Smart Buyers

By

Mike Burkart
Samuel Lee

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Mike Burkart is the Gösta Olson Professor of Finance at the Stockholm School of Economics. He is also a Visiting Professor of Finance at LSE and a Senior Research Associate of the FMG. His research interests cover applied theory in the field of financial contracting and corporate governance. Samuel Lee is Assistant Professor at Stern School of Business, New York University. His research interests include financial market liquidity, corporate finance and corporate governance. Any opinions expressed here are those of the authors and not necessarily of the Financial Markets Group. The research findings reported in this paper are the result of the independent research of the authors and do not necessarily reflect the views of the LSE.
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Mike Burkart† Samuel Lee‡

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Abstract

In many bilateral transactions, the seller fears being underpaid because its outside option is better known to the buyer. We rationalize a variety of observed contracts as solutions to such smart buyer problems. The key to these solutions is to grant the seller upside participation. In contrast, the lemons problem calls for offering the buyer downside protection. Yet in either case, the seller (buyer) receives a convex (concave) claim. Thus, contracts commonly associated with the lemons problem can equally well be manifestations of the smart buyer problem. Nevertheless, the information asymmetries have opposite cross-sectional implications. To avoid underestimating the empirical relevance of adverse selection problems, it is therefore critical to properly identify the underlying information asymmetries in the data.

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†Stockholm School of Economics, London School of Economics, CEPR, ECGI and FMG. E-mail: mike.burkart@hhs.se

‡Stern School of Business, New York University. E-mail: slee@stern.nyu.edu.
1 Introduction

The popular reality television show *American Pickers* films antiques hunters who search for collectibles in homes, barns, and sheds. The show portrays the thrill of discovering items of value unbeknown to their current owners. In similar spirit, the British television show *Bargain Hunt* challenges contestants to make as much profit as possible by buying and reselling antiques.

Buyers outsmart sellers not just on television. In 1981, Microsoft bought the rights to the operating system 86-DOS from Seattle Computer Products (SCP) for just $75,000, and in turn licensed a rebranded version, MS-DOS, to IBM for royalties. IBM made MS-DOS the standard on its personal computers, thus turning Microsoft’s founders into billionaires by 1986. If SCP had grasped the potential of 86-DOS, it would surely have asked for more; but as it had not, Microsoft got a bargain.

Short-changed sellers feel regret, or even indignation. India has recently seen violent protests against public land-purchase programs, because the sellers—mainly rural farmers—felt cheated out of their land. The government, it seems, acted as a straw buyer to buy the land cheaply at the behest of private developers. With hindsight, the indignant sellers want damages in recompense. As for new deals, they demand price appreciation rights, or simply refuse to sell. In July 2011, for example, the Supreme Court of India quashed a deal and returned the land to the villagers who sold it.\(^1\)

These instances are dramatic but by no means unique. Others examples of buyers with private information include pharmaceutical companies buying patents, management teams buying out shareholders, venture capital firms buying into start-ups, producers buying movie rights, and collectors of all sorts, to name a few. The key problem that a better informed buyer faces is that the less informed seller suspects the terms of trade to be unfavorable. Such suspicion of being short-changed creates a trade friction that is the inverse of the Akerlof (1970) lemons problem. In the lemons problem, the seller knows more about the value of the good, so the buyer hesitates because the seller may overstate the value to inflate the price. In the reverse constellation, which we call the “smart buyer problem,” the buyer knows more about the value of the good. Here, the

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\(^1\) For more on this issue, see “Get the government out of land deals,” The Economic Times, 21 September 2010; “Return Noida land to villagers, orders Supreme Court,” The Hindu, 6 July 2011; and “Give share in developed land: Farmers,” The Times of India, 24 July 2011.
seller hesitates because the buyer may *understate* the value to lower the price.

This difference can also be cast in terms of signaling incentives. In the Spence (1973) labor market example, workers have private information about their skills, so employers are concerned about overpaying them. To avoid being underpaid, applicants must therefore signal that they do not demand too much. In some labor market situations, the information advantage is arguably reversed. A music producer signing up a garage band, a sports club scouting young talent, tenured faculty recruiting PhDs out of graduate school, and other human capital buyers are often more adept at appraising an applicant’s skill than the applicant. In such situations, employers must signal that they do not offer too little.

We examine contractual solutions to the smart buyer problem. Our conclusions are threefold: First, the problem is important in practice. We identify trade situations that arguably suffer from the smart buyer problem, and derive solutions that resemble real-world contracts, such as royalties, cash-equity bids, earnouts, debt-equity swaps, “gross points,” and concessions. Second, while the lemons problem and the smart buyer problem call for solutions that are opposite in logic - the former gives the uninformed party downside protection, the latter gives upside participation - the resulting contracts are observationally similar. Thus, a contract commonly associated with the lemons problem could be the solution to the smart buyer problem. Third, even though the two information asymmetries induce identical contract shapes, they can lead to an opposite relationship between contract shape and underlying value. This deceptive similarity can lead to erroneous conclusions in empirical contract studies.

By way of illustration, consider a takeover. Suppose the would-be acquirer is less confident about the target company than its current owners. If the owners are better informed, the acquirer is wary of overpaying for the target. The owners can then signal a high valuation by accepting a larger share of the takeover consideration in equity. This signal is credible because less optimistic owners would not be as willing to expose themselves to the firm’s future performance. By taking on upside exposure, they provide the acquirer with downside protection. By contrast, if the acquirer is better informed, the owners are wary of selling the target too cheaply. The acquirer can then signal a low valuation by giving the current owners a larger equity share. This signal is credible because a more optimistic acquirer would not be as willing to relinquish equity. In so doing, the acquirer grants the owners upside participation. So, equity consideration can be
a sign of either the smart buyer problem or the lemons problem.

However, the owners use equity to signal a higher target value, whereas the acquirer uses equity to signal a lower target value. Neglecting this nuance - identical contract form but contrary cross-sectional predictions - can lead to false conclusions. Imagine an empirical study that relates target valuations, or post-takeover performance, to the share of equity consideration and fails to find a positive relationship. While such evidence contradicts the lemons problem, it does not rule out that information asymmetries shape contract choice (equity consideration). The transactions in the sample may suffer from the smart buyer problem, making the relationship negative. Alternatively, some transactions may suffer from the lemons problems, but others from the smart buyer problem, such that the countervailing effects wash out of the data. In general, confounding the two problems makes asymmetric information appear less relevant for contract design than it actually is.

To study the smart buyer problem, we take a generic lemons problem model and simply reverse the information asymmetry. That is, we assume that the buyer has superior information not only about its own valuation of the good, but also about the common value component. We use the “informed principal” bargaining protocol of Maskin and Tirole (1992), both to shift the equilibrium allocation in favor of the buyer (which facilitates trade) and to reduce equilibrium multiplicity.2 Our analysis focuses on fully revealing equilibria because, unlike pooling equilibria, they always exist in informed principal games. Fully revealing equilibria are also the only ones that remain when we use the Cho and Kreps (1987) intuitive criterion to select between multiple equilibria.

While application of the lemons problem pervades many fields, papers that focus on the smart buyer problem are scarce and isolated.3 Beggs (1992) studies licensing contracts between a privately informed developer and an inventor, and

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2Shifting bargaining power to the seller merely changes the allocation of rents between seller and buyer. The results with respect to optimal contract form and the cross-sectional differences that obtain from a comparison between the smart buyer problem and the lemons problem are, for all intents and purposes, orthogonal to the allocation of bargaining power.

3Riley (2001) and Horner (forthcoming) provide recent reviews of the signaling literature. Most applications cited in these surveys – in marketing, industrial organization, finance, labor markets, politics, and biology – involve lemons problems: firms want to signal high quality to customers, competitors, or investors; workers want to signal high skill levels; politicians want to signal that they are highly attractive to voters; and animals want to signal high levels of fitness. A notable exception is the Banks (1990) (non-trade) model of a political agenda setter who wants to signal that rejecting a proposal, that is, reversion to the status quo, is undesirable. Note that the agenda setter wishes to signal a low outside option.
shows that the developer can reveal information through royalties. In the market microstructure models of Kyle (1985) and Glosten and Milgrom (1985), uninformed dealers face both lemons problems and smart buyer problems, as they trade stocks with informed sellers or informed buyers. Shleifer and Vishny (1986) and Hirshleifer and Titman (1990) analyze tender offers in which the acquirer de facto faces a smart buyer problem, although there is no asymmetric information about common values.\(^4\) In contemporaneous and independent research, Dari-Mattiacci et al. (2010) also consider a bilateral trade model with a buyer who is informed about the seller’s outside option. These authors focus exclusively on pooling equilibria, and show that the buyer’s information advantage pushes low-quality goods out of the market and raises the equilibrium price. Informally, they also discuss legal and contractual remedies for smart buyer problems, such as duties to disclose and buyback options. By contrast, we study both pooling and separating equilibria and examine in detail how the buyer can structure the purchase offer to credibly reveal its private information.

Not every bilateral transaction with a privately informed buyer has a smart buyer problem. The classic monopolistic screening model of Maskin and Riley (1984) considers buyers that have superior information about their own reservation prices - that is, their private valuations. There are two main differences between such a screening problem and the smart buyer problem. First, when private information pertains only to private values, giving the informed buyer full bargaining power restores efficiency. This is not true for a smart buyer problem. Second, the monopolist’s optimal screening schedule typically exhibits “quantity discounts,” that is, larger quantities are sold at lower unit prices. By contrast, incentive-compatible price-quantity schedules in our model match larger quantities with higher unit prices. In fact, the very motivation for trading less is that the buyer gets to pay less per unit.

More broadly, the literature on bilateral bargaining with unilateral asymmetric information considers common (or correlated) values (Muthoo, 1999, Chapter 9.2.2). But again, most applications concern informed sellers. Moreover, the bargaining literature does not focus on optimal contracts, which is the theme of this

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\(^4\) Burkart and Lee (2010) provides a systematic study of such tender offers, in which the bidder is better informed about post-takeover share value improvement. These authors point out that the target shareholders’ free-rider behavior creates a smart buyer problem in this transaction, even though the target shareholders’ (i.e., the seller’s) outside option is commonly known.
The body of literature where informed buyers are the norm is auction theory. In auction models, there are no latent opportunities, as the auctioneer is committed to sell to the winning bidder. Consequently, the seller is concerned about intensifying competition rather than being short-changed. By contrast, one interpretation of the smart buyer problem is that seeking other buyers or organizing auctions is costly, and that the (first) buyer knows more about the (net) benefit of pursuing these “outside options.” Thus, an auction bidder faces actual competition which it must defeat, whereas a “smart buyer” must convince the seller that its offer surpasses any latent alternative.

