Limits of Arbitrage: The State of the Theory

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Limits of Arbitrage: The State of the Theory

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Abstract

We survey theoretical developments in the literature on the limits of arbitrage. This literature investigates how costs faced by arbitrageurs can prevent them from eliminating mispricings and providing liquidity to other investors. Research in this area is currently evolving into a broader agenda emphasizing the role of financial institutions and agency frictions for asset prices. This research has the potential to explain so-called “market anomalies” and inform welfare and policy debates about asset markets. We begin with examples of demand shocks that generate mispricings, arguing that they can stem from behavioral or from institutional considerations. We next survey, and nest within a simple model, the following costs faced by arbitrageurs: (i) risk, both fundamental and non-fundamental, (ii) short-selling costs, (iii) leverage and margin constraints, and (iv) constraints on equity capital. We finally discuss implications for welfare and policy, and suggest directions for future research.

Keywords: limits of arbitrage, market anomalies, liquidity, financial constraints, financial institutions, survey

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1 INTRODUCTION

Standard models of asset pricing assume a representative agent who participates in all markets costlessly. Equilibrium prices are tied to the representative agent’s consumption, which coincides with the aggregate consumption in the economy. The relationship between prices and consumption is summarized in the consumption CAPM, according to which an asset’s expected return in excess of the riskfree rate is proportional to the asset’s covariance with aggregate consumption. Intuitively, assets that correlate positively with consumption add to the risk borne by the representative agent and must offer high expected return as compensation.

The relationship between risk and expected return predicted by standard models appears at odds with a number of stylized facts commonly referred to as market anomalies. Leading anomalies include (i) short-run momentum, the tendency of an asset’s recent performance to continue into the near future, (ii) long-run reversal, the tendency of performance measured over longer horizons to revert, (iii) the value effect, the tendency of an asset’s ratio of price to accounting measures of value to predict negatively future returns, (iv) the high volatility of asset prices relative to measures of discounted future payoff streams, and (v) post-earnings-announcement drift, the tendency of stocks’ earning surprises to predict positively future returns.¹ Reconciling these anomalies with standard models requires explaining variation in asset risk: for example, in the case of short-run momentum, one would have to explain why good recent performance renders an asset riskier and more positively correlated with aggregate consumption. A recent literature pursues explanations along these lines by introducing more general utility functions for the representative agent. Yet, reconciling standard models with all the anomalies, and in a way consistent with their quantitative magnitude remains elusive.

The anomalies listed above concern the predictability of asset returns based on past prices and earnings. An additional set of anomalies concern the relative prices of assets with closely related payoffs. For example, (i) “Siamese-twin” stocks, with claims to almost identical dividend streams, can trade at significantly different prices, (ii) stocks of a parent and a subsidiary company can trade at prices under which the remainder of the parent company’s assets has negative value, and (iii) newly issued “on-the-run” bonds can trade at significantly higher prices than older “off-the-run” bonds with almost identical payoffs.² Anomalies concerning relative prices have been documented


²See, for example, Rosenthal & Young (1990) and Dabora & Froot (1999) for evidence on Siamese-twin stocks,
for a more limited set of assets, partly because of the scarcity of asset pairs with closely related payoffs. At the same time, these anomalies are particularly hard to reconcile with standard models. Indeed, while standard models may offer slightly different predictions as to how risk and expected returns are related, they all imply the law of one price, i.e., assets with identical payoffs must trade at the same price. In the previous examples, however, differences in payoffs seem insignificant relative to the observed price differences.

Understanding why anomalies arise and persist requires a careful study of the process of arbitrage: who are the arbitrageurs, what are the constraints and limitations they face, and why can arbitrage fail to bring prices close to the fundamental values implied by standard models. This is the focus of a recent literature on the limits of arbitrage. This article surveys important theoretical developments in that literature, nests them within a simple model, and suggests directions for future research.

Limits of arbitrage are usually viewed as one of two building blocks needed to explain anomalies. The other building block are demand shocks experienced by investors other than arbitrageurs. Anomalies are commonly interpreted as arising because demand shocks push prices away from fundamental values and arbitrageurs are unable to correct the discrepancies. Such “non-fundamental” shocks to demand are often attributed to investor irrationality. In this sense, research on the limits of arbitrage is part of the behavioral finance agenda to explain market anomalies based on investors’ psychological biases.3

This article departs from the conventional view in two related respects. First, it argues that research on the limits of arbitrage is relevant not only for behavioral explanations of anomalies but also for the broader study of asset pricing. Indeed, non-fundamental demand shocks can also arise because of institutional frictions relating to contracting and agency, as illustrated in the next section. Research on the limits of arbitrage characterizes how non-fundamental demand shocks, whether behavioral or not, impact prices.

In the conventional view, non-fundamental demand shocks concern investors other than arbitrageurs, and therefore can be understood independently of the limits of arbitrage. Our second departure is to argue that many non-fundamental demand shocks can be understood jointly with limits of arbitrage within a setting that emphasizes financial institutions and agency. Indeed, arbitrage is often performed by specialized institutions such as hedge funds and investment banks,

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3Behavioral explanations for the anomalies include Barberis et al. (1998), Daniel et al. (1998), Hong & Stein (1999) and Barberis & Shleifer (2003). See also the survey by Barberis & Thaler (2003).
and the trading strategies of these institutions are constrained by agency frictions. At the same time, financial institutions and agency frictions are the source of many non-fundamental demand shocks. In this sense, financial institutions do not necessarily correct anomalies, but can also cause them. Research on the limits of arbitrage is currently evolving into a broader agenda emphasizing the role of financial institutions and agency frictions for asset prices. This agenda has the potential to explain market anomalies within a fully rational framework.

The emphasis on financial institutions and agency frictions is fruitful for the analysis of welfare and public policy. Crises, including the recent one, show that government intervention can be important for the smooth functioning of financial markets. In standard models, however, there is no scope for such intervention because the equilibrium is Pareto optimal. Research on the limits of arbitrage has the potential to deliver a more useful framework for designing and assessing public policy. Indeed, this research takes a two-tiered view of financial markets: a core of sophisticated arbitrageurs trade against mispricings, and in doing so provide liquidity to a periphery of less sophisticated investors. Under this view, the financial health of arbitrageurs is crucial for the smooth functioning of markets and the provision of liquidity. Understanding how financial health is affected by arbitrageurs’ trading decisions, and whether these decisions are socially optimal, can guide public policy.

This article proceeds as follows. Section 2 presents examples of non-fundamental demand shocks, emphasizing that they often stem from institutional considerations. Section 3 surveys important theoretical developments in the literature on the limits of arbitrage, and nests them within a simple model. It emphasizes the following costs faced by arbitrageurs: (i) risk, both fundamental and non-fundamental, (ii) costs of short-selling, (iii) leverage and margin constraints, and (iv) constraints on equity capital. While these are not the only costs faced by real-life arbitrageurs, they are among the most important and have received significant attention in the literature. Besides examining the implications of each type of cost for asset price behavior, Section 3 sketches how these costs can be integrated into richer models that incorporate multiple assets and dynamics. Such models have the potential to address a variety of anomalies—both those concerning violations of the law of one price, and those concerning return predictability—and are the subject of a rapidly growing literature. Finally, Section 4 discusses implications for welfare and public policy.

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4Risk is a cost when arbitrageurs are not fully diversified and bear a disproportionate share of the risk of arbitrage trades. Under-diversification is related to financial constraints, as we explain in Section 3.4. In this sense, costs (i), (iii) and (iv) are related.
Models of limited arbitrage typically assume that some investors experience demand shocks driving prices away from fundamental values. This section presents examples of such shocks and their price effects. Shocks in the first example are probably best interpreted as arising from behavioral considerations, while in the other examples they more likely arise from institutional frictions.

