Information Aggregation in a Competitive Economy*

by

Rohit Rahi
and
Jean-Pierre Zigrand
Department of Finance
London School of Economics,
Houghton Street, London WC2A 2AE

June 12, 2015

Abstract

We consider the market for a risky asset for which agents have interdependent private valuations. We study competitive rational expectations equilibria under the standard CARA-normal assumptions. Equilibrium is partially revealing even though there are no noise traders. Complementarities in information acquisition arise naturally in this setting, and can lead to multiple equilibria. There may be excessive information gathering in equilibrium. Our framework encompasses the classical REE models in the CARA-normal tradition.

Journal of Economic Literature classification numbers: D82, G14.

Keywords: Private information, rational expectations, information acquisition, complementarity.

*This paper originated from a discussion with Marzena Rostek. We would like to thank Sergei Glebkin, Giovanna Nicodano, Xavier Vives, Liyan Yang, and seminar participants at INSEAD, LSE, Oxford and the University of Kyoto for helpful suggestions and comments.

1 Introduction

We study the market for a single risky asset for which agents have interdependent private valuations. Heterogeneous valuations may arise for various reasons. For example, agents may differ with respect to the uses they have for the asset, their liquidity needs, their investment opportunities, or the regulatory constraints they face. Diversity in valuations can be thought of as an indirect way to capture idiosyncratic preference or endowment shocks.[1] It can also be interpreted in purely behavioral terms – for example, agents could "agree to disagree" about the distribution of the asset payoff, or a subset of traders could be subject to psychological biases or misperceptions. Each trader is uncertain about his own valuation, and has the opportunity to acquire private information about it prior to trade. Equilibrium prices reflect some of this information.

We use a standard setup, with Gaussian shocks and constant absolute risk aversion, that nests the classical models of Grossman and Stiglitz (1980) and Hellwig (1980). Essentially the only difference with respect to the classical framework is that we allow agents' valuations to be imperfectly correlated. This gives us a tractable model of partial revelation without resorting to exogenous noise trade, with a unique linear equilibrium price function. In spite of the simplicity of this model, it delivers a rich set of conclusions, such as the possibility of large price jumps in response to small changes in the environment, and excessive information gathering.

In order to provide a summary exposition of our results, it suffices to consider a symmetric version of our model with two types. Agents of type i ($i = 1, 2$) have valuation θ_i, and a proportion λ_i of these agents has private information about θ_i. For example, one of the types could be pure speculators who care only about the liquidation value of the asset, and acquire information about it, while the other type could have a different valuation due to a hedging motive, and gather information about both the asset payoff and their hedging need. Assume for the moment that the proportions (λ_1, λ_2) are fixed exogenously. Then the equilibrium price is proportional to $\lambda_1\theta_1 + \lambda_2\theta_2$. Since the price depends on the valuations of both types, it does not fully reveal θ_i for either i.

Complementarities in information acquisition arise naturally in this setting. Let ρ be the correlation coefficient between θ_1 and θ_2, $|\rho| < 1$. Suppose first that $\rho < 0$. If $\lambda_2 > 0$, the price conveys some information to type 1 agents about their valuation θ_1 even if none of them acquire information about it ($\lambda_1 = 0$). They may actually learn less from the price if $\lambda_1 > 0$, since then the price becomes a mixed signal about θ_1; for example, a high price can result from a high θ_1 ("good news" for type 1) or from a high θ_2 ("bad news" for type 1). Thus, if $\rho < 0$, a *within-type complementarity* can arise, wherein price informativeness for a given type is lower when more agents of that type acquire information. Now suppose $\rho \geq 0$ and $\lambda_1 > 0$. Then price informativeness for type 1 is decreasing in λ_2. This is an *across-type*

[1]These shocks may depend, for example, on group affiliations or on the geographic location of traders. See Rostek and Weretka (2012) for further discussion and interpretation.

complementarity wherein agents of a given type learn less from the price if more agents of another type acquire information.

Next, we endogenize the information acquisition decision of agents, i.e. the proportions λ_1 and λ_2. A Grossman-Stiglitz type paradox arises if ρ exceeds a certain threshold – prices are so informative that there is no incentive for agents to gather costly information. On the other hand, if ρ is low enough, all agents of both types acquire information.

With more than two types, complementarities in information acquisition can lead to multiple equilibria. It can turn out that there is an equilibrium in which all agents of type i (for some i) acquire information and another equilibrium in which none of these agents do. In fact, in the latter equilibrium, prices can be more informative for all types, including type i.

Finally we analyze the link between price informativeness and welfare. If prices are more informative about the valuation of type i, these agents can make better portfolio decisions. But more informative prices are also closer to their true valuation, reducing profitable trading opportunities. In general, agents who have less to gain from learning from prices, because their private information is relatively precise, are better off if prices are less informative. Agents' incentives to gather information are not necessarily aligned with their objectives, however. They may acquire information even though they would be better off coordinating with other agents to stay uninformed, leading to a Prisoner's Dilemma style suboptimal equilibrium.

Related Literature:

Vives (2014) studies a competitive rational expectations equilibrium model with private valuations and no noise trade. As in our paper, there is no equilibrium with a high correlation of types, and when an equilibrium does exist the price function is partially revealing. However, price informativeness does not depend on the mass of informed agents (as long as this mass is positive) – the price reveals the average type of all agents regardless of how many are informed. This in turn implies that the information acquisition decisions of agents are independent. The parametric specification of the model is also different from ours, featuring risk neutral traders who incur a quadratic trading cost and face an exogenous supply curve.

Our stochastic environment shares some features with that of Rostek and Weretka (2012), insofar as it allows heterogeneity in the correlations between the private valuations of traders. But the aims of their paper are different from ours – they study the effect of market size on information aggregation in a strategic double auction model.

There is a large literature on complementarities in information acquisition. The closest to the present paper are competitive models in which these complementarities arise because prices become less informative as more agents gather information.[2]

[2]Other mechanisms have also been explored in the literature. Complementarities in information acquisition arise in Goldstein and Yang (2014) because agents collect different pieces of information about the asset value (as more agents of one group acquire information, the uncertainty about the

Stein (1987) provides an early example of the entry of informed speculators reducing price informativeness for existing traders, in a setting where agents seek to forecast transitory and permanent shocks to the supply of the underlying in a futures market. In an environment closer to that of Grossman-Stiglitz, Barlevy and Veronesi (2008) find that a complementarity can arise because the asset payoff and noise trader demand are negatively correlated. Their mechanism has a similar flavor to our within-type complementarity which is due to a negative correlation between the valuations of traders from different groups. Price informativeness can be decreasing in the incidence of informed trading in Ganguli and Yang (2009) and Manzano and Vives (2011) because agents have access to two sources of information (about the asset payoff and the asset supply), in Goldstein et al. (2014) because agents with different trading opportunities may trade on the same information in opposite directions, and in Breon-Drish (2012) due to non-normality of shocks in an otherwise standard Grossman-Stiglitz setup.

Most of the rational expectations literature relies on exogenous noise trade and hence does not provide a suitable framework for welfare analysis. Usually a proxy for welfare is employed, such as price informativeness, price volatility or some measure of liquidity. The present paper is well-suited for analyzing welfare and its connection to price informativeness.

The paper is organized as follows. We describe the economy in Section 2. In Sections 3–5 we take the information acquisition decisions of agents as given. We characterize the unique linear equilibrium price function in Section 3. Then, in Section 4, we provide several examples in which this characterization can be employed, including the classical setups of Grossman and Stiglitz (1980) and Hellwig (1980). In Section 5 we analyze the information content of the price for each type, and how this is affected by the (still exogenous) information acquisition decisions of agents. We endogenize information acquisition in Section 6. In Section 7 we discuss complementarities in information gathering and equilibrium multiplicity. Section 8 is devoted to welfare. Section 9 concludes. Most of the proofs are in the Appendix.

2 The Economy

There is a single risky asset in zero net supply, and a riskfree asset with the interest rate normalized to zero. There are N types of agents, $N \geq 2$, and a continuum of agents of each type. Formally, we index agents of any given type by the unit

asset payoff is reduced for the other group, increasing the return from gathering information for them), in Mele and Sangiorgi (2015) because of Knightian uncertainty (as prices become more informative, uninformed agents have a greater incentive to acquire information to resolve the ambiguity and thus "decode" the information contained in prices), in García and Strobl (2011) due to relative wealth concerns (as the proportion of informed agents rises, so does the average wealth of all agents, giving the uninformed an additional incentive to gather information), and in Veldkamp (2006b,a) because of increasing returns to scale in the supply of information (information gets cheaper as more agents acquire information).

interval, endowed with Lebesgue measure. The private valuation for the risky asset of an agent of type i is given by $v_i = \theta_i + \eta_i$. Prior to trade, type i agents can acquire a private signal about θ_i by incurring a cost c_i; for agent n of type i (agent in for short) this signal takes the form $s_{in} = \theta_i + \epsilon_{in}$. In other words, type i agents are distinguished by their valuation v_i, and θ_i is the part of v_i about which they can gather information at some cost; their signals can have some idiosyncratic variation, however.