It is interesting that, despite this difference, auction theory delivers similar predictions on “optimal securities” (or means of payment). A sequence of papers shows that auction sellers prefer to receive bids in “steep” claims (Hansen, 1985; Rhodes-Kropf and Viswanathan, 2000; DeMarzo et al., 2005; and Axelson, 2007); they prefer, for example, equity over cash. Steep claims intensify competition, which induces higher bids. An auction seller who can choose securities thus retains equity-like claims, just like a seller in the smart buyer problem. Bidders, however, prefer to bid in “flat” claims to reduce the seller’s rent; indeed, given the choice, they bid cash (DeMarzo et al., 2005). This contrasts with the smart buyer problem, where the buyer voluntarily offers steep claims to sway the seller, and further, to minimize the seller’s rent.

This paper proceeds as follows. Section 2 outlines the model and gives examples of smart buyer situations. Section 3 examines fully revealing equilibria. Section 4 compares the smart buyer problem to the lemons problem, then discusses competition. Section 5 analyzes pooling equilibria and addresses the issue of equilibrium selection. Our concluding remarks are in Section 6, and the mathematical proofs are in the Appendix.

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5. Che and Kim (2010) show that sellers may prefer flatter securities when bidders have private information about investment cost. Similarly, Gorbenko and Malenko (2011) show that competition among sellers may lead to auctions in flatter securities, as sellers must relinquish rents to attract bidders.

6. Inderst and Mueller (2006) also study security design by an informed investor. These authors consider a better-informed lender who screens loan applicants. Since the lender privately learns the borrower’s inside option (payoff from borrowing), their setting is more akin to the lemons problem. The lender de facto “sells” a product that may or may not be good for the borrower, as in the literature on credence goods.
2 Framework

2.1 Model

A buyer approaches a seller, who possesses one unit of a tradable good. A transaction is characterized by a pair \((x, t)\) where \(x \in \mathcal{X} \subset [0, 1]\) is the traded quantity, and \(t\) is the total (net) cash transfer from the buyer to the seller. The buyer’s and the seller’s payoffs are

\[
V(x; \cdot) = v(x; \cdot) - t \quad \text{and} \quad U(x; \cdot) = u(1 - x; \cdot) + t,
\]

respectively, where \(u\) and \(v\) are differentiable functions with \(u(0; \cdot) = v(0; \cdot) = 0\), \(u_x > 0\), and \(v_x > 0\). To focus on the adverse effects of asymmetric information, we assume that the buyer has no wealth constraints, whereas the seller is penniless.

The buyer’s valuation of the good can be written as the sum of two components,

\[
v(x; \cdot) = [u(1; \cdot) - u(1 - x; \cdot)] + z(x; \cdot).
\]

The first term \(u(1; \cdot) - u(1 - x; \cdot)\) represents the seller’s loss from giving up \(x\) of the good, and the second term \(z(x; \cdot)\) represents the gains from trade. In other words, the function \(v(x; \cdot)\) can be decomposed into a common value component \(u(1; \cdot) - u(1 - x; \cdot)\) and the gains from trade \(z(x; \cdot)\).

For tractability, we let the seller’s valuation be constant per unit of the good, \(u(x) = \theta_x x\) with \(\theta_x \geq 0\). The coefficient \(\theta_x\) reflects the seller’s outside option. For example, \(\theta_x\) could be the (expected) price a latent alternative buyer would pay, in which case \(z(x; \cdot)\) would be the value-added by the present buyer, relative to the latent alternative buyer. This linearity simplifies the analysis but is not critical to the qualitative insights.

Given the decomposition and linearity, the preferences can be written as

\[
\begin{align*}
U(x; \cdot) &= \theta_x (1 - x) + t \quad \text{(Seller)} \\
V(x; \cdot) &= \theta_x x + z(x; \cdot) - t. \quad \text{(Buyer)}
\end{align*}
\]

We assume that \(z = z(x; \theta_x, \theta_z)\). That is, the buyer’s private gains depend on the traded quantity \(x\), factors determining the common value (captured by \(\theta_x\)), and other factors (captured by \(\theta_z\)). For example, \(\theta_x\) is the objective quality of a car as the determinant of its market price, and \(\theta_z\) is the buyer’s idiosyncratic pleasure from driving.

**Assumption 1** \(z \geq 0, z_x \geq 0, z_{\theta_x} \geq 0, z_{\theta_z} \geq 0, z_{x\theta_x} \geq 0, \text{ and } z_{x\theta_z} \geq 0\).
The buyer’s private gains are non-negative and increase in $x$, $\theta_x$, and $\theta_z$. Further, they increase marginally more in $x$ when $\theta_x$ or $\theta_z$ are higher. Assumption 1 implies that the efficient outcome is full trade, $x = 1$.\footnote{This assumption is the opposite of Assumption 5 in Riley (1979); it implies that increasing the level of the “signal” is cheaper for lower, rather than higher, common value types. At the same time, both assumptions serve the same purpose; they ensure that the single-crossing condition is satisfied, and thus allow for separation.}

The parameters are continuously distributed on $\Theta_x \times \Theta_z = [\underline{\theta}_x, \overline{\theta}_x] \times [\underline{\theta}_z, \overline{\theta}_z]$ according to a commonly known distribution. The true parameters $(\theta_x, \theta_z)$ are realized prior to the offer. We now introduce our central assumption.

**Assumption 2** *Only the buyer observes $\theta_x$.***

Put differently, the seller knows less about its outside option than the buyer. As a result, the seller is concerned that the buyer’s offer might be too low. This in turn puts the burden on the buyer to convince the seller that the offer indeed matches, or exceeds, the latter’s true outside option.\footnote{Unless the buyer’s private information is about $\theta_x$, there are no frictions in this model; the buyer would offer $t = E[\theta_x]$ for $x = 1$, which the seller would always accept. That is, being better informed does not matter to the buyer as long as the buyer can guarantee to pay the seller’s outside option. If the seller, rather than the buyer, had private information about $\theta_x$, the buyer would face the lemons problem.}

With respect to $\theta_z$, the bulk of our analysis presumes symmetric information, so that the buyer’s type (information advantage) is one-dimensional and given by $\theta_x \in \Theta_x$. For simplicity, we assume that $\theta_x$ is common knowledge. (Assuming instead that neither party observes $\theta_z$ does not change the qualitative results.) However, in Section 5.2, we assume that $\theta_z$ is also known exclusively by the buyer, in which case the type (information advantage) is two-dimensional and given by $(\theta_x, \theta_z) \in \Theta_x \times \Theta_z$. In either case, the buyer has superior information both about the gains from trade and about the seller’s outside option. The difference is that, in the one-dimensional case, $\theta_x$ is a sufficient statistic for both.

Contracting follows the bargaining protocol of Maskin and Tirole (1992). The informed buyer moves first and makes a take-it-or-leave-it offer to the seller. The offer consists of a menu of contracts. The seller decides whether to accept the menu. If the seller accepts the menu, the buyer chooses one contract from the menu, which is then implemented.

We now introduce two conditions that render our model applicable to a wide range of bilateral trade situations, as the examples in Section 2.2 illustrate.
**Condition V** $v(x)$ is verifiable.

Condition V (for “verifiability”) is satisfied when a third party, such as a court, can verify the buyer’s valuation (only) after trade has taken place. If Condition V holds, the buyer can commit to monetary transfers that are contingent on the buyer’s valuation; otherwise, only fixed monetary transfers are feasible. Note that, under Condition V, the buyer’s valuation is verifiable as a whole, but the individual components of the buyer’s valuation are not. Otherwise, payments could be made contingent on the common value $\theta_x$, which would trivially resolve the smart buyer problem.

**Condition D** $\mathcal{X} = [0, 1]$.

Condition D (for “divisibility”) states that the traded good is (perfectly) divisible. When Condition D is not satisfied, the parties can either trade the entire good or none at all. Unless otherwise stated, Conditions V and D are not satisfied.

### 2.2 Applications

We now present several examples of smart buyer constellations, including some mentioned earlier in the introduction.

**A0: Art collector.** An experienced art collector wants to buy a work from a novice artist. The collector derives non-verifiable (hedonic) utility from complementing an existing collection with this work, but also has more experience in assessing the potential (future) value of the artist’s work. Here, $\theta_z$ captures the collector’s idiosyncratic component from hedonic utility, while $\theta_x$ reflects the latent market value of the work. Neither Condition V nor Condition D is satisfied.

**A1: Securities trading.** A sophisticated investor wants to buy securities from a market maker. The investor gains from the trade partly because it hedges risk exposures that are specific to the investor’s current portfolio. These hedging gains are non-verifiable and cannot be transferred to the market maker. The investor also has private information about the fundamental value of the securities. Here, $\theta_z$ captures the investor’s idiosyncratic hedging demand, while $\theta_x$ reflects the securities’ fundamentals. Condition D is satisfied.
A2: Real estate. A well-known real estate developer wants to buy some property to build a hotel and replace the existing buildings. The future cash flow from operating the hotel can be shared. The current owner is unable to develop the land in the same way. The current owner is also less informed about (valuations in) the real estate market. Here, $\theta_z$ captures the developer’s capabilities, while $\theta_x$ reflects the value of the “location.” Condition V is satisfied.

A3: Patent. A company wants to buy a patent from a scientist to improve its products. The scientist knows less about how valuable the patent is for improving such products. Here, $\theta_z$ captures the company’s product market share, while $\theta_x$ reflects the patent’s latent market value. Condition V is satisfied.

A4: Restructuring. The controlling shareholder of a firm under bankruptcy protection offers to inject new capital in exchange for partial debt forgiveness. While there is consensus that continuation is better, the controlling shareholder has superior information about the going concern value and the liquidation value. The creditors question the proposed terms. Here, $\theta_z$ captures the shareholder’s managerial ability, while $\theta_x$ reflects the firm’s liquidation value. Condition V is satisfied.

A5: Takeover. A small firm with promising ideas is approached by a large industry peer. The target management deems the buy-side valuations of their firm suspiciously low. Here, $\theta_z$ captures acquirer characteristics, while $\theta_x$ reflects target characteristics. Condition V is satisfied.

A6: Venture capital. A seasoned venture capitalist wants to invest in a start-up firm, help develop its business, and finally take it public. The venture capitalist contributes useful experience to the start-up but also knows more about its potential market value. The firm founders fear conceding too large a stake. Here, $\theta_z$ captures the venture capitalist’s experience, while $\theta_x$ reflects the potential of the firm. Condition V is satisfied.

A7: Movie rights. A Hollywood studio wants to buy the movie rights to a series of novels. Compared to the seller (writer and/or publishing company), the studio can better assess the box office potential of the novels. The seller is concerned about giving up a “hidden gem.” Here, $\theta_z$ captures the studio’s movie-making capacities, while $\theta_x$ reflects the novels’ box office potential. Conditions V and D are satisfied.

