Abstract

Executive stock options reward success but do not penalise failure. In contrast, the standard principal-agent model implies that pay is normally monotonically increasing in performance. This paper shows that, under loss aversion, the use of carrots but not sticks is a feature of an optimal compensation contract. Low risk aversion and high loss aversion is particularly propitious to the use of options. Moreover, loss aversion on the part of executives explains the award of at the money options rather than discounted stock or bonus related pay. Other features of stock option grants are also explained, such as resetting or reloading with an exercise price equal to the current stock price.

JEL: F3; F4

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*London School of Economics. Tel.: 44 2079557275; E-mail: d.c.webb@lse.ac.uk. We would like to thank Heskey Bar Isaac and Jean Charles Rochet for useful advise.
1 Introduction

It is a truth universally acknowledged that a CEO in possession of a Fortune 500 company must be in want of a stock-option scheme. Indeed Murphy (2002) reports that in 2000 stock options comprised more than 50% of the median pay of CEOs of the S&P 500 industrial companies. At first sight the explanation is that stock options, by aligning the interests of the CEO and shareholders, mitigate agency costs. Yet this optimal contracting perspective does not entirely fit the facts. Stock options are a very special form of performance pay; they reward success but do not penalise failure. That is, the exercise price of the option is typically the grant date market price, so if the shares do badly the CEO is not actually worse off. The question is therefore why choose a scheme with the property that, over some ranges, the CEOs pay does not increase in the share price. Indeed, protection from downside loss is not implied by the standard principal-agent problem. As shown by Holmstrom (1979) and Grossman and Hart (1983), if the monotone-likelihood property and other weak assumptions are satisfied, pay is everywhere increasing in measured performance.\footnote{For example, Huddart (1994), examines the optimal exercise strategy for executive stock options within an optimisation framework but does not establish the optimality properties of the contract itself.} So stock options do not emerge as the solution to an optimal contracting problem, although the award of stock (i.e. an option with strike price zero) might do so.\footnote{Hall and Murphy (2001) appears to be inconsistent with this claim, but the absence of a regular participation constraint means they do not analyse a standard principal-agent model. There is also a non trivial issue concerning whether the effect on expected utility of raising stock price should be measured at the exercise date or the issue date.}

A rival to the incentive theory is that stock options are an expropriation device. According to this managerial power view, which has grown in prominence following the Enron, WorldCom and other "scandals", CEOs loot the companies over which they rule. Stock options facilitate the robbery since, being nominally performance based (even if typically repriced or “reloaded” when under water) they appear more defensible to investors than would a sky high salary. In the terminology of Bebchuck, Fried and Walker (2001) and Bebchuck (2003) stock options are less likely to violate the share-holders “outrage constraint” than more transparent forms of remuneration including restricted stock.

Both the incentive and expropriation motives are buttressed by accounting and tax rules that do not require stock options to be expensed (e.g. U.S. Internal Revenue Code Section 162 (m)).\footnote{This may be changing, with some companies such as Coca Cola voluntarily electing to expense stock options. In the United Kingdom this is now mandatory, yet stock options remain a common and important form of compensation. See Carter and Lynch (2001) for evidence that firms incur economic costs for favourable treatment of stock options.} This makes it seem as if options are free of charge for they require no cash up front. It is therefore easy to hide from shareholders the true
extent of the theft involved. Even if stock options are not necessarily the best way to create incentives, the tax benefits and the positive effect on short term stock price from their appearance as below-the-line expenditures in company accounts may be enough to offset the second-best incentive properties and dysfunctional behavior thereby induced. Finally, stock-options are excluded from the definition of remuneration in the US “capping” law which subjected payments in excess of $1m to unfavourable tax treatment. Not surprisingly much of the recent growth in the use of options dates from this time.

This paper contributes to the optimal contracting approach by showing that even in the absence of accounting and tax considerations, loss aversion implies that stock options are optimal incentive devices. Risk aversion assigns no role to gains and losses but involves a utility function that is strictly concave in the income level. In contrast, loss aversion implies an asymmetric valuation of gains and losses. This psychological feature was first brought to the attention of economists as one of the elements of Prospect Theory (Kahneman and Tversky, 1979). Empirical tests suggest that people commonly value losses from two to four or more times their valuation of equal gains (for example, Kahneman, Knetsch, and Thaler, 1990). Evidence of such reference effects has come from both experimental and non experimental settings (e.g. Samuelson and Zeckhauser, 1988; Kahneman, Knetsch, and Thaler, 1991). A typical case is Kachelmeier and Shehata’s (1992) finding that subjects in one experiment were willing to pay, on average, $5.60 for a 50 percent chance to win $20, but the same individuals demanded an average of $10.87 to give up the identical chance to win the same $20 prize. The evidence is that the phenomenon is not primarily the result of income or wealth effects, or even of transaction costs (for example, Kahneman, Knetsch and Thaler, 1990; Knetsch, Tang, and Thaler, 2001).

The main finding of our paper is that replacing a Von Neumann- Morgenstern utility function by one with loss aversion implies that the standard principal-agent analysis yields an incentive-pay schedule that can be interpreted as a basic wage plus the grant of a stock option. At first sight it is obvious that if people have a strong dislike to feeling themselves to be down, remuneration schemes should minimise such occurrences. That misses the point that the purpose of performance pay is to change behaviour. So a heavy penalty for failure does seem a potentially

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4 This requires there to be information problems such as signal jamming. See, for example Stein (1989).
5 Murphy (2002) proposes that the prevalence of stock options is best explained by a “perceived cost bias” whereby managers genuinely underestimate the true cost of their award.
6 Hall and Murphy (2003) argue that any theory of stock option compensation must not only explain executive stock options but also options granted to the rank and file. The theory in this paper applies to all types of employee but cannot be considered to add much light on the general pattern of such compensation.
effective scheme. In fact, if realizations involve a stochastic element it turns out to be optimal to use incentives that are regarded as a carrot for good performance rather than a stick for bad.\(^7\)

