An Empirical \((S, s)\) Model of Dynamic Capital Structure *

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Abstract

We develop a general \((S, s)\) model of capital structure that allows us to investigate the relationship between target leverage, refinancing thresholds, and firm characteristics in a dynamic environment. Unlike traditional regressions, our model is capable of distinguishing the target and refinancing thresholds separately. We find that target leverage is positively related to profitability. The thresholds vary predictably with firm characteristics such as the market-to-book ratio, asset tangibility, and research and development expenses.

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Understanding the cross-sectional properties of leverage ratios rests at the heart of empirical capital structure research and constitutes perhaps the most important link between empirical analysis and capital structure theory. From classical early studies by Taggart (1977), Marsh (1982), and Titman and Wessels (1988) to the most recent contributions in the field, linear regressions have been the workhorse for exploring cross-sectional and time-series variation in capital structure. This body of work has provided invaluable insights into the comparative statics predictions of many static models of corporate financial structure.

In recent years, dynamic models of capital structure have substantially extended our understanding of firms' financial decisions. In a dynamic setting, the conditional mean of leverage that is the focus of much of the current empirical literature is a combination of various aspects of the dynamics in capital structure. For example, Strebulaev (2007) finds that linear regressions do not estimate well leverage covariates in a dynamic model of infrequent refinancing and Morellec, Nikolov, and Schürhoff (2012), by deriving the cross-sectional stationary leverage distribution in a similar model, demonstrate that the mean of leverage typically is not target leverage.

In this paper, we propose an alternative empirical model that accounts explicitly for capital structure dynamics. We build and implement a so-called \((S, s)\) model that captures important features of the data generating process: leverage adjustments are infrequent and lumpy.\(^2\) The \((S, s)\) model, developed originally by Arrow, Harris, and Marshak (1951), is not new to economics and has been used previously to study cash balances, inventories, corporate investment decisions, and consumer demand (e.g., Miller and Orr (1966), Caballero (1993), Eberly (1994), Attanasio (2000), Caplin and Leahy (2006; 2010)). The basic economic assumption in developing an \((S, s)\) model

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\(^2\)Stokey (2009) discusses a plethora of economic environments, in which costly adjustments lead to infrequent changes, and provide a broad theoretical treatment of this approach.
for capital structure is shared with other applications of the model: we posit that firms follow a target leverage policy but, because of imperfections, deviate from their target. The existence of the target leverage ratio, $L^*$, assumes the existence of a trade-off between the benefits and costs of debt financing, which leads to an interior solution at the firm level. This model presumes the existence of a certain loss function that specifies the cost incurred if the firm deviates from its preferred target leverage ratio. This loss function can in principle be very generic. In capital structure, the losses would include (but are not limited to) lost tax shields, costs of financial distress such as debt overhang and asset substitution, and other increases in agency costs.

The $(S, s)$ model complements and enriches the existing empirical methods in several ways. First, it enables us to compare the estimates of target leverage determinants obtained by linear regressions and the $(S, s)$ model. To examine these estimates, we apply the $(S, s)$ model to quarterly Compustat data on non-financial firms between 1984 and 2009. We find that in many instances, the estimates of target leverage coefficients have the same sign as the OLS coefficients but are of a very different magnitude. In many important cases, though, the qualitative implications are different. For example, the OLS approach reliably produces a well-documented negative relation between leverage and profitability, while the $(S, s)$ model instead produces a robust positive relation.

Second, the $(S, s)$ approach enables us to study the determinants of refinancing triggers. Our results suggest that, to a certain extent, the differences in estimates on target leverage between linear regression and our $(S, s)$ model are attributed to the fact that leverage in linear regressions captures a combination of the target leverage and refinancing thresholds, as well as the dynamics of leverage in periods of no refinancing. Indeed, in some cases, the thresholds are more sensitive to the covariates than the target capital structure, which suggests that some of the traditional regression results may be driven by the refinancing thresholds rather than the target leverage. For example, the negative relation between earnings volatility and leverage in standard OLS regressions is in fact driven by the upper refinancing threshold, whereas target leverage is unrelated to earnings volatility. In addition, we find that firms allow
leverage to float in a wide range around the target, suggesting that either adjustment costs are high or the loss of value from being away from the target is low, at least for moderate leverage ratios. The thresholds vary predictably with firm characteristics such as the market-to-book ratio, asset tangibility and research and development expenses.

\((S, s)\) models are consistent with the financial policy of the contingent claims capital structure paradigm that has been actively developed in recent years.\(^3\) At the same time, an important advantage of the \((S, s)\) approach is that we do not need to specify the functional form of frictions or the stochastic process of the underlying model’s state variables. The methodology does not rely on the exact economic mechanism, and various conjectures can be comfortably nested together. This benefit comes at a cost, though, because our results are harder to interpret in terms of underlying frictions. It is thus important to stress that this methodology complements rather than substitutes for the model-based structural estimation that has been gaining attention recently (e.g. Hennessy and Whited (2005, 2007), Nikolov and Whited (2011)). For example, with the assumption that firms behave according to a particular dynamic model, a structural estimation should produce a much better estimate if the assumptions prove correct.

Ours is not the first research to enrich traditional empirical methods in capital structure. Strebulaev (2007) shows that popular regressions of profitability on leverage cannot be interpreted as finding the true relation between profitability and target leverage. Morellec et al. (2012) also note that the conditional mean of leverage does not correspond to the target leverage in a contingent claims model of the firm. Instead, they use a simulated maximum likelihood approach to estimate their model. This method exploits the entire conditional distribution of leverage but, as a result, is more structural in nature.

In developing an \((S, s)\) model for capital structure, we modify the standard \((S, s)\) model along a number of dimensions to take into account many additional salient

\(^3\)Some examples include Fischer, Heinkel, and Zechner (1989), Goldstein, Ju, and Leland (2001), Strebulaev (2007), Bhamra, Kuehn, and Strebulaev (2010a;2010b), and Morellec et al. (2012).
features of the leverage data generating process that are absent in other \((S, s)\) applications. For example, the refinancing thresholds may vary over time, and have error components that are not perfectly correlated between the upper and the lower threshold. Also, many leverage measures are naturally bounded (e.g., the market leverage ratio is less than or equal to 1).

At the same time, we wish to stress that, although we present our results only in application to capital structure, the method can be applied to a much broader range of topics for which the dynamic behavior of agents (firms, consumers, investors, governments) leads to an asymmetric cross-sectional distribution of the variables of interest, such as cash flow and payout policies, corporate investment policy with fixed adjustment costs, a government economic policy that intervenes only when a certain indicator (e.g., unemployment) rises above a threshold, or workforce adjustments in the presence of hiring and firing costs (e.g., Pfann and Palm, 1993).

The remainder of the paper is structured as follows. Section I introduces a generic \((S, s)\) model of capital structure. Section II develops our empirical model. In Section III, we describe the data sample, and in Section IV, we discuss our empirical results. Section V concludes.

I An \((S, s)\) Model for Capital Structure

In a general \((S, s)\) model of capital structure, the firm allows leverage to float around without intervention, until either an upper \((S)\) or lower \((s)\) threshold is reached. Upon reaching either threshold, the firm refinances to a “target” capital structure, which we refer to as \(L^*\). In a slight notational deviation from the literature, we refer to the upper and lower thresholds as \(L^u\) and \(L^d\), respectively. An example of a time-series of leverage observations under this policy is illustrated in Figure 1.

The optimality of an \((S, s)\) policy is based on the existence of both a loss function and an adjustment cost function that makes continuous small adjustments suboptimal. The loss function captures the costs that the firm incurs if it deviates from its preferred target leverage ratio. This loss function in principle can be very generic,
although it is typically assumed to be convex. In capital structure, the losses include (but are not limited to) lost tax shields, costs of financial distress such as debt overhang and asset substitution, and increased agency costs. An important type of cost function that we consider here involves fixed adjustment costs. The result that fixed costs lead to an impulse control problem, where the variable of interest follows an exogenously specified stochastic process in the inaction region and is reset when it hits lower and upper triggers, is well-known in stochastic optimal control literature. See Harrison, Sellke, and Taylor (1982) for an early development and Stokey (2009) for a textbook treatment. Leary and Roberts (2005) consider other examples of the adjustment cost function, such as convex and fixed plus linear; it is important to generalize our results to cover these cases as well.

