Strategic Investment, Industry Concentration, 
and the Cross Section of Returns

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Abstract

This paper provides an alternative real options framework to assess how firms’ strategic interaction under imperfect competition affects the industrial dynamics of investment, concentration, and expected returns. When firms have similar production technologies, the cross sectional variation in expected returns is low, firms’ investments are more synchronized, firms’ expected returns co-move positively, and the industry is less concentrated. Conversely, in more heterogeneous industries, the cross sectional variation in expected returns is high, there are leaders and followers whose expected returns co-move negatively, and the industry is more concentrated. The model rationalizes several empirical facts, including: (i) that firms’ returns co-move more positively in less concentrated industries; (ii) that booms and busts in industry returns are more pronounced in less concentrated industries; and (iii) that less concentrated industries earn higher returns on average.

Keywords: expected returns, investment, imperfect competition, industry concentration.

JEL codes: L11, L22, G11, G12, G31.

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Introduction

While the investment based asset pricing literature motivated by Cochrane (1996), Berk, Green and Naik (1999) and Carlson, Fisher and Giammarino (2004) predicts that firms’ investment decisions affect their exposure to systematic risk, few theoretical papers explore how the organization of an industry affects the conditional dynamics of firms’ expected returns through its effect on investment.\(^1\) Meanwhile, recent empirical findings by Hou and Robinson (2006) and Hoberg and Phillips (2010) suggest that industrial organization has a significant impact on the cross section of expected returns. In less concentrated industries, firms’ returns are higher on average; firms’ returns co-move more positively; and average industry returns show a regular pattern of *booms* or periods of high investment and high returns, followed by *busts* or periods of low investment and low returns.\(^2\)

This paper proposes a dynamic model of investment under imperfect product market competition whose asset pricing implications explain these empirical facts. The model considers a real options framework in which firms with heterogeneous production technologies compete in capacity. Each firm in the industry has a single growth option to increase its capacity and decides when and how much to invest. As in Carlson, Fisher and Giammarino (2004), the model predicts that the exposure to systematic risk or *beta* of any firm depends on the relative contribution of its own growth opportunities to its value. In contrast with Carlson, Fisher and Giammarino (2004), however, in imperfectly competitive industries the exposure to systematic risk of any firm also depends on the growth opportunities of its competitors. Under imperfect competition, the expected returns of all firms in the industry are mechanically interrelated.

The core prediction of the model is then that the industrial dynamics of expected returns depend on the underlying distribution of firms’ current and future production technologies. When firms have more similar production technologies, firms are closer competitors whose investments are more synchronized, and firms make similar additions to capacity upon increases in market demand. This leads to less concentrated industries in which firms’ expected returns *co-move positively* over time, and the dynamics of the expected returns of each firm in particular are similar to those of the industry on average. Conversely, when firms have more heterogeneous production technologies, firms with relatively more valuable investment

\(^1\)These papers include Aguerrevere (2009) and Novy Marx (2010). I enlarge on related literature below.
\(^2\)I elaborate on these empirical findings below.
opportunities invest significantly earlier and more than other firms. This leads to more concentrated industries in which the expected returns of leaders and followers co-move negatively over time, and the dynamics of the expected returns of each firm in particular are not representative of those of the industry on average.

The equilibrium dynamics of firms’ investment and expected returns under imperfect competition coincide with those described in the real options literature for idle firms only when firms have more similar production technologies. When firms are close competitors and tacitly coordinate their investments, each firm expects an increase in its market share upon option exercise. This is the same prediction by Dixit and Pyndick (1994) for idle firms. The industry has a common investment threshold at which all firms find it optimal to increase their capacity; firms’ betas jointly increase before investment and decrease upon option exercise. The real options prediction by Carlson, Fisher and Giammarino (2004) that a boom in a given firm’s beta before investment is followed by a bust upon investment also holds on average at the industry level.

Conversely, in more heterogeneous industries with leaders and followers, whenever one firm expects an increase in its market share upon its own investment, the remaining firms expect a corresponding reduction in their own. The expected reduction in market share due to future additions to industry capacity by competitors dampens the beta of any firm in the same industry—yet this occurs at different points in time for each firm. When one firm expects a decrease in its beta upon its own investment, the remaining firms expect an increase in their own betas once their competitors invest. The industry has multiple investment thresholds, the returns of leaders and followers co-move negatively, and the booms and busts in firms’ expected returns are not easily observable on average at the industry level.

The testable implications rationalize the evidence on average industry returns and the Herfindahl Hirshman Index (HHI) of concentration. As in Hoberg and Phillips (2010), the model predicts that firms’ returns co-move more positively in less concentrated industries. The stylized real options prediction that a boom in a given firm’s beta before investment is followed by a bust upon investment holds for average industry returns only in less concentrated industries. Hoberg and Phillips (2010) find that less concentrated industries have more predictable average returns, with periods of high investment and high returns followed by periods of low investment and low returns. Finally, in more concentrated industries,

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3 The US Federal Trade Commission defines the HHI as the sum of the squares of the market shares of the 50 largest firms within the industry.
firms’ betas are dampened by the expected reduction in market share due to future additions to industry capacity. This result can explain the finding by Hou and Robinson (2006) that less concentrated industries earn higher returns on average.

The model also yields testable implications on the relation between the cross sectional variance of industrial returns and industry concentration. The industrial cross sectional variance of returns is higher in more concentrated industries at any point in time. The model predicts that the booms and busts in the industrial cross sectional variance of returns are easily detectable irrespective of industry concentration. Furthermore, the model predicts that the current industrial cross sectional variance in returns has predictability over the future concentration of an industry. In particular, more heterogeneous industries with high cross sectional variance in firms’ betas become highly concentrated.4

The model characterizes how the underlying determinants of market demand affect the industrial dynamics of investment, concentration and expected returns. In industries with low demand elasticity, high demand growth, and high demand uncertainty, firms are more likely to coordinate investments and have more similar investment policies.5 Such industries are more likely to be less concentrated, have higher returns on average, have lower cross sectional variance in returns, and have more predictable booms and busts. Since high demand growth and volatility boost the value of firms’ growth options, the framework explains why Hoberg and Phillips (2010) find more pronounced booms and busts in less concentrated industries with high demand growth and high demand uncertainty.

Finally, the paper elaborates on the effects of a higher number of competitors on industry dynamics. In line with the evidence in Bulan, Mayer and Somerville (2009), and the results in Grenadier (2002) and Aguerrevere (2009), increased competition erodes firms’ growth option values and firms’ expected returns in all industries. This paper adds to these studies as it shows that in industries with heterogeneous firms with higher installed capacity and less productive growth opportunities are more severely hit by increased competition.6 The evidence in Bulan (2005) supports this prediction. The model also adds to Grenadier (2002) as it shows that increased competition need not induce all firms to invest earlier under imperfect competition. A higher number of competitors induces all firms in the same industry to accelerate investment only if they are close competitors, but not necessarily

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4The evidence in Alexander and Thistle (1999) relates to this last prediction.
5This is consistent with the industrial organization literature on tacit collusion. See Ivaldi et al (2003), Motta (2004) and Boyer et al (2001).
To summarize, the main contributions of the paper are three. First, the paper provides an alternative real options framework which predicts how the organization of an industry affects firms’ exposure to systematic risk by focusing on the effects of strategic interaction on firms’ investment decisions. This is in contrast with the existing literature on industrial investment, expected returns and concentration which, by construction, does not elaborate on the equilibrium effects of firms’ strategic interaction. Second, the paper yields several testable implications on the relation between industry concentration and the industrial cross section of returns. The model highlights that industry concentration is not only related to average industry returns, but also to the industrial cross sectional variance of returns, and to the co-movement in firms’ expected returns within the same industry. The third more broader contribution is to bridge the gap between the investment based asset pricing literature and the industrial organization literature.

The related literature includes studies in both finance and economics. The theme of the paper is very closely related to Aguerrevere (2009), who applies a real options model of identical firms under imperfect competition to explain the evidence in Hou and Robinson (2006). Using a different real options framework, Aguerrevere (2009) provides the conditional prediction that less concentrated industries may earn higher returns on average if demand is sufficiently high. This paper proposes the complementary explanation that in more concentrated industries firms’ betas are dampened by the expectation of future additions to industry capacity.

The proposed real options framework is yet more closely related to Fundenberg and Tirole (1985), Grenadier (1996), Weeds (2002) and Mason and Weeds (2010), who consider alternative duopolies in which firms optimally decide when to invest. This paper explores the asset pricing implications of a model in which multiple, heterogeneous firms decide when and how much to invest. The model incorporates incentive compatibility constraints and sorting conditions to endogeneize firms’ incentives to deviate from their strategies, in line with Maskin and Tirole (1988). The approach is consistent Back and Paulsen (2009), who

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7This is because a higher number of competitors may induce firms to invest sequentially. See Section IV.
8Aguerrevere (2009) and Novy Marx (2010) consider industries in which firms invest continuously and simultaneously at each point in time. In this paper, each firm has a single growth option and the ordering of firms’ investments is an equilibrium outcome.
9The implementation also relates to Bustamante (2011), who applies sorting conditions to a real options game with multiple action strategies.
require real option games of imperfect competition to account for firms’ incentives to invest sequentially or simultaneously.

The paper is also closely related to Carlson et al (2009), who predict in a Stackelberg real options duopoly that the beta of the leader is dampened by the expectation of lower future profits once the follower invests. Bena and Garlappi (2011) obtain a similar result in a model of investment timing in R&D, and provide the supporting empirical evidence that race leaders have lower systematic risk. This paper observes that the beta of any operating firm in more concentrated industries is dampened by the expectation of future additions to industry capacity. Boyer et al (2001), Grenadier (2002), Lambrecht and Perraudin (2003), Garlappi (2004), Pawlina and Kort (2006), and Novy Marx (2010) consider alternative mechanisms through which imperfect competition affects firms’ growth options and expected returns.

The paper is organized in five sections. Section I defines firms’ values, expected returns and the industrial return moment conditions. Section II derives firms’ investment strategies in equilibrium. Section III provides the asset pricing implications. Section IV elaborates on comparative statics. Section V concludes.

I Industrial organization and expected returns

A Main assumptions

Consider an industry with \( N \) firms. Each firm \( j = 1, 2, \ldots, N \) has both assets in place and a growth option to invest and increase capacity. Each firm is all equity financed and run by a manager who is the single shareholder. Firms operate at full capacity at any point in time.

Firms compete in capacity and produce an homogeneous good which they sell in the market at a price \( p_t \). The product demand function requires that the market price \( p_t \) equals

\[
p_t = X_t Q_t^{-\frac{1}{2}}
\]

where \( \varepsilon > 1 \) is the elasticity of demand and \( X_t \) is a systematic multiplicative shock, and the industry output \( Q_t \) is the sum of the production at time \( t \).\(^{10}\) The demand shock \( X_t \) follows a geometric Brownian motion with drift \( \eta \), volatility \( \sigma \) and \( X_0 > 0 \).\(^{11}\)

\(^{10}\)Note that firms under imperfect competition do not operate in the range where \( \varepsilon < 1 \).

\(^{11}\)I further assume that \( X_0 \) is strictly lower than any of the optimal thresholds derived in the paper. This ensures that the value of firms’ growth options is positive when they start operating.
Denote \( q_j \) the installed capacity of firm \( j \) before investing in its growth option, and \( q_j \) its total capacity after investment. Firms’ option to increase capacity by \( \Delta q_j = q_j - q_j \) is subject to a fixed cost of investment \( I \). The decision to invest is irreversible such that \( \Delta q_j > 0. \)

Firms differ in their current production technologies, their future production technologies, or both. This is reflected in their instantaneous profit function \( \pi_{jt} \) at time \( t \). Before exercising its own growth option, the instantaneous profit of firm \( j \) is given by

\[
\pi_{jt}^- = (p_t^- - \bar{c} X_t) q_j
\]

where the superscript \(-\) denotes value before option exercise, and \( \bar{c} \) reflects an instantaneous marginal cost of production. Hence firms may differ in their current production technologies via \( q_j \). Upon investment, the instantaneous profits of firm \( j \) are given by

\[
\pi_{jt}^+ = (p_t^+ - c_j X_t) q_j
\]

where the superscript \( + \) denotes value after option exercise. Hence firms may differ in their future production technologies via \( c_j < \bar{c} \).