There is a well known issue in applying Prospect Theory. As utility depends on changes in wealth as well as on its level the question arises as to the period over which gains and losses accrue and the reference point from which they are measured. In our context it seems natural that performance is measured at the end of each year. Choice of reference point is more delicate. Critics regard the absence of a straightforward rule tying this down as a major weakness of the theory. Those sympathetic to the approach are prepared to accept that the particular context often suggests an obvious reference point, the relevance of which can be tested for empirically. In many cases there is a natural candidate for the reference point. Suppose, for example, that a CEO was on a pay schedule yielding $80k with probability 0.2, $100k with probability 0.7 and $120k with probability 0.1. The obvious reference point for coding gains and losses is then $100k. Our generalisation is that the reference point is the modal payoff. Other formulations are possible.\(^8\) The recent history of the individual’s pay will doubtless be important in locating a reference point. The main argument of the paper is though that if there is a references point, stock options plus a basic salary is likely to be the optimal incentive scheme.

The Prospect theory model that we develop here is consistent with a number of number of well documented empirical observations concerning stock options. Because the reference income is unlikely to be below base income, our analysis explains why options are granted at or in the money. This also explains why out of the money options are reloaded. Early exercise of options has been well documented by a number of authors including for example Huddart and Lang (1996) who argue that this is done to lock in option gains relative to the reference income. In this sense our model is a theoretical counterpart to the empirical work based on Prospect Theory of Heath, Huddart and Lang (1999).

\(^7\)There is another potential benefit of stock options. If the utility index is strictly concave over gains and strictly convex over losses, it is convex in the region of losses. Then when people feel they are down they are inclined to gamble to restore their status quo income. In the case of a CEO, reckless gambling may not be in the interest of shareholders. Nick Leeson’s bankrupting of Barings may involve something along these lines. One implication is that there is an incentive to design compensation schemes that reduce the propensity of CEOs to feel down. Stock options have this property, though in convexifying the payoff function may also raise risk taking.\(^8\) Gul (1991) examines disappointment aversion so as to reconcile choice over lotteries with the Allais paradox. He modifies the independence axiom in a simple way that accommodates Allais’ findings. The utility function that emerges exhibits both elation and disappointment. This utility function has one more parameter than the expected utility function and is unique up to an affine transformation, it exhibits both expected utility and Kahneman and Tverskey type preferences as special cases.
2 The Model

We assume that stock price is the only verifiable performance indicator for the CEO. Given the optimal (unobservable) action of the CEO, the end of year stock price is symmetrically distributed. Writing reference income as $Y^*$ and realised income as $Y$, and effort as $e \in \{0, \tau\}$, the utility function of the CEO is:

$$V(Y, Y^*, e) = \begin{cases} 
U(Y) - e & \text{if } Y > Y^* \\
U(Y) - l(Y^* - Y) - e & \text{if } Y < Y^*
\end{cases}$$

We assume that the function $U(Y)$ exhibits risk aversion, so that $U' > 0$ and $U'' < 0$. Also, to ensure that consumption is positive in all states we impose the Inada condition, $\lim_{Y \to 0} [U'(Y)] \to -\infty$. In conformity with Prospect Theory, loss aversion is measured by $l > 0$. Hence, there is a kink in the utility function at the reference income $Y^*$. The linear loss adjustment makes for a tractable analysis but similar results are available if the function is $U(Y) - L(Y^* - Y) - e$ with $L(0) = 0, L'(0) > 0, L''(Y^* - Y) \geq 0$ when $Y < Y^*$.

We assume that managerial effort is unobservable. However, the company’s stock price is stochastically related to the effort level and is the only indicator of the manager’s long-term performance directly observed by the principal. Effort level $\tau$ induces the probability distribution $P$ and that the low effort level of $0$ induces the probability distribution $P^*$. If the principal wanted the manager to apply low effort, this is efficiently achieved with a flat compensation schedule. Matters are more complex if the principal wants to induce the high level of effort. The firm’s owners are risk neutral and seek to minimise the expected payment to the CEO subject to individual rationality (IR) (the CEO has reservation utility $V^*$) and incentive compatibility constraints (IC).

A Three State Example Suppose there are three possible stock-price realisations. The principal chooses how to reward the agent as a function of firm performance. It is though impossible to implement a reward scheme that is anywhere decreasing in firm performance because the agent can easily sabotage output post realisation. Using obvious notation, there are three possible realisations of the stock price $H, M$ and $L$ occurring with probabilities $P_H, P_M, P_L$ when effort is applied, and $P^*_H, P^*_M, P^*_L$ when no effort is applied. The contracted payment to the entrepreneur in these states are $Y_H, Y_M$ and $Y_L$ respectively. Assume that even when effort is applied, either $M$ or $L$ occurs with more than 50% probability, so median income, $Y_M$, is the status quo or reference income. Then
the principal’s problem is:

**Problem 1.**

\[
\min \{ P_H Y_H + P_M Y_M + P_L Y_L \} \tag{1}
\]

\[
st. \ P_H U(Y_H) + P_M U(Y_M) + P_L [U(Y_L) - l(Y_M - Y_L)] - \bar{\sigma} \geq V^* \tag{2}
\]

and \( \Delta P_H U(Y_H) + \Delta P_M U(Y_M) + \Delta P_L [U(Y_L) - l(Y_M - Y_L)] \geq \sigma \tag{3} \]

\[Y_M - Y_L \geq 0 \tag{4}\]

In the incentive compatibility condition \( \Delta P = P - P^* \) is the effect of changes in effort on the probability of the income level in a given state. Note that first order stochastic dominance implies that \( \Delta P_L < 0 \) and the monotone likelihood ratio property (MLRP) implies that \( \frac{\Delta P_L}{\Delta P_M} < \frac{\Delta P_M}{\Delta P_H} \). This means that high levels of the stock price, on which rewards are based, are more likely to be the result of high levels of effort. The final constraint rules out decreasing reward schedules.

