Security-Voting Structure and Bidder Screening

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Abstract

This paper analyzes how non-voting shares affect the takeover outcome in a single-bidder model with asymmetric information and private benefit extraction. In equilibrium, the target firm’s security-voting structure influences the bidder’s participation constraint and in response the shareholders’ conditional expectations about the post-takeover share value. Therefore, the structure can be chosen to discriminate among bidder types. Typically, the socially optimal structure deviates from one share - one vote to promote all and only value-increasing bids. As target shareholders ignore takeover costs, they prefer more takeovers and hence choose a smaller fraction of voting shares than is socially optimal. In either case, the optimal fraction of voting shares decreases with the quality of shareholder protection and increases with the incumbent manager’s ability. Finally, shareholder returns are higher when a given takeover probability is implemented by (more) non-voting shares rather than by (larger) private benefits.
1 Introduction

Dual-class shares in publicly traded firms continue to be controversial. The New York Stock Exchange used to deny listings to firms with multiple share classes but abandoned this requirement in 1986. By contrast, the European Commission currently considers a proposal to mandate the one share - one vote principle, which would ban shares with differential voting rights and voting restrictions. If adopted, these provisions would affect a large number of firms: According to a survey commissioned by the Association of British Insurers, 35 percent of the top 300 European companies deviated in 2005 from the one share - one vote principle. In the US deviations are less frequent but still common. The fraction of listed firms with dual-class shares is about 10 percent (Chemmanur and Jiao, 2005).

Proponents of the one share - one vote rule argue that it is most conducive to an efficient allocation of corporate control. The theoretical foundation of this view is the analysis of Grossman and Hart (1988) and Harris and Raviv (1988). In their framework, the security benefits and private benefits vary across bidders who compete for a dispersely held firm. Since bidders compete for voting shares, one share - one vote prevents that bidders’ willingness-to-pay for control and their ability to create value diverge, thereby ensuring an efficient control allocation. By contrast, deviations from one share - one vote bear the risk that an inefficient bidder with large private benefits outbids more efficient bidders. At the same time, a dual-class share structure may be in the shareholders’ interest as it allows to extract more surplus from the winning bidder.

It has to be noted that in this framework the security-voting structure matters for the control allocation only if the bidders’ ranking according to security benefits differs from their ranking according to private benefits. If the most efficient bidder has the largest security benefits and the largest private benefits, she wins the bidding contest irrespective of the security-voting structure. Moreover, the security-voting structure is immaterial for bid price and shareholder wealth in the absence of (effective) competitors. Due to the target shareholders’ free-rider behavior (Grossman and Hart, 1980), the bid price must in this case match the winning bidder’s security benefits. Nonetheless, one share - one vote is optimal in the sense that no other security-voting structure leads to a more efficient control allocation in this framework.

The present paper revisits the allocational role of the security-voting structure in a model with atomistic target shareholders and absent (effective) competition. Instead, it assumes a single bidder who has private information about her ability to create value. As a result, the bid price is solely determined by the shareholders’ free-rider behavior and must at least equal the expected post-takeover share value. Costly bids are feasible because the bidder can extract part of the value generated by her as private benefits. Within this framework, we show that one share - one vote maximizes the severity of the
free-rider problem, thereby deterring too many value-increasing takeovers. Thus, contrary to the prevailing view, one share - one vote does not ensure an efficient control allocation and is typically inferior to a dual-class share structure.

In our model, all bidders who make a bid in equilibrium offer the same price. While the equilibrium price is fair, i.e., equal to the average post-takeover share value, some (overvalued) bidder types pay more and some (undervalued) types pay less than their true post-takeover share value. In addition, there is a cut-off value, and all and only types who generate less value are deterred as they would make a loss when offering the equilibrium price. These types cannot succeed at a lower price since higher valuation types would mimic them, and shareholders in turn would not tender because the offered price would be less than the average post-takeover share value. Hence, the presence of asymmetric information has two effects. First, it causes redistribution among all bidder types who actually make a bid. Second, it exacerbates the free-rider problem as ceteris paribus more bids fail than under symmetric information.

The paper’s novel insight is that the security-voting structure affects the takeover outcome by altering how shareholders update their expectations about the post-takeover share value conditional on an observed bid price. (More) non-voting shares reduce the fraction of return rights that bidders purchase and therefore render a bid ceteris paribus more profitable for overvalued types. Hence, some formerly frustrated types can earn a profit and now make a bid. In response, shareholders revise their beliefs about the post-takeover share value downward. This in turn lowers the bid price at which shareholders are willing to tender and makes the takeover profitable for further types. That is, deviations from one share - one vote mitigate in the presence of asymmetric information the free-rider problem, thereby promoting takeover activity.

The monotone relationship between the fraction of voting shares and the cut-off value implies that the security-voting structure can be used to discriminate among desirable and undesirable bids. Unless takeover costs are either too high or too low relative to the bidder’s private benefits, the socially optimal structure implements the first-best outcome: all and only bids with value improvements in excess of the takeover cost succeed. Since target shareholders abstract from the takeover cost, they have a higher cost of deterring a bid. Therefore, they choose a lower cut-off value, or equivalently, a higher fraction of non-voting shares.

Either result can be cast in terms of the incumbent management’s quality. When its quality is lower, the proportion of value-increasing types (for a given distribution) is larger, and the optimal fraction of voting shares decreases. Intuitively, more non-voting shares increase ceteris paribus the probability that the incumbent manager is replaced, which is warranted for less able managers. Conversely, if most bidder types are inferior, it is optimal to protect the incumbent management from the takeover threat with the one share - one vote structure. Thus, our model consents with the common perception
that the merit of the one share - one vote structure is to prevent value-decreasing bids (Grossman and Hart, 1988).

Another important determinant of the optimal security-voting structure is the bidder’s ability to extract private benefits. Weak legal shareholder protection (higher extraction rates), like more non-voting shares, promotes takeovers. Thus, the optimal fraction of voting shares increases with private benefits. This result suggests that the case for the one share - one vote structure is strongest in countries with weak investor protection.

Finally, we compare shareholder wealth under the optimal security-voting structure in two regimes with different extraction rates, both implementing the same takeover probability. Promoting value-increasing bids by lowering the fraction of voting shares is cheaper for shareholders than allowing bidders to extract more private benefits. The reason is that extraction is a transfer from shareholders to bidders, whereas the security-voting structure merely affects the extent of redistribution among bidder types. This result stands out against the existing literature which advocates private benefits (higher extraction rates) as a means to mitigate the free-rider problem and the one share - one vote structure as a means to deter value-decreasing bidders (Grossman and Hart, 1980, 1988).