Firms maximize firm value by choosing the optimal investment strategy \( \Gamma_j \equiv \{x_j; \Delta q_j\} \). The investment strategy \( \Gamma_j \) combines a stopping rule specifying the critical value \( x_j \) for the stochastic demand shock \( X_t \) at which firm \( j \) invests, and the amount \( \Delta q_j \) that firm \( j \) adds to its existing capacity upon investment. I elaborate of firms’ strategies in equilibrium and the industry equilibrium concept in Section II.

**B Firms’ values**

Firms’ values are given by the expected present value of their risky profits. To evaluate profits, I assume the existence of a pricing kernel. Using the standard argument in Duffie (1996), I construct a risk neutral probability measure under which the demand shock \( X_t \) follows a geometric Brownian motion with drift \( r - \delta \) and volatility \( \sigma \), where \( \delta \) is a convenience yield. The risk premium associated with the stochastic process \( X_t \) is given by \( \psi \equiv \eta - (r - \delta) \).

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\(^{12}\)The irreversibility of investment implies a commitment by firms not to adjust their capacity upon a reduction in market prices.\(^{13}\)For the sake of tractability, the production cost \( \bar{c} \) is not firm specific, and yet firms may differ in the value of their assets in place since their installed capacities \( q_j \) are firm specific.
Denote by $V_{jt}$ the value of firm $j$ at time $t$ for any investment strategy $\Gamma_j = \{x_j; \Delta q_j\}$. The valuation of firm $j$ before option exercise contains two well known components. The first is the value of a growing perpetuity of cash flows generated by its assets in place. The second is the value of its investment opportunities or growth options. This provides the standard prediction in real options models that the value of firm $j$ depends on its lifestage.\footnote{See, for instance, Dixit and Pyndick (1994).}

The model allows for a third component which reflects the impact of strategic interaction on firm value. In imperfectly competitive industries, the value of firm $j$ depends not only on its own lifestage but also on the lifestage of its competitors. At any point in time, the market price $p_t$ at which firm $j$ sells its production depends the capacity decisions of all firms in the industry. Before firm $j$ invests, the capacity additions of its competitors reduce current and future market prices, and hence lower the current and future profits of firm $j$. After firm $j$ invests, the instantaneous profits of firm $j$ decrease if their competitors make subsequent additions to capacity.

Denote $\Delta \pi_{jt}$ the expected change in instantaneous profits of firm $j$ due to subsequent investments by other firms. The value of firm $j$ for any investment strategy $\Gamma_j = \{x_j; \Delta q_j\}$ is then given by

$$
V_{jt} = \begin{cases} 
\frac{\pi_{jt}}{\delta} + \frac{\Delta \pi_{jt}}{\delta} + \left[ \left( \frac{\Delta \pi_{jt}}{\delta} - \frac{\pi_{jt}}{\delta} \right) \right]_{X_t = x_j} - I \left( \frac{x_t}{x_j} \right)^v & \text{if } X_t \leq x_j \\
\frac{\pi_{jt}}{\delta} + \frac{\Delta \pi_{jt}^+}{\delta} & \text{if } X_t > x_j
\end{cases}
$$

where $v > 1$ is defined in the Appendix.

Expression (4) suggests that $V_{jt}$ depends on firm $j$’s assets in place, its growth opportunities, and the effect of other firms’ investment strategies before or after firm $j$ exercises its own growth option. The fact that each firm’s investment strategy affects the value of its competitors is reflected in the terms $\Delta \pi_{jt}^-$ and $\Delta \pi_{jt}^+$. I characterize $\Delta \pi_{jt}^-$ and $\Delta \pi_{jt}^+$ explicitly in Section II.

C Firms’ expected returns

Firms are subject to a single source of systematic risk given by the demand shock in (1). Given that the riskless rate of return $r$ is exogenously specified, the expected return of firm $j$ at time $t$ equals

$$
R_{jt} = r + \psi \beta_{jt}
$$

(5)
where $\psi$ is the market price of risk, and $\beta_{jt}$ is the beta of firm $j$ which reflects the sensitivity of $V_{jt}$ to systematic risk. In line with other papers in the literature, the market price of risk $\psi$ is constant and exogenously given, and the dynamics of $R_{jt}$ are driven by $\beta_{jt}$.\(^{15}\)

The paper studies the impact of imperfect competition on firms’ expected returns by analyzing firms’ betas. To determine $\beta_{jt}$, I follow Carlson, Fisher and Giammarino (2004) and infer expected returns from replicating hedge portfolios composed of a risk free asset and a risky asset that exactly reproduce the dynamics of firm value.\(^{16}\) The proportion of the risky asset held in the replicating portfolio at any time $t$ equals $\beta \equiv \frac{\delta V}{\delta X} \times \frac{X}{V}$. The beta of firm $j$ is then given by

$$\beta_{jt} = 1 + I_t (v - 1) \left[ 1 - \frac{1}{\delta} \frac{\pi_{jt}}{V_{jt}} \right] \tag{6}$$

where $I_t$ is an indicator function which is equal to 0 if all firms have invested at time $t$ and is equal to 1 otherwise.

Equation (6) highlights the importance of imperfect competition on firms’ exposure to systematic risk. Firm’s relative market power affects profit margins and hence their betas. In less competitive industries, firms equate marginal income to marginal costs, firms’ instantaneous profits are strictly positive, and their betas are strictly lower than $v > 1$ before investment. In more competitive industries, instantaneous profits are closer to zero, and firms’ betas are closer to $v$ before investment. In the extreme case of perfectly competitive industries, instantaneous profits equal zero, there is no option value of waiting to invest and firms’ betas are always equal to 1.

Equation (6) is also consistent with the prediction by Carlson, Fisher and Giammarino (2004) for idle firms that the exposure to systematic risk of a firm depends on the relative contribution of its own growth opportunities to total firm value. In contrast with Carlson, Fisher and Giammarino (2004), however, $\beta_{jt}$ also depends on the growth opportunities of all other firms in the industry. This is because both the current assets in place of firm $j$ and its future value depend on the market share of firm $j$ over time. The expected returns of all firms in the industry are interrelated due to firms’ strategic interaction.

Finally, denote the Lerner index of firm $j$ by $l_{jt}$ and the market share of firm $j$ at time $t$ by $s_{jt}$. Reordering terms in (6), it is straightforward to show that $\beta_{jt}$ equals

$$\beta_{jt} = 1 + I_t (v - 1) \left[ 1 - l_{jt} \times s_{jt} \times \frac{1}{\delta} \frac{p_t Q_t}{V_{jt}} \right] \tag{7}$$

\(^{15}\)See, for instance, Berk, Green and Naik (1999) and Carlson, Fisher and Giammarino (2004).

\(^{16}\)See the proof of Proposition 2 in Carlson, Fisher and Giammarino (2004).
Equation (7) predicts that $\beta_{jt}$ depends mechanically on firm $j$'s Lerner index, its current market share, and its current size relative to that of the product market. This is consistent with the static industrial organization literature on firms’ betas and imperfect competition.\textsuperscript{17} In contrast with this literature, however, (7) shows that firms’ betas depend on both on firms’ current and future relative market power.\textsuperscript{18} Pyndick (1985) already highlights the importance of firms’ future investment opportunities when measuring their market power.

**Proposition 1** Under imperfect product market competition, the exposure to systematic risk of each firm $\beta_{jt}$ depends on the relative contribution of growth opportunities to firm value of all firms in the industry.

### D Industry expected returns

The identities for $\beta_{jt}$ yield expressions for the industrial expected return moment conditions. Denote the average industry beta by $\mu_{\beta,t}$. Then (6) and (7) imply that at any time $t$ the average industry beta depends mechanically on firms’ average profit margins, market shares and relative growth opportunities.

More interestingly, the model characterizes the industrial cross sectional variation of expected returns $\sigma_{\beta,t}$. Notice that in industries in which firms are identical there is no cross sectional variation in expected returns. In industries where firms differ in their production technologies, however, the model predicts that there is cross sectional variation in expected returns such that

$$\sigma_{\beta,t} = I_t (v - 1) \left( \frac{1}{\delta} \sigma_{\nu,t} \right)$$

Equation (8) implies that the cross sectional differences in firms’ production technologies affect mechanically the industrial cross section of returns. While the literature provides investment based explanations on the cross section of expected returns on aggregate,\textsuperscript{19} equation (8) relates firms’ investment policies to the industrial cross section of expected returns, and highlights that $\sigma_{\beta,t}$ is affected by firms’ strategic interaction.

Finally, note that the model provides an alternative expression for $\sigma_{\beta,t}$ such that

$$\sigma_{\beta,t} = I_t (v - 1) (pQ) \left[ \mu_{\beta,t}^2 - \mu_{\beta,t}^2 \mu_{\beta,t}^2 - \sigma_{\beta,t}^2 \left( \rho_{\beta,t}^2 \sigma_{\beta,t}^2 - 2 \mu_{\beta,t} \mu_{\beta,t} \frac{\rho_{\beta,t} \sigma_{\beta,t}}{\sigma_{\beta,t}} \right) \right]^{\frac{1}{2}}$$

\textsuperscript{17}See, for instance, Alexander and Thistle (1999).

\textsuperscript{18}This is because $V_{jt}$ also depends on firms’ future production technologies.

where $\rho_{ls,t}$ denotes the correlation between $\frac{lv_{jt}}{vj_{jt}}$ and $s_{jt}$. Equation (9) then shows that $\sigma_{s,t}$ depends on firms’ Lerner indexes, market shares, and relative growth opportunities. In particular, (9) provides the novel prediction that the industrial cross section of returns depends on industry concentration. Note that the term $\sigma^2_{s,t}$ in (9) relates by construction to the HHI, which measures industry concentration and is the sum of the squares of the market shares of all firms in the industry.

In sum, equations (6)-(9) show that the organization of a given industry affects its distribution of returns over time. This important result obtains under the sole assumptions of investment under uncertainty, imperfect competition and heterogeneity in firms’ production technologies.

II Equilibrium investment strategies

Section I provides identities for the industrial return moment conditions which stem directly from the assumptions of imperfect competition in capacity and the cross sectional differences in firms’ production technologies. These identities, however, hold for any set of investment strategies and hence do not provide any equilibrium implication about the exact relation between an industry’s organization, its investments and expected returns over time. This section characterizes firms’ investment strategies in equilibrium. I elaborate on the equilibrium asset pricing implications of these strategies in Section III.

Firms’ optimal investment strategies are such that firms maximize value by choosing the investment threshold $x_j$ and the increase in capacity $\Delta q_j$. At each point in time, the state of the industry is described by the history of the stochastic demand shock $X_k$. At any point in time $t$, a history is the collection of realizations of the stochastic process $X_s$, $s \leq t$ and the actions taken by all firms in the industry until time $t$. Hence the investment strategy $\Gamma_j$ maps the set of histories of the industry into the set of actions $\{x_j; \Delta q_j\}$ for firm $j$. Before investment, firm $j$ responds immediately to its competitors’ investment decisions. This yields Nash equilibria in state dependent strategies of the closed-loop type. Upon investment, firm $j$ cannot take any other action.

Firms follow Markov strategies such that their actions are functions of the current state $X_t$ only. As discussed in Weeds (2002), other non-Markov strategies may also exist; however, if one firm follows a Markov strategy, the best response of the other firm is also Markov.\(^{20}\)

\(^{20}\)See Fundenberg and Tirole (2001) and Weeds (2002) for a discussion on this point.
Furthermore, I consider the set of subgame perfect equilibria in which each firm’s investment strategy, conditional on its competitor’s strategy, is value maximizing. A set of strategies that satisfies this condition is Markov perfect. The initial demand shock $X_0$ is sufficiently low to focus on equilibria in pure strategies.\footnote{When firms are identical, the equilibrium may involve mixed strategies, whose formulation is complicated by the continuous time nature of the game, as observed in Fundenberg and Tirole (1985) and Weeds (2002). When firms have different production technologies, however, a sufficient assumption to avoid these concerns is that $X_0$ is sufficiently low such that $X_0 < x^*_L$. See Mason and Weeds (2010).}

In a nutshell, the section shows that firms’ relative differences in current and future production technologies predetermine the ordering and magnitude of their investment strategies in Markov perfect equilibria. The distribution of firms’ production technologies presets whether industries are less or more concentrated, and whether firms’ profit margins co-move positively or negatively over time. To ease on exposition, I focus on the main predictions of the model for the specific case of a duopoly in which firms which have the same installed capacities and different future production technologies. The qualitative results for this specific case apply to the more general case in which firms differ in their current and future technologies. I discuss the more general case in the Appendix.