Figure 2 shows an example of the stationary leverage distribution for the case where $L$ follows an arithmetic Brownian motion in the intermediate periods between refinancings. The figure illustrates the basic and yet fundamental result of this section: Leverage distributions are asymmetric around $L^*$. The implications of such asymmetry are profound. The mean of the asymmetric distribution is not equal to $L^*$, barring knife-edge type solutions. Instead, the mean is influenced not only by $L^*$, but also by the thresholds and the characteristics of the stochastic process of leverage (in this specific example, the mean and volatility of the arithmetic Brownian motion). Standard regression coefficients therefore reflect a combination of all these components of the dynamic capital structure.

The target and thresholds can arise endogenously, and the resulting $(S,s)$ policy is consistent with many models in the class of contingent claims structural models, such as Fischer et al. (1989), Goldstein et al. (2001), Strebulaev (2007), and Morellec et al. (2010). The Morellec et al. paper also derives an expression for the stationary distribution of leverage that is consistent with the distribution in Figure 2. This insight is important because it shows that the $(S,s)$ policy can arise endogenously in

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4 Note that for our purposes, the optimality of “target” leverage and other parameters of interest is immaterial. The distinction between various definitions of leverage is not important either and in this section we do not specify the term “leverage ratio” any further.
capital structure models.

An \((S, s)\) model is essentially a reduced-form approach that relies on the generic economic structure of the problem. It thus complements a more structural approach to studying corporate financial decisions, which may imply the same dynamic strategy. One advantage of estimating an \((S, s)\) model is its reliance on fewer assumptions about deep structural parameters that are needed to estimate the dynamic behavior of firms. In particular, we do not make any specific assumptions about the nature or functional form of tax benefits, costs of financial distress, or agency costs. We also make no assumptions about the nature of the driving stochastic process, whereas dynamic contingent claims models rely on particular assumptions (e.g., a geometric Brownian motion for EBIT, an AR(1) process for productivity). For example, our empirical model makes no assumptions about the presence of jumps, whereas Kane, Marcus, and McDonald (1985) show that the implications of jumps in the stochastic process are of first-order importance. In this sense, the strengths of the \((S, s)\) empirical method also helps pinpoint its weaknesses. Although the approach is more general than a given model, it does not provide the rich economic content underlying the mechanisms at work, as specific models do so well, nor can it, of course, replace the need for a better understanding of the factors that drive corporate decisions. Thus, a structural model and an \((S, s)\) model should be viewed as closely connected approaches that lead to the same ultimate goal.

In a dynamic world with adjustment costs, even instrumental variables (IVs) or differences-in-difference regressions from natural experiments may not correctly identify the impact of an exogenous shock on target capital structure. First, the shock may not be large enough for firms to adjust. This is essentially a “weak instruments” problem. Second, and more worrisomely, if the object of interest is the change in a firm’s target leverage, then the magnitude may be biased for the same reasons as the parameter coefficients are biased in regression models, namely, the difference between the true target \(L^*\) and the mean of leverage is not fixed. In other words, the change in the conditional mean of leverage is not the same as the change in \(L^*\). If the natural experiment also changes \(L^d, L^u\), or the shape of the loss function, even the sign of
the IV or differences-in-differences regressions may be wrong, such as would occur if a natural experiment were to change the fixed cost of adjustment.

In reality target and refinancing thresholds may vary over time and across firms. We accommodate this in the empirical model introduced in the next section. It is important to point out that the basic insights from this section are unaffected by such time-series and cross-sectional variation.

II Empirical $(S, s)$ Model of Capital Structure

We estimate the benchmark capital structure $(S, s)$ model, where for firm $i$ at time $t$:

$$L^*_{it} = X'_{it} \beta + u^*_{it},$$

$$L^u_{it} = L^*_{it} + \exp(X'_{it} \theta^u + u^u_{it}),$$

$$L^d_{it} = L^*_{it} - \exp(X'_{it} \theta^d + u^d_{it}).$$

Equation (1) models target leverage, $L^*_{it}$, and appears analogous to the traditional regression model, which is ubiquitous in extant literature. However, as we discuss next, the identification and interpretation of this equation are quite different in the $(S, s)$ model. The exponential terms in (2) and (3) represent the gap between the upper refinancing threshold, $L^u_{it}$, and target leverage, and the gap between the lower threshold, $L^d_{it}$, and target, respectively. The use of the exponential function guarantees that the target leverage is located between the two thresholds. The vector of explanatory variables, $X_{it}$, is assumed to be the same for the target and the two thresholds.

The distribution of the error terms is assumed to be jointly Gaussian:

$$\begin{bmatrix} u^*_{it} \\ u^u_{it} \\ u^d_{it} \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} (\sigma^*)^2 & \rho^{*u} \sigma^* \sigma^u & \rho^{*d} \sigma^* \sigma^d \\ \rho^{*u} \sigma^* \sigma^u & (\sigma^u)^2 & 0 \\ \rho^{*d} \sigma^* \sigma^d & 0 & (\sigma^d)^2 \end{bmatrix}\right).$$

The errors are i.i.d. across firms and time and uncorrelated with $X_{it}$. The likelihood function for this model is derived in Appendix A.
Our empirical model is based on other applications of \((S, s)\) models. For example, Attanasio (2000) uses a similar structure to estimate an \((S, s)\) model for consumers’ automobile purchases as a function of time-invariant household characteristics, while imposing a common error term on the upper and lower thresholds. Applying this methodology to capital structure, however, introduces a number of novel features. First, we need to allow for time-varying covariates, which results in time-variation in the thresholds. Second, because the upper and lower thresholds on capital structure are driven by fundamentally different considerations, we allow for separate error terms in the upper and lower adjustment thresholds. Third, there are natural bounds on the values that leverage may take, which may differ by measure. For example, market leverage is naturally bounded above by 1, and leverage measures gross of cash are bounded below at 0. It is important to recognize these bounds in the estimation.

The parameters of target leverage, \(\beta\), are identified from the first observation of leverage following a refinancing event, when the firm returns to \(L^*\). For example, if \(L^*\) is the mode of the stationary leverage distribution, as in our examples in Section ??I, then \(\beta\) can be interpreted as determining the conditional mode of the leverage distribution as a linear function of \(X\). The error term, \(u^*\), captures explanatory variables that are omitted from \(X\). In addition, in practice leverage is reported only at discrete intervals, and so it typically is not observed by empiricists immediately upon refinancing. Leverage may have moved away from the true \(L^*\) in the period between refinancing and observation, introducing an additional observation error. Furthermore, \(L^*\) may have changed over this period. The estimate of \(\sigma^*\) captures both sources of error.

We can learn about the upper and lower thresholds from the periods of inaction, in which firms do not refinance. For the firms that do not refinance in the current period, leverage is between the thresholds, \(L_{it}^d < L_{it} < L_{it}^u\). This condition yields a lower bound on \(L_{it}^u\) and an upper bound on \(L_{it}^d\). With discrete time, leverage may still move beyond its current value before the actual refinancing event takes place (even if that happens in the next period), and without further assumptions we can only identify a lower bound on \(L_{it}^u\) and an upper bound on \(L_{it}^d\). We estimate the
parameters, $\theta^u$ and $\theta^d$, that characterize these bounds. In the period before levering down, the observed leverage ratio is the best estimate of the lower bound on $L^u_{it}$. Similarly, the leverage in the period before levering up is the best estimate of the upper bound on $L^d_{it}$. With a high frequency of observations, the estimated bounds on $L^u$ and $L^d$ will be close to the true refinancing thresholds. For statistical efficiency, we use all observations of leverage to identify the parameters of these bounds, not just the observations in the period before a refinancing. In the robustness section, we explore an alternative estimation procedure that identifies the thresholds exactly, and show that the empirical results are similar. Note that if we are in a static world or a dynamic world without adjustment costs, our model would not converge, because the coefficients in $\theta^u$ and $\theta^d$ that load on the intercept in $X$ would tend to negative infinity.5

The correlations between $L^u$ and $L^*$, and between $L^d$ and $L^*$ can be identified. In the inaction region, we can learn about $L^u$ (and hence $u^u$) if leverage is above the target (as partly determined by the realization of $u^*$). This reveals information about the correlation between these two error terms. The same argument applies to $u^d$ and $u^*$. However, the correlation between $u^d_i$ and $u^u_i$ in (4) is not identified, because in no-refinancing periods we learn about either $L^u_{it}$ or $L^d_{it}$, not both simultaneously. In the benchmark case, we assume this correlation to be zero.