Even though our paper identifies new drawbacks of (mandating) the one share - one vote structure, it does by no means posit that one share - one vote is generally dominated. Most notably, we consider a setting with asymmetric information and a single bidder. By contrast, previous work establishing the social optimality of the one share - one vote structure relies on a competitive setting. Since neither result invalidates the other, the appropriate conclusion is that the optimal security-voting structure varies considerably across different - equally plausible - settings.

Our results depend on two assumptions. First, we assume dispersed ownership of voting shares. While the results extend to any ownership structure where the majority of voting rights are dispersely held, they do not hold in case of a blockholder who is pivotal for the takeover outcome. Therefore, our model cannot address the problem that dual-class share structures enable owners to control the firm with a minority equity stake. While many dual-class firms have a controlling owner, dispersely held votes are by no means unusual. In the sample of Bennedsen and Nielsen (2004) 57 percent of 1035 European dual-class firms have dispersed control, i.e., no group that comprises every owner with at least 5 percent of the votes holds collectively the majority. For the 500 largest firms, this figure is above 67 percent.\(^1\)

\(^1\)Typically, neither ownership studies nor dual-class studies report ownership separately for dual-class firms. Bennedsen and Nielsen (2004) allow to construct the fraction of dual-class firms which have dispersed control as defined above. Covering 10 European countries, their dataset merges data from Faccio and Lang (2002) and self-collected data for Sweden and Denmark. The total number of firms in their sample is 5162. The numbers of widely-held (total) dual-class firms for each country are: Austria 6 (23), Denmark 14 (70), Finland 29 (47), France 11 (16), Germany 40 (124), Ireland 15 (16), Italy 32 (86), Spain 1 (1), Sweden 59 (185), UK 385 (467). In the sample of Pajuste (2005) the two largest...
Second, the assumed constant extraction rate implies that security benefits and private benefits are positively correlated. As a result, shareholders overvalue in equilibrium bidder types with small private benefits and undervalue those with large private benefits. Increasing the fraction of voting shares therefore discourages types with a low propensity to bid. Yet, this need not be true for alternative correlations. When security benefits and private benefits are inversely related, low private benefit types are undervalued, and redistribution among types encourages takeovers. Thus, our results - like others in this literature - are sensitive to the assumed relationship between security and private benefits. Section 4.3 discusses this issue in more detail.

Several papers analyze takeovers by a single bidder who has private information about the post-takeover security benefits. Shleifer and Vishny (1986) show that the acquisition of a stake prior to the tender offer provides a partial solution to the free-rider problem. Their (partial) pooling equilibrium anticipates the equilibrium outcome in our benchmark case with a single share class. The sole difference is that the source of the bidder’s gains is private benefit extraction rather than toehold acquisition. In a model with two bidder types, Marquez and Yilmaz (2005) demonstrate that uncertainty about the post-takeover security benefits may make bids profitable for the type with high security benefits and low private benefits. Neither paper examines the role of the security-voting structure. Hirshleifer and Titman (1990) and Chowdry and Jegadeesh (1994) analyze models in which takeover outcomes are probabilistic and equilibrium offers reveal the bidder’s type. Given that the post-takeover security benefits become known to the shareholders, the security-voting structure is immaterial in these models. Each type offers a price equal to the post-takeover share value, making her gains independent of the fraction of shares purchased in the tender offer.

As discussed above, Grossman and Hart (1988) and Harris and Raviv (1988) show that forcing a would-be acquirer to purchase all return rights ensures an efficient control allocation in a bidding competition but may not maximize shareholder wealth. Bergström et al. (1997) and Cornelli and Felli (2000) revisit these effects in the context of the mandatory bid rule and the sale of a bankrupt firm. In Burkart et al. (1998), deviations from one share - one vote can be socially optimal. In the single bidder case, non-voting shares reduce the fraction of return rights that the bidder has to purchase. This leads to larger private benefits which may be necessary to make the takeover profitable. In case of bidding contests, non-voting shares intensify competition and force the winning bidder to purchase more return rights, thereby reducing inefficient private benefit extraction. While we also argue that dual-class shares can be optimal, the mechanisms differ. In their model, the fraction of voting shares determines the private benefits as opposed to shareholders own less than 20 percent of the votes in about 24 percent of the dual-class firms.

Simulations by Sercu and Vinaimont (2006) show that for many distributions of bidder types the one share - one vote structure does not maximize ex ante shareholder wealth.
the shareholders’ expectations about the post-takeover security benefits. Gromb (1992) shows in a framework with a finite number of shareholders that non-voting shares mitigate the free-rider problem. Reducing the number of voting shares makes each voting shareholder more pivotal and increases their tendering probability. For the same reason, super-majority rules increase the takeover probability (Holmström and Nalebuff, 1992).

The paper is organized as follows. Section 2 outlines the model and derives the pooling equilibrium in the simple case with a single share class and value-increasing bidders. Section 3 solves the model for a dual-class target and demonstrates that deviations from one share - one vote mitigate the free-riding problem. Section 4 introduces value-decreasing bidders and shows that the security-voting structure can be used to screen bidder types. We derive the socially and the shareholders’ optimal security-voting structure and examine the comparative static properties of these structures. Concluding remarks are in Section 6, and the mathematical proofs are in the Appendix.

2 Model

Consider a widely held firm that faces a single potential acquirer, henceforth the bidder \( B \). If the bidder gains control, she can generate revenues \( V \). While the bidder learns her type prior to making the tender offer, target shareholders merely know that the revenues \( V \) are uniformly distributed on \( [V, \overline{V}] \).

In addition, the bidder can divert part of the revenues as private benefits. The non-contractible diversion decision is modelled as the bidder’s choice of \( \phi \in [0, \overline{\phi}] \), such that security benefits (dividends) are \( X = (1 - \phi)V \) and her private benefits are \( \Phi = \phi V \). Accordingly, the opportunities to extract private benefits increase with the generated revenues, and private benefit extraction does not dissipate value, i.e., is efficient.\(^3\) Furthermore, the upper bound \( \overline{\phi} \in (0, 1) \) is commonly known and identical for all bidder types. The latter assumption will be relaxed in section 4.3 where we allow bidder types to differ in their extraction abilities.

Tender offers are the only admissible mode of takeover, and a successful offer requires that the bidder attracts at least 50 percent of the firm’s voting rights. To illustrate the workings of the model, we first consider the one share - one vote structure and defer the analysis of dual-class shares to subsequent sections. If the takeover succeeds, the bidder incurs a fixed cost \( K \) of administrating the takeover which is independent of her type and common knowledge.

