi, 1, k) \leq \Delta_2^M(\phi, 1, k)$. In this case, divesting a successful start-up is costly, because the VC can reallocate his resources less efficiently from one start-up to the other. Since the VC anticipates the high cost of inefficient termination at the time of choosing the size of the portfolio, he chooses a smaller portfolio. Correspondingly, for this range of parameters, the possibility of active portfolio management shrinks the

set of feasible start-ups compared to the case where the VC cannot engage in portfolio management, yielding $\Delta_1^B(\phi, 1, k) \geq \Delta_1^M(\phi, 1, k)$.

7 Empirical implications

The empirical predictions of our model hinge on the factors affecting the value of critical parameters in our model, especially Δ. In our setting, parameter Δ summarizes the effect of all the variables that may have an impact on the value of a start-up in the VC's portfolio. Thus the value of Δ is affected by factors such as the VC's and the entrepreneur's contribution to the project (such as their skills, knowledge and experience), the quality of the start-up, and so on. These observations lead to the following empirical implications.

(i) Specialized VCs manage smaller portfolios. Experienced and specialized VCs add substantial value to their portfolio companies and, in our paper, are characterized by a greater value of the parameter Δ. Specialized VCs will manage portfolios with a small number of companies, while less specialized VCs manage larger portfolios. This is a novel and directly testable prediction of our model. This prediction also suggests that specialized VCs, since they hold portfolios with a small number of companies, are able to take a more active role in the direct management of their portfolio companies. This prediction is consistent with the findings of Bottazzi, Da Rin and Hellmann (2004) which document that specialized VCs with greater human capital have a more active investment style and are significantly more involved with their portfolio companies.

(ii) Specialized VCs expand their portfolios only if they have access to investment opportunities in the sector of their specialization. A prediction of our model is that specialized VCs invest in focused portfolios. Our model gives that Δ_2^M is an increasing function of ϕ, implying that more experienced and specialized VCs (high Δ) will expand their portfolios only if they can invest in sufficiently related start-ups (high ϕ). Forming a portfolio with a high degree of focus benefits VCs in two ways. First, focus allows them to extract more surplus from their portfolio companies and, second, it facilitates the reallocation of their human capital among portfolio companies, limiting dilution in value. In both cases, managing a focused portfolio increases VCs' investment incentives and rewards their human capital acquisition. This implies that specialized VCs refrain from investing in start-ups that are not related to their core business and that they are more likely to increase their investments only when their industry of specialization experiences a positive technological shock (an increase in investment opportunities). Conversely, investment activity by

VCs with more general expertise is more sensitive to the overall business cycle conditions of the economy. This prediction is consistent with the findings of Gompers, Kovner, Lerner and Scharfstein (2004) which document that venture capital firms with the most industry specific human capital and experience react most to an increase in investment opportunities in the sectors of their specialization. The evidence is explained by the view that it is more difficult for diversified and less specialized VCs to redeploy their human capital from the sectors of their investment to the sector experiencing an increase in investment opportunities.

(iii) High quality start-ups are financed by VCs with small portfolios. High quality start-ups with strong fundamentals are expected to generate greater value if successful, and therefore are characterized by a greater value of Δ. Similarly, high quality entrepreneurs can be characterized by a low cost of effort, k. Our model predicts that this type of firms should be financed by VCs managing smaller and focused portfolios. For high value start-ups with strong fundamentals entrepreneurial incentives are critical to the success of the venture, and therefore VCs prefer to preserve strong entrepreneurial incentives by keeping their portfolios small. Moreover, the cost of a large portfolio in terms of reduction in entrepreneurial incentives is most severe for start-ups with strong fundamentals (high Δ and low k).

(iv) VCs investing in high-risk technologies manage larger and focused portfolios. The VC benefits from investing in related portfolio companies because focus allows a more efficient reallocation of human capital from one start-up to another. Reallocation of resources from one portfolio company to another is more likely when failure probability is high (small Δ and large k). A greater level of focus reduces the ex-post inefficiency associated with spreading the VC's resources across several start-ups, and increases the benefits of ex-post resource reallocation. This implies that focused portfolios are more desirable (all else equal) in the case of risky start-ups that invest in technologies with high uncertainty and failure rates. This prediction is consistent with the findings of Cumming (2004). This paper documents that large portfolios are more likely to be observed for VCs investing in life sciences (rather than other high-tech firms), and argues that this strategy emerges because there are greater complementarities across entrepreneurial firms in the life sciences industry.

(v) More experienced and older VCs manage larger portfolios. The focus parameter ϕ measures the value the VC can add to a start-up when the other start-up fails. Hence, the higher the value of ϕ is, the higher the value adding capability of the VC is. One empirical implication which follows from one of our results is that more experienced and older VCs (characterized by high ϕ), with greater value adding ability, would hold larger portfolios since they are more efficient in reshuffling resources from one start-up

to another.

