Do Reputational Concerns Lead to Reliable Ratings?

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Do Reputational Concerns Lead to Reliable Ratings?

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Abstract

This paper examines to what extent reputational concerns give rating agencies incentives to reveal information. It demonstrates that, in a simple model in which a rating agency has public and private information about a project, it may ignore private information and even contradict public information in an attempt to minimize reputational costs. A monopolistic agency can act conservatively by issuing too many bad ratings when a project is expected to be good based on private and public information. In a competitive setting, an agency becomes bolder and can issue too many good ratings when a project is expected to be bad based on private and public information. The paper provides a reason for why competition in the ratings industry might lead to overly optimistic ratings even in the absence of conflicts of interest.

Keywords: Reputation, rating agencies, competition, conformism, conservatism, boldness.

JEL Classifications: D82, G1, G24

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1 Introduction

The role of rating agencies is to provide information to investors about the ability of firms or other institutions to timely repay their debt obligations. Reputation is rating agencies’ main asset, since it confers credibility to their announcements and consequently makes firms hire their services. The Economist\textsuperscript{1} summarizes the importance of reputation for rating agencies as follows:

“Even more than for accountants and lawyers, they [rating agencies] must trade on their reputations. If bond investors lose faith in the integrity of rating agencies’ judgments, they will no longer pay attention to their ratings; if rating agencies’ opinions cease to affect the price that borrowers pay for capital, companies and governments will not pay their fees. So market forces should make rating agencies careful of their good names”

Therefore, one would expect reputational concerns to be a strong motive for rating agencies to try hard not to make mistakes and to use all available information, both public (e.g. accounting statements) and private (e.g. confidential interviews) when reporting their judgments to investors. Of course these two sources of information need to be balanced if the objective is to avoid mistakes; an accurate rating should incorporate private information, but issuing a rating according to public expectations might be the best strategy for a rating agency whose private information is noisy.\textsuperscript{2} However, this paper shows that reputational concerns can actually destroy this balance and generate too little reliance on private information, and in some circumstances, even on public information.

The aim of the paper is therefore to assess how reliable the information transmitted by rating agencies that worry about reputation can be taking into account the trade-off between private and public information, and also to explore in what way the structure of the ratings industry affects information revelation. It demonstrates that reputational concerns combined

\textsuperscript{1}Use and Abuse of Reputation”, The Economist, page 20, April 6th, 1996.

\textsuperscript{2}Because private information might be incorrect or difficult to interpret or even because the rating agency is unsure about the quality of its risk assessment models.
with poor quality of private information and an environment in which this information is easy to manipulate generate situations in which rating agencies can be overly pessimistic or overly optimistic in their ratings depending on how the ratings industry is organized. The results hold even though the model abstracts from conflicts of interest and repeated relationships between rating agencies and the firms that hire them, or bribes.

Ratings are widely used to mitigate asymmetric information among market participants and also for regulatory purposes. For example, the regulatory regime requires or encourages investors such as broker-dealers, banks, insurance companies or pension funds, to purchase financial instruments that are rated investment grade, and borrowers’ credit ratings are used to calculate capital adequacy ratios of banks. For these reasons, it is of foremost importance to understand how rating agencies behave and which mechanisms can be put into practice to increase the informational content of ratings.

In the model presented below, there is an entrepreneur at each time period who develops a project that can be good or bad and that is fully financed by outside investors using debt. Ratings are mandatory and a rating can be good or bad. Investors demand a good rating to finance the project. There is one rating agency that has both public and private information about the type of the project. Public information is the prior belief that the project is good. Depending on the value of the prior belief a project is defined as expected to be very bad, bad, good or very good. Private information takes the form of a private signal. This private signal can be perfect or noisy which means that a rating agency can be of two types: it can perfectly identify the project’s type or it can make small mistakes. Both investors and the entrepreneur are unsure about the rating agency’s type but attach a subjective probability to a rating agency not making mistakes. I refer to this probability as the rating agency’s reputation. The fee paid by the entrepreneur for rating services will be shown to increase with reputation.

When a project is expected to very good or very bad (and even bad) based on public information, a rating agency that makes mistakes often chooses to conform to public information going against what its private information indicates because of fears of being wrong.
For example, if investors expect the project to be very good (very bad) based on public information and the agency’s private information indicates that the project is bad (good), there are situations in which it chooses to issue a good (bad) rating. I refer to this behavior as *conformism*.

However, when a project is expected to be good based on public information there are situations in which a rating agency that makes mistakes chooses to contradict this information even when the agency’s private information indicates that the project is good, and issues a bad rating. I refer to this behavior as *conservatism*. This happens because the rating agency wants to minimize reputational costs. A project issued with a good rating is undertaken and its outcome is verifiable. In this case, a correct rating boosts the rating agency’s reputation and consequently, the future fee that it can charge; and an incorrect rating reveals the rating agency’s type and lowers this future fee. A project issued with a bad rating is not undertaken as such project does not receive funding from outside investors. Its outcome is therefore not verifiable. This limits the learning process about the rating agency’s type and allows a rating agency that makes mistakes to keep its reputation (and the future fee) reasonably unscathed. Sending a bad rating is therefore seen as a safer option for such a rating agency.

Finally, the paper looks at what happens when two rating agencies compete to rate a project. It will be shown that the rating agency that is hired by the entrepreneur is the one with the highest reputational level. In a competitive setting a rating agency issues good ratings more often. In fact, when a project is expected to be bad based on public information there are situations in which a rating agency that makes mistakes chooses to contradict this information even when the agency’s private information indicates that the project is bad, and issues a good rating. I refer to this behavior as *boldness*. This happens because a rating agency needs to persuade the entrepreneur and investors that it is of the type that does not make mistakes or otherwise the entrepreneur might prefer hiring the competitor rating agency. As in the monopolistic case, a project issued with a good rating allows the rating agency to boost its reputation and to continue being hired by the entrepreneur if this rating is correct. If the rating is incorrect, the rating agency’s type is revealed and the entrepreneur prefers
hiring the competitor. However, a bad rating also entails some risks in a competitive setting. By not being verifiable a bad rating does not necessarily boost reputation and it can even be perceived by investors as a likely mistake, in particular when a project is expected to be good or very good based on public information. The rating agency’s type is not revealed but its chances of continuing being hired by the entrepreneur can also be damaged. Hence, the gains for a rating agency from persuading everyone else that it is of the type that does not make mistakes and keeping the competitor away tend to outweigh the penalization that losing business inflicts on it if a good rating turns out to be a mistake.

**Related Literature**

This paper contributes to the literature on competition and information revelation for financial intermediaries by explicitly considering the role of reputation and the role of public versus private information. Moreover, it is applied to the case of rating agencies which have different features from other financial intermediaries that have been discussed in the literature, as for example investment banks or financial analysts.

A recent contribution to this literature is the paper by Bolton, Freixas and Shapiro (2007) which looks at the conflicts of interest that financial intermediaries face when providing advice about which financial products are best for their customers, and how sufficient competition can overcome such conflicts. Also in this literature, Lizzeri (1999) discusses the strategic manipulation of information by intermediaries who collect information from privately informed agents and then decide what to disclose to the uninformed ones. Both models abstract from the role of public versus private information and from how the outcome of an intermediary’s actions affects its reputation. In Lizzeri’s model an intermediary cannot establish a reputation and in Bolton, Freixas and Shapiro’s model reputation takes the form of a fixed reputational cost, contrarily to the model developed here which deals with reputation using Bayesian updating.

Benabou and Laroque (1992) and Morris (2001) also address the issue of reputation and information revelation. Both papers build on the papers of Crawford and Sobel (1982) and Sobel (1985) by developing repeated cheap talk models in which there is a sender of infor-
mation, which is equivalent to the rating agency in the model presented here, whose type (honest if he always reports truthfully or strategic if he wants to maximize reputation) is unknown to receivers. Benabou and Laroque (1992) assume the honest sender always reports his signal and, because private information is noisy, they conclude that a strategic sender that wants to look honest can manipulate information without risking losing all his credibility as predictions which turn out to be incorrect can always be attributed to an honest mistake. Morris (2001) endogenizes the behavior of the honest sender and shows that this sender can also have incentives to lie in order to enhance reputation. These papers also do not fully explore the trade-off between public and private information. Moreover, in the model developed here a rating agency always acts as a strategic sender that wants to maximize reputation and therefore profits.

Information revelation for investment banks has been covered by Chemmanur and Fulghieri (1994) which focus on the moral hazard problem that investment banks face while setting their evaluation standards and not on the problem of manipulation of information. Morgan and Stocken (2003) develops a static cheap talk model of information revelation for financial analysts whose compensation, paid out by the investment bank, is contingent on the recommendation issued by the financial analyst. This differs from the model develop here because the rating agency’s fee is paid upfront by the entrepreneur that requires rating services and before any assessment has been performed by the agency. This is common practice with rating agencies.

In the literature on rating agencies, Cantor and Packer (1995) provide an extensive survey on the ratings industry and on the theoretical side, Boot, Milbourn and Schmeits (2006) develop a rationale for ratings as a way to coordinate investors and to avoid multiple equilibria.

This paper is also closely related to the theoretical literature on career concerns, whose seminal paper is Holmstrom (1999, 1982). Later developments are for example, Scharfstein and Stein (1990), or Trueman (1994) and Graham (1999) on career concerns and herd behavior of firm managers and financial analysts, respectively, and Boot, Milbourn and Thakor (2005) on the delegation of ideas.
The rest of the paper is organized as follows. Sections 2 describes the basic characteristics of the monopolistic model and section 3 contains the equilibrium analysis and comparative statics. In Section 4 competition is introduced and section 5 concludes.

2 The Model: The Monopolistic Case

In this economy, there are three different classes of risk-neutral agents at each time period: an entrepreneur, one rating agency and a large group of homogeneous investors. The model lasts for two time periods $t$, with $t=\{1,2\}$, and there is no discounting.\footnote{Obviously, rating agencies operate for more than two periods, however a two-period model adds expositional simplicity to the analysis while still capturing the importance of reputation. The implications of this assumption are discussed below.}

2.1 The Project and Public Information

At each time period there is an entrepreneur who wants to implement an idea for a investment project that costs $\frac{1}{2}$. At the end of the period the project yields a cash-flow that equals 1 if the project is good (G) and 0 if the project is bad (B). In this case, the entrepreneur is protected by limited liability and the liquidation value is zero.

The entrepreneur has initial wealth $w$, with $w < \frac{1}{2}$, and derives a non-monetary private benefit of $B$ from undertaking the project. He knows the project’s type but the existence of a private benefit ensures that even an entrepreneur with a bad project wants to undertake it. Investors have access to unlimited funds and want to participate in the project. I abstract from issues of capital structure and assume that the entrepreneur at time period $t$ raises the required financing for the project by issuing debt claims with a promised repayment $D_t$ to be paid at the end of the period. These debt claims need to be rated as the institutional and legal regime establishes that ratings are mandatory.

Both investors and the rating agency do not know the project’s type \textit{ex-ante} but general conditions of the economy determine a common prior belief over it. Therefore, a project is good with probability equal to $\theta$, with $\theta \in (0,1)$. This probability summarizes the public information about the project’s type and for simplicity, it is assumed to be exogenous and
constant over time.

2.2 The Rating Agency’s Objective and Private Information

The rating agency is hired by the entrepreneur who pays it an upfront fee as common practice with rating agencies. This fee is set by the rating agency at the beginning of each time period to maximize profits while leaving the entrepreneur indifferent between hiring a rating agency or not. This implicitly assumes that the rating agency has the bargaining power and appropriates the full value of a rating to an entrepreneur. There are three reasons to ascribe so much power to the rating agency: ratings are mandatory, there is a single rating agency and the project’s payoff together with the private benefit can be assumed to be high enough such that the entrepreneur has more to lose if the project is not undertaken than the rating agency has to lose by not being hired. Alternatively, the rating agency and the entrepreneur could Nash-bargain over the fee. This would add complexity to the model without changing its qualitative results.

The entrepreneur’s initial wealth \( w \) is taken to be high enough to cover this fee but low enough to prevent him from offering bribes to the rating agency in an attempt to influence ratings. This is to eliminate distortions to the behavior of the rating agency that would bias it towards issuing good ratings.\(^4\) The rating agency has no initial wealth and the rating is always made public and at no extra cost for investors.\(^5\)\(^6\)

After being hired, the rating agency collects private information about the project. This private information takes the form of a private signal that can be of two types: \( s_G \) indicates that the project is good and \( s_B \) indicates that the project is bad. The quality of the private signal depends on the rating agency’s ability, denoted by \( a \), which can be high (H) or low (L).