The random variables $\{\theta_i, \eta_i, \{\epsilon_{in}\}_{n\in[0,1]}\}_{i=1,\dots,N}$ are joint normal with mean zero. Let $\boldsymbol{\theta} := (\theta_i)_{i=1}^N$ and $\boldsymbol{\eta} := (\eta_i)_{i=1}^N$. For each type i, the valuation shock η_i is independent of $\boldsymbol{\theta}$, the signal shock ϵ_{in} is independent of $(\boldsymbol{\theta}, \boldsymbol{\eta})$, and the signal shocks across agents, $\{\epsilon_{in}\}_{n\in[0,1]}$, are i.i.d. We adopt the convention that the average of a continuum of i.i.d. random variables with mean zero is zero. Then, the average signal of agents of type i, $\int_n s_{in} dn$, is equal to θ_i.[3] To ensure that the problem is nontrivial, we assume that the covariance matrix of $\boldsymbol{\theta}$ is positive definite.

If agent in buys q_{in} units of the risky asset at price p, his "wealth" is $W_{in} = (v_i - p)q_{in}$. Given his information set $\mathcal{I}_{in}$, which consists of all the random variables that he observes prior to trade, he solves $\max_{q_{in}} E[-\exp(-r_i W_{in})|\mathcal{I}_{in}]$. Agents have rational expectations – they know the price function, a function of the private signals of all agents in the economy, also denoted by p, and condition on the price when making their portfolio decisions. Thus $\mathcal{I}_{in} = \{s_{in}, p\}$ if agent in is informed, and $\mathcal{I}_{in} = \{p\}$ if he is uninformed.

We denote the proportion of agents of type i who choose to become informed by $\lambda_i \in [0,1]$. An equilibrium consists of a vector $\boldsymbol{\lambda} := (\lambda_i)_{i=1}^N$, and a price function p, such that agents optimize and markets clear. Agent optimization requires that each agent is happy with his information acquisition decision (to acquire private information or not) given the price function p, and subsequently, for any realization of p, he chooses an optimal portfolio given his information. Letting $q_i := \int_n q_{in} dn$, the aggregate trade of type i, the market-clearing condition is $\sum_i q_i = 0$.

For random variables x and y, we denote the covariance of x and y by $\sigma_{x,y}$, the variance of x by σ_x^2, and the conditional variance of x given y by $\sigma_{x|y}^2$.

3 The Equilibrium Price Function

In this section we solve for a rational expectations equilibrium price function for given $\boldsymbol{\lambda}$. We conjecture a linear price function of the form

$$p = \sum_{i=1}^{N} a_i \theta_i, \tag{1}$$

for some coefficients $(a_1, \dots, a_N)$, not all zero. Thus the private signals of type i agents are reflected in the price only through their average signal, which is equal

[3] See the technical appendix of Vives (2008) for a discussion of the use of the strong law of large numbers in this context. For ease of exposition, we drop the qualifier "almost surely".

to θ_i. Given the linear-normal structure of the model, agents have a mean-variance objective function. The optimal portfolio of agent in is:

$$q_{in} = \frac{E(v_i|\mathcal{I}_{in}) - p}{r_i \mathrm{Var}(v_i|\mathcal{I}_{in})}. \tag{2}$$

To calculate agents' portfolios, we use the standard projection theorem for normals.[4] Let $\beta_i := \sigma_{\theta_i,p}/\sigma_p^2$. Then

$$\sigma^2_{\theta_i|p} = \sigma^2_{\theta_i} - \beta_i \sigma_{\theta_i,p}. \tag{3}$$

We proceed under the provisional assumption that p does not (fully) reveal θ_i for any i, i.e. $\sigma^2_{\theta_i|p} > 0$. We will show later, in the proof of Proposition 3.2, that this assumption is in fact satisfied at any equilibrium. We use the superscripts I and U to distinguish between the portfolios of informed and uninformed agents.

Lemma 3.1 (Optimal portfolios) *Suppose $\sigma^2_{\epsilon_i}$ and $\sigma^2_{\eta_i}$ are not both zero, and $\sigma^2_{\theta_i|p} > 0$. Then the optimal portfolios of type i agents are given by*

$$\begin{aligned} q^I_{in} &= \frac{1}{r_i} \cdot \frac{\sigma^2_{\theta_i|p} s_{in} - \big[\sigma^2_{\theta_i|p} + (1-\beta_i)\sigma^2_{\epsilon_i}\big] p}{(\sigma^2_{\theta_i|p} + \sigma^2_{\epsilon_i})\sigma^2_{\eta_i} + \sigma^2_{\theta_i|p}\sigma^2_{\epsilon_i}}, \\ q^U_{in} &= -\frac{1}{r_i} \cdot \frac{1-\beta_i}{\sigma^2_{\theta_i|p} + \sigma^2_{\eta_i}} p. \end{aligned}$$

From Lemma 3.1, the aggregate trade of type i agents is

$$q_i = \frac{\lambda_i}{r_i} \cdot \frac{\sigma^2_{\theta_i|p}\theta_i - \big[\sigma^2_{\theta_i|p} + (1-\beta_i)\sigma^2_{\epsilon_i}\big] p}{(\sigma^2_{\theta_i|p} + \sigma^2_{\epsilon_i})\sigma^2_{\eta_i} + \sigma^2_{\theta_i|p}\sigma^2_{\epsilon_i}} - \frac{1-\lambda_i}{r_i} \cdot \frac{1-\beta_i}{\sigma^2_{\theta_i|p} + \sigma^2_{\eta_i}} p, \tag{4}$$

which is linear in θ_i and p. We can now solve for the price function using the market-clearing condition, $\sum_i q_i = 0$. Before proceeding with this task, we impose some further assumptions which will stay in force for the remainder of the paper:

A1. $\lambda_k > 0$ and $\lambda_\ell > 0$ for at least two types k and ℓ.

A2. The equilibrium trade of agent in is measurable with respect to his information $\mathcal{I}_{in}$.

[4] Consider random vectors $\mathbf{x}_1$ and $\mathbf{x}_2$, $(\mathbf{x}_1, \mathbf{x}_2) \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, and partition $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$ as follows:

$$\boldsymbol{\mu} := \begin{pmatrix} \boldsymbol{\mu}_1 \\ \boldsymbol{\mu}_2 \end{pmatrix}, \qquad \boldsymbol{\Sigma} := \begin{pmatrix} \boldsymbol{\Sigma}_{11} & \boldsymbol{\Sigma}_{12} \\ \boldsymbol{\Sigma}_{12} & \boldsymbol{\Sigma}_{22} \end{pmatrix},$$

where $\boldsymbol{\mu}_i := E(\mathbf{x}_i)$ and $\boldsymbol{\Sigma}_{ij} := \mathrm{Cov}(\mathbf{x}_i, \mathbf{x}_j)$, $i, j = 1, 2$. If $\boldsymbol{\Sigma}_{22}$ is nonsingular, we have

$$(\mathbf{x}_1|\mathbf{x}_2) \sim N\big(\boldsymbol{\mu}_1 + \boldsymbol{\Sigma}_{12}\boldsymbol{\Sigma}_{22}^{-1}(\mathbf{x}_2 - \boldsymbol{\mu}_2), \boldsymbol{\Sigma}_{11} - \boldsymbol{\Sigma}_{12}\boldsymbol{\Sigma}_{22}^{-1}\boldsymbol{\Sigma}_{12}\big).$$

A3. For any type i, one of the following information structures applies:

(a) *Asymmetric information:* $\sigma^2_{\epsilon_i} = 0$ and $\sigma^2_{\eta_i} > 0$; or

(b) *Differential information:* $\sigma^2_{\epsilon_i} > 0$ and $\sigma^2_{\eta_i} = 0$.

Assumption A1 is only provisional – we endogenize $\boldsymbol{\lambda}$ in Section 6 where we show that, in any equilibrium, λ_i is indeed positive for at least two types (see Lemma 6.3). Assumption A2 rules out some trivial equilibria. Assumption A3 is for tractability, and gives us two canonical information structures that have been employed in the literature. Under information structure (a), type i is asymmetrically informed in the sense that the informed agents of type i know θ_i while the uninformed of that type do not. Under information structure (b), type i is differentially informed in the sense that the informed agents of type i have conditionally i.i.d. signals about θ_i; moreover, the restriction $\sigma^2_{\eta_i} = 0$ implies that $v_i = \theta_i$, so that their pooled information reveals their type.[5] Note that Assumption A3 allows some types to be asymmetrically informed and others to be differentially informed.