8 Conclusions

This paper investigates the optimal size and focus of a VC's portfolio. We have identified three main effects of portfolio size on the VC's and entrepreneurs' incentives. The first one is the rent extraction effect: The VC's rent extraction ability increases in portfolio size since the VC can extract higher rents in a larger portfolio by using his ability to reallocate his limited resources from one start-up to another. This effect, everything else constant, leads to stronger incentives for the VC and weaker incentives for the entrepreneurs. The second effect is the resource allocation effect: The VC benefits from investing in a large number of start-ups because this increases (all else equal) the probability that at least one of the start-ups will be successful and thus the VC will have greater chances to earn a return from his investment. As we have argued, this effect depends on the VC's ability to reallocate ex-post his resources from one start-up to another, after observing whether they are successful or not. This effect has a positive impact both on the VC's investment incentives and entrepreneurial incentives. The third effect is the value dilution effect: A larger portfolio requires the VC to spread his limited resources across a large number of start-ups, diluting his value-adding role, with a negative impact on both the VC's and entrepreneurial incentives.

Our paper has several implications for the ways VCs form their portfolios. A small portfolio, by providing better incentives to entrepreneurs, is desirable for start-ups with strong fundamentals to promote a high level of effort from the entrepreneurs. Small size is also optimal when the average relatedness of start-ups in the economy is low, limiting the benefits of a large portfolio. Finally we show that the VC optimally chooses a small portfolio when his bargaining power is low as the rent extraction advantage of a large portfolio is small when the VC has a low bargaining power.

Portfolio focus impacts both entrepreneurial and VC's incentives. A high level of focus increases the VC's ability to reallocate his resources across start-ups more efficiently, since start-ups in focused portfolios are more related to each other. This is both good news and bad news for the start-ups. It is good news because if other start-ups in the portfolio fail, the VC can reallocate the resources to the surviving start-up at a minimum loss if the overall portfolio is more focused. It is bad news as a high level of focus between start-ups increases the rent extraction ability of the VC, intensifying competition between start-ups and impacting entrepreneurial incentives adversely. Whether the VC is better off with a more focused portfolio is determined by comparing the benefits and costs of focus. We find that specialized VCs with superior

skills and value adding potential increase the size of their portfolios only if they have access to start-ups in related industries and can form focused portfolios. However, VCs with lower value adding potential, or generalized VCs choose to hold larger portfolios with a lower degree of focus.

We have also shown that entrepreneurs may benefit from belonging to a large portfolio, rather than a small one, even if this means that the VC can extract more surplus from them. This happens when a large portfolio is the only way to enable the VC to make the start-up specific investments necessary for the success of the start-ups in his portfolio. We show that this possibility emerges when entrepreneurs have start-ups of moderate value and when the VC holding small portfolios cannot extract sufficient surplus to compensate him for his initial investment.

Finally, we show that the VC engages in portfolio management by divesting some start-ups early to extract higher surplus from the remaining start-ups. We find that the VC benefits from portfolio management when the relatedness of the start-ups in his portfolio is high and his bargaining power is low. A high degree of relatedness results in more efficient resource reallocation from the divested start-up to the remaining one, reducing the cost of inefficient start-up termination. The VC is more likely to engage in early divestiture when he has low bargaining power as the rent extraction benefit of early divestiture is most desirable at low levels of the VC's bargaining power.

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Appendix

Proof of Proposition 1

The first order condition of (3) with respect to p is $\frac{2\Delta}{1+\lambda} = kp$, which, if solved for p, gives (5). Substituting (5) into the entrepreneur's and the VC's objective functions, provided by (3) and (4) respectively, gives (6) and (7).

Proof of Lemma 1

Differentiating the VC's expected profits, (7) with respect to λ gives $\frac{\partial \pi^*_{VC}}{\partial \lambda} = \frac{4\Delta^2(1-\lambda)}{(1+\lambda)^3 k}$. It can immediately be verified that $\frac{\partial \pi^*_{VC}}{\partial \lambda} \geq 0$ if and only if $\lambda \leq 1$.

Proof of Lemma 2

The VC will make the investment if and only if his expected profits are nonnegative. It is straightforward to show that the VC's profits given by (7) are nonnegative if and only if $\Delta \geq \Delta_m(\lambda) = \frac{(1+\lambda)}{2}\sqrt{\frac{ck}{\lambda}}$.

Proof of Lemma 3

Solving the system of equations (11) and (12) for $l^i_{VC}(SS)$ and noting that $l_{EN,i}(SS) = \Delta - l^i_{VC}(SS), i = 1, 2$ give (13) and (14).

Proof of Lemma 4

Direct inspection of (13) immediately reveals that $l_{VC}(SS)$ is an increasing function of Δ and ϕ. Taking the partial derivative of $l_{VC}(SS)$ with respect to λ gives that $\frac{\partial l_{VC}(SS)}{\partial \lambda} = \frac{(2+\lambda)^2+\phi(2-\lambda^2)}{(2+\lambda)^2(1+\lambda)^2} 2\Delta$. It is easy to see $(2+\lambda)^2 + \phi(2-\lambda^2) = \lambda^2(1-\phi) + 4(1+\lambda) + 2\phi > 0$ since $\phi \leq 1$, which implies that $\frac{\partial l_{VC}(SS)}{\partial \lambda} > 0$. Consider now $l_{EN,i}(SS)$. Direct inspection of (14) reveals that $l_{EN,i}(SS)$ is an increasing function of Δ and a decreasing function of ϕ. Taking the partial derivative of $l_{EN,i}(SS)$ with respect to λ gives that $\frac{\partial l^M_{EN,i}(SS)}{\partial \lambda} = -\frac{1}{2}\frac{(2+\lambda)^2+\phi(2-\lambda^2)}{(2+\lambda)^2(1+\lambda)^2} 2\Delta$, which is always negative, since we have just shown that $(2+\lambda)^2 + \phi(2-\lambda^2) = \lambda^2(1-\phi) + 4(1+\lambda) + 2\phi > 0$.

