Default, Liquidity and the Yield Curve

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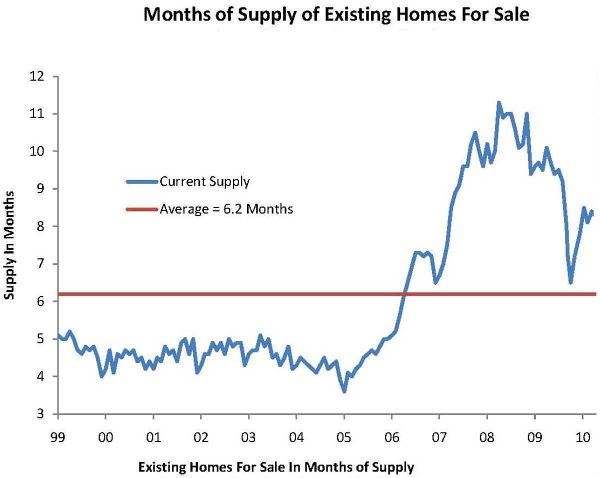
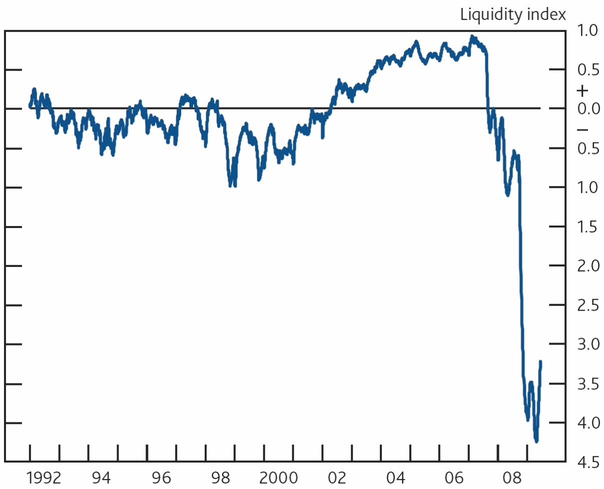
Real estate 'liquidity'

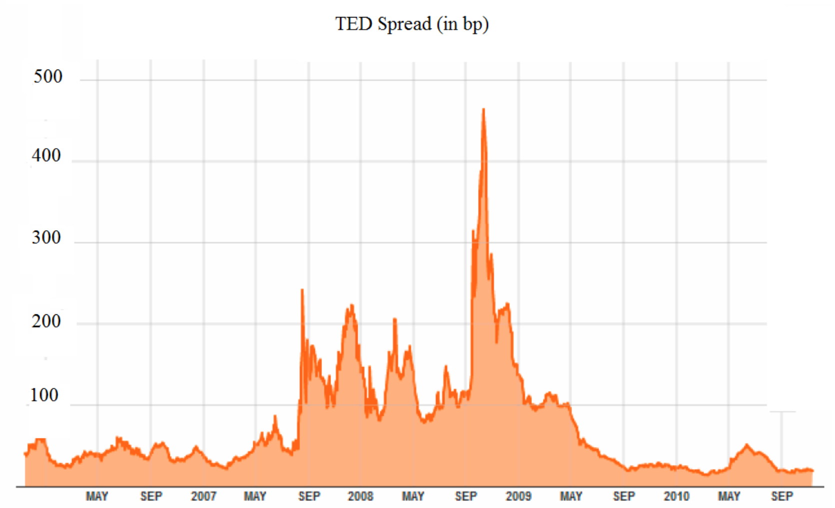


Market liquidity index (Bank of England)



source: Bank of England, 2009.

Counterparty risk and interbank rates: the TED spread



The crisis raises several questions

1. How does illiquidity in asset/commodity markets affect the demand for money and the role of monetary policy?
2. How does default risk affect the demand for money and the role of regulation and monetary policies?
3. What are the implications of liquidity and default risk for activity, asset prices and the yield curve?

To answer these questions, we need

1. A monetary model, i.e. a model where money is needed to conduct transactions
2. A model where *endogenous* default is allowed in the interbank market
3. A model with heterogeneous agents, so that illiquidity has an effect on activity and asset prices
4. Illiquidity and default risk should be uninsurable; we assume this is the only uninsurable risk

Results

1. Illiquidity in asset and commodity markets increases the demand for money
2. Default risk increases the short-term interest rate and the elasticity of money demand to short-term rate is lower the higher the default penalty.
3. Higher interest rates lower trade activity, and the more so for illiquid commodities.
4. Even in absence of aggregate uncertainty, uncertainty in funding costs generate a risk-premium in the yield curve.

