Liquidity shocks, roll-over risk and debt maturity

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Goal:

Analyzing link between maturity mismatch & roll-over risk at individual-bank & market level in model with rich dynamic structure

How:

- Infinite horizon equilibrium model in which banks decide principal, interest rate & maturity of non-tradeable debt
- Why relevant at all?
  - Savers' time-preferences are subject to shocks $\rightarrow$ rationale for maturity transformation
  - Systemic liquidity risk $\rightarrow$ "normal" refinancing strategies fail and banks need *bridge financing* (BF)

Main result: Equilibrium maturity is inefficiently short

[Some BF constraints generate pecuniary externalities]

Key ingredients of the model

- Simplifying assumptions $\rightarrow$ small set of state variables
- Two opposite forces behind maturity (& leverage) decisions
  - Short maturity reduces savers' disutility from delaying consumption when desired
  - Crises force banks to rely on expensive BF

$$
\begin{tikzcd}
\text{E[Cost of liquidity in crises]} \arrow[r] & \text{Optimal debt structures} \arrow[d] \\
\begin{array}{c}\text{Refinancing market}\\ \text{frictions}\end{array} \arrow[u] & \begin{array}{c}\text{Refinancing needs}\\ \text{in crisis}\end{array} \arrow[l]
\end{tikzcd}
$$

- Externality: Coordinated increase in debt maturity (intensive margin) can decrease cost of funds, allowing banks to expand maturity transformation on the extensive margin

Related literature

- Closest connections:
  - Diamond-Dybvig (83): preference shocks, but we move beyond 3-period setup
  - He-Xiong (09a): random maturity of contracts
  - He-Xiong (09b): maturity decision with debt overhang in model à la Leland (1994)
    [our inefficiency is not due to "within the firm" conflict of interest but "across firms" externality]
- Other approaches to funding maturity in corporate finance
  - Disciplinary effects (Flannery 94)
  - Flexibility to improve terms when good information made public (Diamond 91)

Outline of the presentation

I. The model

II. Characterizing equilibrium (5 steps)

III. Analysis of efficiency

IV. Conclusions

I. THE MODEL

- $t = 0, 1, 2, ...$
- Two classes of long-lived risk neutral agents: savers & experts
  - Enter in OLG fashion, endowed with 1 unit of funds
  - Different in sophistication and intertemporal preferences
- Banks posses long-lived assets & sell claims on their returns
- Information structure & contractual constraints

  $\rightarrow$ look at each of them in order

Savers, normally born patient (discount rate ρ_P):

- Unsophisticated: only saving alternative is bank debt
- May randomly and *irreversibly* become impatient ($\rho_I > \rho_P$)
  - Idiosyncratically, with pr. γ per period
  - In *systemic liquidity crises*, with pr. 1

$$\Pr[s_{t+1} = C \mid s_t = N] = \varepsilon, \Pr[s_{t+1} = C \mid s_t = C] = 0$$

[1 period = duration of a crisis]

- Plan consumption prior to learning s_{t+1} & rectifying has cost $\varkappa$

Experts, always impatient (ρ_I):

- May become bankers *or* invest in private project (at entry)
- Projects have heterogenous NPV $z \in [0, \overline{\phi}]$:

$$F(\phi) = \text{Measure of entering experts with } z \leq \phi$$

$$[F'(\phi) > 0,\ F(0) = 0,\ F(\overline{\phi}) = \overline{F},\ \text{suff. large}]$$

- Entering experts with $z < \varkappa$ are natural "bridge financiers" in crises

Banks, measure-one continuum of them:

- Own potentially-perpetual illiquid assets
  - Per period cash flow $\mu > 0$, if continuously managed
  - Residual liquidation value L, otherwise
- 100% owned & managed by experts (called *bankers*)
- Can place *some* classes of securities among other agents
  - Maintained assumption:
    Banks cannot offer contracts contingent on idiosyncratic/aggregate preference shocks (or unilaterally postpone payments)
  - We further restrict set of possible funding structures to have a dynamic problem with simple state space

Normal times funding

- Bankers hold 100% of equity
- Continuum of non-tradeable debt

$$
\begin{array}{cccccc}
(& D, & & \delta, & & r \quad) \\
 & \uparrow & & \uparrow & & \uparrow \\
 & \text{principal} & & \text{pr of maturing} & & \text{interest rate} \\
 & \text{(issued at par)} & & \text{(per period)} & & \text{(per period)}
\end{array}
$$