2.1 Palm-3Com

Lamont & Thaler (2003) study the sale of Palm by its parent company 3Com and the behavior of the two companies’ stock prices around this event. On March 2, 2000, 3Com sold 5% of its stake in Palm through an initial public offering (IPO). 3Com also announced that it would spin off its remaining stake in Palm to 3Com shareholders before the end of the year. Under the terms of the spin off, 3Com shareholders would receive 1.525 shares of Palm for each share of 3Com they owned.

The law of one price implies that prior to the spin off, 3Com shares should have been trading at a price exceeding 1.525 times the price of Palm shares. This is because one 3Com share was equivalent to 1.525 shares of Palm plus an equity claim on 3Com’s remaining (non-Palm) assets, and the latter claim had non-negative value because of limited liability. The law of one price was, however, violated for a period of approximately two months starting from the IPO. For example, on the day of the IPO, Palm closed at $95.06 per share, implying a lower bound of $145 for the share price of 3Com. 3Com, however, closed at $81.81 per share, having dropped from $104.13 on the previous day. Under these prices the implied value of 3Com’s non-Palm assets was -$22 billion, implying an economically significant violation of the law of one price.

The mispricing between Palm and 3Com is a challenge to standard asset pricing models. One must explain, in particular, why some investors were willing to buy Palm shares for $95.06, while they could acquire them at a lower cost by buying 3Com shares. The most plausible explanations are based on investors’ psychological biases and cognitive limitations. Investors buying Palm were possibly not sophisticated enough to appreciate the opportunity of buying it through 3Com. Moreover, because Palm was a manufacturer of a relatively new product (handheld devices), it was possibly associated with the “new economy” to an extent higher than 3Com. This might have led investors overly optimistic about the new economy to be willing to pay a disproportionately high price for Palm.
2.2 Index Effects

Index effects stem from a stock’s addition to or deletion from prominent market indices. Starting with Harris & Gurel (1986) and Shleifer (1986), a number of papers document that addition to the Standard and Poor (S&P)’s 500 index raises the price of a stock, while deletion lowers its price. For example, Chen et al. (2004) find that over 1989-2000, a stock’s price would increase by an average 5.45% on the day of the announcement that the stock would be added to the index, and a further 3.45% from the announcement day to the day of the actual addition. Conversely, a stock’s price would decrease by an average 8.46% on the day of the announcement that the stock would be deleted from the index, and a further 5.97% from the announcement day to the deletion day. These effects are economically significant.

Standard models can account for index effects only if additions and deletions convey information about assets’ fundamental values. Even if one is to accept, however, that S&P has an informational advantage relative to the market, it is hard to explain why this advantage (i) grew larger after 1989, which is when index effects became the most significant, and (ii) can be large enough to account for the observed index effects. A more plausible explanation is that index additions and deletions trigger changes in the demand by mutual funds. Indeed, passively managed mutual funds track indices mechanically, and actively managed funds benchmark their performance against indices. If, therefore, a stock is added to the S&P500 index, funds that track or are benchmarked against the index are eager to buy the stock, and this can raise the stock’s price. The institutional explanation is consistent with the growth of index effects in recent decades since this parallels the growth of institutional investing, index tracking and benchmarking.

Boyer (2007) provides further evidence consistent with the institutional explanation. He focuses on the BARRA value and growth indices, which consist of value and growth stocks, respectively. Unlike the S&P500 index, BARRA indices are constructed using publicly disclosed rules, so additions and deletions do not signal any private information. Boyer finds that “marginal value” stocks, defined as those that just switched from the growth into the value index, comove significantly more with the value than with the growth index, while the reverse is true for “marginal growth” stocks. These effects are hard to explain within standard models because marginal value stocks have very similar characteristics to marginal growth stocks. On the other hand, it is plausible that these effects arise from shifts in demand by mutual funds. For example, inflows into funds tracking the value index trigger purchases of all stocks in that index, and this can raise the prices of these stocks simultaneously. The institutional explanation is further strengthened by Boyer’s finding that the
effects appear only after 1992, which is when the BARRA indices were introduced.\footnote{Earlier evidence linking index membership to comovement is in Vijh (1994) and Barberis et al. (2005). Both papers focus on the S&P 500 index.}

### 2.3 Fire Sales by Mutual Funds

Mutual funds respond to outflows by selling stocks in their portfolios. Coval & Stafford (2007) study the behavior of stock prices around sales driven by large outflows. They define fire sales as the sales by the 10% of funds that experience the largest outflows within a given quarter. For each stock, they compute the fraction of average volume generated by fire sales, and they focus on the 10% of stocks for which this fraction is largest. These stocks exhibit a V-shaped price pattern. During the fire-sale quarter and the quarter immediately preceding, their average cumulative abnormal return is -7.9%. This price decline is followed by a recovery: during the year following the fire-sale quarter, the average cumulative abnormal return is 6.1%. This return rises to 9.7% during the 18 months following the fire-sale quarter.\footnote{Related evidence suggesting that fund outflows have large price effects is in Anton & Polk (2008), Jotikasthira et al. (2009), Greenwood & Thesmar (2009) and Lou (2009).}

The slow and predictable price recovery represents a challenge to standard models. Indeed, these models can account for an increase in expected return only through an increase in the co-variance with aggregate consumption. Explaining why this covariance increases for stocks sold by distressed mutual funds, and why such an increase can account for an annual abnormal return of 6% is difficult. A more plausible explanation is that sales by distressed mutual funds generate price pressure, pushing prices below fundamental values and raising expected returns going forward. This explanation is related to institutional frictions: distressed sales are likely to be triggered by investors who lose confidence in the quality of managers running underperforming funds.

### 2.4 U.K. Pension Reform and the Term Structure

Demand shocks are likely to also affect prices outside the U.S. and for assets other than stocks, as our last example illustrates. It concerns the impact of the U.K. pension reform on the term structure of interest rates, an episode described in greater detail in Greenwood & Vayanos (2010a). A major objective of U.K. pension reform over the past twenty years has been to ensure the transparency and solvency of pension funds; indeed, the reform was motivated partly by the failure of the Maxwell pension fund in the early 1990s. The reform stipulated that pension funds had to meet a minimum ratio of assets to liabilities. Pension-fund assets, such as stocks and bonds, are publicly traded and
can be valued using market prices. On the other hand, pension-fund liabilities are not traded, and their valuation requires a suitable discount rate. Under the Pensions Act of 2004, this discount rate had to be the yield on long-term inflation-linked government bonds, on the grounds that pension liabilities are long term and indexed to inflation.

Pension funds responded to the reform by buying large amounts of long-term bonds. Indeed, because long-term bonds were providing the discount rate to calculate the value of pension liabilities, they were also the best hedge for these liabilities. Pension-fund purchases had a significant impact on the term structure, especially at the long end. For example, in late 2003, the inflation-indexed bonds maturing in 2016 and 2035 had approximately the same yield. During 2004 and 2005, however, the yield of the 2035 bond fell relative to that of the 2016 bond, with the spread reaching an all-time low of -0.49% in January 2006. At that time, the 2035 and 2055 bonds had yields of 0.72% and 0.48%, respectively, which are extremely low relative to the historical average of 3% of long real rates in the U.K. In accordance with the generally-held view that long yields had decreased because of demand by pension funds, the government agreed in 2005 to issue bonds with maturities of up to 50 years, while also shifting the overall mix of maturities towards the long term.