If the takeover does not materialize, the incumbent manager remains in control. The incumbent can generate revenues \( V^I \) which are known to all shareholders. Like the bidder, she can extract a fraction \( \phi \in [0, \overline{\phi}] \) of the revenues. Hence, shareholders obtain

\(^3\)Inefficient private benefit extraction would not alter the qualitative results, but induce a successful bidder to acquire as few (voting) shares as necessary to gain control (Burkart et al., 1998).
\[ X^I = (1 - \phi)V^I \] in the absence of a takeover. Initially, we restrict attention to value-increasing bids and set \( V^I = V \). The sequence of events unfolds as follows.

In stage 1, the bidder learns her type \( V \) and decides whether to make a take-it-or-leave-it, conditional, unrestricted cash tender offer. If she does not make a bid, the game moves directly to stage 3. If she makes a bid, she offers to purchase all shares for a total price \( P \), provided that at least 50 percent of the shares (voting rights) are tendered. Throughout the paper, we assume that the bidder can neither credibly disclose her true \( V \) nor make the offer terms contingent on \( V \) by e.g. a security-exchange offer.\(^4\)

In stage 2, the target shareholders non-cooperatively decide whether to tender their shares. Shareholders are homogeneous and atomistic and do not perceive themselves as pivotal for the tender offer outcome.

In stage 3, if at least 50 percent of the shares are tendered, the bidder gains control and pays the price \( P \) and the cost \( K \). Otherwise, the incumbent manager remains in control. In either case, the controlling party chooses which fraction \( \phi \) of the revenues to divert as private benefits, subject to the constraint \( \phi \leq \bar{\phi} \).

Solving the tender offer game by backward induction, we begin with the stage 3 diversion decision. Given that private benefit extraction entails no deadweight loss, the optimal revenue allocation is straightforward. Unless a successful bidder has acquired all shares in the tender offer, in which case she is indifferent between any \( \phi \in [0, \bar{\phi}] \), she extracts the upper bound \( \bar{\phi} \). That is, setting \( \phi = \bar{\phi} \) is a successful bidder’s (weakly) dominant strategy, and the post-takeover security benefits are independent of the size of the bidder’s final stake. If the bid fails or does not materialize, the incumbent manager chooses likewise the maximum extraction rate \( \bar{\phi} \) as she owns no equity.

Since shareholders are atomistic, each of them tenders at stage 2 only if the offered price at least matches the expected security benefits. Shareholders condition their expectations on the offered price \( P \), the known takeover cost \( K \) and the anticipated extraction decision \( \phi = \bar{\phi} \). Hence, a successful tender offer must satisfy the free-rider condition

\[ P \geq E(X|P, K, \phi) = (1 - \bar{\phi})E(V|P, K). \]

For simplicity, we assume that shareholders tender if they are indifferent. This assumption eliminates mixed strategies and all equilibrium outcomes in which a bid succeeds and the bidder owns less than all (voting) shares. In addition, it rules out failure as an equilibrium outcome for bid prices that satisfy the free-rider condition.

\(^4\)In the present setting, a successful security-exchange offer would have to grant all return rights to the shareholders due to the free-rider behaviour. Otherwise, retaining the initial claim on the revenues would be a weakly dominant strategy for each shareholder. Thus, a security-exchange offer would result in a complete separation of voting rights and return rights. As this outcome is equivalent to a simple replacement of management, allowing for security-exchange offers would obviate the need for a takeover in the first place.
At stage 1, the bidder is willing to offer at most \( V - K \) as a successful offer attracts all shares. Thus, the bidder’s participation constraint is simply \( V - K \geq P \).

To avoid trivial outcomes, we impose a joint restriction on takeover cost, maximum extraction rate and the support of bidder types.

**Assumption 1** \( \bar{\phi}V < K < \bar{\phi}V \).

These restrictions ensure that some high bidder types but not all low bidder types can make a profitable bid when paying a price equal to their respective post-takeover security benefits. This in turn excludes outcomes where either all or no bidder types make an offer.\(^5\)

In any Perfect Bayesian Equilibrium, the bidder must have correct expectations about which bid prices are acceptable and prefer the smallest successful offer, provided her participation constraint is satisfied. Given shareholders by assumption tender when they are indifferent, this immediately rules out equilibria in which offers succeed at different prices. As there can only be a single equilibrium price \( P^* \), shareholders infer from observing a bid that it may come from any type who makes a non-negative profit at that price. Thus, the shareholders’ conditional expectations about the post-takeover security benefits are

\[
E(X|P^*, K) = (1 - \bar{\phi})E(V|V \geq P^* + K).
\]

Given bidder types are uniformly distributed on \([V, \bar{V}]\), a bid is therefore made and succeeds in equilibrium if the bidder’s participation constraint

\[
V - P^* \geq K
\]

and the free-rider condition

\[
P^* \geq (1 - \bar{\phi})(\bar{V} + P^* + K)/2
\]

hold.

There exists a continuum of prices that satisfy condition (2) and so constitute Perfect Bayesian Equilibria of the tender offer game. Following Shleifer and Vishny (1986), we select the minimum bid equilibrium which is the unique equilibrium satisfying the credible beliefs criterion of Grossmann and Perry (1986). All other equilibria require shareholders to believe that bidders generate, on average, security benefits that are smaller than the offered equilibrium price. (Details of the equilibrium selection are provided in the Appendix.) Imposing the credible beliefs criterion implies that shareholders do not

\(^5\)The condition \( \bar{\phi}V < K \) is more restrictive than required to obtain deterrence of some bidder types in equilibrium. The weaker constraint \( [\bar{\phi}(\bar{V} + V) - (\bar{V} - V)]/2 < K \) is sufficient but less intuitive, and the choice is inconsequential for the results.
reject a bid consistent with the free-rider condition (2), and that bidders hence choose the minimum price

\[ P^* = (1 - \bar{\phi}) (\bar{V} + P^* + K) / 2. \]

**Proposition 1 (Minimum bid equilibrium)** Given that the target firm has a one share - one vote structure, only types \( V \in [V^c, \bar{V}] \) make a bid and offer the same price \( P^* \) where

\[ V^c = \frac{(1 - \bar{\phi}) (\bar{V} + 2K)}{1 + \bar{\phi}} \quad \text{and} \quad P^* = \frac{(1 - \bar{\phi}) (\bar{V} + K)}{1 + \bar{\phi}}. \]

Since a bidder can appropriate part of the revenues as private benefits, some value-increasing bids succeed in equilibrium despite the target shareholders’ free-rider behavior. Nonetheless, all bidder types below the cut-off value \( V^c \) are frustrated.\(^6\) Asymmetric information aggravates the free-rider problem which becomes most apparent when considering the bidder’s participation constraint \( \bar{\phi} V \geq [(P^* - X) + K] \). In a full information setting, free-riding would imply \( P^* = X \), and all bidder types with \( \bar{\phi} V \geq K \) would make a successful bid. Under asymmetric information, \( P^* = X \) holds on average but not for each individual bidder type. Instead, some types pay more and others less than their respective post-takeover security benefits. The mispricing deters some types whose private benefits are sufficient to cover the takeover cost. That is, the cut-off value \( V^c \) under asymmetric information exceeds \( K/\bar{\phi} \) (the cut-off value under full information).\(^7\)

Moreover, bidder types \( V \in [K/\bar{\phi}, V^c) \) cannot succeed with a lower offer because all types \( V \geq V^c \) would then make the same offer, and target shareholders would on average be offered less than the post-takeover security benefits. Hence, the shareholders’ free-riding behavior, exacerbated by asymmetric information, prevents some bids, even though they would be value-increasing.