Summary

2 -Period Model

Money Market Equilibrium and Default

Positive Interest Rate

for a given interest rate there exists a unique money demand

positive buying and selling price wedge

Nominal Determinacy

Financing Costs

Summary

Nominal Determinacy

Financing Costs

$$\frac{u'(c_i)}{u'(c_0)} = \frac{p_i \theta_i}{(1+r_i)p_0}$$

Determinacy of State Prices

$$\left(\theta_i = \pi_i \frac{M_0}{M_i}\right)$$

Nominal variable changes
generate real effects

Real payoffs of securities
are a function of 'liquidity'

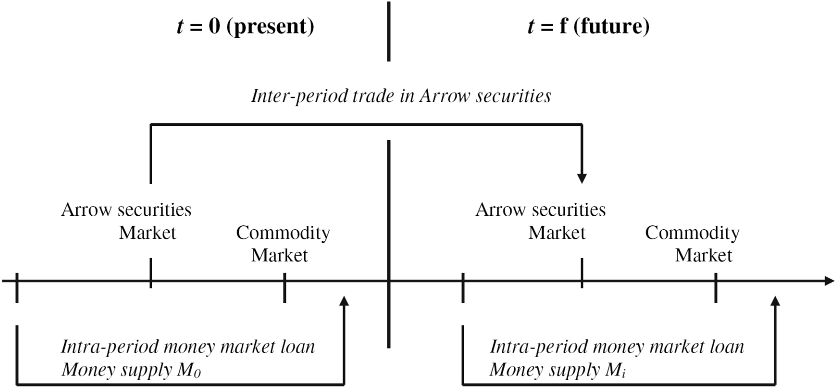
Uncertainty and Complete Markets

The model is an exchange economy with cash-in-advance constraints and incomplete markets: liquidity risk is uninsurable. All other risks are insurable.

There are two periods. The first period is indexed by 0. The second period has S states, indexed by $s = 1, \ldots, S$.

There are two agents: $i = \alpha,\ \beta$, and a consolidated banking system. α is the borrower. β is the lender.

Cash-in-Advance Constraints



Maximization problem

$$\max_{q_0,(q_s,\mu_s,h_s,d_s)_{s\in\{1\dots S\}}} u^{\alpha}(q_0) + \frac{1}{S}\left(\sum_{s=1}^{S} u^{\alpha}(e_s - q_s) - \delta \max(d_s\mu_s;0)\right)$$

$$\text{s.t.} \quad p_0 q_0 \leq \sum_{1\leq s\leq S} \theta_s h_s$$

$$\forall s \in \{1,\dots,S\} \quad h_s \leq \frac{1}{1+r_s}\mu_s$$

$$\mu_s(1-d_s) \leq p_s q_s$$

Quantity Theory of Fiat Money

Quantity Theory of Fiat Money

$$\forall s \in \{0, \ldots, S\} \;\; p_s q_s = M_s$$

Intuition:
Money is used for transaction purposes. All money available is used in liquidity-constrained economies.

Velocity and liquidity are set to 1.

We will come back to this later.

Value of Money and Default

Value of Money

$$\forall s \in \{0, \ldots, S\} \quad r_s \approx d_s = 1 - \delta M_s$$

Proof:

Outcome of four equilibrium conditions:

1. marginal cost of default = marginal cost of repaying

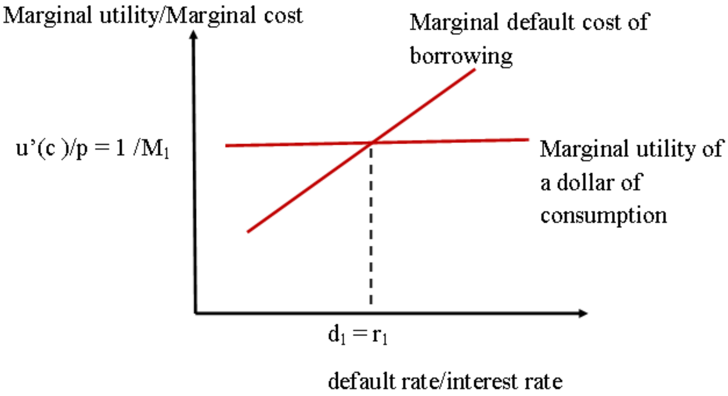
$$\Longrightarrow \delta = u'^{\alpha}(c_s)/p_s$$