  - Maturity is independent within & between banks
  - Expected time to maturity $= 1/\delta$

    [same effect as staggered fixed-maturity contracts]

- Implied burden per period: paying $rD +$ “refinancing” δD
- Normal periods: free cash flow $\mu - rD \Rightarrow$ dividends

Refinancing during crises

- Bankers learn after having consumed normal dividends
- Experts can be offered equity stake α to refinance δD
  - If δD is not refinanced, bankruptcy & liquidation
  - Assume avoiding bankruptcy is optimal $\rightarrow$ BF constraint: (BF)
- Competitive market determines α:
  - Marginal provider of BF has $z = \phi$
  - Market clearing condition: $F(\phi) = \delta D$

$$\Rightarrow \phi = F^{-1}(\delta D) \equiv \Phi(\delta D)$$

[inverse supply of *crisis liquidity*, with $\Phi' > 0$]

What do we have?

- Prima facie case for *banking*:

  Delegated fund management + Maturity transformation

- Intuitive trade-off for funding maturity decisions:
  - Offering savers who turn impatient an early exit
  - Avoiding refinancing costs in a systemic crisis

- Equilibrium decisions (as shown next):
  - achieve both only partially
  - are not efficient $\rightarrow$ social planner subject to same constraints might do it better

II. CHARACTERIZING EQUILIBRIUM (5 steps)

Equilibrium Tuple $((D^e, r^e, \delta^e), \phi^e)$ such that:

1. Patient savers accept
2. Normal-period bank owners' value is maximized s.t. (BF) constraint
3. Market for crisis liquidity clears

1. Determining savers' required maturity premium

- Consider valuation of debt with $(1, r, \delta)$
- Valuation depends on whether saver is patient (P) or impatient (I)

$$U_P = \frac{1}{1+\rho_P}\{r + \delta + (1-\delta)[(1-\varepsilon)(1-\gamma)U_P + ((1-\varepsilon)\gamma + \varepsilon)U_I]\}$$
$$U_I = \frac{1}{1+\rho_I}[r + \delta + (1-\delta)U_I]$$

[U_i: value just after current interest was paid; no credit risk]

- Let $\pi \equiv (1-\varepsilon)\gamma + \varepsilon$ (unconditional pr. of P turning I), then

$$U_P(r,\delta) = \frac{r+\delta}{\rho_I+\delta}\frac{\rho_I+\delta+(1-\delta)\pi}{\rho_P+\delta+(1-\delta)\pi}$$

$$U_I(r,\delta) = \frac{r+\delta}{\rho_I+\delta}$$

- Properties:

  1. $\rho_P < \rho_I \Rightarrow U_P(r,\delta) > U_I(r,\delta)$
  2. Increasing in r
  3. For $r < \rho_I$, increasing in δ

- Contract acceptability

  – Equivalent to $U_P(r,\delta) \geq 1$

  – Banks make it binding $\Rightarrow r(\delta) = \frac{\rho_I\rho_P+\delta\rho_P+(1-\delta)\pi\rho_I}{\rho_I+\delta+(1-\delta)\pi}$

Proposition 1 *Minimal acceptable interest rate is $r(\delta)$, strictly decreasing & convex, with $r(0) = \rho_I \frac{\rho_P+\pi}{\rho_I+\pi} \in (\rho_P, \rho_I)$ & $r(1) = \rho_P$.*