**Corollary 1** *The takeover probability decreases with the takeover cost and the variance of bidder types but increases with private benefits.*

The ex ante probability of a takeover corresponds to the probability that a bidder type exceeds the cut-off value \( V^c \). Under the uniform distribution, this probability is equal to \( (\bar{V} - V^c) / (\bar{V} - \bar{V}) \). Accordingly, any change in the cut-off value or in the support of the bidder types affects the takeover probability.

When the takeover cost increases, any bidder who can still break even must on average generate higher revenues. As a bid signals higher post-takeover security benefits, target shareholders only tender at a higher price. This increases the cut-off value \( V^c \), thereby decreasing the takeover probability.

\(^6\)In an extension with private benefit extraction, Chowdry and Jeegadesh (1994) derive an equilibrium in which a subset of types also offer an uninformative bid price.

\(^7\)The inequality \( V^c - K/\bar{\phi} > 0 \) follows from the condition \( \bar{\phi} \bar{V} > K \) which must hold for some bids to succeed in equilibrium (Assumption 1).
A higher variance reduces the fraction of types above the cut-off value, because the cut-off value exceeds the unconditional mean \((\overline{V} + \underline{V}) / 2\). In addition, a larger support \((\overline{V} - \underline{V})\) increases on average the post-takeover security benefits generated by those types who make bid. Accordingly, target shareholders demand a higher price, and the cut-off value increases.

By contrast, larger private benefits (\(\bar{\phi}\)-values) not only enable bidders to recoup the takeover cost more easily but also lower the post-takeover share value. Both effects induce target shareholders to revise their expectations about the post-takeover share value downward. This lowers the equilibrium bid price and cut-off value.

### 3 Non-Voting Shares and Takeover Activity

We now explore the impact of dual-class shares on the takeover outcome. More specifically, the target firm has a fraction \(s \in (0, 1]\) of voting shares entitled to the same (pro-rata) cash flow rights as the \(1 - s\) non-voting shares. Here we treat the fraction \(s\) as a parameter and analyze its optimal choice in the next section. While we assume dispersed ownership of (voting) shares, the results also hold in the presence of a minority block, or more precisely, for any ownership structure where the majority of the shares (voting rights) are held by atomistic shareholders.

The takeover bid and the decision to tender proceed under the same premises as before. In addition, the tender offer may discriminate between share classes but not within the same class. Thus, the bidder may quote different prices for voting and non-voting shares. However, if she submits a price for a certain share class, she has to buy all tendered shares from that class, conditional upon a control transfer.\(^8\)

To derive the equilibrium, we initially assume that the bidder only offers to buy voting shares. As we will show below, this is (part of) the optimal bidding strategy. Since either all or none of the voting shareholders tender in equilibrium, a bidder has to pay \(sP\) to gain control. Her willingness-to-pay for all voting shares is equal to \(sX + \bar{\phi}V - K\) and increases with her private benefits and the fraction of voting shares. Hence, the bidder’s participation constraint is

\[
\bar{\phi}V - s(P - X) \geq K. \tag{3}
\]

Upon observing a bid, shareholders infer that the bidder can make a profit when buying all \(s\) voting shares at that price. Consequently, they condition their expectations about the post-takeover share value on the subset of types for whom the participation constraint is satisfied. Given that \(V\) is uniformly distributed on \([\underline{V}, \overline{V}]\), these expectation

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\(^8\) The assumption that a bid has to be unrestricted for a given class is not crucial. Indeed, one can easily replicate the analysis of intra-class restricted bids by redefining \(s\). For example, restricted offers for half of the voting shares are equivalent to unrestricted offers for all \(s' = s/2\) voting shares.
\begin{align}
E \left[ X \right| \bar{\phi}V - s (P - X) \geq K \right] &= \frac{1 - \bar{\phi}}{2} \left[ V + \frac{sP + K}{\bar{\phi} + s (1 - \bar{\phi})} \right]. \quad (4)
\end{align}

In equilibrium, the expectations must at least be matched by the bid price. As before, we impose the credible beliefs criterion to select the minimum bid equilibrium.

**Lemma 1** Given \( s \in [0, 1] \), only types \( V \in [V^c(s), \bar{V}] \) make a bid \( sP^*(s) \) for the voting shares where

\[ V^c(s) = \frac{s \left( 1 - \bar{\phi} \right) \bar{V} + 2K}{2\bar{\phi} + s \left( 1 - \bar{\phi} \right)} \quad \text{and} \quad P^*(s) = \frac{\left( \bar{\phi} + s \left( 1 - \bar{\phi} \right) \right) \bar{V} + K}{\left( 2\bar{\phi} + s \left( 1 - \bar{\phi} \right) \right) / (1 - \bar{\phi})}. \]

Lemma 1 replicates Proposition 1 for a target firm with dual-class shares. All types who make a bid offer the same price which is equal to their average post-takeover security benefits. Hence, a given type purchases the \( s \) voting shares either at a premium \( (P^*(s) > \bar{X}) \) or at a discount \( (P^*(s) < \bar{X}) \), but the gains of the undervalued types are exactly offset by the losses of the overvalued types. Furthermore, all types \( V < V^c(s) \) abstain from bidding because the cost of purchasing overpriced (voting) shares exceeds their private benefits net of the takeover cost.

Since non-voting - like voting - shareholders only tender if the bidder offers at least the post-takeover security benefits, a bidder needs to offer the same price to purchase the non-voting shares. Accordingly, only undervalued types have an incentive to extend the offer \( P^* \) to non-voting shares. As shareholders are aware of this, they would reject bids for all shares. Thus, acquiring only voting shares is optimal for all types who make a bid in equilibrium: Overvalued types limit the loss on the shares purchased in the offer, while undervalued types avoid to reveal that they purchase the voting shares at a discount.