A Markov perfect equilibria

Consider a firm $L$ and another firm $F$ which have the same installed capacity $q$ but differ in their future production technologies such that $c_L < c_F$. Subgame perfection requires that each firm’s strategy maximizes its value conditional on its competitor’s strategy. Anticipating, there are two possible Markov perfect industry equilibria with $N = 2$: a simultaneous equilibrium in which firms invest simultaneously and the industry is less concentrated, and a sequential equilibrium with leaders and followers and the industry is more concentrated. The type of equilibrium that emerges depends on how firm $L$’s value under sequential investment compares with the value of the simultaneous investment strategy.

Firms’ values are given by (4), and yet the exact formula for $\Delta \pi_{jt}$ depends on whether firms invest simultaneously or sequentially in equilibrium. Denote $\Gamma^c_j$ the investment strategy of firm $j$ in the simultaneous equilibrium. Under simultaneous investment, firms’ values are given by (4) with $\Delta \pi^c_{jt} \equiv 0$. Hence strategic interaction does not affect the dynamics of firms’ values in equilibrium. Denote $\Gamma^s_j$ the investment strategy of firm $j$ in the sequential equilibrium. When firms invest sequentially, the model predicts $\Delta \pi^s_{jt} \leq 0$ since both firms
expect a reduction in profits at different points in time.\textsuperscript{22} Hence strategic interaction has \textit{equilibrium effects} on firms’ values when firms invest sequentially.

The \textit{sorting conditions} of the game suggest that the ordering and magnitude of firms’ investment decisions in equilibrium are essentially determined by the value maximizing strategy for the more efficient firm $L$. I prove in the Appendix the sorting condition of the multiple action strategy $\{x_j; \Delta q_j\}$ is such that more efficient firms find it less costly to invest earlier and more, namely

$$\frac{\partial}{\partial c_j} \left[ \frac{\partial V_j}{\partial x_j} \right] > 0, \quad \frac{\partial}{\partial c_j} \left[ \frac{\partial V_j}{\partial q_j} \right] < 0$$  \hspace{1cm} (10)

The sorting conditions in (10) have important implications for firms’ strategic behavior. First, since firm $L$ has a comparative advantage to invest earlier and more, firm $F$ does not become a leader in equilibrium even if it has incentives to preempt firm $L$. This implies that firm $L$ is the only potential leader if firms invest sequentially in equilibrium. Second, if firm $L$ does not have an incentive to become a leader, neither does firm $F$, whose ability to invest earlier and more is comparatively lower. This implies that both firms invest simultaneously if firm $L$ has incentives to do so.\textsuperscript{23}

In sum, the model predicts that firm $L$ has the \textit{real option to become a leader}, and firm $L$ exercises this option only if the early monopoly rents acquired as a leader are relatively higher than the shadow cost of preventing firm $F$ to invest earlier. Firm $L$ becomes a leader when firm $F$ is not a close competitor, such that $\sigma_c \equiv c_F - c_L$ is larger than some lower bound $\sigma_\underline{\bar{c}}$. Hence the industry has leaders and followers and are more concentrated if $\sigma_c > \sigma_\underline{\bar{c}}$, whereas firms invest simultaneously and the industry is less concentrated if $\sigma_c \leq \sigma_\underline{\bar{c}}$. I elaborate on these predictions below.

\section*{A.1 Sequential equilibrium}

Consider first the \textit{sequential equilibrium} in which the more efficient firm $L$ invests earlier and more, and the less efficient firm $F$ invests later and less. This result resembles that of Stackelberg games.\textsuperscript{24} In contrast with these games, however, the sequential ordering of investment decisions is an equilibrium outcome.

\textsuperscript{22}I characterize $\Delta \pi^*_j$ below.

\textsuperscript{23}I elaborate on this in the Appendix.

\textsuperscript{24}Note that the referred Stackelberg games include the real option frameworks that take as given the ordering of firms’ investment decisions. These include Trigeorgis (1986) and Carlson et al (2009).
To obtain the ordering of firms’ investment decisions endogenously, consider the incentives to both firms to become leaders. Sequential equilibria occur when firm $L$ has incentives to lead such that $\sigma_c > \bar{\sigma}_c$.\textsuperscript{25} Meanwhile, firm $F$ has incentives to lead whenever the early monopoly rents obtained as a leader are higher than the implied cost of investing earlier with a less efficient technology. Denote $\Gamma^*_j$ the strategy of firm $j$ in a Stackelberg game in which firm $L$ invests first. Firm $F$ has incentives to preempt firm $L$ whenever

$$\tilde{V}_F \bigg|_{X_t = x^*_L} \geq V^*_F \bigg|_{X_t = x^*_L}$$

(11)

where $\tilde{V}_j$ indicates deviation in both timing and capacity for $j = L, F$.\textsuperscript{26} I assume that (11) holds throughout the paper.\textsuperscript{27}

Consider the equilibrium strategy $\Gamma^*_F$ chosen by firm $F$ when firm $L$ invests earlier. Using the approach in Dixit and Pyndick (1994), the optimal investment threshold $x^*_F$ equals

$$x^*_F = \frac{I \delta v}{1 - v} \left[ (Q^s)^{\frac{1}{2}} - c_F \right] q^*_F - \left[ (q + q^*_L)^{\frac{1}{2}} - c_i \right] q$$

(12)

where $Q^s \equiv q^*_L + q^*_F$. Meanwhile, firm $F$ maximizes its capacity $\Delta q^*_F$ such that

$$c_F = (Q^s)^{\frac{1}{2}} \left( 1 - \frac{1}{\varepsilon Q^s} q^*_F \right)$$

(13)

Conditions (12) and (13) reflect that firm $F$ invests later and less if firm $L$ has a larger capacity $q_L$.

Consider now the equilibrium strategy of the leading firm $\Gamma^*_L$. Given (11), firm $L$ maximizes its value subject to the additional complementary slackness condition

$$\lambda \left[ V^*_F - \tilde{V}^*_F \right] \bigg|_{X_t = x^*_L} = 0$$

(14)

where the Lagrange multiplier $\lambda > 0$ reflects the shadow cost of preemption for firm $L$.\textsuperscript{28}

The Lagrange multiplier $\lambda$ therefore relates to Posner (1975) and measures to what extent

\textsuperscript{25}I prove this formally later on.

\textsuperscript{26}Due to the sorting conditions, the strategy to deviate in timing only is dominated by the strategy to deviate in both timing and capacity.

\textsuperscript{27}Condition (11) provides an upper bound $\bar{\sigma}_c$ such that firm $F$ has no incentives to preempt firm $L$ if $\sigma_c > \bar{\sigma}_c$. Firms’ optimal equilibrium strategies when $\sigma_c > \bar{\sigma}_c$ correspond to those of a standard Stackelberg game in which firm $L$ invests first, namely $\Gamma^*_j \equiv \Gamma^*_j (\lambda = 0)$. The core implications of this paper yet relate to the equilibrium effects of firms’ strategic behavior when $\sigma_c \leq \bar{\sigma}_c$. I thus assume $\sigma_c > \bar{\sigma}_c$ for simplicity.

\textsuperscript{28}The solution approach relates to Maskin and Tirole (1988) and Bustamante (2011).
the contest for monopoly power hinders the early monopoly profits of firm $L$. The sorting conditions ensure that (14) is binding when (11) holds.\(^{29}\)

I show in the Appendix that the investment threshold $x^s_L$ that solves the constrained optimization problem of firm $L$ is then given by

$$x^s_L = \frac{I \delta v}{\left( q + q^s_L \right)^{-\frac{1}{2}} q^s_L - \varphi c_L} [q^s_L - (2q)^{-\frac{1}{2}} - \bar{c}] q$$

where $\varphi = \frac{c_L - \lambda c_F}{c_L (1 - \lambda)} < 1$. In line with Dixit and Pyndick (1994), the optimal investment threshold in (15) is the ratio between the fixed cost of investment $I$ and the present value of the expected net profits from increasing capacity, adjusted for the option value of waiting to invest $\frac{v}{1-v}$. Meanwhile, the optimal capacity $q^s_L$ for firm $L$ satisfies

$$(1 - \kappa^s) (1 - \frac{1}{\bar{v}}) x^s_L (q_L + q)^{-\frac{1}{2}} + \kappa^s x^s_L (Q^s)^{-\frac{1}{2}} \left[ 1 - \frac{1}{\bar{v}} \frac{q^s_L}{q_L} \left( 1 + \frac{\partial q^s_L}{\partial q_L} \right) \right] = \varphi c_L x^s_L$$

where all prices are evaluated at $x^s_L$ such that $\kappa^s = \left( \frac{x^s_L}{x^*} \right)^{v-1}$. The right hand side of (16) reflects the marginal benefits of increasing capacity. Firm $L$ may increase its monopoly rents, and may deter firm $F$’s investment by either reducing its future market share or inducing $F$ to invest later. The left hand side of (16) reflects firm $L$’s marginal costs of increasing capacity.

Notably, both (15) and (16) reflect that the net gains from investment are subject to the shadow cost of preemption $\lambda > 0$. The shadow cost of preemption $\lambda > 0$ makes firm $L$ behave as if it had a lower production cost since $\varphi c_L < c_L$. The denominator in (15) is increasing in $\lambda$, since firm $L$ behaves more aggressively to secure its position as a leader. Equation (16) also reflects how firm $L$’s optimal capacity choice is affected by its strategic concerns. Firm $L$ determines its optimal capacity as if it were more efficient since $\varphi c_L < c_L$.

Finally, recall the expected reductions in profits $\Delta \pi_{jt}$ defined in Section I. These expected reductions in profits capture the impact of strategic interaction on firms’ values in equilibrium. When firms invest sequentially, firms’ expected reductions in profits are such that $\Delta \pi_{jt}^s \leq 0$. The equilibrium ordering of capacity additions implies $\Delta \pi_{F,t}^s = 0$ and $\Delta \pi_{L,t}^s = 0$.

\(^{29}\)See Appendix for the derivation of the sorting conditions.
Firm $F$ experiences a reduction in market prices once firm $L$ invests adds capacity. The expected reduction in instantaneous profits $\Delta \pi_{F,t}^-$ is then

$$\Delta \pi_{F,t}^- = \left[ (q_L^* + q)^{-\frac{1}{z}} - (2q)^{-\frac{1}{z}} \right] x_L^* g \left( \frac{X_t}{x_L^*} \right)^v < 0 \quad (17)$$

for $X_t \leq x_L^*$. Firm $L$ also expects a reduction in market prices once firm $F$ adds capacity. The corresponding reduction in instantaneous profits $\Delta \pi_{L,t}^+$ yields

$$\Delta \pi_{L,t}^+ = \left[ Q^{-\frac{1}{z}} - (q_L^* + q)^{-\frac{1}{z}} \right] x_F^* g \left( \frac{X_t}{x_F^*} \right)^v < 0 \quad (18)$$

for $x_L^* < X_t \leq x_F^*$. Table I illustrates numerically firms’ expected reductions in profits $\Delta \pi_{jt}^*$ at $X_t = X_0$.

### A.2 Simultaneous equilibrium

The *simultaneous equilibrium* of an industry with $N = 2$ is such that both firms invest at the investment threshold of firm $L$, and firms’ capacity increases are analogous to those of Cournot duopolies. In contrast with Cournot games, however, the simultaneous equilibrium obtains in a real options set-up in which both firms attain a higher value by coordinating their investments.\(^{30}\)

To see this, consider the stage in which neither firm has invested, and assume further that the leader optimizes value by investing simultaneously since $\sigma_c < \sigma_c^*$. The sorting conditions in (10) imply that if firm $L$ attains a higher value under simultaneous investment, so does firm $F$, whose ability to lead is relatively less profitable. I show in the Appendix that for any set of sequential and simultaneous strategies, if firm $L$ prefers simultaneous investment, firm $F$ also does. Hence both firms find it more profitable to invest simultaneously when $\sigma_c < \sigma_c^*$.\(^{31}\)

Consider then the equilibrium investment strategies $\Gamma_j^c$. The first order condition on $V_j^c$ for the optimal capacity $q_j^c$ of firm $j$ when firms invest simultaneously yields

$$c_j = (Q^c)^{-\frac{1}{z}} \left( 1 - \frac{1}{\varepsilon} \frac{q_j^c}{Q^c} \right) \quad (19)$$

\(^{30}\)This is in line with Weeds (2002) and Mason and Weeds (2010).

\(^{31}\)Both Fundenberg and Tirole (1985) and Weeds (2002) argue that if one equilibrium Pareto-dominates all others, it is the most reasonable outcome to expect. This is the case when $\sigma_c < \sigma_c^*$. 