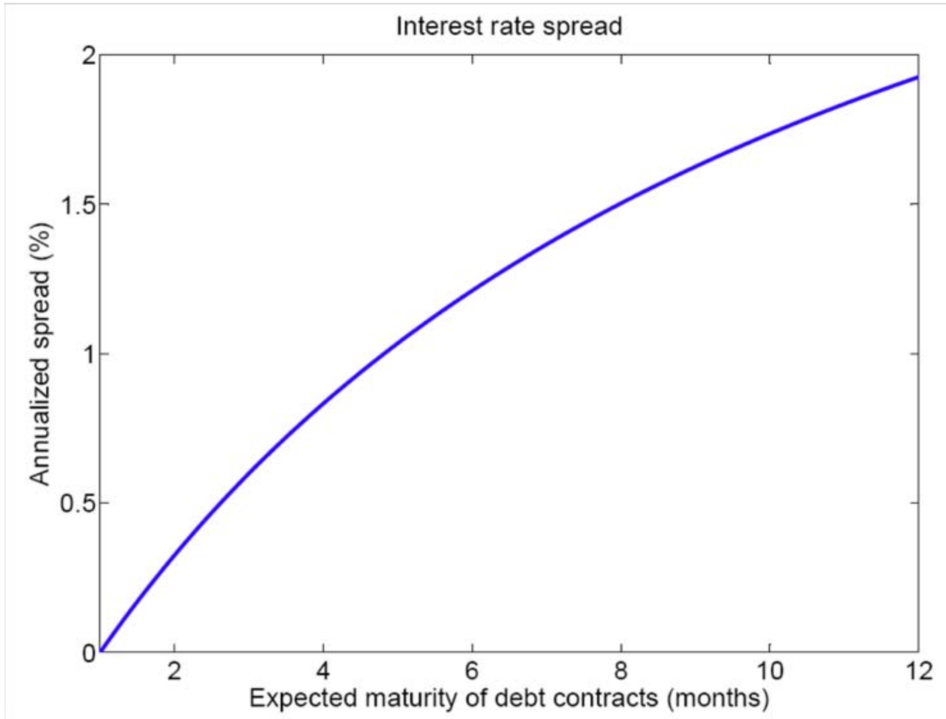

Fig. 1: Spread $r(\delta) - r(1)$ as a function of $1/\delta$

Period	ρ_P	ρ_I	μ	γ	ε	$\Phi(x)$
1 m	2%	6%	4%	$1/12$	$1/120$	$0.5x$

2. Value of a bank in normal times

- Set $r = r(\delta) \Rightarrow$ funding structures described as (D, δ)
- Initial bankers maximize total market value

$$V(D, \delta; \phi) = D + E(D, \delta; \phi), \text{ where}$$

$$\begin{aligned} E(D, \delta; \phi) \; = \; & \tfrac{1}{1+\rho_I}\{(\mu - rD) + (1-\varepsilon)E(D, \delta; \phi) + \\ & + \varepsilon(1-\alpha)\tfrac{1}{1+\rho_I}[\mu - (1-\delta)rD + \delta D + E(D, \delta; \phi)]\} \end{aligned}$$

- 3rd term = equity value after receiving BF in crisis
  - Payoffs + continuation values one period ahead (so discounted)
  - Dividends inflated by having lower debt: $\mu - (1-\delta)rD$
  - Reissuing "bridge financed" debt yields δD
  - Original debt structure $\Rightarrow$ original equity value restored!

3. Determination of α and (BF)

- Setting α:

$$\alpha[\mu - (1-\delta)rD + \delta D + E(D,\delta;\phi)] = (1+\rho_I)(1+\phi)\delta D \quad (1)$$

$$[\alpha \times \text{3rd term} = (1+\rho_I) \times \text{BF opportunity cost}]$$

- Imposing $\alpha \leq 1$ yields *bridge financing constraint*:

$$\mu + E(D,\delta;\phi) \geq [(1+\rho_I)(1+\phi)\delta + (1-\delta)r - \delta]D \qquad \text{(BF)}$$

- Plugging (1) in Bellman equation and solving for $E(D,\delta;\phi)$:

$$E(D,\delta;\phi) = \tfrac{1}{\rho_I}\left[\mu - r(\delta)D - \tfrac{\varepsilon}{1+\rho_I+\varepsilon}\{[(1+\rho_I)\phi+\rho_I] - r(\delta)\}\delta D\right]$$

  – Unlevered cash flow μ
  – Normal debt cost $r(\delta)D$
  – Differential cost due to crisis (proportional to δD)

- Crisis cost term (per unit of δD) is product of
  - Net present value multiplier: $\frac{\varepsilon(1+\rho_I)}{1+\rho_I+\varepsilon}$
  - Excess BF costs: $\frac{\varepsilon}{1+\rho_I+\varepsilon}\{[(1+\rho_I)\phi+\rho_I]-r(\delta)\}$
- Implied total market value

$$V(D,\delta;\phi)=\frac{\mu}{\rho_I}+\frac{\rho_I-r(\delta)}{\rho_I}D-\frac{\varepsilon\{[(1+\rho_I)\phi+\rho_I]-r(\delta)\}}{1+\rho_I+\varepsilon}\delta D$$

  - value of unlevered bank
  - value of financing the bank with debt claims if no crises
  - costs due to refinancing problems during systemic crises
    - * zero under $\delta=0$, but $r(0)<\rho_I$ so debt is source of value
    - * maturity transformation can generate even more value