This problem will choose the reward in the median state, \( Y_M \). Note that if the alternative to working for the principal is to obtain a certain outside income evaluated with the managers utility function, then because of constraint (2) the solution will set \( Y_M \) equal to the certainty equivalent. This will show up more clearly in our numerical examples. Then our formulation has the kinked indifference curve property at the certainty equivalent level of income in Gul’s (1991) formulation of Kahneman and Tversky type preferences as a special case of disappointment aversion.

In solving Problem 1 let \( \gamma \geq 0, \mu \geq 0 \) and \( \phi \geq 0 \) denote the multipliers on constraints (IR), (IC) and the non-negativity condition respectively. Define the Lagrangian:

\[
\Lambda = \{ [P_H Y_H + P_M Y_M + P_L Y_L] + \gamma (V^* - P_H U(Y_H) - P_M U(Y_M) -
\]

\[
P_L (U(Y_L) - l(Y_M - Y_L)) + \mu \sigma - \Delta P_H U(Y_H) - \Delta P_M U(Y_M) -
\]

\[
\Delta P_L (U(Y_L) - l(Y_M - Y_L)) + \phi [Y_L - Y_M] \}
\]
Then the payments $Y_H$, $Y_M$ and $Y_L$ must satisfy the following Kuhn-Tucker conditions:

\[
\frac{\partial \Lambda}{\partial Y_H} = P_H - \gamma P_H U''(Y_H) - \mu \Delta P_H U''(Y_H) \geq 0
\]

\[
\frac{\partial \Lambda}{\partial Y_M} = P_M - \gamma [P_M U''(Y_M) - P_L l] - \mu [\Delta P_M U''(Y_M) - \Delta P_L l] - \phi \geq 0
\]

\[
\frac{\partial \Lambda}{\partial Y_L} = P_L - \gamma [P_L U''(Y_L) + P_L l] - \mu [\Delta P_L U''(Y_L) + \Delta P_L l] + \phi \geq 0
\]

\[
\frac{\partial \Lambda}{\partial \gamma} \leq 0, \frac{\partial \Lambda}{\partial \mu} \leq 0, \frac{\partial \Lambda}{\partial \phi} \leq 0
\]

\[
\frac{\partial \Lambda}{\partial Y_H} Y_H = 0, \frac{\partial \Lambda}{\partial Y_M} Y_M = 0, \frac{\partial \Lambda}{\partial Y_L} Y_L = 0
\]

\[
\frac{\partial \Lambda}{\partial \gamma} \gamma = 0, \frac{\partial \Lambda}{\partial \mu} \mu = 0, \frac{\partial \Lambda}{\partial \phi} \phi = 0
\]

\[
Y_H \geq 0, Y_M \geq 0, Y_L \geq 0; \gamma \geq 0, \mu \geq 0 \text{ and } \phi \geq 0.
\]

That these conditions are necessary and sufficient for a minimum is shown in the Appendix. Our assumptions on the utility function mean that at an optimum $Y_H$, $Y_M$ and $Y_L$ are strictly positive so that (7), (8) and (9) hold as strict equalities.

**Proposition 1a.** Assuming risk aversion, there exists a value of loss aversion denoted by $l^*$ such that $l \geq l^*$ is necessary and sufficient for $Y_M = Y_L$.

**Proof.** Consider the case $Y_M = Y_L$. By adding (8) and (9) and dividing by $P_L + P_M$,

\[
\frac{1}{U''(Y_M)} = \gamma + \mu \frac{\Delta (P_M + P_L)}{P_M + P_L} = \gamma - \mu \frac{\Delta P_H}{1 - Y_H}
\]

Also (7) as an equality can be written as
\[ \frac{1}{U'(Y_H)} = \gamma + \mu \frac{\Delta P_H}{P_H} \] (14)

If the multiplier \( \gamma = 0 \), then since \( \Delta P_H > 0 \), (14) implies that \( U'(Y_M) < 0 \). But this is impossible, so that the solution must have \( \gamma > 0 \). Next consider the possibility that the multiplier \( \mu = 0 \). In this case (14) and (15) imply that the managers reward schedule is flat so that the manager would always supply the lowest level of effort. Hence, \( \mu > 0 \). Next, note that given the optimal values of \( Y_M \) and \( Y_H \), conditions (14) and (15) determine the values of \( \gamma \) and \( \mu \) independently of \( l \).

Then rewrite (9)

\[ 1 + \frac{\phi}{P_L} = (\gamma + \mu \frac{\Delta P_L}{P_L})(U'(Y_L) + l) \] (15)

If \( \phi = 0 \), \( l = l^* = \frac{1}{\gamma + \mu \frac{\Delta P_L}{P_L}} - U'(Y_L) \) is the value of \( l \) consistent with the choice of \( Y_L = Y_M \) in the problem without the constraint \( Y_L \geq Y_M \). It then follows that if \( \phi > 0 \), \( l > l^* \). Hence, \( Y_L = Y_M > 0 \implies l \geq l^* \). (Note that if we imposed MLRP at this stage then from (14) \( U'(Y_L) > \gamma + \mu \frac{\Delta P_L}{P_L} \) so that \( l^* > 0 \)).

Now we need to show that \( l \geq l^* \) implies \( Y_L = Y_M \). If \( l = l^* \), then \( Y_L = Y_M \), because this is the optimal choice at this value of \( l \) in the unconstrained problem. Finally, rewrite condition (8) as

\[ 1 - \frac{\phi}{P_M} = (\gamma + \mu \frac{\Delta P_M}{P_M})U'(Y_M) - (\gamma + \mu \frac{\Delta P_L}{P_L}) \frac{P_L}{P_M} l \] (16)

then from (16) and (17), if it were the case that \( l > l^* \) and \( \phi = 0 \), then \( U'(Y_M) > U'(Y_L) \) and so risk aversion implies that \( Y_L > Y_M \). But this is not possible, so that \( \phi > 0 \) and \( Y_L = Y_M \). Hence, \( Y_L = Y_M > 0 \iff l \geq l^* \).