The inversion at the long end of the U.K. term structure is hard to rationalize within standard representative-agent models. Indeed, in these models the interest rate for maturity $T$ is determined by the willingness of the representative agent to substitute consumption between times 0 and $T$. Therefore, these models would attribute the drop in the 30-year interest rate to the unlikely scenario that the pension reform signalled a drop in aggregate consumption 30 years into the future. In a similar spirit, the expectations hypothesis of the term structure would attribute the drop in the 2035-2016 yield spread to expectations about short-term interest rates past 2016 decreasing sharply during 2004 and 2005. A more plausible explanation is that the reform triggered high demand for long-term bonds by pension funds, and that this generated price pressure.

2.5 Summary

The examples in this section describe a variety of demand shocks that had significant and long-term price effects. A natural question is why arbitrageurs are unable to absorb such shocks and bring prices back to fundamental values. For example, why don’t arbitrageurs eliminate the abnormally high expected returns following fire sales by buying the stocks in question? And why were arbitrageurs unable to eliminate the Palm-3Com mispricing by shorting Palm and buying 3Com? Using a simple model, we next examine the constraints faced by arbitrageurs and the implications
for asset prices.

3 A SIMPLE MODEL

3.1 Cross-Asset Arbitrage and Intertemporal Arbitrage

We consider an economy where assets are traded in Period 1 and pay off in Period 2. There are two risky assets, $A$ and $B$, paying off $d_A$ and $d_B$, respectively. We denote by $\bar{d}_i$ and $\sigma_i$, respectively, the mean and standard deviation of $d_i$, $i = A, B$, and by $\rho$ the correlation between $d_A$ and $d_B$. For tractability, we assume that $d_A$ and $d_B$ are jointly normal. We examine how shocks to the demand for asset $A$ affect that asset’s equilibrium price $p_A$ in Period 1. For simplicity, we take as exogenous both the price $p_B$ of asset $B$ in Period 1, and the riskless rate. We set the former equal to asset $B$’s expected payoff $\bar{d}_B$, and the latter to zero.

There are two types of agents: outside investors and arbitrageurs. Outside investors’ demand for asset $A$ is inelastic and equal to $u$ shares (with $u$ positive or negative). We refer to $u$ as the demand shock: it is a constant for now, but stochastic in Section 3.2. Arbitrageurs are competitive, risk averse and maximize expected utility of wealth $W_2$ in Period 2. For tractability, we assume that utility is exponential. By possibly reinterpreting the demand shock $u$ as net demand, we normalize the supply of asset $A$ to zero. This normalization ensures that absent a demand shock ($u = 0$), the price of asset $A$ equals the asset’s expected payoff $\bar{d}_A$.

A demand shock $u \neq 0$ can push the price of asset $A$ away from the expected payoff. Arbitrageurs trade to profit from this discrepancy. Doing so, they also provide liquidity to outside investors. Suppose, for example, that $u$ is positive, in which case outside investors wish to buy asset $A$. Arbitrageurs provide liquidity because they take the opposite side of this transaction, shorting the asset and limiting the price rise. Liquidity is high when the demand shock’s price impact is small: if, for example, $u$ is positive, high liquidity means that the price rise is small.

The model has two interpretations, capturing different but closely related real-life arbitrage situations. In the “cross-asset arbitrage” interpretation, assets $A$ and $B$ are different and arbitrageurs use asset $B$ to hedge their position in asset $A$. In the “intertemporal arbitrage” interpretation, arbitrageurs exploit discrepancies between the prices of the same asset at different points in time. In that interpretation, the arbitrageurs’ positions in assets $A$ and $B$ represent trades that arbitrageurs execute in the same asset in different periods. The two interpretations yield the same basic insight:
the price effects of demand shocks depend on the risk aversion of arbitrageurs and on the risk they cannot hedge away.

Cross-Asset Arbitrage

In Period 1, the arbitrageurs choose positions \( x_A \) and \( x_B \) in assets \( A \) and \( B \) to maximize expected utility

\[
-E_1 [\exp(-\alpha W_2)]
\]

subject to the budget constraint

\[
W_2 = W_1 + x_A(d_A - p_A) + x_B(d_B - p_B),
\]

where \( \alpha \) denotes the risk aversion of arbitrageurs and \( W_1 \) their wealth in Period 1.\(^7\) Substituting (2) into (1), and using normality and the assumption that \( p_B = \bar{d}_B \), we find that arbitrageurs maximize the mean-variance objective

\[
x_A(\bar{d}_A - p_A) - \frac{\alpha}{2} (x_A^2 \sigma_A^2 + x_B^2 \sigma_B^2 + 2x_Ax_B\rho \sigma_A \sigma_B).
\]

The optimal demand for asset \( B \) is \( x_B = -\rho \sigma_A / \sigma_B x_A \), i.e., arbitrageurs choose a position in asset \( B \) to hedge that in asset \( A \). Substituting the optimal \( x_B \) into (3), we can write the arbitrageurs’ objective in a way involving only \( x_A \):

\[
x_A(\bar{d}_A - p_A) - \frac{\alpha}{2} x_A^2 \sigma_A^2(1 - \rho^2).
\]

The optimal demand for asset \( A \) is

\[
x_A = \frac{\bar{d}_A - p_A}{\alpha \sigma_A^2(1 - \rho^2)}.
\]

Asset \( A \) being in zero supply, market clearing requires

\[
x_A + u = 0.
\]

Combining (5) and (6), we find the equilibrium price of asset \( A \):

\[
p_A = \bar{d}_A + \alpha \sigma_A^2(1 - \rho^2)u.
\]

\(^7\)Our analysis of cross-asset arbitrage is related to Wurgler & Zhuravskaya (2002). As in our model, they take the price of asset \( B \) to be exogenous and equal to the asset’s expected payoff. Unlike in our model, however, they restrict arbitrageurs’ aggregate dollar investment in assets \( A \) and \( B \) to be zero.
Eq. (7) highlights a number of properties. First, larger demand shocks have a larger price impact ($\partial p_A/\partial u > 0$) because arbitrageurs must be compensated for bearing more risk. Second, a given demand shock $u$ has a larger price impact when arbitrageurs are more risk averse (large $\alpha$) and asset $A$ has a more uncertain payoff (large $\sigma_A$) because in both cases arbitrageurs require more compensation for bearing its risk. The shock’s impact is also larger when the payoff of asset $A$ is less correlated with that of asset $B$ (small $|\rho|$) because arbitrageurs are less able to hedge their position in asset $A$ using asset $B$. Thus, assets with higher idiosyncratic risk and fewer substitutes are more sensitive to demand shocks. Note that in the special case where assets $A$ and $B$ have identical payoffs ($d_A = d_B$, i.e., $\rho = 1$), the demand shock has no effect and the two assets trade at the same price. Arbitrageurs are able to align the price of asset $A$ fully with that of asset $B$ because they bear no risk in exploiting price discrepancies between the two assets.

**Intertemporal Arbitrage**

In cross-asset arbitrage, arbitrageurs exploit discrepancies between the prices of different assets at a given point in time. We next consider intertemporal arbitrage, where arbitrageurs exploit discrepancies between the prices of the same asset at different points in time. To interpret our model as one of intertemporal arbitrage, we split Period 1 into subperiods $1A$ and $1B$, and assume that positions $x_A$ and $x_B$ in assets $A$ and $B$ represent trades in the same asset in subperiods $1A$ and $1B$, respectively. The model so derived is a simplified version of Grossman & Miller (1988).