III Data

To assemble the main sample, we start with quarterly data from Compustat between 1984 (the year quarterly equity issuance and repurchase data first became available) and 2009. We exclude utilities (SIC codes 4900–4999) and financial firms (SIC codes 6000–6999) to avoid companies whose financial policies are largely driven by government regulation. We also exclude companies with less than $10m in book assets, in year 2000 dollars. We use leverage measures commonly employed in prior literature,

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5 Note that our model is more informative than simply identifying the refinancing thresholds from quantile regressions of leverage on the covariates (or the extremes of a firm’s leverage ratios), as doing so would ignore the actual refinancing decisions.
which we define as follows:

- Market leverage: book debt (the sum of short-term debt (DLCQ) and long-term debt (DLTTQ)), divided by the sum of book debt and the market value of equity (the product of shares outstanding (CSHOQ) and price per share (PRCCQ)).


The explanatory variables we employ are standard and are defined as follows:

- PROF: Profitability, measured as EBITDA (OIBDPQ) divided by book assets.

- MB: Market-to-book ratio, defined as the sum of the market value of equity and book debt divided by book assets.

- PPE: Property, plant, and equipment (PPENTQ) divided by book assets.

- DEPR: Depreciation (DPQ) divided by book assets.

- RD: R&D expense (XRDQ) divided by book assets.

- RDdum: An indicator variable that equals 1, if the firm had a non-zero R&D expense in the past quarter, and 0 otherwise.


- VOL: Volatility of quarterly profitability over the past 10 years.

To minimize the influence of outliers, we trim net market and book leverage at −1, as well as profitability, depreciation, R&D, and earnings volatility at the 99th percentile; furthermore, we trim profitability at the 1st percentile and the market-to-book ratio at 10.
Because an \((S, s)\) model is inherently a model of dynamic capital structure, we restrict the sample further to exclude firms with fewer than eight quarters of consecutive data. The final sample is an unbalanced panel of 5,252 firms spanning 160,305 firm-quarters. Table II reports summary statistics, and Figure 4 depicts histograms for our leverage measures. Requiring at least two years of data inevitably introduces a survivorship bias. Yet, compared with the full sample, there is no material difference in leverage, though in our sample firms are slightly larger and more profitable, and more firms have non-zero R&D expenditures (results not reported).

We define refinancing in a manner consistent with prior studies (e.g., Hovakimian, Opler, and Titman (2001), Korajczyk and Levy (2003), Hovakimian (2004), Leary and Roberts (2005), and Frank and Goyal (2009)). A firm is classified to have increased its leverage in a given quarter if net debt issuance (debt issuance (DLTISY) minus debt repurchases (DLTRY)) minus net equity issuance (equity issuance (SSTKY) minus repurchases (PRSTKCY)) over the quarter is greater than 5% of the beginning-of-quarter book value of assets.\(^6\) Conversely, if net equity issuance minus net debt issuance is greater than 5% of book assets, the firm is designated to have reduced its leverage. This definition of refinancing captures not only public but also private debt issuance, the most prevalent form of debt financing (e.g., Houston and James (1996), Bradley and Roberts (2004)). The 5% threshold is chosen to capture financing decisions that are intended to change capital structure, and it excludes most “mechanical” issuance and repurchases, such as equity issuance due to the planned exercise of executive stock options. As Leary and Roberts (2005) note, even though this measurement may result in some misclassifications (such as the calling of convertible debt or the transfer of equity accounts from subsidiaries to parents), the 5% classification scheme produces results similar to using new debt and equity issuance data from SDC (see also Hovakimian et al. (2001), Korajczyk and Levy (2003)).

\(^6\)Compustat reports debt and equity issuance and repurchase variables on a cumulative basis throughout the fiscal year, so for each variable we first obtain its net contribution for a given quarter.
Panel C of Table II reports summary statistics for refinancing activity. Firms increase their leverage in 8.1% of the observed firm-quarters, and the median firm increases leverage twice during the sample. When they increase leverage, firms issue net debt in excess of net equity of 16% of book assets on average (median 10%). Leverage decreases occur in 5.6% of the firm-quarters. The median firm reduces leverage once and issues net equity in excess of net debt of 16% on average (median 9%). The distribution of refinancings is skewed: 15.9% of firms do not refinance in the sample period, 15.0% refinance only once, and 10.3% of firms undergo 10 or more refinancings. However, these numbers may overstate the number of true refinancings, because some leverage rebalancings span more than one quarter. In the robustness section, we show that our results are robust to grouping consecutive refinancing events together.

IV Empirical Analysis of the \((S, s)\) Model

In this section, we discuss the results of the empirical analysis of the \((S, s)\) model of leverage. We first concentrate on target leverage, then turn to the results on the refinancing thresholds. Finally, we explore the robustness of these results.

A Target leverage

Table III reports the parameter estimates of the \((S, s)\) model, where we choose net market leverage (i.e, leverage net of cash) as our benchmark proxy, as well as the coefficients of a standard OLS regression of leverage on the covariates. Although we concentrate first on the results for target leverage and their comparisons with those derived from the OLS approach, it is important to stress that one of the critical differences between the \((S, s)\) and OLS modeling approaches is that the latter allows the firm to adjust only leverage in response to changing covariates, whereas the former provides the firm with two additional levels of control, namely, adjustments to the upper and lower refinancing thresholds. Thus, the OLS approach, if taken as a structural model, corresponds to a static world, in which adjustment costs are zero.
(and thus, firms are always at the optimum). Any explanation in differences between
the results on target leverage should thus emphasize the additional flexibility of the
($S, s$) approach.

The $\beta$ coefficients in Table III show that target leverage is statistically significantly
higher for larger, more profitable firms with lower market-to-book ratios, higher tan-
gible assets, lower depreciation, and zero research and development outlays. Although
these coefficients have the same sign as the OLS estimates in most cases, the implied
economically meaningful magnitudes are vastly different. The most striking result,
though, involves the profitability coefficient. In standard OLS regressions, profitabil-
ity reliably has a strong negative relationship to leverage, consistent with prior studies
(see, e.g., Fama and French (2002)). In contrast, the ($S, s$) model yields a positive
relation between target leverage and profitability. The coefficient is statistically sig-
nificant at the 1% level and, as robustness tests show, is consistent across various
measures of leverage, although its economic significance is rather low, as Figure 5
indicates. Overall, the positive loading on profitability is consistent with a dynamic
trade-off model with adjustment costs. In that model, positive (negative) shocks
to profitability mechanically lower (increase) leverage, driving the negative OLS co-
efficient, even though the target leverage ratio is positively related to profitability
(Strebulaev (2007)).

To gain insight into the economic significance of the $\beta$ estimates, in Figure 5 we
plot comparative statics of fitted target leverage, $L^*$ (marked with an “o”), along
with the OLS fitted values (marked with an “x”). For example, the top-left plot
shows target leverage for the three values of profitability at the 20th, 50th, and 80th
percentiles of the sample data, leaving all the other covariates at their sample median
values. The corresponding numerical values are in Table IV. The fitted target leverage
from the ($S, s$) model in Figure 5 is larger than the OLS point estimate across all the
covariates. The difference is economically large: The median firm’s target leverage is
0.23, compared with the OLS point estimate of 0.19. This difference suggests that the
underleverage result, widely documented in prior literature, may be less severe than
previously thought. As firms grow, the presence of adjustment costs induces them
to optimally allow leverage to drift down, resulting in lower leverage on average, compared with their target leverage upon refinancing.