2. $(1 - d_s)(1 + r_s)M_s = M_s \Longrightarrow r_s \approx d_s$
3. marg. default cost of borrow. = value of \$1 to release liquidity constraint

$$\Longrightarrow \delta(1 + r_s) \approx \frac{u'^{\beta}(q_s)}{p_s} = \frac{1}{p_s q_s} = \frac{1}{M_s}$$

Higher money supply decreases default and interest rates

Intution: Inflation $\Longrightarrow$ higher cost of default. $\Longrightarrow$ default and interest rates decrease



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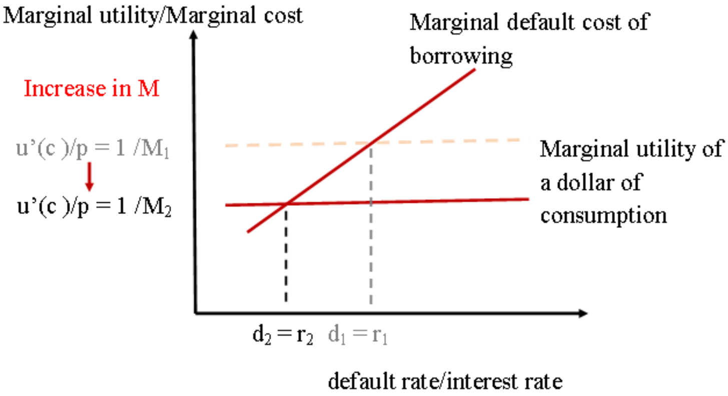
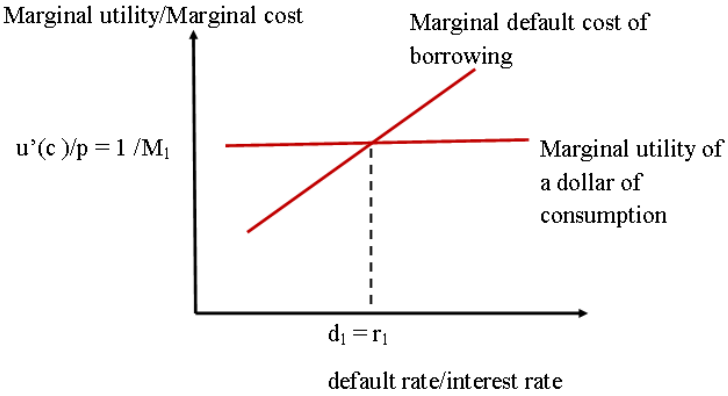


"Slutsky" decomposition of effect of regulation: 1

Tighter regulation (harsher penalties) increases the utility cost of default $\Longrightarrow$ reduces the default rate and therefore the interest rate.



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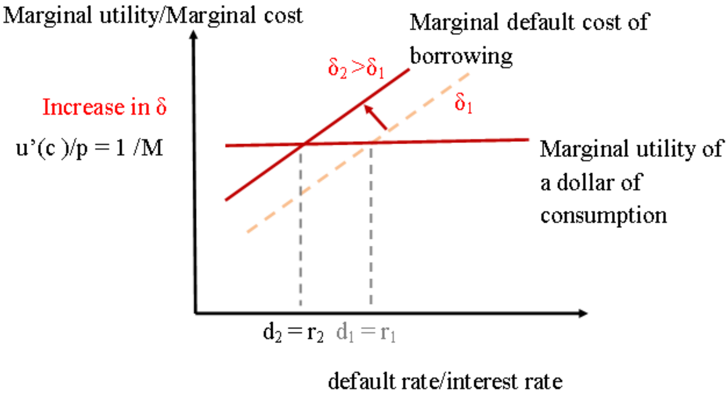
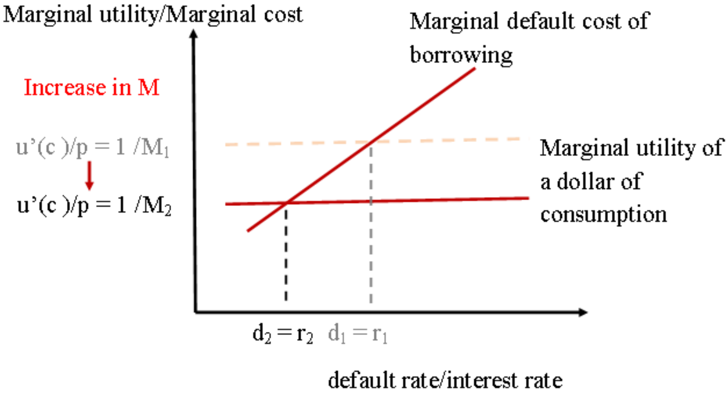


"Slutsky" decomposition of effect of regulation: 2

Tighter regulation (harsher penalty) increases the cost of default due to interest rates $\Longrightarrow$ reduces the elasticity of interest rates to money supply.