4. Bankers' value maximization

Bankers' problem:

$$\max_{D,\,\delta} V(D,\delta;\phi) = D + E(D,\delta;\phi)$$

$$\text{s.t.} \quad E(D,\delta;\phi) \geq 0 \qquad \text{(LL)}$$

$$\mu + E(D,\delta;\phi) - [(1+\rho_I)(1+\phi)\delta + (1-\delta)r - \delta]D \geq 0 \quad \text{(BF)}$$

$$[\text{(BF)} \Rightarrow \text{(LL)} \Rightarrow \mu - r(\delta)D \geq 0]$$

A1: $\overline{\phi} \leq 2\frac{1+\rho_P}{1+\rho_I} - 1$

A2: $\pi < \frac{1-\rho_I}{2}$

Proposition 2 *Bankers' problem has a unique solution* (D^*, δ^*)

1. *(BF) is binding, so* $\alpha^e = 1$
2. *Increasing* ϕ *increases* $1/\delta^*$ *& decreases* $\delta^* D^*$

- Intuitions:
  - Banks are interested in max D
  - $\uparrow \phi$ reduces value of maturity transformation
  - In numerical examples D^* also decreases with ϕ

5. Equilibrium

- $((D^e, r^e, \delta^e), \phi^e)$ exists & is unique (**Prop 3**)
- $\uparrow \Phi(x) \Rightarrow \downarrow \delta^e, \downarrow \delta^e D^e, \uparrow r^e, \uparrow \phi^e$ (**Prop 4**)
- Graphs with comparative statics:
  - 1st column: Changing a in $\Phi_a(x) = ax$
    - crises become more expensive
    - $\uparrow$maturity, $\downarrow$leverage, $\uparrow \phi^e$
  - 2nd column: Increasing time to systemic shocks ($1/\varepsilon$)
    - BF is less costly and (BF) is relaxed
    - $\downarrow$maturity, $\uparrow$leverage, $\uparrow \phi^e$
  - 3rd column: Increasing time to idiosyncratic shocks ($1/\gamma$)
    - maturity is lengthen, BF costs fall
    - $\uparrow$maturity, $\uparrow$leverage, $\downarrow \phi^e$