A8: Hiring talent. A music producer wants to sign up a new band. The contract would confer exclusive rights to produce several records with the band.
While the band is inexperienced, the producer has a track record of developing new talent. The band members have reservations about some of the contract terms and wonder whether they could get a better deal elsewhere or later. Here, $\theta_z$ captures the producer’s capability, while $\theta_x$ reflects the potential of the band. Conditions V and D are satisfied.

A9: Legal counsel. A defendant (or plaintiff) considers hiring a specialist attorney, as opposed to seeking standard counsel. The specialist attorney claims that the chances of winning the case are slim without the specialist attorney’s help. The defendant is unsure about the validity of this claim, particularly given the high(er) legal fees. Here, $\theta_z$ reflects the quality of the specialist lawyer, while $\theta_x$ reflects the chances of winning the case with a non-specialist attorney. Condition V is satisfied.

3 Persuasive purchase offers

An actual trade contract takes the form

$$ C = [x, t_0, \tau(v)] $$

where $x$ is the quantity traded, $t_0$ is a fixed monetary transfer, and $\tau(v)$ is a monetary transfer contingent on the ex post realization of $v$. Thus, the total payment is $t = t_0 + \tau(v)$. When Condition V is violated, $\tau(v) = 0$, whereas $x = 1$ for any non-trivial buy offer when Condition D is violated. Let $C_0 \equiv [0,0,0]$ denote the null contract. A buy offer consists of a set of contracts $C$, henceforth referred to as a contract menu.

In the absence of gains from trade ($z = 0$), the unique equilibrium outcome is the absence of trade. This result follows directly from the no-trade theorem (e.g., Milgrom and Stokey, 1982). Without gains from trade, any buy offer reveals that the buyer deems the common value of the asset (weakly) higher than the offered price. Hence, rejecting the buyer’s offer is the dominant strategy for any offered contract (menu). Trade is feasible in equilibrium only if there are gains from trade.

In this section, we focus on fully revealing Perfect Bayesian equilibria. (We later show that, contrary to pooling equilibria, such an equilibrium always exists and survives the Intuitive Criterion.) In a fully revealing equilibrium, the
buyer offers a contract menu \( C \) such that, if the menu is accepted, the buyer’s selection from the menu fully reveals the buyer’s type. That is, the menu can be represented by a function, \( C(\theta_x) \), that attributes a distinct contract to each buyer type. Without loss of generality, we assume that \( C(\theta_x) \) is differentiable (Mailath and von Thadden, 2010), and that every buyer type submits the same menu.

### 3.1 Upward-sloping supply

Suppose Conditions V and D are violated, the good is indivisible, and transfers cannot be contingent. Thus, the buyer can acquire the good only through a contract of the form \( C = [1, t, 0] \). It is straightforward to see that such contracts rule out any separating offer, since all bidder types would select the contract with the lowest \( t \) from any given contract menu.

To construct separating equilibria, we need to allow for stochastic contracts. Under a stochastic contract \( \tilde{C} \), a deterministic contract \( C \) is randomly implemented according to a probability distribution, \( g(C) \).

For a buyer of type \( \theta_x \), the expected payoff from a stochastic contract \( \tilde{C} \) is

\[
\Pi(\tilde{C}; \theta_x) = p[\theta_x + z (1; \theta_x, \theta_z) - t^1] - (1 - p)t^0
\]

where \( p \equiv \Pr (x = 1) \), \( t^1 \equiv E(t | x = 1) \), \( t^0 \equiv E(t | x = 0) \), and \( \tilde{t} \equiv E(t) \) under the probability distribution \( g \). The payoff-relevant characteristics of \( \tilde{C} \) are thus summarized by \( p \) and \( \tilde{t} \), which allows us to express a stochastic contract in reduced form as \( \tilde{C} = [p, \tilde{t}] \).

**Proposition 1 (Trade failures)** Suppose Conditions V and D are violated. No deterministic fully revealing equilibrium exists. There exist stochastic fully revealing equilibria, in all of which buyer type \( \theta_x \in \Theta_x \) trades with probability

\[
p(\theta_x, \theta_z) = \exp \left[ - \int_{\theta_x}^{\Theta_x} [z(1; s, \theta_z)]^{-1} ds \right]
\]

and the expected transfer to the seller is \( \tilde{t}(\theta_x, \theta_z) = p(\theta_x, \theta_z)\theta_x \).

Both success probability \( p(\theta_x, \theta_z) \) and expected transfer \( \tilde{t}(\theta_x, \theta_z) \) are increasing in buyer type \( \theta_x \). A lower-valued buyer can credibly reveal its type by
accepting a higher risk of trade failure. With less to gain, lower-valued types are less keen on trading and so bid less aggressively. Effectively, this implies a (stochastic) upward-sloping supply curve; the seller is more willing to supply the good, the higher the price.

The expected transfer equals the seller’s true reservation price. So, in expectation, the buyer appropriates no part of the common value, which implies that the buyer does not signal its type by forgoing common value through its contract choice. Rather, it is through relinquishing expected private gains that the buyer reveals its type when accepting a higher risk of trade failure.

A reduction in $\theta_z$ reduces the private gains of all buyer types, and hence decreases the trade probability of all but the highest type, that is, $\partial p(\theta_x, \theta_z)/\partial \theta_z > 0$. Intuitively, a lower $\theta_z$ means that the buyer - or, more precisely, its capacity to derive utility from the good - is less “special,” which increases the seller’s suspicion that the buyer is merely after a good bargain.

Example 1 (Art). Consider the art collector setting (A0). An experienced art collector approaches a young artist to buy a painting. The painting is indivisible, and the collector’s hedonic pleasure from owning the painting is non-verifiable.

Under the stochastic contract

$$\hat{C} = \begin{cases} [1, \theta_x, 0] & \text{with probability } p(\theta_x, \theta_z) \\ C_0 & \text{with probability } 1 - p(\theta_x, \theta_z) \end{cases}, \quad (3)$$

the collector commits to a mechanism that results in trade under the deterministic contract $[1, \theta_x, 0]$ with probability $p(\theta_x, \theta_z) \in [0, 1]$, and in no trade with probability $1 - p(\theta_x, \theta_z)$. By construction, $\hat{t}(\theta_x, \theta_z) = p(\theta_x, \theta_z)\theta_z$, and provided $p(\theta_x, \theta_z)$ satisfies (2), the stochastic contract $\hat{C}$ implements a stochastic fully revealing equilibrium. The collector’s personal interest in the work ($z > 0$) facilitates trade. If the collector’s intentions were predominantly commercial ($\theta_z \to \theta_0$), the artist would be more prone to reject low bids.

Key to signaling is that private gains can be forgone in a way that reveals information about the common value. Stochastic contracts allow the buyer to forgo private gains by accepting failure with a self-selected probability. As we shall see, such randomization is no longer necessary for signaling when Condition V or Condition D holds.
Now suppose only Condition D holds; while the good can be divided, contingent transfers are still not possible. For a buyer of type $\theta_x$, the payoff from a deterministic contract $C = [x, t, 0]$ is

$$\Pi(C; \theta_x) = x\theta_x + z(x; \theta_x, \theta_z) - t. \quad (4)$$

**Proposition 2 (Trade rationing)** Suppose only Condition D is satisfied. There exists a unique deterministic fully revealing equilibrium in which buyer type $x\in \Theta_x$ acquires quantity $x(x, \theta_z)$ at price $t_0(x, \theta_z) = x(x, \theta_z)\theta_x$, where $x(x, \theta_z)$ satisfies the differential equation

$$\frac{x'(x, \theta_z)}{x(x, \theta_z)} = [z_x(x(x, \theta_z); \theta_x, \theta_z)]^{-1} \quad (5)$$

and the boundary condition $x(\bar{\theta}_x, \theta_z) = 1$.

The trade quantity $x(x, \theta_z)$ and the unit price $t(x, \theta_z)/x(x, \theta_z)$ are increasing in the buyer type $\theta_x$. The buyer signals a lower valuation by trading a smaller quantity. Quantity rationing is a means to relinquish gains from trade, analogous to lowering trade probability in the stochastic separating equilibrium. The exact quantity schedule $x(x, \theta_z)$ depends on the trade surplus function $z(x; \theta_x, \theta_z)$, which we illustrate using a linear example.

Again, trade is de facto characterized by an upward-sloping supply curve; the seller is willing to supply more of the good when the price is higher. In contrast, the lemons problem leads to a downward-sloping demand curve. In Duffie and DeMarzo (1999), for example, the securities market suffers illiquidity in the form of downward-sloping demand. This provides the backdrop for our next example.

**Example 2 (Liquidity).** Consider a simple two-period model of financial trade (A1). There are a buyer and a seller, both of whom are endowed with (zero-interest) cash to support trade. In addition, the seller is endowed with one unit of a security that yields an uncertain payoff $\bar{\theta}_x \in \Theta_x$ later at date 1, where $\Theta_x = (1, \bar{\theta}_x]$.

The seller’s and the buyer’s consumption utilities are

$$u(c) = c_0 + c_1 \quad \text{and} \quad v(c) = c_0 + (1 + \theta_z)c_1,$$

respectively; $c_t$ denotes date-$t$ consumption, and $\theta_z \in \{-\theta_z, \theta_z\}$ is a consumption
preference shock. If $\theta_z = -\theta_z$, the buyer is impatient and prefers consumption at date 0. If $\theta_z = \theta_z$, the buyer is patient and prefers consumption at date 1. By contrast, the seller is indifferent with respect to the timing of consumption.

When $\theta_z = -\theta_z$, the buyer uses her wealth to consume at date 0, and there is no demand for trading the security. However, when $\theta_z = \theta_z$, the buyer would like to invest some of its wealth in the security to increase date 1 consumption. If both knew the realization of $\theta_x$ at date 0, the buyer would simply offer $t = \theta_x$ and would enjoy additional benefits of $z(1; \theta_x, \theta_z) = \theta_x \theta_x$. When only the buyer learns the true return $\theta_x$, there exists a unique deterministic fully revealing equilibrium characterized by Proposition 2.

To determine the equilibrium quantity schedule, note that (5) becomes

$$\frac{x'(\theta_x, \theta_z)}{x(\theta_x, \theta_z)} = [\theta_z \theta_x]^{-1},$$

since $z(1; \theta_x, \theta_z) = \theta_x \theta_x x$ in this example. Integrating on both sides, and using $x(\theta_x, \theta_z) = 1$ to determine the integration constant, yields

$$x(\theta_x, \theta_z) = \left(\frac{\theta_x}{\theta_x}\right)^{1/\theta_z}. \quad (7)$$

Since the equilibrium per-unit price is $\theta_z$, (7) also describes an upward-sloping supply curve. One can invert (7) to derive an equilibrium price function

$$P = \theta_x x^{\theta_z}.$$  

The slope of this function $\partial P/\partial x = \theta_x \theta_x x^{\theta_z-1}$ reflects the price impact of a given quantity order, similar to Kyle’s (1985) λ, though not a constant.