Proof of Proposition 2

Since the reaction functions of the two entrepreneurs are symmetric, the Nash-equilibrium of the effort choice subgame is obtained by setting $p_j \equiv p_i$ in the first-order condition (17), and then solving for p_i, giving (18). Substituting the Nash-equilibrium level of effort (18) into the entrepreneurs' objective function, (15), and in the VC's objective function, (16), gives the VC's profits (19) and the entrepreneurs' profits (20).

Proof of Lemma 5

Differentiating the Nash-equilibrium level of effort (18) with respect to Δ gives that $\frac{\partial p^*(2)}{\partial \Delta} = \frac{k(2+\lambda)^2(1+\phi)}{(1+\lambda)((2+\lambda)k+2\Delta\phi)^2} >$

0. Differentiating (18) with respect to ϕ gives that $\frac{\partial p^*(2)}{\partial \phi} = \frac{(2+\lambda)\Delta(k(2+\lambda)-2\Delta)}{(1+\lambda)((2+\lambda)k+2\Delta\phi)^2} > 0$, since, from (5), $p^* < 1$ requires that $2\Delta < k(1+\lambda)$. Differentiating (18) with respect to λ gives that $\frac{\partial p^*(2)}{\partial \lambda} = -\frac{(1+\phi)\Delta(k(2+\lambda)^2+2\Delta\phi)}{(1+\lambda)^2((2+\lambda)k+2\Delta\phi)^2} < 0$. Finally, direct inspection of (18) reveals that $p^*(2)$ is a decreasing function of k.

Proof of Lemma 6

Consider first the VC's profits $\pi^*_{VC}(2)$, given by (19). Differentiating (19) with respect to Δ gives that

$$\frac{\partial \pi^*_{VC}(2)}{\partial \Delta} = 2\lambda\left(2+\lambda\right)\left(1+\phi\right)^2 \Delta \frac{\Delta\phi\left(2\Delta\phi + 3k\left(2+\lambda\right)\right) + 2k^2\left(2+\lambda\right)^2}{\left(1+\lambda\right)^2\left((2+\lambda)k+2\Delta\phi\right)^3}$$

It is immediate to verify that $\frac{\partial \pi^*_{VC}(2)}{\partial \Delta} > 0$. Differentiating (19) with respect to ϕ gives that

$$\frac{\partial \pi^*_{VC}(2)}{\partial \phi} = 2\lambda\left(2+\lambda\right)\Delta^2\left(1+\phi\right)\frac{k\left(2+\lambda\right)\left(2k\left(\lambda+2\right)\right) + \Delta\phi\left(3k\left(\lambda+2\right) - (1-\phi)2\Delta\right)}{\left(1+\lambda\right)^2\left((2+\lambda)k+2\Delta\phi\right)^3}.$$

From (5) we know that $p^* < 1$ requires that $\Delta < \frac{1}{2}\left(1+\lambda\right)k$, which implies that $2\Delta(1-\phi) < \left(1+\lambda\right)k(1-\phi)$. We obtain

$$\begin{aligned} k\left(2+\lambda\right)\left(2k\left(\lambda+2\right)\right) + \Delta\phi\left(3k\left(\lambda+2\right) - (1-\phi)2\Delta\right) &> \\ k\left(2+\lambda\right)\left(2k\left(\lambda+2\right)\right) + \Delta\phi\left(3k\left(\lambda+2\right) - \left(1+\lambda\right)k(1-\phi)\right) &= \\ 2\left(2+\lambda\right)^2 k^2 + \Delta\phi\left(\phi(1+\lambda) + 2\lambda + 5\right)k &> 0. \end{aligned}$$

Therefore, the numerator of $\frac{\partial \pi^*_{VC}(2)}{\partial \phi}$ is always positive. It is easy to see that the denominator of $\frac{\partial \pi^*_{VC}(2)}{\partial \phi}$ is also always positive, giving that $\frac{\partial \pi^*_{VC}(2)}{\partial \phi} > 0$. Differentiating (19) with respect to k gives that

$$\frac{\partial \pi^*_{VC}(2)}{\partial k} = -\frac{k2\lambda\left(2+\lambda\right)^3\left(1+\phi\right)^2\Delta^2}{\left(1+\lambda\right)^2\left((2+\lambda)k+2\Delta\phi\right)^3} < 0.$$

Finally, differentiating $\partial \pi^*_{VC}(2)$ with respect to λ gives that

$$\frac{\partial \pi^*_{VC}(2)}{\partial \lambda} = 2\left(1+\phi\right)^2\Delta^2\frac{\left(2\Delta\phi + k\left(2+\lambda\right)\left(1-\lambda\right)\right)\left(2\Delta\phi + k\left(2+\lambda\right)^2\right)}{\left(1+\lambda\right)^3\left((2+\lambda)k+2\Delta\phi\right)^3}.$$

By direct inspection it is easy to see that $\frac{\partial \pi^*_{VC}(2)}{\partial \lambda} > 0$ if and only if $2\Delta\phi + k\left(\lambda+2\right)\left(1-\lambda\right) > 0$, which is the case if and only if $0 \leq \lambda < \frac{\sqrt{(9k^2+8k\Delta\phi)}-k}{2k}$.