The magnitude and sign of the differences between the $\beta$ and OLS coefficients are driven by multiple factors. First, the drift and volatility of the leverage process between refinancings may vary with the covariates. Table V shows the coefficient estimates of regressions of firm-level leverage drift and volatility on covariates, where drift is measured across non-refinancing periods. Most covariates are statistically significantly related to the drift of leverage (the first column of the table). For example, highly profitable firms have a faster growing market value of equity, so their market leverage tends to drift down faster than that of less profitable firms. Keeping the refinancing thresholds constant, this tendency shifts a greater mass of the leverage distribution towards the lower values, decreasing the mean relative to the target leverage. Coefficients on R&D and earnings volatility reveal the same pattern (but not firm size). By the same token, the drift of leverage is higher for firms with higher market-to-book ratios, tangible assets, depreciation, and R&D expenses. For all of these covariates, apart from $PPE$ and the R&D dummy, the OLS coefficients in Table III are indeed higher than the corresponding values of $\beta$.

The volatility of the leverage process, absent refinancings, may also vary with the covariates. The second column in Table V shows that most covariates are strongly related to the volatility of the leverage process. All else being equal, a lower volatility of the leverage process implies that the density of leverage has more mass concentrated around the target leverage ratio, leading the OLS estimates to approach the $\beta$ values. For example, firms with a higher market-to-book ratio tend to have lower leverage volatility. Figure 5 shows that the OLS fitted leverage is closer to the target leverage when $MB$ is high. The results for all covariates are consistent with this phenomenon, except for profitability, depreciation, and R&D.

The second factor underlying the large differences between the $\beta$ and OLS coefficients is that these coefficients may be affected by the location of the refinancing thresholds. If the thresholds vary systematically with the covariates, the conditional mean of leverage, and thus the OLS estimates, may be pulled in the direction of
the threshold (in Figure 2, we see how the distribution, and thus the mean, of the leverage distribution shifts if the thresholds shift). For example, the coefficient on earnings volatility for target leverage in Table III is virtually zero, whereas the OLS coefficient is $-0.831$. At the same time, the upper refinancing threshold loading on earnings volatility is $-1.202$, and the lower threshold loading is virtually zero. Because the upper refinancing threshold is closer to target leverage for higher levels of earnings volatility, the mass of the leverage distribution shifts towards the lower leverage ratios, and therefore conditional mean leverage is pulled down relative to target leverage. This mechanism results in a lower coefficient on earnings volatility in OLS regressions relative to the corresponding $\beta$ coefficient.

An important conclusion that can be made from these considerations is that it is difficult to interpret the OLS fitted leverage ratios in a structural manner, because they represent a simultaneous combination of forces related to the leverage diffusion process, the target leverage, and the refinancing thresholds.

B Refinancing thresholds

The $(S, s)$ approach enables us to introduce direct estimates of the refinancing boundaries into the capital structure literature and explore their dependence on covariates, as well as their empirical relation to target leverage estimates. Table IV reveals that the inaction region between the two refinancing thresholds is large: Firms allow their leverage ratios to vary between the lower threshold of around $0.41$ and an upper threshold around $0.75$. Although the magnitude of the gap is driven to a significant extent by our use of a measure of the net of cash leverage, it still is notably large for gross of cash measures. Unreported, for (gross) market leverage measure, the corresponding lower (upper) thresholds are approximately $0.04$ ($0.57$).

The width of the inaction region may be driven by either high costs of adjustment or a low loss of value for being away from target leverage (these are not mutually

\footnote{Recall that the “upper” threshold is a leverage-reducing trigger, at which firms opt to refinance when leverage is too high, whereas the “lower” threshold is a leverage-increasing trigger, at which firms opt to refinance when leverage is too low.}
exclusive explanations). In particular, the large negative lower threshold indicates that firms are not overly concerned with running large cash balances. This result is consistent with Korteweg (2010), who finds that the present value of the net benefits to leverage is quite flat for a large range of leverage ratios around the target. Korteweg also documents that the size of issuance and repurchase transaction costs alone are not enough to explain the deviations from the target observed in the data. Other costs of adjustment, not captured by pure transaction costs, may include asymmetric information costs of selling undervalued securities and management time spent raising capital rather than running the firm (which may be especially high for distressed firms).

The size of the inaction region may appear surprising, considering that the median firm appears to refinance once a year, as documented in Table II. However, the distribution of refinancings is skewed, with 16% of firms choosing not to refinance at all over the sample period, and another 15% refinancing only once. Only a small fraction of firms rebalance frequently. Moreover, single rebalancing events are often implemented and reported over two consecutive quarters, likely overstating the true number of refinancing actions, although the quantitative impact of “quarter stretching” is not immediately available to us. Finally, any deliberate temporary deviations from \((S, s)\) policies, such as those discussed by DeAngelo, DeAngelo, and Whited (2011), may overstate the frequency of refinancings relative to the \((S, s)\) model we study here.

As Table IV shows, the upper and lower refinancing thresholds have different sensitivities to the covariates. The upper refinancing threshold is decreasing in profitability, whereas the lower threshold is not significantly related to it. Moreover, the upper threshold is an order of magnitude more sensitive to profitability than the target leverage. Overall, the results suggest that firms are most concerned about their capital structure situation when leverage is high and the likelihood of financial distress is non-trivial; they are less preoccupied with the decision to lever up (e.g., because of asymmetry in the value function, according to Korteweg (2010), who finds that it is relatively flat for low but drops sharply for very high leverage ratios).
Table IV shows that several other variables, such as the market-to-book ratio, tangibility, the R&D dummy, and size, are important drivers of both the upper and lower refinancing thresholds. Many of these relationships are intuitive and in line with well-known economic mechanisms. For example, the relationship between the market-to-book ratio and the refinancing thresholds is consistent with the debt overhang mechanism, where firms keep leverage in a lower range when they have substantial growth opportunities. This mechanism also can explain the evidence regarding the R&D dummy.

Tangible assets, as measured by \( PPE \), are related strongly to both higher target leverage and higher refinancing thresholds, implying that firms with a large proportion of tangible assets find it advantageous to have high leverage. This result is consistent with the lower costs of financial distress for such firms (e.g., due to lower fire-sale costs). It is also consistent with a credit rationing story, in which firms with more pledgeable assets are less constrained by creditors in taking on more debt. Both thresholds and target leverage also increase with firm size, so large firms exhibit significantly higher leverage ratios than small firms.

In a standard trade-off model, firms with higher earnings volatility have lower leverage to avoid the negative consequences of financial distress, which occurs with higher likelihood for more volatile firms (all else being equal). The negative coefficient on earnings volatility in the OLS specification in Table III supports this explanation. The \((S, s)\) model estimates show, however, that the effect of earnings volatility works mainly through the upper refinancing threshold: Firms with highly volatile earnings reduce debt at lower leverage ratios than firms with stable earnings, whereas the target leverage is not affected by earnings volatility. This mechanism affects leverage parameters through financial distress, so it is not surprising that the lower refinancing threshold is not related to earnings volatility. The same explanation can be applied to research and development expenses.

Because the economic significance results in Table IV are based on comparative statics, it is not clear how much variation in capital structure policies the model actually captures. To explore this question, Figure 6 shows histograms of the target
capital structure and the refinancing thresholds across 56 two-digit SIC code industries (excluding financials and any industries with fewer than 1,000 firm-quarter observations). For each industry, we calculate median industry characteristics and feed them into the model using the parameter estimates of Table IV. The figure reveals substantial variation not only in the target leverage ratio but also in the refinancing thresholds across industries. The upper threshold is as low as 0.55 for plastics and chemicals (SIC code 28), an industry characterized by high market-to-book and R&D expenses, many intangible assets, and volatile earnings; it is as high as 0.88 for railroads (SIC code 40), an industry with large firms mostly comprised of tangible assets, few growth opportunities, and stable earnings. The lower threshold ranges from -0.60 for lab equipment (SIC code 38), an R&D-intensive industry with few tangible assets, to -0.17 for railroads (SIC code 40).