The traded quantity $x(\theta_x, \theta_z)$ is strictly increasing in $\theta_z$ for all $\theta_x \in \Theta_x \setminus \{\theta_x\}$. (Recall that $x(\theta_x, \theta_z) = 1$ regardless of $\theta_z$.) When non-informational trade motives become less important ($\theta_z \to 0$), the seller becomes more reluctant to trade, which translates here into less liquidity: trade quantity decreases, and price impact increases.

### 3.2 Upside participation

Now suppose only Condition V holds; while the good cannot be divided, transfers can be made contingent on $v(\cdot)$. Feasible deterministic contracts now take the
form $C = [1, t_0, \tau(v)]$. We allow for more general contracts further below but begin with a simple category of contingent transfers: linear sharing rules where $	au(v) = (1 - \alpha)v$.

For a buyer of type $\theta_x$, the payoff from a revenue sharing contract is

$$\Pi(C; \theta_x) = \alpha[\theta_x + z(1; \theta_x, \theta_z)] - t_0. \quad (8)$$

It is straightforward to see that (8) is isomorphic to (1), and Proposition 1 therefore applies.

**Proposition 3 (Revenue sharing)** Suppose only Condition $V$ is satisfied and contingent transfers are restricted to linear sharing rules. There exists a unique deterministic fully revealing equilibrium in which buyer type $\theta_x \in \Theta_x$ acquires the good in exchange for a fixed transfer $t_0(\theta_x, \theta_z) = \alpha(\theta_x, \theta_z)\theta_x$ and a fraction $1 - \alpha(\theta_x, \theta_z)$ of the buyer’s total revenues, where

$$\alpha(\theta_x, \theta_z) = \exp \left[ - \int_{\theta_x}^{\theta_x} z(1; s, \theta_z)^{-1} ds \right]. \quad (9)$$

When the good has a low(er) common value, the buyer offers a large(r) fraction of revenues. The intuition behind the inverse relationship is that the buyer signals a low common value through granting the seller upside participation. Buyers do not want to mimic lower-valued types because the gains from paying a lower price are (more than) offset by the cost of conceding more revenues. Conversely, overstating the value of the good is not profitable since the gains from a larger share of revenues do not compensate for the higher cash price.

Note that the buyer relinquishes exactly the same amount of (expected) private gains as in the stochastic fully revealing equilibrium (Proposition 1), yet through revenue sharing rather than through trade failure. The above equilibrium is efficient (since trade always occurs), and Pareto-dominates the stochastic fully revealing equilibrium (since the seller is strictly better off).

**Example 3 (Debt-equity swap).** Consider an owner-managed firm in financial distress (A4). Everyone agrees that continuation is efficient, and the owner-manager is willing to inject fresh capital in exchange for partial debt forgiveness. Such a transaction amounts to “buying” back control from the creditors.
The problem is that the owner-manager is in a better position to assess both the firm’s liquidation value \( \theta_x \) and its continuation value \( v(1; \theta_x, \theta_z) \). This creates disagreement: On one hand, the creditors question the low liquidation value estimates. On the other hand, the owner-manager deems creditor demands too high. One solution is to “settle” the debt not only in cash but also in equity, whereby creditors benefit from a cash infusion \( t_0(\theta_x, \theta_z) \) and receive a \( 1-\alpha(\theta_x, \theta_z) \) equity stake in the restructured firm.

A standard explanation for the use of debt-equity-swaps in financial distress is debt overhang. The current example shows that smart buyer problems provide an alternative explanation for debt-equity-swaps. In fact, while debt overhang problems can be resolved by means of a debt buyback for cash only, this is not true for smart buyer problems.\(^9\)

In addition to the cash-equity offers discussed in the introduction, patent royalties are another example. Suppose a company wants to purchase a patent from a scientist to develop new products. If the company can better appraise (the latent value of products based on) the patent, the scientist is wary of an outright sale for fear of giving up a gem. Beggs (1992) shows that the company can allay this fear by granting the scientist upside participation through royalties.

We now relax the restriction of linear sharing rules and let the buyer choose the form of the contingent transfers \( \tau(\cdot) \). Consequently, a buyer of type \( \theta_x \) chooses \( C = [1, t_0, \tau(v)] \) to maximize

\[
\Pi(C; \theta_x) = \theta_x + z(x; \theta_x, \theta_z) - t_0 - E[\tau(v)|\theta_x].
\]

Given that \( \theta_z \) is commonly known, the buyer knows exactly how large the contingent payment will be. In fact, \( E[\tau(v)|\theta_x] = \tau[v(1; \theta_x, \theta_z)] \). This simplifies the optimal contracting problem greatly.

**Proposition 4 (Security design)** Suppose Condition \( V \) is satisfied, and contingent transfers are unrestricted. There exists a deterministic fully revealing equilibrium in which buyer type \( \theta_x \) acquires the good in exchange for a fixed

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\(^9\)The restructuring of Marvel in the mid-1990s provides anecdotal evidence of smart buyer problems in bankruptcy. Ron Perelman, as debtor-in-possession, plays the role of a smart buyer. His attempt to regain control of the financially distressed Marvel company at too low a price is eventually thwarted.
transfer $t_0(\theta_x, \theta_z) = \theta_x$ and a contingent transfer

$$\tau(v) = \begin{cases} 
0 & \text{if } v \leq v(1; \theta_x, \theta_z) \\
 v - v(1; \theta_x, \theta_z) & \text{if } v > v(1; \theta_x, \theta_z)
\end{cases}$$

thereby retaining the entire trade surplus, as under symmetric information.

For $x = 1$ and a given $\theta_z$, the buyer’s total valuation $v$ is a one-to-one mapping from $\Theta_x$ to $[v(1; \theta_x, \theta_z), v(1; \theta_x, \theta_z)]$. Moreover, $v$ is strictly increasing in $\theta_x$. Consequently, one can use a simple scheme to punish the buyer for understating $\theta_x$: Let the buyer’s reported type be $\hat{\theta}_x$. Then, the buyer incurs a penalty if and only if the realized $v$ is larger than the announced $v(1; \hat{\theta}_x, \theta_z)$. The penalty specified in Proposition 4 satisfies limited liability ($\tau \leq v - t_0$); it requires the buyer to pay the seller the difference between actual total valuation and total valuation implied by $\hat{\theta}_x$. This amounts to a call option that allows the seller to extract more from the buyer when $v$ is high. Thus, the penalty scheme grants the seller upside participation.$^{10}$

Example 4 (“20-against-20”). Consider an example of “hiring talent” (A8). A film studio wants an actor for a lead role in a new movie. The studio is better informed about industry factors that determine the actor’s latent outside options ($\theta_x$), and it can better estimate the movie’s box office potential ($v(1; \theta_x, \theta_z)$). A producer and a director are already signed up, both well-known and experienced (high $\theta_z$).

The actor bargains for a high salary ($t'_0$), otherwise reluctant to commit to the project in hopes of better options. Finding the actor’s demands too high ($t'_0 > \theta_x$), the studio pays the larger of a cash salary $t_0$ and a fraction of the revenues $\alpha v$; $\max \{ t_0, \alpha v \}$. In essence, this compensation package amounts to a fixed salary supplemented by a fraction of revenues, provided that the revenues exceed a certain threshold.

Such convex salaries exist in the film industry. One better-known example is the so-called “20-against-20” contract, whereby an actor effectively receives the

$^{10}$The penalty need neither have this specific form nor be payable to the buyer to ensure incentive compatibility. Also, a similar penalty scheme is effective even when $\theta_z$ is unobserved by either party. Under Assumption 1, the support of $v$ would differ across buyer types. In fact, the maximum of the support $\pi(\theta_x, \theta_z)$ would be strictly increasing in $\theta_x$. But even if the support of $v$ were identical for all types, an incentive-compatible penalty scheme exists if the distributions $\{ f_{\theta_x}(v) \}_{\theta_x \in \Theta_x}$ satisfy the monotone likelihood property (Burkart and Lee, 2010).
larger of $20 million and 20 percent of the movie’s gross revenues.\textsuperscript{11} This creates upside participation; the payoff is flat until the revenues reach $100 million but thereafter increases linearly with further revenues. Weinstein (1998) argues that one explanation for such contracts is that the studio is better informed than the star.\textsuperscript{12}

The contingent payments in a “20-against-20” contract are, in a way, non-linear royalties. Such royalties also exist in other intellectual property transactions. Publishing contracts often contain so-called escalation clauses, whereby the publisher pays the author a royalty rate that is increasing in sales. For example, a higher royalty rate may kick in once sales surpass a pre-specified target.

In the context of acquisitions of privately held firms, non-linear contingent payments are referred to as contingent value rights (CVRs). Consistent with our theory, practitioners argue: “The conditional nature of CVRs makes them valuable in breaking logjams in price negotiations. When a would-be acquirer’s offer is spurned as too low by a target corporation, the deal can be sweetened by using CVRs to promise future rewards.”\textsuperscript{13} For instance, earnout clauses specify supplementary payments when the target’s operational or financial performance exceeds pre-determined threshold levels within a given time period after the acquisition. Similarly, “anti-embarrassment” clauses specify supplementary payments when the buyer on-sells the asset at a higher price within a specific period. Such payments amount to an adjustment of the original price, and prevent the original seller from being “embarrassed” by having sold the asset at too low a price.

Another example is start-up financing. A salient concern of entrepreneurs is that they might let investors buy-in at bargain prices. The entrepreneurs resolve this smart buyer problem by retaining the (most) convex claim. Indeed, entrepreneurs of venture capital financed firms obtain more cash flow rights as

\textsuperscript{11}For example, Tom Cruise signed a 20-against-20 contract for \textit{Valkyrie}.

\textsuperscript{12}Goetzmann et al. (2007) study pricing and contracts in sales of screenplays. In our view, their evidence is consistent with the notion that film studios know more than inexperienced screenwriters. In particular, they show that experienced screenwriters more often receive fixed payments, and that studios forecast box office success well. Further, their motivating example, studios offering contingent contracts when less optimistic, and the cross-sectional evidence, better scripts coinciding with higher prices and less contingent payments, are indicative of the smart buyer problem (cf. Section 4.2 below).