Consider now the entrepreneurs' profits, $\pi^*_{EN1} = \pi^*_{EN2}$, given in (20). Differentiating (20) with respect

to Δ gives that

$$\frac{\partial \pi^*_{EN1}}{\partial \Delta} = \frac{\Delta k^2 (2+\lambda)^3 (1+\phi)^2}{(1+\lambda)^2 ((2+\lambda)k + 2\Delta\phi)^3} > 0.$$

Differentiating (20) with respect to ϕ gives that

$$\frac{\partial \pi^*_{EN1}}{\partial \phi} = \frac{((2+\lambda)k - 2\Delta) k (2+\lambda)^2 (1+\phi) \Delta^2}{(1+\lambda)^2 ((2+\lambda)k + 2\Delta\phi)^3}.$$

Again, from (5), we have that $p^* < 1$ implies that $2\Delta < (1+\lambda) k$, which gives that $\frac{\partial \pi^*_{EN1}}{\partial \phi} > 0$. Differentiating (20) with respect to k gives that

$$\frac{\partial \pi^*_{EN1}}{\partial k} = -\frac{1}{2} \frac{((2+\lambda)k - 2\Delta\phi) (2+\lambda)^2 (1+\phi)^2 \Delta^2}{(1+\lambda)^2 ((2+\lambda)k + 2\Delta\phi)^3}$$

Again, from (5), we have that $p^* < 1$ implies that $2\Delta < (1+\lambda) k$, which implies that $2\Delta\phi - (2+\lambda)k < 0$ as $\phi < 1$. Therefore, $\frac{\partial \pi^*_{EN1}}{\partial k} < 0$. Finally, differentiating (20) with respect to λ gives that

$$\frac{\partial \pi^*_{EN1}}{\partial \lambda} = -\frac{k \left(k(2+\lambda)^2 + 2\Delta\phi\right) (2+\lambda) (1+\phi)^2 \Delta^2}{(1+\lambda)^3 ((2+\lambda)k + 2\Delta\phi)^3} < 0.$$

Proof of Lemma 7

Direct comparison of $p^*(1)$ in (5) with $p^*(2)$ in (18) reveals that $p^*(1) > p^*(2)$ if and only if

$$\frac{2}{k} > \frac{1+\phi}{k + \frac{2\Delta\phi}{2+\lambda}},$$

which is always true for $\phi \leq 1$. From (5) and (18) we have that

$$p^*(1) - p^*(2) = \Delta \frac{k (1-\phi) (2+\lambda) - 4\Delta\phi}{(1+\lambda) k (2k + k\lambda + 2\Delta\phi)}. \tag{31}$$

Differentiating (31) with respect to Δ gives that

$$\begin{aligned} \frac{\partial (p^*(1) - p^*(2))}{\partial \Delta} &= \frac{\partial \left(\frac{2\Delta}{(1+\lambda)k} - \frac{(2+\lambda)(1+\phi)\Delta}{(1+\lambda)((2+\lambda)k+2\Delta\phi)}\right)}{\partial \Delta} \\ &= \frac{k^2 (2+\lambda)^2 (1-\phi) + 8\Delta\phi (\Delta\phi + k\lambda + 2k)}{(1+\lambda) k (2k + k\lambda + 2\Delta\phi)^2} > 0. \end{aligned}$$

Differentiating (31) with respect to ϕ gives that

$$\frac{\partial(p^*(1)-p^*(2))}{\partial\phi} = -\Delta\frac{(2+\lambda)(k\lambda-2\Delta+2k)}{(1+\lambda)(2k+k\lambda+2\Delta\phi)^2} < 0,$$

since $p^*(1) < 1$ implies that $2\Delta < k + k\lambda$. Differentiating (31) with respect to k yields that

$$\frac{\partial(p^*(1)-p^*(2))}{\partial k} = \Delta\frac{k^2(2+\lambda)^2(-1+\phi)-8\Delta\phi(\Delta\phi+2k+k\lambda)}{(1+\lambda)k^2(2k+k\lambda+2\Delta\phi)^2} < 0.$$

Finally, differentiating (31) with respect to λ gives that

$$\frac{\partial(p^*(1)-p^*(2))}{\partial\lambda} = \Delta\frac{-(2+\lambda)^2(1-\phi)k^2-2\Delta\phi(7-\phi+4\lambda)k-8\Delta^2\phi^2}{(1+\lambda)^2k(2k+k\lambda+2\Delta\phi)^2} < 0$$

.

