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# The Value of Benchmarking

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## Abstract

We consider the provision of venture capital in a dynamic model with multiple research stages, where time and investment needed to meet each benchmark are unknown. The allocation of funds is subject to moral hazard. The optimal contract provides for incentive payments linked to attaining the next benchmark, which must be increasing in the funding horizon of each stage. Benchmarking reduces agency costs, directly by shortening the agent's guaranteed funding horizon, and indirectly via an implicit incentive effect of information rents in future financing rounds.

The ex ante need to provide incentives and the venture capitalist's desire to cut information rents ex post create a hold-up conflict, which can be overcome by providing all funds in every stage in a single up-front payment. Empirical patterns of the evolution of financing rounds and research intensity over the lifetime of a project are explained as optimal choices: the optimal capital allocated and the funding horizon are increasing from one stage to the next. This emphasizes the notion that early stages are the riskiest in an innovative venture.



*Key words:* venture financing, optimal stopping, benchmarking, stage financing, abandonment option.

*JEL Classification:* D83, D92, G24, G31.

# 1. Introduction

## 1.1. Motivation

The venture capital industry, which has become the main source of financing of innovative projects, offers unique insights into how financiers and managers of innovative start-ups align their interest and resolve agency conflicts. The most frequently cited agency problems in venture capital contracting are that entrepreneurs may invest into efforts that have high personal return (scientific recognition, investment in human capital, etc.) but add little or no value to the venture, and the tendency of the entrepreneurs to continue their projects beyond the efficient stopping time. The importance of the latter problem has arguably been reaffirmed by the slow and expensive wind-down of many cash-burning internet start-ups after March 2000.

Stage financing stands out as "the most potent control mechanism a venture capitalist can employ" (Gompers and Lerner (1999), p. 139). Venture capitalists do not commit to future financing rounds, but will only agree to future financing rounds if their intermediate evaluation of the project is positive. By staging their financing, venture capitalists retain the real option to abandon the project periodically. Often, explicit benchmarks - technological or financial in nature - are written into the contracts, giving the venture capitalists additional contingent control rights that can be exercised if the benchmarks are missed, including the rights to change the management of the venture or to initiate liquidation procedures. Typically, the estimated cash need for the entire stage is injected at the beginning of the round, putting a large cash reserve at the disposal of the firm that is gradually drawn down.

Empirical research has revealed that the precise use of staging instruments depends on the risk and the characteristics of the project. The riskier is the project, the less information venture capitalists have about the venture, or the larger is the discretion of the entrepreneurs[1], the shorter are the staging intervals, and hence the more frequently are venture capitalists reevaluating the project and pondering the abandonment option. The larger is the total funding received of a venture, the more financing rounds are used. There is further empirical evidence on typical patterns of stage financing over the lifetime of a project. Typically, the duration of stages are increasing from one stage to the next. Also, the amount of cash injected per round is increasing over time. The rate of return seems to be highest in the early stages of a project, both measured by the internal valuations estimated at the start of every financing round, as by market-based exit valuations.[2] Practitioners apply considerably higher discount rates in

[1] Kaplan and Stromberg (2001)(2002), Gompers and Lerner (1999).

[2] See for these observations Cochrane (2001), Das et. al. (2002), Gompers and Lerner (1999), p. 139.

early stages, reflecting a perceived higher failure risk there than in later stage investments.

While it has long been recognized in the literature that stage financing is a tool to mitigate agency conflicts, explicit dynamic studies on how projects are benchmarked, and how the optimal staging policy interacts with the typical conflicts in the financier-entrepreneur relationship, are surprisingly rare. Agency considerations are, however, an important determinant of the optimal funding policy of an innovative project. They influence the research intensity, research lay-out and the research budget. This paper aims to provide a more detailed understanding of this link by looking at the role of benchmarking.

We propose a simple model of a venture project over multiple stages to analyze this interaction. The venture capitalist controls the investment opportunity but she needs a wealth constrained entrepreneur to run it. The project consists of several stages, each characterized by a benchmark, and the successful completion of the project requires that every benchmark is met. Time and money needed to meet each benchmark are subject to uncertainty, since in each period, the research effort can either make progress or fail. As the project continues to receive financing without achieving the next benchmark, the investor gets closer to the point where she wishes to abandon the entire project. When one of the benchmarks is attained, the probability of the entire project jumps upwards.

The investment effort is unobservable to the investor and the entrepreneur

can divert the funds to his private ends. The entrepreneur's control of the fund allocation introduces a conflict meant to capture in a stylized way the two main agency conflicts in the venture financing cited earlier, namely self-serving investments and the bias towards inefficient continuation. In each period, the solution of the agency conflict has to take into account the intertemporal incentives for the entrepreneur. If the entrepreneur diverts the capital flow for private purposes, she knows that she continues to receive funding for sure. In contrast, if she invests the funds, she knows that with a certain probability she is successful and the funding in the current stage will end. The longer the funding horizon of the current stage, the larger is this option value of the diversion.

### 1.2. Results and empirical implications

We first analyze the optimal funding when the venture capitalist cannot observe whether the intermediate benchmarks have been attained. In this case, the venture capitalist can only define a total funding horizon for the entire project, and make sure that the reward to the entrepreneur in case of completion of the last benchmark provides sufficient incentives to invest.

This solution is inefficient compared with benchmarking for three reasons: first, there is no abandonment if the early benchmarks are not completed in time, adding to the entrepreneur's discretion and information rent. Second, if the early stages take longer than expected, the remaining budget for the last stages is inefficiently small. Finally, since the venture capitalist is in a position of asymmetric information with respect to the number of benchmarks that have already been met, the incentive payments must be tailored to fit several possible "types" of the entrepreneur, which again increases the information rent.

We then consider the case where the benchmarks are observable. The optimal contract uses stage financing, the conclusion of a new contract upon reaching each benchmark. This offers the advantage of exploiting the value of the real option to abandon the project over time. The necessary incentive payments are an increasing function of the entrepreneur's discretion over the funds, and thus of the funding horizon of each stage. It reduces agency costs, because the agent's guaranteed funding horizon is reduced by the introduction of intermediate benchmarks. Agency costs are also reduced by the fact that the informal promise of information rents in future financing rounds acts as an implicit incentive device. A hold-up problem emerges between the ex ante incentive potential of implicit contracts and the venture capitalist's desire to cut information rents ex post. The supply of excess cash to the venture, as implied by providing all funds in a given stage in a lump-sum payment at the beginning of each stage, is a commitment device to overcome this problem.

We find that the optimal funding horizon is increasing from one stage to the next. This effect is exacerbated by the impact of the agency costs, and by the implicit incentive effect of future information rents. Thus, our model shows that the principal stylized facts of the evolution of funding over time can be explained as optimal choices: the research intensity is lower for early stages, explaining that a smaller budget is allocated to them, that their duration is shorter, and their success probability smaller. This is turn explains why the research risk is larger in early stages, and thus the observed return, conditional on success in early stages, is larger.

### 1.3. Related Literature and Overview

While the importance of stage financing has been widely documented in the empirical literature on venture capital contracting,[3] only a small number of theoretical papers have explicitly tried

[3] Kaplan and Stromberg (2001), (2002), Kaplan et al. (2002), Gompers and Lerner (1999), Cochrane (2001) and Das et al. (2002).

to provide a rationale for the use of benchmarks in venture finance. Cornelli and Yosha (1999) analyze the problem of an entrepreneur manipulating short-term results for purposes of "window-dressing". Neher (1994) shows that stage financing can serve as an instrument to reduce the bargaining power of an opportunistic entrepreneur who can repudiate her financial obligations. Berk et al. (2000) distinguish between purely technical risk in early stages and a diverse sources of risk in later stages. They show that the systematic risk component is strongest in early stages, justifying a larger risk premium. Elitzur and Gavious (2001) have a model with several stages, where the probability to attain each benchmark is determined by the entrepreneur's one shot effort choice. In their setting, optimal incentives contracts give rewards only upon completion the last stage, in contrast to our results.

The basic set-up of our model closely follows our earlier papers on venture funding and the financing of innovation (Bergemann and Hege (1998) and (2002)), where we studied the dynamics of the optimal contract, the role of hard claims, the impact of time consistency on the stopping decision, and distinguished between arm's length and relationship financing. The innovation in this paper is the inclusion of intermediate benchmarks.