We denote by $d$ the asset’s payoff in Period 2, by $\bar{d}_A$ and $\bar{d}_B$ the expectations of $d$ as of subperiods $1A$ and $1B$, respectively, and by $\sigma_A$ and $\sigma_B$ the respective standard deviations of $d$. The expectation $\bar{d}_B$ is random as of subperiod $1A$ if new information arrives between subperiods $1A$ and $1B$, and has variance $\sigma_A^2 - \sigma_B^2$.\(^8\)

Since in subperiod $1B$ arbitrageurs can trade the asset at a price equal to its expected payoff, they are not compensated for bearing risk. Therefore, their position in subperiod $1B$ is zero, which means that their trade $x_B$ in subperiod $1B$ is the opposite of their trade $x_A$ in subperiod $1A$. Using $d_A = d_B = d$, $p_B = \bar{d}_B$ and $x_A = -x_B$, we can write (2) as

$$W_2 = W_1 + x_A(\bar{d}_B - p_A).$$

(8)

Substituting (8) into (1), we find that arbitrageurs choose their position $x_A$ in subperiod $1A$ to

\(^8\)This follows from $\text{Var}(d|1A) = E[\text{Var}(d|1B)|1A] + \text{Var}[E(d|1B)|1A]$. 
maximize the mean-variance objective

\[ x_A (\bar{d} - p_A) - \frac{\alpha}{2} x_A^2 (\sigma_A^2 - \sigma_B^2). \]  

(9)

This objective is identical to (4), derived under cross-sectional arbitrage, provided that we set \( \rho \equiv \sigma_B/\sigma_A < 1 \). The parameter \( \rho \), equal to the correlation between assets \( A \) and \( B \) under cross-sectional arbitrage, reflects the informational similarity between subperiods \( \bar{1}A \) and \( \bar{1}B \) under intertemporal arbitrage: when, for example, \( \rho = 1 \), the information available in the two subperiods is identical. Maximizing (9) yields the demand (5), and imposing market clearing yields the equilibrium price (7). As in the case of cross-sectional arbitrage, the demand shock has a larger price impact when arbitrageurs are more risk averse (large \( \alpha \)) and bear more risk. In the case of intertemporal arbitrage, the relevant risk is that between subperiods \( \bar{1}A \) and \( \bar{1}B \), and is measured by \( \sigma_A^2 - \sigma_B^2 = \sigma_A^2 (1 - \rho^2) \). This is because arbitrageurs offset their risky position in subperiod \( \bar{1}A \) by an opposite position in subperiod \( \bar{1}B \). In the extreme case where subperiods \( \bar{1}A \) and \( \bar{1}B \) coincide, \( \rho = 1 \), and the demand shock has no effect. As Grossman & Miller emphasize, the time between subperiods \( \bar{1}A \) and \( \bar{1}B \) can be interpreted as the time it takes for enough risk-bearing capacity to arrive in the market and fully eliminate the effect of the demand shock.

The two interpretations of our model correspond to different real-life arbitrage situations and, accordingly, to different strands of empirical studies. From an asset-pricing theory viewpoint, however, they are isomorphic. From now on, we focus on the cross-asset arbitrage interpretation of the model, and enrich it in ways that illustrate developments in the literature on the limits of arbitrage. The results we derive carry through to the intertemporal arbitrage interpretation.

### 3.2 Non-Fundamental Risk

In our baseline model, the risk borne by arbitrageurs is “fundamental risk,” i.e., the risk that the two legs of the arbitrage do not yield the same payoff. An additional type of risk stems from demand shocks when these affect prices. DeLong et al. (DSSW 1990) label this type of risk “noise-trader risk.” We use instead the term “non-fundamental risk” to emphasize that while demand shocks may be unrelated to asset payoffs, they can arise from rational behavior as Section 2 illustrates. To introduce non-fundamental risk in our model, we assume that the variables \( \bar{d}_A, \bar{d}_B \) and \( u \), which

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\(^9\)A recent literature derives the slow arrival of new investors from search costs. See, for example, Weill (2007), Duffie et al. (2008), Duffie & Strulovici (2009) and Lagos et al. (2009). See also He and Xiong (2008), who derive capital immobility and segmentation from agency frictions: preventing traders from moving across assets can provide their employer with a better signal of their effort.
are constant as of Period 1, are random as of an initial Period 0. Fundamental risk arises in Period 0 because prices in Period 1 depend on the realizations of \( \bar{d}_A \) and \( \bar{d}_B \). Non-fundamental risk arises because asset A’s price in Period 1 depends on the realization of \( u \). We assume that fundamental risk in Period 0 is the same as in Period 1, i.e., the standard deviation of \( \bar{d}_i, i = A, B, \) is \( \sigma_i \), and the correlation between \( \bar{d}_A \) and \( \bar{d}_B \) is \( \rho \). We denote the standard deviation of \( u \) by \( \sigma_u \), and assume \( u \) to be independent of \( \bar{d}_A \) and \( \bar{d}_B \). We maintain the notation \( p_i \) for the price of asset \( i \) in Period 1.

Eq. (7) implies that as of Period 0, the standard deviation of asset A’s price in Period 1 is

\[
\sigma_A \sqrt{1 + \alpha^2 \sigma_A^4 (1 - \rho^2)^2 \sigma_u^2}.
\]  

Eq. (10) shows that non-fundamental risk \( \sigma_u \) increases asset A’s volatility. The effect is through the second term inside the square root, which is the ratio of non-fundamental variance \((\alpha^2 \sigma_A^4 (1 - \rho^2)^2 \sigma_u^2)\) to fundamental variance \((\sigma_A^2)\). This ratio is larger when the demand shock is less predictable (large \( \sigma_u \)) and when a given demand shock \( u \) has a larger price impact in Period 1. Consistent with (7), price impact is larger when arbitrageurs are more risk averse (large \( \alpha \)), and the payoff of asset A is more uncertain (large \( \sigma_A \)) and less correlated with the payoff of asset B (small \(|\rho|\)).

Eqs. (7) and \( p_B = \bar{d}_B \) imply that as of Period 0, the correlation between the prices of assets A and B in Period 1 is

\[
\frac{\rho}{\sqrt{1 + \alpha^2 \sigma_A^4 (1 - \rho^2)^2 \sigma_u^2}}.
\]  

Eq. (11) shows that non-fundamental risk lowers the correlation between assets A and B. This is because it increases the volatility of asset A without affecting asset B.

Suppose next that there is a demand shock in Period 0, and an arbitrageur responds to the shock by taking a position in assets A and B. Since non-fundamental risk increases asset A’s volatility and lowers its correlation with asset B, it increases the volatility of the arbitrageur’s return in Period 1. This volatility matters when the arbitrageur has a short horizon and must close his position in Period 1. Price volatility caused by demand shocks in Period 1 deters the arbitrageur from absorbing demand shocks in Period 0. DSSW build on this idea to show that non-fundamental risk can be self-fulfilling. They assume a discrete-time infinite-horizon economy with an exogenous riskless rate \( r \) and an asset paying a constant dividend \( r \) in each period. Since one share of the asset yields the same payoff as an investment of one dollar in riskless rate, the law of one price implies that the asset’s price should be one. DSSW show, however, that when arbitrageurs have a one-period
horizon an equilibrium exists in which this price is stochastic. Intuitively, if arbitrageurs expect the price to be stochastic, demand shocks have an effect, and this renders the price stochastic.