Proof of Proposition 3

Consider first the case in which the VC selects a portfolio with only one start-up, $\eta = 1$. In this case, from (7) and Lemma 2, we have that the VC earns positive expected profits if and only if $\Delta \geq \Delta_m(\lambda) \equiv \frac{1+\lambda}{2}\sqrt{\frac{ck}{\lambda}}$. Consider now the case in which the VC selects a portfolio with two start-ups, $\eta = 2$. Define $\hat{\Delta}^M(\phi,\lambda,k)$ implicitly by setting $\pi^*_{VC}(2) = 0$ in (19). Note that, from Lemma 6, we know that $\frac{\partial\pi^*_{VC}(2)}{\partial\Delta} > 0$ and $\frac{\partial\pi^*_{VC}(2)}{\partial\phi} > 0$. Thus, by the implicit function theorem, we have that $\frac{\partial\hat{\Delta}^M(\phi,k,\lambda)}{\partial\phi} < 0$. Also, from $\frac{\partial\pi^{*M}_{VC}(2)}{\partial\Delta} > 0$, the VC earns positive expected profits only if $\Delta > \hat{\Delta}^M(\phi,\lambda,k)$. Part (i) of the proposition is obtained by setting $\Delta_1^M(\phi,\lambda,k) \equiv min\{\Delta_m(\lambda); \hat{\Delta}^M(\phi,\lambda,k)\}$. Note that $\frac{\partial\Delta_1^M(\phi,\lambda,k)}{\partial\phi} \leq 0$ since $\frac{\partial\hat{\Delta}^M(\phi,k,\lambda)}{\partial\phi} < 0$ and $\Delta_m(\lambda)$ is independent of ϕ.

Define now ϕ^M as the unique solution to $\pi^*_{VC}(2) = 0$ at $\Delta = \Delta_m(\lambda)$, and note that at $\phi = \phi^M$ we have that $\pi^*_{VC}(2) = \pi^*_{VC}(1) = 0$.

Let now $\phi^M \leq \phi < \bar{\phi}$. In this case, if $\Delta_1^M \leq \Delta < \Delta_m(\lambda)$, the VC earns positive profits if he invests in two start-up companies (since $\Delta \geq \Delta_1^M$), but he earns negative expected profits if he invests in one start-up only (since $\Delta < \Delta_m(\lambda)$). Thus, in this case, the VC optimally selects two start-ups. Let now $\Delta \geq \Delta_m(\lambda)$. In this case, the VC earns positive expected profits whether he invests in one or two start-ups, and both choices are feasible. Therefore, the optimal choice of whether to invest in one or two start-ups is made by the VC by comparing expected profits with one start up only, $\pi^*_{VC}(1)$, with the expected profits

with two start-ups, $\pi^*_{VC}(2)$. Using (7) and (19), we have that the VC invests in one start-up if and only if

$$\frac{4\Delta^2\lambda}{(1+\lambda)^2 k} > \frac{2\lambda(2+\lambda)((2+\lambda)k+\Delta\phi)(1+\phi)^2\Delta^2}{(1+\lambda)^2((2+\lambda)k+2\Delta\phi)^2}. \tag{32}$$

Rearranging the inequality, we obtain that $\pi^*_{VC}(1) > \pi^*_{VC}(2)$ if and only if

$$P_1 \equiv 16\lambda(1+\lambda)^2\phi^2\Delta^2 - 2\lambda k\phi\left(-7+2\phi+\phi^2\right)(2+\lambda)(1+\lambda)^2\Delta$$
$$-2\lambda k^2\left(-1+2\phi+\phi^2\right)(2+\lambda)^2(1+\lambda)^2 > 0.$$

Note that P_1 is a convex parabola in Δ with two roots, $\tilde{\Delta}_1^M$and $\tilde{\Delta}_2^M$, given by

$$\begin{aligned}
\tilde{\Delta}_1^M &\equiv \left(-\frac{7}{16}+\frac{1}{16}\phi(2+\phi)-\frac{1}{16}\sqrt{(\phi^2+2\phi+17)(\phi+1)^2}\right)k\frac{2+\lambda}{\phi},\\
\tilde{\Delta}_2^M &\equiv \left(-\frac{7}{16}+\frac{1}{16}\phi(2+\phi)+\frac{1}{16}\sqrt{(\phi^2+2\phi+17)(1+\phi)^2}\right)k\frac{2+\lambda}{\phi}.
\end{aligned}$$

It is straightforward to show that $\tilde{\Delta}_1^M < 0$ for all $0 < \phi < 1$, $k > 0$, and $\lambda > 0$. We also have that $\tilde{\Delta}_2^M > 0$ for $\phi > \phi^M$. This can be seen by noting that at $\phi = \phi^M$ we have that $\tilde{\Delta}_2^M(\phi^M,\lambda,k) = \Delta_1^M(\phi^M,\lambda,k) = \Delta_m(\lambda) > 0$ and

$$\begin{aligned}
\frac{\partial\tilde{\Delta}_2^M}{\partial\phi} &= \frac{1}{16}\left(\phi\left(\phi^2+\phi-1\right)-17+\sqrt{(\phi^2+2\phi+17)}\left(\phi^2+7\right)\right)k\frac{2+\lambda}{\sqrt{(\phi^2+2\phi+17)}\phi^2}\\
&> \frac{1}{16}\left(7\sqrt{17}-17\right)k\frac{2+\lambda}{\sqrt{(\phi^2+2\phi+17)}\phi^2} > 0,
\end{aligned}$$

since $\left(\phi\left(\phi^2+\phi-1\right)-17+\sqrt{(\phi^2+2\phi+17)}\left(\phi^2+7\right)\right)$ is increasing in ϕ.