\(^4\)The fact that there is no repeated interaction between the rating agency and the entrepreneur in this model makes the offer of bribes quite ineffective. If the rating agency pockets the bribe before the rating is announced, once it receives the money it has no incentives to report differently from its first best anymore. But if the entrepreneur waits until the rating is made public, then the rating agency knows that an entrepreneur with a good rating is going to fail on the bribe payment.

\(^5\)The zero wealth assumption rules out negative fees, i.e. situations in which the rating agency pays the entrepreneur to rate the project at time period 1 in an attempt to boost reputation and recover this money at time period 2.

\(^6\)All that is needed is for investors to known that the entrepreneur hired a rating agency. Whenever a rating is not made public they can infer that this rating must have been bad as it is in the entrepreneur’s best interest to always make a good rating available to investors.
An H rating agency collects a private signal that reveals the project’s type, while an L rating agency collects a noisy private signal. The probability $\alpha_t$ represents the subjective belief that a rating agency is of type H at time period t, with $\alpha_1 \in (0, 1)$ and $\alpha_2$ derived endogenously. I refer to this probability as the rating agency’s reputation at time period t. The rating agency learns its type after being hired at time period 1 and the other agents learn about it over time.\footnote{This can be interpreted as having a rating agency employing a credit analyst. At the time the analyst is hired the rating agency is unsure about how good she is or whether she is going to conduct due diligence investigations on the bonds she is rating, and learns about it by working with her. Or, it can refer to a situation in which a rating agency faces the task of assigning a rating to a new financial instrument and is unsure about how good its credit model is to assess this instrument. This assumption allows the paper to focus on the most interesting case in which a rating agency cannot use the fee charged to the entrepreneur as a signal to reveal its type. If this was the case, reputation would not play any role.}

Therefore, an H rating agency always identifies the project’s type which means that:

$$\Pr (s_G \mid G, H) = \Pr (s_B \mid B, H) = 1,$$

and

$$\Pr (s_G \mid B, H) = \Pr (s_B \mid G, H) = 0. \quad (1)$$

An L rating agency makes small mistakes which means that:

$$\Pr (s_G \mid G, L) = \Pr (s_B \mid B, L) = \varepsilon$$

and

$$\Pr (s_G \mid B, L) = \Pr (s_B \mid G, L) = 1 - \varepsilon. \quad (2)$$

where $\varepsilon \in \left[\frac{1}{2}, 1\right)$ measures the quality of the private signal. It is assumed that $\varepsilon > 1 - \theta$ which will later be shown to be necessary to make requesting a rating valuable for the entrepreneur. The costs of collecting the private signal are assumed to be arbitrarily small and the effort exerted to collect this signal is assumed to be observable and verifiable. This ensures that a rating agency requires a positive fee for assigning a rating to a project and allows the paper to abstract from moral hazard issues.
The rating agency uses this private signal and the prior belief about the project’s type to assign a rating to a project. There are two types of ratings: a good rating \( r_G \) and a bad rating \( r_B \). A bad rating can be interpreted as a non-investment grade rating and a good rating can be interpreted as an investment grade rating. A bad rating implies that no funding is provided for the project or that this funding is provided by different sources, like bank loans or venture capital, on terms that are not comparable. This captures the fact that in reality some of the largest purchasers of fixed-income securities, for example institutional investors, are often governed by investment rules that require them to invest only in securities that are rated investment grade. This makes it much harder, or even impossible, for an entrepreneur issuing securities that are rated non-investment grade to be successful at raising funds.

The rating agency can issue a rating that does not coincide with the private signal but whenever this happens it incurs a cost denoted by \( c \). This cost is assumed to be independent of the rating agency’s type and of the quality of the private signal and can be arbitrarily small. One way to interpret this cost is related to the process of assigning a rating. During this process, rating agencies assign an analyst to evaluate a project and to come up with a rating report, but the analyst’s evaluation is subject to a review by a committee formed inside the rating agency which can request changes to the rating report (and to the rating itself) before it becomes public. This requires time and effort to write a new report and to make sure that such rating is justifiable in case it is challenged or the rating agency is subject to an inquiry or a lawsuit. The equilibrium probabilities that at each time period a rating agency with ability \( a \) issues a rating that does not coincide with the private signal, i.e. \( \Pr(r_G \mid s_B, a) \)

---

8 In this setting unsolicited ratings are meaningless as these ratings would be based on public information and would not provide new information to investors.

9 This assumption could be relaxed and it could be derived within the model when a project is in fact undertaken for a given rating. It would still be the case that a bad rating would lead, in most circumstances, to no funding being provided. And if not, all that is needed to generate the qualitative results presented below is that the rating agency’s reputation is less sensitive to the outcome of bad rating than to the outcome of a good rating. There is plenty of anecdotal evidence that supports the fact that it is more damaging for a rating agency’s reputation when investors lose money after investing in a security which turns out to be overrated than the opposite. And when a project financed with debt issued a bad rating does fail it is very difficult to say if the project was indeed bad or if the failure was not simply precipitated by the rating itself, which makes it hard to evaluate the rating agency’s work. This is because bad ratings are often associated with higher borrowing costs, difficulties to raise financing or problems to convince creditors to roll-over existing debt obligations and consequent sale of the firm’s assets to raise funds, all of which make a failure much more likely to occur.
and \( \Pr(r_B \mid s_G, a) \), are denoted by \( \gamma_t \) and \( \gamma_h \) respectively.

### 2.3 Investors’ Objective

Investor use the rating, their subjective belief that the rating is correct, and the other variables that are common knowledge to evaluate a project. They decide whether to invest in the project and which promised repayment \( D_t \) to require for doing so. Only when they invest the project is undertaken. The cash-flows of the project are realized and publicly observed at the end of the period. At this point, all claims are settled and investors update their belief about the rating agency’s type, by comparing the rating to the outcome of the project. This updated belief, which is generally denoted by \( \alpha_2 \), is equal to one of the following three values: \( \alpha_2(r_G, G) \), \( \alpha_2(r_G, B) \) or \( \alpha_2(r_B) \). Therefore, \( \alpha_2(r_G, G) \) denotes the posterior belief that the rating agency is of type \( H \) given the it correctly assigns a good rating to a good project, i.e. \( \Pr(H \mid r_G, G) \); \( \alpha_2(r_G, B) \) denotes the posterior belief that the rating agency is of type \( H \) given that it incorrectly assigns a good rating to a bad project, i.e. \( \Pr(H \mid r_G, B) \); and finally, \( \alpha_2(r_B) \) denotes the posterior belief that the rating agency is of type \( H \) given that it issues a bad rating, i.e. \( \Pr(H \mid r_B) \).

The game is then repeated with a new project and a new entrepreneur who knows the outcome at time period 1. The sequence of events and the notation presented so far are summarized in Figure 1 and in Table 1 respectively.

### 3 Equilibrium Analysis

An equilibrium consists of the optimal choices at each time period by the rating agency of \( \gamma_t \) and \( \gamma_h \) that maximize profits and of investors on whether to invest. Investors determine the promised repayment \( D_t \), and the fee that the entrepreneur is willing to pay for rating services is calculated based on \( D_t, \theta, \alpha_t, \varepsilon, \gamma_t, \gamma_h \).
The entrepreneur hires the rating agency and agrees on the fee.

The rating agency collects the private signal and issues a rating.

Investors observe the rating and decide whether to invest.

The project’s cash-flows are realized if investment has taken place and all claims are settled.

Investors update beliefs about the rating agency’s type.

The game is repeated with a new entrepreneur and project.

Figure 1: Sequence of Events

3.1 The Promised Repayment to Investors and the Rating Agency Fee

Without a rating the entrepreneur cannot undertake the project and keeps the initial wealth \( w \). If he asks for a rating he uses \( w \) to pay for the rating agency’s fee and by assumption has funds left that he can use to invest in the project. These funds represent a cheaper source of financing than outside funds. But given that the rating agency can set the fee, it fully appropriates the expected value in excess of \( w \) generated by the project. So there is no advantage for the entrepreneur from investing his own funds in the project. For analytical simplicity suppose that the entrepreneur does indeed raise \( \frac{1}{2} \) in debt. Using the assumptions that the fee is paid upfront before the rating is known, a bad rating implies no investment and there is limited liability, an entrepreneur with a good project at time period \( t \) pays a fee equal to:

\[
\Pr(r_G \mid G, t) (1 - D_t),
\]

where \( \Pr(r_G \mid G, t) \) is the probability that a good rating is assigned to a good project at time period \( t \). An entrepreneur with a bad project knows that the expected value generated by his project is going to be zero however he wants to enjoy the private benefit, therefore he is willing to pay the same fee that an entrepreneur with a good project pays in order not to
Table 1: Summary of notation

<table>
<thead>
<tr>
<th>$B$</th>
<th>Entrepreneur’s private benefit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w$</td>
<td>Entrepreneur’s initial wealth</td>
</tr>
<tr>
<td>$t$</td>
<td>time, $t \in {1, 2}$</td>
</tr>
<tr>
<td>$D_t$</td>
<td>Promised repayment to investors at time period $t$</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Prior probability of a G project, $\theta \in (0, 1)$</td>
</tr>
<tr>
<td>$s_G, s_B$</td>
<td>Good and bad private signal</td>
</tr>
<tr>
<td>$r_G, r_B$</td>
<td>Good and bad rating</td>
</tr>
<tr>
<td>$a$</td>
<td>Rating agency’s ability, $a={H, L}$</td>
</tr>
<tr>
<td>$\alpha_t$</td>
<td>Probability of an H rating agency at time period $t$</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>Quality of an L rating agency’s private signal, $\epsilon \in [\frac{1}{2}, 1]$</td>
</tr>
<tr>
<td>$c$</td>
<td>Cost of misreporting the private signal</td>
</tr>
<tr>
<td>$\alpha_2(r_G, G)$</td>
<td>Probability of an H rating agency at time period 2 when $r_G$ is assigned to a G-project</td>
</tr>
<tr>
<td>$\alpha_2(r_G, B)$</td>
<td>Probability of an H rating agency at time period 2 when $r_G$ is assigned to a B-project</td>
</tr>
<tr>
<td>$\alpha_2(r_B)$</td>
<td>Probability of an H rating agency at time period 2 when $r_B$ is issued</td>
</tr>
<tr>
<td>$\gamma_t$</td>
<td>Probability of deviating from a bad private signal at time period $t$</td>
</tr>
<tr>
<td>$\gamma_t$</td>
<td>Probability of deviating from a good private signal at time period $t$</td>
</tr>
</tbody>
</table>

reveal his type.$^{10}$ Note that when investors contribute with the investment funds it is because $D_t < 1$ and the fee paid to the rating agency is positive.

The investors’ market is competitive and risk neutral, hence the promised repayment to investors is determined at each time period using the investors’ participation constraint defined as follows:

$$\Pr(G \mid r_G, t) D_t = \frac{1}{2}.$$  

Using Bayes’ rule, it can be derived that:

$$D_t = \frac{\Pr(r_G \mid t)}{2 \Pr(G) \Pr(r_G \mid G, t)}.$$  

(4)

The entrepreneur has ex-ante the same information about the rating agency’s type as investors have when they are confronted with a rating. Hence, $\Pr(r_G \mid G, t)$ in expressions (3) and (4)

$^{10}$One implicit assumption is that the private benefit more than compensates the entrepreneur (in utility terms) for the value of the fee.
coincide. Given this, substituting (4) in (3) the fee at time period t becomes:

\[
\frac{\Pr(G) \Pr(r_G \mid G, t) - \Pr(B) \Pr(r_G \mid B, t)}{2 \Pr(G)}.
\]

(5)

3.2 Time Period 2 (No Reputational Concerns)

When hired at time period 2 the rating agency receives the fee for this time period and subsequently collects the private signal. Then, it decides on the rating to issue. It takes this decision to maximize future profits which are equal to zero as the game ends after time period 2. And it costs \( c \) to deviate from the private signal which implies that a rating agency minimizes costs by always issuing the private signal as a rating regardless of its type. Therefore, both \( \gamma_2 \) and \( \gamma_2' \) are equal to zero.