Proposition 3.2 (Equilibrium price function) *There is a unique linear equilibrium price function given by*

$$p = k \sum_i \gamma_i \theta_i, \qquad k \neq 0, \tag{5}$$

where

$$\gamma_i = \begin{cases} \lambda_i (r_i \sigma^2_{\eta_i})^{-1} & \text{if type } i \text{ is asymmetrically informed,} \\ \lambda_i (r_i \sigma^2_{\epsilon_i})^{-1} & \text{if type } i \text{ is differentially informed.} \end{cases}$$

The price function does not reveal θ_i for any i.

From Lemma 3.1, we see that the coefficient on the private signal in the optimal trade of an informed agent of type i is $(r_i \sigma^2_{\eta_i})^{-1}$ in the asymmetric information case, and $(r_i \sigma^2_{\epsilon_i})^{-1}$ in the differential information case. We can think of this as the "trading intensity" of an informed agent. Thus the coefficient of θ_i in the price function is proportional to the trading intensity of the informed agents of type i, times their mass λ_i.

It is instructive to compare the revelation properties of our price function with that of Vives (2014). Vives assumes that there is a continuum of types, with the result that the price reveals the average type, which for any trader is a sufficient statistic for the information of all *other* traders. Thus every trader effectively has access to the pooled information of all traders in the economy (Vives calls this a

[5] While the terms "asymmetric" and "differential" are a useful way to distinguish between one-sided and multifaceted private information, we should point out that our usage is somewhat loose – we say that type i is asymmetrically informed even if all type i agents know θ_i, and we refer to a type as differentially informed even though, strictly speaking, this label only applies to the informed agents of that type.

"privately revealing" equilibrium). In our model, on the other hand, there are finitely many types, with a continuum of each type. While the price does not reflect any idiosyncratic variation *within* types, it is affected by idiosyncratic variation *across* types. An agent of type i, who seeks to learn θ_i, knows θ_i in equilibrium only if his private signal already tells him what θ_i is. If he does not observe θ_i directly, how much he learns from the price depends on the mass of informed agents of every type. Price informativeness in Vives' model is the same regardless of the mass of informed agents (as long as this mass is positive).

4 Examples

In this section we provide a number of examples of our general framework. These examples include the economy in Grossman and Stiglitz (1980) with noisy aggregate supply, and the competitive limit of the economy in Hellwig (1980) with noise traders. In both cases, the noise is easily endogenized as optimizing trade arising from liquidity or hedging considerations. It can also be interpreted in purely behavioral terms. All the random variables in these examples are joint normally distributed with mean zero. In each case, we use Lemma 3.1 and Proposition 3.2 to determine optimal portfolios and the equilibrium price function.

Example 4.1 (Grossman-Stiglitz) The asset payoff is $v = \theta + \eta$. There is a unit mass of investors, of whom a proportion $\lambda \in (0,1)$ privately observes θ. In addition, there is a unit mass of "noise traders" whose private valuation is $u + \eta$, of which they privately observe the component u. The random variables θ, η, and u are mutually independent. This fits into our model with two types, both of which are asymmetrically informed: $\lambda_1 = \lambda, \lambda_2 = 1, \theta_1 = \theta, \theta_2 = u$, $\eta_1 = \eta_2 = \eta$, and $\sigma^2_{\epsilon_1} = \sigma^2_{\epsilon_2} = 0$. The optimal portfolios are

$$q^I_{1n} = \frac{\theta - p}{r_1 \sigma^2_\eta}, \qquad q^U_{1n} = \frac{E(v|p) - p}{r_1 \mathrm{Var}(v|p)}, \qquad q^I_{2n} = \frac{u - p}{r_2 \sigma^2_\eta}.$$

The price function is

$$p = k \left[\frac{\lambda}{r_1 \sigma^2_\eta} \theta + \frac{1}{r_2 \sigma^2_\eta} u \right],$$

which takes the same form as in Grossman and Stiglitz (1980), with u playing the role of the random aggregate supply or noise trade.

Notice that our noise traders do not trade an exogenous amount, as they are typically assumed to do in the noisy rational expectations literature. They can be thought of as "sentiment traders", with u being the sentiment shock, as in Mendel and Shleifer (2012) (whose model is a variant of the above example), or as investors who trade on noise as though it were information, as in Banerjee and Green (2015) or Peress (2014). $\|$

Example 4.2 (Grossman-Stiglitz with optimizing liquidity traders) Rather than mimic the standard assumption of independent noise trade, suppose we replace the type 2 traders in Example 4.1 with optimizing "liquidity traders". These traders perceive the asset value to be $v = \theta + \eta$, just like the type 1 traders, but also have an endowment, which is the product of two normal random variables, x and e.[6] The random variable x is independent of e and η, is not perfectly correlated with θ, and is privately observed by (all) liquidity traders prior to trade; it can be interpreted as the size of their liquidity shock. The covariance $\sigma_{\eta e}$ is nonzero. Conjecturing that a liquidity trader can infer θ from the equilibrium price, given that he knows the realization of his liquidity shock x,[7] his optimal portfolio is given by

$$q_{2n}^I = \frac{\theta - r_2 \sigma_{\eta e} x - p}{r_2 \sigma_\eta^2}. \tag{6}$$

This fits into our model as in Example 4.1, except that here $\theta_2 = \theta - r_2 \sigma_{\eta e} x$. The price function is

$$\begin{aligned} p &= k \left[\frac{\lambda}{r_1 \sigma_\eta^2} \theta + \frac{1}{r_2 \sigma_\eta^2} (\theta - r_2 \sigma_{\eta e} x) \right] \\ &= \frac{k}{\sigma_\eta^2} \left[(\lambda r_1^{-1} + r_2^{-1}) \theta - \sigma_{\eta e} x \right]. \end{aligned}$$

which confirms our conjecture that the liquidity traders learn θ from the price. If x is independent of θ, the price function is of the same form as in Grossman and Stiglitz (1980). ‖

Example 4.3 Consider an extension of Example 4.2 to multiple categories of informed traders with varying liquidity needs. The asset payoff is $v = \theta + \eta$. There are N types of agents. All agents of the first $N-1$ types are informed. In particular, an agent of type i, $i = 1, \ldots, N-1$, has endowment $x_i e_i$, a product of two normal random variables, and he privately observes both θ and x_i. The random variable x_i is independent of e_i and η, the covariance matrix of $(\theta, x_1, x_2, \ldots, x_{N-1})$ is positive definite, and $\sigma_{\eta e_i} \neq 0$. Agents of type N have no endowment and a proportion λ of them privately observe θ.

This example fits into our model as follows: For $i = 1, \ldots, N-1$, $\lambda_i = 1$, and $\theta_i = \theta - r_i \sigma_{\eta e_i} x_i$. For type N, $\lambda_N = \lambda$, and $\theta_N = \theta$. Also, $\sigma_{\epsilon_i}^2 = 0$ and $\eta_i = \eta$, for all

[6] A number of papers in the CARA-normal REE literature have used such a specification of the endowment to generate a hedging motive for trade. See, for example, Rahi (1996). We can also interpret the random endowments of the risky asset in Diamond and Verrecchia (1981) as an example of this specification, with e perfectly correlated with the asset payoff .

[7] Alternatively, we could assume that the liquidity traders directly observe θ prior to trade, in addition to x.

i. The optimal portfolios are

$$q^I_{in} = \frac{\theta - r_i \sigma_{\eta e_i} x_i - p}{r_i \sigma^2_\eta}, \qquad i = 1, \ldots, N-1,$$
$$q^I_{Nn} = \frac{\theta - p}{r_N \sigma^2_\eta},$$
$$q^U_{Nn} = \frac{E(v|p) - p}{r_N \mathrm{Var}(v|p)}.$$

The price function is

$$p = k \left[\sum_{i=1}^{N-1} \frac{1}{r_i \sigma^2_\eta} (\theta - r_i \sigma_{\eta e_i} x_i) + \frac{\lambda}{r_N \sigma^2_\eta} \theta \right]$$
$$= \frac{k}{\sigma^2_\eta} \left[\left(\sum_{i=1}^{N-1} r_i^{-1} + \lambda r_N^{-1} \right) \theta - \sum_{i=1}^{N-1} \sigma_{\eta e_i} x_i \right].$$

The "noise" in the price from the perspective of an uninformed agent of type N is a linear combination of the shocks $x_i, i = 1, \ldots, N-1$. These shocks may be correlated with each other and with θ. $\qquad \|$

Example 4.4 (Hellwig) The asset payoff is θ. There is a unit mass of differentially informed agents who receive conditionally i.i.d. signals about θ. In addition, there is a unit mass of "noise traders" who are differentially informed about their private valuation u, which is independent of θ. There are no uninformed agents. This is a special case of our general setup with two types, both of which are differentially informed: $\lambda_1 = \lambda_2 = 1, \theta_1 = \theta, \theta_2 = u$, and $\sigma^2_{\eta_1} = \sigma^2_{\eta_2} = 0$. The optimal portfolios are:

$$q^I_{1n} = \frac{E(\theta|s_{1n}, p) - p}{r_1 \mathrm{Var}(\theta|s_{1n}, p)}, \qquad q^I_{2n} = \frac{E(u|s_{2n}, p) - p}{r_2 \mathrm{Var}(u|s_{2n}, p)}.$$

The price function is

$$p = k \left[\frac{1}{r_1 \sigma^2_{\epsilon_1}} \theta + \frac{1}{r_2 \sigma^2_{\epsilon_2}} u \right].$$

This is essentially the limiting equilibrium in Hellwig (1980), as the number of informed traders goes to infinity, with u playing the role of the exogenous noise trade as in Example 4.1.