C Robustness

As is true of any empirical study, it is important to verify whether our results are driven by specific assumptions about variable construction or the estimation procedure. As a first robustness check on our results, we run an alternative estimation procedure that identifies the thresholds exactly, by assuming that the firm exceeded the threshold in the period prior to refinancing. In this case, $\theta^u$ and $\theta^d$ have a different interpretation: They specify the exact threshold rather than a bound on the thresholds. This exact identification comes at the cost of assuming that firms do not immediately refinance after hitting the threshold, which is contrary to the intuition of the $(S,s)$ model (though it is possible to rationalize this behavior through, for example, infrequent capital structure evaluation by management). It is, however, reassuring that the results under the two identification strategies turn out to be quantitatively very similar.\footnote{Attanasio (2000) uses a similar identification strategy to the one we employ in our robustness tests.}

Second, we set the threshold for refinancing at 3\% and 7\% of book assets, instead of the 5\% used in the main results. The results are largely unaffected. The most
A noteworthy change for the 7% threshold case is that the $\beta$-coefficient on $DEPR$ loses significance. For the 3% threshold case, the $\beta$-coefficient on $PROF$ falls to 5% significance (but remains positive), the $\theta^u$ loading on $LN(TA)$ flips sign from negative and significant to positive and significant, and the $\theta^d$ loading on $RD$ becomes statistically significant at the 1% level (from a statistically insignificant loading in the benchmark case).

For our third robustness test, we grouped consecutive refinancings together to account for cases in which, for example, an equity issue and debt repurchase were split across quarters. Most of our results are unchanged qualitatively, except for the $\beta$-loading on $DEPR$, which becomes insignificant, and the $\theta^d$-coefficients on $R&D$ and $LN(TA)$ switch sign and become positive and significant at the 1% level.

Fourth, we included short-term debt changes in the definition of refinancing events. Our main results use long-term debt issuance and repurchase only, since short-term debt is often seasonal, and is missing in many cases. Including short-term debt reduces the sample size in half, because of missing observations, but our results remain close to the ones we reported in the previous subsections.

Fifth, we considered measures of leverage different from the benchmark net market leverage that we used in the prior subsections. Using net book leverage gives very similar results. The only notable differences are that the $\beta$-loading on $DEPR$ and the $\theta^u$-loading on $PROF$ become positive and significant. Using book leverage (gross of cash) does not change our results qualitatively, unless we drop zero leverage firms, in which case the $\beta$-coefficient on $PROF$ turns negative. Using market leverage (gross of cash) gives similar results to book leverage.

To summarize, we find that our main results are robust to using a different identification strategy, to using different measures of leverage, and to other variations on data definitions.
V Conclusion

We propose and develop a general \((S, s)\) model of capital structure, with which we investigate the relationship between target leverage and its covariates in a dynamic environment. Unlike traditional empirical capital structure methods, the \((S, s)\) model takes into account the non-linear dynamics of the leverage data generating process and therefore can be more informative about multiple issues at the core of capital structure research, such as target leverage behavior. The empirical results show substantial differences between the traditional mean-based and \((S, s)\) models. In particular, the target leverage in the \((S,s)\) model is higher than the fitted value from the traditional regressions, and the target leverage increases with profitability, in contrast with the traditional regression results that suggest a negative relation between leverage and profitability. We also offer several new results regarding the refinancing thresholds. We show that firms allow leverage to float in a wide band around the target, and we find that some of the standard OLS regression results are driven by the refinancing thresholds rather than the target leverage – such as the negative coefficients on research and development outlays and earnings volatility.

Because this study is the first to estimate dynamic \((S, s)\) models for capital structure, we have deliberately chosen a simple specification. Obviously, our \((S, s)\) model should be viewed as only the first step in building the next generation of dynamic empirical models in capital structure and elsewhere in corporate finance. For example, we assume the existence of only fixed costs of adjustment. In the presence of both fixed and variable costs, the return points of target leverage differ between upper and lower refinancing triggers. We also do not allow for predetermined financing events (e.g., when outstanding debt becomes due) or investment-driven financing. The \((S, s)\) method can be extended to include these and many other enriching features in capital structure and, more generally, empirical corporate finance. We believe this methodology is an important avenue for future research.
References


Appendix A: Likelihood function

The log-likelihood for leverage for firm $i$ at time $t$, $L_{it}$, is

$$
\log f(L_{it}) = \mathbb{I}_{\text{refi}} \cdot \log f_{\text{refi}}(L_{it}) + \mathbb{I}_{\text{refiup}} \cdot \log f_{\text{refiup}}(L_{it}) + \mathbb{I}_{\text{refidown}} \cdot \log f_{\text{refidown}}(L_{it}) + \mathbb{I}_{\text{norefi}} \cdot \log f_{\text{norefi}}(L_{it}).
$$

Each firm-year falls into one of four cases:

1. Refinancing in this period (“refi”).
2. No refinancing in this period and levering up next period (“refi up”).
3. No refinancing in this period and levering down next period (“refi down”).
4. No refinancing in this period or the next (“no refi”).

We show the log-likelihood of each case using our main identification strategy, where $L_{it}$ is bounded above at 1, and the period before refinancing is the best estimate of the threshold. Imposing an additional lower bound for gross of cash leverage measures, or assuming that the firm refinances only after crossing the boundary, is straightforward.

**Case 1: Refinancing this period.** The observed leverage, $L_{it}$, is a noisy observation of the target, $L_{it}^*$, so we have an observation of $u_{it}^* = L_{it} - X_{it}' \beta$. Taking into account that $L_{it}^* \leq 1$, the likelihood is the truncated pdf of $u^*$,

$$
f_{\text{refi}}(L_{it}) = \frac{1}{\sigma^*} \phi \left( \frac{L_{it} - X_{it}' \beta}{\sigma^*} \right) \cdot \mathbb{I}_{\{1 - X_{it}' \beta\}} \Phi \left( \frac{1 - X_{it}' \beta}{\sigma^*} \right),
$$

where $\phi(\cdot)$ is the pdf of a standard Normal distribution and $\Phi(\cdot)$ is its cdf.

**Case 2: No refinancing in this period and levering up next period.** We observe $L_{it} = L_{it}^d$, implying $u_{it}^d | u^* = \log (X_{it}' \beta + u^* - L_{it}) - X_{it}' \theta^d$. The gap between $L_{it}^*$ and $L_{it}$ must be non-negative, and therefore $u_{it}^* \geq L_{it} - X_{it}' \beta$. Given the upper bound, $L_{it}^* \leq 1$, we also enforce $u_{it}^* \leq 1 - X_{it}' \beta$. The likelihood is

$$
f_{\text{refiup}}(L_{it}) = \int_{L_{it} - X_{it}' \beta}^{1 - X_{it}' \beta} \frac{1}{\sigma^d} \phi \left( \frac{u^*}{\sigma^*} \right) \cdot \frac{1}{\hat{\sigma}^d} \phi \left( \frac{\log (X_{it}' \beta + u^* - L_{it}) - X_{it}' \theta^d - \hat{\mu}^d}{\hat{\sigma}^d} \right) du^*.
$$

The distribution of $u^d$ inside the integral is conditional on $u^*$, so $\hat{\mu}^d = \frac{\sigma^d}{\sigma^*} \rho^d u^*$, and $\hat{\sigma}^d = \sigma^d \sqrt{1 - (\rho^d)^2}$.