The agency problem in our paper is also related to papers emphasizing the role of hard budget constraints in the funding of innovation, like Ambec and Poitevin (2001) and Qian and Xu (1998). Finally, a large literature has investigated capital structure design, and in particular the use of convertible securities, as a tool for the venture capitalist to force abandonment of unprofitable projects, and thus as an alternative or complementary instrument to staged financing. Recent papers have frequently looked at two-sided moral hazard situations between entrepreneur and venture capitalist, e.g. Casamatta (2000), Repullo and Suarez (2000) and Schmidt (2002).

The paper is organized as follows. The model is presented in Section 2

. The single stage project is reviewed in Section 3. The structure and efficiency of multi-stage projects without benchmarking is examined in Section 4. We then consider stage financing with benchmarking in Section 5. Section 6 discusses possible extensions and concludes.

## 2. The Model

We consider a project with uncertain return that needs continuous financing over several stages and that can be undertaken by an entrepreneur or agent with zero wealth. The project is financed by a venture capitalist providing up the necessary funds. The entrepreneur and venture capitalist are both risk-neutral and have a common discount rate $r > 0$. We introduce first the

technological characteristics of stage financing before turning to the contracting environment.

### 2.1. Project and Stages

The innovative project needs to go through $N$ sequential stages, which we denote by $n = 1, 2..., N$, to be successful. At the end of each stage, there is a discernible output, or *benchmark*. This may be a first research result, a key module, a prototype or a beta version, a product ready for mass production, and finally the production, distribution and marketing facilities necessary for the launch of operations. The stages are sequential in the sense that the successful completion of the stage $n-1$ is a technological prerequisite for entering into the stage $n$.

If the last stage is completed, the output is verifiable and a gain of $R$ is realized. The value of an incomplete project is zero (discussed in Section 6). We assume that it is worthwhile to undertake the project for at least one period, $R > c$.

The uncertainty of the project is resolved over time by a *discovery process*. In every stage, experimentation is needed to preserve the chances that the benchmark is eventually met, and experimentation requires time and money. If experimentation is undertaken in a given period $t$, then the stage of the project is successfully completed with probability $\lambda$, and costs $c\lambda$ to undertake. Therefore, the probability of completion of each stage per period is either $\lambda$ (if there is investment), or 0 (if there is none). These conditions are the same for every stage. The nature of uncertainty in our model essentially is about the time and investment needed within each stage.

The investment only influences the conditional probability of success in every period and independent of time. In particular, the investment flow does not influence the value of the successful realization, $R$.

As the experimentation process unfolds over time, agent and venture capitalist learn more about the prospects of the project. Suppose then that for each stage, experimentation is undertaken for a total of $T^n$ periods, where $T^n = T^1, T^2, ...T^N$ denotes the maximum duration or *horizon* for the completion of each of the stages $n = 1, 2, ...N$. The ex ante probability of successful completion of the first stage is thus $1 - (1 - \lambda)^{T^1}$. Since the completion of each earlier stage is required to move on, if there is no success in the $n^{th}$ stage within the horizon of $T^n$ periods, it means that the entire project is abandoned. The ex ante probability that the entire project will be successful is then

$$p_0^1 = \prod_{n=1}^{N} \left(1 - (1 - \lambda)^{T^n}\right) ,$$

We next determine the evolution of the posterior beliefs. Denote by $p_t^n$ the jointly held belief that the project will eventually be a success, held in the $t^{th}$ period of the $n^{th}$ stage based on continuous experimentation prior to $t$:

$$p_t^n = \left(1 - (1-\lambda)^{T^n - t}\right) \prod_{i=n+1}^{N} \left(1 - (1-\lambda)^{T^i}\right)$$

Thus, as long as the entrepreneur continues to invest in any given stage, the belief about the project's success is gradually diminishing as a result of the shorter number of chances of a yet successful experiment, $T^n - t$. However, once stage $n$ is successfully completed, the belief discontinuously jumps to a higher level. We get a stochastic seesaw pattern of the evolution of the following belief: the belief slopes down within each stage but has an upwards trend overall, representing the improvement in beliefs as the projects nears completion of the final stage. The timing of the jumps are stochastic due to the uncertain nature of each stage. (See *Figure 1*).

### 2.2. Moral Hazard and Financing

Entrepreneur and venture capitalists have initially the same assessment about the likelihood of success, which is given by the prior belief $p_0^1$. The funds are supplied by the venture capitalist, but they can only be allocated by the agent to generate the desired success $R$. The venture capitalist, however, cannot observe whether the funds are correctly applied to the experiment, and thus a moral hazard problem arises between financier and entrepreneur. The entrepreneur can in fact "shirk" and decide to divert the capital flow to her private ends, gaining a utility of $c\lambda$ in the process.[4] In contrast, the successful completion of any stage $n = 1, ..., N-1$ is observable and verifiable.

The venture capitalist proposes a contract to the entrepreneur which can be contingent on time, the capital provided by the investor, as well as on new agreements between the two parties. Then the entrepreneur accepts or rejects the proposal, implying that the venture capitalist captures the entire surplus of the project.[5] Because of the moral hazard problem of the financing, however, the contract cannot be made contingent on the use of the funds. The

[4] An equivalent interpretation of the moral hazard problem is that running the experiments requires effort, which is costly for the agent. By reducing the effort, the agent also reduces the probability of success and hence the efficiency of the employed capital.

[5] The opposite assumption about the distribution of bargaining power is made in Bergemann and Hege (1998) and (2002).

design of the contract has to ensure that incentive compatibility and individual participation constraints are satisfied.

We will argue below that *stage contracts* of the following form are the optimal arrangements in this environment. The venture capitalist proposes such a stage contract at the entry into each new stage $n$. The contract specifies the maximal stage duration $T^n$ to complete the stage $n$, and a dynamic schedule of monetary reward payments $s_t^n$ that the agent receives in the event of meeting the benchmark of the $n^{th}$ stage after $t$ periods.

The contract contains provisions effectively inhibiting the continuation of the project once $T^n$ periods have lapsed; the project will have to be irrevocably abandoned. In other words, we assume that the venture capitalist can choose $T^n$ and *commit* to the following horizon.[6]

## 3. A Single Stage

We prepare the ground by looking at the simple case where there is just a single possible stage, $N = 1$. This case is a version of the model analyzed in our earlier papers (Bergemann and Hege (1998)and (2002), where details for the expression of this Section can be obtained) with a simplified belief process, but with two important differences: first, the venture capitalist has the bargaining power; and second, the number of periods is determined by the venture capitalist's profit maximization objective rather than efficiency.

*Value of the Venture.* We denote by $V_t(T)$ the value of the project in stage $t$ if the total horizon comprises $T$ periods. Suppose the optimal number of financing periods is fixed at $T$ which we assume for now (and show later) to be finite. Note that in the first best, $T = \infty$ since we have assumed that $\lambda R - c\lambda \geq 0$. If the project should be funded once, it should receive funds indefinitely since the problem is stationary. Hence we obtain the value of the venture in the terminal period $T$ as $V_T(T) = \lambda R - c\lambda$, and in earlier periods recursively via the following dynamic programming equation:

$$V_t(T) = \max_{i_t \in \{0,\lambda\}} \left\{ i_t R - c i_t + \frac{1 - i_t}{1 + r} V_{t+1}(T) \right\} , \tag{3.1}$$

where $i_t \in \{0, \lambda\}$ is the venture capitalist's allocation of funds. Clearly, the linear form of the value function (3.1) indicates that it is optimal to invest at the level of $c\lambda$ for as long as $t \leq T$.

We consider the transition to the continuous time limit of our model, as in Bergemann and Hege (2002), from where details can be gleaned. Let $\Delta$ denote the time elapsed between two

[6] See Bergemann and Hege (2002) for an extensive discussion of renegotiation in this game.

periods $t$ and $t+1$. With this notation, (3.1) can be rewritten as:

$$V_t(T) = R\Delta\lambda - c\Delta\lambda + \frac{1-\Delta\lambda}{1+\Delta r} V_{t+\Delta}(T)$$

Letting $\Delta \to 0$ and solving yields the continuous-time expression of the project's value function at $t = 0$:

$$V_0(T) = \frac{1 - e^{-(r+\lambda)T}}{r+\lambda} (R - c)\,\lambda \tag{3.2}$$

The value function $V_0(T)$ offers an intuitive explanation. $c\lambda$ is invested in each period, and the prize $R$ is obtained with probability $\lambda$, conditional on no earlier discovery. The value of the project is then discounted with an effective factor of $r + \lambda$, compounding pure time, discounting $r$ and the probability of success $\lambda$ in each period. Total project uncertainty is then captured by the first term in (3.2), which can be understood as a stochastic discount factor over $T$ period, discounting the uncertain arrival time of the risky success.