The stochastic equilibrium of DSSW hinges on a number of assumptions. One is that arbitrageurs have short horizons: if they were infinitely lived, they would enforce the law of one price through buy-and-hold strategies. Short horizons can be viewed as a reduced form for financial constraints, as we show in Sections 3.4 and 3.5. A second critical assumption is that of an infinite horizon: with a finite horizon, the law of one price would hold.\footnote{Our model confirms this: if assets $A$ and $B$ have the same payoff, the correlation $\rho$ is one, and the non-fundamental risk in (10) and (11) disappears.} Lowenstein & Willard (2006) show that even under these two assumptions, the law of one price would hold in DSSW if interest rates were endogenized, prices were precluded from becoming negative, or borrowing limits were imposed. But while DSSW’s result on the failure of the law of one price may not be robust, their broader point that non-fundamental risk is an impediment to arbitrage remains important.\footnote{Related to non-fundamental risk, which stems from demand shocks, is a risk stemming from the actions of other arbitrageurs. Abreu & Brunnermeier (2002; 2003) assume that arbitrageurs learn about a profitable investment opportunity privately at different points in time, so the opportunity’s existence is not common knowledge. Arbitrageurs find it profitable to trade on the opportunity if they expect other arbitrageurs to trade on it in the near future. Lack of common knowledge, however, prevents such coordination, and can cause the opportunity to persist even after all arbitrageurs have become aware of its existence. For coordination frictions in arbitrage, see also Zigrand (2005).}

### 3.3 Short-Selling Costs

In our analysis so far, the only cost arbitrageurs face is risk. Additional costs, however, stem from the way arbitrageur positions are established and financed. Arbitrageurs often establish their positions in the repo market. For example, an arbitrageur wishing to establish a long position in an asset can borrow some of the needed cash by posting the asset as collateral—a repo transaction. Conversely, an arbitrageur wishing to establish a short position in an asset can borrow (to subsequently sell) the asset by posting cash as collateral—a reverse repo transaction. The interest rate earned on the cash, known as the repo rate, can differ across assets and this can be a source of arbitrage costs. For example, shorting an asset that carries a low repo rate relative to other assets is costly because the cash collateral posted by the short-seller earns a below-market interest rate.

Costs involved in establishing and financing positions are often referred to as holding costs. Tuckman & Vila (1992, 1993) introduce exogenous holding costs and show that they prevent arbitrageurs from eliminating mispricings. Arbitrageurs trade against mispricings only when these are large enough to compensate them for the holding costs they incur. Dow & Gorton (1994) show that holding costs can have disproportionately large effects when they are incurred by a sequence
of short-horizon arbitrageurs.

Short-selling costs are holding costs associated with short positions. We introduce short-selling costs in our model by assuming that shorting asset $A$ involves a cost $c$ per share. Arbitrageurs maximize the objective

$$x_A(d_A - p_A) - \frac{\alpha}{2} \left( x_A^2 \sigma_A^2 + x_B^2 \sigma_B^2 + 2 x_A x_B \rho \sigma_A \sigma_B \right) - c|x_A|1_{\{x_A < 0\}},$$

(12)

where $1_S$ is equal to one if condition $S$ is satisfied and zero otherwise. Eq. (12) is derived from (3) by subtracting the cost $c|x_A|$ of holding a short position $x_A$ in asset $A$. Solving for the optimal demand for asset $A$ and combining with the market-clearing condition (6), we find that the equilibrium price of asset $A$ is given by

$$p_A = d_A + \alpha \sigma_A^2 (1 - \rho^2) u \quad \text{if} \quad u \leq 0,$$

(13a)

$$p_A = d_A + \alpha \sigma_A^2 (1 - \rho^2) u + c \quad \text{if} \quad u > 0.$$

(13b)

Short-selling costs affect the price only when the demand shock $u$ is positive because this is when arbitrageurs hold a short position. The price increases by an amount equal to the short-selling cost $c$, so that arbitrageurs are compensated for incurring $c$.

Short-selling costs have an effect even when the assets have identical payoffs, and in that case they cause the law of one price to be violated. Setting $\rho = 1$ in (13b), we find $p_A = \bar{d} + c$, where $\bar{d}$ denotes the expected payoff of the two assets ($\bar{d} = \bar{d}_A = \bar{d}_B$). Therefore, when the demand shock $u$ is positive, the price of asset $A$ exceeds that of the identical-payoff asset $B$ by an amount equal to the short-selling cost $c$. This analysis fits well with the Palm-3Com example: $c$ may be the cost of shorting Palm and $u$ the demand by investors eager to hold Palm over 3Com. D’Avolio (2002) and Lamont & Thaler (2003) report that $c$ was large and prevented arbitrageurs from exploiting the mispricing.

In our model, the short-selling cost $c$ is an exogenous deadweight loss. A number of papers seek to endogenize $c$ based on frictions in the repo market. In Duffie (1996), short-sellers have the choice between two assets with identical payoffs but different exogenous transaction costs. They prefer to short the low transaction cost asset, and their demand to borrow that asset in the repo market lowers the repo rate, driving up the short-selling cost. The friction in the repo market is that asset owners must incur an exogenous transaction cost to lend their asset. Unlike in our model, the short-selling cost $c$ is not a deadweight loss, but accrues to asset owners. Therefore, it is an additional payoff earned from holding the asset and increases the asset price—even in the
absence of any positive demand shock $u$. Krishnamurthy (2002) uses a similar model to show that a price premium arising from short-selling costs can coexist with a premium arising from an asset’s superior liquidity.

Duffie et al. (2002) model the repo market as a search market in which asset borrowers and lenders negotiate bilaterally. The search friction generates a short-selling cost, which increases in the demand for short-selling. Vayanos & Weill (2008) show that the combination of a search spot market and a search repo market yields violations of the law of one price—even in the absence of any exogenous differences in transaction costs. Short-sellers concentrate on the more liquid asset, and their activity is what renders the asset more liquid. The more liquid asset carries a price premium both because of its superior liquidity and because high demand for short-selling drives up short-selling costs.

In the extreme case where short-selling costs are infinite, they amount to short-sale constraints, whose implications for asset prices are examined in a number of papers. Miller (1977) shows that when short-sales are not allowed, pessimistic investors are unable to trade and prices reflect the valuation of the most optimistic investors. Harrison & Kreps (1978) show that with multiple trading periods, prices even exceed the valuation of investors who are currently the most optimistic. Indeed, these investors have the option to resell the asset should other investors become more optimistic in future periods. Scheinkman & Xiong (2003) value the resale option in a continuous-time model where differences in beliefs stem from overconfidence. Hong et al. (2006) show that overpricing and the value of the resale option are highest for assets with low float. Diamond & Verrecchia (1987) show that short-sale constraints do not cause overpricing when differences in beliefs stem from private signals rather than an agreement to disagree. Indeed, the market adjusts rationally for the fact that investors with negative private signals are unable to trade. Bai et al. (2006) show that short-sale constraints can cause underpricing because they generate uncertainty about the extent of negative private information. Hong & Stein (2003) show that the occasional release of negative private information can be the source of market crashes.

3.4 Leverage Constraints

In our analysis so far, there is no explicit role for arbitrageur capital: while portfolio decisions and asset prices depend on arbitrageur risk aversion, they are independent of arbitrageur wealth. Wealth does not matter because of the simplifying assumption that arbitrageurs have exponential utility, i.e., their coefficient of absolute risk aversion is independent of wealth. Yet, the capital
available to real-life arbitrageurs appears to be an important determinant of their ability to eliminate mispricings and provide liquidity to other investors.

The study of arbitrageur capital is related to that of financial constraints. Indeed, an important theme in the literature on the limits of arbitrage is that arbitrageurs are sophisticated traders, better able to identify mispricings than other, less sophisticated investors. Since capital in the hands of arbitrageurs can earn higher return, other investors can gain by investing their capital with arbitrageurs. If, however, arbitrageurs could access external capital frictionlessly, they would be able to eliminate mispricings, and asset prices and allocations would be as in standard models. Thus, while arbitrageurs can access external capital, this access is limited by financial constraints. Note that in limiting access to external capital, financial constraints cause arbitrageurs to be under-diversified and bear a disproportionate share of the risk of arbitrage trades.