Just as we replaced the noise traders in Example 4.1 with optimizing liquidity traders in Example 4.2, we can do that here as well. The optimal portfolio of type 2 agents is then given by (6). We assume that ϵ_{1n} is independent of x. This fits into our general model as follows: $\lambda_1 = \lambda_2 = 1, \eta_1 = 0, \eta_2 = \eta, \sigma^2_{\epsilon_1} = \sigma^2_\epsilon, \sigma^2_{\epsilon_2} = 0, v_1 = \theta_1 = \theta,$

and $v_2 = \theta_2 + \eta$, and $\theta_2 = \theta - r_2\sigma_{\eta e}x$.[8] The price function is

$$p = k\left[\frac{\theta}{r_1\sigma_\epsilon^2} + \frac{\theta - r_2\sigma_{\eta e}x}{r_2\sigma_\eta^2}\right]$$
$$= k\left[\left(\frac{1}{r_1\sigma_\epsilon^2} + \frac{1}{r_2\sigma_\eta^2}\right)\theta - \frac{\sigma_{\eta e}}{\sigma_\eta^2}x\right].$$

Notice that, unlike the noise traders in the first part of this example, who are differentially informed, all the liquidity traders have the same information. ‖

5 Price Informativeness

In this section we develop some results on the informativeness of the equilibrium price function for given proportions of informed agents $\boldsymbol{\lambda}$. Letting $\boldsymbol{\gamma} := (\gamma_i)_{i=1}^N$, we can write the price function (5) as $p = k\boldsymbol{\gamma}^\top\boldsymbol{\theta}$.[9] We denote the covariance matrix of $\boldsymbol{\theta}$ by $\mathbf{V}$, assumed to be positive definite, and the i'th column of $\mathbf{V}$ by $\mathbf{V}_i$. Due to the symmetry of $\mathbf{V}$, the i'th row of $\mathbf{V}$ is $\mathbf{V}_i^\top$. Then we have

$$\sigma_p^2 = k^2\boldsymbol{\gamma}^\top\mathbf{V}\boldsymbol{\gamma}, \qquad \text{and} \qquad \sigma_{\theta_i,p} = k\mathbf{V}_i^\top\boldsymbol{\gamma}.$$

For the uninformed agents of type i, we use the following measure of price informativeness:

$$\mathcal{V}_i := \frac{\sigma_{\theta_i}^2 - \sigma_{\theta_i|p}^2}{\sigma_{\theta_i}^2}. \tag{7}$$

Clearly, $\mathcal{V}_i \in [0,1)$. Substituting from (3), we get

$$\mathcal{V}_i = \frac{\sigma_{\theta_i,p}^2}{\sigma_{\theta_i}^2\sigma_p^2} = \frac{1}{\sigma_{\theta_i}^2}\cdot\frac{(\mathbf{V}_i^\top\boldsymbol{\gamma})^2}{\boldsymbol{\gamma}^\top\mathbf{V}\boldsymbol{\gamma}}. \tag{8}$$

Notice that $\mathcal{V}_i$ is homogeneous of degree zero in $\boldsymbol{\gamma}$: price informativeness depends only on the relative values of the γ_i's.

If $\sigma_{\epsilon_i}^2 > 0$, the informed agents of type i also learn from the price. For these agents, the corresponding measure of price informativeness is:

$$\mathcal{V}_i^I := \frac{\mathrm{Var}(\theta_i|s_{in}) - \mathrm{Var}(\theta_i|s_{in}, p)}{\mathrm{Var}(\theta_i|s_{in})}.$$

Lemma 5.1 *Suppose $\sigma_{\epsilon_i}^2 > 0$. Then $\mathcal{V}_i^I \in [0,1)$ and is a strictly increasing function of $\mathcal{V}_i$.*

[8] For example, we can think of the asset payoff being $\theta + \eta$, with type 1 agents having the ability hedge the risk η.

[9] All vectors are taken to be column vectors unless transposed.

In view of this result, we will use $\mathcal{V}_i$ as our measure of price informativeness for agents of type i, whether or not they observe a noisy private signal in addition to the price. If type i is differentially informed, all type i agents make inferences from the price. If type i is asymmetrically informed, on the other hand, only the uninformed agents learn from the price; the informed already know θ_i.

We now investigate the effect of changing the proportions of informed agents on the informativeness of the price. In particular, we are interested in conditions under which a greater incidence of informed trading reduces price informativeness. Under the assumption that $\sigma^2_{\theta_i} = \sigma^2_\theta$ for all i, we can characterize $\mathcal{V}_i$ and its derivatives in terms of the correlation matrix $\mathbf{R} := (\sigma^2_\theta)^{-1}\mathbf{V}$, with ij'th element $\rho_{ij} := \mathrm{Corr}(\theta_i, \theta_j)$. Let $\mathbf{R}_i$ be the i'th column of $\mathbf{R}$. We write $x \propto y$ to indicate that x and y have the same sign.

Proposition 5.2 (Price informativeness) *Suppose $\sigma^2_{\theta_i} = \sigma^2_\theta$ for all i. Then price informativeness for type i is given by*

$$\mathcal{V}_i = \frac{(\mathbf{R}_i^\top \boldsymbol{\gamma})^2}{\boldsymbol{\gamma}^\top \mathbf{R} \boldsymbol{\gamma}}.$$

Furthermore,

$$\frac{\partial \mathcal{V}_i}{\partial \lambda_i} \propto \mathbf{R}_i^\top \boldsymbol{\gamma}.$$

Finally, if $\lambda_i > 0$ and $\partial \mathcal{V}_i / \partial \lambda_i > 0$, then $\partial \mathcal{V}_i / \partial \lambda_j < 0$ for some $j \neq i$.

For type i, we say that there is a *within-type complementarity* if $\partial \mathcal{V}_i / \partial \lambda_i < 0$, and an *across-type complementarity* if $\partial \mathcal{V}_i / \partial \lambda_j < 0$ for some $j \neq i$. Proposition 5.2 tells us that, for any i, at least one kind of complementarity arises. A necessary condition for within-type complementarity for type i is that $\rho_{ij} < 0$ for some j.

We say that the economy is *symmetric* if $r_i, \sigma^2_{\theta_i}, \sigma^2_{\eta_i}$, and $\sigma^2_{\epsilon_i}$ are the same for all i (in which case, we drop the subscript i on these parameters), and all types are either asymmetrically informed or differentially informed. For a symmetric economy, $\boldsymbol{\gamma}$ is proportional to $\boldsymbol{\lambda}$ by Proposition 3.2, so that Proposition 5.2 gives us:

Corollary 5.3 (Price informativeness in a symmetric economy) *For a symmetric economy, price informativeness for type i is given by*

$$\mathcal{V}_i = \frac{(\mathbf{R}_i^\top \boldsymbol{\lambda})^2}{\boldsymbol{\lambda}^\top \mathbf{R} \boldsymbol{\lambda}}. \tag{9}$$

Furthermore,

$$\frac{\partial \mathcal{V}_i}{\partial \lambda_i} \propto \mathbf{R}_i^\top \boldsymbol{\lambda}. \tag{10}$$

For most of the results in the rest of the paper we restrict ourselves to symmetric economies, since this simplifies the analysis and provides us with the clearest intuitions. We often invoke the further assumption that $\rho_{ij} = \rho$, for all $i \neq j$. We then need to impose a lower bound on ρ due to

Lemma 5.4 (Lower bound on ρ) *Suppose $\rho_{ij} = \rho$, for all $i \neq j$. Then the correlation matrix $\mathbf{R}$ is positive definite if and only if*

$$\rho > \rho_{\min} := -\frac{1}{N-1}.$$

We denote by $\mathcal{E}$ the set of symmetric economies, with $\rho_{ij} = \rho > \rho_{\min}$, for all $i \neq j$. Specializing Corollary 5.3 to such an economy, we obtain:

Corollary 5.5 (Price informativeness for an economy in $\mathcal{E}$) *For an economy in $\mathcal{E}$, price informativeness for type i is given by*

$$\mathcal{V}_i = \frac{\left[(1-\rho)\lambda_i + \rho \sum_k \lambda_k\right]^2}{(1-\rho)\sum_k \lambda_k^2 + \rho\left(\sum_k \lambda_k\right)^2}. \tag{11}$$

Furthermore,

$$\frac{\partial \mathcal{V}_i}{\partial \lambda_i} \propto \lambda_i + \rho \sum_{k \neq i} \lambda_k. \tag{12}$$

We see from (12) that ρ must be negative for a within-type complementarity to arise for any type. A particularly tractable case, that will be the basis for several results to follow, is obtained when $\lambda_i = 1$ for all $i \neq j$. Let $\mathcal{V}_j(\lambda_j)$ denote the dependence of $\mathcal{V}_j$ on λ_j, fixing $\lambda_i = 1$ for $i \neq j$, and let $\lambda^* := -\rho(N-1)$.