*Incentive Contracts.* Since successful completion of the project is the only verifiable evidence on the agent's effort, the incentives provided to the entrepreneur should maximally discriminate with respect to the signal $R$. With the wealth constraint of the entrepreneur, the optimal contract is a share contract, where the agent receives a positive reward $s_t \geq 0$ if the project was a success and nothing otherwise. is The *minimal reward* $s_t$ of the entrepreneur is chosen so that she truthfully carries out the proposed investment policy. We consider only optimal share contracts from the venture capitalist's point of view, with full scope for intertemporal transfers, i.e. long-term contracts.

We start from the incentive compatibility constraint for the entrepreneur in the last period, which immediately leads to the last period requirement on the entrepreneur's reward, $\lambda s_T \geq c\lambda$, and hence an expected value to the entrepreneur of $E_T(T) = c\lambda$. Moving backwards in time, we obtain a sequence of value functions, denoted by $E_t(T)$, and characterized recursively by the incentive problem:

$$E_t(T) = \min_{s_t} \left\{ \Delta\lambda s_t + \frac{1-\Delta\lambda}{1+\Delta r} E_{t+1}(T) \right\} , \tag{3.3}$$

where $s_t$ is the minimum cash reward satisfying the entrepreneur's incentive constraint.[7] We notice the intertemporal structure of the problem. The incentives to divert for the agent

[7] The incentive constraint takes the form

$$\Delta\lambda s_t + \frac{1-\Delta\lambda}{1+\Delta r} E_{t+1}(T) \geq c\Delta\lambda + \frac{1}{1+\Delta r} E_{t+1}(T).$$

arise *(i)* via a contemporaneous effect, namely the utility from the diverted funds $c\lambda$, and *(ii)* via the dynamic effect that the contract continuation into the next period becomes more likely.

The solution $s_t$ of the minimization problem (3.3) delivers the expected value $E_t(T)$ the entrepreneur receives for a given funding policy, taking in to account the sequence of incentive constraints. Taking again limits as $\Delta \to 0$, the value function $E_t(T)$ of the entrepreneur and his reward in case of success is given by:

$$E_t(T) = s_t = c\lambda \frac{1 - e^{-r(T-t)}}{r}. \tag{3.4}$$

The compensation $s_t$ ensures that the entrepreneur employs the capital in every period towards the discovery process. The behavior of the shares $s_t$ over time is thus determined by an underlying *option problem.* The value of this particular option is determined as any regular option by the volatility of the underlying state variable (represented by the conditional probability $\lambda$) and the maturity (the remaining length of the funding, $T - t$). Therefore, two forces help to realign the interest of the entrepreneur with the ones of the investor: (*i*) sufficiently strong discounting and (*ii*) shares are decreasing over time and hence penalize late discovery.

*Optimal Stopping.* As the market for venture capital is competitive, in equilibrium the net value of the project will belong entirely to the venture capitalist. Prior to stopping the project, the venture capitalist will decide to fully fund the project; therefore, the decision on the optimal stopping time $T$ sufficiently summarizes the venture capitalist's investment policy. The venture capitalist's initial problem is then given by:

$$\max_{\{T,(s_t)\}} \{V_t(T) - E_t(T)\} \tag{3.5}$$

We have already characterized the optimal function of rewards $s_t$ in (3.4). To determine the optimum stopping time $T$, we investigate the maximum of (3.5) and obtain the following solution (from the first-order condition):

$$T = -\frac{\ln \frac{c}{R-c}}{\lambda}. \tag{3.6}$$

Thus, unlike the first best solution, which always would be to choose $T = \infty$, the presence of agency costs implies a reduction in the horizon that maximizes the venture capitalist's profits.

## 4. Funding without Benchmarks

We first consider the case where the venture capitalist has no benchmarking technology, i.e. no capacity to observe or verify the completion of earlier stages. Only the realization of the

last stage remains verifiable. Completion of all stages is indispensable for the project to create value, namely the prize $R$. Therefore, in this case, the venture capitalist can budget only for a single investment stage of total length $T$. For simplicity, we restrict this discussion to the case where there are just two stages, with $N = 2$, which is sufficient to analyze the structure of the solution. For this analysis, we make the additional assumption that

$$R > c\left(2 + \frac{1}{\lambda}\right). \tag{4.1}$$

This assumption guarantees that the venture capitalist is willing to offer a share $s_T$ that is sufficient to ensure the entrepreneur's incentives in period $T - 1$, even if the entrepreneur has not yet completed the first benchmark.

### 4.1. Value of the Venture

Let us then consider the value of the firm in this problem. Assume that the entrepreneur has successfully completed the first stage. We use the superscript $i \in \{1, 2\}$ for the value functions to indicate that the entrepreneur knows to be in stage $i = 1, 2$. If the entrepreneur knows to be in the second stage, the value of the venture is obtained recursively by the dynamic programming equation:

$$V_t^2(T) = \max_{i_t}\left\{\Delta i_t R - c\Delta i_t + \frac{1 - \Delta i_t}{1 + \Delta r} V_{t+\Delta}^2\right\}, \tag{4.2}$$

where $i_t \in \{0, \lambda\}$ indicates the funding policy of the venture capitalist. We know that the entrepreneur may try to complete the second stage for at most $T$ periods, and the first stage for $T$ periods. If the last stage is a failure in all $T$ periods, the final prize will be zero. Again, maximum investment $i_t = \lambda$ will be optimal in all periods. Thus,

$$V_t^2(T) = \frac{1 - e^{-(r+\lambda)(T-t)}}{r + \lambda}(R - c)\,\lambda\,.$$

Similarly, the agent's value function in the first stage is,[8]

$$V_t^1(T) = \max_{i_t \in \{0,\lambda\}}\left\{\frac{\Delta i_t}{1 + \Delta r} V_{t+\Delta}^2(T) - c\Delta i_t + \frac{1 - \Delta i_t}{1 + \Delta r} V_{t+\Delta}^1(T)\right\}. \tag{4.3}$$

Solving recursively, and considering the limit as $\Delta \to 0$ (see the Appendix for a derivation),

[8] Whether $V_t^2(T)$ or $V_t^1(T)$ is the true value is private knowledge of the entrepreneur, since he alone observes the first benchmark.

$$V_0^1(T) = \frac{\lambda}{r+\lambda}\left[\frac{1-e^{-(r+\lambda)T}}{r+\lambda} - Te^{-(r+\lambda)T}\right](R-c)\,\lambda \tag{4.4}$$

In this expression, the second term in the square bracket indicates the increasing loss from the following unconditional temporal limit: the later is the entry $t$ into the second stage, the shorter will be the remaining time $T-t$ to successfully complete this final round. The first two terms in expression (4.4) represent the stochastic discounting of the final value, which occurs over the two stages. Finally, the last term $Te^{-(r+\lambda)T}$ expresses loss from suboptimal exploitation of second stage.

### 4.2. Information Rent and Optimal Stopping

We turn then to the entrepreneur's rent in this case. The optimal continuation contract at the entry into the last stage is exactly as the contract would be for a single stage problem with a maximum of $T$ periods. That is, the entrepreneur can secure herself at least a rent of

$$E_t^1(T) = c\lambda\frac{1-e^{-r(T-t)}}{r} \tag{4.5}$$

Since only the entrepreneur observes whether the first benchmark has been attained or not, asymmetric information between venture capitalist and entrepreneur emerges as the project is undertaken - the entrepreneur knows whether he is of "type 1" - still trying to meet the first benchmark - or already of "type 2" - i.e., advanced to the second stage -, while the venture capitalist must design a contract that is incentive-compatible for one or for both types. Clearly, the project cannot succeed if it is not incentive compatible at least initially for the type 1, and later for type 2. It is intuitive that, as the funding horizon $T$ draws to a close, it is easier to ensure the incentives of type 2 than of type 1, who is in a more remote position, i.e. still two benchmarks away from final success. The critical question is, therefore, whether the optimal contract will provide incentives for both types throughout, or whether it will abandon the type 1 entrepreneur at some point and only provide incentives to the more advanced type 2.