Given this, the conditional probabilities \( \Pr(r_G \mid G, 2) \) and \( \Pr(r_G \mid B, 2) \) are equal to \( \alpha_2 + (1 - \alpha_2) \varepsilon \) and \( (1 - \alpha_2)(1 - \varepsilon) \) respectively and the fee at time period 2 is derived using (5). To make the dependence of the rating agency’s fee on its reputation explicit this fee is denoted by \( F(\alpha_2) \) and is given by:

\[
F(\alpha_2) = \frac{\alpha_2 (1 - \varepsilon) + \theta - (1 - \varepsilon)}{2 \theta}.
\]

(6)

The fee is increasing in the reputational level \( \alpha_2 \). It is easier to understand why this happens by looking at expression (3). As the reputational level increases the repayment amount \( D_t \) that investors demand after a good rating becomes lower as this rating becomes less likely to represent a mistake from an L rating agency. And for the same reason \( \Pr(r_G \mid G, t) \) increases. Both effects act to increase the value created by a rating to the entrepreneur and therefore increase the fee. As a result, a rating agency acts to maximize reputation in order to maximize fees.

This fee is always positive because \( \varepsilon > 1 - \theta \). This means that even if the entrepreneur and investors know that they are dealing with an L rating agency at time period 2, the quality of the private signal is high enough to make the entrepreneur with a good project reasonably confident that the rating agency is going to correctly identify it, and to make
investors contribute with the funds after a good rating. Consequently, rating services are valuable to the entrepreneur. This assumption is not necessary to solve the model but it simplifies it and allows the paper to to abstract from the hiring decision of the entrepreneur and to focus on the most interesting part, which is to derive the behavior of the rating agency. By doing so, it also facilitates the comparison of the results derived here in section 3 to the results derived for a duopolistic ratings industry in section 4.

3.3 Time Period 1 (Reputational Concerns)

3.3.1 Rating Agency Optimal Behavior

When hired at time period 1 the rating agency receives the fee for this time period, collects the private signal and then decides on the rating to issue. It takes this decision to maximize future profits. If a rating agency with ability \(a\) and a good private signal issues the private signal as a rating it will receive \(F (\alpha_2 (r_G, G))\) at time period 2 if this rating is correct and the project turns out to good, and \(F (\alpha_2 (r_G, B))\) otherwise. Its future profit is given by the expected fee at time period 2 which is equal to:

\[
Pr (G \mid s_G, a) F (\alpha_2 (r_G, G)) + Pr (B \mid s_G, a) F (\alpha_2 (r_G, B)).
\]

If the rating agency does not issue the private signal as a rating its future profit is certain and equal to \(F (\alpha_2 (r_B)) - c\), where \(F (\alpha_2 (r_B))\) is the fee at time period 2 and \(c\) is the cost of misreporting the private signal at time period 1. The rating agency issues the rating that generates the highest future profit. That is to say that, in equilibrium, the sign of:

\[
Pr (G \mid s_G, a) F (\alpha_2 (r_G, G)) + Pr (B \mid s_G, a) F (\alpha_2 (r_G, B)) - (F (\alpha_2 (r_B)) - c),
\] (7)

is positive (negative) when the rating agency follows (contradicts) the private signal and (7) is equal to zero when the rating agency is indifferent. In this case the equilibrium is in mixed strategies.

Likewise for the case of a bad private signal. If the rating agency issues the private signal
as a rating, the future profit is equal to $F(\alpha_2(r_B))$. But if the rating agency does not issue the private signal as a rating the future profit is given by the expected fee at time period 2 and the cost of misreporting the private signal at time period 1, which is equal to:

$$\Pr(G \mid s_B, a) F(\alpha_2(r_G, G)) + \Pr(B \mid s_B, a) F(\alpha_2(r_G, B)) - c.$$  

In equilibrium, the sign of:

$$\Pr(G \mid s_B, a) F(\alpha_2(r_G, G)) + \Pr(B \mid s_B, a) F(\alpha_2(r_G, B)) - c - F(\alpha_2(r_B)) \tag{8}$$

is negative (positive) when the rating agency follows (contradicts) the private signal and (8) is equal to zero when the rating agency is indifferent.

The results when the cost of misreporting tends to zero can be generalized by Proposition 1. Figure 2 gives a graphical representation of this proposition.

**Proposition 1** The behavior of a rating agency at time period 1 is such that:

1. An H rating agency always issues the private signal as a rating: it issues a good rating whenever it collects a good private signal, and it issues a bad rating whenever it collects a bad private signal.

2. There are $\theta$ and $\overline{\theta}$, with $\frac{1}{2} < \theta < \varepsilon < \overline{\theta}$, such that, for $\theta \in [\theta, \overline{\theta}]$, an L rating agency always issues the private signal as a rating. Otherwise, the rating agency behaves as follows:

   - For $\theta \in (\overline{\theta}, 1)$, it issues a good rating whenever it collects a good private signal, and it issues a good rating with probability $0 < \gamma_1 < 1$ and a bad rating with probability $1 - \gamma_1$ whenever it collects a bad private signal.

   - For $\theta \in (1 - \varepsilon, \theta)$ it issues a bad rating whenever it collects a bad private signal, and it issues a bad rating with probability $0 < \gamma_1 \leq 1$ and a good rating with probability $1 - \gamma_1$ whenever it collects a good private signal.
Figure 2: Equilibrium behavior of an L rating agency with reputational concerns as a function of $\theta$

Region (a): When the private signal is $s_B$, the rating agency issues $r_B$; when the private signal is $s_G$, it issues $r_B$ with probability $\gamma_1$ and $r_G$ otherwise. Between $\bar{\theta}$ and $\tilde{\theta}$ it issues the private signal as a rating. Region (b): When the private signal is $s_G$, it issues $r_G$; when the private signal is $s_B$, it issues $r_G$ with probability $\gamma_1$ and $r_B$ otherwise.

The full proof of this proposition is relegated to section 6.1 of the Appendix but the main intuition is presented next. For a better exposition of the results I characterize a project in the following way:

**Definition 1** A project is expected to be good (or very good) based on public information for $\theta > \frac{1}{2}$ and close to $\frac{1}{2}$ (for $\theta > \frac{1}{2}$ and close to 1) and it is expected to be bad (or very bad) for $\theta < \frac{1}{2}$ and close to $\frac{1}{2}$ (for $\theta < \frac{1}{2}$ and close to 0).\textsuperscript{11}

The starting point is to show that there cannot be an equilibrium in which an H rating agency issues a rating that goes against the private signal. To prove this result note that at time period 2 a rating agency always issues the private signal as a rating regardless of its type. Therefore, because investors and entrepreneurs value a correct rating, the fee at time period 2 increases the more likely it is that the rating agency is of type H. This gives an L rating agency incentives to mimic the behavior of an H rating agency at time period 1. From here, it is straightforward to show the result. This is done by contradiction. If, for example, an H rating agency always issues a good (bad) rating regardless of the private signal, then an L rating agency would also always issue a good (bad) rating. But this would not constitute an equilibrium as the H rating agency would then prefer to issue a bad (good) rating to try to distinguish itself from an L rating agency and reveal its true type. Situations in which an H rating agency plays a mixed strategy can also be ruled out following a similar argument.

Using this result, it is shown that the behavior of an L rating agency is characterized by **conformism** and **conservatism**. When investors expect the project to be very good (very bad

\textsuperscript{11}Note that $\theta \to 0$ when $\varepsilon \to 1$.}
or bad) based on public information and the rating agency’s private information indicates that the project is bad (good), there are situations in which it chooses to issue a good (bad) rating. I refer to this behavior as conformism.\footnote{This result is directly related to the issue of conformity. A number of papers such as Bikhchandani, Hirshleifer and Welch (1992, 1998) and Brandenburger and Polak (1996) discuss this topic.} This happens because when the project is expected to be very good (very bad or bad) based on public information the L rating agency knows that, had it been an H rating agency, it would have been more likely to have collected a good (bad) private signal.

But when a project is expected to be good based on public information there are situations in which an L rating agency chooses to contradict this information even when the agency’s private information indicates that the project is good, and issues a bad rating. I refer to this behavior as conservatism. This happens because the L rating agency wants to minimize reputational costs, measured in terms of the fee at time period 2. If an L rating agency that collects a good private signal chooses to issue a good rating and this rating is correct, the rating agency manages to boost reputation and consequently, the fee at time period 2. But if such rating is incorrect, the rating agency’s type is revealed and the fee at time period 2 is very low. So the rating agency attempts to eliminate this possibility by hiding its true type behind a bad rating which is not verifiable. In this way, the rating agency’s reputation might not be boosted but it will definitely remain reasonably unscathed. The same happens to the fee at time period 2.\footnote{Conservatism is discussed in Zwiebel (1995), although the application here is quite different. The idea in Zwiebel’s paper is that reputational concerns may lead managers to refrain from undertaking innovations that are first order stochastically dominant because of the downside risk of being fired.}

### 3.3.2 Comparative Statics

A number of interesting results can be derived when performing comparative statics in the equilibrium values of $\pi_1$ and $\gamma_1$. The derivations are in section 6.2 of the Appendix.

**Proposition 2** The equilibrium probabilities $\pi_1$ and $\gamma_1$ are monotonic and:

1. always increasing and decreasing, respectively, in the value of the prior belief $\theta$;
2. *decreasing in the quality of the private signal* $\varepsilon$, *for most values of the prior belief* $\theta$;

3. *always decreasing and increasing, respectively, in the initial reputational level* $\alpha_1$;

4. *always decreasing in the cost of misreporting* $c$.

Point 1. means that the more extreme the public information is, the higher the probability of conforming with this information: the worse (better) the public information is, the higher the probability of sending a bad (good) rating when facing a good (bad) private signal.

Point 2. is an intuitive result: increasing the quality level of the private signal lowers the rating agency’s incentive to ignore it. This always happens for the case of $\gamma_1$. However, it cannot be excluded that $\gamma_1$ increases in the quality of the private signal for high values of $\theta > \frac{1}{2}$ and close to the threshold $\theta$.\textsuperscript{14} In this case, a project is expected to be good based on public information and a more precise private signal decreases the ex-ante probability that an L rating agency is going to collect a bad private signal. Other things equal, an L rating agency is now expected to issue bad (good) ratings less (more) frequently than before. This in turn means that investors believe they are more (less) likely to be dealing with an H (L) rating agency when they see a bad rating and are less able to differentiate between an H and an L rating agency when they see a correct good rating. The L rating agency, whose objective is to mimic the behavior of an H rating agency, reacts by issuing bad ratings more frequently, i.e. by increasing $\gamma_1$.

An increase in the quality level of the private signal can take many forms in practice, for example through a policy towards making the agencies models of risk assessment more robust combined with increased efforts to train and retain qualified staff.

Point 3. means that the behavior of the L rating agency is such that as the initial reputational level increases, it tends to issue bad ratings more often than good ones. An increase in the initial reputational level has a positive impact in the reputational level at time period 2, both after a good or a bad rating have been issued at time period 1, and therefore it has

\textsuperscript{14}If this effect exists it is going to be for a very restricted interval of the prior belief $\theta$. But because it is not possible to provide closed-form solutions for $\gamma_1$ and the several parameters can be combined in many different ways, it is not possible to exclude that such region does exist.
a positive impact in the fee. But the increase in the latter is certain and the increase in the former only occurs if the good rating is correct. Otherwise, the rating agency is identified as a type L, the reputational level at time period 2 is equal to zero and the fee decreases accordingly. As the initial reputational level increases, the reputational cost that a rating agency faces from having its type revealed, in terms of the fees that it forgoes, increases. In view of this, a more reputable agency has less incentives to gamble by issuing a good rating.

Point 4. is a straightforward result: as deviating from the private signal becomes more costly, issuing a rating that does not coincide with this signal becomes less frequent. In reality, it is the case that different agencies look at different criteria to define and assign a rating. This gives scope to different possibilities to interpret and to use the information collected about the firms and debt instruments that are being rated, which means that c is in practice very low. By being clearer about the meaning of ratings and about what exactly a rating should reflect, regulators would make the ratings process more rigorous and rating agencies more accountable for their ratings and easier to monitor.