In this section we study constraints on arbitrageurs’ ability to lever up by raising margin debt. We assume that assets $A$ and $B$ have identical payoffs. This isolates the effects of leverage constraints from those of risk: arbitrageurs bear no risk when exploiting a price discrepancy between assets $A$ and $B$, and such a discrepancy can arise solely because of leverage constraints. To model leverage constraints, we focus on the mechanics of collateral in the repo market, following Geanakoplos (1997; 2003). We also assume that arbitrageurs must collateralize their positions in each asset separately. The model so derived is a simplified version of Gromb & Vayanos (2002; 2010).

Consider an arbitrageur wishing to establish a long position $x_i$ in asset $i$. The arbitrageur can borrow some of the needed cash by posting asset $i$ as collateral. The borrowed cash is typically less than the market value $x_i p_i$ of the asset collateral; otherwise, a drop in the asset price would cause the value of the collateral to drop below that of the loan. To determine the size of the loan, we assume for simplicity that margin loans have to be riskless and that competitive lenders set the rate equal to the riskless rate (which is zero). Since riskless loans are not feasible with normally distributed asset payoffs, we assume instead that $d$ is distributed symmetrically over the bounded support $[\bar{d} - \epsilon, \bar{d} + \epsilon]$, where $\bar{d} \geq \epsilon$. An arbitrageur wishing to establish a long position $x_i$ in asset $i$ can thus borrow a maximum of $x_i (\bar{d} - \epsilon)$, and must pay the remainder

$$x_i \left( p_i - \bar{d} + \epsilon \right) = x_i m_i^+. \quad (14)$$

out of his wealth. The parameter $m_i^+$ is the margin (or haircut) for a long position in asset $i$.\(^\text{12}\)

Consider next an arbitrageur wishing to establish a short position $x_i$ in asset $i$. The arbitrageur

\(^{12}\)An alternative to requiring margin loans to be riskless is to impose an upper bound on the default probability. This would yield a constraint of the same form as (14), which would be interpreted as a value-at-risk constraint.
can borrow (and subsequently sell) asset \( i \) posting cash as collateral. The cash collateral typically exceeds the proceeds \( |x_i|p_i \) from the sale of the borrowed asset; otherwise, an increase in the asset price would cause the value of the loan to rise above that of the collateral. As for long positions, we assume that margin loans have to be riskless and that competitive asset lenders pay the riskless rate on the cash collateral.\(^{13}\) An arbitrageur wishing to establish a short position \( x_i \) in asset \( i \) must post \( |x_i| (d + \epsilon) \) units of cash as collateral. Selling the asset yields \( |x_i|p_i \) units, and the remainder
\[
|x_i| (d + \epsilon - p_i) \equiv |x_i|m_i^-.
\] (15) must be drawn from the arbitrageur’s wealth. The parameter \( m_i^- \) is the margin (or haircut) for a short position in asset \( i \). Note that the margin requirements \( m_i^+ \) and \( m_i^- \) are increasing in the parameter \( \epsilon \), which is a measure of the volatility of asset \( i \). Indeed, volatility increases the maximum loss that a long or short position can experience.

From (14) and (15), the positions \( x_A \) and \( x_B \) of an arbitrageur with wealth \( W_1 \) must satisfy the following leverage constraint:
\[
W_1 \geq \sum_{i=A,B} |x_i| (m_i^+1_{\{x_i>0\}} + m_i^-1_{\{x_i<0\}}).
\] (16)
Arbitrageurs maximize expected utility (1) subject to the budget constraint (2) and the leverage constraint (16).

When can arbitrageurs eliminate price discrepancies between assets \( A \) and \( B \), thus enforcing the law of one price? The market-clearing condition (6) requires that arbitrageurs absorb the demand shock \( u \), i.e., take a position \( x_A = -u \). If, in addition, assets \( A \) and \( B \) trade at a price equal to their expected payoff \( \bar{d} \), then it is optimal for arbitrageurs not to bear risk and hold an offsetting position \( x_B = u \) in asset \( B \). Since for \( p_A = p_B = \bar{d} \) the margins \( m_i^+ \) and \( m_i^- \) are equal to \( \epsilon \), the leverage constraint (16) is satisfied if
\[
W_1 \geq 2|u|\epsilon.
\] (17)
Arbitrageurs can enforce the law of one price if their wealth \( W_1 \) is large relative to the demand shock \( u \) and the margin requirement \( \epsilon \). In that case, the demand shock has no effect on the price of asset \( A \) and the market is perfectly liquid. Intuitively, arbitrageurs can provide perfect liquidity when their wealth is large because the leverage constraint is not binding. When, however, arbitrageur wealth is small, the leverage constraint is binding and liquidity is imperfect.

\(^{13}\)This assumption eliminates the short-selling costs of Section 3.3.
Leverage constraints can give rise to amplification, whereby the effects of an exogenous shock are amplified through changes in arbitrageur positions. Amplification can be derived in our model by assuming that arbitrageurs enter Period 1 with a position from Period 0, and that outside investors’ demand is elastic. If, for example, arbitrageurs enter Period 1 with a long position, then a negative demand shock $\Delta u$ lowers the price $p_A$, thus lowering arbitrageur wealth $W_1$ and tightening the leverage constraint (16). This can force arbitrageurs to liquidate positions, amplifying the price drop. Since arbitrageurs drive the price further away from fundamental value, their activity is price-destabilizing rather than stabilizing. Moreover, since they liquidate positions following a negative demand shock, they consume rather than provide liquidity. The liquidity providers are the outside investors, and their demand must be assumed elastic so that they are willing to buy from arbitrageurs.

Leverage constraints and amplification have been studied in macroeconomic settings, starting with Bernanke & Gertler (1989) and Kiyotaki & Moore (1997). In these papers, an adverse shock to economic activity depresses collateral values, and this can amplify the drop in activity. Hart & Moore (1994; 1995) show that uncertainty about assets’ liquidation values impairs agents’ ability to borrow—a close parallel to the result that the margins in (14) and (15) increase with volatility. Shleifer & Vishny (1992) endogenize liquidation values and the ability to borrow in market equilibrium. Geanakoplos (1997; 2003) defines the concept of collateral equilibrium, in which margin contracts are endogenous, and shows that amplification can arise because margins increase following adverse shocks.

In a financial market context, Gromb & Vayanos (2002) study how leverage constraints affect the ability of arbitrageurs to eliminate mispricings and provide liquidity to outside investors. They show, within a dynamic setting, that liquidity increases in arbitrageur capital and that arbitrageurs can amplify exogenous shocks. Brummermeier & Pedersen (2009) and Gromb & Vayanos (2009a) extend this analysis to multiple assets in a static and dynamic setting, respectively. With multiple assets, leverage constraints generate not only amplification, but also contagion, whereby shocks to one asset are transmitted to otherwise unrelated assets through changes in arbitrageur positions. Pavlova & Rigobon (2008) derive a contagion result in an international-economy model with portfolio constraints, of which leverage constraints are a special case. Kondor (2009) shows that amplification can arise even in the absence of exogenous shocks, purely as a consequence of arbitrage activity. Indeed, if a price discrepancy between two assets were to remain constant or

\footnotesize{Weill (2007) derives a link between arbitrageur capital and liquidity provision in a search model. See also Duffie & Strulovici (2009), who use a search model to study the gradual flow of arbitrage capital across trading opportunities.}
decrease over time, arbitrageurs would exploit it and reduce it to a level from which it could increase. Garleanu & Pedersen (2009) show that all else equal, assets with lower margin requirements can trade at higher prices. Related results are derived in Cuoco (1997), Basak & Croitoru (2000; 2006) and Geanakoplos (2003). Other dynamic models with leverage constraints include Aiyagari & Gertler (1999), Allen & Gale (2000), Anshuman & Viswanathan (2005), Geanakoplos & Fostel (2008), Adrian et al. (2009), Chabakauri (2009), Danielsson et al. (2009) and Rytchkov (2009).  