**Case 3: No refinancing in this period and levering down next period.** We observe $L_{it} = L_{it}^u$, and therefore $u_{it}^u | u^* = \log (L_{it} - X_{it}' \beta - u^*) - X_{it}' \theta^u$. Note that the upper
bound, $L_{it}^u \leq 1$, is inconsequential because $1 \geq L_{it} \geq L_{it}^u$. Since the gap between $L_{it}$ and $L_{it}^*$ must be non-negative, we also enforce $u_{it}^* \leq L_{it} - X_{it}' \beta$. This subsumes the bound $L_{it}^* \leq 1$. The likelihood is

$$f_{\text{refidown}}(L_{it}) = \int_{-\infty}^{L_{it}} \frac{1}{\sigma^*} \phi \left( \frac{u^*}{\sigma^*} \right) \cdot \frac{1}{\tilde{\sigma}^u} \phi \left( \frac{\log \left( L_{it} - X_{it}' \beta - u^* \right) - X_{it}' \theta^u - \tilde{\mu}^u}{\tilde{\sigma}^u} \right) \, du^*,$$

where $\tilde{\mu}^u = \frac{\sigma^u}{\sigma^*} \rho^* u^*$, and $\tilde{\sigma}^u = \sigma^u \sqrt{1 - (\rho^*)^2}$.

**Case 4: No refinancing in this period or next period.** We observe $L_{it}^d < L_{it} < L_{it}^u$, so $u_{it}^d | u^* > \log \left( X_{it}' \beta + u^* - L_{it} \right) - X_{it}' \theta^d$, and $u_{it}^u | u^* > \log \left( L_{it} - X_{it}' \beta - u^* \right) - X_{it}' \theta^u$. Note that only one of the two conditions can bind for a given $u^*$. The bounds on leverage ratios also require $u_{it}^d | u^* \leq \log \left( 1 - X_{it}' \beta - u^* \right) - X_{it}' \theta^u$ and $u_{it}^* \leq 1 - X_{it}' \beta$. The likelihood is

$$f_{\text{norefi}}(L_{it}) = \int_{-\infty}^{1-X_{it}' \beta} \frac{1}{\sigma^*} \phi \left( \frac{u^*}{\sigma^*} \right).$$

$$\left\{ 1 - \mathbb{I}_{\{X_{it}' \beta + u^* - L_{it} > 0\}} \cdot \left[ 1 - \Phi \left( \frac{\log \left( X_{it}' \beta + u^* - L_{it} \right) - X_{it}' \theta^d - \tilde{\mu}^d}{\tilde{\sigma}^d} \right) \right] - \right.$$

$$\left. - \mathbb{I}_{\{X_{it}' \beta + u^* - L_{it} < 0\}} \cdot \left[ \Phi \left( \frac{\log \left( 1 - X_{it}' \beta - u^* \right) - X_{it}' \theta^u - \tilde{\mu}^u}{\tilde{\sigma}^u} \right) \right] - \Phi \left( \frac{\log \left( L_{it} - X_{it}' \beta - u^* \right) - X_{it}' \theta^u - \tilde{\mu}^u}{\tilde{\sigma}^u} \right) \right \} \, du^*.$$
Figure 1

Example of time-series of leverage

This figure illustrates the rebalancing of leverage at the boundaries $L^d$ and $L^u$. Upon hitting one of the boundaries, the firm rebalances to $L^*$.
Figure 2
Stationary leverage distribution: Arithmetic Brownian motion

This figure shows the stationary distribution of leverage, where, between refinancings, the leverage process follows an arithmetic Brownian motion with drift. $L^*$ is target leverage $L^*$, $L^d$ is the lower refinancing threshold, and $L^u$ is the upper refinancing threshold.
Figure 3  
Stationary distribution and mean leverage estimates: Structural models

This figure shows the distribution of leverage and estimates of means versus target leverage in various structural models. Each row of plots represents a model, where KS (in Panel A) indicates the structural model in this paper, GJL (in Panel B) refers to Goldstein, Ju, and Leland’s (2001) model, and MNS (in Panel C) refers to the model by Morellec, Nikolov, and Schürhoff (2012). For each model, we simulate a panel data set with 10,000 firms (each with its own set of randomly drawn parameters as shown in Table I), for 100 years of monthly data. The left-hand side plots are the stationary distributions of leverage for each model. The right-hand side plots are scatterplots of target leverage ($L^*$, on the horizontal axis) versus firms’ average leverage (vertical axis), where each dot is a separate firm. The straight line in each plot is the 45 degree line that runs through the origin.
Figure 4
Histograms of leverage

This figure depicts histograms of various measures of leverage across all firm-quarters in the sample. The leverage measures are as defined in Table II.
Economic significance: Net market leverage

This figure depicts the economic significance of the \((S,s)\) model estimates of Table III. For each covariate, the figure plots the value of target leverage \((L^*)\), denoted by “o”, and the OLS fitted leverage, denoted by “x”, for the values of the covariate at the 20th, 50th and 80th percentile of the pooled empirical distribution, keeping all other covariates at their median values. The covariates are described in Table II. The variable \(RDdum\) can only take the value of 0 (no R&D expenses) or 1 (non-zero R&D outlays). For \(RD\), we set \(RDdum = 1\) and consider the percentiles of the distribution of positive R&D outlays only.
Figure 6
Capital structure policy across industries: Net market leverage

This figure depicts histograms of $L^*$, $L^u$, and $L^d$ for 56 two-digit SIC code industries with at least 1,000 firm-quarter observations, excluding financials (two-digit codes 60 through 69), using median industry characteristics and the model estimates from Table III.
Table I

Model parameters for simulations

This table shows the parameters ranges used for simulating structural models from the literature. In order of model complexity, KS indicates the structural model in this paper, GJL stands for the Goldstein, Ju, and Leland (2001) model, and MNS refers to the model by Morellec, Nikolov, and Schürhoff (2012). For each model, we simulate a panel of 100 years of monthly data for 10,000 firms. Each firm has a different set of parameters, drawn from a Uniform distribution between the upper and lower bounds shown in this table. We use the parameter notations from the text and, where possible, we use the original notation from the GJL and MNS models.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>KS lower</th>
<th>KS upper</th>
<th>GJL lower</th>
<th>GJL upper</th>
<th>MNS lower</th>
<th>MNS upper</th>
<th>Description</th>
</tr>
</thead>
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<tr>
<td>$\mu$</td>
<td>0</td>
<td>0.05</td>
<td>0</td>
<td>0.05</td>
<td>0</td>
<td>0.05</td>
<td>Expected cash flow growth under $Q$</td>
</tr>
<tr>
<td>$\mu^P$</td>
<td>$\mu$</td>
<td>$\mu + 0.05$</td>
<td>$\mu$</td>
<td>$\mu + 0.05$</td>
<td>$\mu$</td>
<td>$\mu + 0.05$</td>
<td>Expected cash flow growth under $P$</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.2</td>
<td>0.5</td>
<td>0.2</td>
<td>0.5</td>
<td>0.2</td>
<td>0.5</td>
<td>Volatility of cash flow growth</td>
</tr>
<tr>
<td>$r$</td>
<td>$\mu + 0.02$</td>
<td>$\mu + 0.05$</td>
<td>$\mu + 0.02$</td>
<td>$\mu + 0.05$</td>
<td>$\mu + 0.02$</td>
<td>$\mu + 0.05$</td>
<td>Risk-free rate</td>
</tr>
<tr>
<td>$\tau_c$</td>
<td>0.1</td>
<td>0.25</td>
<td>0.1</td>
<td>0.25</td>
<td>0.1</td>
<td>0.25</td>
<td>Corporate tax rate</td>
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<tr>
<td>$\tau_d$</td>
<td>-</td>
<td>-</td>
<td>0</td>
<td>0.2</td>
<td>0</td>
<td>0.2</td>
<td>Dividend tax rate</td>
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<tr>
<td>$\tau_i$</td>
<td>-</td>
<td>-</td>
<td>0</td>
<td>0.2</td>
<td>0</td>
<td>0.2</td>
<td>Interest income tax rate</td>
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<tr>
<td>$\alpha$</td>
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<td>0.5</td>
<td>0.1</td>
<td>0.5</td>
<td>$\kappa$</td>
<td>$\kappa + 0.5$</td>
<td>Proportional bankruptcy cost</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0</td>
<td>0.25</td>
<td>Proportional cost of renegotiation</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0</td>
<td>0.02</td>
<td>0</td>
<td>0.02</td>
<td>0</td>
<td>0.02</td>
<td>Proportional cost of debt issuance</td>
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<tr>
<td>$\phi$</td>
<td>-</td>
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<td>-</td>
<td>-</td>
<td>0</td>
<td>0.05</td>
<td>Cost of control challenge</td>
</tr>
<tr>
<td>$\eta$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.2</td>
<td>0.8</td>
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<tr>
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<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0</td>
<td>0.5</td>
<td>Managerial stock ownership fraction</td>
</tr>
</tbody>
</table>