Proposition 5.6 *Consider an economy in $\mathcal{E}$. Suppose there is a type j such that $\lambda_i = 1$ for all $i \neq j$. Then we have:*

i. *$\mathcal{V}_i$ is strictly decreasing in λ_j, for $i \neq j$;*

ii. *$\partial \mathcal{V}_j / \partial \lambda_j \propto \lambda_j - \lambda^*$. If $\rho < 0$, $\mathcal{V}_j(\lambda^*) = 0$;*[10]

iii. *$\mathcal{V}_j(1) > \mathcal{V}_j(0)$ if and only if $\rho > 0.5\,\rho_{\min}$.*

Thus information acquisition by type j agents reduces price informativeness for all other types, regardless of the value of ρ. If ρ is non-negative, information acquisition by type j makes prices more informative for type j itself. However, if ρ is negative, $\mathcal{V}_j$ is not monotonic in λ_j. Figure 1 depicts the 3-type case, graphing $\mathcal{V}_3(\lambda_3)$ for ρ equal to -0.2 and -0.3 (the points a b and c are candidate equilibria that we will discuss later, in the context of Proposition 7.1).[11] Notice that $\mathcal{V}_3$ achieves its minimum value of 0 at $\lambda_3 = \lambda^*$. The value of ρ for which $\mathcal{V}_3(1) = \mathcal{V}_3(0)$ is $0.5\,\rho_{\min} = -0.25$. If ρ is less than -0.25, then $\mathcal{V}_3(1) < \mathcal{V}_3(0)$: type 3 agents learn less from the price when all of them acquire information relative to the case where none of them do.

[10] Note that $\lambda^* \in (0,1)$ if and only if $\rho \in (\rho_{\min}, 0)$.

[11] Due to symmetry, the shape of $\mathcal{V}_j(\lambda_j)$ is the same for all j, given that $\lambda_i = 1$ for $i \neq j$.

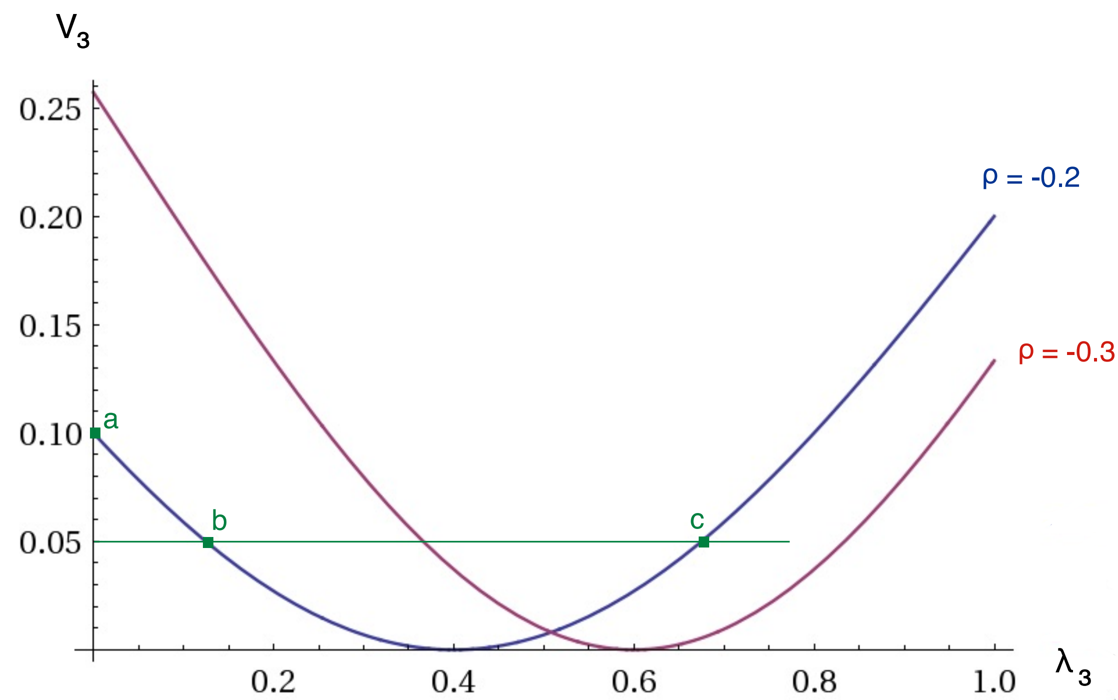

Figure 1: Price informativeness $\mathcal{V}_3$ as a function of λ_3, given $\lambda_1 = \lambda_2 = 1$.

6 Endogenous Information Acquisition

So far we have taken the allocation of private information to be exogenous. We now endogenize information acquisition. In order to become informed, a type i agent must pay a positive cost c_i. He takes as given the vector $\boldsymbol{\lambda}$ and the corresponding price function $p = k\boldsymbol{\gamma}^\top\boldsymbol{\theta}$. We wish to find $\boldsymbol{\lambda}$ such that, for any type, both the informed and uninformed find their decision with regard to information acquisition optimal.

It is convenient to use the following monotonic transformation of ex ante expected utility:

$$\mathcal{U}_{in} := \left(E[\exp(-r_i\hat{W}_{in})]\right)^{-2},$$

where $\hat{W}_{in} = W_{in} - c_i$ if agent in acquires information, and $\hat{W}_{in} = W_{in}$ if he does not. Using superscripts I and U for informed and uninformed agents respectively, we have

Lemma 6.1 (Utilities) *For given* $\boldsymbol{\lambda}$*,*

$$\mathcal{U}^I_{in} = e^{-2r_ic_i} \cdot \frac{(\sigma^2_{\theta_i|p} + \sigma^2_{\epsilon_i})\sigma^2_{v_i-p}}{(\sigma^2_{\theta_i|p} + \sigma^2_{\epsilon_i})\sigma^2_{\eta_i} + \sigma^2_{\theta_i|p}\sigma^2_{\epsilon_i}}, \qquad \textit{and} \qquad \mathcal{U}^U_{in} = \frac{\sigma^2_{v_i-p}}{\sigma^2_{\theta_i|p} + \sigma^2_{\eta_i}}.$$

Since the utility of an agent depends only on his type, and on whether he is informed or uninformed, we shall henceforth drop the subscript n. Thus $\mathcal{U}_i^I$ will denote the utility of all informed agents of type i, and $\mathcal{U}_i^U$ the utility of all uninformed agents of this type. An equilibrium $\boldsymbol{\lambda}$ is characterized by

$$\frac{\mathcal{U}_i^I}{\mathcal{U}_i^U} \quad \text{is} \quad \begin{cases} \geq & 1 \quad \text{for} \quad \lambda_i = 1 \\ = & 1 \quad \text{for} \quad \lambda_i \in (0,1) \\ \leq & 1 \quad \text{for} \quad \lambda_i = 0. \end{cases} \tag{13}$$

Notice that, if $\lambda_i \in (0,1)$, the ex ante expected utility of an informed agent of type i (after paying the cost c_i) must be equal to the ex ante expected utility of an uninformed agent of that type. We now compute the utility ratio:

Lemma 6.2 (Utilities of informed vs uninformed) *For given* $\boldsymbol{\lambda}$*,*

$$\frac{\mathcal{U}_i^I}{\mathcal{U}_i^U} = e^{-2r_i c_i}\left[1 + \frac{\sigma_{\theta_i}^2}{\sigma_{\eta_i}^2}(1 - \mathcal{V}_i)\right],$$

if type i is asymmetrically informed. If type i is differentially informed, we get the same expression with $\sigma_{\eta_i}^2$ replaced by $\sigma_{\epsilon_i}^2$.

For both information structures, the utility ratio is decreasing in $\mathcal{V}_i$. As one would expect, the incentive to become informed is lower if prices are more informative.