We will show in the Appendix that under assumption (4.1), the lower bound of the entrepreneur's value in (4.5) represents at the same time the value function of the agent in the optimal incentive-compatible contract, that provides incentive compatibility for *both* types for the longest time possible, namely for the first $T-1$ periods. Since maximal incentive compatibility can be ensured with a contract that costs no more than the lower bound (4.5) of the entrepreneur's value, this contract must be optimal.

Taking the difference of expressions (4.4) and (4.5) yields the venture capitalist's objective function as:

$$V_0^1(T) - E_0^1(T) = \frac{\lambda}{r+\lambda}\left[\frac{1-e^{-(r+\lambda)T}}{r+\lambda} - Te^{-(r+\lambda)T}\right](R-c)\,\lambda - c\lambda\frac{1-e^{-rT}}{r}$$

The first-order condition yields the solution, which is given as the solution $T$ of

$$Te^{-\lambda T} = \frac{c}{\lambda\,(R-c)} \tag{4.6}$$

There are then two cases to be considered: either the project has a positive value for the venture capitalist and the optimal project horizon corresponds to the larger solution of (4.6) which is the only candidate for a maximum. Or, if the project is "poor", i.e. $R$ is small relative to $c$, it may be optimal to choose $T = 0$.

## 5. Stage Financing

In this Section, we assume that the venture capitalist is able to observe and verify the completion of the first stage. The optimal contract is then a succession of stage financing contracts: only after successful completion of the first stage will the contract for the second stage be drafted. Contingent stopping, after failure in the first stage, is a valuable, real option in this case (as we will show), ensuring that continuing finance is only taking place if there is success in the preceding stage.

### 5.1. Value of the Venture and Agency

Proceeding again in a backwards fashion from the last stage, the value of the venture in the last stage, $V_t^N(T^N)$, corresponds exactly to the value in the single stage problem expressed in, where the optimal funding horizon in the stage $N$ will now be denoted as $T^N$.

Consider then the value function in the penultimate stage. This value is a function of both the current duration $T^{N-1}$ and the last stage duration $T^N$. To keep the notation short, let $\mathbf{T}^n = (T^n, T^{n+1}, ...T^N)$ denote the vector of the durations of the remaining stages in stage $n$. The value function is recursively determined as

$$V_t^{N-1}(\mathbf{T}^{N-1}) = \max_{i_t\in\{0,\lambda\}}\left\{\Delta i_t V_0^N(T^N) - c\Delta i_t + \frac{1-\Delta i_t}{1+\Delta r}V_{t+\Delta}^{N-1}(\mathbf{T}^{N-1})\right\} \tag{5.1}$$

Taking again limits as $\Delta \to 0$ and solving recursively, this value can be expressed as

$$V_0^{N-1}(\mathbf{T}^{N-1}) = \frac{1-e^{-(r+\lambda)T^{N-1}}}{r+\lambda}\left(V_0^N(T^N)-c\right)\lambda \tag{5.2}$$
$$= \frac{1-e^{-(r+\lambda)T^{N-1}}}{r+\lambda}\left(\frac{1-e^{-(r+\lambda)T^{N}}}{r+\lambda}(R-c)\lambda - c\right)\lambda\,,$$

where the last equation is obtained after using the expression for the last stage derived in Section 3. More generally, the value function for $n$ stages can be stated as:

$$V_0^1(\mathbf{T}^1) = \left(\prod_{n=1}^{N}\lambda\frac{1-e^{-(r+\lambda)T^n}}{r+\lambda}\right)(R-c) - \sum_{i=1}^{N}\left(\prod_{n=i}^{N}\lambda\frac{1-e^{-(r+\lambda)T^n}}{r+\lambda}\right)c \tag{5.3}$$

Clearly, as we construct recursively the value function of the multi-stage problem $V_0^{(n)}(T^n, ..., T^N)$, we find that the value function exhibits the following time pattern: with the completion of one stage, synonymous to entry into the next stage, the value experiences a discontinuous jump upwards; but within each stage, as the time runs towards the horizon set for its completion, the value is decreasing. Thus, the value function follows a stochastic seesaw pattern with an upwards drift just as the belief function does.

We turn then to the entrepreneur's rent at entry into the last stage. The optimal continuation contract at the (stochastic) entry point into the last stage is exactly as the contract would be for a single stage problem. That is, the entrepreneur can secure herself at least a rent of $E_0^{N)}(T^N) = c\lambda\frac{1-e^{-rT^N}}{r}$, where $T^N$ is the funding horizon of the last stage. In fact, this rent is independent of the exact time when the last stage is reached. Thus, since the problem at the beginning of the last period is isomorphic to the single stage problem investigated above, the venture capitalist's preferred solution will be the same. The venture capitalist designs this contract to maximize $V_0^N(T^N) - E_0^N(T^N)$, and the solution will be as in (3.6), $T^N = -\frac{1}{\lambda}\ln\frac{c}{R-c}$.

The situation becomes more complicated though as we move backwards in time. Consider the penultimate stage. Since the successful completion of this stage is verifiable, the agent can be paid a reward upon meeting the benchmark of this stage, and this reward can be conditional on the timing of the success. We will denote this time-contingent reward by $s_t^{N-1}$, where $t$ is the period within the second to last stage where the agent meets the benchmark. Moreover, the agent knows that success carries with it the implicit compensation of moving on to the last stage, with its information rent $E_0^N(T^N)$. This information rent has an incentive effect in the second to last period. As it turns out, the size of this incentive effect, relative to the required incentive payment schedule within the stage, leads to an important distinction in the construction of the agent's value function.

### 5.2. Always Immediate Incentives

If $T^{N-1}$ periods have passed without discovery, the project is liquidated, and the agent gets nothing, i.e. $E^{N-1}_{T^{N-1}+\Delta}(\mathbf{T}^{N-1}) = 0$. Therefore, and considering again the continuous-time case in the last period of stage 1, the entrepreneur expects a rent of

$$E^{N-1}_{T^{N-1}}(\mathbf{T}^{N-1}) = \lambda \left[ s^{N-1}_{T^{N-1}} + E^{N}_{0}(T^{N}) \right] \tag{5.4}$$

In fact, incentive compatibility requires that this expected value be larger than $c\lambda$, which is the entrepreneur's option value from shirking. Hence if $\lambda E^{N}_{0}(T^{N}) < c\lambda$, then $s^{N-1}_{T^{N-1}} > 0$ is required. This condition can be rewritten as:

$$\lambda < \hat{\lambda} \equiv \frac{r}{1 - e^{rT^{N}}} \tag{5.5}$$

Note that $\lambda > \hat{\lambda}$ will generally hold if *(i)* $\lambda > r$ and if *(ii)* $T^N$ is large enough. In essence, the promise of the minimum of future information rents suffices by itself to guarantee incentive compatibility, and further contemporaneous incentives are not needed.

We consider then first the case where $\lambda < \hat{\lambda}$, since this case is easier to analyze, leaving the complementary case to the next subsection. With this condition, a positive reward payment is needed whenever the agent successfully completes one of the stages. We denote by $s^{n}_{t} > 0$ the *minimum* reward required upon completion of the $n^{th}$ stage in period $t$ of stage $n$. The following simple observation is important to understand the structure of feasible contracts if $\lambda < \hat{\lambda}$:

**Lemma 1.** *Feasible stage financing contracts require that the agent be paid with an immediate cash reward of at least* $s^{n}_{t}$.

*Proof: See Appendix.*

In particular, it is not sufficient to pay the agent with equity or other contingent claims on $R$, that can only be cashed in if all stages are successfully completed. *Immediacy* of incentive rewards is the key observation here, and the reason for this immediacy is that a hold-up problem would arise otherwise: if the payments to the agent were contingent on achieving further benchmarks, they would have incentive effect in the next stage. As a result, the investor would cut back the rewards offered ex post, at entry into the subsequent stage. Anticipating this reduction ex ante, the agent would find it more attractive to shirk rather than to work. The investor needs to *commit* to the level of incentive payments required ex ante, and pledging

immediate rewards that are not contingent on further achievements are the obvious way to do it.