\textsuperscript{13}“Shadowy Shares: The Dark Side of Contingent Value Rights,” Forbes, May 9, 2011.
company performance improves, while the venture capitalists have cash flow rights that are senior to those held by the entrepreneur when the firm performs poorly (Kaplan and Stromberg, 2003).\textsuperscript{14} Moreover, entrepreneurs often prefer debt over venture capital because they are afraid of “giving away equity too quickly or cheaply.”\textsuperscript{15}

Propositions 3 and 4 illustrate the two effects that the verifiability of $v$ (Condition V) and the use of contingent transfers have on the solution to the smart buyer problem. First, trade becomes efficient, since revenue sharing replaces more wasteful means of relinquishing gains from trade, such as rationing. Second, the buyer appropriates more of the trade surplus in the absence of restrictions on the contract form, since security design enhances the buyer’s ability to commit to truthful behavior.

A noteworthy proviso is that both results rely on the (implicit) assumption that contingent transfers do not affect the trade surplus. This is debatable in some applications. CVRs, for example, can dampen the acquirer’s incentives to increase the target’s post-takeover value, creating tension between signaling and incentive provision.

4 Fooled buyers or short-changed sellers?

4.1 Observationally equivalent contracts

In the smart buyer problem, the buyer knows the seller’s outside option better than the seller. By contrast, the lemons problem arises when the seller has superior knowledge of the buyer’s inside option. These opposite points of departure become manifest in the signaling incentives: An informed buyer wants to convey a low value, whereas an informed seller wants to convey a high value.

This difference is also reflected in the solutions to these problems. Signaling a high value calls for \textit{downside protection}, whereby the uninformed party is recompensed if expectations are not met. Signaling a low value calls for \textit{upside participation}, whereby the uninformed party is recompensed if expectations are surpassed. This is evident in the security design solution of Propo-

\textsuperscript{14}Nevertheless, venture capitalists’ cash flow claims may be convex in overall firm performance due to third-party debt financing.

\textsuperscript{15}“The Wisest Entrepreneurs Know How to Preserve Equity,” New York Times, November 15, 2011.
Figure 1: The graph illustrates the security design solution (Proposition 4) under limited liability. It plots the value of contingent claims granted to the buyer and the seller (vertical axis) as functions of total realized value $v$ (horizontal axis). The seller receives equity (blue line), and the buyer receives debt (red line).

Proposition 4, which provides the uninformed seller with the convex claim $\tau(v) = \max\{0, v - v(1; \theta_x, \theta_z)\}$. As $v$ increases from zero, the payoff from this claim is flat until $v = v(1; \theta_x, \theta_z)$ and then increases linearly. The buyer’s payoff is accordingly concave, increasing linearly until $v = v(1; \theta_x, \theta_z)$ and flat thereafter. As Figure 1 illustrates, these claims represent standard securities, debt for the buyer and equity for the seller.

These results are compelling insomuch as the same claim structure is optimal in security design models with a better-informed seller (DeMarzo and Duffie, 1999). This is important for empirical studies as it means that lemons problems and smart buyer problems in practice can lead to observationally equivalent contracts. For example, debt issuance is usually linked to private information of the issuer, as in Myers and Majluf (1984). Their pecking order theory posits that debt best protects less informed investors from buying overvalued securities. As we show, however, debt is also optimal when the issuer deals with better informed investors. Debt protects the issuer from selling undervalued securities.

There are many other examples. Consider cash-equity payments in mergers and acquisitions. The standard explanation for the use of equity is based on the assumption that target shareholders have private information about the target
(Hansen, 1987; Fishman, 1989; Eckbo et al., 1990). However, the use of equity is, as we show, just as rational when the acquirer has private information about the target. Similarly, it has been shown that royalties can convey not only information from licensors to licensees (Gallini and Wright, 1990), but also vice versa (Beggs, 1992). Analogous arguments apply to other contractual provisions, such as earnouts and “20-against-20.” Empirical tests built on the presumption that any of the above contracts - say, debt - is (only) a sign of better-informed sellers is therefore incorrectly designed.

The observational equivalence can be attributed to the fact that, as the mode of signaling switches between downside protection and upside participation, so does the identity of the informed principal. The eventual claim structure is the same regardless of whether the seller grants the buyer downside protection or the buyer grants the seller upside participation. The more fundamental reason is that both the lemons problem and the smart buyer problem can be reduced to asymmetric information about a common value component. Crucially, conditions for incentive compatibility and separation depend on neither the identity of the informed party nor whether it wants to overstate or understate the value. Conditional on separation, the informed principal’s identity affects only the surplus division (between buyer and seller and across common value types).

4.2 Opposite cross-sectional predictions

Because of identical contractual solutions, real-world contracts are unlikely to be sufficient to identify empirically the underlying information problem. One may have to look beyond the contract shape and take into account the division of surplus. For example, signaling costs are borne by the seller in the lemons problem, whereas they are borne by the buyer in the smart buyer problem. Hence, identifying the party willing to pay for third-party verification, such as due diligence or fairness opinions, can help to discriminate between the two information problems. However, in practice, it is difficult to attribute such

\footnote{An exception is Berkovitch and Narayanan (1990), where the seller's outside option is to wait for a competing bid, the value of which depends on the initial buyer's privately observed quality relative to potential competitors. Our smart buyer framework parsimoniously subsumes the Berkovitch and Narayanan setting in reduced form (cf. our discussion of latent competition in Section 4.4).}

\footnote{For example, Datar et al. (2001) argue that earnouts resolve the lemons problem, and Weinstein (1998) acknowledges the possibility that non-linear actor salaries might reflect a lemons problem.}
Figure 2: The two graphs illustrate that the smart buyer problem (informed purchase) and the lemons problem (informed sale) can imply opposite predictions about the relationship between the underlying common value and the signaling instrument. The left graph shows this for trade rationing, while the right graph shows it for linear sharing rules.

expenses to one or the other party, because they may be laid out by one party but accounted for in the transaction price, or because of differences in bargaining power.

Alternatively, one can study how contracts relate to (revealed) common value, which reflects the distribution of rents across common value types. This relation changes with the identity of the informed party. Let us extend the financial trade application (Examples 2 and 5) by giving the informed party an endowment that it wants to sell in case of impatience, thereby introducing a lemons problem. Figure 2a depicts the relation between trade quantity \( x \), which is the signaling instrument in this setting, and common value \( \theta_x \). The relation is positive when the informed party wants to buy (green line) but negative when it wants to sell (grey line). Identifying trade “direction” is therefore important, as commonly done in market microstructure research (e.g., Lee and Ready, 1991).

Accounting for trade direction also matters in other settings. Figure 2b illustrates this point for linear sharing rules, where the seller’s equity stake \( \alpha \) is the signaling instrument. In smart buyer problems, \( \alpha \) and \( \theta_x \) are inversely related (green line); sellers receive more equity when common values are lower. In lemons problems, they are positively related (grey line); sellers retain more equity when common values are higher. For instance, consider a firm that wants to issue equity. Leland and Pyle (1977) presume that the issuer is better informed than the investors, who are therefore unwilling to pay a high(er) price unless the
issuer retains a large(r) stake. However, it can also be that (some) investors know more than the issuer, who therefore wants a high(er) price if the investors ask for a large(r) stake. Uninformed investors buy less for more; an uninformed issuer sells less for less. The relations between quantity and price are opposite.

Similarly, consider again cash-equity offers in mergers and acquisitions. A would-be acquirer who is less informed than the current owners is unwilling to pay a high(er) price unless the owners are willing to accept a large(r) part of the consideration in equity. Conversely, an acquirer who is more informed than the current owners must either pay a high(er) price or offer a large(r) part of the consideration in equity. Similarly, the relations between equity consideration and (total) price are opposite. Confounding, or incorrectly classifying, the lemons problem and the smart buyer problem can hence lead to erroneous conclusions. Suppose a takeover study searches for evidence of the lemons problem, but finds that takeover premia (or post-takeover performance) are not increasing in the share of equity consideration. This does not warrant the conclusion that asymmetric information is negligible for choice of payment. In fact, the evidence could be consistent with the smart buyer problem; or the average effect could be weak because both information asymmetries with their countervailing effects are present in the data. Incidentally, the existing evidence on contractual signaling in external financing, or in mergers and acquisitions, is rather mixed. The above discussion suggests one possible reason why past empirical studies may have been inconclusive.\footnote{Interestingly, Chang (1998) documents that stock offerings, on average, have a positive announcement effect on the bidder when the target company is privately held but a negative one when the target is publicly held. One possible explanation, suggested also by Chang, is that the two settings have opposite information asymmetries.}

4.3 Identification through intermediary contracts

In some markets, information frictions are mitigated by expert intermediaries, who buy and resell the goods. The role of such intermediaries is not entirely obvious. In particular, how can the presence of another, say, smart buyer resolve an uninformed seller’s fear of being short-changed?

We argue that this is indeed possible, and that intermediary contracts can help infer the underlying information problem. Suppose a person wants to sell an inherited antique but lacks expertise to assess its value ($\theta_i$). To evade a bargain
hunter, the seller enters into a contract with a specialized antiques dealer: The
dealer buys the antique for \(\theta_x\) and makes supplementary payments if the antique
is resold for more than \(\theta_x + \varepsilon\). Since the dealer is smart, the ultimate buyer
will not be able to buy the antique for less than \(\theta_x\). The increment \(\varepsilon\) can be
interpreted as a dealer commission. In fact, a similar deal can be implemented
through a percentage commission.

This example illustrates that informed intermediation helps for two reasons.
First, the intermediary does not buy the good for its own consumption. Oth-
erwise, the seller is back to the original problem. Second, the resale generates
verifiable information about \(\theta_x\). This allows for contracts that resemble the se-
curity design solution or revenue sharing, thereby obviating inefficient signaling
through trade failures. In practice, such informed intermediation is provided by
agents (for performing artists), galleries (for visual artists), market-makers (in
stock exchanges), or underwriters (in capital markets).

Relevant to identification of the underlying information problem is the fact
that the intermediary enters into different contracts with the informed and the
uninformed parties. In a smart buyer problem, the intermediary enters into a
contingent contract with the seller, such as a commission, while engaging in a
simple cash transaction with the buyer. By contrast, in a lemons problem, the
intermediary enters into a contingent contract with the buyer, such as a warranty,
while engaging in a simple cash transaction with the seller. This asymmetry in
the way the intermediary interacts with both sides of the market allows to infer
the underlying information problem.

4.4 Robustness to competition

One may argue that the smart buyer problem is resolved through competition,
and therefore lacks relevance in practice. In the model, the problem would indeed
disappear if there were a second informed buyer; bidding competition would
ensure that the winning bid matches at least the common value. However, there
are several reasons why this may often not be the case in practice.