From Proposition 3.2 and Lemma 6.2, it is apparent that the two cases of type i being asymmetrically or differentially informed are formally identical as far as equilibrium is concerned. We will present our equilibrium results for asymmetric information. If type i is differentially informed, the corresponding results are obtained simply by replacing $\sigma_{\eta_i}^2$ by $\sigma_{\epsilon_i}^2$.

We assume that agents have an incentive to acquire information if they cannot learn anything from the price. Letting

$$\bar{c}_i := \frac{1}{2r_i}\log\left[1 + \frac{\sigma_{\theta_i}^2}{\sigma_{\eta_i}^2}\right],$$

this assumption is equivalent to the following condition (from Lemma 6.2):

A4. For each type i, $c_i < \bar{c}_i$.

This will be a standing assumption (along with assumptions A1–A3 imposed in Section 3) for the rest of the paper. It says that the cost c_i is low relative to the signal-to-noise ratio $\sigma_{\theta_i}^2/\sigma_{\eta_i}^2$. This leads us to the following result, which rules out fully revealing equilibria:

Lemma 6.3 (Partial revelation) *An equilibrium vector* $\boldsymbol{\lambda}$ *has at least two elements that are strictly positive.*

Proof If $\lambda_i = 0$ for all i, the price does not reveal any information to any type. By Assumption A4, all types have an incentive to acquire information, a contradiction. If $\lambda_j > 0$, and $\lambda_i = 0$ for all $i \neq j$, the price fully reveals θ_j, so that there is no incentive for type j agents to engage in costly information acquisition in the first place. $\square$

We now specialize the discussion to symmetric economies, i.e. those for which the risk aversion coefficients r_i, and the shock variances $\sigma^2_{\theta_i}$, $\sigma^2_{\epsilon_i}$ and $\sigma^2_{\eta_i}$, are the same for all i. Let

$$\alpha_i := 1 - (e^{2rc_i} - 1)\frac{\sigma^2_\eta}{\sigma^2_\theta}.$$

Then, from (13) and Lemma 6.2, together with (9), we have:

Lemma 6.4 (Price informativeness vs cost) *For a symmetric economy, an equilibrium $\boldsymbol{\lambda}$ is characterized by*

$$\mathcal{V}_i = \frac{(\mathbf{R}_i^\top \boldsymbol{\lambda})^2}{\boldsymbol{\lambda}^\top \mathbf{R} \boldsymbol{\lambda}} \quad \textit{is} \quad \begin{cases} \leq & \alpha_i \quad \textit{for} \quad \lambda_i = 1 \\ = & \alpha_i \quad \textit{for} \quad \lambda_i \in (0,1) \\ \geq & \alpha_i \quad \textit{for} \quad \lambda_i = 0. \end{cases}$$

The parameter α_i lies in the interval (0,1), due to Assumption A4, and is a strictly decreasing function of the cost c_i. Whether agents of type i acquire information or not depends on the magnitude of c_i, or equivalently of α_i, relative to the informativeness of the price $\mathcal{V}_i$. The indifference condition is $\mathcal{V}_i = \alpha_i$. Without loss of generality, we order the types so that the c_i's are in ascending order ($c_1 \leq c_2 \leq \ldots \leq c_N$), or equivalently the α_i's are in descending order ($\alpha_1 \geq \alpha_2 \geq \ldots \geq \alpha_N$).

We say that an equilibrium $\boldsymbol{\lambda}$ is *stable* if $\partial\mathcal{V}_i/\partial\lambda_i \geq 0$ for all i satisfying $\mathcal{V}_i = \alpha_i$. If this condition is violated, i.e. if $\mathcal{V}_j = \alpha_j$ and $\partial\mathcal{V}_j/\partial\lambda_j < 0$ for some j, a small increase in λ_j will lead to prices being less informative about θ_j, making the increase in λ_j self-fulfilling; if $\lambda_j = 1$, we can apply the same logic for a small decrease in λ_j. No condition is needed if $\mathcal{V}_i \neq \alpha_i$ since this inequality continues to hold for a small change in λ_i. The stability property has a simple characterization:

Lemma 6.5 (Stability) *For a symmetric economy, an equilibrium $\boldsymbol{\lambda}$ is stable if and only if $\mathbf{R}_i^\top \boldsymbol{\lambda} > 0$ for all i such that $\mathcal{V}_i = \alpha_i$.*[12]

We will only be concerned with stable equilibria in this paper. We shall therefore drop the adjective "stable" in what follows, without any risk of confusion – henceforth, whenever we refer to an equilibrium, it is implied that it is stable.

For the remainder of the paper, we shall restrict ourselves to economies in the set $\mathcal{E}$. Recall that these are symmetric economies satisfying the additional assumption that $\rho = \rho_{ij}$ for $i \neq j$.

Our first equilibrium characterization result says that types with a positive mass of informed agents can be ranked by price informativeness: for a lower cost type, the proportion of informed agents is higher, which implies that price informativeness is higher as well.

[12] If $\mathcal{V}_i \neq \alpha_i$ for every i, the stability restriction is vacuous.

Proposition 6.6 (Ranking by price informativeness) *Consider an economy in $\mathcal{E}$. Suppose λ_i and λ_j are nonzero and not both equal to 1. Then the following statements are equivalent: (a) $c_i < c_j$, (b) $\lambda_i > \lambda_j$, and (c) $\mathcal{V}_i > \mathcal{V}_j$. The following statements are also equivalent: (a) $c_i = c_j$, (b) $\lambda_i = \lambda_j$, and (c) $\mathcal{V}_i = \mathcal{V}_j$.*

Notably missing from the proposition is a ranking of types for which no agent acquires information relative to types for which some agents do. Later, in Proposition 7.2, we show that it is possible to have $\lambda_i = 0$ and $\lambda_j = 1$ even though $c_i < c_j$. This counterintuitive situation arises because of complementarities in information gathering.

The incentive to acquire information depends on the value of the common correlation coefficient ρ. Our next result characterizes the values of ρ for which either all agents acquire information or none do. More precisely, in the latter case, all agents have the incentive to free ride on the information gathering of others, and hence there is no equilibrium.

Proposition 6.7 (Information acquisition: polar cases) *Consider an economy in $\mathcal{E}$. There is no equilibrium if $\rho \geq \sqrt{\alpha_2}$.*[13] *There is an equilibrium with $\lambda_i = 1$ for all i if*

$$\rho \leq \frac{\alpha_N N - 1}{N - 1}.$$

If $\rho \geq \sqrt{\alpha_2}$, then in fact $\rho \geq \sqrt{\alpha_i}$ for all $i \geq 2$ (since $\alpha_2 \geq \ldots \geq \alpha_N$). Due to the high correlation of types, agents of type i, $i \geq 2$, have an incentive to free ride on the information revealed through the price. But we know from Lemma 6.3 that in equilibrium there must be at least two types with a positive mass of informed agents. Hence there is no equilibrium. Notice that the cutoff value $\sqrt{\alpha_2}$, beyond which the correlation of types induces too much free riding, is decreasing in the cost of information c_2 and the noise-to-signal ratio $\sigma_\eta^2/\sigma_\theta^2$. Evidently, agents are more prone to free ride on others' information if the relative gain from acquiring their own information is small. The second part of Proposition 6.7 says that if ρ is sufficiently small, price informativeness is low enough to sustain an equilibrium in which all agents of all types acquire information.

The nonexistence result for $\rho \geq \sqrt{\alpha_2}$ shows how the Grossman-Stiglitz paradox can arise in an economy with correlated types, in both the asymmetric information and the differential information cases (the Grossman and Stiglitz (1980) setting with no noise in the aggregate supply can be seen as a limiting case of our model with asymmetrically informed types as $\rho \to 1$).

7 Complementarity and Multiplicity

Recall that a within-type complementarity, say for type i, arises when an increase in the mass of type i agents acquiring information results in prices becoming less

[13] It can be shown that this bound is tight, in the sense that if $\rho \in (\rho_{\min}, \sqrt{\alpha_2})$, there are parameter values such that an equilibrium exists.

informative for type i, inducing even more agents of type i to become informed. In this section we show how the presence of such a complementarity can lead to multiple equilibria. By Assumption A4, the vector of costs across types, $(c_i)_{i=1}^N$, lies in the set C, where C is the Cartesian product of the open intervals $(0, \bar{c}_i)$, $i = 1, \ldots, N$. Our results hold for economies in $\mathcal{E}$ corresponding to an open subset of C. We also compare price informativeness across equilibria. Given two equilibria A and B, we say that A *informationally dominates* B if price informativeness is strictly higher at A for every type.