NB: called $q$ in GJL and $\lambda$ in MNS.
Table II
Summary statistics

This table reports summary statistics for leverage measures (Panel A), explanatory variables (Panel B), and refinancing activity (Panel C). The sample consists of 160,305 firm-quarters for 5,252 firms between 1984 and 2009. Market leverage is defined as book debt (the sum of short and long-term debt) divided by the sum of book debt and the market value of equity. Net market leverage uses book debt net of cash in the numerator. Book leverage is book debt divided by book assets, and net book leverage is book debt net of cash divided by book assets. Interest coverage is defined as EBITDA divided by interest expense over the prior quarter. PROF is EBITDA divided by book assets; MB is the market-to-book ratio, the sum of the market value of equity and book debt divided by book assets; PPE is property, plant, and equipment divided by book assets; DEPR is depreciation divided by book assets; RD is R&D expense divided by book assets; RDdum is an indicator variable that equals one if the firm had non-zero R&D expenses in the past quarter; LN(TA) is the natural log of book assets; and VOL is the volatility of quarterly profitability over the past 10 years. For RD we report the summary statistics for the subset of firms that report positive R&D expenses. All flow variables are reported on a quarterly basis. A leverage increase (decrease) occurs when the net debt (equity) issuance minus net equity (debt) issuance exceeds 5% of book assets. The median duration is the median number of quarters between refinancing events of the same type (e.g., for leverage increases it is the median number of quarters between leverage increases). Median adj per firm is the median number of refinancings (of a given type) on a per firm basis. Issuance amount is the amount of net debt minus net equity issued in the refinancing, scaled by beginning-of-period book assets.

<table>
<thead>
<tr>
<th>Panel A: Leverage Variables</th>
<th>Mean</th>
<th>St. Dev.</th>
<th>10</th>
<th>50</th>
<th>90</th>
</tr>
</thead>
<tbody>
<tr>
<td>Net market leverage</td>
<td>0.15</td>
<td>0.30</td>
<td>-0.20</td>
<td>0.13</td>
<td>0.56</td>
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<tr>
<td>Market leverage</td>
<td>0.25</td>
<td>0.24</td>
<td>0.00</td>
<td>0.19</td>
<td>0.62</td>
</tr>
<tr>
<td>Net book leverage</td>
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<td>0.32</td>
<td>-0.31</td>
<td>0.15</td>
<td>0.48</td>
</tr>
<tr>
<td>Book leverage</td>
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<td>0.20</td>
<td>0.00</td>
<td>0.22</td>
<td>0.52</td>
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</table>

<table>
<thead>
<tr>
<th>Panel B: Explanatory variables</th>
<th>Mean</th>
<th>St. Dev.</th>
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<th>50</th>
<th>90</th>
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<tr>
<td>PROF</td>
<td>0.03</td>
<td>0.04</td>
<td>-0.00</td>
<td>0.03</td>
<td>0.07</td>
</tr>
<tr>
<td>MB</td>
<td>1.41</td>
<td>1.04</td>
<td>0.61</td>
<td>1.10</td>
<td>2.56</td>
</tr>
<tr>
<td>PPE</td>
<td>0.34</td>
<td>0.24</td>
<td>0.07</td>
<td>0.28</td>
<td>0.71</td>
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<td>DEPR</td>
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<td>0.01</td>
<td>0.00</td>
<td>0.01</td>
<td>0.02</td>
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<tr>
<td>RD</td>
<td>0.00</td>
<td>0.01</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>RDdum</td>
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<td>0.30</td>
<td>0.00</td>
<td>0.00</td>
<td>1.00</td>
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<tr>
<td>LN(TA)</td>
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<td>1.81</td>
<td>3.64</td>
<td>5.65</td>
<td>8.35</td>
</tr>
<tr>
<td>VOL</td>
<td>0.03</td>
<td>0.03</td>
<td>0.01</td>
<td>0.02</td>
<td>0.05</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Panel C: Refinancing activity</th>
<th>Number of adjustments</th>
<th>Percent of quarters</th>
<th>Median duration</th>
<th>Median adj per firm</th>
<th>Issuance amount</th>
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<tr>
<td>type</td>
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<tr>
<td>Leverage change</td>
<td>21944</td>
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<td>4</td>
<td>3</td>
<td>0.02</td>
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<tr>
<td>Leverage increase</td>
<td>12965</td>
<td>8.09</td>
<td>5</td>
<td>2</td>
<td>0.16</td>
</tr>
<tr>
<td>Leverage decrease</td>
<td>8979</td>
<td>5.60</td>
<td>6</td>
<td>1</td>
<td>-0.16</td>
</tr>
</tbody>
</table>

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Table III
Parameter estimates: Net market leverage

This table shows the Maximum Likelihood parameter estimates of the (S,s) model. For firm i at time t,

\[ L^*_{it} = X'_i \beta + u^*_{it}, \]  
(Target leverage)

\[ L^u_{it} = L^*_{it} + \exp(X'_i \theta^u + u^u_{it}), \]  
(Upper refinancing threshold)

\[ L^d_{it} = L^*_{it} - \exp(X'_i \theta^d + u^d_{it}), \]  
(Lower refinancing threshold)

where \( \sigma^*, \sigma^u, \) and \( \sigma^d \) are the standard deviations of \( u^*, u^u, \) and \( u^d, \) respectively, and the \( \rho \)'s refer to their correlations (the correlation between \( u^u \) and \( u^d \) is not identified). The covariates are as defined in Table II. The OLS column shows OLS regression coefficients, where the standard deviation of OLS residuals is shown under \( \sigma^*. \) Standard errors are in brackets, and are adjusted for heteroscedasticity and autocorrelation for OLS. ***, **, and * denote significance at the 1%, 5%, and 10% levels, respectively.

Dependent variable: Net market leverage

<table>
<thead>
<tr>
<th></th>
<th>( \beta )</th>
<th>( \theta^u )</th>
<th>( \theta^d )</th>
<th>OLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>PROF</td>
<td>0.111</td>
<td>-0.525</td>
<td>-0.004</td>
<td>-0.425</td>
</tr>
<tr>
<td></td>
<td>(0.017)***</td>
<td>(0.071)***</td>
<td>(0.020)</td>
<td>(0.037)***</td>
</tr>
<tr>
<td>MB</td>
<td>-0.076</td>
<td>-0.001</td>
<td>-0.067</td>
<td>-0.060</td>
</tr>
<tr>
<td></td>
<td>(0.001)***</td>
<td>(0.002)</td>
<td>(0.002)***</td>
<td>(0.001)***</td>
</tr>
<tr>
<td>PPE</td>
<td>0.296</td>
<td>-0.311</td>
<td>0.003</td>
<td>0.294</td>
</tr>
<tr>
<td></td>
<td>(0.003)***</td>
<td>(0.007)***</td>
<td>(0.021)</td>
<td>(0.007)***</td>
</tr>
<tr>
<td>DEPR</td>
<td>-0.482</td>
<td>0.009</td>
<td>-0.009</td>
<td>-0.018</td>
</tr>
<tr>
<td></td>
<td>(0.057)***</td>
<td>(0.195)</td>
<td>(0.098)</td>
<td>(0.165)</td>
</tr>
<tr>
<td>RD</td>
<td>-0.041</td>
<td>-33.636</td>
<td>-0.109</td>
<td>-6.120</td>
</tr>
<tr>
<td></td>
<td>(0.284)</td>
<td>(0.753)***</td>
<td>(0.146)</td>
<td>(0.538)***</td>
</tr>
<tr>
<td>RDDum</td>
<td>-0.081</td>
<td>0.000</td>
<td>0.030</td>
<td>-0.093</td>
</tr>
<tr>
<td></td>
<td>(0.001)***</td>
<td>(0.006)</td>
<td>(0.002)***</td>
<td>(0.003)***</td>
</tr>
<tr>
<td>LN(TA)</td>
<td>0.020</td>
<td>-0.008</td>
<td>-0.009</td>
<td>0.023</td>
</tr>
<tr>
<td></td>
<td>(0.000)***</td>
<td>(0.001)***</td>
<td>(0.000)***</td>
<td>(0.001)***</td>
</tr>
<tr>
<td>VOL</td>
<td>0.000</td>
<td>-1.202</td>
<td>-0.000</td>
<td>-0.831</td>
</tr>
<tr>
<td></td>
<td>(0.023)</td>
<td>(0.099)***</td>
<td>(0.030)</td>
<td>(0.061)***</td>
</tr>
<tr>
<td>Intercept</td>
<td>0.117</td>
<td>-0.484</td>
<td>-0.362</td>
<td>0.071</td>
</tr>
<tr>
<td></td>
<td>(0.001)***</td>
<td>(0.003)***</td>
<td>(0.001)***</td>
<td>(0.007)***</td>
</tr>
</tbody>
</table>