First, competition may not arise, especially in a common value environment,
when it is costly to become informed. Once there is an informed bidder, any
potential rival does not incur the cost of acquiring information unless it expects
to have substantially larger gains from trade. In fact, the potential buyer with
the largest expected gains from trade has the strongest incentive to become informed first so as to preempt competition.

Second, competition may only be latent. A seller may have to wait or search for a second buyer to get a competing offer. This is costly and has benefits unknown to the seller. The expected (net) return from turning latent into actual competition is one interpretation of the seller’s outside option in our model.

Third, the smart buyer problem persists even with actual competition if the bidders have heterogeneous information. In such a setting, the price at which the (last) competitor withdraws from the auction may be below the winner’s best estimate of the common value. Since the seller cannot fully infer the winner’s best estimate from the bidding process, the two parties find themselves in a smart buyer constellation once the bidding is completed.\footnote{In other words, the smart buyer problem arises in common value auctions with heterogeneous bidders when the seller is not committed to sell.} Thus, competition eliminates the smart buyer problem only if the buyers are homogeneous. By the way, homogeneous competition also eliminates the lemons problem: The buyer’s informational disadvantage becomes immaterial once it faces two equally informed sellers offering identical goods.

5 Pooling outcomes

In this section, we consider equilibrium outcomes that are not fully revealing. Dari-Mattiacci et al. (2010) show that in pooling outcomes, a market with informed buyers collapses from the bottom, unlike a market with informed sellers, which collapses from the top. That is, informed buyers find it impossible to buy low-quality assets at low prices, while informed sellers find it impossible to sell high-quality goods at high prices.

In our framework - as in all signaling games with the Maskin-Tirole bargaining protocol - the fully revealing equilibrium always exists. It is also the unique equilibrium unless there exists a pooling equilibrium in which all buyer types are (weakly) better off than in the fully revealing outcome, which requires that the distribution of buyer types is sufficiently skewed towards low types. When we use the intuitive criterion to reduce multiplicity, only the fully revealing equilibrium survives.

At the same time, once we extend our framework such that the buyer has ad-
ditional private information about the gains from trade, full revelation becomes impossible. Signaling devices, such as revenue sharing and security design, become ineffective because the seller can no longer infer the common value from the offer terms.

5.1 Equilibrium multiplicity

Since the security design solution in Proposition 4 achieves the first-best outcome, we must exclude it to create a role for pooling outcomes. We thus abstract from verifiability and focus on trade rationing contracts; only Condition D is satisfied, and contracts take the form $C = [x, t, 0]$. In this setting, an equilibrium is efficient if all buyer types trade the full quantity $x = 1$ and must therefore be uninformative.

To analyze pooling equilibria, we focus without loss of generality on binary contract menus that contain two elements; $C_0$ and some contract $C_P \neq C_0$ such that, if the menu is accepted, the buyer selects either $C_0$ or $C_P$ for all $\theta \in \Theta$. In an uninformative equilibrium, every type $\theta \in \Theta$ submits the same binary offer $C_P = \{C_P, C_0\}$ and selects the same contract from this menu. In a partially revealing equilibrium, offers may, but need not, contain more than two elements. The defining feature is that not all buyer types choose the same contract, although some contracts are chosen by more than one type. For example, an equilibrium in which all types submit the same offer $C_P = \{C_P, C_0\}$ and both contracts are sometimes chosen is partially revealing.

A binary contract offer $C_P = \{C_P, C_0\}$ must meet both parties’ participation constraints. Clearly, the element $C_0$ trivially satisfies this condition. By contrast, $C_P$ meets the buyer’s participation constraint only when the latter’s type $\theta \in \Theta$ satisfies

$$x_p \theta_x + z(x_p; \theta_x, \theta_z) \geq t_p. \quad (12)$$

For a given $C_P$, let $\Theta^P$ denote the subset of buyer types for whom (12) holds.

Similarly, $C_P$ satisfies the seller’s participation constraint if and only if

$$t_p \geq x_p E[\theta_x | C]. \quad (13)$$

In words, the seller must deem the fixed transfer larger than the forgone common value, given beliefs that are conditional on the observed offer.
Two considerations determine how the expectations in (13) are formed. First, the seller must conjecture which subset of $\Theta$ would make such an offer $C_P = \{C_P, C_0\}$. Let $\Theta_P^1$ denote this subset. Second, the seller must infer which subset of $\Theta_P^1$ prefers $C_P$ over $C_0$. Let $\Theta_P^2$ denote this subset. Suppose the seller conjectures $\Theta_P^1 = \Theta$, that is, it believes that all types $\theta \in \Theta$ make the observed offer (as must be true in an uninformative equilibrium). Then, $\Theta_P^2 = \Theta_P$ and (13) can be written as

$$t_P \geq x_P E[\theta_x | \theta \in \Theta_P^P].$$

(14)

An uninformative equilibrium offer $C_P$ must satisfy (14) and further requires out-of-equilibrium beliefs that prevent deviations from $C_P$.

**Lemma 1** Suppose only Condition D is satisfied. In the absence of restrictions on the offer form,

- There always exists one fully revealing equilibrium.
- There is a unique threshold value $\mu_x \in (\bar{\theta}_x, \theta_x)$ such that an uninformative equilibrium exists if and only if $E(\theta_x) \leq \mu_x$.
- A partially revealing equilibrium exists if and only if there exists a pooling offer $C_P = [x_P, t_P, 0]$ such that $t_P \geq x_P E[\theta_x | \theta \in \Theta^+_x(C_P)]$.

As is common in signaling games, the equilibrium can, but need not, be unique. The separating outcome of Proposition 2 always exists. Hence, any unique equilibrium is fully revealing. Since the seller’s participation constraint is binding for every type in this outcome, any alternative offer that some types find more attractive, if attributed to the highest of those types, is unacceptable to the seller.

By contrast, uninformative equilibria do not exist, unless every buyer type weakly prefers some uninformative offer over the separating offer. This is only the case when the seller’s average outside option, $E(\theta_x)$, is so low as to warrant a sufficiently low pooling price. Indeed, skewing the probability distribution toward type $\bar{\theta}_x$ raises every type’s pooling payoff but leaves their separating payoff unchanged.

Finally, there can be partially revealing equilibria. For example, given some offer $C_P$, let $\Theta^+_x(C_P) \subset \Theta^+_x$ denote the subset of buyer types that prefers $C_P$ over the separating outcome. If $C_P$ is acceptable to the seller under the premise that
it is made by all types in $\Theta_x^+(C_P)$, there is an equilibrium in which all types in $\Theta_x^+(C_P)$ submit $C_P$ and all other types submit the separating offer schedule.

The proclivity for pooling increases as the seller’s average outside option decreases, that is, as pooling becomes more lucrative (less expensive) for high (low) types. Indeed, all buyer types are weakly better off in partially revealing or uninformative equilibria than in the separating equilibrium; otherwise, some type(s) would deviate to the separating offer, which invariably succeeds. In the limit, as $E(x) \to \bar{x}$, every buyer type prefers the efficient (uninformative) equilibrium over any other equilibrium. An appealing conjecture is that, under such conditions, standard refinement criteria select the efficient uninformative equilibrium. Yet, this is not the case. On the contrary, the intuitive criterion uniquely selects the fully revealing equilibrium, even though it is every buyer type’s least preferred.

**Proposition 5** Only the fully revealing equilibrium survives the intuitive criterion.

In any equilibrium that is not fully revealing, some types pay more, and others less, than their respective common values. There always exists a deviating offer with a lower price and smaller quantity that is attractive only to the overpaying types. Under the intuitive criterion, such a deviation can be attributed only to these types, thereby eliminating pooling outcomes.

Put differently, pooling equilibria are not robust to the intuitive criterion unless overpaying types cannot deviate to a lower quantity due to an exogenous constraint. Imposing a minimum quantity is, however, not sufficient. In general, similar restrictions must be imposed on any signaling instrument such as trading probabilities or, if applicable, revenue sharing.

### 5.2 Inscrutable private information

We now consider the extension with two-dimensional private information: The buyer observes both $x$ and $z$, but the seller observes neither. First, we note that the proof of the uninformative equilibrium in Lemma 1 is still valid, except that the condition for the existence of efficient uninformative equilibria becomes $\theta_x + z(1; \theta_x, \theta_z) \geq E(\theta_z)$. At the same time, one expects the additional private information about $\theta_z$ to make separating offers more difficult. The reasons
for this is that all aforementioned signaling devices - stochastic offers, trade rationing, revenue sharing, and security design - exploit the relation between $\theta_x$ and $z$. Being less informed about $\theta_z$, and therefore about this relation, impairs the seller’s ability to infer the common value from the gains from trade.

To illustrate this point in a stark way, we demonstrate how two-dimensional private information affects the revenue sharing and security design solutions.

**Proposition 6** Suppose only Condition $V$ is satisfied and contingent transfers are unrestricted. If the buyer has private information about both $\theta_x$ and $\theta_z$, there exists no equilibrium in which $\theta_x$ is fully revealed to the seller.

It is instructive to consider why the specific signaling mechanisms in Propositions 3 and 4 collapse. Linear sharing rules enable the buyer to signal its type by relinquishing a particular fraction of $v = \theta_x + z$. The willingness to do so is informative because there is symmetric information about $\partial z/\partial \theta_z$. Private information about $\partial z/\partial \theta_z$ on the part of the buyer obstructs the seller’s inference: It is unclear to what extent the relinquished part of $v$ represents gains from trade or common value.

The collapse of the security design solution illustrates the problem even more starkly. With $\theta_z$ known, there is a one-to-one mapping from common value $\theta_x$ to total value $v(1; \theta_x, \theta_z)$. This allows the buyer to signal $\theta_x$, and to offer $t = \theta_x$, by accepting a commensurate penalty in the event of $v > v(1; \theta_x, \theta_z)$. When less informed about $\theta_z$, the seller must be wary of another type $(\theta_x', \theta_z')$ with larger common value $\theta_x' > \theta_x$ but identical total value $v(1; \theta_x', \theta_z') = v(1; \theta_x, \theta_z)$. Under the above contract, type $(\theta_x', \theta_z')$ would earn (precisely $\theta_x' - \theta_x$) more than under full information, and hence more than under a fully revealing contract. In fact, since these types are ex post indistinguishable, there is no penalty scheme that can discriminate between them based on ex post information.\(^{20}\)

\(^{20}\)While the same problem undermines signaling via stochastic offers, signaling via trade rationing can remain feasible. Unlike the other signaling devices, rationing quantity need not reduce common value and private benefits in the same proportions for all types, since $\partial z/\partial x$ can vary across types. Due to this variation, trade rationing retains discriminatory power under two-dimensional private information. Still, full revelation can break down as, for instance, when $\partial v(x; \theta_x, \theta_z)/\partial x > \partial v(x; \theta_x', \theta_z')/\partial x$ for some $\theta_z < \theta_z'$ and $\theta_x > \theta_x'$.