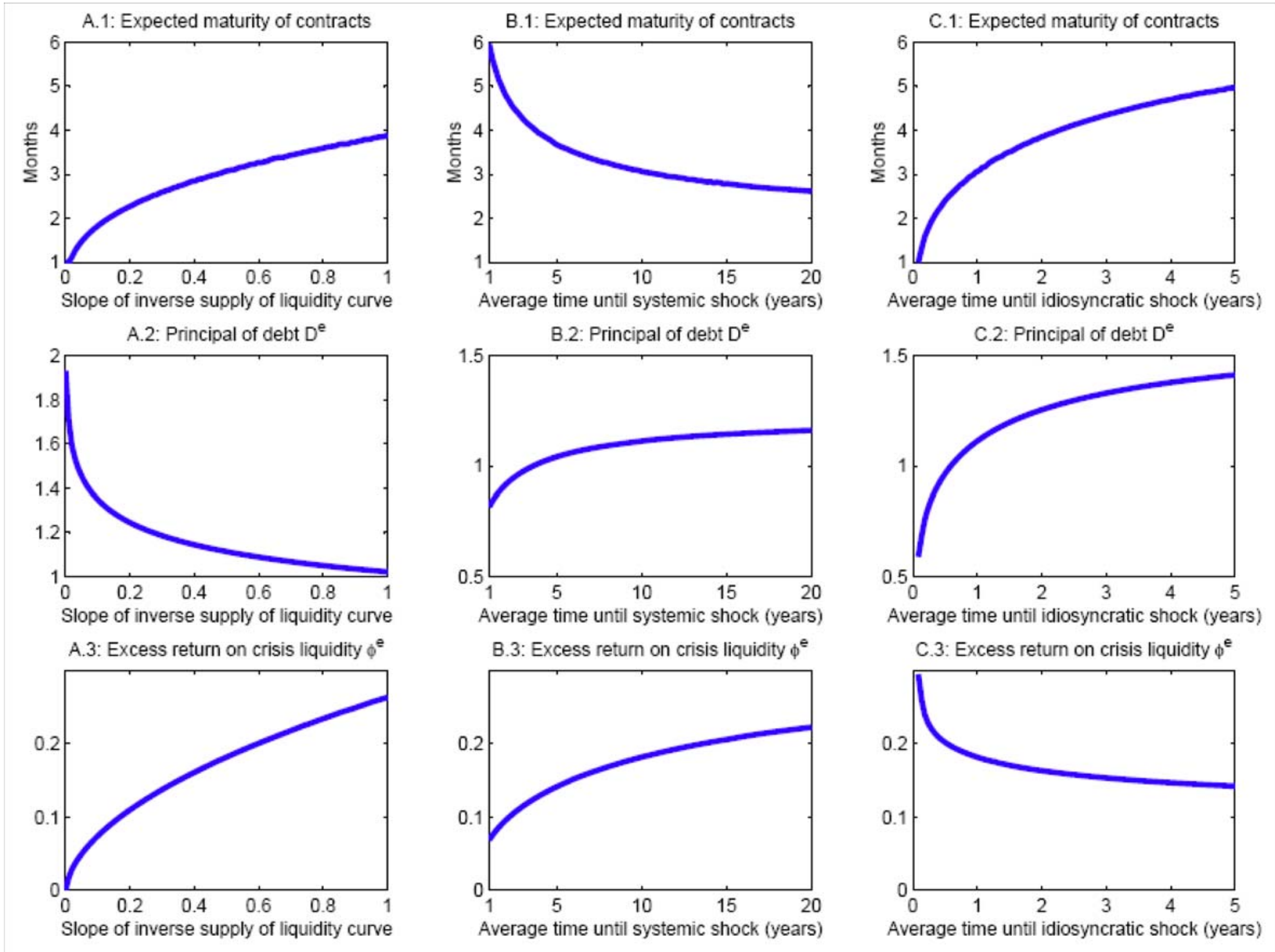

Fig. 2: Comparative statics

III. ANALYSIS OF EFFICIENCY

Do equilibrium decisions coincide with the decisions a social planner (SP) facing the same frictions & constraints would make?

- Suppose SP can regulate D and δ
- Only initial and "would be" bankers obtain surplus = Welfare
- In crises, entering experts obtain $\phi - z$

$$u(D,\delta) = \int_0^{D\delta} (\Phi(\delta D) - \Phi(x))\, dx = \delta D \Phi(\delta D) - \int_0^{D\delta} \Phi(x) dx$$

- In expected present value:

$$U(D,\delta) = \frac{1}{\rho_I} \frac{(1+\rho_I)\varepsilon}{1+\rho_I+\varepsilon} u(D,\delta)$$

• Social planner's problem:

$$\max_{D,\,\delta,\,\phi} \; W(D,\delta) = V(D,\delta;\phi) + U(D,\delta)$$

s.t. $\quad \mu + E(D,\delta;\phi) - [(1+\rho_I)(1+\phi)\delta + (1-\delta)r - \delta]D \geq 0$ (BF')

$\quad \phi = \Phi(\delta D)$ (MC)

[SP internalizes effect on excess cost of crisis liquidity]

$$\Rightarrow W(D,\delta) = \frac{\mu}{\rho_I} + \frac{\rho_I - r(\delta)}{\rho_I} D - \frac{\varepsilon\{[(1+\rho_I)\phi + \rho_I] - r(\delta)\}}{1+\rho_I+\varepsilon}\delta D$$
$$-\frac{1}{\rho_I}\frac{(1+\rho_I)\varepsilon}{1+\rho_I+\varepsilon}\int_0^{D\delta} \Phi(x)dx$$

Proposition 5 *With exogenously fixed D, equilibrium is efficient*

[Changing $\delta =$ pure redistribution of value between BF & initial bankers]

Proposition 6

If $\delta^e \in (0,1)$ *a social planner can increase social welfare by choosing* $\delta^s < \delta^e$. *Specifically:*

$$\left.\frac{dW(D^s(\delta),\delta)}{d\delta}\right|_{\delta=\delta^e} < 0, \quad \left.\frac{dD^s(\delta)}{d\delta}\right|_{\delta=\delta^e} < 0, \quad \left.\frac{d(\delta D^s(\delta))}{d\delta}\right|_{\delta=\delta^e} > 0$$

- Intuition:

  SP internalizes that reducing δ reduces ϕ and relaxes (BF), implying

  – reduction in surplus for future bridge financiers

  – even larger increase in value for existing bankers

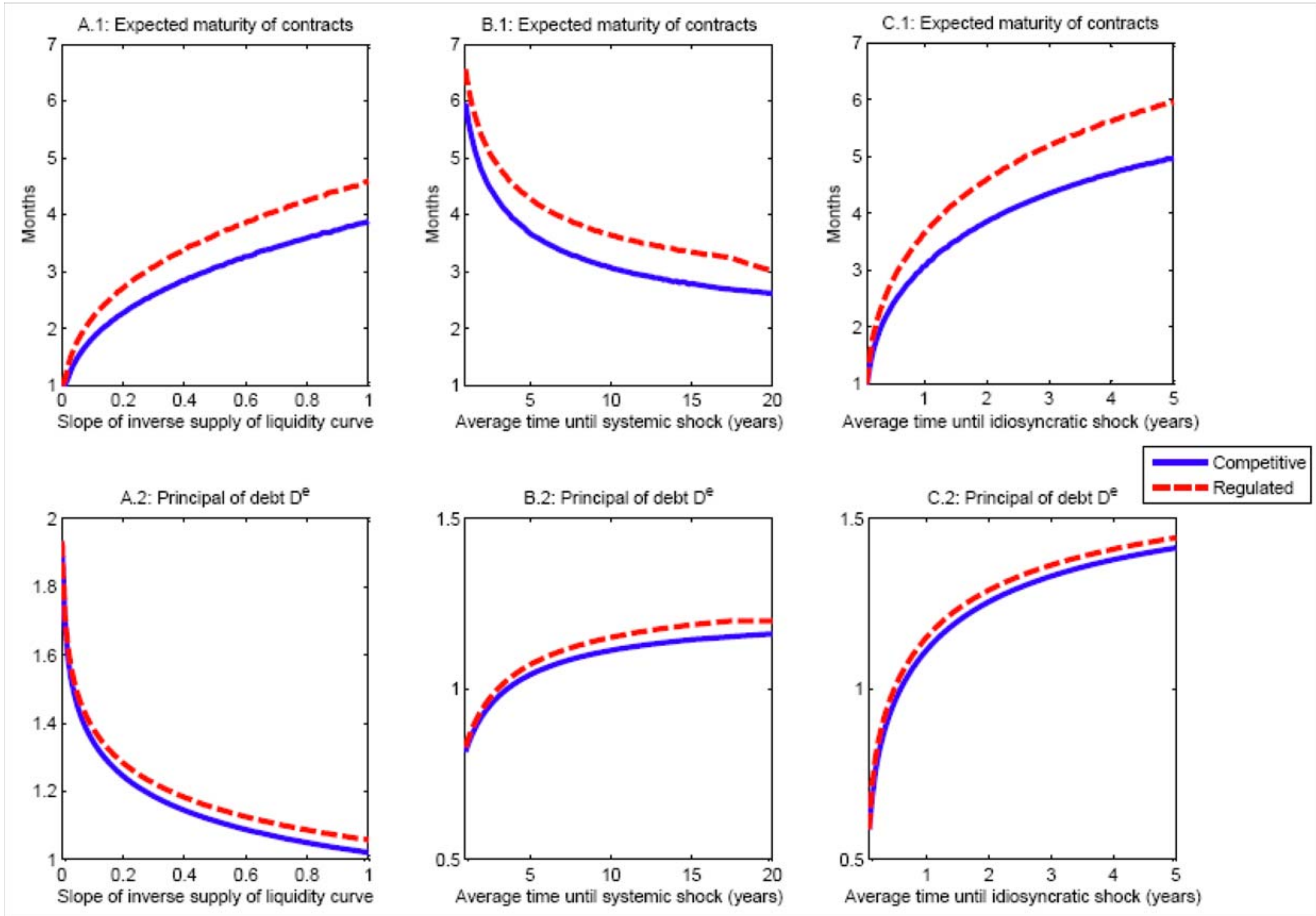

Fig. 3: Equilibrium vs. optimal funding structures

IV. CONCLUSIONS

- Infinite horizon equilibrium model with
  - Diamond-Dybvig type microfoundations for savers' preference for short maturities
  - Events in which banks' normal financing channels fail
- Pecuniary externality renders the unregulated competitive equilibrium socially inefficient...

  Due to combination of:
  - ex-post competitive pricing of funds during crises
  - constraints banks have to satisfy ex-ante in order to guarantee their refinancing capacity during crises
- Result similar to inefficiency of private provision of liquidity in presence of aggregate shocks obtained in other banking models