This observation is interesting because in principle, one would expect that the *highest power of incentives* could be attained by making all rewards *contingent on completion of the final benchmark:* in our nested model, the last stage has the highest information value on effort, since its completion means that the agent has truthfully invested in all prior stages. By contrast, success in any earlier stage gives information regarding the agent's effort only up to that benchmark. Although, incentives contingent on meeting the final benchmark appear to be the cheapest device for the investor, since actual reward payments will have to be paid only if the entire project is successful. Indeed, Gavious and Elitzur (2001) obtain a result that incentives should be based on the accomplishing of the last benchmark in a different model of stage financing, which shares with our model the feature that stages are nested.[9]

The fact that we obtain a different result here clearly points to a trade-off in our model, a trade-off between the advantage high-powered incentives (which pleads in favor of postponing compensation) and the need to be time-consistent. An important practical implication of our immediacy result is:

**Corollary 1.** *Contracts can guarantee immediate incentives by providing all the funding needs for any stage up-front.*

Up-front financing in the current stage, and hence the build-up of potentially important cash reserves, is not the only way to achieve the required immediate rewards; other forms are possible as well. But it is the way that is frequently observed in practice. Stage financing typically implies that cash is raised discontinuously, at the beginning of each stage, while the cash outflow from the venture is often much smoother. Our analysis interprets this discontinuous evolution in the venture's cash position as a deliberate choice to guarantee the agent's required information rents. In our view, the cash paid up-front is cash at the discretion of the agent, and this discretion protects the agent against any possible hold-up by the investor. Notice that a lower bound of the entrepreneurs' information rent at any given moment is the remaining stream of cash in-flows up to the maximum horizon of the current stage; and any excess cash that the entrepreneur holds in the current stage, after reaching the benchmark early, will be used in the subsequent stage since the new can easily provide the right incentives for mutual advantage.

[9] The logic of making compensation contingent on the most informative output is also reminiscent of Innes (1990) moral hazard model.

We will continue with the analysis of the optimal contract in the case of $\lambda < \hat{\lambda}$, and in particular determine the optimal horizon $T^n$ in a typical stage $n$. Starting from condition (5.4) in period $T^n$ and moving backwards in time, suppose the agent accomplishes the benchmark of this stage in period $t < T^n$. When discovery is made, the entrepreneur will get a future rent that is at least equal to $E_0^{n+1}(T^{n+1}, T^{n+2}, ...)$, and this regardless of the period in which discovery occurs. We assume that immediate incentives are always needed, i.e. $s_t^n > 0$ is necessary throughout to satisfy this incentive constraint. Taking this into account, the continuous time incentive constraint at any time must satisfy

$$E_t^n(\mathbf{T}^n) = \lambda \left[ s_t^{(n)} + E_0^{n+1}(\mathbf{T}^{n+1}) \right] + (1-\lambda) E_t^n(\mathbf{T}^n) \geq c\lambda + E_t^n(\mathbf{T}^n) \tag{5.6}$$

Since $s_t^n > 0$, clearly the inequality in (5.6) will be binding. Hence, after solving recursively,

$$E_t^n(\mathbf{T}^n) = \lambda c \lambda \frac{1 - e^{-rT^n}}{r} \tag{5.7}$$

As we would expect, the entrepreneur's rent is monotonically decreasing within the first stage, i.e. $E_t^n(\mathbf{T}^n) > E_\tau^n(\mathbf{T}^n)$ for $\tau > t$.

We can then determine the optimal budgeting decision in this case. The investor recursively solves the contract design problem and chooses the optimal funding horizon $T^n$ by maximizing her net value $V_0^n(\mathbf{T}^n) - E_0^n(\mathbf{T}^n)$. Substituting and solving the maximization problem (7.7) yields the following results:

**Proposition 1.** *Suppose immediate rewards are needed in every period, i.e. $s_t^n > 0$ for all $n$ and $t$. Then the optimal horizon in stage $n$ is given by:*

$$T^n = -\frac{1}{\lambda} \ln \left( \frac{c}{V_0^{n+1}(\mathbf{T}^{n+1}) - c} \right) \tag{5.8}$$

*The optimal horizon $T^n$ is strictly increasing in $n$.*

*Proof: See Appendix.*

### 5.3. Implicit Incentives

We turn now to the case where $\lambda > \hat{\lambda}$. Consider the entrepreneur's incentives in the last period of the penultimate stage. If the benchmark is accomplished, the entrepreneur will get a future rent that is at least equal to

$$E_0^N(T^N) = c\lambda \frac{1 - e^{-rT^N}}{r}.$$

Thus, the entrepreneur's current value can be evaluated as

$$E^{N-1}_{T^{N-1}}(\mathbf{T}^{N-1}) = \max\left\{c\lambda, \lambda c\lambda\frac{1-e^{-rT^N}}{r}\right\}.$$

But in the case $\lambda > \hat{\lambda}$, we know the entrepreneur's implicit incentives given by the prospect of moving on the last stage exceed the minimal incentives required, and hence

$$E^{N-1}_{T^{N-1}}(\mathbf{T}^{N-1}) = \lambda c\lambda\frac{1-e^{-rT^N}}{r},$$

meaning that $s^{N-1}_{T^{N-1}} = 0$ is sufficient.

Taking this into account, clearly incentive compatibility will be satisfied in the last period of stage $N-1$. Moving backwards in time, in each prior period of this stage incentive compatibility requires that:

$$E^{N-1}_t(\mathbf{T}^{N-1}) = \lambda\left[s^{N-1}_t + c\lambda\frac{1-e^{-rT^N}}{r}\right] + (1-\lambda)E^{N-1}_t(\mathbf{T}^{N-1}) \geq c\lambda + E^{N-1}_t(\mathbf{T}^{N-1}) \quad (5.9)$$

Inequality (5.9) reveals that $s^{N-1}_t = 0$ is sufficient as long as

$$\lambda\left(c\lambda\frac{1-e^{-rT^N}}{r} - E^{N-1}_t(\mathbf{T}^{N-1})\right) \geq c\lambda \quad (5.10)$$

Let us consider any general funding stage $n$. Now, as we would expect, the entrepreneur's rent is monotonically decreasing *within* each stage, i.e. $E^n_t(\mathbf{T}^n) > E^n_\tau(\mathbf{T}^n)$ for $\tau > t$. To see this, note that $E^n_t(\mathbf{T}^n) \leq \frac{c\lambda}{r}$ for all $t \in [0, T^n]$, i.e. the agent's rent will never exceed the value of a perpetual stream of funding of $c\lambda$ (since this is the maximum rent she can divert). Thus, there will be at most a single transition period where inequality (5.10) switches from being violated (and hence $s^n_t > 0$ for all periods prior to the transition period) to being satisfied (hence $s^n_t = 0$). We denote this transition period by $\hat{t}^n$. That is, $\hat{t}^n$ is the first period where implicit incentives are wholly sufficient to guarantee incentive compatibility. We can then express the value of the information rent as

$$E^n_t(\mathbf{T}^n) = \begin{cases} \lambda\left(\frac{1-e^{-(r+\lambda)(t-\hat{t}^n)}}{r+\lambda}\right)E^{n+1}_0(\mathbf{T}^{n+1}) & \text{if } t \geq \hat{t}^n \\ c\lambda\frac{1-e^{-r(\hat{t}^n-t)}}{r} + e^{-r(\hat{t}^n-t)}\lambda\left(\frac{1-e^{-(r+\lambda)(T^n-\hat{t}^n)}}{r+\lambda}\right)E^{n+1}_0(\mathbf{T}^{n+1}) & \text{if } t < \hat{t}^n \end{cases} \quad (5.11)$$

An investigation of the investor's problem allows us to establish the following key insight:

**Proposition 2.** *The optimal stopping time $T^n$ will be such that the reward function $s^{(n)}_t$ is initially strictly positive, for an initial interval of $t \geq 0$.*

*Proof: See Appendix.*

The observation that the rewards $s_t^n$ are initially strictly positive is equivalent to saying that the transition point $\hat{t}^n$ in equation (5.12) is strictly positive. This observation allows us to write the

agent's value, at entry into stage $n$, as:

$$E_0^n(\mathbf{T}^n) = c\lambda \frac{1-e^{-r\hat{t}^n}}{r} + e^{-r\hat{t}^n}\lambda\left(\frac{1-e^{-(r+\lambda)(T^n-\hat{t}^n)}}{r+\lambda}\right)E_0^{n+1}(\mathbf{T}^{n+1}) \tag{5.12}$$

In other words, the experimentation will be long enough for the agent initially to receive enough contingent compensation via information rents in future stages. As time is running out in the current stage, however, the option value that the agent obtains by deviating diminishes, and with it the need to compensate for the loss in this value. In other words, once $\hat{t}_1$ is passed, the experimentation in the current stage comes essentially free for the investor, since the implicit promise of future rents is sufficient; but that second phase cannot be prolonged without increasing the first, costly phase.