\(^{21}\)When $v$ is a random variable drawn from a conditional distribution $h(v|\theta_x, \theta_z)$, differences in $h(v|\theta_x, \theta_z)$ across types with identical $E(v|\theta_x, \theta_z)$ may facilitate separation. Still, if (a subset of) different types have identical $h(v|\theta_x, \theta_z)$, the above result holds. For example, this is the case when $v$ is normal, and $\theta_x$ and $\theta_z$ only affect the conditional mean. If $h(v|\theta_x, \theta_z)$ is different for each type, types can be described by a single parameter. Though this need
Intuitively, Proposition 6 can be explained as follows. The smart buyer problem arises because the seller is less informed about the common value. The buyer can overcome this problem contractually by sharing trade surplus, provided that the seller knows how trade surplus relates to common value. Yet, this signaling mechanism breaks down when the informational disadvantage is so severe that the seller, for any given common value, cannot fathom the buyer’s ability to add value; that is, communicating private information becomes more difficult for the buyer when the seller does not “understand the business.”

Example 8 (Merger). Consider a takeover of a small firm by a larger industry peer (A5). The acquirer paints a bleak picture of the target’s standalone future (low estimates of $\theta_x$), but a rosy one of the potential merger synergies (high estimates of $\theta_z$). With the target being wary of low cash offers, the acquirer considers an offer that includes an equity stake in the merged firm.

The problem is that the post-merger value ($v$) also depends on the quality of the acquirer’s assets ($\theta_z$), about which the target is not well-informed. Suspicious again, the target demands a large stake to be on the safe side. This demand in turn makes equity payments less attractive to the acquirer. The acquirer is caught in a dilemma: It needs to concede equity to overcome the smart buyer problem (private information about $\theta_x$), but issuing equity suffers from the lemons problem (private information about $\theta_z$).

6 Concluding remarks

Our analysis of bilateral trade frictions, and their contractual resolution, premises that the buyer is better informed about the seller’s outside option. This outside option, which we posit in reduced form, could be the seller’s (counterfactual) payoff either when retaining the good indefinitely or when seeking out alternative buyers to eventually sell the good. In the latter case, our implicit assumption is that searching for alternative buyers is costly, and that the initial buyer has private information about the costs and benefits of doing so. Clearly, a natural extension is to embed the current model into a search market, in which participants on one side of the market are informed about each other’s valuations, whereas participants on the other side of the market know only their individual not ensure full revelation; within the “redefined” type space, separation may still be infeasible because the single-crossing property need not hold.
valuations. In such a setting, every meeting between potential trading partners results in a smart buyer problem, since one has private information about the other’s outside option. What contracts would arise in equilibrium, and how would they depend on the (severity of the) search frictions?

Another promising avenue, touched upon in Section 4.3, is to explore the role of intermediaries in brokering trade. In practice, laypeople frequently employ experts as agents to negotiate trades with the other (better informed) side of the market, often motivated by the fear of otherwise being short-changed. Conversely, better-informed parties sometimes use “front men” to trade on their behalf in order to avoid suspicion. This use of third parties by both buyers and sellers has possibly interesting implications for market structure, intermediary contracts, and firm boundaries. These issues as well as more specific applications of the smart buyer framework are left for future research.
7 Appendix

7.1 Proof of Proposition 1

The non-existence of a deterministic separating equilibrium is explained in the text. It remains to show that a stochastic separating equilibrium must satisfy the properties stated in the proposition.\textsuperscript{22}

Given that the buyer’s expected payoff function satisfies the single-crossing property, a fully revealing equilibrium is the solution to the maximization problem

\[
\max_{\theta_x} p(\hat{\theta}_x)[\theta_x + z (1; \theta_x, \theta_z)] - \bar{v}(\hat{\theta}_x)
\]

s.t. \( \bar{v}(\hat{\theta}_x) \geq p(\hat{\theta}_x)\theta_x \) (PC)

\( p'(\theta_x)[\theta_x + z (1; \theta_x, \theta_z)] = \bar{v}(\theta_x) \) (FOC)

\( p'(\theta_x) \geq 0 \) (M)

where \( \hat{\theta}_x \) is the buyer’s self-reported type (Baron and Myerson, 1982).

The first-order condition (FOC) and the monotonicity condition (M) are necessary and sufficient for incentive compatibility.

\textbf{Necessity:} Consider two arbitrary buyer types, \( \theta^h_x \) and \( \theta^l_x < \theta^h_x \). The downstream incentive compatibility constraint is

\[
p(\theta^h_x)[\theta^h_x + z (1; \theta^h_x, \theta_z)] - \bar{v}(\theta^h_x) \geq p(\theta^l_x)[\theta^h_x + z (1; \theta^h_x, \theta_z)] - \bar{v}(\theta^l_x).
\]

Similarly, the upstream incentive compatibility constraint is

\[
p(\theta^l_x)[\theta^l_x + z (1; \theta^l_x, \theta_z)] - \bar{v}(\theta^l_x) \geq p(\theta^h_x)[\theta^l_x + z (1; \theta^l_x, \theta_z)] - \bar{v}(\theta^h_x).
\]

Rearranging and combining these constraints yields

\[
[p(\theta^h_x) - p(\theta^l_x)] [\theta^h_x + z (1; \theta^h_x, \theta_z)] \geq \bar{v}(\theta^h_x) - \bar{v}(\theta^l_x)
\]

\[
\geq [p(\theta^h_x) - p(\theta^l_x)] [\theta^l_x + z (1; \theta^l_x, \theta_z)].
\]

By Assumption 1, \( \theta_x + z (1; \theta_x, \theta_z) \) is increasing in \( \theta_x \). Hence, the above condition can hold only if \( p(\theta_x) \) is non-decreasing in \( \theta_x \). Dividing by \( (\theta^h_x - \theta^l_x) \) and taking

\textsuperscript{22} We thank Vladimir Vladimirov for comments that helped to greatly shorten this proof.
the limit \((\theta^b_x - \theta_x^*) \to 0\) implies (FOC) and (M).

**Sufficiency:** Using (FOC) to substitute for \(\bar{p}(\hat{\theta}_x)\) in the first derivative of the objective function and rearranging yields

\[
p'(\hat{\theta}_x)[\theta_x - \hat{\theta}_x + z(1; \theta_x, \theta_z) - z(1; \hat{\theta}_x, \theta_z)]
\]

(15)

Since \(z_{\theta_x}(x; \theta_x, \theta_z) \geq 0\) (Assumption 1), condition (M) implies that (15) is positive for all \(\hat{\theta}_x \leq \theta_x\) but negative for all \(\hat{\theta}_x \geq \theta_x\). That is, the buyer’s objective function is quasi-concave.

To construct the cheapest mechanism for the buyer, we impose that the seller’s participation constraint (PC) binds for every buyer type. Differentiating on both sides with respect to \(x\) and substituting in (FOC) yields the differential equation \(p'(\theta_x) = p(\theta_x)[z(1; \theta_x, \theta_z)]^{-1}\), which implies

\[
p(\theta_x) = p(\bar{\theta}_x) \exp \left[ - \int_{\theta_x}^{\bar{\theta}_x} [z(1; s, \theta_z)]^{-1} ds \right].
\]

Since the buyer’s expected payoff, \(p(\theta_x)z(1, \theta_x, \theta_z)\), increases in \(p(\theta_x)\), it is immediately clear that \(p(\bar{\theta}_x) = 1\). Note that \(p(\theta_x)\) is differentiable, meets the properties of a probability, \(p(\theta_x) \in [0, 1]\), and satisfies (M).

### 7.2 Proof of Proposition 2

Parallel to Proposition 1, a fully revealing equilibrium is the solution to the maximization problem

\[
\max_{\hat{\theta}_x} x(\hat{\theta}_x) \theta_x + z(x(\hat{\theta}_x); \theta_x, \theta_z) - t(\hat{\theta}_x)
\]

s.t. \(t(\hat{\theta}_x) \geq x(\hat{\theta}_x) \theta_x\) \hspace{1cm} (PC)

\(x'(\theta_x) \theta_x + z(x; \theta_x, \theta_z) = t'(\theta_x)\) \hspace{1cm} (FOC)

\(x'(\theta_x) \geq 0\) \hspace{1cm} (M)

where \(\hat{\theta}_x\) is the buyer’s self-reported type. As above, (FOC) and (M) are necessary and sufficient for incentive compatibility.

To construct the cheapest mechanism for the buyer, we impose again that the seller’s participation constraint (PC) binds for every buyer type. Differentiating on both sides with respect to \(\theta_x\) and substituting in (FOC) yields the differential
equation $x'(\theta_x) = x(\theta_x)[z_x(x(\theta_x); \theta_x, \theta_z)]^{-1}$. Note that (M) is satisfied. Since the principal’s expected payoff increases in $x(\theta_x)$, it is immediate that $x(\theta_x) = 1$. ■

7.3 Proof of Proposition 3

It is straightforward to see that (8) is isomorphic to (1), and Proposition 1 therefore applies.

7.4 Proof of Proposition 4

A buyer of type $\theta_x$ receives the payoff $z(1; \theta_x, \theta_z)$ when making a truthful offer with the fixed transfer $\theta_x$. Now consider the buyer’s payoff when mimicking a lower-valued type $\theta'_x < \theta_x$. By Assumption 1, $v(1; \theta'_x, \theta_z) < v(1; \theta_x, \theta_z)$. Hence, when mimicking type $\theta'_x$, type $\theta_x$ would incur a penalty $\tau > \theta_x - \theta'_x$ and its payoff would be less than $z(1; \theta_x, \theta_z)$. Now consider the payoff from mimicking any type $\theta''_x > \theta_x$. By Assumption 1, $v(1; \theta''_x, \theta_z) > v(1; \theta_x, \theta_z)$. Hence, mimicking would not trigger a penalty, but type $\theta_x$ would pay a fixed transfer of $\theta''_x$, which is higher than the fixed transfer $\theta_x$ under its truthful offer. ■

7.5 Proof of Lemma 1

Existence of single fully revealing equilibrium: Consider the fully revealing equilibrium in Proposition 2. We show that there exist seller beliefs such that any deviation to another (not fully revealing) contract (menu) is rejected.