3.5 Constraints on Equity Capital

In addition to constraints on margin debt, arbitrageurs often face constraints in raising equity. For example, the equity of a mutual fund is determined by the flow of investors into the fund, and could be lower than the level at which the fund’s profitable investment opportunities are fully exploited. Moreover, poor returns by the fund could trigger outflows by investors, and so render the fund more constrained. Shleifer & Vishny (1997) study the implications of constraints on equity capital for arbitrageurs’ ability to exploit mispricings. They show that constraints give rise to amplification, via a mechanism akin to that for leverage constraints. Suppose that arbitrageurs hold long positions in an asset. A negative shock to the asset lowers its price and triggers outflows by investors in arbitrage funds. As a consequence, arbitrageurs are forced to reduce their positions, amplifying the price drop. These results can be derived in our model by assuming that (i) part of arbitrageur wealth belongs to other investors, and is withdrawn following poor returns in Period 1, (ii) arbitrageurs cannot borrow, (iii) arbitrageurs enter Period 1 with a position from Period 0, and (iv) outside investors’ demand is elastic.

Shleifer & Vishny show additionally that constraints have an effect not only when they are binding, but also because of the possibility that they might bind in the future. Indeed, suppose that arbitrageurs with ample capital in Period 0 identify an underpriced asset. Investing heavily in that asset exposes them to the risk of large outflows by investors were the underpricing to worsen in Period 1. This would deprive arbitrageurs of capital when they need it the most since this is when the underpricing is the most extreme. As a consequence, arbitrageurs could refrain from investing heavily in the underpriced asset in Period 0, keeping instead some capital in cash. Arbitrageurs’

\[ \text{See also Yuan (2005) for a model in which leverage constraints hamper the revelation of private information, and Grossman & Vila (1992), Liu & Longstaff (2004), Jurek & Yang (2007) and Milbradt (2009) for partial-equilibrium models of portfolio choice by leverage-constrained investors. Amplification and contagion can also be derived in models without explicit leverage constraints but where arbitrageur risk aversion depends on wealth. This is done in Kyle & Xiong (2001) and Xiong (2001), who endow arbitrageurs with logarithmic utility, under which the coefficient of absolute risk aversion decreases in wealth. Following adverse shocks, arbitrageurs reduce their positions because they become more risk averse and not because they hit leverage constraints.} \]
strategy of seeking to match capital with profitable investment opportunities amounts to risk management as in Froot et al. (1993). Risk management by arbitrageurs is also derived in models with leverage constraints, e.g., Gromb & Vayanos (2002), Liu & Longstaff (2004), Brunnermeier & Pedersen (2009) and Kondor (2009), and is further emphasized in Acharya et al. (2009) and Bolton et al. (2009). Holmstrom & Tirole (2001) explore the implications of risk management by firms for the cross-sectional pricing of assets. They show that assets paying off in states where firms’ financial constraints bind are more expensive than assets paying off in other states because they provide capital when it is needed the most.

A number of papers integrate constraints on the equity capital available to arbitrage funds—and especially its dependence on past performance—into dynamic settings. In Vayanos (2004), fund managers face the constraint that their fund will be liquidated following poor performance, and this makes them unwilling to hold illiquid assets at times of high volatility. In He & Krishnamurthy (2008; 2009), the capital available to fund managers cannot exceed a fixed multiple of their personal wealth—a constraint derived from optimal contracting under moral hazard. When managers underperform, the constraint becomes binding and causes volatility and risk premia to increase. In Vayanos & Woolley (2008), investors withdraw capital following underperformance by fund managers because they infer rationally that managers might be inefficient. If, in addition, withdrawals are constrained to be gradual, they generate short-run momentum and long-run reversal in asset returns. Dasgupta et al. (2008) derive short-run momentum and long-run reversal from herding by fund managers, caused by reputation concerns. Guerrieri & Kondor (2009) show that reputation concerns generate amplification effects.\(^\text{16}\)

### 3.6 Summary and Next Steps

Sections 3.1-3.5 show how an array of costs faced by arbitrageurs limit their ability to eliminate mispricings and provide liquidity to outside investors. This section sketches what we view as two important next steps in this research agenda: (i) derive the financial constraints of arbitrageurs within an optimal contracting framework, and (ii) develop richer limits-of-arbitrage models that incorporate multiple assets and dynamics and that can be used to address empirical puzzles.

As Sections 3.4 and 3.5 emphasize, arbitrageurs face financial constraints stemming from agency

\(^{16}\)Reputation concerns are also explored in Dasgupta & Prat (2008), and Malliaris & Yan (2009). Other papers studying general equilibrium implications of delegated portfolio management include Cuoco & Kaniel (2009), Petajisto (2009), Basak & Pavlova (2010) and Kaniel & Kondor (2010).
problems with their providers of capital. Most of the papers referenced in these sections do not fully endogenize the constraints. For example, papers on leverage constraints typically rule out equity and papers on equity constraints typically rule out debt. Deriving financial constraints within an optimal contracting framework is an important next step. Indeed, while the constraints studied in the literature take a variety of forms, they tend to generate some similar results, e.g., amplification, contagion, and risk management by arbitrageurs. Endogenizing the constraints would help identify whether the common results are driven by a single underlying friction, or whether the constraints are fundamentally different. Identifying the frictions that underlie the constraints would also be useful for policy analysis, as it would clarify what a regulator can and cannot do to alleviate the effects of the constraints.

Starting with Kehoe and Levine (1993), a macroeconomic literature explores the asset pricing implications of financial constraints when these derive from limited commitment by borrowers. Limited commitment is also the source of the constraints in Kiyotaki and Moore (1997), while Holmstrom & Tirole (2001) derive the constraints from moral hazard. Constrained agents in these papers are real-economy firms having access to productive technologies not available to other investors. The interpretation of constrained agents as financial firms is made explicit in He & Krishnamurthy (2008; 2009), who derive constraints on fund managers uniquely able to invest in a subset of traded assets. The constraints stem from moral hazard and contracts are restricted to be static.\footnote{See also Acharya & Viswanathan (2009), who derive constraints from asset substitution, and Hombert and Thesmar (2009) and Stein (2009), who study arbitrageurs’ choice between short- and long-term financing.} Extending this line of research to dynamic contracts, while retaining the tractability that is necessary to compute asset prices in general equilibrium, would be an important step forward. A recent literature on optimal dynamic financial contracting in continuous time, e.g., Biais et al. (2007) and DeMarzo & Fishman (2007), could be useful in this respect. The result of that literature that investors punish underperforming managers by scaling down their firms fits with the idea that investors reduce their stakes in underperforming funds.\footnote{Bolton & Scharfstein (1990) and Gromb (1994) derive this result in finite-horizon discrete-time settings, and Heinkel & Stoughton (1994) derive the result in a two-period fund-management setting. See also Biais et al. (2010), who show the converse result that investors reward overperforming managers by scaling up their firms.}