3.4 Value of a Rating and Reputational Concerns

It was derived above that a monopolistic rating agency can ignore the private signal when it issues a rating at time period 1, which is the time period for which the rating agency has reputational concerns. To determine to what extent this is generated by these concerns, the results from Proposition 1 should be contrasted to what a rating agency would do if its sole concern was to maximize the value created by assigning a rating to a project at time period 1. Given the bargaining assumptions considered between the entrepreneur and the rating agency, this value is given by the fee at time period 1 which is defined by expression (5). The fee can take one of the following forms. First, when the rating agency deviates from a good private signal \( \Pr (r_G \mid G, 1) = \alpha_1 + (1 - \alpha_1) \varepsilon (1 - \gamma_1) \) and \( \Pr (r_G \mid B, 1) = (1 - \alpha_1) (1 - \varepsilon) (1 - \gamma_1) \).

As a result the fee charged at time period 1, denoted by \( F(\alpha_1) \), is equal to:

\[
F(\alpha_1) = \frac{\alpha_1 (1 - \varepsilon) + \theta - (1 - \varepsilon) - (1 - \alpha_1) (\theta - (1 - \varepsilon)) \gamma_1}{2 \theta}.
\]  
(9)
Second, when a rating agency deviates from a bad private signal \( \Pr(r_G \mid G, 1) = \alpha_1 + (1 - \alpha_1) (\varepsilon + (1 - \varepsilon) \gamma_N) \) and \( \Pr(r_G \mid B, 1) = (1 - \alpha_1) (1 - \varepsilon + \varepsilon \gamma_N) \). It follows that the fee at time period 1 is given by:

\[
F(\alpha_1) = \frac{\alpha_1 (1 - \varepsilon) + \theta - (1 - \varepsilon) + (1 - \alpha_1) (\theta - \varepsilon) \gamma_N}{2\theta}.
\]  

(10)

Third, when a rating agency does not deviate from the private signal, the fee at time period 1 is calculated by setting \( \gamma_N = 0 \) in expression (9) or \( \gamma_N = 0 \) in expression (10).\(^\text{15}\)

The fees are maximized when \( \gamma_N = 0 \) for \( \theta > 1 - \varepsilon \) and when \( \gamma_N = 0 \) for \( \theta < \varepsilon \). But for \( \theta > \varepsilon \), expression (10) is maximized when \( \gamma_N = 1 \). This means that in order to maximize the value created by assigning a rating to a project, an L rating agency should issue the private signal as a rating for \( 1 - \varepsilon < \theta < \varepsilon \) but it should issue a good rating regardless of the private signal for \( \theta > \varepsilon \). Note that this is the result that would have been obtained had the rating agency just wanted to minimize mistakes. If this was the case, an H rating agency would always issue the private signal as a rating. An L rating agency would issue a good rating after a good private signal if \( \Pr(G \mid s_G, L) \geq \Pr(B \mid s_G, L) \). Consequently, the rating agency would issue a good rating for \( \theta > 1 - \varepsilon \), and a bad rating otherwise. Similarly, an L rating agency would issue a bad rating after a bad private signal if \( \Pr(B \mid s_B, L) \geq \Pr(G \mid s_B, L) \) or equivalently, the rating agency would issue a bad rating for \( \theta < \varepsilon \), and a good rating otherwise. See Figure 3 for an illustration of these results.

![Figure 3: Equilibrium behavior of an L rating agency that maximizes the value created by assigning a rating as a function of \( \theta \)](image)

Region (b): When the private signal is \( s_G \), the rating agency issues \( r_G \); when the private signal is \( s_B \), it issues \( r_G \) with probability \( \gamma_N \) equal to 1. Between \( 1 - \varepsilon \) and \( \varepsilon \), it issues the private signal as a rating.

\(^\text{15}\)Note that even though the fee at time period 2 is always positive by assumption, the fee given by expression (9) does not have to be positive for the parameter values considered by the model. This implies that the rating agency would not be hired at time period 1 and if hired at time period 2 it would not have reputational concerns.
Comparing figures 2 and 3, it can be seen that the fact that an L rating agency with reputational concerns conforms to public information to the detriment of private information when a project is expected to be very good based on public information can be justified as an attempt of the rating agency to minimize mistakes. However, it conforms less to public information than a rating agency that wants to minimize mistakes. Moreover, the rating agency ignores private information that indicates that a project is good when a project is expected to be bad based on public information more often than a rating agency that wants to minimize mistakes and acts conservatively by ignoring both private and public information in situations in which a project is expected to be good. This difference in behavior measures the effect of reputational concerns which often reduce the value created by assigning a rating to a project.

**Corollary 1** An L rating agency with reputational concerns issues an excessive number of bad ratings relative to a rating agency that just wants to maximize the value created by assigning a rating to a project (or to minimize mistakes).

### 3.5 More Time Periods

In a model with another time period, for example time period 0, the rating agency would choose the rating to issue at time period 0 based on the expected fee at time period 1 and on the expected fee at time period 2 given the optimal action at time period 1, both generally increasing in reputation. Then a correct good rating at time period 0 would increase the future profit but an incorrect good rating at time 0 means that the rating agency’s type would already be revealed at time 0 and that the future profit would be permanently low. Issuing a bad rating even when faced with a good private signal at time period 0 could still be at least as good as issuing a good rating, in particular for a rating agency whose quality of the private information is low and/or the initial reputational level is high, as this would be the agency that benefits the most from hiding its true type.
4 The Competitive Setting

The focus of the paper so far has been on the strategic information revelation of a monopolistic rating agency. However, it would be interesting to contrast this situation to what happens when there is competition in the ratings industry. Competition is modeled in a very stylized manner. Consider the framework developed in section 2 but with two rating agencies that at each time period compete to rate the project. The fact that only one rating agency is hired at each time period makes the competitive setting quite aggressive even with only two rating agencies in the industry. Not allowing for the two rating agencies to be sequentially hired in the same time period simplifies the model but without significantly changing the qualitative results. In this case it is likely that there would herding in the sense that, when in doubt the rating agency that issues the second rating would choose to issue a rating that coincides with the first rating that was issued.\(^\text{16}\) But in this case, it is the behavior of the rating agency that issues the first rating that is crucial, and there is no reason to expect this rating agency to behave differently from the rating agency that is hired in the model presented here.\(^\text{17}\)

There are many ways to motivate a one-rating scenario within the model. The entrepreneur may be facing a situation in which investment timing is crucial to guarantee the success of the project. In this case, the entrepreneur faces a time constraint and waiting for a second rating could effectively mean that the second rating agency is assessing a different and considerably worse project, which may be unworthy to pursue. Alternatively, the entrepreneur may be credit constrained and unable to pay for a second rating.

Each rating agency, denoted by \(i\) and \(j\), has an initial reputational level that for simplicity and to facilitate the comparison to the monopolistic case, is the same for the two agencies and equal to the initial reputational level of the rating agency in Section 2, i.e. \(\alpha_{i1} = \alpha_{j1} = \alpha_1\). In this case, the two rating agencies are ex-ante identical and the entrepreneur is equally likely to hire any of them at time period 1. At time period 2, the reputational level of the rating

\(^{16}\)Or eventually a more risky strategy of anti-herding in an attempt to discredit the competitor. See for example Effinger and Polborn (2001) or Trueman (1994) and Hong, Kubik and Solomon (2000) for an analysis of herding and anti-herding behavior in the context of firms and financial analysts.

\(^{17}\)They would both base their ratings on public information and on the private signal and they would both want to maximize reputation to maximize fees.
agency that is hired at time period 1 is updated as before, and the reputational level of the competitor remains equal to $\alpha_1$. The subscripts $i$ and $j$ are added to the variables defined above to identify the rating agency. Table 3 revises the notation for rating agency $i$.

### 4.1 The Promised Repayment to Investors and the Rating Agency Fee

The promised repayment to investors is given by expression (4) as before. However, the fee is calculated in a different way: rating agencies make take-it-or-leave-it offers to the entrepreneur. The process that generates the fee is similar to the process that generates the price in a Bertrand model. Let $i$ be the rating agency that is hired at time period $t$. Then, the entrepreneur makes an expected profit of $\Pr(r_{iG} \mid G, t) (1 - D_{it}) - F(\alpha_{it})$. However, for the entrepreneur to accept the offer from rating agency $i$, this profit needs to be at least as high as what the entrepreneur would have made had he accepted the offer from rating agency $j$. This is given by $\Pr(r_{jG} \mid G, t) (1 - D_{jt}) - F(\alpha_{jt})$. So for rating agency $i$ to be hired it has to be that $\Pr(r_{iG} \mid G, t) (1 - D_{it})$ exceeds $\Pr(r_{jG} \mid G, t) (1 - D_{jt})$ such that even if rating agency $j$ sets a zero fee, the entrepreneur still prefers to hire rating agency $i$. In this case the fee is simply equal to $\Pr(r_{iG} \mid G, t) (1 - D_{it}) - \Pr(r_{jG} \mid G, t) (1 - D_{jt})$ which is the difference between the fees that $i$ and $j$ would have charged the entrepreneur had they been in a monopolistic setting. Using expression (5) this fee at time period $t$ can be written as follows:
\[
\frac{\Pr(G) [\Pr(r_{iG} | G, t) - \Pr(r_{jG} | G, t)] - \Pr(B) [\Pr(r_{iG} | B, t) - \Pr(r_{jG} | B, t)]}{2 \Pr(G)}.
\]

Since the two rating agencies are ex-ante identical, then \( \Pr(r_{iG} | G, 1) = \Pr(r_{jG} | G, 1) \) and \( \Pr(r_{iG} | B, 1) = \Pr(r_{jG} | B, 1) \), which means that the fee at time period 1 is arbitrarily small as it only needs to covers the costs of collecting the private signal.\(^{18}\) The fee at time period 2 is derived next.

### 4.2 Time Period 2 (No Reputational Concerns)

Let \( i \) be the rating agency that is hired at time period 2. Following the same argument as in the monopolistic case, this rating agency always issues the private signal as a rating regardless of its type. Therefore the fee at time period 2 simplifies to:

\[ F(\alpha_{i2}) = \frac{(1 - \varepsilon)(\alpha_{i2} - \alpha_{j2})}{2\theta}. \]  \hspace{1cm} (11)

Note that now, not only does a rating agency want to maximize reputation but it also needs a reputational level that is higher than the competitor’s reputational level to be hired by the entrepreneur at time period 2.

### 4.3 Time Period 1 (Reputational Concerns)

Let \( i \) also be the rating agency that is hired at time period 1. This is without loss of generality. As in the monopolistic case, the rating agency is going to compare the future profits after issuing a good rating to the future profits after issuing a bad rating. The results for the case in which the cost of misreporting tends to zero are derived in section 6.3 in the Appendix

\(^{18}\)Negative fees are ruled by the assumption that rating agencies do not have initial wealth. However the results would not change if this was not the case. First, note that no one would set a negative fee at time period 2 as this is the last period. At time period 1 however, rating agencies could start a race to decrease fees below zero attempting to be hired by the entrepreneur. But this would be of no use because since rating agencies do not know their type until they start working, they are ex-ante identical, and they would both be willing to pay exactly the same amount to have the chance to be hired at time period. But they would still be regarded as identical by the entrepreneur.
and can be summarized by Proposition 3. Figure 4 gives a graphical representation of this proposition.

**Proposition 3** The behavior of rating agency $i$ is such that:

1. An $H$ rating agency always issues the private signal as a rating: it issues a good rating whenever it collects a good private signal, and it issues a bad rating whenever it collects a bad private signal.

2. There are $\theta_c$ and $\overline{\theta}_c$, with $1 - \varepsilon < \frac{\theta_c}{\overline{\theta}_c} < \frac{1}{2}$, such that for $\theta \in [\theta_c, \overline{\theta}_c]$, an $L$ rating agency always issues the private signal as a rating. Otherwise, the rating agency behaves as follows:

   - For $\theta \in (\overline{\theta}_c, 1)$, it issues a good rating whenever it collects a good private signal, and it issues a good rating with probability $0 < \gamma_{i1} < 1$, with $\gamma_{i1} > \gamma_1$, and a bad rating with probability $1 - \gamma_{i1}$ whenever it collects a bad private signal.

   - For $\theta \in (1 - \varepsilon, \theta_c)$ it issues a bad rating whenever it collects a bad private signal, and it issues a bad rating with probability $0 < \gamma_{i1} < 1$, with $\gamma_{i1} < \gamma_1$, and a good rating with probability $1 - \gamma_{i1}$ whenever it collects a good private signal.