QED.

Since \( l \geq l^* \iff Y_L = Y_M > 0 \), the next proposition follows by completeness.

**Proposition 1b.** With risk aversion and MLRP, \( l < l^* \) is necessary and sufficient for \( Y_M > Y_L \).

**Proof.** Since from Proposition 1a \( Y_M = Y_L \) if and only if \( l \geq l^* \), then \( l < l^* \) if and only if \( Y_M \neq Y_L \). Then (8) and (9) and MLRP mean that \( Y_M > Y_L \).

Finally, from (7) and (8), risk aversion and MLRP imply that \( Y_H > Y_M \).
3 Multiple states

When there are multiple states, intermediate levels of loss aversion may result in the optimal scheme having flats in the region of the reference income but penalties for lower levels of performance. Only when loss aversion is sufficient does the full stock option solution emerge. To see the possibilities it is enough to extend the previous case to four realisations of firm performance, $H, M, L$ and $S$ but still restrict attention to only two effort levels. The principal’s problem is:

**Problem 2.**

$$\min\{P_H U(Y_H) + P_M U(Y_M) + P_L U(Y_L) + P_S U(Y_S)\} \quad (17)$$

$$st. P_H U(Y_H) + P_M U(Y_M) + P_L[U(Y_L) - l(Y_M - Y_L)]$$
$$+ P_S[U(Y_S) - l(Y_M - Y_S)] - \tau \geq V^* \quad (18)$$

and $\Delta P_H U(Y_H) + \Delta P_M U(Y_M) + \Delta P_L[U(Y_L) - l(Y_M - Y_L)]$
$$+ \Delta P_S[U(Y_S) - l(Y_M - Y_S)] \geq \tau \quad (19)$$

$$Y_M - Y_L \geq 0 \quad (20)$$

$$Y_M - Y_S \geq 0 \quad (21)$$

Let $\gamma, \mu, \phi_L$ and $\phi_S$ be the multipliers associated with the four constraints on the problem. Define the Lagrangian:
\[ \hat{\Lambda} = \{ [P_H Y_H + P_M Y_M + P_L Y_L + P_S Y_S] + \]

\[ \gamma [V^* - P_H U(Y_H) - P_M U(Y_M) - P_L U(Y_L) - l(Y_M - Y_L)] - P_S (U(Y_S) - l(Y_M - Y_S)) + \bar{\sigma} \]

\[ + \mu [\Delta P_H U(H) - \Delta P_M U(M) - \Delta P_L U(L) - l(Y_M - Y_L)] - \Delta P_S (U(Y_S) - l(Y_M - Y_S)) \]

\[ + \phi_L [Y_L - Y_M] + \phi_S [Y_S - Y_M] \}

The first-order conditions are:

\[ \frac{\partial \hat{\Lambda}}{\partial Y_H} = P_H - \gamma P_H U'(Y_H) - \mu \Delta P_H U'(Y_H) \geq 0 \] (23)

\[ \frac{\partial \hat{\Lambda}}{\partial Y_M} = P_M - \gamma [P_M U'(Y_M) - P_L l - P_S l] - \mu [\Delta P_M U'(Y_M) + \Delta P_L l + \Delta P_S l] - \phi_L - \phi_S \geq 0 \] (24)

\[ \frac{\partial \hat{\Lambda}}{\partial Y_L} = P_L - \gamma [P_L U'(Y_L) + P_L l] - \mu [\Delta P_L U'(Y_L) + \Delta P_L l + \phi_L] \geq 0 \] (25)

\[ \frac{\partial \hat{\Lambda}}{\partial Y_S} = P_S - \gamma [P_S U'(Y_S) + P_S l] - \mu [\Delta P_S U'(Y_S) + \Delta P_S l + \phi_S] \geq 0 \] (26)

\[ \frac{\partial \hat{\Lambda}}{\partial \gamma} \geq 0, \frac{\partial \hat{\Lambda}}{\partial \mu} \geq 0, \frac{\partial \hat{\Lambda}}{\partial \phi_L} \geq 0, \frac{\partial \hat{\Lambda}}{\partial \phi_S} \geq 0 \] (27)

\[ \frac{\partial \hat{\Lambda}}{\partial Y_H} Y_H = 0, \frac{\partial \hat{\Lambda}}{\partial Y_M} Y_M = 0, \frac{\partial \hat{\Lambda}}{\partial Y_L} Y_L = 0, \frac{\partial \hat{\Lambda}}{\partial Y_S} Y_S = 0 \] (28)

\[ \frac{\partial \hat{\Lambda}}{\partial \gamma} \gamma = 0, \frac{\partial \hat{\Lambda}}{\partial \mu} \mu = 0, \frac{\partial \hat{\Lambda}}{\partial \phi_L} \phi_L = 0, \frac{\partial \hat{\Lambda}}{\partial \phi_S} \phi_S = 0 \] (29)
\( Y_H \geq 0, Y_M \geq 0, Y_L \geq 0 \text{ and } Y_S \geq 0; \gamma \geq 0, \mu \geq 0, \phi_L \geq 0 \text{ and } \phi_S \geq 0. \) \quad (30)

Then we have the following generalisation of Proposition 1.

**Proposition 2.** Granted risk aversion and MLRP, there exist \( l_S^* \geq l_L^* \geq 0 \) such that \( l \geq l_s^* \iff Y_S = Y_M = Y_L \) and \( l_s^* \geq l \geq l^*_L \iff Y_L = Y_M \) but \( Y_S < Y_L \), and \( l < l^*_L \iff \) the standard solution \( Y_H > Y_M > Y_L > Y_S \).