Denote the preferred contract of type $\theta'_x$ in the fully revealing equilibrium by $C' = [x', t', 0]$. Suppose that the deviation offer contains a contract $C'^d = [x'^d, t'^d, 0]$ that type $\theta'_x$ strictly prefers to $C'$ where $x'^d \leq x'$. Given $z_x \geq 0$ (Assumption 1), the total surplus is weakly smaller under $C'^d$ than under $C'$. Since type $\theta'_x$ strictly prefers $C'^d$ over $C'$, the seller’s payoff must be smaller under $C'^d$ than under $C'$ if chosen by type $\theta'_x$. Given that the seller breaks even under $C'$, beliefs that assign the deviation offer to type $\theta'_x$ cause the seller to reject the deviation offer.

Now suppose that the deviation offer contains a contract $C'^d = [x'^d, t'^d, 0]$ that type $\theta'_x$ strictly prefers to $C'$ where $x'^d > x'$. In this case, $x'^d$ must be equal to the quantity that some higher type $\theta''_x \in (\theta'_x, \theta_x]$ trades in the fully revealing outcome. Denote the preferred contract of type $\theta''_x$ in the fully revealing equilibrium by
\[ C'' = [x'', t'', 0] \] where \( x'' = x_d^2 \). Since type \( \theta_x'' \) prefers \( C' \) over \( C'' \) (by incentive compatibility), it also prefers \( C_2^d \) over \( C'' \) (by transitivity). Because \( x'' = x_d^2 \), this implies \( t'' > t^d \) so that type \( \theta_x'' \) also prefers \( C_2^d \) over \( C'' \). This in turn implies that the seller’s payoff is smaller under \( C_2^d \) than under \( C'' \) if chosen by type \( \theta_x'' \). Given that the seller breaks even under \( C'' \), beliefs that assign the deviation offer to type \( \theta_x'' \) cause the seller to reject the deviation offer.

Defining off-equilibrium beliefs in this manner, one can deter all potential deviations of every buyer type to support the unique fully revealing equilibrium of Proposition 2.

**Existence of an uninformative equilibrium:** An uninformative equilibrium exists when a binary offer \( \mathcal{C}_P = \{C_P, C_0\} \) (a) satisfies the seller’s participation constraint and (b) yields a higher payoff for every type than in the fully revealing equilibrium. Therefore, all buyer types must participate in an uninformative equilibrium, and the lowest acceptable price is \( P_P = E(\theta_x) = \mu_x \). The payoff of type \( \theta_x' \) in such an equilibrium is

\[ \Pi_P(\theta_x') = x_P\theta_x' + z(x_P; \theta_x', \theta_x) - \mu_x. \]

Unlike the buyer’s payoff in the fully revealing equilibrium, \( \Pi_P(\theta_x') \) depends on the probability distribution of \( \theta_x \) over \( \Theta_x \), in particular \( \partial \Pi_P(\theta_x')/\partial \mu_x < 0 \) for all \( \theta_x' \in \Theta_x \). Hence, for sufficiently large \( x_P \), condition (b) can always be satisfied by letting \( \mu_x \to \bar{\theta}_x \).

For example, consider the limit \( \lim_{\mu_x \to \bar{\theta}_x} \Pi_P(\theta_x') \) for \( x_P = 1 \):

\[ \lim_{\mu_x \to \bar{\theta}_x} \Pi_P(\theta_x') = \theta_x' + z(1; \theta_x', \theta_x) - \bar{\theta}_x. \]

For every \( \theta_x' \in \Theta_x \), this limit is weakly larger than the buyer’s payoff under full information, and hence also larger than in the fully revealing equilibrium. Thus, there exists some threshold \( \mu_x \in (\theta_x, \bar{\theta}_x] \) such that, for \( \mu_x < \mu_x \), some binary offer makes every type better off than in the fully revealing equilibrium.

Finally, we show that there exist seller beliefs such that any deviation from such a binary offer is rejected. Deviations to any other contract menu that contains some contract with \( P < \bar{\theta}_x \) can be deterred by beliefs attributing this offer to type \( \bar{\theta}_x \). Offers that contain only contracts with \( P \geq \bar{\theta}_x \) yield lower payoffs than the payoffs in the fully revealing equilibrium, which by construction are lower than those under the binary offer.
Existence of partially revealing equilibrium: A partially revealing equilibrium can exist only if there is a binary offer that (a) satisfies the seller’s participation constraint and (b') yields a higher payoff than in the fully revealing equilibrium for a subset of buyer types. For any binary offer \( \mathcal{C}_P = \{C_P, \emptyset\} \), define a subset \( \Theta_x(C_P) \subset \Theta_x \) that contains all buyer types whose payoff is larger under \( C_P = [x_P, t_P, 0] \) than in the fully revealing equilibrium. There always exist some \( C_P \) such that \( \Theta_x(C_P) \) is non-empty, and hence satisfy condition (b'). Such an offer also satisfies condition (a) if and only if

\[
t_P \geq x_P E \left[ \theta_x | \theta_x \in \Theta_x(C_P) \right].
\]

Whenever there exists a binary offer with non-empty \( \Theta_x(C_P) \) that satisfies (16), there exists a partially revealing offer of the following kind: All types \( \theta_x \in \Theta_x(C_P) \) make the binary offer, whereas all other types make some other offer. Beliefs associated with any deviation are chosen as in the proof of the uninformative equilibrium.

7.6 Proof of Proposition 5

In any equilibrium other than the fully revealing one, there is a subset \( \Theta^P \) with at least two types that choose the same contract \( \mathcal{C}_P = [x_P, x_P P_P, 0] \). To satisfy the seller’s participation constraint, \( P_P \geq E(\theta_x | \theta_x \in \Theta^P) \). Denote the lowest type in that subset by \( \theta_{P}^P \equiv \min \Theta^P \). Clearly, \( P_P > \theta_{P}^P \).

Consider the contract \( \mathcal{C}^d = [x^d, x^d P_d, 0] \), with \( x^d = x_P - \delta \). A given type \( \theta_x \) prefers \( \mathcal{C}^d \) over \( \mathcal{C}_P \) if and only if

\[
x_P P_P - x^d P_d > v(x_P; \theta_x, \theta_z) - v(x^d; \theta_x, \theta_z).
\]

Since the right-hand side of the inequality increases in \( \theta_x \) (Assumption 1), if the inequality holds for some type \( \theta_x \), then it also holds for all lower types. Hence, we can adjust \( P_d \) such that the inequality holds only for \( \theta_x \leq \theta_{P}^P \). For very small \( \delta \), this requires a small change in \( P_d \) such that \( P_d \geq \theta_{P}^P \). Under the intuitive criterion, the seller assigns the deviation \( \mathcal{C}^d \) to types \( \theta_x \leq \theta_{P}^P \). Given \( P_d \geq \theta_{P}^P \), the seller therefore never rejects the contract. Thus, any equilibrium other than the fully revealing equilibrium does not survive the intuitive criterion.
7.7 Proof of Proposition 6

For the proof, it is convenient to define buyer types in the \( \theta_x-z \)-space. We proceed as follows: (I) We characterize necessary conditions for separating types with different \( \theta_x \) but the same total valuation \( v \). (II) We then characterize necessary conditions for separating types with different \( v \). (III) We demonstrate that the conditions in (I) and (II) cannot be reconciled with each other.

(I) Define \( \Theta_v \equiv \{ (\theta_x, z) : \theta_x + z = v_i \} \), which contains some type \( (\bar{\theta}_x, \bar{z}) \). Using \( z = v - \theta_x \), we can express any type in \( \Theta_v \) in terms of \( \theta_x \) only, namely \( (\theta_x, v - \theta_x) \). Let \( \mathcal{C}_v(\theta_x) = \{ [1, t_v(\theta_x), [1 - \alpha_v(\theta_x)]v] \} \) denote an offer that separates all types in \( \Theta_v \), where \( \alpha_v(\theta_x) \) is a claim contingent on the realized \( v \).

For a given contract in this offer, type \( \theta_x \)'s payoff is

\[
\Pi(\theta_x) = E[\alpha_v(v; \theta_x) v | v] - t_v(\theta_x) = \alpha_v(v; \theta_x) v - t_v(\theta_x), \tag{17}
\]

which is deterministic because the buyer knows its \( v \). To achieve separation in \( \Theta_v \), the payoff must satisfy the invariance condition

\[
\Pi(\theta_x) = \bar{\Pi} \quad \text{for all } \theta \in \Theta_v. \tag{18}
\]

Otherwise, some types in \( \Theta_v \) would be mimicked by other types in the set. Further, note that \( \bar{\Pi} = \bar{z} \). Otherwise, type \( (\bar{\theta}_x, \bar{z}) \) would deviate to the contract \( [1, \bar{\theta}_x, 0] \). Using this, merging (17) and (18), and simplifying yields

\[
t_v(\theta_x) = \bar{\theta}_x - \beta_v(v; \theta_x) v \tag{19}
\]

where \( \beta_v(v; \theta_x) v \equiv [1 - \alpha_v(v; \theta_x)]v \) denotes the contingent claim paid to the seller. For \( \partial \beta_v(v; \theta_x) / \partial \theta_x \neq 0 \), (19) characterizes all offers \( \mathcal{C}_v(\theta_x) \) that achieve separation within \( \Theta_v \), that is, separation of all types that generate the same total value \( v \).

The invariance condition pins down a buyer’s payoff as a function of total valuation. This function is given by, with slight abuse of notation, \( \Pi(v) = \bar{z} = v - \bar{\theta}_x \). That buyer profits follow this function across \( \Theta_v \)s is a necessary condition for separation within \( \Theta_v \)s. Importantly, note that the function is linear in \( v \), that is,

\[
\partial \Pi / \partial v = 1. \tag{20}
\]
(II) We now consider separation across different $v$, which is a necessary condition for achieving separation across all $\theta_x \in \Theta_x$. (If marginally different $v$ are not separated, some $\theta_x$-types are pooled.) A direct mechanism that separates different $v$ must yield \( \arg \max_{\hat{v}} \{ E [\alpha (v; \hat{v}) v | v] - t (\hat{v}) \} = v \) for all $v$. The corresponding first-order condition is

\[
\frac{\partial \alpha (v; v)}{\partial v} v = \frac{\partial t (v)}{\partial v}.
\] (21)

By the envelope theorem, separation across $v$ requires that equilibrium payoffs must vary across $v$ according to

\[
\partial \Pi / \partial v = \alpha (v; v).
\] (22)

(III) Conditions (20) and (22) can hold simultaneously only if $\alpha (v; v) = 1$. This already shows that separation cannot simultaneously hold within each $\Theta_v$ and across $v$. Indeed, substituting $\alpha (v; v) = 1$ — more precisely, $\partial \alpha (v; v) / \partial v = 0$ — into (21) yields $\partial t (v) / \partial v = 0$, which in turn implies that $t (v) = K$ where $K$ is some constant. It is obvious that $\alpha (v; v) = 1$ and $t (v) = K$ cannot achieve separation across $v$. 

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References