Even though non-voting shareholders are excluded from the offer, both classes of shareholders realize the same expected payoff. Conditional on a takeover, voting shareholders receive the bid price in cash, whereas non-voting shareholders retain shares of uncertain value. In equilibrium, the mispricing cancels out on average such that the expected post-takeover share value equals the cash amount paid to the voting shareholders.\(^9\)

The comparative-static properties of the minimum bid equilibrium are key to our subsequent analysis of the optimal security-voting structure. Lemma 1 shows that each security-voting structure \( s \) maps into a unique minimum bid equilibrium. In particular, the equilibrium price \( P^*(s) \) and cut-off value \( V^c(s) \) are continuously increasing in the fraction \( s \) of voting shares. This has a straightforward implication for the takeover probability.

**Proposition 2** Non-voting shares promote takeovers.

\(^9\)This result is specific to the minimum bid equilibrium. In any other Perfect Bayesian equilibrium, tendering (voting) shareholders receive on average more than the expected post-takeover security benefits.
In equilibrium, the marginal type who makes a bid ($V = V^c(s)$) purchases the voting shares at a loss that is exactly offset by her private benefits net of the takeover cost. A lower fraction of voting shares enables her to earn a positive profit as she has to purchase fewer overvalued shares. In addition, fewer voting shares render a bid feasible for some previously deterred types whose participation constraint (3) is now satisfied. More generally, any type is now willing to pay more per voting share because her private benefits have to be spread over fewer shares. Hence, a higher fraction of non-voting shares induces more types to bid, even if the price were to remain unchanged.

Shareholders infer that less exposure to mispricing extends the pool of types making a bid. They revise their expectations accordingly and are willing to tender at a lower price.\(^\text{10}\) This in turn further relaxes the participation constraint (3) of overvalued types and induces additional types to bid, thereby reinforcing the reduction in the minimum acceptable bid price.

Thus, as the fraction of voting shares decreases, bidders are willing to pay more but need to pay less for each voting share. This enlarges the pool of types that can make a profit and promotes takeover activity.

## 4 Optimal Security-Voting Structure

To highlight how the security-voting structure affects the tendering decision of poorly informed shareholders, we have abstracted from value-decreasing bids. As has been shown, a tender offer can succeed in equilibrium even though target shareholders incur a loss (e.g., Bebchuk, 1985). Confronted with a value-decreasing offer, dispersed shareholders face a pressure-to-tender problem: tendering may be individually rational to avoid being in a less favorable minority position.

Value-decreasing bids are also relevant in the present context. The quality of a security-voting structure is determined by the extent to which it frustrates value-decreasing bids but encourages value-increasing bids. To examine both dimensions, we relax our earlier restriction $V^I = \underline{V}$ and let $V^I \in [\underline{V}, \overline{V}]$, thereby introducing value-decreasing bidder types.

The presence of value-decreasing bidder types does, however, not affect our preceding analysis and results. Indeed, the share value under the incumbent management does not matter for the takeover outcome. Each shareholder tenders if the offered bid price matches the conditional expected post-takeover security benefits. Similarly, the decision to make a tender offer depends for a given price solely on the bidder’s type, i.e., her ability to generate revenues. Hence, Lemma 1 continues to hold for any $V^I \in [\underline{V}, \overline{V}]$.

\(^{10}\)The derivative of the up-dating function (4) with respect to $s$ is positive if $\phi P(s) > (1 - \phi)K$ which holds in equilibrium. As $P^*(s) > (1 - \phi)V^c(s)$, it suffices to show that $\phi(1 - \phi)V^c(s) > (1 - \phi)K$. Inserting the explicit expression for $V^c(s)$, this inequality becomes $\phi \overline{V} > K$ which is satisfied (Assumption 1).
Consequently, success of a value-decreasing bid is an equilibrium outcome in our setting whenever $V^I > V \geq V^c(s)$.

It has to be noted that failure of a conditional tender offer - whether value-decreasing or increasing - can in general be supported as an equilibrium outcome. However, our assumption that indifferent shareholders tender eliminates failure as an equilibrium if the bid satisfies the free-rider condition. That is, when success and failure of a given bid can be supported as equilibrium outcomes, we select success even if the bidder is inferior to the incumbent manager.

Alternatively, one may assume that shareholders reject all bids below the current share value. This amounts to shareholders playing weakly dominated strategies whenever a bid is lower than the current share value but higher than the post-takeover security benefits. More broadly, this selection criterion abstracts from coordination problems among dispersed shareholders, such as the pressure-to-tender problem, and resulting undesirable takeover outcomes. But these are precisely the major issues in the literature on optimal takeover regulation (e.g., Bebchuk and Hart, 2001). Sharing these concerns, we select success as the equilibrium outcome and analyze how the security-voting structure can help to overcome coordination problems.

Given our selection criterion, $V^I$ affects neither takeover probability nor takeover outcome. Nonetheless, being the revenues when the takeover fails, $V^I$ matters for the choice of the security-voting structure. To analyze this choice, we assume that the social planner decides on the fraction $s \in (0,1]$ of voting shares, knowing the current share value, the takeover cost $K$, the upper bound $\bar{\delta}$ and the distribution of bidder types. Later (section 4.2), we also derive the shareholders’ preferred security-voting structure under the same informational assumptions.

4.1 **Social Planner’s Choice**

From a social perspective, the takeover cost is a deadweight loss, while it is immaterial how the revenues are shared between shareholders and bidder or incumbent manager. Hence, the expected social welfare is

$$W = (1 - \Pr(V \geq V^c))V^I + \Pr(V \geq V^c) (E[V|V \geq V^c] - K).$$

Takeovers are socially desirable if they increase revenues by more than the takeover cost. That is, the socially optimal cut-off value is equal to $V^I + K$. Indeed, inserting the takeover probability converts the social welfare into

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11Given a bid is conditional, a shareholder who believes the bid to fail is indifferent between tendering and retaining. Imposing this belief on all shareholders and breaking the indifference in favour of retaining supports failure as an equilibrium outcome, irrespective of the offered price (Burkart et al., 2006).
\[ W = V^I + \frac{(V - V^c)}{(V - \bar{V})} \int_{V^c}^{V} \frac{(V - K - V^I)}{(V - V^c)} dV, \]

and the first-order condition with respect to \( V^c \) yields

\[ V^c_{soc} = V^I + K. \] (5)

While derived for uniformly distributed bidder types, this result holds for any distributional assumption. Implementing the optimal cut-off value is straightforward in view of the inverse relationship between \( s \) and \( V^c \) (Lemma 1).

**Proposition 3** Each firm has a unique socially optimal fraction of non-voting shares which decreases with the revenues generated by the incumbent manager.