![Figure 4: Equilibrium behavior of an L rating agency in competition as a function of $\theta$](image)

Region (a): When the private signal is $s_{iB}$ the rating agency issues $r_{iB}$; when the private signal is $s_{iG}$ it issues $r_{iB}$ with probability $\gamma_{i1}$ and $r_{iG}$ otherwise. Between $\theta_c$ and $\overline{\theta}_c$, it issues the private signal as a rating. Region (b): When the private signal is $s_{iG}$, it issues $r_{iG}$; when the private signal is $s_{iB}$ it issues $r_{iG}$ with probability $\gamma_{i1}$ and $r_{iB}$ otherwise.

Comparing Figures 2 and 4 it is clear that in a competitive setting a rating agency still maintains some of the conformist behavior of the monopolistic rating agency. However, it issues good ratings more often than before. There are some differences with respect to the monopolistic case that explain these results. Issuing a good rating is risky for an L rating agency.
agency because it allows investors and entrepreneurs to compare the rating to the outcome of the project. If the outcome of the project differs from the rating, the rating agency is identified as being of type L and its reputational level becomes zero. In a competitive setting, and contrarily to what happened in a monopoly, the rating agency is not going to be hired at time period 2 since the entrepreneur prefers hiring the most reputable rating agency and the competitor has a reputational level equal to $\alpha_1$. However, issuing a bad rating can also be risky. Since a bad rating is not verifiable, reputation is not necessarily boosted after such rating. And when a project is expected to be good or very good based on public information, issuing a bad rating can actually worsen the rating agency’s reputation as investors believe that the rating agency is likely to be mistaken or/and hiding its true type. As a result, the reputational level at time period 2 also does not necessarily exceed the competitor’s reputational level $\alpha_1$.

Given this, in a competitive setting an L rating agency that collects a good private signal needs to consider two effects. First, if it issues a good rating it now faces a higher reputational cost of a mistake as it is not going to be hired at time period 2. This effect is stronger discouraging the revelation of a good private signal than in the monopolistic case but it is capped at zero as the rating agency is only penalized through the loss of business. Second, there is also a reputational cost associated to a bad rating as a rating agency can be perceived as being of type L. This effect is stronger encouraging the revelation of a good private signal than in the monopolistic case.

When a project is expected to be good or very good based on public information, the second effect dominates. In order to be hired at time period 2 a rating agency needs to persuade investors that they are likely to be dealing with an H rating agency. Hence, a rating agency that collects a good private signal always chooses to issue a good rating and there are situations in which even a rating agency that collects a bad private signal chooses to ignore it and issue a good rating.

When a project is expected to be bad based on public information, the second effect does not exist but it turns out that the first effect is not enough to keep the conservative behavior that characterizes a monopolistic rating agency. This is because the reputational cost of a
mistake is too low as it is capped at zero. Consequently, an L rating agency has more to gain if it manages to persuade investors and entrepreneurs that they are likely to be dealing with an H rating agency than to lose if a mistake reveals its true type. Hence, a rating agency that collects a good private signal still chooses to issue a good rating and the behavior of a rating agency that collects a bad private signal is characterized by boldness. This means that when a project is expected to be bad based on public information there are situations in which an L rating agency chooses to contradict this information even when the agency’s private information indicates that the project is bad, and issues a good rating.

The comparative statics results to the equilibrium values of $\gamma_{IL}$ and $\gamma_{HI}$ yield similar results to the monopolistic case. The derivations are in the Appendix.

4.4 Value of a Rating and Reputational Concerns

It would be interesting to contrast the behavior of a rating agency in a competitive setting with the results derived in Section 3.4. Note that now the value created by assigning a rating to a project is given by the same expressions as in the competitive case but these expression no longer represent the fee at time period 1. The value created by assigning a rating to a project at time period 1 is now appropriated by the entrepreneur.

In a competitive setting an L rating agency with reputational concerns also attempts to minimize mistakes (or to maximize the value created by assigning a rating to a project) by conforming to public information when a project is expected to be very good or very bad based on this information. However, for a wide range of values of the prior belief $\theta$ the rating agency issues too many good ratings relative to an agency that wants to minimize mistakes. This means that:

**Corollary 2** An L rating agency with reputational concerns issues an excessive number of good ratings relative to a rating agency that just wants to maximize the value created by assigning a rating to a project (or to minimize mistakes) for a wide range of values of the prior belief $\theta$.

It is also not the case that competition represents an improvement relative to the mo-
nopolistic case. When public information is extreme, competition forces a rating agency to behave less conservatively which improves the value created by assigning a rating to a project and minimizes the possibility of a mistake. But when public information is not extreme, for example for $\theta$ close to $\frac{1}{2}$, it is not clear which situation is better: if to be bold by issuing too many good ratings or to be conservative by issuing too many bad ratings.

4.5 More Time Periods

If there was a time period 0, the rating agency would choose the rating to issue at this period based on the expected fee at time period 1 and on the expected fee at time period 2 given the optimal action at time period 1, both generally increasing with reputation. Then a correct good rating at time period 0 would increase the future profit and an incorrect good rating would make the rating agency’s future profit permanently equal to zero which means that as before, the reputational cost of a mistake is capped at zero. When a project is expected to be bad based on public information, it would still possible that an L rating agency would have more to gain by issuing a good rating and managing to persuade investors and entrepreneurs that they were likely to be dealing with an H rating agency than to lose if a mistake revealed its true type. And when a project is expected to be good or very good based on public information, issuing a bad rating at time period 0 would also mean that investors would perceive the rating agency as being an L type and would replace it by its competitor. The future profit would be equal to zero unless the competitor was either revealed or also perceived as being of type L in the subsequent time periods. Hence, issuing a good rating even when faced with a bad private signal at time period 0 could also still be at least as good as issuing a bad rating, in particular for a rating agency whose quality of the private information is low and/or the initial reputational level is low, as this is the agency that has the least to lose from issuing a good rating.
5 Concluding Remarks

This paper studies the behavior of rating agencies, in particular it looks at their incentives to issue a rating that is not justified by the private information collected about the project they are rating in a framework in which they value reputation. The model finds that reputational concerns are not enough to prevent deviations from the private signal, in fact these concerns might end up being the driving force behind these deviations. A rating agency whose private signal is perfect issues this private signal as a rating but a rating agency can make mistakes may end up ignoring the private signal and issuing the rating that minimizes reputational costs. Despite its simplicity, the model can motivate several patterns of behavior. In the monopolistic setting, a rating agency is conservative in the sense that it issues too many bad ratings ignoring private and even public information that indicates that the project is good. Competition forces rating agencies to be more aggressive to make sure that they continue being hired and are not replaced by the competitor. Hence, reputational concerns combined with competition originate boldness as rating agencies issue too many good rating ignoring private and even public information that indicates that the project is bad.

The model clearly illustrates how reputation and informational issues can distort ratings. Competition might not solve the incentive problems faced by rating agencies unless it is combined with better models of risk assessment, which would improve the quality of rating agencies assessments, more transparency in that rating’s procedures, and measures to improve monitoring and accountability in the ratings industry.

6 Appendix

6.1 Proof of Proposition 1

This proposition proves the existence of a unique equilibrium in which a rating agency is rewarded for correct ratings and punished otherwise. For this reason, \( \alpha_2 (r_G, G) \) exceeds \( \alpha_2 (r_G, B) \), and hence reputation and fee are expected to increase when the rating agency is correct and to decrease otherwise. Of course, this need not be the only equilibrium if one
allows for different patterns of reward/punishment actions. However, this is the the only reasonable equilibrium in the setup considered in the paper.

Define $\tau \in \{H_G, H_B, L_G, L_B\}$ as the set of possible types, where $H$ and $L$ indicate the rating agency’s type, and $G$ and $B$ indicate the private signal, e.g. $H_G$ is an $H$ rating agency with a good private signal. The set of possible actions is binary: issue a good rating ($r_G$) or issue a bad rating ($r_B$). The proof is divided in two main steps: first, it is shown that in equilibrium the $H$ rating agency must issue the private signal as a rating and second, the equilibrium behavior of the $L$ rating agency is derived.

6.1.1 Agency’s Posterior Beliefs about the Project’s Type

The rating agency forms the posterior belief about the project’s type using the prior about the project’s type and (1) or (2) depending on its type. Hence:

$$\Pr(G \mid s_G, H) = 1, \quad \Pr(G \mid s_B, H) = 0$$

and

$$\Pr(G \mid s_G, L) = \frac{\varepsilon \theta}{\varepsilon \theta + (1 - \varepsilon)(1 - \theta)}, \quad \Pr(G \mid s_B, L) = \frac{(1 - \varepsilon) \theta}{(1 - \varepsilon) \theta + \varepsilon (1 - \theta)}.$$  

(13)

Note that

$$\Pr(G \mid s_G, H) > \Pr(G \mid s_G, L) > \Pr(G \mid s_B, L) > \Pr(G \mid s_B, H).$$

(14)

6.1.2 Auxiliary Lemmas

**Lemma 1** An $H$ rating agency always has less of an incentive to deviate from the private signal than an $L$ rating agency.

If an $H_G$ rating agency issues $r_B$ with positive probability then it is because:

$$F(\alpha_2(r_B)) - c \geq F(\alpha_2(r_G, G)).$$

(15)
Given that $F(\alpha_2(r_G, G))$ exceeds $F(\alpha_2(r_G, B))$ and using condition (14), then an $L_G$ rating agency has even more incentives to deviate from the private signal. By doing so it also receives $F(\alpha_2(r_B)) - c$ and by non-deviating it expects to receive less than what the $H_G$ rating agency receives, i.e.:

$$F(\alpha_2(r_B)) - c \geq F(\alpha_2(r_G, G)) > \Pr(G | s_G, L) F(\alpha_2(r_G, G)) + \Pr(B | s_G, L) F(\alpha_2(r_G, B)).$$

And when an $H_B$ rating agency issues $r_G$ with positive probability it is because:

$$F(\alpha_2(r_G, B)) - c \geq F(\alpha_2(r_B)).$$

Following the same reasoning as before, an $L_B$ rating agency is also going to deviate from the private signal because it expects to receive more than what the $H_B$ rating agency receives by doing so, i.e.:

$$\Pr(G | s_B, L) F(\alpha_2(r_G, G)) + \Pr(B | s_B, L) F(\alpha_2(r_G, B)) > F(\alpha_2(r_G, B)) - c \geq F(\alpha_2(r_B)).$$

Consequently, an $L$ rating agency deviates from the private signal whenever an $H$ rating agency does so.

**Lemma 2** Whenever an $L_G$ ($L_B$) rating agency deviates from the private signal with positive probability, then an $L_B$ ($L_G$) rating agency does not deviate from the private signal. Likewise for an $H$ rating agency.

If an $L_G$ rating agency deviates from the private signal with positive probability it is because:

$$F(\alpha_2(r_B)) - c \geq \Pr(G | s_G, L) F(\alpha_2(r_G, G)) + \Pr(B | s_G, L) F(\alpha_2(r_G, B)).$$

But then using condition (14) it follows that:

$$F(\alpha_2(r_B)) > \Pr(G | s_B, L) F(\alpha_2(r_G, G)) + \Pr(B | s_B, L) F(\alpha_2(r_G, B)) - c,$$
which means that an $L_B$ rating agency is necessarily going to issue the private signal as a rating. On the other hand, if an $L_B$ rating agency issues $r_G$ with positive probability then

$$\Pr(G \mid s_B, L) F(\alpha_2(r_G, G)) + \Pr(B \mid s_B, L) F(\alpha_2(r_G, B)) - c \geq F(\alpha_2(r_B))$$  \hspace{1cm} (18)

and consequently, using condition (14):

$$\Pr(G \mid s_G, L) F(\alpha_2(r_G, G)) + \Pr(B \mid s_G, L) F(\alpha_2(r_G, B)) > F(\alpha_2(r_B)) - c$$  \hspace{1cm} (19)

which means that an $L_G$ rating agency issues the private signal as a rating. These results follow as long as $\Pr(G \mid s_G, a) > \Pr(B \mid s_G, a)$ and $\Pr(G \mid s_B, a) < \Pr(B \mid s_B, a)$. In particular, the same results follow for an $H$ rating agency when applying the corresponding conditional probabilities.