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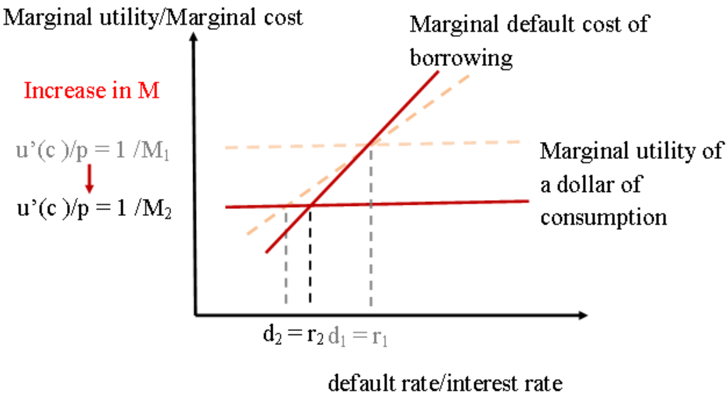


Endogenous State Prices

Theorem

Assume RRA ≥ 1.

$$\forall s \neq s', \; r_s > r_{s'} \Longleftrightarrow \theta_s > \theta_{s'} \Longleftrightarrow \hat{\pi}_s > \hat{\pi}_{s'}$$

Intuition: Risk-neutral probabilities are higher in states of nature with tighter liquidity constraints.

1. The financing cost acts like a tax and decreases trade.
2. Higher interest rate means lower trade
3. Lower trade implies higher marginal utility for the buyer...
4. ...and higher demand for the asset $\Longrightarrow$ higher state price.

Proof

For the buyer

$$\frac{\theta_s}{\theta_{s'}} = \frac{u'(q_s)/p_s}{u'(q_{s'})/p_{s'}} \tag{1}$$

For the seller

$$\frac{\theta_s}{\theta_{s'}} = \frac{u'(e_s - q_s)(1 + r_s)/p_s}{u'(e_s - q_{s'})(1 + r_{s'})/p_{s'}} \tag{2}$$

Combining FOCs:

$$1 = \frac{(1 + r_s)u'(e_s - q_s)/u'(q_s)}{(1 + r_{s'})u'(e_s - q_{s'})/u'(q_{s'})} \tag{3}$$

$u'(e - q)/u'(q)$ is increasing in q. Hence, $r_s > r_{s'} \Rightarrow q_s < q_{s'}$.

Proof

For the buyer

$$\frac{\theta_s}{\theta_{s'}} = \frac{u'(q_s)/p_s}{u'(q_{s'})/p_{s'}} \tag{4}$$

For the seller

$$\frac{\theta_s}{\theta_{s'}} = \frac{u'(e_s - q_s)(1 + r_s)/p_s}{u'(e_s - q_{s'})(1 + r_{s'})/p_{s'}} \tag{5}$$

Combining FOCs and quantity theory of money :

$$\frac{\theta_s}{\theta_{s'}} = \frac{u'(q_s)q_s/p_s q_s}{u'(q_{s'})/p_{s'}q_{s'}} = \frac{u'(q_s)q_s/M_s}{u'(q_{s'})q_{s'}/M_{s'}} \tag{6}$$

q is decreasing in r. $u'(q)q$ is decrasing in $q \iff RRA \geq 1$.

Example: CRRA

Comparing state prices (or risk-neutral probabilities) across states of nature s and s'

$$\frac{\theta_s}{\theta_{s'}} = \overbrace{\left(\frac{e^{\alpha}_{s'}}{e^{\alpha}_{s}}\right)^{\rho-1}}^{aggregate\ uncertainty} \overbrace{\frac{M_{s'}}{M_s}}^{inflation} \overbrace{\left(\frac{q^{\alpha}_{s}/e^{\alpha}_{s}}{q^{\alpha}_{s'}/e^{\alpha}_{s'}}\right)^{1-\rho}}^{heterogeneity}$$

where

$$\underbrace{\frac{q^{\alpha}_{s}/e^{\alpha}_{s}}{q^{\alpha}_{s'}/e^{\alpha}_{s'}} = \frac{\left(\frac{1+r'_s}{1+r_0}\right)^{\frac{1}{\rho}} + \frac{e^{\beta}_{0}}{q^{\beta}_{0}} - 1}{\left(\frac{1+r_s}{1+r_0}\right)^{\frac{1}{\rho}} + \frac{e^{\beta}_{0}}{q^{\beta}_{0}} - 1}}_{higher\ interest\ rate\ decreases\ trade}$$