The revenues generated by a firm’s incumbent manager plus the takeover cost determine the unique optimal cut-off value \( V^I + K \). Due to the inverse relationship between \( s \) and \( V^c \), there is in turn a unique fraction of voting shares that implements this cut-off value (or the closest achievable value). Thus, each firm as defined by its \( V^I \) has a unique socially optimal security-voting structure which increases in \( V^I \). As long as the optimal security-voting structure includes both voting and non-voting shares \((0 < s^* < 1)\), it achieves the first-best control allocation: It frustrates all and only value-decreasing bids. This does not hold for the two corner solutions \( s^* = 1 \) and \( s^* = 0 \). If \( V^I \) is sufficiently high, the one share - one vote structure is constrained optimal in the sense that not all, though as many as possible, value-decreasing bids are frustrated. Similarly, for sufficiently low \( V^I \) complete separation of cash flow rights and voting rights does not ensure that all value-increasing bids succeed. Thus, when one share - one vote or complete separation are optimal, they only implement the second-best control allocation.

Variations in the optimal fraction of non-voting shares across firms translate into different degrees of control contestability. Low values of \( V^I \) go together with high fractions of non-voting shares. Such stark deviations from one share - one vote are necessary to elicit bids from the many bidder types that can generate higher revenues (net of the takeover cost) than the incumbent manager. In the case of inept incumbent managers \((V^I < (1 - \bar{\phi})K/\bar{\phi})\), complete separation of return rights and voting rights is optimal.\(^{12}\) It minimizes the free-rider problem and is most conducive for a value-increasing takeover. Conversely, one share - one vote is optimal if the firm is run by a sufficiently competent manager. Since most bidder types are in this case less competent, the optimal takeover barrier is set high. In all other cases, one share - one vote offers incumbent managers too much protection from takeovers. Thus, we find that deviations from one share - one

\(^{12}\) As the optimal security-voting structure is derived under the assumption of dispersely held voting rights, this extreme outcome is not meant to be taken literally. Voting rights without any cash flow entitlement yield no economic benefit to small owners, making dispersed ownership an unlikely outcome.
vote are in many cases socially optimal. At the same time, our theory concurs with the argument that one share - one vote is effective in deterring value-decreasing bids (Grossman and Hart, 1988).

In recent years, many dual-class firms have unified their shares into a single class. In some cases the abolition of dual-class shares has been a voluntary decision, in others it has been a response to an anticipated new regulation. Proposition 3 pertains to stock reunifications when undertaken by firms with dispersely held votes.

**Corollary 2** Share-class reunifications are always in the managers’ interest but need not be socially efficient.

The one share - one vote structure minimizes the threat (probability) of a takeover. While this is always in the interest of the incumbent manager, it is socially desirable only if the incumbent manager is of high quality. The abolition of dual-class shares can have a sizeable entrenchment effect even for an average manager as the following example illustrates. Suppose $V$ is $5M and bidder types are uniformly distributed between $4M$ and $6M. In addition, let $K$ be $1M and $\theta = .05$. The socially optimal security-voting structure in this case, $s^* \approx .36$, leads to value-improving takeovers with a probability of 45 percent. Once the firm unifies its shares, the takeover probability is reduced to 19 percent. Or put differently, one share - one vote frustrates about 58 percent of all potential value-improving takeovers.

Thus, our theory implies that announcements of share-class reunifications may lead to both negative or positive stock price reactions. By contrast, empirical studies on the abolition of dual-class share structures report significant positive announcement returns (e.g., Hauser and Lauterbach, 2004; Dittmann and Ulbricht, 2004.) These studies do, however, not distinguish between sample firms with dispersed votes and (the majority of sample) firms with a controlling shareholder. A proper test of our prediction would require to analyze the subsample of dispersely held firms separately.\footnote{Interestingly, Dittmann and Ulbricht (2004) find that the proportion of voting shares not owned by blockholders has a negative impact on the announcement returns.}

Proposition 3 argues that dispersely held firms should have a dual-class share structure, unless the incumbent manager is sufficiently competent. Importantly, deviations from one share - one vote promote takeovers only if the bidder can discriminate between voting and non-voting shares. The requirement to make voting and non-voting shareholders the same offer replicates the one share - one vote structure, which may deter some value-increasing bids.\footnote{Since 1987 the Toronto Stock Exchange requires any firm which newly issues shares with superior voting rights to include a provision that obliges would-be acquirers to extend the offer at the same terms to all classes of shares (Allaire, 2006).} This so-called “coattail” provision has the same impact on takeovers as banning restricted bids for targets with a one share - one vote structure. Like non-voting shares, restricted bids reduce the fraction of shares that the bidder has...
to purchase when gaining control.\textsuperscript{15} Thus, these rules need not be socially optimal nor in the target shareholders interest, even though they lead to higher takeover premia. To fully assess whether these rules are beneficial, one would need to observe how many value-increasing takeovers are being frustrated.

Besides the incumbent manager’s quality, the takeover cost $K$ and the maximum extraction rate $\bar{\phi}$ are further determinants of the socially optimal security-voting structure. Hence, variations in these parameters also alter the optimal fraction of voting shares.

The size of the takeover cost has two opposite effects on the optimal security-voting structure. On the one hand, higher costs raise the socially optimal cut-off value, as the revenues generated by the bidder must exceed current revenues by a larger margin. On the other hand, higher costs require larger private benefits to break even. This deterrence effect is reinforced by the adjustment of the shareholders’ expectations about the post-takeover security benefits. Therefore, the latter effect dominates and the optimal fraction of non-voting shares increases with the takeover cost.

Higher extraction rates enable bidders to recoup the takeover costs more easily and lower the shareholders’ expectations about post-takeover share value. Hence, higher extraction rates and non-voting shares are substitutes: Both promote takeovers. As a result, more voting shares are required to implement a given cut-off value, when other governance mechanisms put weaker constraints on private benefit extraction.

**Proposition 4** The optimal fraction of non-voting shares increases in the quality of shareholder protection.

The result suggests that the rationale for one share - one vote is strongest in countries with weak shareholder protection whereas deviations may be desirable in environments where extraction is limited by strong institutions. Furthermore, it weakens the case for regulatory harmonization across diverse governance systems.

### 4.2 Shareholders’ Choice

As the equilibrium bid price always equals the expected post-takeover share value, voting and non-voting shareholders have homogeneous preferences. Hence, we can describe their collective and individual preferences by the aggregate wealth function

$$\Pi = (1 - \phi) V^I + \frac{(V - V^c)}{(V - \bar{V})} \int_{V^c}^V (1 - \phi) \frac{(V - V^I)}{(V - \bar{V})} dV.$$  \textsuperscript{(6)}

Simplifying and deriving the first-order condition with respect to $V^c$ yields

$$V^c_{sh} = V^I.$$ 

\textsuperscript{15}Note that a mandatory bid rule matters in our model only for the likelihood of a takeover. Conditional upon a bid, the rule is irrelevant for the division of takeover gains among target shareholders.
Proposition 5 Target shareholders choose a higher fraction of non-voting shares than the social planner.