Note that this lemma does not exclude the possibility that both an $L_G$ and an $L_B$ rating agency do not deviate from the private signal: expression (16) can hold with the reverse sign while expression (17) is still valid. Likewise for expressions (18) and (19).

**Lemma 3** There cannot be an equilibrium in which an $H$ rating agency does not issue the private signal as a rating.

This is proved by contradiction using lemmas 1 and 2. Assume that in equilibrium an $H_G$ ($H_B$) rating agency issues $r_B$ ($r_G$) with positive probability. Then, an $L_G$ ($L_B$) rating agency always issues $r_B$ ($r_G$) by lemma 1 as well as an $H_B$ ($H_G$) and an $L_B$ ($L_G$) rating agency by lemma 2. But then an $H_G$ ($H_B$) rating agency prefers to deviate and issue $r_G$ ($r_B$) as by doing so it reveals its type and maximizes reputation.

### 6.1.3 Investors’ and Entrepreneur’s Posterior Beliefs about the Rating Agency’s Type

In order to show that an equilibrium exists, the way reputation evolves between the two time periods needs to be examined. Consider the first case in which an $L_G$ rating agency deviates
from the private signal with probability \( \gamma \). Then by lemmas 2 and 3: 1) an \( L_B \) rating agency issues \( r_B \), and 2) an \( H \) rating agency always issues its private signal as a rating. Hence, if \( r_B \) is issued a rating agency’s reputation at time period 2 is given by:

\[
\alpha_2 (r_B) = Pr(H | r_B) = \frac{\alpha_1 (1-\theta)}{\alpha_1 (1-\theta) + (1-\alpha_1) (\varepsilon (1-\theta) + (1-\varepsilon) \theta + ((1-\varepsilon) (1-\theta) + \varepsilon \theta) \gamma)}
\]

(20)

There are several situations that could give rise to this event: (H and B) with probability \( \alpha_1 (1-\theta) \), (L, \( s_B \) and \( r_B \), and G or B) with probability \( (1-\alpha_1) (\varepsilon (1-\theta) + (1-\varepsilon) \theta) \), and (L, \( s_G \) and \( r_B \), and G or B) with probability \( (1-\alpha_1) ((1-\varepsilon) (1-\theta) + \varepsilon \theta) \gamma \). And if the rating agency issues \( r_G \), its reputation at time period 2 varies depending on whether the project pays off 1 or 0. These two reputational levels are given by:

\[
\alpha_2 (r_G, G) = Pr(H | r_G, G) = \frac{\alpha_1}{\alpha_1 + (1-\alpha_1) \varepsilon (1-\gamma)}
\]

(21)

and

\[
\alpha_2 (r_G, B) = Pr(H | r_G, B) = 0
\]

respectively. Moreover, \( \alpha_2 (r_G, G) > \alpha_2 (r_B) > \alpha_2 (r_G, B) \) but \( \alpha_2 (r_B) \) does not exceed \( \alpha_1 \) unless \( \theta \) is low enough, and definitely for a range of \( \theta < \frac{1}{2} \). Obviously, investors need to be very convinced that the project is bad to believe in a rating that is not verifiable. This is going to be important for the competition case.

Consider the second case in which an \( L_B \) rating agency deviates from the private signal with probability \( \overline{\gamma} \). Then by lemmas 2 and 3: 1) an \( L_G \) rating agency issues \( r_G \), and 2) an \( H \) rating agency always issues its private signal as a rating. When \( r_B \) is issued, a rating agency’s reputation at time period 2 is given by:

\[
\overline{\alpha_2} (r_B) = Pr(H | r_B) = \frac{\alpha_1 (1-\theta)}{\alpha_1 (1-\theta) + (1-\alpha_1) (\theta (1-\varepsilon) + \varepsilon (1-\theta)) (1-\overline{\gamma})}
\]

(22)
And if the rating agency issues $r_G$ its reputation is

$$
\overline{\alpha}_2 (r_G, G) = Pr (H \mid r_G, G) = \frac{\alpha_1}{\alpha_1 + (1 - \alpha_1) (\varepsilon + (1 - \varepsilon) \overline{\alpha}_1)}
$$

(23)

or

$$
\overline{\alpha}_2 (r_G, B) = Pr (H \mid r_G, B) = 0,
$$

depending on whether the project pays off 1 or 0 respectively. In this case $\overline{\alpha}_2 (r_G, G)$ does not necessarily exceed $\overline{\alpha}_2 (r_B)$ and $\overline{\alpha}_2 (r_B)$ exceeds $\alpha_1$ when $\theta < \frac{1}{2}$ and it is lower than $\alpha_1$ otherwise, depending on the parameter values. Note that it is indeed the case that $F (\alpha_2 (r_G, G)) > F (\alpha_2 (r_G, B))$ as $\alpha_2 (r_G, B) = 0$.

6.1.4 The Equilibrium Behavior of an L rating agency

This section derives the equilibrium values for the probabilities $\gamma_1$ and $\overline{\gamma}_1$ taking into account lemmas 1-3. The cost of misreporting is taken to be arbitrarily small and is therefore ignored. The effect of a non-arbitrarily small cost of misreporting is discussed in section 6.2.4.

(i) $L_B$ deviates from the private signal with probability $\overline{\gamma}_1$: Let an $L_B$ rating agency issue $r_G$ with probability $\overline{\gamma}_1$ and $r_B$ otherwise. An $L_B$ rating agency decides to misreport at time period 1 by looking at

$$
Pr (G \mid s_B, L) F (\overline{\alpha}_2 (r_G, G)) + Pr (B \mid s_B, L) F (\overline{\alpha}_2 (r_G, B)) - F (\overline{\alpha}_2 (r_B)).
$$

(24)

The fee and the conditional probabilities are given by expressions (6) and (13) respectively. The expression simplifies to:

$$
\frac{1 - \varepsilon}{2 \theta} \left[ \frac{\theta (1 - \varepsilon)}{\theta (1 - \varepsilon) + (1 - \theta) \varepsilon} \overline{\alpha}_2 (r_G, G) - \overline{\alpha}_2 (r_B) \right]
$$
that using (22) and (23), implies the following:

\[
\frac{1 - \varepsilon}{2\theta} \left[ \frac{\theta (1 - \varepsilon)}{\theta (1 - \varepsilon) + (1 - \theta) \varepsilon \alpha_1 + (1 - \alpha_1) (\varepsilon + (1 - \varepsilon) \gamma_1)} - \frac{\alpha_1 (1 - \theta)}{\alpha_1 (1 - \theta) + (1 - \alpha_1) (\theta (1 - \varepsilon) + \varepsilon (1 - \theta) (1 - \gamma_1)^{-1})} \right].
\] (25)

This expression is strictly decreasing in \( \gamma_1 \) and it is easily shown to be strictly increasing in \( \theta \) for the parameter values considered in the model and for \( 0 \leq \gamma_1 < 1 \) by looking at derivative with respect to \( \theta \). The first term of this derivative is \( -\frac{1 - \varepsilon}{2\theta^2} \) multiplied by the expression in square brackets. This expression is equal to zero when the equilibrium is in mixed strategies and it is negative if in equilibrium \( \gamma_1 = 0 \). This makes this term equal to zero or positive for \( 0 \leq \gamma_1 < 1 \). The second term of the derivative is given by the following expression which is always positive:

\[
\frac{1 - \varepsilon}{2\theta} \left[ \frac{(1 - \varepsilon) \varepsilon}{(\theta (1 - \varepsilon) + (1 - \theta) \varepsilon)^2 \alpha_1 + (1 - \alpha_1) (\varepsilon + (1 - \varepsilon) \gamma_1)} - \frac{-\alpha_1 (1 - \varepsilon) (1 - \alpha_1) (1 - \gamma_1)}{(\alpha_1 (1 - \theta) + (1 - \alpha_1) (\theta (1 - \varepsilon) + \varepsilon (1 - \theta)) (1 - \gamma_1))^2} \right].
\]

For \( \gamma_1 = 1 \) expression (25) is always negative meaning that the rating agency would be better off deviating and reporting the private signal. And for \( \gamma_1 = 0 \), the value of expression (25) is negative only when \( \theta \) is sufficiently low. It is otherwise positive meaning that the rating agency would better off deviating and misreporting the private signal. Hence, when \( \theta \) is low enough, for example \( \theta \to \varepsilon \), expression (25) simplifies to:

\[
\frac{1 - \varepsilon}{4\varepsilon} \frac{\alpha_1}{\alpha_1 + (1 - \alpha_1) (\varepsilon + (1 - \varepsilon) \gamma_1)} - \frac{\alpha_1}{\alpha_1 + (1 - \alpha_1) 2\varepsilon (1 - \gamma_1)}
\]

which is always negative and in equilibrium \( \gamma_1 = 0 \). But there is a threshold \( \overline{\theta} > \varepsilon \) such that for \( \theta \in (\overline{\theta}, 1) \) there cannot be an equilibrium in pure strategies for any value of the parameters. Because expression (25) is decreasing in \( \gamma_1 \), it follows that in equilibrium in this
interval $0 < \gamma_1 < 1$.

(ii) $L_G$ deviates from the private signal with probability $\gamma_1$: This proof mirrors the previous arguments. Let an $L_G$ rating agency issue $r_B$ with probability $\gamma_1$ and $r_G$ otherwise. An $L_G$ rating agency's decision at time period 1 is taken by looking at:

$$\text{Pr}(G \mid s_G, L) F(\alpha_2(r_G, G)) + \text{Pr}(B \mid s_G, L) F(\alpha_2(r_G, B)) - F(\alpha_2(r_B)).$$  (26)

As before (6), (13), (20) and (21) are used to rewrite the expression which simplifies to,

$$\frac{1 - \varepsilon}{2\theta} \left[ \frac{\theta \varepsilon}{\theta \varepsilon + (1 - \theta)(1 - \varepsilon)} \frac{\alpha_1}{\alpha_1 (1 - \theta) + (1 - \alpha_1)(\varepsilon (1 - \theta) + (1 - \varepsilon)(1 - \theta) + \varepsilon \theta)} \right].$$  (27)

Expression (27) is strictly increasing in $\gamma_1$ and it is strictly increasing in $\theta$ for any $0 < \gamma_1 \leq 1$ as shown by its derivative with respect to $\theta$. The first term of this derivative is $-\frac{1 - \varepsilon}{2\theta \gamma_1}$ multiplied by the expression in square brackets which is equal to zero when the equilibrium is in mixed strategies and it is negative if in equilibrium $\gamma_1 = 1$. The second term of the derivative is the following expression which is always positive:

$$\frac{1 - \varepsilon}{2\theta} \left[ \frac{(1 - \varepsilon) \varepsilon}{(\theta \varepsilon + (1 - \theta)(1 - \varepsilon))^2} \frac{\alpha_1}{\alpha_1 (1 - \theta) + (1 - \alpha_1)(\varepsilon (1 - \theta) + (1 - \varepsilon)(1 - \theta) + \varepsilon \theta)} \right].$$

Then, observe that for $\theta$ sufficiently high, expression (27) is always positive regardless of the value of $\gamma_1$. This means that the rating agency is better off reporting the private signal for high values of $\theta$ and in equilibrium $\gamma_1 = 0$. Since the derivative of (27) is difficult to sign for
\( \gamma_1 = 0 \) it cannot be immediately concluded that in this case (27) decreases as \( \theta \) decreases. However, since the term in square brackets in (27) is always strictly increasing in \( \theta \) and \( \frac{1-\varepsilon}{2\theta} \) is always positive for \( 0 < \theta < 1 \), then it is the case that as \( \theta \) decreases the whole expression becomes negative exactly at the point at which the term in the square brackets becomes negative. When this happens the rating agency would be better off deviating from the private signal. And for \( \gamma_1 = 1 \) expression (27) is negative (meaning that this is indeed an equilibrium) or positive (meaning that the equilibrium is in mixed strategies because (27) is increasing in \( \gamma_1 \)) depending on the initial reputational level \( \alpha_1 \). Take for example \( \theta = \frac{1}{2} \). In this case expression (27) becomes:

\[
(1 - \varepsilon) \left[ \frac{\varepsilon \alpha_1}{\alpha_1 + (1 - \alpha_1) \varepsilon (1 - \gamma_1)} - \frac{\alpha_1}{\alpha_1 + (1 - \alpha_1) (1 + \gamma_1)} \right] \tag{28}
\]

and it is straightforward to show that it is negative for \( \gamma_1 = 0 \) but its sign depends on \( \alpha_1 \) for \( \gamma_1 = 1 \). When \( \alpha_1 \to 1 \) the expression is negative and \( \gamma_1 = 1 \) in equilibrium.\(^{19}\) But when \( \alpha_1 \to 0 \), (28) becomes positive and \( 0 < \gamma_1 < 1 \) in equilibrium. This results can be generalized for any value of \( \alpha_1 \) because it can be shown that the derivative of (28) with respect to \( \alpha_1 \) is negative for \( 0 < \gamma_1 < 1 \). This derivative is equal to:

\[
\frac{1 - \varepsilon}{2\theta} \left\{ \frac{\varepsilon}{\alpha_1 + (1 - \alpha_1) \varepsilon (1 - \gamma_1)} - \frac{1}{\alpha_1 + (1 - \alpha_1) (1 + \gamma_1)} \right. \\
+ \alpha_1 \left[ \frac{-\varepsilon (1 - \varepsilon (1 - \gamma_1))}{(\alpha_1 + (1 - \alpha_1) \varepsilon (1 - \gamma_1))^2} - \frac{\gamma_1}{(\alpha_1 + (1 - \alpha_1) (1 + \gamma_1))^2} \right] \right\}
\]

and for \( 0 < \gamma_1 < 1 \) expression (28) is equal to zero which means that the first two terms of the derivative disappear.\(^{20}\) Hence, for \( \theta \geq \theta > \frac{1}{2} \) an \( L_G \) rating agency always issues the private signal as a rating and for \( \theta < \theta \) there exists \( 0 < \gamma_1 \leq 1 \) such that the rating agency is either indifferent between misreporting and not misreporting the private signal or always misreports the private signal, depending on \( \alpha_1 \).