Proposition 7.1 (Multiple equilibria) *Consider an economy in $\mathcal{E}$. Suppose $N \geq 3$, and $\rho < 0$. Then, for an open subset of C, there exists an equilibrium with $\lambda_i = 1$ for $i \neq N$ and $\lambda_N = 0$, and another equilibrium with $\lambda_i = 1$ for $i \neq N$ and $\lambda_N > \lambda^*$. The equilibrium with $\lambda_N > \lambda^*$ is informationally dominated by the equilibrium with $\lambda_N = 0$.*

Recall that $\lambda^* := -\rho(N-1)$. The two equilibria differ in the proportion of informed agents of the highest cost type, type N. The equilibrium in which $\lambda_N > \lambda^*$ exhibits "excessive" information gathering: price informativeness would be higher for everyone if no agent of type N acquires information, which is the case in the other equilibrium.

As we know from (12), ρ must be negative for a within-type complementarity to arise. In order to understand how complementarity drives multiplicity, it is instructive to take a detailed look at the 3-type case, depicted in Figure 1. We can ensure that $\lambda_1 = \lambda_2 = 1$ by choosing sufficiently low values of c_1 and c_2. Consider first the plot of $\mathcal{V}_3(\lambda_3)$ for $\rho = -0.2$. There is a unique equilibrium for $\alpha_3 \geq 0.1$: λ_3 lies in the interval $[0.8, 1]$ and is increasing in α_3 (recall that α_3 is a strictly decreasing function of c_3). If $\alpha_3 = 0.1$, we have $\mathcal{V}_3 = \alpha_3$ at $\lambda_3 = 0$. However, this does not qualify as an equilibrium by our definition since it is not stable. The open set for which Proposition 7.1 applies corresponds to $\alpha_3 < 0.1$. The case of $\alpha_3 = 0.05$ is shown in the figure. There are two equilibria,[14] indicated by points a and c (b is unstable).[15] Perversely, type 3 agents learn more from the price when none of them acquire information (point a). By Proposition 5.6, agents of types 1 and 2 also learn more from the price at a than at c.

Suppose we are initially at point a, with α_3 just below 0.1. Consider an increase in α_3. As α_3 crosses 0.1, there is a discontinuous jump in λ_3 from 0 to 0.8. A small decrease in the cost of information sets off a "frenzy" of information gathering for type 3 agents, with the proportion of informed agents jumping from 0 to 80%. As soon as α_3 exceeds 0.1, the cost of information is low enough to justify acquiring

[14]Equilibrium multiplicity is generated by the non-monotonicity of price informativeness as a function of the proportion of informed traders. This is in contrast to Ganguli and Yang (2009) where multidimensional information leads to two equilibrium price functions for any given allocation of private information. For one price function price informativeness is monotonically increasing in the proportion of informed traders, while for the other price function it is monotonically decreasing.

[15]Note that for $\alpha_3 < 0.1$, $\lambda_3 = 0$ is a stable equilibrium. If λ_3 increases by a small amount from 0, price informativeness $\mathcal{V}_3$, which is continuous in λ_3, still remains above α_3.

it. But as more agents acquire the information, prices become less informative, inducing even more agents to acquire information. The same discontinuous jump in information acquisition arises if α_3 is just below 0.1, and there is a small increase in the uncertainty facing uninformed agents, as measured by the σ_θ^2. This has the effect of reducing $|\rho|$, shifting the curve downwards.[16]

Now consider the plot for $\rho = -0.3$. Again start with $\lambda_3 = 0$ with $\alpha_3 < 0.257$. As α_3 increases beyond this cutoff value, the equilibrium jumps to $\lambda_3 = 1$. A small decrease in the cost of gathering information (or a small increase in σ_θ^2), leads to *all* agents of type 3 acquiring information. In this case, there is also a discontinuous downward jump in price informativeness for type 3 agents. Of course, at $\lambda_3 = 1$, the information revealed by the price is only relevant in the differential information case (in the asymmetric information case, the agents' private signals already tell them what θ_3 is). With differential information, it is indeed possible that each type 3 agent learns less about his valuation when all type 3 agents acquire information, even when he combines his private signal with the information contained in the price.

Next, we show that, under a tighter condition on ρ, equilibrium multiplicity can be much more pronounced than is suggested by Proposition 7.1. For this result, we focus on equilibria that are "extremal" in the sense that λ_i is either 0 or 1 for all i. We say that type i is uninformed if $\lambda_i = 0$ and informed if $\lambda_i = 1$.

Proposition 7.2 (Multiple equilibria II) *Consider an economy in $\mathcal{E}$. Suppose $N \geq 3$, and $\rho < -N^{-1}$. Then, for an open subset of C, and for all integers m satisfying $N/2 < m \leq N$, there exists an equilibrium in which m types are informed and the remaining $(N - m)$ types are uninformed. At any such equilibrium, price informativeness for the uninformed types is strictly higher than price informativeness for the informed types. Furthermore, the equilibrium in which all types are informed is informationally dominated by any equilibrium in which some types are uninformed.*

The proposition says that, in the presence of a sufficiently strong complementarity, there is a plethora of equilibria. The equilibrium in which all agents of all types acquire information is actually the worst in terms of price informativeness: prices would be more informative for everyone if one or more types switch to not acquiring any information. The allocation of types to the informed and uninformed groups is arbitrary. Thus there are equilibria in which the types that acquire information have a higher cost than the types that do not.

The results of this section rely on a negative correlation between agent valuations. We focus on the more tractable case where all pairwise correlations are the same, but this is not an essential assumption. Negative correlations can arise due to hedging motives, which can easily be incorporated in our model as in Examples 4.2 and 4.3. The complementarity result in Goldstein et al. (2014), in a segmented markets setting, has a similar flavor: it is driven by a sufficiently strong hedging motive that makes a subset of informed investors trade in the opposite direction to others

[16] The magnitude of ρ can also be affected by a public signal about $\boldsymbol{\theta}$, or by market size as in Rostek and Weretka (2012).

who only have a speculative motive. Another plausible scenario that can generate negatively correlated valuations, described by Barlevy and Veronesi (2008), is one where some agents have access to a private technology the returns on which are higher in good times, when the asset fundamental is also high. These agents sell the asset in order to free up resources for other projects.

8 Welfare

The literature on market efficiency typically takes higher price informativeness to be a desirable goal in itself. One rationale is that more informative prices lead to better real investment decisions. However, most REE models (including the present one) do not explicitly take this into account. Absent such a feedback to the real sector, the question remains as to how price informativeness relates to the welfare of agents within the model. Most models in the literature are not equipped to answer this question, given their reliance on exogenous noise trade. Our framework, on the other hand, is free from these difficulties and is well-suited for welfare analysis.[17]

We consider the effect of information acquisition by agents of a given type on their own welfare as well as on the welfare of agents of other types. The effect on their own welfare clearly depends on their cost of gathering information. In addition, the welfare of all types depends on the informativeness of prices. The effect of price informativeness on welfare is not unambiguous, however. On the one hand, higher price informativeness for agents of type i (a higher $\mathcal{V}_i$) leads to better portfolio decisions for these agents. On the other hand, it brings prices closer to their valuation v_i, thus reducing the gains from trade that they can exploit.[18] As we shall see, the welfare of type i agents is increasing in $\sigma^2_{v_i-p}$. While $\mathcal{V}_i$ and $\sigma^2_{v_i-p}$ are not bound by a tight functional relationship, these two variables tend to be inversely related. We will show that if agents have sufficiently precise private information, so that learning from prices is relatively unimportant for them, they prefer prices to be less revealing. Conversely, if there is a lot of noise in their signals, they prefer prices to be more informative. Incentives to gather information may be misaligned with their own objectives, however: agents may choose to gather information even though they are worse off in the ensuing equilibrium.

In order to simplify matters, we consider an economy in $\mathcal{E}$ in which there is a type j such that $\lambda_i = 1$ for $i \neq j$, and compare welfare for λ_j equal to 0 or 1. It is convenient to write all variables as functions of λ_j (where λ_j is either 0 or 1). From Proposition 5.6, $\mathcal{V}_i(1) < \mathcal{V}_i(0)$ for $i \neq j$. By acquiring information, type j agents exert a negative informational externality on the other types. For type j

[17]We carry out a conventional welfare analysis under the assumption that agents' objective functions are a faithful representation of their welfare. This may not be the case if we give a behavioral interpretation to their heterogeneous valuations.

[18]We can see from the optimal portfolio of type i, given by (2), that these agents tend to go long when $v_i > p$ and short when $v_i < p$; there are gains from trade for type i only to the extent that the price does not perfectly reflect their valuation.

itself, Proposition 5.6 tells us that price informativeness may go up or down. If $\rho \in (\rho_{\min}, 0.5\rho_{\min})$, type j agents learn more from the price if none of them gather information (as in the case of $\rho = -0.3$ in Figure 1). But apart from this interval, which shrinks as N increases, $\mathcal{V}_j(1) > \mathcal{V}_j(0)$.