**Proof.** Consider the case \( Y_M = Y_L = Y_S \). By adding (25), (26) and (27) and dividing by \( P_M + P_L + P_S \)

\[
\frac{1}{U(Y_M)} = \gamma + \mu \frac{\Delta(P_M + P_L + P_S)}{P_M + P_L + P_S} = \gamma - \mu \frac{\Delta P_H}{1 - P_H} \quad (31)
\]

Also (24) can be written as

\[
\frac{1}{U(Y_H)} = \gamma + \mu \frac{\Delta P_H}{P_H} \quad (32)
\]

Conditions (32) and (33) then determine \( \gamma \) and \( \mu \), which are seen to be independent of \( l \). Using the same argument as in Proposition 1a, these multipliers can be shown to be positive. Condition (27) yields

\[
1 + \frac{\phi_S}{P_S} = (\gamma + \mu \frac{\Delta P_S}{P_S})(U(Y_S) + l) \quad (33)
\]

Then form this condition \( \phi_S = 0 \iff l = l_S^* = \frac{1}{\gamma + \mu \frac{\Delta P_S}{P_S}} - U(Y_S) \) and \( \phi_S > 0 \iff l > l_S^* \). Similarly \( \phi_L = 0 \iff l = l_L^* = \frac{1}{\gamma + \mu \frac{\Delta P_L}{P_L}} - U(Y_L) \) and \( \phi_L > 0 \iff l > l_L^* \). That \( l \geq l_S^* \) implies \( Y_S = Y_M \) and \( l \geq l_L^* \) implies \( Y_L = Y_M \) follows along the same lines as in the proof of Proposition 1a. Then MLRP implies that \( l \geq l_S^* > l_L^* = \frac{1}{\gamma + \mu \frac{\Delta P_L}{P_L}} - U(Y_L) \) so that \( Y_S = Y_M \) and \( Y_L = Y_M \).

Now consider the complement. By completeness, risk aversion and MLRP mean that \( l < l_S^* \iff Y_M > Y_S \) and \( l < l_L^* \iff Y_M > Y_L \).

Finally MLRP implies that if it is optimal to set \( Y_L = Y_M \), it is not necessary that \( Y_S = Y_M \) is also optimal. QED.

In the absence of loss aversion, the optimal compensation scheme pays more for outcomes that are statistically
relatively more likely to occur under $e = \tau$ than under $e = 0$. Similarly it offers less compensation for outcomes that are relatively more likely when $e = 0$ is chosen. In general this compensation package need not be monotonic or linear. Moreover, the compensation scheme must in general offer the manager an expected level of income in excess of that under full information to compensate for the risk borne.

A scheme satisfying IC and for which IR binds is flat for some realisations below the reference level and may be flat for all values. This can be implemented by a stock option. Firm performance is then a one sided bet for the manager. When loss aversion is high the reward schedule is flat everywhere but for lower degrees of loss aversion, it is only for realisations immediately below the reference level that the optimal reward scheme is flat.

4 Numerical Examples

Proposition 1 assume that implementability of the compensation scheme requires that payoffs are nowhere decreasing in performance and that there are three states. Then, for $l$ sufficiently high, it is optimal that pay is equal in the two lowest states. The intuition for this result revolves around the relative cost of creating incentives by motivating agents to avoid losses (the costs of regret, plan disruption and so forth) or just to escape low pay itself. Both factors create a reason for the agent to apply effort by lowering the utility associated with poor performance. The penalties must though be compensated by higher pay elsewhere in the schedule. There is though a difference. Creating regret has no direct effect on the principal’s outlay, whereas cutting pay does. So it is the fact that regret is a real cost not just a transfer which is the basis for wanting to design an incentive scheme that minimises its occurrence.

Here are some illustrations of this principle.

Three-state Examples The manager’s utility function is given by $U = Y^{0.5}$, with relative risk aversion of 0.5. There are three states, $H$, $M$ and $L$ which occur with probabilities 0.25, 0.5, 0.25 if effort is high. Without effort probabilities are 0.15, 0.5, 0.35, so MLRP is satisfied by this formulation. The cost of effort the effort needed to induce the superior distribution is 0.1. The alternative for the manager is to receive a riskless income at an alternative effort level normalised at zero. The objective is to minimise the cost of compensation subject to fixed a reservation level of utility from income plus the utility cost of effort of $U^* + \tau = 3.1$, and subject to the incentive
constraint that effort is applied.

Without loss aversion the problem is:

\[
\min \{0.25Y_H + 0.5Y_M + 0.25Y_L\} \\
\text{st. } 0.25Y_H^{0.5} + 0.5Y_M^{0.5} + 0.25Y_L^{0.5} = 3.1, \\
0.1Y_H^{0.5} - 0.1Y_L^{0.5} = 0.1
\] (34)

Minimum cost is 9.735, achieved by setting \(Y_H = 12.96, Y_M = 9.61, Y_L = 6.76.\)\(^9\)

Now introduce loss aversion by setting \(l = 0.1.\) Evaluated at \(Y_M = 9.61\) this puts the marginal utility of income gains at three quarters of income losses.\(^{10}\) Ignoring the constraint that \(Y_M \geq Y_L,\) the problem is now

\[
\min \{0.25Y_H + 0.5Y_M + 0.25Y_L\} \\
\text{st. } 0.25Y_H^{0.5} + 0.5Y_M^{0.5} + 0.25Y_L^{0.5} - 0.025(Y_M - Y_L) = 3.1, \\
0.1Y_H^{0.5} - 0.1Y_L^{0.5} + 0.1(0.1(Y_M - Y_L)) = 0.1
\] (35)

and the optimal scheme is \(Y_L = 8.65, Y_M = 6.44, Y_H = 17.32.\)\(^{11}\) As this solution involves \(Y_L > Y_M\) it violates the monotonicity requirement. The next step is therefore to impose the constraint \(Y_L = Y_M.\) The optimal program is now \(Y_L = Y_M = 8.12, Y_H = 14.82,\) a stock option solution. Though the pattern of rewards is considerably different to that in the absence of loss aversion, minimised cost is very little higher at 9.8. So a stock option eliminates the utility cost of loss aversion at very little financial cost.