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which is independent of the demand threshold \( x^c \). The system of equations implied by (19) for both firms determines the equilibrium quantities \( q^c_L \) and \( q^c_F \) irrespective of the choice of \( x^c \).\(^{32}\)

Given the asymmetry in firms' production technologies, each firm strictly maximizes its value at different investment thresholds. In principle, this would yield a range of potential equilibrium thresholds \( x^c \). The lowest demand threshold \( x^c \) would correspond to that of the more efficient firm \( L \); conversely, the upper bound \( \bar{x}^c \) would correspond to the optimal demand threshold for the less efficient firm \( F \). However, firm \( L \) has no incentives to wait further than its own optimal threshold \( x^c \). Meanwhile, firm \( F \) still has incentives to invest at \( \bar{x}^c \) not to become a follower. The equilibrium investment threshold \( x^c \) for all firms in the industry is then

\[
x^c = \frac{I^{\delta v}}{1-v} \left[ (Q^c)^{-\frac{1}{\theta}} - c_L \right] q^c_L - \left[ (2q)^{-\frac{1}{\theta}} - c \right] q
\]

which is the optimal demand threshold for firm \( L \) given the capacity choices in (19).

Table I illustrates the equilibrium investment strategies \( \Gamma_j^c \) numerically. In the simultaneous equilibrium, firm \( L \) invests more and attains a higher value than firm \( F \). This is in line with Cournot games and reflects the asymmetry in production technologies between both firms. Table I shows that when firms invest simultaneously industries are less concentrated than when firms invest sequentially. Compared to sequential equilibria, firm \( L \) invests later and less (i.e. \( x^c_L < x^c \) and \( q^c_L < q^c_s \)), while firm \( F \) invests earlier and more (i.e. \( x^c_L < x^c_L \) and \( q^c_L < q^c_s \)). Finally, note that in Table I the expected reduction in profits \( \Delta \pi^c_j \) equals zero for all firms at any point in time. This is because firms invest simultaneously in equilibrium.

**B Equilibrium outcome**

To fully characterize the equilibrium outcome, consider the incentives of firm \( L \) to invest sequentially or simultaneously. Firm \( L \) may become a market leader, enjoy early monopoly rents and yet pay the shadow cost of preemption. Alternatively, firm \( L \) may allow the follower to invest simultaneously, attain lower duopoly rents from the start and yet avoid any cost of preemption. Firm \( L \)'s real option to become a leader is therefore given by

\[
V^\text{max} \big|_{x_1=x_L^1} = \max \left\{ V^s_L \big|_{x_1=x_L^1} ; V^c_L \big|_{x_1=x_L^1} \right\}
\]

\(^{32}\)This is because firms' instantaneous profits are linear in \( q_j \). The equilibrium capacity choices implied by (19) are hence comparable to those which obtain in Cournot duopolies.
where (21) is evaluated at $x_I^s$ since the sequential equilibrium requires earlier exercise (i.e. $x_I^s < x^c$).\textsuperscript{33} The lower bound $\underline{c}$ at which firm $L$ is indifferent between leading and clustering obtains when $V_L^s = V_L^c$ at $X_t = x_L^c$.

In sum, there are two subgame perfect industry equilibria depending on the dispersion in firms’ production technologies $\sigma_c$. When $\sigma_c < \underline{c}$, firms are close competitors, neither firm has incentives to lead, and a simultaneous equilibrium obtains in which firms invest simultaneously and the industry is less concentrated. When $\sigma_c > \underline{c}$ and firms are more distant competitors, firm $L$ has incentives to lead, and a sequential equilibrium emerges in which firm $L$ becomes the leader and the industry is less concentrated. Firm $F^0$’s incentives to lead determine the magnitude of the shadow cost of preemption $\lambda$ on firm $L$.

Table I and Figure 1 illustrate these predictions numerically. Table I compares the sequential and simultaneous equilibria for the same $\sigma_c$. In the example, firm $L$ attains a higher value in the simultaneous investment since $\sigma_c < \underline{c}$, and firms invest simultaneously in equilibrium. Figure 1 compares firm values in equilibrium as a function of $\sigma_c$. While firm $F$ is always better off in the simultaneous equilibrium, firm $L$ is better off under simultaneous investment when $\sigma_c$ is low. Concentration upon investment is higher in sequential equilibria. The shadow cost of preemption $\lambda$ decreases with $\sigma_c$.

More importantly, firms’ investment strategies in equilibrium predict how the underlying distribution of firms’ production technologies $\sigma_c$ affects the dynamics of firms’ profits over time. When $\sigma_c < \underline{c}$, both firms add capacity simultaneously and strategic interaction has no equilibrium effects since $\Delta \pi_{jt}^c = 0$. When $\sigma_c > \underline{c}$, firms invest sequentially and strategic interaction affects firms’ profits in equilibrium since $\Delta \pi_{jt}^s \leq 0$. Each firm expects a reduction in its profits precisely when its competitor invests and increases its own. These dynamics have important asset pricing implications which I discuss in Section III.

**Proposition 2** The subgame perfect industry equilibrium of the model is such that firms follow strategies $\Gamma_j^c$ if $\sigma_c < \underline{c}$ and $\Gamma_j^s$ otherwise, and hence

- When $\sigma_c < \underline{c}$, firms invest simultaneously, the industry is less concentrated, and $\Delta \pi_{jt}^c = 0$;

\textsuperscript{33} Note that firm $L$ always chooses between the sequential equilibrium that maximizes its value and the simultaneous equilibrium in which all firms invest at the optimal threshold for firm $L$. All other equilibria are dominated for firm $L$.  

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• When \( \sigma_c \geq \sigma_{e^*}, \) industries have leaders and followers, are highly concentrated, and \( \Delta \pi_{jt}^* \leq 0. \)

I show in the Appendix that the qualitative predictions of Proposition 2 also apply to the more general case in which firm type is given by both \( q_j \) and \( c_j \). In imperfectly competitive industries with heterogeneous firms, firms with lower installed capacity \( q_j \) and lower future costs of production \( c_j \) have a comparative advantage to invest earlier and more than other firms.\(^{34}\)

### III  Equilibrium expected returns

This section shows how the equilibrium investment strategies described so far serve as an input to obtain testable predictions on how firms’ strategic interaction in imperfectly competitive industries affect their exposure to systematic risk. The asset pricing implications rationalize the recent empirical evidence by Hoberg and Phillips (2010) and Hou and Robinson (2006).

As a remark, note that Section II characterizes firms’ investment dynamics in imperfectly competitive industries for the special case of \( N = 2 \). A natural question to ask is whether these results also hold in a more general framework with \( N \) firms. Concentration measures such as the HHI depend both on the cross sectional differences in market shares and the number of firms in the industry.

The number of firms \( N \) influences the equilibrium outcome. When \( N > 2 \), some firms within the same industry may invest sequentially, while some others may cluster instead. However, Table II illustrates that the relevant takeaways of Section II still hold for the case of \( N > 2 \) firms.\(^{35}\) Firms invest simultaneously if they are close competitors, and industries have leading firms which invest earlier otherwise. Whenever firms invest sequentially, concentration is higher, and all firms expect a reduction in their profits due to increases in capacity by their competitors.

To ease on exposition, I focus on the asset pricing implications for the case of \( N = 2 \) to elaborate on the relation between the industrial cross section of returns and concentration. The case of \( N = 2 \) associates more (less) concentrated industries with industries

\(^{34}\)See Appendix for discussion.  
\(^{35}\)See Appendix for discussion.
with high (low) heterogeneity in production technologies in which firms invest sequentially (simultaneously). I provide the comparative statics for $N > 2$ in Section IV.

A Expected returns’ dynamics and co-movement

In the same way in which the dynamics of investment are different in less and more concentrated industries, the dynamics of firms’ expected returns also differ. In more homogeneous industries in which firms invest simultaneously, firms’ betas co-move positively. Conversely, in more heterogeneous industries, the betas of leaders and followers co-move negatively.\textsuperscript{36} Hoberg and Phillips (2010) provide the corresponding empirical evidence on this result, as they find that firms’ returns co-move more positively in industries with lower HHI.

To understand the mechanism behind this prediction, consider first the dynamics of firms’ betas $\beta_{jt}$ when $\sigma_c < \overline{\sigma_c}$. In less concentrated industries in which firms invest simultaneously, the value of each firm is larger than the value of its assets in place before investment, and equal to the value of its assets in place thereafter. This implies that $\beta_{jt}^c$ is higher than one before investment and equal to one thereafter. Figure 2 illustrates this result for the case of $\sigma_c = \overline{\sigma_c}$.

More importantly, the dynamics of $\beta_{jt}$ in less concentrated industries are in line with those that obtain in a more standard real options set ups in which firms do not invest strategically, since $\Delta \pi_{jt}^c \equiv 0$.\textsuperscript{37} In both cases, firms’ betas reflect whether their own investment option is in the money. Strategic interaction has no equilibrium effects on the industry’s dynamics when firms are close competitors, and hence $\beta_{jt}^c$ behaves as if firm $j$ were an idle firm.

Consider now the dynamics of $\beta_{jt}$ when $\sigma_c > \overline{\sigma_c}$. In more concentrated industries with leaders and followers, the equilibrium effect of strategic interaction on firms’ profits is such that $\Delta \pi_{jt}^s \leq 0$ at different points in time. Whenever one firm expects to improve its profit margin and market share upon investment, the other firm expects a reduction in its own due to the capacity addition by its competitor. Hence firms’ betas co-move negatively in more concentrated industries.

The predicted dynamics of $\beta_{Lt}^s$ in more concentrated industries are consistent with those

\textsuperscript{36}As a remark, note that in the general case of more concentrated industries with $N$ firms, the betas of leaders and followers comove negatively over time. However, the betas of all firms in general only comove less positively (instead of negatively) since the betas of various followers may comove positively once a leading firm invests.

of the leading firm in Carlson et al (2009) and Bena and Garlappi (2011). Before investment, \( \beta_{Lt}^s \) is higher than one and higher than that \( \beta_{Ft}^s \) since the growth option of firm \( L \) is more valuable. Upon investment, however, the leader becomes a mature firm which expects a reduction \( \Delta \pi_{Lt}^s < 0 \) in its future profits. This pushes \( \beta_{Lt}^s \) below one and below \( \beta_{Ft}^s \) until all firms exercise their growth option.

This paper adds to the literature as it observes that the beta of any operating firm in more concentrated industries is dampened by the expectation of future additions to industry capacity. In duopolies, strategic interaction affects both the dynamics of \( \beta_{Lt}^s \) and \( \beta_{Ft}^s \) in equilibrium. Since both firms are already operating in the industry before firm \( L \) invests, firm \( F \) also expects a reduction in profits \( \Delta \pi_{Ft}^s < 0 \) up until firm \( L \) invests. Figure 2 shows how this expected reduction in the future profits by firm \( F \) affects the dynamics of its expected returns. In particular, \( \beta_{Ft}^s \) is lower than one until firm \( L \) invests and higher or equal to one thereafter.

**Proposition 3** The equilibrium dynamics of firms’ betas under imperfect competition depend on \( \sigma_c \) such that

- When \( \sigma_c < \sigma_c \), industries are less concentrated and firms’ betas co-move positively;

- When \( \sigma_c \geq \sigma_c \), industries are more concentrated and the betas of leaders and followers co-move negatively.

**B Average industry expected returns**

The model characterizes the relation between average industry expected returns \( \mu_{\beta,t} \) and industry concentration. Consider first the case of less concentrated industries. When \( \sigma_c < \sigma_c \), firms are close competitors and strategic interaction has no equilibrium effects on the dynamics of firms’ betas. In line with other real options models with idle firms, firms’ betas are all higher than one before investment and lower than one thereafter. Hence the average industry expected return is such that \( \mu_{\beta,t}^c > 1 \) for \( x_e \leq X_t \) and \( \mu_{\beta,t}^c = 1 \) thereafter.

In contrast, in more concentrated industries strategic interaction does affect the equilibrium dynamics of firms’ betas, and firms’ betas co-move negatively over time. The average industry beta \( \mu_{\beta,t}^s \) is pushed down by the expectation of a future reduction in profits by either firm \( F \) (up to \( X_t \leq x_F^e \)) or firm \( L \) (up to \( X_t \leq x_F^e \)). The degree to which these expected reductions in profits push the equally weighted average industry return \( \mu_{\beta,t}^s \) below
one depends on parameter values. However, the value weighted average industry return is such that \( \mu_{\beta,t}^* < 1 \) until all firms invest, and \( \mu_{\beta,t}^* = 1 \) thereafter.\(^{38}\)

The result that the value weighted average expected returns of more concentrated industries \( \mu_{\beta,t}^* \geq 1 \) are lower than those of less concentrated industries \( \mu_{\beta,t}^* \leq 1 \) provides a rationale to the finding by Hou and Robinson (2006) that less concentrated industries earn higher returns on average. Figure 3 illustrates this result numerically. The premium \( \mu_{\beta,t}^* - \mu_{\beta,t}^* \) is strictly positive up \( X_t = x^c \) using any type of weights, and is strictly positive until all firms invest in both industries using value weights. The mechanism relies on the equilibrium effects of strategic interaction.