\(^{19}\)Expression (27) is increasing in \( \theta \) for \( \gamma_1 = 1 \).

\(^{20}\)This derivative is always negative for any value of \( \theta \) as shown below in section 6.2.2.
6.1.5 Establishing the Distinct $\theta$ Ranges

Take $\theta = \bar{\theta}$ defined above as the value of $\theta$ for which (25) holds with equality for $\gamma_{\bar{\theta}} = 0$ and $\theta = \bar{\theta}$ as the value of $\theta$ for which (27) holds with equality for $\gamma_{\bar{\theta}} = 0$. The derivatives of these expressions with respect to $\theta$, and to $\gamma_{\bar{\theta}}$ and $\gamma_{\bar{\theta}}$ respectively, are presented above. It can be easily demonstrated that $\frac{\partial \gamma_{\bar{\theta}}}{\partial \theta} > 0$ and $\frac{\partial \gamma_{\bar{\theta}}}{\partial \theta} < 0$ by the implicit function theorem.

Moreover, as $\theta \rightarrow 1$, $\gamma_{\bar{\theta}} = 1$, and as $\theta \rightarrow 0$, $\gamma_{\bar{\theta}} = 0$. Thus, for $\theta \in (\bar{\theta}, 1)$ then $\gamma_{\bar{\theta}} > 0$ and for $\theta \in (1 - \varepsilon, \bar{\theta})$ then $\gamma_{\bar{\theta}} > 0$. It remains to be shown that $\theta < \bar{\theta}$. Only then can be stated that there is a region $[\theta, \bar{\theta}]$ for which there is no deviation from the private signal by the L rating agency. Expression (27) evaluated at $\theta = \frac{1}{2}$ (or $\gamma_{\bar{\theta}} = 0$) is identical to expression (25) when this is evaluated at $\theta = \bar{\theta}$ (or $\gamma_{\bar{\theta}} = 0$), except for the probabilities $Pr(G | s_G, L)$ and $Pr(G | s_B, L)$. Since $Pr(G | s_G, L) > Pr(G | s_B, L)$, then expression (27) would exceed (25) if $\bar{\theta} = \frac{1}{2}$. Since both expressions are increasing in $\theta$, equality in (25) and in (27) requires that $\theta < \bar{\theta}$.

6.2 Proof of Proposition 2

6.2.1 Proof of Proposition 2.1

Point 1. is shown in section 6.1.4.

6.2.2 Proof of Proposition 2.2

The proof is done by implicit differentiation. By straightforward differentiation, and using the fact that $\theta > \bar{\theta} > \frac{1}{2}$, it can be easily shown that $\frac{\partial \gamma_{\bar{\theta}}}{\partial \varepsilon}$ is negative because expression (25) is decreasing in $\varepsilon$ and in $\gamma_{\bar{\theta}}$. Also note that (25) is equal to zero in an equilibrium in mixed strategies which means that the first term of the derivative of (25) with respect to $\varepsilon$, which is equal to the derivative of $\frac{1 - \varepsilon}{2\varepsilon}$ times (25), is also equal to zero. It is not so simple to show what happens with $\frac{\partial \gamma_{\bar{\theta}}}{\partial \varepsilon}$. Expression (27) is increasing in $\gamma_{\bar{\theta}}$ and in $\varepsilon$ for $\theta < \frac{1}{2}$ but can be
decreasing in $\varepsilon$ otherwise. The derivative of (27) with respect to $\varepsilon$ is equal to:

$$\frac{1 - \varepsilon}{2\theta^2} \left[ \frac{\theta}{\theta \varepsilon + (1 - \theta) (1 - \varepsilon)} + \frac{\varepsilon (1 - 2\theta)}{(\theta \varepsilon + (1 - \theta) (1 - \varepsilon))^2} \right] - \frac{\theta \varepsilon}{\theta \varepsilon + (1 - \theta) (1 - \varepsilon)} \frac{(1 - \alpha_1) (1 - \gamma_1)}{\alpha_1 + (1 - \alpha_1) \varepsilon (1 - \gamma_1)} \alpha_2 (r_G, G) \left( 1 + \frac{(1 - 2\theta)}{\theta \varepsilon + (1 - \theta) (1 - \varepsilon)} \right) \alpha_2 (r_B).$$

For $\theta < \frac{1}{2}$ this expression is positive as the term inside the square brackets simplifies to:

$$\frac{\theta \varepsilon}{\theta \varepsilon + (1 - \theta) (1 - \varepsilon)} \left\{ \frac{\alpha_1}{(\alpha_1 + (1 - \alpha_1) \varepsilon (1 - \gamma_1))} + \frac{(1 - 2\theta)}{\theta \varepsilon + (1 - \theta) (1 - \varepsilon)} \right\}.$$

However, for $\theta > \frac{1}{2}$ it is difficult to determine the signal of this expression. The expression inside the curly brackets is always decreasing in $\theta$. It is positive when $\theta \to \frac{1}{2}$ (which makes the derivative of (27) with respect to $\varepsilon$ positive) and it is negative when $\theta \to 1$ (which makes the derivative of (27) with respect to $\varepsilon$ negative). Of course $\theta < \varepsilon < 1$, but because it is not possible to compute closed-form solutions to $\theta$ and $\gamma_1$, it cannot be excluded that there is small region of $\theta$ for which the overall derivative (27) is decreasing in $\varepsilon$. Thus, $\frac{\partial \gamma_1}{\partial \varepsilon}$ is negative when $\theta$ is not too high and can be positive otherwise.

6.2.3 Proof of Proposition 2.3

Ignore $\frac{1 - \varepsilon}{2\theta}$. The derivative of (25) with respect to $\alpha_1$ is found to be negative. This can be proved by straightforward derivation:

$$\frac{\theta (1 - \varepsilon)}{\theta (1 - \varepsilon) + (1 - \theta) \varepsilon} \left[ \frac{\alpha_2 (r_G, G)}{\alpha_1} - \frac{\alpha_2 (r_G, G) (1 - (\varepsilon + (1 - \varepsilon) \gamma_1))}{\alpha_1 + (1 - \alpha_1) (\varepsilon + (1 - \varepsilon) \gamma_1)} \right] - \frac{\alpha_2 (r_B)}{\alpha_1} \left[ \frac{\alpha_2 (r_B) (1 - \theta - \theta (1 - \varepsilon) + \varepsilon (1 - \theta)) (1 - \gamma_1)}{\alpha_1 (1 - \theta) + (1 - \alpha_1) (\theta (1 - \varepsilon) + \varepsilon (1 - \theta)) (1 - \gamma_1)} \right],$$

summing and subtracting $\frac{(1 - \varepsilon) \theta}{(1 - \varepsilon) \theta + (1 - \theta) \varepsilon} \alpha_2 (r_G, G) \left( \frac{1 - \theta - ((1 - \varepsilon) \theta + \varepsilon (1 - \theta)) (1 - \gamma_1)}{\alpha_1 (1 - \theta) + (1 - \alpha_1) ((1 - \varepsilon) \theta + \varepsilon (1 - \theta)) (1 - \gamma_1)} \right)$ and making use of the fact that in a mixed strategies equilibrium condition (25) is equal to zero and
that \( \alpha_2(r_B) < \alpha_2(r_G, G) \), i.e. \((\varepsilon + (1 - \varepsilon) \gamma_1) (1 - \theta) < (\theta (1 - \varepsilon) + \varepsilon (1 - \theta)) (1 - \gamma_1)\). As (25) varies negatively with \( \gamma_1 \), then \( \frac{\partial \gamma_1}{\partial \alpha_1} \) is negative.

The derivative of (27) with respect to \( \alpha_1 \) is found to be negative. This is shown by computing the derivative:

\[
\frac{\theta \varepsilon}{\theta \varepsilon + (1 - \theta) (1 - \varepsilon)} \left[ \frac{\alpha_2(r_G, G)}{\alpha_1} - \frac{\alpha_2(r_G, G) (1 - \varepsilon (1 - \gamma_1))}{\alpha_1 + (1 - \alpha_1) \varepsilon (1 - \gamma_1)} \right] - \frac{\alpha_2(r_B)}{\alpha_1}
\]

\[
+ \frac{\alpha_2(r_B) (1 - \theta - (\varepsilon (1 - \theta) + (1 - \varepsilon) \theta + ((1 - \varepsilon) (1 - \theta) + \varepsilon \theta) \gamma_1))}{\alpha_1 (1 - \theta) + (1 - \alpha_1) (\varepsilon (1 - \theta) + (1 - \varepsilon) \theta + ((1 - \varepsilon) (1 - \theta) + \varepsilon \theta) \gamma_1)},
\]

summing and subtracting \( \frac{\theta \varepsilon}{\theta \varepsilon + (1 - \theta) (1 - \varepsilon)} \alpha_2(r_G, G) \frac{1 - \theta - (\varepsilon (1 - \theta) + (1 - \varepsilon) \theta + ((1 - \varepsilon) (1 - \theta) + \varepsilon \theta) \gamma_1)}{\alpha_1 (1 - \theta) + (1 - \alpha_1) (\varepsilon (1 - \theta) + (1 - \varepsilon) \theta + ((1 - \varepsilon) (1 - \theta) + \varepsilon \theta) \gamma_1)} \)

and making use of the fact that when the equilibrium is in mixed strategies (27) is equal to zero and \( \alpha_2(r_B) < \alpha_2(r_G, G) \), i.e. \((1 - \theta) \varepsilon (1 - \gamma_1) < \varepsilon (1 - \theta) + (1 - \varepsilon) \theta + ((1 - \varepsilon) (1 - \theta) + \varepsilon \theta) \gamma_1 \).

As (27) varies positively with \( \gamma_1 \), then \( \frac{\partial \gamma_1}{\partial \alpha_1} \) is positive.