Consider now the comparative statics of the optimisation problem as the utility cost of effort, \(e,\) is varied. The procedure here is to suppose that the alternative occupation involves no effort but a fixed monetary payment (since the coefficient of relative risk aversion remains 0.5 income is 9 ) Increasing the cost of effort thus involves raising the RHS of the incentive constraint but not of the participation constraint (similar results obtain if the higher cost of effort also affects the alternative occupation) The table illustrates five cases. It shows that for effort

\(^9\)Note that \(Y_M = 9.61\) is the certainty equivalent income, that is \((9.61)^{0.5} = 3.1.\)

\(^{10}\)Note that \(9.61 = 3.1^2\)

\(^{11}\)Cost is minimised at 9.71, less than previously due to the exploitation of "gain attraction", but this ignores the one-way loss aversion function.
levels \( e \in \{0.1, 0.15, 0.18, 0.19\} \) and \( l = 0.1 \), a stock option obtains, but at the higher effort levels \( e \in \{0.2, 0.21\} \), the compensation scheme is strictly increasing in performance. The pattern is that with a greater cost of effort the power of incentives must be raised. When effort is sufficiently costly it becomes worth incurring the deadweight cost of loss aversion to use the standard penalty of low income for poor performance.

<table>
<thead>
<tr>
<th>cost of effort</th>
<th>total cost</th>
<th>( Y_L )</th>
<th>( Y_M )</th>
<th>( Y_H )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( e = 0.1 )</td>
<td>9.78</td>
<td>8.12</td>
<td>8.12</td>
<td>14.82</td>
</tr>
<tr>
<td>( e = 0.15 )</td>
<td>10.34</td>
<td>7.7</td>
<td>7.7</td>
<td>18.28</td>
</tr>
<tr>
<td>( e = 0.18 )</td>
<td>10.72</td>
<td>7.45</td>
<td>7.45</td>
<td>20.52</td>
</tr>
<tr>
<td>( e = 0.19 )</td>
<td>10.85</td>
<td>7.37</td>
<td>7.37</td>
<td>21.3</td>
</tr>
<tr>
<td>( e = 0.2 )</td>
<td>10.99</td>
<td>7.24</td>
<td>7.46</td>
<td>21.78</td>
</tr>
<tr>
<td>( e = 0.21 )</td>
<td>11.13</td>
<td>7.1</td>
<td>7.57</td>
<td>22.27</td>
</tr>
</tbody>
</table>

It is also natural to consider the role of risk aversion in the choice of optimal incentives although it is not completely straightforward how to undertake the comparative statics. Changing the curvature of the utility of income function also changes its height almost everywhere. This matters in this context since effort is a utility not a monetary cost. Hence, changing risk aversion is an amalgam of varying preferences with respect to income fluctuation and altering the cost of effort relative to the benefit of consumption. The particular methodology here is to use CRRA utility function \( U = Y^{1-\sigma}, 1 \geq \sigma > 0 \) and to suppose that the income available in the other occupation remains at 9.\(^{12}\) This tends to neutralise the income effect of changing \( \sigma \). Other procedures can be devised but the pattern of results is similar.

Table 2 shows results when the coefficient of relative risk aversion is lowered from 0.5 to 0.4. When the cost of effort is high (0.2) the effect of greater risk toleration is that stock options become optimal. Creating incentives in the regular way by raising the size of the reward for the low probability of outstanding performance is then not very costly and the deadweight cost of incurring "losses" is best avoided. When the cost of effort is more modest (0.1) stock options are optimal for both levels of risk aversion. Greater risk toleration narrows the spread

\(^{12}\)The general form of the iso-elastic utility function is utility function in the more familiar form \( U = AY^{1-\sigma}/(1-\sigma), A > 0, \sigma \leq 0 \), where the denominator ensures strict concavity for all values of \( \sigma \). However, as \( \sigma > 0 \), in the neighbourhood of our comparative statics, the function \( U = Y^{1-\sigma} \) is strictly concave.
between the rewards in the two low states and that in the high state (this is also true of the optimal schemes when the cost of effort is high). At first sight this is surprising, in that it might be expected that the risk tolerant take on more risk. In fact the only reason for taking on any risk is to provide an effort incentive. Under this formulation, greater risk tolerance means the less the marginal utility of income declines as income increases. Thus the marginal utility of income is relatively high for the risk tolerant so less of a bonus is needed to cover the cost of effort. The message to take from the simulations is that risk tolerance makes it more attractive to avoid loss aversion by means of options.

\[
\sigma \quad e \quad \text{total cost} \quad Y_L \quad Y_M \quad Y_H
\]
\[
\begin{align*}
\sigma = 0.5 & \quad 0.1 \quad 9.8 \quad 8.12 \quad 8.12 \quad 14.82 \\
\sigma = 0.4 & \quad 0.1 \quad 9.47 \quad 8.4 \quad 8.4 \quad 12.66 \\
\sigma = 0.5 & \quad 0.2 \quad 10.99 \quad 7.24 \quad 7.45 \quad 20.81 \\
\sigma = 0.4 & \quad 0.2 \quad 10.07 \quad 7.83 \quad 7.83 \quad 16.82
\end{align*}
\]

**Four-State Examples**

**Example 1** Now suppose that there are four states. When effort is applied the probabilities of the states are 0.25, 0.5, 0.15 and 0.1 respectively. When no effort is applied these are 0.15, 0.5, 0.19, and 0.16. First consider an example in which a full option arises when the loss-aversion parameter is set at \( l = 0.1 \). The utility function takes the same form as above with the outside option set at 9 and utility of effort at 0.1, so that \( U^* + \tau = 3.1 \) a little bit lower than in the three state example above.

Without loss aversion the problem is

\[
\begin{align*}
\min \{0.25Y_H + 0.5Y_M + 0.15Y_L + 0.1Y_S\} \\
\text{st.} \quad 0.25Y_H^{0.5} + 0.5Y_M^{0.5} + 0.15Y_L^{0.5} + 0.1Y_S^{0.5} = 3.1, \\
\quad \text{and} \quad 0.1Y_H^{0.5} - 0.04Y_L^{0.5} - 0.06Y_S^{0.5} = 0.1.
\end{align*}
\]
Cost is minimised at $9.73$ by the scheme $Y_S = 5.8, Y_L = 7.8, Y_M = 9.61, Y_H = 12.68$.