Therefore, $\pi^*_{VC}(1) > \pi^*_{VC}(2)$ for $\Delta > \tilde{\Delta}_2^M$ and $\pi^*_{VC}(1) < \pi^*_{VC}(2)$ for $\Delta_m(\lambda) \le \Delta < \tilde{\Delta}_2^M$. Define then $\Delta_2^M(\phi,\lambda,k) \equiv \tilde{\Delta}_2^M(\phi,\lambda,k)$ giving part (iii).

Part (ii) of the proposition is easily verified by noting again that $\Delta_2^M(\phi^M,\lambda,k) = \Delta_1^M(\phi^M,\lambda,k) = \Delta_m(\lambda) > 0$ at $\phi = \phi^M$. Thus, for $0 \le \phi < \phi^M$, $\Delta \ge \Delta_m$ implies that $\Delta > \tilde{\Delta}_2^M$ since $\frac{\partial\tilde{\Delta}_2^M}{\partial\phi} > 0$ and $\Delta_2^M(\phi^M,\lambda,k) = \Delta_1^M(\phi^M,\lambda,k)$ at $\phi = \phi^M$, and hence, $\pi^*_{VC}(1) > \pi^*_{VC}(2)$, concluding the proof.

Proof of Proposition 4

i) From the proof of Proposition 3, note that Δ_2^M is a linear function of λ, and positive for $\phi > \phi^M$. Since we have $\lambda > 0$, it follows that $\frac{\partial\Delta_2^M}{\partial\lambda} > 0$ for all $\phi > \phi^M$.

ii) From the proof of Proposition 3, note that Δ_2^M is a linear function of k, and positive for $\phi > \phi^M$.

Since we have $k > 0$, it follows that $\frac{\partial \Delta_2^M}{\partial k} > 0$ for all $\phi > \phi^M$.

Proof of Proposition 5

By direct calculation, comparing the VC's payoff from bilateral bargaining $l_{VC}^B(SS)$, given by (22), and that from multilateral bargaining $l_{VC}^M(SS)$, given by (13), reveals that $l_{VC}^M(SS) \geq l_{VC}^B(SS)$ if and only if $\phi \leq \overline{\phi} \equiv \frac{\lambda(2+\lambda)}{\lambda^2+2\lambda+2}$. Similarly, $l_{VC}^M(SS) \geq l_{VC}^B(SS)$ if and only if $\lambda \geq \ \bar{\lambda} \equiv \frac{\sqrt{1-\phi^2}-(1-\phi)}{(1-\phi)}$.

Proof of Proposition 6

Since the reaction functions of the two entrepreneurs are symmetric, the Nash-equilibrium of the effort choice subgame is obtained by setting $p_j = p_i^B$ in the first-order condition (27) and solving for p_i^B, giving (28). Substituting the Nash-equilibrium level of effort (28) into the entrepreneurs' objective function (25) and VC's objective function, (26), gives (29) and (30).

Proof of Lemma 8

Consider first the VC's profits, π_{VC}^{*B}, given by (29). Differentiating (29) with respect to Δ gives that

$$\frac{\partial \pi_{VC}^{*B}}{\partial \Delta} = 4\lambda(1+\lambda)(1+\phi)^2 \Delta \frac{(1+\phi)^2(1+2\lambda)\Delta^2 + 6k(1+\lambda)^2(1+\phi)\Delta + 8k^2(1+\lambda)^3}{(2k(1+\lambda)^2 + \Delta(1+\phi)(1+2\lambda))^3} > 0.$$

Differentiating (29) with respect to ϕ gives that

$$\frac{\partial \pi_{VC}^{*B}}{\partial \phi} = 4\lambda(1+\lambda)\Delta^2(1+\phi) \frac{(1+\phi)^2(1+2\lambda)\Delta^2 + 6k(1+\lambda)^2(1+\phi)\Delta + 8k^2(1+\lambda)^3}{(2k(1+\lambda)^2 + \Delta(1+\phi)(1+2\lambda))^3} > 0.$$

Differentiating (29) with respect to k gives that

$$\frac{\partial \pi_{VC}^{*B}}{\partial k} = -\frac{8\lambda(2k(1+\lambda)^2 + \Delta(1+\phi))(1+\lambda)^2(1+\phi)^2\Delta^2}{(2k(1+\lambda)^2 + \Delta(1+\phi)(1+2\lambda))^3} < 0.$$

Differentiating (29) with respect to λ gives that

$$\frac{\partial \pi_{VC}^{*B}}{\partial \lambda} = 4(1+\phi)^2\Delta^2 \frac{(\Delta(1+\phi) + 2k(1+3\lambda))\left(\Delta(1+\phi) + 2k(1-\lambda^2)\right)}{(2k(1+\lambda)^2 + \Delta(1+\phi)(1+2\lambda))^3}.$$

By direct inspection it is easy to see that $\frac{\partial \pi_{VC}^{*B}}{\partial \lambda} > 0$ if and only if $\left(\Delta(1+\phi) + 2k(1-\lambda^2)\right) > 0$; which is the case if and only if $0 \leq \lambda < \sqrt{1 + \frac{(1+\phi)\Delta}{2k}}$.