In contrast to the social planner, shareholders do not internalize the takeover cost $K$ that is privately borne by the bidder. They are only concerned about the security benefits. Since these are proportional to the revenues, shareholders prefer the management that creates more value, irrespective of the takeover cost. Thus, shareholders prefer a lower cut-off value than socially optimal and choose an accordingly lower fraction of voting shares. When a bidder with $V \in (V^f, V^f + K)$ succeeds, the private benefits of the incumbent manager are not fully compensated by the takeover gains net of takeover costs.

The privately and socially optimal security-voting structure only coincide when both are corner solutions, i.e., when either complete separation is privately optimal or one share - one vote is socially optimal. In all other cases, shareholders would allow too many takeovers. From this viewpoint, a mandated one share - one vote structure may be seen as a possibly overshooting correction of inefficient private incentives.

Like the social planner, shareholders have an interest to protect competent managers. Since they want to admit only bidder types that create more value than the status quo, their preferred level of control contestability, and hence the optimal fraction of non-voting shares, decreases in the incumbent’s ability.\footnote{The optimal $V^c_{sh}$ could also be achieved for any $s$ through the coordination of shareholders, i.e., the appropriate equilibrium selection when success and failure can be supported as equilibrium outcomes. To support the failure of all bidder types $V < V^c_{sh}$ requires, however, that shareholders play weakly dominated strategies.}

Given that non-voting shares and extraction rates are substitutes, the question arises which combination of $s$ and $\phi$ target shareholders prefer. To this end, we compare two regimes implementing the same takeover probability. More precisely, consider the alternatives $\{\phi', s'\}$ and $\{\phi'', s''\}$ where $\phi' < \phi''$, $s' < s''$ and $V^c|_{s', \phi'} = V^c|_{s'', \phi''}$.

Proposition 6 For a given takeover probability, shareholder return is higher in the regime with less extraction and more non-voting shares.

This result reverses the role commonly attributed to the security-voting structure and private benefit extraction (e.g., Grossman and Hart; 1980, 1988). In our setting, shareholders do not choose a high $\phi$ to promote takeovers and a high $s$ to frustrate inefficient bids. Instead, a low $\phi$ is used to deter undesirable bidders and a low $s$ is used to encourage the others. The security-voting structure affects the redistribution among bidder types. More specifically, reducing the fraction of voting shares promotes takeovers by reducing the gains that high bidder types earn from mimicking low bidder types. By contrast, the extraction rate affects how the takeover surplus is split between shareholders and bidders. When using $\phi$ to encourage bids, shareholders bribe bidders out of their own
pockets. From a social perspective, the regimes are equivalent as they both implement the same takeover probability and hence the same control allocation.

4.3 Heterogenous Extraction Abilities

So far, we assumed that the security and private benefits are positively correlated. In our view, this is a plausible assumption because it reflects circumstances in which private benefits are primarily determined by firm characteristics and the institutional environment rather than the bidder’s identity. At the same time we agree that theoretical reasoning alone does not preclude alternative correlations. As in other models, our optimal security-voting structure and its comparative-static properties are sensitive to the posited relationship between security and private benefits.

When security benefits and private benefits are assumed to be inversely related, non-voting shares have the opposite effect and discourage takeovers. Suppose that all bidder types create the same revenues $V$ but differ in their extraction abilities $\tilde{\phi}$, which are distributed on the unit interval. When the target has a single share class, all types bid $P^* = \mathbb{E}[1 - \tilde{\phi}] V = V/2$ in equilibrium, provided that the takeover cost is not too large ($V > 2K$). Since all shareholders tender in a successful bid, every type enjoys the same surplus $V - P^* - K$. In this equilibrium, no bid is frustrated while all types $\tilde{\phi} < K/V$ would fail under symmetric information.

In the setting with negative correlation, the free-rider problem is mitigated rather than exacerbated by asymmetric information. The pooling price leads to a transfer from high extraction types to low extraction types, thereby subsidizing those types whose private benefits do not cover the takeover cost. More non-voting shares decrease the subsidy which low extraction types receive and discourage these types. Complete separation eliminates all subsidies, and - as under full information - only types whose private benefits exceed the takeover cost make a bid. Hence, either one share - one vote or complete separation are the socially optimal structure, depending on the current revenues $V^I$. For $V > V^I + K$ one share - one vote is optimal. Otherwise, complete separation is optimal without preventing value-decreasing bids by all types $\tilde{\phi} \in (K/V, 1]$. In Grossman and Hart (1988), one share - one vote is also optimal when security and private benefits are inversely related (across bidders). However, the merit of one share - one vote is the opposite: Bidders with large private benefits face the greatest deterrence. Here, one share - one vote is the structure with the weakest deterrence.

In the general case where bidder types differ both in their ability to generate revenues and extract private benefits, non-voting shares will exhibit both effects: they will encourage some bidder types but at the same time deter others. Which effect dominates and, in

\footnote{This result suggests that more information about the bidder type may not always be desirable, as shown in a two-type model by Marquez and Yilmaz (2005). They also briefly analyse the positive effect of supermajority rules on takeover activity. This is equivalent to the effect that voting shares have here.}
particular, which security-voting structure is socially optimal will depend on the extent of the assumed uncertainty about $V$ and $\tilde{\varphi}$. For convex type spaces, an optimal security-voting structure will in general not perfectly discriminate between value-increasing and value-decreasing types. For any optimal cut-off value there will be higher $V$-types who are deterred because their extraction rates are too low. Conversely, there will be lower $V$-types who bid because they can extract more private benefits. While a full analysis is beyond the scope of this paper, the mechanism identified in the preceding analysis carries over to any specification. The security-voting structure will affect the takeover outcome through the same channel: The fraction of voting shares alters the bidders’ participation constraint and hence shareholders’ updating about the post-takeover security benefits.

5 Conclusions

This paper identifies a new mechanism by which the security-voting structure matters for the outcome of a takeover bid in the absence of competition. When the bidder has private information about her ability to generate value, the fraction of non-voting shares affects both the bidder’s participation constraint and the shareholders’ conditional expectations. These interacting effects can be exploited to discriminate between desirable and undesirable bids. We find that the one share - one vote rule is in general not socially optimal.

In accordance with earlier theoretical work, our analysis demonstrates that the optimal security-voting structure is sensitive to the model assumptions, notably the competitive environment and the assumed relationship between security benefits and private benefits. One challenge for future research is to assess the empirical relevance of the various mechanisms identified in the literature.