With loss aversion but no monotonicity constraint:

$$\min \{0.25Y_H + 0.5Y_M + 0.15Y_L + 0.1Y_S\} \quad (37)$$

$$\text{st. } 0.25Y_H^{0.5} + 0.5Y_M^{0.5} + 0.15Y_L^{0.5} + 0.1Y_S^{0.5}$$

$$-0.015(Y_M - Y_L) - 0.01(Y_M - Y_S) = 3.1,$$

and $0.1Y_H^{0.5} - 0.04Y_L^{0.5} - 0.06Y_S^{0.5} + 0.004(Y_M - Y_L) + 0.006(Y_M - Y_S) = 0.1$

yielding $Y_S = 3.42, Y_L = 15.67, Y_M = 5.96, Y_H = 15.42$, which has the property that $Y_L > Y_M$. Imposing the constraint $Y_L = Y_M$ we obtain the solution $Y_H = 15.37, Y_M = 7.85, Y_L = 7.85, Y_S = 8.55$. As $Y_S > Y_L = Y_M$ we now impose the constraint $Y_S = Y_L = Y_M$ and find that optimality requires that $Y_S = Y_L = Y_M = 8.12, Y_H = 14.82$. Thus taking one way constraints into account, we obtain a full option. Cost is now minimised at 9.8, so once again using a stock option to eliminate loss aversion is almost costless for the principal.

**Example 2.** Now consider an example with $Y_M = Y_L > Y_S$ (i.e. a partial flat) achieved by setting the loss aversion parameter at $l = 0.06$.

It is optimal to have $Y_M = Y_L > Y_S$ because without loss aversion $Y_L$ is closer to $Y_M$ than is $Y_S$. Without this constraint:

$$\min \{0.25Y_H + 0.5Y_M + 0.15Y_L + 0.1Y_S\} \quad (38)$$

$$\text{st. } 0.25Y_H^{0.5} + 0.5Y_M^{0.5} + 0.15Y_L^{0.5} + 0.1Y_S^{0.5}$$

$$-0.009(Y_M - Y_L) - 0.006(Y_M - Y_S) = 3,$$

and $0.1Y_H^{0.5} - 0.04Y_L^{0.5} - 0.06Y_S^{0.5} + 0.0024(Y_M - Y_L) + 0.0036(Y_M - Y_S) = 0.1$

yielding $Y_H = 14.87, Y_M = 7.16, Y_L = 12.54, Y_S = 5.27$ with minimised cost of 9.71. The monotonicity constraint
is clearly violated. With the monotonicity constraint $Y_M = Y_L$ added to the above problem, the solution is $Y_S = 7.67, Y_L = Y_M = 8.36, Y_H = 14.39$ at a cost to the principal of 9.8. Cost is hardly increased relative to the no loss risk aversion case. So, the optimal reward scheme has a partial flat. The penalty for very poor performance could be dismissal. It would though make almost no difference to the principal’s payoff to introduce a double flat, a full option.

The information in examples 2 and 3 can be combined to show the effect of loss aversion on the reward schedule for the manager. The picture (figure 1) illustrates a monotonic reward schedule for $l = 0$, another with a partial flat between states $M$ and $L$ when $l = 0.06$, and a full flat for states $M$, $L$ and $S$ when $l = 0.1$. Observe also, that as loss aversion increases and sticks are replaced with carrots, the reward for good performance, the difference between states $M$ and $H$ increases. The picture also illustrates clearly how loss aversion first leads to a flattening of the reward schedule to the immediate left of the reference level of income an how as the degree of loss aversion increases the reward schedule is flattened everywhere.

Figure 1 here

5 Discussion

Our formulation assumes that individuals have a reference income from which they weight losses more heavily than gains. What we show is that with enough loss aversion the optimal schedule must then be flat below some threshold, an option like feature. Moreover, little loss aversion is needed for this result. Flats in the remuneration schedule do not obtain in the standard expected utility model with risk aversion. Were income the same in two adjacent states, a small transfer from the lower performance level to the higher has vanishingly small risk cost but a first-order incentive benefit. With loss aversion, the first observation is no longer true, so flats can arise.

To establish the use of options as optimal it is further necessary to show that the option contract is not dominated by discounted equity or discretionary bonuses. First, consider bonus pay. In principle they can replicate the option based contract or even in improve on it by eliminating noise. However, to the extent the payment option remains in the hands of the firm so that reneging may be an issue. This problem may be mitigated
through a bridging contract or diminished in a repeated offer setting where firms have reputations to protect.

Now consider discounted restricted equity. The restriction is that there is a minimum vesting period. Suppose that managers can take equity at a discount to the current market price, the reference point. Moreover, let the discount equal the difference in the valuation between the manager’s and firm’s values of the ‘at the money option’, so the cost of the two schemes are the same to the firm. Because the stock position exposes the manager to losses that the option does not, the manager will prefer the option. To get the manager to take the stock, the discount must be increased. The firm can only benefit if there is a more than compensating increase in effort incentives. However, with Prospect Theory preferences the utility cost of increased loss exposure will exceed the gain in effort incentives. This is the main result of our paper.