In more concentrated industries, firms’ expected reduction in profits due to future additions to industry capacity pushes the average industry return downwards. Meanwhile, in less concentrated industries, there is no such expectation. Figure 3 illustrates the dynamics of \( \mu_{\beta,t}^* \) for \( N = 2 \) and \( \sigma_c = \sigma_{cc} \). Whenever the growth opportunity of one firm becomes more in the money, the beta of this firm increases, whereas the beta of its competitor decreases due to the corresponding expected reduction in profits upon investment. This second effect has always a relatively larger impact on the value weighted \( \mu_{\beta,t}^* \).\(^{39}\)

The model therefore provides an alternative rationale to the finding by Hou and Robinson (2006) relative to that proposed by Aguerrevere (2009). Aguerrevere (2009) derives the asset pricing implications of an investment model of oligopoly in which all firms are identical, and invest continuously and simultaneously at any point in time. Aguerrevere (2009) provides the conditional prediction that less concentrated industries earn higher returns if market demand is sufficiently low.\(^{40}\) In contrast, this paper explores industries with a cross section of firms which decide when and how much to invest, and highlights that in more concentrated industries the industrial average beta is lower due to firms’ strategic interaction.

**Proposition 4** In more concentrated industries, firms’ betas are dampened by the expectation of reduced profits due to future additions to industry capacity. Hence more concentrated industries may earn lower returns on average than less concentrated industries.

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\(^{38}\)I prove that \( \mu_{\beta,t}^* < 1 \) until \( X_t = x_F^* \) using value weights in the Appendix.

\(^{39}\)The expected reduction in prices by firm \( L \) when firm \( F \) invests pushes \( \mu_{\beta,t}^* \) downwards since firm \( L \) has much more installed capacity than firm \( F \). Similarly, the expected reduction in prices by firm \( F \) when firm \( L \) invests pushes \( \mu_{\beta,t}^* \) downwards since firm \( L \) expects to operate with low prices and low installed capacity for a long period of time.

\(^{40}\)Note that this finding also implies that more competitive industries earn higher returns during recessions. Hoberg and Phillips (2010) interpret the result in Aguerrevere (2009) as a time series implication.
C Industry booms and busts

The dynamics of firms’ expected returns are such that firms undergo periods of high expected returns before exercising their own investment opportunity, and periods of low expected returns after exercising their option. This prediction is consistent with Carlson, Fisher and Giammarino (2004). This paper adds to Carlson, Fisher and Giammarino (2004) as it shows that these patterns are more easily observable on average at the industry level in less concentrated industries.

Consider first the case of less concentrated industries in which $\sigma_c < \sigma_c$. When firms coordinate their investments in equilibrium, $\mu_{\beta,t}^c$ follows the pattern of high expected returns before all firms invest and low expected returns thereafter. The threshold $x^c$ constitutes a common reference point for all firms in the industry at which they trigger their investments. As a consequence, the average returns of the industry decrease simultaneously upon investment. This is illustrated in Figure 4.

Denote by $\Delta \mu_{\beta,t} = \frac{1}{N} \sum \beta_{jt}^+ - \beta_{jt}^-$ the equally weighted average change in industry betas at time $t$. When firms invest simultaneously, the model predicts a decrease in the returns of each firm $j$ when the growth options of all firms in the industry become assets in place. This implies

$$\Delta \mu_{\beta,t} \big|_{X_t=x^c} = 1 - \frac{1}{N} \sum \beta_{jt}^c < 0$$

(22)

since $\beta_{jt}^c < 1$.

Consider now the more concentrated industries in which $\sigma_c \geq \sigma_c$. When firms invest sequentially, firms’ betas are negatively correlated, and booms and busts are firm-specific. The industry has multiple investment thresholds at which firms add capacity, and all of these thresholds matter in explaining the dynamics of $\mu_{\beta,t}$. At $X_t = x^L$, firm $L$ invests, its growth option becomes assets in place, and hence $\beta^L_t$ decreases. Meanwhile, the growth option if firm $F$ becomes more valuable once it no longer expects a reduction in profits, such that $\beta^F_t$ increases. Conversely, at $X_t = x^F$, firm $F$ invests and $\beta^F_t$ decreases, whereas $\beta^L_t$ increases up to 1 as firm $L$ no longer expects a reduction in profits.

The actual sign of the discrete changes in $\mu_{\beta,t}$ upon investment is illustrated in Figure 4. Using equally weighted average industry betas, the sign of $\Delta \mu_{\beta,t}^c$ at $x^L$ is positive and the sign of $\Delta \mu_{\beta,t}^s$ at $x^F$ is negative. Hence the econometrician would only identify $\Delta \mu_{\beta,t}^s$ at $x^F$ as a bust. In more concentrated industries, $x^L$ is the only investment threshold at
which boom or period of high average valuation is followed a bust or decrease in $\mu_{\beta,t}^s$ once firm $F$ invests.\footnote{Conversely, using value weighted averages, the sign of $\Delta \mu_{\beta,t}^s$ at $x_L^s$ is negative, whereas the sign of $\Delta \mu_{\beta,t}^c$ at $x_F^c$ is positive. Hence industry effects are driven by the changes in the beta of the more valuable firm $L$.} Figure 4 further illustrates that the magnitude of the bust in $\Delta \mu_{\beta,t}^s$ at $x_F^s$ is strictly smaller than that of $\Delta \mu_{\beta,t}^c$ at $x_c^c$. This is because firms’ betas co-move negatively in less concentrated industries.

The empirical evidence in Hoberg and Phillips (2010) is highly consistent with the predictions of the model on the dynamics of expected returns. The model predicts that the discrete change in the average industry returns is easily observable to the econometrician since firms’ expected returns co-move positively in equilibrium. Hoberg and Phillips (2010) find that less concentrated industries have more predictable average industry returns, with periods of high valuation, high expected returns and high investment or booms are followed by periods of lower valuation and lower expected returns or busts.\footnote{Notably, Hoberg and Phillips (2010) state that their results are consistent with multiple firms in the same industry making investment decisions based on a "common industry signal". In the context of the model, such "signal" is the common investment threshold $x^c$.} They also find no significant booms and busts in the average industry returns of highly concentrated industries.

**Proposition 5** The stylized real options prediction that a boom in a given firm’s beta before investment is followed by a bust upon investment only holds for average industry returns in less concentrated industries.

\textbf{D Industrial cross sectional variance in expected returns}

While both Hou and Robinson (2006) and Hoberg and Phillips (2010) focus on the relation between average industry returns and industry concentration, this model predicts that the relation between industry returns and concentration also extends to second order moments.

The first prediction on second order moments is that the current cross sectional variance of expected returns of an industry has predictability over its future investment dynamics. Note that before all firms invest, $\beta_{j,0}$ reflects the ability of firm $j$ to invest earlier than its competitors; firms with a higher $\beta_{j,0}$ can invest earlier and more. When $\sigma_{\beta,0}^c$ is high, firms invest sequentially, concentration is high. Conversely, when $\sigma_{\beta,0}^c$ is low, firms invest simultaneously, concentration is low. Hence industries with high cross sectional variance in expected returns become highly concentrated. Figure 5 illustrates this result numerically.\footnote{Alexander and Thistle (1999) provide supporting evidence on this result.}
The model also provides the analog implications of the findings by Hoberg and Phillips (2010) and Hou and Robinson (2006) for second order moments. Consider first the implications on the predictability of $\sigma_{\beta,t}$ over time. In contrast with the findings by Hoberg and Phillips (2010) on first order moments, the model predicts that the booms and busts in $\sigma_{\beta,t}$ are observable for the econometrician irrespective of the industry concentration. In all industries, the cross sectional variance in betas $\sigma_{\beta,t}$ increases when firms’ growth options become more in the money, and decreases when firms exercise their options. This is illustrated in Figure 3.

Consider now the comparison between $\sigma_{\beta,t}^s$ and $\sigma_{\beta,t}^c$ at any point in time. Denote $\sigma_{\beta,t}^c - \sigma_{\beta,t}^s$ the difference in the cross sectional dispersion in returns between less and more concentrated industries. In contrast with the findings by Hou and Robinson (2006) for average industry returns, Figure 3 shows that $\sigma_{\beta,t}^c - \sigma_{\beta,t}^s$ is strictly negative at any point in time. This is because more concentrated industries have firms with more heterogenous production technologies.

Proposition 6 $\sigma_{\beta,t}$ is mechanically related to industry concentration such that:

- Industries become less (more) concentrated when $\sigma_{\beta,0}$ is low (high);
- Booms and busts in $\sigma_{\beta,t}$ are easily detectable irrespective of industry concentration;
- More concentrated industries have higher $\sigma_{\beta,t}$ than less concentrated industries.

IV Comparative statics

For the sake of tractability, Sections I-III keep all exogenous parameters related to the organization of the industry constant, with the exception of $\sigma_c$. This section complements the analysis as it shows how demand growth $\mu$, demand volatility $\sigma$, demand elasticity $\varepsilon$, and the number of firms $N$ affect industry dynamics. The comparative statics of the model are consistent with several empirical and theoretical findings in the literature. The main results on industry dynamics in Sections I-III also hold when less and more concentrated industries differ in more dimensions than $\sigma_c$.

A Demand growth, volatility and elasticity

An important feature of the model is that demand shocks follow a diffusion process with drift $\eta$ and volatility $\sigma$. This already suggests that both demand growth and demand
volatility affect industry dynamics. The model shows that firm $L$ is more likely to choose simultaneous investment if either demand growth $\eta$ or demand volatility $\sigma$ are sufficiently high. This implies that $\sigma_c$ is increasing in $\eta$ and $\sigma$ and is illustrated in Figure 6.

The result that firms invest simultaneously in industries with high $\eta$ and high $\sigma$ is consistent with the industrial organization literature on tacit collusion. Ivaldi et al (2003) suggest that for a fixed number of firms tacit collusion is easier to sustain in growing markets, in which current profits are low relative to future profits. Boyer et al (2001) suggest that demand uncertainty induces coordination as it boosts the growth option value for both firms.

The corresponding asset pricing implication is that industries with high demand growth and high demand volatility are more likely to be less concentrated (Figure 6), have more pronounced booms and busts (Figure 7), and have low cross sectional variance in returns (Figure 8). Furthermore, Hoberg and Phillips (2010) provide complementary evidence that the industrial booms and busts are more pronounced in less concentrated industries with high demand growth and high demand uncertainty. The lower panel in Figure 7 illustrates the same result in the context of the model. Given that both $\eta$ and $\sigma$ boost the expected returns of all firms before investment, the decrease in average industry betas $|\mu_{ij,t}|$ upon investment is increasing in $\eta$ and $\sigma$.

Demand elasticity also affects industry dynamics. When demand elasticity is relatively low such that $\varepsilon < \bar{\varepsilon}$, the model predicts that firm $L$ faces a higher shadow cost of preemption and allows simultaneous investment. This is illustrated in Figure 6 and implies that $\sigma_c$ is decreasing in $\varepsilon$. The corresponding testable implication is that those industries with low demand elasticity are less concentrated (Figure 6), have more pronounced booms and busts (Figure 7), and have low cross sectional variance in returns (Figure 8). Furthermore, Figure 7 illustrates how booms and busts are particularly more pronounced in industries with lower demand elasticity. Since a decrease in demand elasticity increases the level of market prices, it boosts firms’ expected returns before investment.

**Proposition 7** Industries with high demand growth $\eta$, high demand volatility $\sigma$ and low demand elasticity $\varepsilon$ are more likely to be less concentrated, have more pronounced booms and busts in industry returns, and have lower cross sectional variance in returns.

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44 The works by Ivaldi et al (2003) and Motta (2004) on tacit collusion provide a similar prediction.
B Competition and preemption

When applied to the more general case of an industry with $N$ firms, the proposed framework in this paper characterizes the joint effects of $\sigma_c$ and $N$ on investment timing and capacity choice. Tables I-II show that an increase in the number of firms $N$ erodes the growth option value of all firms in the industry. This is in line with Grenadier (2002), and the empirical evidence in Bulan, Mayer and Somerville (2009). The paper is also in line with Aguerrevere (2009) who observes that a higher number of firms erodes firms’ betas in imperfectly competitive industries.