The paper makes several contributions to the recurrent debate about the optimality of one share - one vote. Most importantly, deviations from one share - one vote can improve efficiency by promoting value-increasing takeovers. In addition, our analysis casts doubt on the merits of mandating a uniform security-voting structure across firms or countries. The reason is that the optimal structure depends on the quality of both the incumbent manager and the governance mechanisms that constrain managerial self-dealing.

Finally, it has to be emphasized that our analysis applies to widely held firms. In particular, we do not address the issue that controlling (minority) owners can block a takeover even if it were efficient. Instead, we point out that one share - one vote can help to entrench professional managers. Thus, share reunification programs in dispersely held firms may neither be socially optimal nor in the shareholders’ best interest.
Appendix

Proof of Proposition 1

Denote \( f(P) \equiv \left( (1 - \bar{\phi}) (V + P + K) \right) / 2 \). As \( f'(P) = (1 - \bar{\phi}) / 2 \in (0, 1) \), there exists a unique fixed point \( P^* \) such that the free-rider condition, \( P \geq f(P) \), is satisfied for any price above \( P^* \). Thus, any price \( P^+ \geq P^* \) can be supported as a Perfect Bayesian Equilibrium by imposing appropriate out-of-equilibrium beliefs, e.g., \( \mathbb{E}[V | P] = \bar{V} \) for all \( P \neq P^+ \). The pooling equilibrium in which all types bid requires \( K \leq \bar{V} - (1 - \bar{\phi}) \mathbb{E}[V] = \bar{\phi} \bar{V} - \left[ (1 - \bar{\phi}) (\bar{V} - \bar{V}) \right] / 2 \). Assumption 1 \( (K > \bar{\phi} \bar{V}) \) rules this out. Existence of Perfect Bayesian Equilibria in which some bids succeed require \( \bar{V} > V^c \equiv P^* + K \) which is implied by Assumption 1.

The credible beliefs criterion imposes that target shareholders believe a deviating (out-of-equilibrium) bid to come only from types that would want the bid to succeed. For any Perfect Bayesian Equilibrium with \( P^+ > P^* \), denote the set of types whose participation constraint is satisfied for \( P^* \) by \( D = \{ V \in [\bar{V}, \bar{V}] : V \geq P^* + K \} \) and its complement by \( D^C \). No bidder type in \( D^C \) would want to bid \( P^* \) and succeed, whereas bidder types in \( D \) would want to bid and succeed. Consequently, the credible beliefs criterion imposes that shareholders believe \( \Pr[V \in D | P = P^*] = 1 \). Given such beliefs and sequential rationality, shareholders would accept the deviation bid \( P^* \). Hence, no Perfect Bayesian Equilibrium with \( P^+ > P^* \) survives the credible beliefs refinement. If \( P^+ = P^* \), there exists no bid price to which any bidder would like to deviate as any lower price is rejected. Hence, \( P^+ = P^* \) is the unique Perfect Sequential Equilibrium.

Proof of Lemma 1

The bidder’s participation constraint is now \( sX + \Phi \geq sP + K \) which can be written as

\[
V \geq \frac{sP + K}{s(1 - \bar{\phi}) + \bar{\phi}}.
\]

Shareholders’ expectations about the post-takeover share value conditional upon observing a bid \( P \) are

\[
g(P) = \mathbb{E}\left[ X \left| V \geq \frac{sP + K}{\bar{\phi} + s(1 - \bar{\phi})} \right. \right] = (1 - \bar{\phi}) \int_{sP+K \over \bar{\phi}+s(1-\bar{\phi})}^{\bar{V}} \frac{V}{\bar{V} - {sP+K \over \bar{\phi}+s(1-\bar{\phi})}} dV = \frac{1 - \bar{\phi}}{2} \left[ \bar{V} + {sP + K \over \bar{\phi} + s(1 - \bar{\phi})} \right]
\]

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The fixed point of $g(P)$ is

$$P^* = \frac{(\phi + s (1 - \bar{\phi})) \bar{V} + K}{2\phi + s (1 - \bar{\phi}) / (1 - \phi)},$$

and the equilibrium cut-off value $V^c$ is

$$sP + K \bigg|_{s = \frac{s \bar{V} + 2K}{2\phi + s (1 - \bar{\phi})}} = s \frac{(1 - \bar{\phi}) \bar{V} + 2K}{2\phi + s (1 - \bar{\phi})}.$$ 

Existence and uniqueness are proven as in Proposition 1.

**Proof of Proposition 3**

$W$ is continuous in $V^c$. Since

$$\frac{\partial^2 W}{\partial (V^c)^2} = -\frac{1}{\bar{V} - V},$$

$W$ is strictly concave in $V^c$. Since $V^c$ is a continuous and monotonic function of $s$, $W$ is also continuous and strictly concave in $s$. Moreover, since $s \in [0, 1]$, it follows that the problem \(\max_s W(V^c(s))\) has a unique solution.

By Lemma 1 and equation (5),

$$V^I + K = \frac{s (1 - \bar{\phi}) \bar{V} + 2K}{2\phi + s (1 - \bar{\phi})}.$$ 

Solving for $s$, we get

$$s^* = \frac{2 (\bar{\phi} V^I - (1 - \bar{\phi}) K)}{(1 - \bar{\phi}) (\bar{V} - V^I - K)}.$$

Corner solutions are discussed in the text.

**Proof of Proposition 4**

The partial derivative of $s^* \in (0, 1)$ with respect to $\bar{\phi}$ is

$$\frac{\partial s^*}{\partial \bar{\phi}} = \frac{2V^I}{(1 - \bar{\phi})^2 (\bar{V} - V^I - K)} > 0 \quad \text{for} \quad \bar{V} > V^I + K.$$ 

Note that, if $\bar{V} < V^I + K$, then one share - one vote is optimal because any takeover would be inefficient.
Proof of Proposition 6

Let $\phi' < \phi''$ and choose $s'$ and $s''$ such that $V^c|_{s',\phi'} = V^c|_{s'',\phi''} = v$, where $v \in [\underline{V}, \overline{V}]$. By Lemma 1, $v = V^c$ translates into

$$s = \frac{2v\phi - 2K}{(1 - \phi)(\overline{V} - v)}.$$  

Thus, $\phi' < \phi''$ implies $s' < s''$. Comparing shareholder wealth (6) across the two regimes and noting that the cut-off value is identical,

$$(1 - \phi') \left[ V^I + \int_{\underline{V}}^{\overline{V}} \frac{V - V^I}{(V - v)} dV \right] > (1 - \phi'') \left[ V^I + \int_{\underline{V}}^{\overline{V}} \frac{V - V^I}{(V - v)} dV \right],$$

proves the result.
References