Note that the distance $T^n - \hat{t}^n$ is determined in a recursive fashion, via condition of (5.10), and is therefore independent of $T^n$. We adopt the notation $I^n \equiv T^n - \hat{t}^n$ for the duration of this second phase. The agent's value function $E^n$ follows again the, by now familiar stochastic seesaw pattern, decreasing within each stage and upwards jumping at the entry into a new stage (see *Figure 2*). After substituting (5.2) and (5.12) and transiting to continuous time, the investor's problem of maximizing $V_0^n(\mathbf{T}^n) - E_0^n(\mathbf{T}^n)$ can be analyzed. We find:

**Proposition 3.** (i) *Suppose $\lambda > \hat{\lambda}$. Then there is at least one stage, namely the penultimate stage, where implicit incentives are eventually sufficient.*

(ii) *In a stage where implicit incentives are eventually sufficient, the optimal total horizon $T^n$ is given by:*

$$T^n = I^n + \frac{1}{\lambda}\ln\frac{V_0^{n+1}(\mathbf{T}^{n+1})e^{-(r+\lambda)I^n}}{c - rE_0^{n+1}(\mathbf{T}^{n+1})\left(\frac{1-e^{-(r+\lambda)I^n}}{(r+\lambda)}\right)}, \tag{5.13}$$

*where $I^n$, the duration of the phase where implicit incentives are sufficient, is:*

$$I^n = -\frac{1}{r+\lambda}\ln\left(\frac{c\,(r+\lambda) - rE_0^{n+1}(\mathbf{T}^{n+1})}{\lambda E_0^{n+1}(\mathbf{T}^{n+1})}\right). \tag{5.14}$$

(iii) *$I^n$ is increasing in $n$, and $I^n > 0$ in the final stages of the project (except for the last stage), but not necessarily in the early stages. If $I^n > 0$ in stage $n$, then the funding horizon $T^n$ is strictly larger compared with the same stage with only immediate incentives ($\lambda < \hat{\lambda}$).*

*Proof: See Appendix.*

The important insight of this analysis is that the availability of implicit incentives will indeed increase the funding horizon. Since the funding horizon is always too short compared to the first-best (where it is infinite), and since the funding horizon is in principle increasing as more stages are completed and the overall value of the project increases, this is a welcome mechanism to overcome agency-driven capital budgeting constraints. Recall that the funding horizon in the last stage $T^N$, will be the same with immediate or implicit incentives. Thus, a longer horizon in earlier stages is unambiguously good news. Intuitively, this increase in the funding horizon is due to the fact that part of the compensation in the current stage need not be provided contemporaneously. It is implied by the continuation values, making an extension of the current round less costly in terms of information rents. This effect may be so strong that the total funding horizon is not monotonically increasing from one stage to the next.

It is also intuitive why $I^n$ is increasing over time. Since the agent's value must be strictly increasing from stage to stage (otherwise the incentive constraint would be violated), the potential power of implicit incentives is also increasing over time. Thus, frequently implicit incentives will be prevalent in the last stages of a project (while still requiring immediate cash incentives in the early periods of each stage, according to Lemma 2). But in the early stages, it is more likely that the compensation must rely on immediate cash incentives in every period.

### 5.4. Synopsis

We can summarize our observations as follows: if the investor cannot use benchmarks, then the capital budget allocated to the project will be severely curtailed. The information rent has to be compounded over the entire horizon. Moreover, the experimentation horizon is defined for the project as a whole, and not fine-tuned to every stage. This will often lead to unwanted distortions in time allocations between various stages. For example, if most of the horizon has elapsed but the agent did not yet succeed in meeting the first benchmark, he will nevertheless continue to run the experiments for the first stage. The problem of continuing for too long in the first stage is made worse by the fact that the horizon for the subsequent stages will automatically be shortened, since the continuation budget becomes history-dependent.

Benchmarking introduces a sequential real option to abandon. In the context of our dynamic agency model, the option value of these imbedded real options is comprised of the following four effects:

First, since the project is abandoned once a benchmark is not met within its pre-defined horizon, the information rent of the agent is dramatically reduced. In the simplest and perhaps most instructive case (immediate incentives), the compounding period of the information rent is shortened to the maximal duration of a single stage, rather than the maximal duration of the entire project.

Second, benchmarking makes it possible to define optimal and intertemporally consistent research budgets (research horizons) for every single stage. An important advantage is that these budgets will be independent of the history of delays and cost overruns in past financing rounds.

Third, the optimal research horizon increases from one stage to the next. Early stages should stop relatively rapidly because the chance for an overall success is remote. As more benchmarks are realized, the value of the project increases, and it becomes rational to persevere for longer. This finding explains that the first steps in a research project are the riskiest (and deserve the application of a higher risk-adjusted discount rate). Importantly, this decreasing trend in research risk is explained as an endogenous decision in a dynamic model, and not by technological characteristics. We would obtain a similar result if, say, $\lambda$ was larger in early stages, making them a priori more likely to succeed.

A fourth, and more subtle, option value of benchmarking is that it permits the use of implicit incentives comprised by the relational promise of future contingent financing rounds if earlier rounds are successfully completed. The promise of future information rents serves as a powerful incentive device in earlier stages, making the extension of the funding horizon in earlier stages cheaper. The power of implicit incentives will notably be strong if the project's success probability is high relative to the time discount effect ($\lambda > r$). This is a welcome effect from a social point of view since the presence of agency costs means that all research horizons are too short when compared with the first best. The interdependence between the optimal sequencing of research horizons and the implicit incentive effect is perhaps the most intriguing finding of our analysis.

The benefits of benchmarking are reflected in the following comparative findings on the project duration:

**Proposition 4.** (i) *The total research horizon over all stages will always be strictly larger if stage financing is used compared with funding without benchmarking.*

(ii) *If the project is relatively poor, $\frac{R}{c} < 1 + e$, then the last stage of stage financing alone will have a longer funding horizon than the entire horizon if there is no benchmarking.*

*Proof: See Appendix.*

Thus, for a given research budget amounting to the total expected outlays at the beginning (real investments and compensations for the agent), the research horizon and the success probability will be the larger, the better defined, and the better monitored the intermediate benchmarks are. Its initial value and return to investors, as well as the value appreciation of the portfolio company from one financing round to the next, should be increasing functions of the benchmarking intensity.

## 6. Robustness and Conclusion

This paper investigated the provision of venture capital in a research venture with sequential development stages. The binary outcome of each stage of the project is uncertain, and a steady investment flow is needed to safeguard the chances for success in each stage. The entrepreneur controls the application of the funds which are provided by the venture capitalist.

The optimal compensation of the entrepreneur is akin to a nested sequence of option contracts. The options express the value of the intertemporal incentive constraint, and the relational promise of future options works to alleviate the pressure to provide contemporaneous performance-related cash incentives.

A natural extension is to consider what happens if there are intermediate values of the project, that is if upon realization of the intermediate benchmark $n$, a positive value $R_n$ is realized if the project is unsuccessful in the next stage and hence abandoned after $n$ stages. In this case, the combined result of the current and all previous intermediate stages, worth $R_n$, will be sold to outsiders for a cash payment of $R_n$ at the time of the sale. The project will then be irreversibly terminated, since neither the incumbent entrepreneur nor the acquiring outsiders will have a possibility to complete the missing stages. We assume of course that the successive intermediate values of the project satisfy, $R_1 < R_2 < .... < R_{N-1} < R_N$, and that in a perfect world, every stage until the final stage $N$ is worthwhile undertaking.

It is easy to see that this generalization has no impact on our finding whatsoever: As argued in Section 5, in principle the optimal incentive instrument would exploit the highest incentive power possible and grant a rent to the agent only upon completion of the last stage. But this leads frequently to a time consistency conflict between the ex ante level of required investments, and immediate cash compensations are needed to overcome the hold-up problem. Nothing changes in the structure of the optimal incanting contracts, when intermediate results are

introduced. The only effect is in the investor's objective function, and indeed the appreciation of the values $V^n$ from one stage to the next may be substantially reduced, and with it the increase in the optimal funding horizons form one stage to the next.