6.2.4 Proof of Proposition 2.4

When the cost of misreporting \( c \) is not arbitrarily small expressions (25) and (27) become:

\[
\frac{1 - \varepsilon}{2 \theta} \left[ \frac{\theta (1 - \varepsilon)}{\theta (1 - \varepsilon) + (1 - \theta) (1 - \varepsilon) \alpha_1 + (1 - \alpha_1) (\varepsilon (1 - \gamma_1))} \frac{\alpha_1}{\alpha_1 (1 - \theta)} - \frac{\alpha_1 (1 - \theta)}{\alpha_1 (1 - \theta) + (1 - \alpha_1) (\theta (1 - \varepsilon) + \varepsilon (1 - \theta)) (1 - \gamma_1)} \right] - c
\]

and

\[
\frac{1 - \varepsilon}{2 \theta} \left[ \frac{\theta \varepsilon}{\theta \varepsilon + (1 - \theta) (1 - \varepsilon) \alpha_1 + (1 - \alpha_1) \varepsilon (1 - \gamma_1)} \frac{\alpha_1}{\alpha_1 (1 - \theta)} - \frac{\alpha_1 (1 - \theta)}{\alpha_1 (1 - \theta) + (1 - \alpha_1) (\varepsilon (1 - \theta) + (1 - \varepsilon) \theta + ((1 - \varepsilon) (1 - \theta) + \varepsilon \theta) \gamma_1)} \right] + c
\]

respectively. Deviating from the private signal becomes more costly for both an \( L_B \) and an \( L_G \) rating agency, therefore the interval \( [\bar{\theta}, \bar{\theta}] \) for which there is no deviation from the private signal by the \( L \) rating agency increases. Now when evaluated at \( \gamma_1 = 0 \) and at
\( \gamma_1 = 0 \) respectively, expression (27) exceeds exceeds (25) by even more than in section 6.1.6. Moreover, it is straightforward to show that both \( \frac{\partial \gamma_1}{\partial c} \) and \( \frac{\partial \gamma_1}{\partial c} \) are negative.

### 6.3 Proof of Proposition 3

The analysis here follows the same reasoning as in section 6.1. The competitive fees at time period 2 are given by expression (11) whenever a rating agency is hired or are equal to zero when an entrepreneur chooses to hire the rating agency’s competitor. Lemmas 1-3 still apply as these lemmas do not depend on the form that the fee takes and rest on a single and very general assumption that is still reasonable in this case.\(^{21} \) Consequently, the posterior beliefs that define the reputational level of a rating agency at time period 2 are as defined in section 6.1.3. The cost of misreporting is taken to be arbitrarily small and is therefore ignored. Let rating agency \( i \) be hired at time period 1.

(i) \( L_B \) deviates from the private signal with probability \( \bar{\gamma}_1 \): Let an \( L_B \) rating agency issue \( r_{iG} \) with probability \( \bar{\gamma}_1 \) and \( r_{iB} \) otherwise. An \( L_B \) rating agency decides to misreport at time period 1 by looking at:

\[
\Pr(G \mid s_{iB}, L) F(\bar{\alpha}_2(r_{iG}, G)) - F(\bar{\alpha}_2(r_{iB})).
\]

A correct good rating from rating agency \( i \) implies that \( \bar{\alpha}_2(r_{iG}, G) > \alpha_1 \), which means that the rating agency is hired after a correct good rating. However, it is shown in section 6.1.3 that the rating agency reputational level can actually decrease after a bad rating. In this case the competitor is hired at time period 2 and \( F(\bar{\alpha}_2(r_{iB})) = 0 \). For \( \theta < \frac{1}{2} \) it is the case that \( \bar{\alpha}_2(r_{iB}) > \alpha_1 \). Then, both \( F(\bar{\alpha}_2(r_{iG}, G)) \) and \( F(\bar{\alpha}_2(r_{iB})) \) are given by expression (11) and the probability by expression (13). The previous expression simplifies to:

\[
\frac{1 - \varepsilon}{2\theta} \left[ \frac{\theta (1 - \varepsilon)}{\theta (1 - \varepsilon) + (1 - \theta) \varepsilon} (\bar{\alpha}_2(r_{iG}, G) - \alpha_1) - (\bar{\alpha}_2(r_{iB}) - \alpha_1) \right]
\]

and substituting \( \bar{\alpha}_2(r_{iG}, G) \) and \( \bar{\alpha}_2(r_{iB}) \) by expressions (23) and (22) respectively, it is ob-

\(^{21}\)This assumption is that reputation (and fee) is expected to increase when the rating agency is correct and to decrease otherwise.
tained that:

\[
\frac{1 - \varepsilon}{2\theta} \left[ \frac{\theta (1 - \varepsilon)}{\theta (1 - \varepsilon) + (1 - \theta) \varepsilon} \left( \frac{\alpha_1}{\alpha_1 + (1 - \alpha_1) \varepsilon} - \alpha_1 \right) - \left( \frac{\alpha_1 (1 - \theta)}{\alpha_1 (1 - \theta) + (1 - \alpha_1) (\theta (1 - \varepsilon) + \varepsilon (1 - \theta)) (1 - \tau_i) - \alpha_1 \right) \right].
\]  

(29)

Note that this expression is still strictly decreasing in \(\tau_{i1}\) and increasing in \(\theta\) (because \(\bar{\alpha}_2 (r_i G, G)\) exceeds \(\alpha_1\) for any \(0 \leq \tau_{i1} < 1\). For \(\tau_{i1} = 1\) the expression is negative and the rating agency would be better off deviating and reporting the private signal. For \(\tau_{i1} = 0\) the expression becomes:

\[
\frac{1 - \varepsilon}{2\theta} \left[ \frac{\theta (1 - \varepsilon)}{\theta (1 - \varepsilon) + (1 - \theta) \varepsilon} \left( \frac{\alpha_1}{\alpha_1 + (1 - \alpha_1) \varepsilon} - \alpha_1 \right) - \left( \frac{\alpha_1 (1 - \theta)}{\alpha_1 (1 - \theta) + (1 - \alpha_1) (\theta (1 - \varepsilon) + \varepsilon (1 - \theta))} - \alpha_1 \right) \right].
\]

The expression tends to \(-\infty\) as \(\theta \to 0\) and \(\tau_{i1} = 0\) in equilibrium. But its value becomes positive when \(\theta\) is sufficiently high. In particular, for \(\theta = \frac{1}{2}\) it takes the value of \((1 - \varepsilon)^2 \left( \frac{\alpha_1}{\alpha_1 + (1 - \alpha_1) \varepsilon} - \alpha_1 \right) > 0\). As expression (29) decreases with \(\tau_{i1}\), it follows that there exists a threshold \(\bar{\theta}_c < \frac{1}{2}\), such that in equilibrium \(0 < \tau_{i1} < 1\) for \(\theta > \bar{\theta}_c\).

For \(\theta \geq \frac{1}{2}\), \(\bar{\alpha}_2 (r_i B)\) may not exceed \(\alpha_1\). This is not the case for \(\tau_{i1} = 1\). The expression is negative and the rating agency would be better off deviating and reporting the private signal. But for \(\tau_{i1} = 0\), \(\bar{\alpha}_2 (r_i B)\) is lower than \(\alpha_1\), which means that only a good rating generates positive expected fees. Hence, \(0 < \tau_{i1} < 1\) and \(\bar{\alpha}_2 (r_i B) > \alpha_1\).

(ii) \(L_G\) deviates from the private signal with probability \(\tau_{i1}\) : Let an \(L_G\) rating agency issue \(r_i B\) with probability \(\tau_{i1}\) and \(r_i G\) otherwise. An \(L_G\) rating agency’s decision at time period 1 is taken by looking at:

\[
\Pr (G | s_{iG}, L) F (\bar{\alpha}_2 (r_i G, G)) - F (\bar{\alpha}_2 (r_i B))
\]

For \(\theta < \frac{1}{2}\), \(\bar{\alpha}_2 (r_i B)\) may exceed \(\alpha_1\). In this case, (11), (13), (20) and (21) are used to rewrite
the expression which simplifies to,

\[
\frac{1 - \varepsilon}{2\theta} \left[ \frac{\theta\varepsilon}{\theta\varepsilon + (1 - \theta)(1 - \varepsilon)} \left( \frac{\alpha_1 (r_G, G) - \alpha_1}{\alpha_1 + (1 - \alpha_1) \varepsilon (1 - \gamma_{\bar{I}I})} - \alpha_1 \right) \right] - \left( \frac{\alpha_1 (1 - \theta)}{\alpha_1 (1 - \theta) + (1 - \alpha_1) (e (1 - \theta) + (1 - \varepsilon)(1 - \theta) + \varepsilon\theta) \gamma_{\bar{I}I}} - \alpha_1 \right) \right]
\]

or

\[
\frac{1 - \varepsilon}{2\theta} \left[ \frac{\theta\varepsilon}{\theta\varepsilon + (1 - \theta)(1 - \varepsilon)} \left( \frac{\alpha_1 (r_G, G) - \alpha_1}{\alpha_1 + (1 - \alpha_1) \varepsilon (1 - \gamma_{\bar{I}I})} - \alpha_1 \right) \right] - \left( \frac{\alpha_1 (1 - \theta)}{\alpha_1 (1 - \theta) + (1 - \alpha_1) (e (1 - \theta) + (1 - \varepsilon)(1 - \theta) + \varepsilon\theta) \gamma_{\bar{I}I}} - \alpha_1 \right) \right]
\]

The expression is strictly increasing in \( \gamma_{\bar{I}I} \) and in \( \theta \) for \( 0 < \gamma_{\bar{I}I} \leq 1 \). For \( \gamma_{\bar{I}I} = 1 \), then \( \alpha_2 (r_{iB}) < \alpha_1 \) and \( F (\alpha (r_{iB})) = 0 \) and the expression is always positive meaning that the rating agency prefers to deviate. And for \( \gamma_{\bar{I}I} = 0 \), then \( \alpha_2 (r_{iB}) > \alpha_1 \) and \( F (\alpha_2 (r_{iB})) > 0 \) and the expression simplifies to:

\[
\frac{1 - \varepsilon}{2\theta} \left[ \frac{\theta\varepsilon}{\theta\varepsilon + (1 - \theta)(1 - \varepsilon)} \left( \frac{\alpha_1 (r_G, G) - \alpha_1}{\alpha_1 + (1 - \alpha_1) \varepsilon (1 - \gamma_{\bar{I}I})} - \alpha_1 \right) \right] - \left( \frac{\alpha_1 (1 - \theta)}{\alpha_1 (1 - \theta) + (1 - \alpha_1) (e (1 - \theta) + (1 - \varepsilon)(1 - \theta) + \varepsilon\theta) \gamma_{\bar{I}I}} - \alpha_1 \right) \right]
\]

This expression is positive when \( \theta \) is sufficiently high and it follows that in equilibrium \( \gamma_{\bar{I}I} = 0 \), but it is negative otherwise. This is because \( \frac{1 - \varepsilon}{2\theta} > 0 \) for any \( \theta > 0 \), and the expression in square brackets is increasing in \( \theta \). It is positive when \( \theta = \frac{1}{2} \) as \( F (\alpha_2 (r_{iB})) = 0 \) but when \( \theta \rightarrow 1 - \varepsilon \), it can be shown by straightforward manipulation that it becomes \( \frac{1}{2} \left( \frac{\alpha_1 (r_G, G) - \alpha_1}{\alpha_1 + (1 - \alpha_1) \varepsilon} - \alpha_1 \right) - \frac{1}{2} \left( \frac{\alpha_1 (r_G, G) - \alpha_1}{\alpha_1 + (1 - \alpha_1) \varepsilon} - \alpha_1 \right) < 0 \). As (30) is increasing in \( \gamma_{\bar{I}I} \), it follows that there is a \( 1 - \varepsilon < \theta < \frac{1}{2} \) such that in equilibrium \( 0 < \gamma_{\bar{I}I} < 1 \) for \( \theta < \theta_c \).

For \( \theta > \frac{1}{2} \), \( \alpha_2 (r_{iB}) \) never exceeds \( \alpha_1 \) and \( F (\alpha_2 (r_{iB})) = 0 \). In this case, an \( L_G \) rating agency always reports the private signal and \( \gamma_{\bar{I}I} = 0 \). Then it is indeed the case that for an \( L_B \) rating agency in equilibrium \( 0 < \gamma_{\bar{I}I} < 1 \), as shown above.

(iii) **Establishing the Distinct \( \theta \) regions:** It than be shown following the same reasoning as in the monopolistic case that \( 1 - \varepsilon < \theta_c \).
(iv) Comparative Statics

It is clear that the sign of the derivatives of $\gamma_{11}$ and $\gamma_{11}$ with respect to $\theta$, $\alpha$, $\varepsilon$ and $c$ are not going to change relative to the monopolistic case. It is also the case that the intensity of misreporting a good private signal is higher with competition, i.e. $\gamma_{11} > \gamma_{11}$ for a given $\theta$, and the intensity of misreporting a bad private signal is lower, i.e. $\gamma_{11} < \gamma_{11}$. This can be shown by comparing (29) and (25) in the first case, and (30) and (27) in the second: there is extra positive term that makes issuing a good rating more attractive in competition than in the monopolistic case.

7 References