Our understanding of the costs faced by arbitrageurs has greatly benefited from papers exploring these costs in relatively simple and stylized settings. An important next step is to integrate limits-of-arbitrage ideas more squarely into asset pricing theory by developing tractable dynamic multi-asset models that can address empirical puzzles. Work along these lines could take constraints as given and so proceed in parallel with work on optimal contracting—although an important objective should remain that the two lines of research eventually merge.
A number of papers explore dynamic multi-asset equilibrium settings under the assumption that the only cost faced by arbitrageurs is risk. In Greenwood (2005) and Hau (2009a), arbitrageurs absorb demand shocks of index investors following index redefinitions. In Gabaix et al. (2007), arbitrageurs are uniquely able to hold mortgage-backed securities, and while they can hedge interest-rate risk in the government bond market, they bear prepayment risk. In Garleanu et al. (2009), arbitrageurs absorb demand shocks in the options market, and while they can hedge delta risk in the stock market, they bear jump and volatility risk. In Vayanos & Vila (2009) and Greenwood & Vayanos (2010b), arbitrageurs absorb shocks to the demand and supply of government bonds with specific maturities, and hedge by trading bonds with other maturities. A common theme in these papers is that arbitrageurs transmit shocks to the demand for one asset to other assets, with the effects being largest for assets that covary the most with the original asset. This has implications for phenomena as diverse as index effects in the stock market, the pricing of prepayment risk in the mortgage market, the behavior of implied volatility surfaces in the options market, and the behavior of risk premia in the government bond market. Hau (2009b), Jylha & Suominen (2009) and Plantin & Shin (2009) pursue similar themes for the foreign exchange market, and Naranjo (2009) does so for the futures market.

Other papers explore dynamic multi-asset equilibrium settings under the assumption that arbitrageurs face financial constraints. These papers are referenced in the last paragraph of Section 3.4 for the case of leverage constraints and Section 3.5 for the case of constraints on equity capital. They constitute a rapidly growing literature which has the potential to explain a variety of market anomalies based on limits of arbitrage and institutional frictions more broadly.

4 WELFARE AND POLICY

Financial crises, including the recent one, provide a painful reminder that government intervention can be important for the smooth functioning of financial markets. Arguments for intervention often center around the idea that failing financial institutions can cause disruptions in asset markets and the effects can propagate to other institutions. Standard models of asset pricing cannot be used to evaluate the merits of such arguments because they abstract away from financial frictions and institutions. In these models equilibrium is Pareto optimal and there is no scope for intervention.19 Research on the limits of arbitrage has the potential to deliver a more useful framework for designing

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19The same is true for models in which arbitrageur wealth effects derive from wealth-dependent risk aversion rather than explicit financial constraints (e.g., Kyle & Xiong (2001) and Xiong (2001)).
and assessing public policy. Indeed, this research emphasizes the role of specialized institutions in providing liquidity in asset markets. Understanding how the financial health of these institutions is affected by their trading decisions, and whether trading decisions are socially optimal, can guide public policy. Despite its relevance, the welfare analysis of asset markets with limited arbitrage is still in its infancy. In this section we survey the existing work and highlight what we view as the main issues, challenges and promises.

We start by clarifying some methodological issues. Several papers study whether constrained arbitrageurs stabilize or destabilize asset prices, i.e., decrease or increase volatility. This is only indirectly a welfare question; assessing welfare by means of a utility-based criterion, such as Pareto, is preferable. Given a welfare criterion, a natural question is whether constraints lower efficiency, i.e., is unconstrained arbitrage better than constrained arbitrage?20 This question, however, is of limited relevance for assessing policy. Indeed, if arbitrageurs face constraints, one should not assume that a regulator could remove or even relax them. A more suitable criterion is constrained efficiency: can a regulator increase welfare relative to the equilibrium by choosing new allocations that are nevertheless subject to the constraints? At this juncture, the literature has taken two routes, which we discuss next.

The first route is to retain a traditional dynamic equilibrium asset pricing model, but not fully endogenize the constraints. This is done in Gromb & Vayanos (2002), who study how leverage constraints affect arbitrageurs’ ability to provide liquidity. The main result in terms of welfare is that equilibrium can fail to be constrained efficient and a reduction in arbitrageur positions can make all agents better off. The intuition is as follows. Following an adverse shock, arbitrageurs incur capital losses and are forced to liquidate positions because their leverage constraints tighten. As a result, they find themselves more constrained and less able to provide liquidity—at a time when liquidity is low and its provision profitable. Ex-ante, arbitrageurs account for this possibility and engage in risk management by keeping some capital in cash to exploit episodes of low liquidity. However, they fail to account for the impact of their liquidations on other arbitrageurs during such episodes. Indeed, liquidation by one arbitrageur hurts other arbitrageurs because it lowers the price at which they can liquidate. This can hurt not only arbitrageurs but also outside investors because of the reduced liquidity that they receive.

The second route has been followed by papers in macroeconomics and international economics, e.g., Caballero & Krishnamurthy (2001), Lorenzoni (2008), Acharya et al. (2009), Hombert (2009),

20 A related question is whether the presence of arbitrageurs is beneficial, i.e., is constrained arbitrage better than no arbitrage at all? One would expect the two questions to generally have a positive answer: relaxing constraints should be desirable because arbitrageurs provide liquidity.
Korinek (2009). These papers derive financial constraints endogenously from contracting frictions in three-period settings. The constraints, however, concern real-economy firms having access to productive technologies, rather than financial firms such as arbitrageurs. Inefficiencies in these papers arise because of a similar mechanism as in the previous paragraph: firms do not account for the impact of their liquidations on the prices at which other firms can liquidate.\footnote{21}

The mechanism causing the inefficiencies is a pecuniary externality operating through price changes and the redistribution of wealth that these generate. A redistribution of wealth cannot be Pareto improving when markets are complete because marginal rates of substitution across time and states of nature are identical for all agents. However, when markets are incomplete, as is the case with financial constraints, marginal rates of substitution differ and Pareto improvements are possible. Geanakoplos & Polemarchakis (1986) show this mechanism in a general incomplete-markets setting.\footnote{22}

This constrained inefficiency of limited arbitrage opens the door for an analysis of policy (see Gromb & Vayanos (2002; 2009b) for a discussion). Suppose, for example, that arbitrageurs take excessive risk. A regulator might decrease their risk-taking incentives by tightening their financial constraints with a risk-based capital requirement. Alternatively, better risk-management could be enforced directly by taxing arbitrageurs in good times and possibly subsidizing them in bad times, in effect managing part of their resources for them. Subsidies in bad times can be implemented through lender of last resort policies or asset purchase programs.\footnote{23} Last, imperfect competition among arbitrageurs might lead them to internalize part of the price effects of their investment decisions, and so adopt more socially desirable investment policies.\footnote{24} This research agenda has the potential to inform debates on systemic risk, macro-prudential regulation, and lending of last resort, topics that are highly relevant in the context of financial crises.

\footnote{21}{See also Biais & Mariotti (2009) for a similar inefficiency in a labor economics context, and Nikolov (2009) for a calibration exercise. }

\footnote{22}{See also Stiglitz (1982), who shows that in incomplete financial markets, the competitive equilibrium may fail to be constrained efficient. }

\footnote{23}{For a discussion of such policies, see Krishnamurthy (2009) and the references therein. Krishnamurthy argues that such policies are desirable because they can move the market to a more efficient equilibrium. }

\footnote{24}{Much of the literature on the limits of arbitrage, including this article, assumes that arbitrageurs are perfectly competitive. Imperfect competition constitutes an additional limit of arbitrage since arbitrageurs with market power would require a larger price concession to absorb a demand shock. The industrial organization of arbitrage activity is a largely unexplored area of research. It is related to financial constraints since that literature assumes (often implicitly) that barriers to entry prevent the flow of new capital into the arbitrage industry. For models with imperfectly competitive arbitrageurs see, for example, Attari and Mello (2006), Zigrand (2006) and Oehmke (2009).}
Literature Cited