The paper focuses on the financing of venture projects, but the problem analyzed here is present in the financing of R&D in general. We show that the optimal funding horizon or research intensity in each of the sequential stages are derived endogenously, and identify the key determinants, namely future project risks and information rents and their interaction in the current stage. The present work contributes to the understanding of how agency costs and optimal research policy interact.

## 7. Appendix

**Derivation of Equation (4.4):** In discrete time, the value function can be written as follows (where we use the notation $\delta = \frac{1}{1+r}$ for the discount factor):

$$\begin{aligned} V_0^1(T) &= \delta\lambda V_1^2(T) + \delta(1-\lambda)V_1^1(T) = \delta\lambda \frac{1-\delta^{T-1}(1-\lambda)^{T-1}}{1-\delta(1-\lambda)} (R-c)\,\lambda + \\ & \quad \delta(1-\lambda)\left[\frac{1-\delta^{T-2}(1-\lambda)^{T-2}}{1-\delta(1-\lambda)} + \delta(1-\lambda)\frac{1-\delta^{T-3}(1-\lambda)^{T-3}}{1-\delta(1-\lambda)} + ....\right]\delta\lambda\,(R-c)\,\lambda \\ &= \frac{\delta\lambda}{1-\delta(1-\lambda)}\left[\frac{1-\delta^{T-1}(1-\lambda)^{T-1}}{1-\delta(1-\lambda)} - T\delta^{T-1}(1-\lambda)^{T-1}\right](R-c)\,\lambda \end{aligned}$$

For the transition to continuous time, we introduce again the notation $\Delta$, hence replace $\delta = \frac{1}{1+r}$ by $\frac{1}{1+\Delta r}$ and $\lambda$ by $\Delta\lambda$. Taking the limit as $\Delta \to 0$, (4.4) obtains.

**Derivation of Equation (4.5):** We develop the argument in discrete time. We denote by $E_t^i$ the entrepreneur's value function in $t$ if he is of type $i = 1, 2$, and drop the argument $T$ for simplicity. Suppose the entrepreneur is of type 1 in period $T-1$, i.e. he still has not completed the first stage. To ensure incentive compatibility for the last two periods, the last period reward $s_T$ must at least satisfy

$$E_{T-1}^1 = \delta\lambda^2 s_T + \delta(1-\lambda)c\lambda \geq c\lambda(1+\delta) \tag{7.1}$$

Note that the venture capitalist is willing to offer this compensation since, from assumption (4.1), the minimum reward $s_T$ satisfying (7.1) is such that $R - c > s_T$, so the net expected profit of the venture capitalist is positive. The value function of the type 1 entrepreneur in period $t \leq T-1$ can also recursively be expressed as:

$$E_t^1 = \delta\lambda E_{t+1}^2 + \delta(1-\lambda)E_{t+1}^1 \geq c\lambda + \delta E_{t+1}^1, \tag{7.2}$$

where $E_T^1 = c\lambda$ is the value that a type 1 entrepreneur receives in the last period (only diverting the capital can be rational). Hence, to satisfy incentive compatibility of type 1,

$$E_t^2 \geq E_t^1 + \frac{c\lambda}{\delta\lambda} \tag{7.3}$$

is required, and the sequence $s_t, s_{t+1}$ must be chosen so as to satisfy (7.3). Note also that

$$s_T = c\lambda + \frac{c\lambda}{\delta\lambda} \tag{7.4}$$

satisfies this condition (7.3) in period $T-1$ and, hence, also (7.1) with equality. Consider then incentive compatibility for type 2 which requires that

$$E_t^2 = \lambda s_t + \delta(1-\lambda)E_{t+1}^2 \geq c\lambda + \delta E_{t+1}^2 \Leftrightarrow s_t \geq c + \delta E_{t+1}^2\,. \tag{7.5}$$

Using (7.4) and (7.3), it is possible to construct recursively a sequence of rewards $s_{T-1}, s_{T-2}, ...$ such that the incentive compatibility constraints (7.2) and (7.5) for types 1 and 2 hold with equality, everywhere for $t \leq T-1$ and for type 2 at $t = T$. Then, by substituting (7.1) (holding with equality) into (7.2) (also holding with equality), we can recursively solve for the type 1 value function as

$$E_t^1 = c\lambda \frac{1-\delta^{T-t}}{1-\delta}, \tag{7.6}$$

and since (7.6) corresponds to the lower bound (4.5) and guarantees incentive compatibility for both types in the maximum number of periods, this contract must be optimal.

**Proof of Lemma 1:** Suppose that the reward with expected value of $s_t^n$ is paid only conditional on termination of the next benchmark $n+1$, for example, by paying a cash payment of $\hat{s}^{n+1} = \frac{r+\lambda}{1-e^{-(r+\lambda)T^{n+1}}} s_t^n$ if the benchmark of stage $n+1$ is completed. Recall that $E_0^{n+1}(\mathbf{T}^{n+1})$ is the minimum incentive compatible value of the agent upon entry in the subsequent stage. Suppose that, in the contract signed at entry in stage $n+1$, the investor pledges new incentive compatible rewards worth $E_0^{n+1}(\mathbf{T}^{n+1})$ . But since $s_t^n$ is paid only conditional on success at least in the new stage $n+1$, this reward has incentive power in stage $n+1$ as well: the effective value that the entrepreneur expects when never deviating in the new stage is worth $s_t^n + E_0^{n+1}(\mathbf{T}^{n+1})$. It follows that the investor can propose another contract for stage $n+1$, where every success reward is reduced by $\hat{s}^{n+1}$, and yet this contract is incentive-compatible ex post. Thus, incentive payments worth $s_t^n + E_0^{n+1}(\mathbf{T}^{n+1})$ are required ex ante, but only $E_0^{n+1}(T^{n+1})$ will be offered ex post, showing a contradiction.

Finally note that $\lambda > \hat{\lambda}$ is the critical condition: this condition ensures that immediate compensation is required in every period of the last stage, and since $E^{n-1} < E^n$ and $T^{n-1} < T^n$, it follows immediately that the same holds in all prior stages. **QED.**

**Proof of Proposition 1:** The optimal stopping horizon $T^n$ is obtained by substituting (5.3) and (5.7) into the optimization problem:

$$\max_{T^n} V_0^n(\mathbf{T}^n) - E_0^n(\mathbf{T}^n) \tag{7.7}$$

and solving for the first-order condition. The fact that $T^n$ is strictly increasing in $n$ is an immediate consequence of the fact that $V_0^{n+1} > V_0^n$. **QED.**

**Proof of Proposition 2:** Assume to the contrary that $s_t^n = 0$, implying $\hat{t}^n = 0$. Then, according to (5.11), the problem (**??**) becomes:

$$\max_{T^n} \left( \frac{1-e^{-(r+\lambda)T^n}}{r+\lambda} \right) \lambda \left( V_0^{n+1}(\mathbf{T}^{n+1}) - E_0^{n+1}(\mathbf{T}^{n+1}) \right) \tag{7.8}$$

and since $V_0^{n+1}(\mathbf{T}^{n+1}) - E_0^{n+1}(\mathbf{T}^{n+1}) > 0$ as a consequence of recursive optimization, there is no finite solution $T^n$ of (7.8). But then observe that, as $T^n \to \infty$, the agent can secure a perpetual rent of $c\lambda$ simply by always deviating. Thus, incentive compatibility of the contract in stage 1 requires that

$$\lim_{T^n \to \infty} E_0^n(\mathbf{T}^n) \geq \frac{c\lambda}{r}. \tag{7.9}$$

Finally, note that $E_0^n(\mathbf{T}^n) \geq \frac{c\lambda}{r}$ in (5.12) requires that $\hat{t}^n \to \infty$ as well, contradicting our assumption that $\hat{t}^n = 0$. **QED.**

**Proof of Proposition 3:** In a stage where $I^n > 0$, the investor's initial problem can be written as (dropping bracket arguments for simplicity):

$$\max_{\hat{t}^n} \left( \frac{1 - e^{-(r+\lambda)I^n} e^{-(r+\lambda)\hat{t}^n}}{r+\lambda} \right) \lambda V_0^{n+1} - \left( \frac{1 - e^{-r\hat{t}^n}}{r} \right) c\lambda - e^{-r\hat{t}^n} \left( \frac{1 - e^{-(r+\lambda)I^n}}{r+\lambda} \right) \lambda E_0^{n+1} \tag{7.10}$$