In contrast with the symmetric oligopolies in Grenadier (2002) and Aguerrevere (2009), however, competition does not erode the values and the betas of all firms evenly. When firms have different production technologies, an increase in $N$ affects more severely the betas of those firms with less efficient growth options for any strategy $\Gamma_j$. In Tables I-II, firms’ values and firms’ betas decrease more significantly for firm $F$. This is consistent with Bulan (2005), who observes empirically that larger firms with more market power better preserve their growth option value.

The model also discriminates between effects of preemption and competition on firms’ option exercise strategies. The model shows that preemption and competition erode the value and betas of all firms. This is illustrated in Tables I-II. The model also shows that preemption and competition may have opposite effects on firms’ optimal investment timing. In both Grenadier (2002) and the simultaneous equilibrium in this paper, an increase in $N$ gives all firms an incentive to accelerate investment. Meanwhile, in sequential equilibria, an increase in the shadow cost of preemption $\lambda > 0$ gives followers (leaders) an incentive to delay (accelerate) investment instead.

The model also predicts that a higher number of competitors induces all firms in the same industry to accelerate investment if $\sigma_c$ is sufficiently low, but not necessarily otherwise. This is illustrated in Tables I-II. When firms invest sequentially, an increase in $N$ may also magnify the effects of preemption- leaders may behave more aggressively with a larger number of followers, such that it takes longer until all firms exercise their growth options.\footnote{The shadow cost of preemption for firm $L$ is higher in Table II than in Table I.}

Proposition 8 In asymmetric oligopolies, preemption and competition erode the returns of all firms, affect more severely the returns of those firms with less valuable growth opportunities, and may have opposite effects on firms’ incentives to accelerate investment.
C Robustness checks

The comparative statics described so far characterize less concentrated industries as industries with more homogeneous technologies (low $\sigma_c$), high demand growth (high $\eta$), high demand uncertainty (high $\sigma$), low market power (low $\varepsilon$), and a large number of firms (high $N$). Conversely, the model relates more concentrated industries to industries with highly heterogeneous technologies (high $\sigma_c$), low demand growth (low $\eta$), low demand uncertainty (low $\sigma$), high market power (high $\varepsilon$), and a low number of firms (low $N$). These results are all consistent with the industrial organization literature on imperfect competition.

A reasonable concern is yet whether the mechanical relation between the industry return moment conditions and industry concentration derived in Section VI still remains when less and more concentrated industries differ not only in $\sigma$ but also in all of these dimensions. Consider first the comparative statics with respect to $\eta$, $\sigma$ and $\varepsilon$. Since firms’ returns co-move more positively in less concentrated industries (Figure 7), irrespective of parameter choice booms and busts in $\mu_{\beta,t}$ are more pronounced in less concentrated industries and are always detectable in $\sigma_{\beta,t}$ (Figure 8).

Consider now the comparison between less concentrated industries with high $N$, and more concentrated industries with low $N$. If the only difference between these industries were given by the number of firms $N$, the model would not rationalize the evidence in Hou and Robinson (2009) and Hoberg and Phillips (2010). As shown in Aguerrevere (2009) and Tables II-III, all else equal an industry with higher $N$ has lower returns on average. Furthermore, for a given $\sigma_c$ in both industries, the intra-industry co-movement in firms’ betas is the same, and booms and busts in industry returns are equally detectable.

As a consequence, the robustness check when industries differ on $N$ compares an industry with low $N$ and high $\sigma_c$ with an industry with high $N$ and low $\sigma_c$. Figure 9 illustrates the premium $\mu^c_{\beta,t} - \mu^s_{\beta,t}$ between a less concentrated industry with high $N$ and $\sigma_c \approx 0^+$ and a more concentrated industry with $N = 2$ and $\sigma_c > \sigma_c$. As $N$ increases in the less concentrated industry, the equally weighted premium decreases but is strictly positive until $X_t = x^c$. The value weighted premium is also strictly positive until all firms in both industries exercise their growth options. The predictions on co-movement remain the same.

**Proposition 9** Less concentrated industries with low $\sigma_c$, high $\eta$, high $\sigma$, and high $N$ have lower cross sectional variance in returns, more positive co-movement in firms’ returns, more

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pronounced booms and busts, and higher returns on average than more concentrated industries with high $\sigma_c$, low $\eta$, low $\sigma$, and low $N$.

V Conclusion

This paper proposes an alternative real options approach that fully endogeneizes the ordering of investment decisions in oligopolies, and characterizes the equilibrium dynamics of firms’ expected returns under imperfect competition. The model considers asymmetries in firms’ production technologies and multiple firms. Firms optimize both on investment timing and capacity. The solution approach quantifies the costs of preemption, and predicts mutually exclusive equilibria depending on the organization of the industry. The model proves that the mechanical relation between the firms’ expected returns and industry concentration depends on the underlying distribution of firms’ production technologies.

The model demonstrates that an industry’s mean and the cross sectional variance of expected returns depend mechanically on firms’ Lerner indexes, firms’ current market shares, and firms’ relative growth opportunities. When firms have similar production technologies, the cross sectional variation in expected returns is low, firms invest simultaneously, firms’ returns co-move positively, and the industry concentration upon investment is low. When firms have more heterogeneous production technologies, the cross sectional variation in expected returns is high, firms invest sequentially, firms’ returns co-move less positively, and the industry concentration upon investment is high. The predicted relation between expected returns, investment and industry concentration rationalizes several empirical facts, including: (i) that firms’ returns co-move more positively in less concentrated industries; (ii) that booms and busts in industry returns are more pronounced in less concentrated industries; and (iii) that less concentrated industries earn higher returns on average.

The more general conclusion to extract from this paper is yet that an industrial organization model with asset pricing implications can rationalize several empirical findings in the finance literature, whose approach rarely elaborates on the impact of industrial organization on firms’ expected returns. Alternatively, the asset pricing implications of the investment model in this paper provide an alternative mechanism to test the predictions of dynamic games of strategic interaction, which are regularly discussed as economic theories and yet are hardly testable as such. The framework can be extended in many ways.
References


[34] Trigeorgis, L., 1996. Real Options. *MIT Press* (Boston, MA.)

Table I: Industry equilibrium when \( N = 2 \)

<table>
<thead>
<tr>
<th>Strategies</th>
<th>Benchmark Stackelberg ((a))</th>
<th>Sequential Investment ((b))</th>
<th>Simultaneous Investment ((c))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_j )</td>
<td>( 0.032 ) ( 0.125 )</td>
<td>( 0.019 ) ( 0.655 )</td>
<td>( 0.090 ) ( 0.090 )</td>
</tr>
<tr>
<td>( q_j )</td>
<td>( 98.884 ) ( 50.887 )</td>
<td>( 162.074 ) ( 25.684 )</td>
<td>( 64.415 ) ( 60.962 )</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>( 0.000 ) ( 0.000 )</td>
<td>( 0.939 ) ( 0.000 )</td>
<td>( 0.000 ) ( 0.000 )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Valuation at ( X_0 )</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Assets in Place</td>
<td>( 0.258 ) ( 0.258 )</td>
<td>( 0.258 ) ( 0.258 )</td>
<td>( 0.258 ) ( 0.258 )</td>
</tr>
<tr>
<td>Firm Value</td>
<td>( 0.585 ) ( 0.202 )</td>
<td>( 0.270 ) ( 0.060 )</td>
<td>( 0.433 ) ( 0.395 )</td>
</tr>
<tr>
<td>( \Delta \pi_{j,0} )</td>
<td>( -0.405 ) ( -0.169 )</td>
<td>( -0.119 ) ( -0.205 )</td>
<td>( 0.000 ) ( 0.000 )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Market at ( X_t = x_F^s )</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( Q )</td>
<td>( 149.771 ) ( 149.771 )</td>
<td>( 187.758 ) ( 187.758 )</td>
<td>( 125.377 ) ( 125.377 )</td>
</tr>
<tr>
<td>( q_j/Q )</td>
<td>( 0.660 ) ( 0.340 )</td>
<td>( 0.863 ) ( 0.137 )</td>
<td>( 0.514 ) ( 0.486 )</td>
</tr>
<tr>
<td>( HHI )</td>
<td>( 0.551 ) ( 0.551 )</td>
<td>( 0.764 ) ( 0.764 )</td>
<td>( 0.501 ) ( 0.501 )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Firms’ Betas</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>At ( X_0 )</td>
<td>( 1.176 ) ( 0.923 )</td>
<td>( 1.021 ) ( 0.012 )</td>
<td>( 1.129 ) ( 1.112 )</td>
</tr>
<tr>
<td>At ( x_L^c )</td>
<td>( 0.828 ) ( 1.291 )</td>
<td>( 0.943 ) ( 1.313 )</td>
<td>( 1.157 ) ( 1.139 )</td>
</tr>
<tr>
<td>At ( x_F^c )</td>
<td>( 0.706 ) ( 1.296 )</td>
<td>( 0.879 ) ( 1.311 )</td>
<td>( 1.000 ) ( 1.000 )</td>
</tr>
<tr>
<td>At ( x_F^L )</td>
<td>( 1.000 ) ( 1.000 )</td>
<td>( 1.000 ) ( 1.000 )</td>
<td>( 1.000 ) ( 1.000 )</td>
</tr>
</tbody>
</table>

This table illustrates the industry equilibrium when \( N = 2 \) and firms differ in their future production technologies. Firm \( L \) behaves more aggressively and invests earlier and more in the sequential equilibrium relative to Stackelberg games when \( \sigma_c < \sigma_F \). The market share of firm \( L \) is the highest and market concentration the highest in the sequential equilibrium. The value and the expected return of firm \( F \) are always higher under simultaneous investment. In the sequential equilibrium, \( \beta_{F,t}^L \) is lower than one before firm \( L \) invests since it expects a reduction in market prices at \( X_t = x_L^c \). Similarly, \( \beta_{L,t}^F \) is lower than one upon investment since it expects a reduction in market prices at \( X_t = x_F^L \).
Table II: Industry equilibrium when $N = 3$

<table>
<thead>
<tr>
<th>Strategies and Valuation</th>
<th>Sequential Investment $(a)$</th>
<th>Simultaneous Investment $(b)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>L</td>
<td>F</td>
</tr>
<tr>
<td>$x_j$</td>
<td>0.014</td>
<td>0.093</td>
</tr>
<tr>
<td>$q_j$</td>
<td>113.296</td>
<td>46.812</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.959</td>
<td>0.920</td>
</tr>
<tr>
<td>$HHI$</td>
<td>1.000</td>
<td>0.586</td>
</tr>
<tr>
<td>$Firm Value at X_0$</td>
<td>0.237</td>
<td>0.069</td>
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</table>

Table III: Other investment strategies when $N = 3$

<table>
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<tr>
<th>Strategies and Valuation</th>
<th>Mixed Cases</th>
<th>L leads, F and M follow</th>
<th>L and F lead, M follows</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>L</td>
<td>F</td>
<td>M</td>
</tr>
<tr>
<td>$x_j$</td>
<td>0.023</td>
<td>1.113</td>
<td>1.113</td>
</tr>
<tr>
<td>$q_j$</td>
<td>159.453</td>
<td>19.192</td>
<td>12.740</td>
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<tr>
<td>$\lambda$</td>
<td>0.918</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>$HHI$</td>
<td>0.699</td>
<td>0.018</td>
<td>0.018</td>
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<tr>
<td>$Firm Value at X_0$</td>
<td>0.314</td>
<td>0.054</td>
<td>0.049</td>
</tr>
</tbody>
</table>

These tables illustrate the potential equilibrium outcomes with $N = 3$ when firms have the same parameters as those in Table I. All else equal, competition erodes firm values and betas for all firms. $\lambda$ is higher for firm $L$ than form firm $F$, and also higher than in Table I. Firm $L$ would maximize its value by becoming a leader with two followers. However, this is not an equilibrium outcome since firm $F$ only invests simultaneously if firm $L$ invests simultaneously as well. The equilibrium outcome is the simultaneous equilibrium in Table II.
Figure 1: The effect of $\sigma_c$ on firms’ investment strategies