Contrasting our analysis with that of Hall and Murphy (2001) is worthwhile. They assume that the manager cannot short-sell stock in their own company and hence cannot hedge exposure, although the importance of this constraint is disputed by Carpenter (1998) and explored by Core and Guay (2001). They then show that with standard von Neumann-Morgenstern preferences the manager’s valuation of the stock option is less than the firm’s valuation. In particular, this wedge will vary with the extent to which the option is in the money. So, from the firm’s point of view, it is cheaper to issue ‘at the money’ rather than ‘in the money’ options. This though does not establish that options are optimal. In fact it is shown that if the participation constraint binds, as in the standard principal-agent problem, restricted stock Pareto dominates stock options as an incentive device. Only by ignoring the participation constraint does there emerge a possible preference for stock options as cheaper to the principal than stock. Non binding participation constraints may emerge if entrenched CEOs are in a position to capture rents. This view is argued in very robust terms by Bebchuck, Fried and Walker (2001) who claim that rent extraction takes place subject to a shareholder “outrage constraint”. This constraint depends upon the strength of corporate governance, including the existence of genuinely independent remuneration committees and active large shareholders. Some evidence for this rent extraction view is provided by Yermack (1995) and Bertrand and Mullainathan(2000)). Yet the use of stock options requires more than a non-binding participation constraint. Since there is still room for efficient bargaining over the form of CEO pay incentives would still be

\[ \text{---For a given expected outlay by the firm, a risk averse manager would prefer shares to a stock option plus the low marginal utility of income in the success states diminishes the incentive power of stock options.} \]
monotonically increasing in performance. Hall and Murphy do not explain why the Coase theorem fails in this setting where small numbers are involved so efficient bargaining is to be expected. Bebchuck, Fried and Walker might counter with the claim that stock options are less outrageous than other “smash and grab” instruments including restricted stock. Our analysis shows that under Prospect Theory stock options may be optimal so will appear whether or not the participation constraint binds.

A common criticism of Prospect Theory is that it provides no comprehensive theory of the determination of the reference level of income. This is true but not enough to invalidate the theory. Finding that behaviour can be rationalised by a reference income may explain puzzles even if the actual reference income was not predicted. An analogy is that the assumption of stable preferences is a potentially useful, falsifiable theory even if there is no theory of the determinants of preferences. In the case of the reference income in the stock option context, Heath, Huddart and Lang (1999) provide some evidence. A number of researchers have experimented with simple models based on recent observations of the relevant variable (see, for example Barberis, Huang and Santos (2000)). In our theory we assume that managers regard themselves as in the loss space if their income falls below its median level, which as we noted, under some assumptions will be the certainty equivalent of the prospect. Moreover, managers agree with the stock market’s assessment of the firm’s prospects and the equilibrium stock price is the median value. As a result, options are issued at the money.

If a manager has had a history of earning money from the exercise of stock options in which case the reference income will be higher and the option should be issued in the money. At all events, whatever determines the reference income, our results imply that if there is one it is often optimal to use stock options to prevent managers falling into the loss space.

Finally, the analysis presented in this paper has applications beyond CEO pay. It potentially explains why it is more common to observe bonuses for success than penalties for failure. One application is to the structure of financial claims issued by firms. Our theory implies that decision makers or managers should have option-like claims that protect them from downside risk. This protection, or limited liability, on the downside is provided by financiers. Financiers hold the residual claims on the firm and insure the managers in poor states. In effect

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\textsuperscript{14}Reload options are compound American options. They give the holder the right to exercise the option for stock and the granting of new at-the-money reload options. The valuation of these claims is discussed in Hemmer, Matsumaga and Shevlin (1998) and Dybvig and Lowenstein (2003). However, neither paper addresses the optimality properties of these instruments as remuneration schemes.
they have issued a put option on the company's shares to the managers, in much the same way as the holders of corporate bonds have issued a put option on the company's assets to the shareholders. In this sense the current paper provides a complement to the moral hazard model of debt finance in Innes (1990). He shows under certain assumptions that in a risk neutral environment with ex ante non-contractible effort, the optimal financial contract is the one that is least sensitive to the realizations of the entrepreneur's effort. This is a debt contract with full recovery of value by the financier in the event of default. Hence, the entrepreneur is most efficiently incentivised by holding an equity claim protected by limited liability, which can be thought of as a call option on the company's assets.

6 Conclusion

This paper has shown that loss aversion, unlike Expected Utility Theory, can account for the use of stock options in executive remuneration schemes. Using a principal-agent framework with unobservable effort we showed that with sufficiently loss-averse managers, the optimal compensation package comprises a fixed wage plus a stock option. Subjecting a manager to the discomfort of loss relative to their reference income is too costly a stick to efficiently induce good performance. The analysis in the paper also allows us to understand how the properties of preferences and the technological opportunities interact to determine the structure of the compensation package, such as the steepness of rewards for success. Low risk aversion and high loss aversion is particularly propitious to the use of options. Of course the efficacy of corporate governance, financial reporting standards as well as cultural factors must be considered in gaining a full understanding of general patterns of executive compensation both within and between countries. Our contention is that loss aversion is also important.

7 Appendix

Following Grossman and Hart (1982) and Laffont and Martimort (2002) Chapter 4. To establish that the problem satisfies the Kuhn-Tucker necessary and sufficient conditions for an optimum we can restate the problem in utility space. Let \( H = k(U(H)) \) etc., then the programming problem can be written as:

\[ \text{Problem 1'.} \]
\[
\min \{ P_H k(U(Y_H)) + P_M k(U(Y_M)) + P_L k(U(Y_L)) \} \tag{A1}
\]

\[
st. P_H U(Y_H) + P_M U(Y_M) + P_L [U(Y_L) - \tilde{I}(U(Y_M) - U(Y_L))] - \pi \geq V^* \tag{A2}
\]

and \[
\Delta P_H U(Y_H) + \Delta P_M U(Y_M) + \Delta P_L [U(Y_L) - \tilde{I}(U(Y_M) - U(Y_L))] - \pi \geq 0 \tag{A3}
\]

\[
U(Y_M) - U(Y_L) \geq 0 \tag{A4}
\]

where \( \tilde{I} = I(Y_M - Y_L)/(U(Y_M) - U(Y_L)) \). The objective function is convex in utility space and the constraints are linear. Hence this is a standard convex programming problem, so that by the Kuhn-Tucker Theorem the first-order conditions are necessary and sufficient for a minimum to exist.

References


[29] Murphy, K (2002) ’Explaining Executive Compensation:Managerial Power vs. the Perceived Cost of Stock Options” University of Chicago Law Review (Summer)