Maximizing the objective function (7.10) and solving for $T^n$ gives (5.14). The solution of $I^n$ is obtained by solving the equivalent condition for (5.10) for an arbitrary stage $n$, and evaluated with equality at $t = \hat{t}^n$,

$$\lambda \left( E_0^{n+1} - \lambda \left( \frac{1 - e^{-(r+\lambda)I^n}}{r+\lambda} \right) E_0^{n+1} \right) = c\lambda \tag{7.11}$$

Next, $E_0^n(\mathbf{T}^n)$ must be strictly increasing in $n$, since otherwise the incentive constraint could not hold at the beginning of each stage. Inspection of (5.14) shows then that $I^n$ is strictly increasing as well. Then consider the first-order condition of problem (**??**):

$$e^{-(r+\lambda)T^n} \lambda V_0^{n+1} = e^{-r(T^n - I^n)} \left( c\lambda - r \frac{(1 - e^{-(r+\lambda)I^n})}{r+\lambda} \lambda E_0^{n+1} \right) \tag{7.12}$$

Since we are looking at a stage where $I^n > 0$, it must be the case that

$$e^{-rI^n} \left( \frac{1 - e^{-(r+\lambda)I^n}}{r+\lambda} \right) \lambda E_0^{n+1} > e^{-rI^n} \frac{1 - e^{rI^n}}{r} c\lambda \tag{7.13}$$

which implies that, for a given $T^n$,

$$e^{-r(T^n - I^n)} \left( c\lambda - r \frac{(1 - e^{-(r+\lambda)I^n})}{r+\lambda} \lambda E_0^{(n+1)} \right) < e^{-rT^n}. \tag{7.14}$$

But then compare the first-order condition (7.13) to condition (7.7) in the case of immediate incentives. Taking into account (7.14) then implies that for the same stage $n$, with identical

$V_0^{n+1}$ on the left hand side, the optimal $T^n$ must be larger in the case of implicit incentives compared with the case of immediate incentives. **QED.**

**Proof of Proposition 4:** *(i)* Assume that a project with $N = 2$ will be stage financed, but contains the following contract provision: the duration of the second stage is a 1:1 decreasing function of the effective length of the first stage. That is, if the duration of stage 2 is $T^2$ when discovery of the first benchmark is immediate at $t = 0$, then the duration of stage 2 will be shortened to $T^2 - \tau$ if the first-stage discovery is alone made at $\tau > 0$.

Assume that $\lambda < \hat{\lambda}$. The value function of the project is identical to (4.4), and because of $\lambda < \hat{\lambda}$, the agent's value function is identical to (5.7). Let $T = T^1 + T^2$. The objective function is then:

$$\frac{\lambda}{r+\lambda}\left(\frac{1-e^{-(r+\lambda)T}}{r+\lambda} - Te^{-(r+\lambda)T}\right)(R-c)\,\lambda - c\lambda\frac{1-e^{-r(T-T^1)}}{r} \tag{7.15}$$

and differentiation with respect to $T$ leads to the following first-order condition:

$$\lambda T e^{-\lambda T} = \frac{c}{R-c}e^{rT^1} \tag{7.16}$$

Comparison of (7.16) and (4.6) clearly shows that the total funding horizon $T$ is larger than without benchmarking. It is easy to extend this argument to $N > 2$. Moreover, note that the total funding horizon is weakly larger if $\lambda > \hat{\lambda}$, so the result holds a fortiori in this case.

*(ii)* As for the comparison of the length of the last stage under benchmarking, note that comparing (3.6) and (4.6) shows that

$$e^{-\lambda T^1} = \lambda T^2 e^{-\lambda T^2} = \frac{c}{R-c} \tag{7.17}$$

where $T^1$ is the solution of (3.6). Hence $T^2 > T^1$ iff $\lambda T^2 > 1$ in (4.6). Thus, for $\lambda T^1 = 1$, $T^2 = T^1$, and from (3.6), $\lambda T^1 = 1$ implies that $\frac{c}{R-c} = \frac{1}{e}$. Then assume $\lambda T^1 > 1$, implying $e^{-\lambda T^1} = \frac{c}{R-c} < \frac{1}{e}$. Hence also $\lambda T^2 e^{-\lambda T^2} = \frac{c}{R-c} < \frac{1}{e}$. Consider then the inequality $\lambda T^2 e^{-\lambda T^2} < \frac{1}{e}$. Rearranging and taking logs,

$$\lambda T^2 > \ln \lambda T^2 + 1$$

which can only hold if $\lambda T^2 > 1$, hence $T^2 > T^1$. **QED**.

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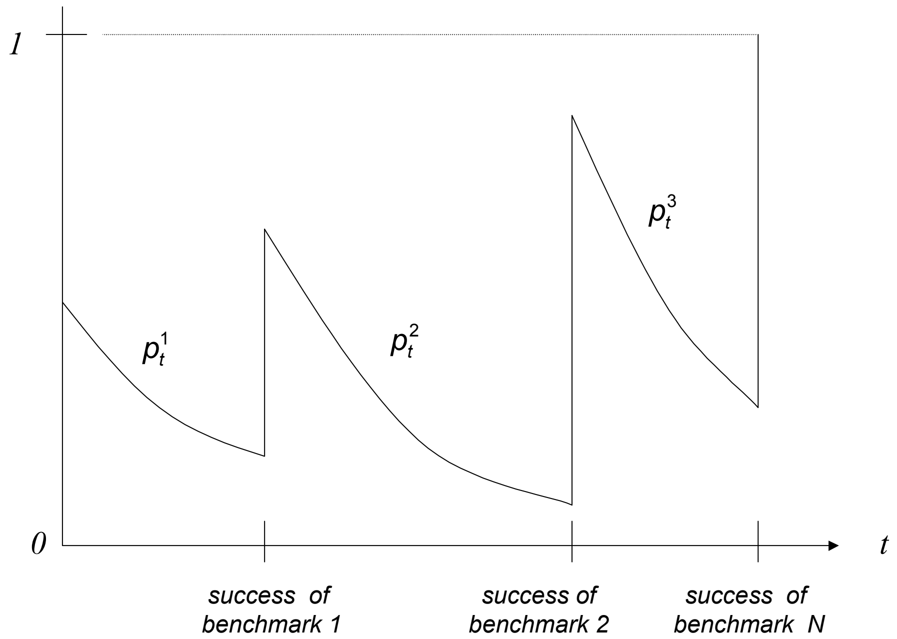


Figure 1: Evolution of final success probability. The success times of intermediate benchmarks are random.

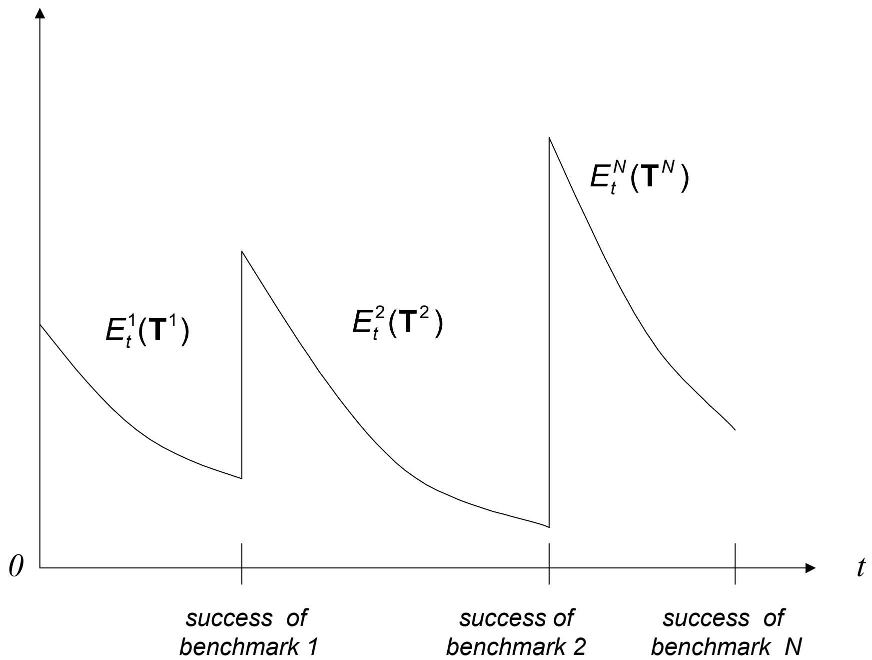


Figure 2: Evolution of the entrepreneur's value function. The success times of intermediate benchmarks are random.