This figure illustrates the predictions on Proposition 2 of how $\sigma_c$ affects firms’ investment strategies in equilibrium. The red (resp. blue) color relates to $\Gamma^*_j$ (resp. $\Gamma^*_F$). The black dotted line reflects firms’ investment strategies in equilibrium. Firm $L$ is more valuable under $\Gamma^*_j$ when $\sigma_c < \sigma_{c*}$, and is more valuable under $\Gamma^*_F$ otherwise. The threshold $\sigma_{c*}$ obtains when firm $L$ is indifferent between $\Gamma^*_j$ and $\Gamma^*_F$. Firm $F$ is always more valuable under $\Gamma^*_F$. The shadow cost of preemption $\lambda$ decreases with $\sigma_c$. The HHI increases with $\sigma_c$. 
Figure 2: The dynamics of $\beta_{jt}$ in less and more concentrated industries

This figure shows the dynamics of $\beta_{jt}$ for $\sigma_c = \sigma_{cc}$. The red lines relate to sequential investment. The solid line corresponds to firm $L$ and the dashed line to firm $F$. The blue lines depict $\beta_{jt}$ for firms $L$ and $F$ under simultaneous investment. Firms’ betas co-move positively (negatively) in less (more) concentrated industries.
This figure illustrates the dynamics of $\mu_{\beta,t}$ and $\sigma_{\beta,t}$ in less and more concentrated industries when $\sigma_c = \sigma_{\beta}$. The solid red (blue) line shows the return moment conditions under sequential (simultaneous) investment when using equally weighted industry returns. The dashed red (blue) line shows the return moment conditions using value weighted industry returns. The solid black line shows the premium $\mu_{\beta,t}^c - \mu_{\beta,t}^s$ and also $\sigma_{\beta,t}^c - \sigma_{\beta,t}^s$ using equal weights. The dashed black line shows the corresponding series using value weights. Using value weights, $\mu_{\beta,t}^c - \mu_{\beta,t}^s$ is strictly positive at any time. Using equal weights, $\mu_{\beta,t}^c - \mu_{\beta,t}^s$ is always positive until all firms invest in the less concentrated industry. $\sigma_{\beta,t}^c - \sigma_{\beta,t}^s$ is always negative using any type of weights.
Figure 4: Changes in average industry betas $\Delta \mu_{\beta,t}$ upon additions to industry capacity

$\sigma_c \leq \sigma_e$

$\Delta \mu_{\beta,t}^c < 0$ at $X_t = x^c$

$\Delta \mu_{\beta,t}^s \geq 0$ at $X_t = x^s_L$

$\Delta \mu_{\beta,t}^s \geq 0$ at $X_t = x^s_L$

This figure illustrates why booms and busts in industry returns are more easily observable in less concentrated industries. In less concentrated industries, firms’ returns co-move more positively, and industry returns decrease when firms adds capacity to the industry. In more concentrated industries, the expected returns of leaders and followers co-move negatively. As a result, the equally or value weighted average industry returns may increase or decrease when the industry capacity increases.

Figure 5: $\sigma_{\beta,0}$ and industry concentration

This figures shows that industries with high $\sigma_{\beta,0}$ become highly concentrated upon investment. The red (blue) color relates to the more concentrated industries in which firms invest sequentially (simultaneously). The positive correlation between $\sigma_{\beta,0}$ and the HHI holds for any level of $\sigma_e$. 

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This figure illustrates how the underlying determinants of market demand affect firms’ investment strategies in equilibrium. The red (blue) color relates to the sequential (simultaneous) investment strategies. The black dotted line reflects the investment strategies in equilibrium. Firm $L$ prefers simultaneous investment in more homogeneous industries with low $\varepsilon$, high $\eta$ and high $\sigma$. This explains why $\sigma_{\varepsilon}$ is decreasing in $\varepsilon$ and increasing in $\eta$ and $\sigma$. 

Figure 6: The effect of market demand on industry equilibria
Figure 7: Comparative statics for discrete changes in betas upon investment

<table>
<thead>
<tr>
<th></th>
<th>$\sigma_c \leq \sigma_e$</th>
<th>$\sigma_c &gt; \sigma_e$</th>
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<tr>
<td></td>
<td>$\bar{X}_t = x^c$</td>
<td>$\bar{X}_t = x^L$</td>
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<tr>
<td>$\xi$</td>
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<tr>
<td>$\eta$</td>
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</tr>
<tr>
<td>$\sigma$</td>
<td><img src="image7" alt="Graph" /></td>
<td><img src="image8" alt="Graph" /></td>
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</table>

This figure illustrates $\Delta \beta_j$ when firms invest in less and more concentrated industries. The dotted (dashed) lines correspond to firm $L$ ($F$). The solid lines correspond to the equally weighted average industry returns. In more homogeneous industries, firms’ betas and the average industry beta increase up to $x^c$ and decrease sharply upon investment. This sharp decrease upon investment is larger in industries with low $\varepsilon$, high $\eta$ and high $\sigma$. In more heterogeneous industries, $\Delta \beta^L$ and $\Delta \beta^F$ have opposite signs both at $x^L$ and $x^F$, and hence booms and busts are not easily detectable at the industry level irrespective of $\varepsilon$, $\eta$ or $\sigma$. 

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Figure 8: Comparative statics on $\sigma_{\beta,t}$

<table>
<thead>
<tr>
<th>$\sigma_{\beta,t}$ when $\sigma_c &lt; \sigma_{-c}$</th>
<th>$\sigma_{\beta,t}$ when $\sigma_c &gt; \sigma_{-c}$</th>
<th>$\sigma_{\beta,t}^c - \sigma_{\beta,t}^s$</th>
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<tr>
<td><img src="image1.png" alt="Graph 1" /></td>
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<tr>
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<tr>
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<td><img src="image20.png" alt="Graph 20" /></td>
<td><img src="image21.png" alt="Graph 21" /></td>
</tr>
</tbody>
</table>

This figure illustrates the cross sectional variation in industry betas when less (more) concentrated industries have lower (higher) $\varepsilon$, and higher (lower) $\mu$ and $\sigma$. The dashed lines correspond to the base case for $\sigma_c = \sigma_{-c}$. The solid lines correspond to the case in which the industry has the indicated parameter either 10% lower or higher relative to the dashed line. The premium $\sigma_{\beta,t}^c - \sigma_{\beta,t}^s$ is strictly negative irrespective of parameter choice.
Figure 9: The premium $\mu_{\beta,t}^c - \mu_{\beta,t}^s$ when industries differ in $\sigma_c$ and $N$

This figure compares the equally weighted (solid line) and value weighted (dashed line) average industry returns of a more concentrated industry with $N = 2$ and $\sigma_c > \sigma_e$ and a less concentrated industry with high $N$ and $\sigma_c \approx 0^+$. As $N$ increases in the less concentrated industry, the equally weighted premium $\mu_{\beta,t}^c - \mu_{\beta,t}^s$ decreases but remains strictly positive until all firms invest in the less concentrated industry.

A Appendix

A Sorting conditions when $q_j = q$ and $c_L < c_F$

The strategy pursued by firms is a multiple-action pair such that $\Gamma_j = \{x_j; \Delta q_j\}$. The proof follows Bustamante (2011) and consists of two steps. The first step is to show that if the value function $V_j(X_t)$ complies the conditions in Cho and Sobel (1990), then the sorting condition of the action pair $\Gamma_j$ corresponds to the sorting conditions of each action in isolation. The second step is to derive the sorting conditions for each action. I first consider the case of $N = 2$. The value function $V_j(X_t)$ firm $j$ given the set of actions $\Gamma_j$ is such that

$$V_j(X_t) = \hat{D}(q) X_t \frac{q}{\delta} + \left[\left(\hat{D}(q_j) - c_j\right) x_j \frac{q_j}{\delta} - I - \hat{D}(q) x_j \frac{q_j}{\delta}\right] \left(\frac{X_t}{x_j}\right)^v$$

(23)

where the $x_j$ and $q_j$ may take any value given $x_j > 0$ and $q_j > 0$. The function $\hat{D}(q_j) x_j$ is the expected present value of the demand prices (1) of firm $j$ when its capacity is $q_j$.

In line with Cho and Sobel (1990), $V_j(X_t)$ is continuous in $S_j$ and for any type $j$. The value function $V_j(X_t)$ is increasing in $\hat{D}(q_j) x_j$ for any type $j$. Finally, if $x_L < x_F$ and $q_L > q_F$, then it must be the case that $V_F(X_t) \leq \tilde{V}_F(X_t)$ implies $V_L(X_t) > \tilde{V}_L(X_t)$. This last condition ensures that if firm $F$ has incentives to deviate, firm $L$ will pay a cost to ensure incentive compatibility. Replacing $V_j(X_t)$ by (23) and operating, the condition $V_F(X_t) \leq \tilde{V}_F(X_t)$ implies $c_F \leq \Omega_c$ where $\Omega_c$ is given by

$$\Omega_c = \frac{\hat{D}(q_L) q_L - \kappa \hat{D}(q_F) q_F + \hat{D}(q) \frac{q}{q_F} (1 - \kappa) - F \delta (x_L^{-1} - x_F^{-1})}{(q_L - \kappa q_F)}$$
Similarly, the condition $V_L(X_t) > \tilde{V}_L(X_1)$ implies $c_L \leq \Omega_c$. Therefore if $c_L < c_F$ and $c_F \leq \Omega_c$, it holds that $c_L < \Omega_c$ for any parameter value.

Consider now the sorting condition for each action $x_j$ and $q_j$ separately. The sorting condition for $x_j$ reflects that, all else equal, more efficient firms find it less costly to invest earlier, namely

$$\frac{\partial}{\partial c_j} \left[ \frac{\partial V_j}{\partial x_j} \right] = -\left(1 - \frac{x_j}{\delta} \right)^v > 0$$

(24)

The sorting condition with respect to $\Delta q_j$ is such that, all else equal, more efficient firms find it less costly to invest more in capacity, namely

$$\frac{\partial}{\partial c_j} \left[ \frac{\partial V_j}{\partial q_j} \right] = -\left(1 - \frac{x_j}{\delta} \right)^v < 0$$

(25)

Conditions (24) and (25) ensure that the incentive compatibility constraint of the follower is binding and that there exists an incentive compatible sequential equilibrium for the duopoly game.

The general case with $N > 2$ is proved similarly. The sufficiency conditions for Cho and Sobel (1990) apply for games with $N$ types, and all conditions above hold when $c_L < c_j < ... < c_N$.

### B Sequential investment strategies

Given that firm $L$ has already invested, the problem faced by firm $F$ is to maximize

$$rV_F^* = \mu X + \frac{\sigma^2}{2} X^2 \frac{\partial^2 V_F^*}{\partial^2 X} + \left[ \left( \frac{2q}{q} \right)^{-\frac{1}{2}} - \bar{c} \right] qX_t$$

subject to the conditions

$$V_F^*|_{X_t = x_F^*} = \frac{1}{\delta} \left[ x_F Q^{-\frac{1}{2}} q_F - q_F c_F \right] - I$$

(27)

$$\frac{\partial V_F^*}{\partial X_t}|_{X_t = x_F^*} = \frac{1}{\delta} \left[ Q^{-\frac{1}{2}} q_F - c_F q_F \right]$$

(28)

$$\frac{\partial V_F^*}{\partial \Delta q_F}|_{X_t = x_F^*} = 0$$

(29)

where (26) equates the required rate of return of the firm to the expected return on the option to invest $v$ is the positive root of (26). Furthermore, (27) requires that the value of firm before investment equals, upon option exercise, the gross profit that the manager extracts from investment net of operating costs and the fixed cost of investment $I$; (28) ensures that the option to invest is exercised along the optimal path; and (29) requires that $\Delta q_F$ maximizes firm value. The strategy that solves (27)-(29) yields (12)-(13).

Since firm $F$ has to ensure sequential investment, firm $F$ maximizes

$$rV_L^* = \mu X + \frac{\sigma^2}{2} X^2 \frac{\partial^2 V_L^*}{\partial^2 X} + \left[ \left( q_L^* + q \right)^{-\frac{1}{2}} - \bar{c} \right] qX_t$$

(30)
subject to the alternative conditions

\[
\begin{align*}
V_L^t |_{X_t = x_L} & = \frac{1}{\delta} \left[ (1 - \kappa^s) q_L^{-\frac{1}{2}} + \kappa^s Q^{-\frac{1}{2}} - c_L \right] x_L q_L - I - \lambda \left[ \frac{V_F^t}{\partial x_L} - \frac{V_F^t}{\partial q_L} \right] \tag{31} \\
\frac{\partial V_L^t}{\partial X_t} |_{X_t = x_L} & = \frac{1}{\delta} \left[ (1 - \nu \kappa^s) q_L^{-\frac{1}{2}} + \nu \kappa^s Q^{-\frac{1}{2}} - c_L \right] q_L - \lambda \left[ \frac{\partial V_F^t}{\partial x_L} - \frac{\partial V_F^t}{\partial q_L} \right] \tag{32} \\
\frac{\partial V_F^t}{\partial \Delta q_L} |_{X_t = x_L} & = 0 \tag{33}
\end{align*}
\]

where \( \lambda \) is the Lagrange multiplier of (14) and conditions (31)-(32) for firm \( L \) have similar interpretations to those of firm \( F \) in (27)-(28). The strategy that solves (31)-(33) is given by (15)-(16). The parameter \( \nu > 1 \) is the root of the ODEs in (26) and (30) and is the same for both firms \( L \) and \( F \).