Upward Term Structure

Puzzle

The risk premium in the term structure (the term premium) is above what would be predicted in a Lucas-type economy

$$p_t = \mathbb{E}[u'(c_{t+1})y_{t+1}/u'(c_t)]$$

Explanations of Puzzle

1. Lucas' model fails in tests from Backus *et al.* (1989) and Grossman *et al.* (1987)
2. Liquidity (Hicks, Lutz, 1940)
3. Preferred Habitat (Modigliani and Sutch, 1967), etc.
4. Uninsurable income risk, as in Weil (1990) and Ayagari (1994), would increase precautionary savings and decrease rates, depending on the shape of the utility function
5. In Elul (1997), with general incomplete market structure, there is no general result

Our term premium

1. Uninsurable monetary costs...
2. ...reduce trade, for any given aggregate income.
3. With any concave utility function, lower trade implies higher marginal utilities and state prices.
4. This generates a risk premium in the term structure.
5. The result holds in an economy with multiple commodities, as long as one commodity is always purchased by agent α, across all states of nature.

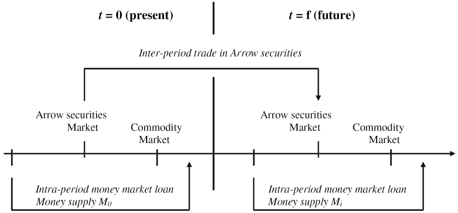
Another look at liquidity: max. problem for agent β

$$\sum_{s\in\{1\ldots S\}} \theta_s h_s^\beta \leq \Lambda_0 p_0 q_0^\beta + \frac{\mu_0^\beta}{1+r_0} + m_0^\beta$$

$$\mu_0^\beta \leq (1-\Lambda_0) p_0 q_0^\beta$$

$$\forall s \in \{1,\ldots,S\}, \;\; p_s q_s^\beta \leq \frac{\mu_s^\beta}{1+r_s} + \lambda_s h_s^\beta + m_s^\beta$$

$$\mu_s^\beta \leq (1-\lambda_s) h_s^\beta$$



Cash-in-Advance Constraints

Quantity theory of Money

$$p_s q_s = \frac{M_s + m_s^{\beta}}{1 - \Lambda_s + (1 - \lambda_s)(\Lambda_s + \frac{1-\Lambda_s}{1+r_s})}$$

- When $\lambda_s = 1,\ p_s q_s = \frac{M_s + m_s^{\beta}}{1-\Lambda_s}$. When $\Lambda_s = 1,\ p_s q_s = \frac{M_s + m_s^{\beta}}{1-\lambda_s}$
- Link with $PQ = vM = 1/k\ M$ where k is the liquidity of money
- $1 - \Lambda$ is the illiquidity of commodities, or the relative liquidity of money.
- Λ can also be thought of as the period in which the asset/commodity is unsold.

Trade and asset prices

$$\frac{u'(e_s - q_s)}{u'(q_s)(\Lambda_s + (1-\Lambda_s)/(1+r_s))} = \frac{u'(e_s - q_{s'})}{u'(q_{s'})(\Lambda_{s'} + (1-\Lambda_{s'})/(1+r_{s'}))}$$

Slope of the Yield Curve

- The transaction cost is decreasing in Λ and increasing in the interest rate.
- Another interpretation: The cost of the inefficiency is increasing in the period in which the asset/commodity is unsold and is increasing in the cost of time.
- Trade is lower the lower M_s, the lower λ_s and the lower Λ_s.
- $\Rightarrow$ State prices (risk neutral probabilities) are lower the higher expected ‘liquidity’ of assets and commodities
- The slope of the yield curve is also determined by expected liquidity and liquidity risk.

This is on top of the other determinants:

1. lower expected inflation (nominal effect)
2. lower inflation volatility (asset payoff volatility)
3. Reduction in the volatility of real activity (SDF volatility)
4. Others (change in preferences, regulations, government supply, demography, etc.)

Concluding Remarks

We showed how:

- Default risk is sufficient to generate nominal determinacy
- Liquidity in asset/commodities affect the demand for money
- Changes in default penalties (regulation) shift money demand and affect the elasticity of money demand to interet rates.
- Asset prices are a function of money supply, asset liquidity and trade efficiencies
- Liquidity risk in endowments and assets matter for the slope of the yield curve