\( \sigma^* \) 0.290  
\( \sigma^u \) 0.173  
\( \sigma^d \) 0.289  
\( \rho^{su} \) -0.602  
\( \rho^{sd} \) -0.013
### Table IV

**Economic significance: Net market leverage**

This table shows the economic significance of the \((S, s)\) model estimates of Table III. For each covariate, it reports the value of target leverage \(L^*\), the upper and lower refinancing thresholds \(L^u\) and \(L^d\), and the OLS fitted leverage \(\hat{L} (\text{OLS})\) for the values of the covariate at the 20th, 50th, and 80th percentile of the pooled empirical distribution, keeping all other covariates at their median values. The covariates are as described in Table II. The variable \(RDdum\) can only take the value of zero (no R&D expenses) or one (non-zero R&D outlays), and the outcomes are shown under the 20th and 50th percentiles, respectively. For \(RD\), we set \(RDdum = 1\) and consider the percentiles of the distribution of positive R&D outlays only.

<table>
<thead>
<tr>
<th></th>
<th>20</th>
<th>50</th>
<th>80</th>
<th>20</th>
<th>50</th>
<th>80</th>
<th>20</th>
<th>50</th>
<th>80</th>
<th>20</th>
<th>50</th>
<th>80</th>
</tr>
</thead>
<tbody>
<tr>
<td>PROF</td>
<td>0.224</td>
<td>0.228</td>
<td>0.232</td>
<td>0.760</td>
<td>0.754</td>
<td>0.749</td>
<td>-0.417</td>
<td>-0.413</td>
<td>-0.409</td>
<td>0.205</td>
<td>0.190</td>
<td>0.176</td>
</tr>
<tr>
<td>MB</td>
<td>0.265</td>
<td>0.228</td>
<td>0.118</td>
<td>0.791</td>
<td>0.754</td>
<td>0.643</td>
<td>-0.397</td>
<td>-0.413</td>
<td>-0.463</td>
<td>0.219</td>
<td>0.190</td>
<td>0.103</td>
</tr>
<tr>
<td>PPE</td>
<td>0.165</td>
<td>0.228</td>
<td>0.353</td>
<td>0.727</td>
<td>0.754</td>
<td>0.815</td>
<td>-0.476</td>
<td>-0.413</td>
<td>-0.288</td>
<td>0.127</td>
<td>0.190</td>
<td>0.314</td>
</tr>
<tr>
<td>DEPR</td>
<td>0.231</td>
<td>0.228</td>
<td>0.222</td>
<td>0.757</td>
<td>0.754</td>
<td>0.749</td>
<td>-0.410</td>
<td>-0.413</td>
<td>-0.418</td>
<td>0.190</td>
<td>0.190</td>
<td>0.190</td>
</tr>
<tr>
<td>RD</td>
<td>0.147</td>
<td>0.147</td>
<td>0.147</td>
<td>0.673</td>
<td>0.672</td>
<td>0.656</td>
<td>-0.513</td>
<td>-0.513</td>
<td>-0.513</td>
<td>0.097</td>
<td>0.096</td>
<td>0.091</td>
</tr>
<tr>
<td>RDdum</td>
<td>0.228</td>
<td>0.147</td>
<td>-</td>
<td>0.754</td>
<td>0.674</td>
<td>-</td>
<td>-0.413</td>
<td>-0.513</td>
<td>-</td>
<td>0.190</td>
<td>0.097</td>
<td>-</td>
</tr>
<tr>
<td>LN(TA)</td>
<td>0.188</td>
<td>0.228</td>
<td>0.281</td>
<td>0.723</td>
<td>0.754</td>
<td>0.796</td>
<td>-0.464</td>
<td>-0.413</td>
<td>-0.344</td>
<td>0.143</td>
<td>0.190</td>
<td>0.252</td>
</tr>
<tr>
<td>VOL</td>
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<td>0.228</td>
<td>0.228</td>
<td>0.761</td>
<td>0.754</td>
<td>0.735</td>
<td>-0.413</td>
<td>-0.413</td>
<td>-0.413</td>
<td>0.199</td>
<td>0.190</td>
<td>0.164</td>
</tr>
</tbody>
</table>
Table V
Leverage drift and volatility regressions
This table reports cross-sectional OLS regressions results, with the drift (left column) and volatility (right column) of net market leverage as the dependent variable. Drift is calculated as the firm-level average change in leverage in non-refinancing periods. Volatility is calculated as the firm-level standard deviation of changes in leverage across non-refinancing periods. The explanatory variables are firm-level medians, as defined in Table II. Standard errors are in brackets, and adjusted for heteroscedasticity and autocorrelation. ***, **, and * denote significance at the 1%, 5%, and 10% levels, respectively.

<table>
<thead>
<tr>
<th></th>
<th>Drift</th>
<th>Volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>PROF</td>
<td>-0.249</td>
<td>-0.618</td>
</tr>
<tr>
<td></td>
<td>(0.022)***</td>
<td>(0.047)***</td>
</tr>
<tr>
<td>MB</td>
<td>0.002</td>
<td>-0.016</td>
</tr>
<tr>
<td></td>
<td>(0.000)***</td>
<td>(0.001)***</td>
</tr>
<tr>
<td>PPE</td>
<td>0.006</td>
<td>-0.037</td>
</tr>
<tr>
<td></td>
<td>(0.002)***</td>
<td>(0.004)***</td>
</tr>
<tr>
<td>DEPR</td>
<td>0.165</td>
<td>0.514</td>
</tr>
<tr>
<td></td>
<td>(0.062)***</td>
<td>(0.110)***</td>
</tr>
<tr>
<td>RD</td>
<td>-0.980</td>
<td>-0.228</td>
</tr>
<tr>
<td></td>
<td>(0.155)***</td>
<td>(0.416)</td>
</tr>
<tr>
<td>RDdum</td>
<td>0.003</td>
<td>0.004</td>
</tr>
<tr>
<td></td>
<td>(0.001)***</td>
<td>(0.001)***</td>
</tr>
<tr>
<td>LN(TA)</td>
<td>-0.001</td>
<td>-0.004</td>
</tr>
<tr>
<td></td>
<td>(0.000)***</td>
<td>(0.000)***</td>
</tr>
<tr>
<td>VOL</td>
<td>-0.035</td>
<td>0.207</td>
</tr>
<tr>
<td></td>
<td>(0.015)***</td>
<td>(0.035)***</td>
</tr>
<tr>
<td>Intercept</td>
<td>0.006</td>
<td>0.130</td>
</tr>
<tr>
<td></td>
<td>(0.001)***</td>
<td>(0.003)***</td>
</tr>
<tr>
<td>R²-adj</td>
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<td>0.234</td>
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<tr>
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<td>5,099</td>
</tr>
<tr>
<td>F</td>
<td>50.574***</td>
<td>195.736***</td>
</tr>
</tbody>
</table>