C Simultaneous investment strategies

As in Fundenberg and Tirole (1985) and Weeds (2002), a simultaneous equilibrium exists when both firms are better off by investing simultaneously. In this model, simultaneous investment is Pareto optimal for both firms when firm \( L \) attains a higher value under simultaneous investment. To show this, consider two alternative strategies \( \Gamma = \{x; q\} \) and \( \Gamma = \{\bar{x}; \bar{q}\} \) for any firm \( j \) such that \( x < \bar{x} \) and \( q > \bar{q} \). \( \Gamma \) corresponds to any sequential equilibrium in which the firm invests earlier and more than under simultaneous investment. \( \Gamma \) corresponds to any simultaneous equilibrium in which the firm invests later and less than as a leader. The condition that firm \( L \) attains a higher value by investing simultaneously with its competitor requires

\[
V_L^s (X_t; S) < V_L^s (X_t; \bar{S}) \tag{34}
\]

\[
\frac{1}{\delta} [(1 - \kappa) p + \kappa \tilde{p}] q - \left[ I + \left( \frac{2q}{2(2q)} - \frac{2}{3} \right) \frac{q^{\kappa}}{q^{\kappa}} \right] \frac{1}{2} \left[ 1 - \left( \frac{\tilde{q}}{q} \right)^{\nu} \right]
\]

Therefore if \( c_L > \Theta \) where \( \Theta \) is given by

\[
\Theta = \frac{1}{\delta} \left[ (1 - \kappa) p + \kappa \tilde{p} \right] q - \left[ I + \left( \frac{2q}{2(2q)} - \frac{2}{3} \right) \frac{q^{\kappa}}{q^{\kappa}} \right] \frac{1}{2} \left[ 1 - \left( \frac{\tilde{q}}{q} \right)^{\nu} \right]
\]

The equilibrium investment timing \( x^c \) maximizes the value of firm \( L \) and is solved as in Dixit and Pyndick (1994). The optimal capacity choice of both firms is solved as in Cournot games.

D Competition in both \( q_j \) and \( c_j \)

Consider first the special case in which \( c_L = c_F \) and \( q_L < q_F \) such that firm type is given by the initial installed capacity of firms \( q_j \). The total capacity once the growth option is exercised relates to the initial capacity since \( q_j = \Delta q_j + \frac{q_j}{x_j} \). The value function \( V_j (X_t) \) firm \( j \) given the set of actions \( \Gamma_j \) yields

\[
V_j (X_t) = \tilde{D} \left( q_j \right) \frac{X_t}{\delta} + \left[ \tilde{D} (q_j) - \tilde{D} \left( \frac{q_j}{x_j} \right) - c \right] \frac{1}{\delta} x_j q_j - F \left( \frac{X_t}{x_j} \right)^{\nu} \tag{34}
\]

In line with Cho and Sobel (1990), the framework requires that leaders have incentives to pay a cost to induce incentive compatibility when followers find it profitable to deviate. If \( x_L < x_F \) and \( q_L > q_F \), then
\( V_F (X_t) \leq \tilde{V}_F (X_t) \) should imply \( V_L (X_t) > \tilde{V}_L (X_t) \). Using (34), the condition \( V_F (X_t) \leq \tilde{V}_F (X_t) \) implies \( q_F \leq \Omega_q \) where \( \Omega_q \) is given by
\[
\Omega_q = \frac{\hat{D} (q_L) q_L - \kappa \hat{D} (q_F) q_F - c (q_L - \kappa q_F) - F \delta (x_L^{-1} - x_F^{-1})}{\hat{D} (q_L) (1 - \kappa)}
\]
Similarly, \( V_L (X_t) > \tilde{V}_L (X_1) \) implies \( q_L \leq \Omega_q \). Hence if \( q_L < q_F \) and \( q_F \leq \Omega_q \), \( q_L < \Omega_q \) for any parameter value.

The sorting conditions for \( x_j \) and \( \Delta q_j \) reflect that, all else equal, firms with larger installed capacity find it more costly to invest earlier and more, namely
\[
\frac{\partial}{\partial q_j} \left[ \frac{\partial V_j}{\partial x_j} \right] > 0 \quad \text{and} \quad \frac{\partial}{\partial q_j} \left[ \frac{\partial V_j}{\partial \Delta q_j} \right] < 0 \quad (35)
\]
where the rationale behind (35) resembles that in Boyer et al (2001). Whenever firms are already operating and one of them invests, an increase in capacity reduces market prices and hence the instantaneous profits for both firms. Since both firms have the same growth opportunities, the expected reduction in profits is less pronounced for the firm with lower installed capacity.

The joint implication of (10) and (35) is then that firms with lower current installed capacity \( q_j \) and lower future production costs \( c_j \) have a comparative advantage to invest earlier and more. The sorting conditions when firm type is given by the pair \( \{ q_j; c_j \} \) are given by
\[
\frac{\partial}{\partial q_j} \left[ \frac{\partial V_j}{\partial x_j} \right] + \frac{\partial}{\partial q_j} \left[ \frac{\partial V_j}{\partial \Delta q_j} \right] > 0, \quad \frac{\partial}{\partial q_j} \left[ \frac{\partial V_j}{\partial x_j} \right] + \frac{\partial}{\partial q_j} \left[ \frac{\partial V_j}{\partial \Delta q_j} \right] < 0 \quad (36)
\]
The sufficiency conditions described for \( N = 2 \) also apply for \( N > 2 \). The methodology to solve for firms’ investment strategies in equilibrium is the same as in the case of \( q_j = q \) and \( c_L < c_F \).

**E  The low concentration premium**

This is the proof that the value weighted premium \( \mu_{\beta,t}^c - \mu_{\beta,t}^s > 0 \) for \( X_t \in [X_0, x_F^p] \). Given (6), it is possible to show that
\[
\mu_{\beta,t}^c - \mu_{\beta,t}^s = (v - 1) \left( \sum_j \frac{\pi_{j,t}}{\delta} - \sum_j \frac{\bar{\pi}_{j,t}}{\delta} \right) \left( \frac{1}{\sum_j V_{j,t}^s} - \frac{1}{\sum_j V_{j,t}^c} \right) \quad (37)
\]
Since the second term in brackets in (37) is always lower or equal to one under simultaneous investment for \( X_t \leq x_F^p \), the proof that \( \mu_{\beta,t}^c - \mu_{\beta,t}^s \) consists in showing that the first term in brackets in (37) is always higher than one for \( X_t \leq x_F^c \). Consider first the interval \( X_t \in (x_L^c, x_F^c) \). Using the results in the paper, the first term in brackets in (37) equals
\[
\frac{I}{v - 1} \left( \frac{X_t}{x_F^c} \right)^v + \frac{\Delta \pi_{Lt}^c}{\delta} < 0 \quad (38)
\]
Denote $s^*_L = \frac{q^*_F}{Q}$ the market share of firm $L$ when both firms have invested. Using the definition of $q^*_F$ in (13) and given $s^*_L = \frac{q^*_F}{Q}$, a sufficient condition such that (38) holds is

$$
\left[ \frac{1}{\varepsilon} (1 - s^*_L)^2 - \left( s^*_L + \frac{q^*_F}{Q} \right)^{-\frac{1}{2}} (1 - s^*_L) \right] < \left[ \left( s^*_L + \frac{q^*_F}{Q} \right)^{-\frac{1}{2}} - 1 \right] s^*_L v
$$

Reordering terms,

$$
\left( s^*_L + \frac{q^*_F}{Q} \right)^{\frac{1}{2}} \left[ \frac{1}{\varepsilon} (1 - s^*_L)^2 + s^*_L v \right] < 1 + (v - 1) s^*_L
$$

(39)

which is true for any choice of parameter choice since the left hand side of (39) is lower than one. This is because $s^*_L < 1$ and each of the factors on the left hand side of (41) lower than one, since

$$
s^*_L + \frac{q^*_F}{Q} < 1 \quad \text{and} \quad s^*_L > 0 > 1 - v \varepsilon
$$

where $v > 1$ and $\varepsilon > 1$. Hence $\mu^*_{\beta,t} - \mu^*_{\beta,t} > 0$ is always true for $X_t \in (x^*_L, x^*_F)$.

Consider next the interval $X_t \in [X_0, x^*_L]$. Using the results in the paper, the expression above equals

$$
\frac{I}{v - 1} \left( \frac{X_t}{x^*_F} \right)^v + \frac{\Delta \pi^+_L}{\delta} + \frac{I}{v - 1} \left( \frac{X_t}{x^*_L} \right)^v + \frac{\Delta \pi^-_F}{\delta} < 0
$$

(40)

Since (38) proves that the first two terms in (40) are negative, a sufficient condition for (40) to hold is that the sum of the second two terms is also negative. Denote $s^*_F = \frac{q^*_L}{2q^*_F + q^*_L}$ the market share of firm $F$ during $X_t \in (x^*_L, x^*_F)$. Using the fact that $q^*_L > q^*_F$, a sufficient condition such that the second two terms in (40) are negative is then

$$
\left[ \frac{1}{\varepsilon} (1 - s^*_F)^2 - \left( \frac{q^*_L + q^*_F}{q^*_L + q^*_F} \right)^{-\frac{1}{2}} (1 - s^*_F) \right] < \left[ \left( \frac{q^*_L + q^*_F}{q^*_L + q^*_F} \right)^{-\frac{1}{2}} - 1 \right] v s^*_F
$$

Reordering terms,

$$
\left( \frac{q^*_F + q^*_L}{q^*_F + q^*_L} \right)^{\frac{1}{2}} \left[ \frac{1}{\varepsilon} (1 - s^*_F)^2 + s^*_F v \right] < 1 + (v - 1) s^*_L
$$

(41)

where, once again, $s^*_F < 1$ and both terms on the right hand side of (41) are strictly lower than one.

F Parameter choice in numerical examples

The parameters in Tables I-III are $r = 65\%$, $\sigma = 25\%$, $\varepsilon = 2.35$, $I = 1$, $X_0 = 0.01$, $c_L = 0.1$, $c_F = c_L + \sigma c$, $v = 0.1105$, $q = 1$ and $\sigma c = 0.00175$. The technology of firm $M$ is such that $c_M = c_L + 2 * \sigma c$. The parameters in Figures 1 – 9 are the same with exception of $\sigma c = 0.0077$. 

44
G The case of N firms

The solution approach for the case of N firms is similar to the duopoly case. Each firm cares about its closest (and strongest) competitor. As in (11), the working assumption is that each firm has incentives to imitate its closest and stronger competitor. The framework further assumes that each firm has no incentives to imitate other stronger competitors.

The sorting conditions (36) facilitate the solution of sequential equilibria for any number of firms. The sorting conditions constrain the possible equilibrium outcomes to those in which the firm with the more efficient growth option invests first. The only binding incentive compatibility constraint for each firm is that of its strongest competitor.

Tables II-III illustrate the equilibria that may arise in pure strategies when \( N = 3 \). Firms may invest sequentially or simultaneously as in the case of \( N = 2 \), but there also other two equilibria in which two of the three firms cluster and the remaining firm either leads or follows. Using the same underlying parameters as in Table I, the equilibrium outcome for \( N = 3 \) in Tables II-III is that firms invest simultaneously.

The example for \( N = 3 \) in Tables II-III highlights the two important features of the model when \( N > 2 \). First, \( \sigma_c \) is itself a function of \( N \), and may either decrease or increase if the distribution of firms’ production technologies is not uniform. In the example, \( \sigma_c \) is the same as in the case of \( N = 2 \): all firms are equally distant competitors. Second, the equilibrium outcome for \( N > 2 \) depends on the incentives of both firm \( L \) and \( F \) to invest earlier than their closest competitor. Firm \( L \) would be better off having \( M \) as a follower (Table III), but firm \( F \) is better off investing simultaneously with \( M \) (Table II).