Consider now the entrepreneurs' profits, π_{ENi}^{*B}, given in (30). Differentiating (30) with respect to Δ

gives that

$$\frac{\partial \pi_{ENi}^{*B}}{\partial \Delta} = \frac{8k^2 (1+\lambda)^2 (1+\phi)^2 \Delta (1+\lambda)^2}{\left(2k (1+\lambda)^2 + \Delta (1+\phi)(1+2\lambda)\right)^3} > 0.$$

Differentiating (30) with respect to ϕ gives that

$$\frac{\partial \pi_{ENi}^{*B}}{\partial \phi} = \frac{8k^2 (1+\lambda)^2 (1+\phi) \Delta^2 (1+\lambda)^2}{\left(2k (1+\lambda)^2 + \Delta (1+\phi)(1+2\lambda)\right)^3} > 0.$$

Differentiating (30) with respect to k gives that

$$\frac{\partial \pi_{ENi}^{*B}}{\partial k} = 2(1+\lambda)^2 (1+\phi)^2 \Delta^2 \frac{\left(-2k (1+\lambda)^2 + \Delta (1+\phi)(1+2\lambda)\right)}{\left(2k (1+\lambda)^2 + \Delta (1+\phi)(1+2\lambda)\right)^3}$$

From (5), we have that $p^* < 1$ implies that $\Delta < \frac{1}{2}(1+\lambda)k$. Using this inequality, we obtain that

$$\begin{aligned}
\left(-2k (1+\lambda)^2 + \Delta (1+\phi)(1+2\lambda)\right) &< \left(-2k (1+\lambda)^2 + \frac{1}{2}(1+\lambda) k (1+\phi)(1+2\lambda)\right) \\
&= \frac{1}{2} k (1+\lambda)(-2\lambda(1-\phi) - 3 + \phi) < 0.
\end{aligned}$$

Thus $\frac{\partial \pi_{ENi}^{*B}}{\partial k} < 0$. Finally, differentiating (30) with respect to λ gives that

$$\frac{\partial \pi_{ENi}^{*B}}{\partial \lambda} = -\frac{4k(1+\lambda)(1+\phi)^2 \Delta^2 \left(2k + 4k\lambda + 2k\lambda^2 + \Delta + \Delta\phi\right)}{\left(2k (1+\lambda)^2 + \Delta (1+\phi)(1+2\lambda)\right)^3} < 0.$$

Proof of Proposition 7

The proof of this proposition is similar to the proof of Proposition 3. Consider the case in which the VC selects a portfolio with two start-ups, $\eta = 2$. From (29) define $\hat{\Delta}^B(\phi, \lambda, k)$ implicitly by setting $\pi_{VC}^{*B}(2) = 0$ in (29). Note that, from Lemma 7, we know that $\frac{\partial \pi_{VC}^{*B}(2)}{\partial \Delta} > 0$ and $\frac{\partial \pi_{VC}^{*B}(2)}{\partial \phi} > 0$. Thus, by the implicit function theorem, we have that $\frac{\partial \hat{\Delta}^B(\phi,k,c,\lambda)}{\partial \phi} < 0$. Also, from $\frac{\partial \pi_{VC}^{*B}(2)}{\partial \Delta} > 0$, the VC earns positive expected profits only if $\Delta > \hat{\Delta}^B(\phi, \lambda, k)$. Define $\overline{\Delta}$ such that $\pi_{VC}^{*B}(2) = \pi_{VC}^{*}(1)$ at $\phi = \bar{\phi}$. Thus, from (7) there is a $\bar{c}(\lambda, k)$ such that if $c < \bar{c}(\lambda, k)$ we have that $\Delta_m(\lambda) < \overline{\Delta}$. Since $\frac{\partial \hat{\Delta}^B(\phi,k,c,\lambda)}{\partial \phi} < 0$ it follows that $\hat{\Delta}^B(\phi, \lambda, k) < \Delta_m(\lambda)$ for all $\bar{\phi} \le \phi \le 1$. Thus, define now $\Delta_1^B(\phi, \lambda, k) \equiv \hat{\Delta}^B(\phi, \lambda, k)$. The proposition is then proved as follows.

For $0 \le \Delta < \Delta_1^B(\phi, \lambda, k)$, we have that $\pi_{VC}^{*B}(2) < 0$ and $\pi_{VC}^{*}(1) < 0$, giving part (i) of the proposition.

For $\Delta_1^B(\phi, \lambda, k) \le \Delta < \Delta_m(\lambda)$, we have that $\pi_{VC}^{*B}(2) \ge 0$ but $\pi_{VC}^{*}(1) < 0$. Thus the VC optimally invests in two start-ups, $\eta^* = 2$. For $\Delta \ge \Delta_m(\lambda)$ we have that $\pi_{VC}^{*B}(2) \ge 0$ and $\pi_{VC}^{*}(1) \ge 0$, and investment

in both one or two start-ups is feasible. Therefore, the optimal choice of whether to invest in one or two start-ups is made by the VC by comparing expected profits with one start up only, $\pi_{VC}^{*}(1)$, with the expected profits with two start-ups, $\pi_{VC}^{*B}(2)$. Using (7) and (29), we have that the VC invests in one start-up if and only if

$$\frac{4\Delta^2\lambda}{(1+\lambda)^2 k} > \frac{4\lambda(1+\lambda)(2k(1+\lambda)+(1+\phi)\Delta)(1+\phi)^2\Delta^2}{(2k(1+\lambda)^2+(1+2\lambda)(1+\phi)\Delta)^2}$$

Rearranging the inequality results in $\pi_{VC}^{*}(1) > \pi_{VC}^{*B}(2)$ if and only if

$$P_2 \equiv 4\lambda(1+2\lambda)^2(1+\phi)^2\Delta^2 - 4\lambda k(1+\phi)(1+\lambda)^2\left(-7\lambda+2\lambda\phi+\lambda\phi^2-3+2\phi+\phi^2\right)\Delta$$
$$-8\lambda k^2\left(-1+2\phi+\phi^2\right)(1+\lambda)^4 > 0.$$

Note that P_2 is convex in Δ, and has two roots, $\tilde{\Delta}_1^B$ and $\tilde{\Delta}_2^B$, given by

$$\begin{aligned}
\tilde{\Delta}_1^B &\equiv \frac{1}{2(1+2\lambda)^2}\left(\phi(2+\phi)(1+\lambda)-7\lambda-3-\sqrt{A}\right)\frac{(1+\lambda)^2 k}{1+\phi}, \\
\tilde{\Delta}_2^B &\equiv \frac{1}{2(1+2\lambda)^2}\left(\phi(2+\phi)(1+\lambda)-7\lambda-3+\sqrt{A}\right)\frac{(1+\lambda)^2 k}{1+\phi};
\end{aligned}$$

where $A \equiv (1+\phi)^2\left(\phi(1+\lambda)^2(2+\phi)+1+10\lambda+17\lambda^2\right)$. It is straightforward to show that $\tilde{\Delta}_1^B$ is always negative. $\tilde{\Delta}_2^B$ is positive for $\phi > \bar{\phi}$ since for $c \geq \bar{c}(\lambda,k)$ we have that $\tilde{\Delta}_2^B > \Delta_m(\lambda) > 0$ for all $1 > \phi > \bar{\phi}$ and

$$\frac{\partial\tilde{\Delta}_2^B}{\partial\phi} = \frac{(1+\lambda)^2 k}{2(1+2\lambda)^2}\frac{(\phi(2+\phi)(\lambda+1)+9\lambda+5)\sqrt{A}+(1+\phi)^4(1+\lambda)^2}{\sqrt{A}(1+\phi)^2} > 0.$$

Hence it follows that $\pi_{VC}^{*}(1) \geq \pi_{VC}^{*B}(2)$ if $\Delta \geq \tilde{\Delta}_2^B$ and $\pi_{VC}^{*}(1) < \pi_{VC}^{*B}(2)$ if $\Delta_m(\lambda) \leq \Delta < \tilde{\Delta}_2^B$. Define then $\Delta_2^B(\phi,\lambda,k) \equiv \tilde{\Delta}_2^B(\phi,\lambda,k)$, giving part (ii).

Proof of Proposition 8

Set $\lambda = 1$ and note that $\bar{\phi} = \frac{3}{5}$ at $\lambda = 1$. It is possible to show that $\Delta_2^B(\phi,1,k)$ and $\Delta_2^M(\phi,1,k)$ intersects only at $\widehat{\phi} = .93791$ for $\bar{\phi} \leq \phi < 1$. Comparing $\Delta_2^B(\phi,1,k)$ and $\Delta_2^M(\phi,1,k)$ at $\phi = 1$ yields that $\Delta_2^B(\phi=1,1,k) = \frac{4}{9}\left(-1+\sqrt{10}\right)k \geq \Delta_2^M(\phi=1,1,k) = \frac{3}{4}\left(\sqrt{5}-1\right)k$, which in turn implies that $\Delta_2^B(\phi,1,k) \leq \Delta_2^M(\phi,1,k)$ if and only if $\phi \leq \widehat{\phi} = .93791$. From the definition of $\Delta_2^B(\phi,1,k)$ and $\Delta_2^M(\phi,1,k)$, $\Delta_2^B(\phi,1,k) = \Delta_2^M(\phi,1,k)$ at $\phi = \widehat{\phi}$ implies that $\pi_{VC}^{*B}(2) = \pi_{VC}^{*M}(2)$ at $\phi = \widehat{\phi}$, and which, in turn, implies that $\Delta_1^B(\phi,1,k)$ and $\Delta_1^M(\phi,1,k)$ intersect only once at $\phi = \widehat{\phi}$ for $\bar{\phi} \leq \phi < 1$, since

$\Delta_1^B(\phi, 1, k)$ and $\Delta_1^M(\phi, 1, k)$ are defined as solutions to $\pi_{VC}^{*B}(2) - c = 0$ and $\pi_{VC}^{*M}(2) - c = 0$ respectively. Calculating $\Delta_1^B(\phi, 1, k)$ and $\Delta_1^M(\phi, 1, k)$ at $\phi = 1$ yields that $\Delta_1^B(\phi = 1, 1, k) < \Delta_1^M(\phi = 1, 1, k)$, which in turn implies $\Delta_1^B(\phi, 1, k) \geq \Delta_1^M(\phi, 1, k)$ if and only if $\phi \leq \widehat{\phi}$ given that $\Delta_1^B(\phi, 1, k)$ and $\Delta_1^M(\phi, 1, k)$ intersect only once for $\bar{\phi} < \phi < 1$.