For an economy in $\mathcal{E}$, all types are either asymmetrically informed or differentially informed. We start with the asymmetric information case ($\sigma^2_{\epsilon_i} = 0$, for all i). From Lemma 6.1:

$$\mathcal{U}_i^I = e^{-2rc_i} \cdot \frac{\sigma^2_{v_i - p}}{\sigma^2_\eta}, \tag{14}$$

$$\mathcal{U}_i^U = \left[1 + \frac{\sigma^2_\theta}{\sigma^2_\eta}(1 - \mathcal{V}_i)\right]^{-1} \frac{\sigma^2_{v_i - p}}{\sigma^2_\eta}. \tag{15}$$

The utility of agents of type i, $i \neq j$, is given by (14), since they are informed regardless of the value of λ_j. Price informativeness is actually irrelevant for these agents; they observe the true value of θ_i and hence have no need to make inferences from prices. What matters to them is how far the price is from their valuation, as measured by $\sigma^2_{v_i - p}$. The utility of agents of type j is given by (15) when $\lambda_j = 0$, and by (14) when $\lambda_j = 1$. Like the other types, their utility is higher if $\sigma^2_{v_j - p}$ increases. Unlike the other types, they also care about price informativeness when they are uninformed ($\lambda_j = 0$), but whether price informativeness is higher or lower compared to the case where $\lambda_j = 1$ is irrelevant. The following result summarizes the effect of the value of λ_j on $\sigma^2_{v_i - p}$:

Lemma 8.1 *Consider an economy in $\mathcal{E}$ in which all types are asymmetrically informed. Suppose there is a type j such that $\lambda_i = 1$ for all $i \neq j$. Then $\sigma^2_{v_i - p}(1) > \sigma^2_{v_i - p}(0)$ for $i \neq j$, and $\sigma^2_{v_j - p}(1) < \sigma^2_{v_j - p}(0)$.*

The price function places a nonzero weight on θ_j when $\lambda_j = 1$, and consequently the price is closer to v_j and further away from v_i, $i \neq j$. Lemma 8.1 and equation (14) taken together give us the following result:

Proposition 8.2 (Welfare: asymmetric information I) *Consider an economy in $\mathcal{E}$ in which all types are asymmetrically informed. Suppose there is a type j such that $\lambda_i = 1$ for all $i \neq j$. Then $\mathcal{U}_i^I(1) > \mathcal{U}_i^I(0)$ for $i \neq j$.*

Thus information acquisition by type j results in a positive welfare externality on all the other types. As we saw earlier (Proposition 5.6), the informational externality is negative, but of course this is irrelevant for the welfare of these agents as they have nothing to learn from the price.

Agents of type j may be better or worse off when $\lambda_j = 1$ depending on their cost of gathering information. It turns out that this cost may be low enough to give each type j agent an incentive to acquire information, even though they are worse off in the resulting equilibrium. Moreover, since j can be an arbitrary type, we have an equilibrium in which all agents of all types gather information, but agents of any given type would be better off if they could coordinate on remaining uninformed.

Proposition 8.3 (Welfare: asymmetric information II) *Consider an economy in $\mathcal{E}$ in which all types are asymmetrically informed. For an open subset of C, there exists an equilibrium in which (a) $\lambda_i = 1$ for all i; and (b) for any type j, $\mathcal{U}_j^I(1) < \mathcal{U}_j^U(0)$.*

Condition (b) in the proposition says that agents of type j are better off at $\lambda_j = 0$ than at $\lambda_j = 1$, given that $\lambda_i = 1$ for all $i \neq j$.

To summarize our results for the asymmetric information case, information acquisition by type j agents lowers price informativeness for all other types but makes these types better off. Type j agents themselves may be worse off even if the cost of information is low enough to give them an incentive to acquire it.

We now turn to the case where all types are differentially informed ($\sigma^2_{\eta_i} = 0$, for all i). From Lemma 6.1, ex ante utilities are:

$$\mathcal{U}_i^I = e^{-2rc_i}\left[\frac{\sigma_\theta^2}{\sigma_\epsilon^2} + (1-\mathcal{V}_i)^{-1}\right]\frac{\sigma^2_{v_i-p}}{\sigma^2_\theta}, \tag{16}$$

$$\mathcal{U}_i^U = (1-\mathcal{V}_i)^{-1}\cdot\frac{\sigma^2_{v_i-p}}{\sigma^2_\theta}. \tag{17}$$

Comparing (14) and (16), we see that price informativeness has a direct impact on the welfare of informed agents in the differential information case. This is because private information is noisy and hence the price provides additional information. Given that the price is less informative for types $i \neq j$ when $\lambda_j = 1$, one might conjecture that they are better off when $\lambda_j = 1$ (as in the asymmetric information case) if the signal-to-noise ratio $\sigma^2_\theta/\sigma^2_\epsilon$ is high, so that there is not much to learn from the price. This is indeed the case:

Proposition 8.4 (Welfare: differential information I) *Consider an economy in $\mathcal{E}$ in which all types are differentially informed. Suppose there is a type j such that $\lambda_i = 1$ for all $i \neq j$. Then there is a $\zeta^* > 0$, which depends only on ρ and N, such that*

i. $\mathcal{U}_i^I(1) > \mathcal{U}_i^I(0)$ *for all $i \neq j$ if and only if $\sigma^2_\theta/\sigma^2_\epsilon > \zeta^*$; and*

ii. $\mathcal{U}_i^I(1) < \mathcal{U}_i^I(0)$ *for all $i \neq j$ if and only if $\sigma^2_\theta/\sigma^2_\epsilon < \zeta^*$.*

Proposition 8.3 extends verbatim to the differential information case:

Proposition 8.5 (Welfare: differential information II) *Consider an economy in $\mathcal{E}$ in which all types are differentially informed. For an open subset of C, there exists an equilibrium in which (a) $\lambda_i = 1$ for all i; and (b) for any type j, $\mathcal{U}_j^I(1) < \mathcal{U}_j^U(0)$.*

In this equilibrium all agents gather information, but any given type would be better off if all agents of that type were to stay uninformed. Moreover, from Proposition 8.4, if the signal-to-noise ratio is low, *all* types are better off if agents of one type stay uninformed. Formally,

Corollary 8.6 (Excessive information gathering) *Consider an economy in $\mathcal{E}$ in which all types are differentially informed. Suppose $\sigma_\theta^2/\sigma_\epsilon^2 < \zeta^*$. Then, for an open subset of C, there exists an equilibrium in which (a) $\lambda_i = 1$ for all i; and (b) for any type j, $\mathcal{U}_i^I\big|_{\lambda_j=1} < \mathcal{U}_i^U\big|_{\lambda_j=0}$ for all i.*

In Section 7, we had seen that information gathering may be excessive in the sense of reducing price informativeness for all agents. Corollary 8.6 tells us that information gathering may also be excessive in the sense of reducing everyone's welfare. Moreover, if $\rho > 0.5\,\rho_{\min}$, we know from Proposition 5.6 that $\mathcal{V}_j(0) < \mathcal{V}_j(1)$, so that type j agents have an even greater incentive to acquire information when type j is uninformed than when it is informed. Hence the Pareto-improving "collusion" outcome of $\lambda_j = 0$, and $\lambda_i = 1$, for $i \neq j$, cannot be sustained as an equilibrium. In this sense, if $\rho > 0.5\,\rho_{\min}$, the suboptimality of the equilibrium in Corollary 8.6 is of the Prisoner's Dilemma type.

Note that while our results on multiplicity of equilibria in Section 7 depend on a within-type complementarity, and hence require that valuations be negatively correlated across types, our welfare results do not require any such assumption.

9 Concluding Remarks

We provide a framework for studying competitive rational expectations equilibria that encompasses the classical REE models. It admits a simple characterization of the (unique) linear equilibrium price function. Due to the heterogeneity in the private valuations of agents, the price function is partially revealing. There are equilibria in which information acquisition by some agents reduces the informativeness of prices for all. Using price informativeness as a social objective can be misleading, however. More informative prices can reduce welfare. Agents may have an incentive to acquire information even though they would be better off if they could coordinate on staying uninformed.

Appendix

Proof of Lemma 3.1 The assumption that $\sigma^2_{\theta_i|p} > 0$ ensures that the covariance matrix of (s_{in}, p) is nonsingular even if $\sigma^2_{\epsilon_i} = 0$. The conditional expectations of v_i, given $\{s_{in}, p\}$ and p, respectively, are:

$$\begin{aligned}
E(v_i|s_{in},p) &= [\sigma^2_{\theta_i} \quad \sigma_{\theta_i,p}] \begin{bmatrix} \sigma^2_{\theta_i}+\sigma^2_{\epsilon_i} & \sigma_{\theta_i,p} \\ \sigma_{\theta_i,p} & \sigma^2_p \end{bmatrix}^{-1} \begin{bmatrix} s_{in} \\ p \end{bmatrix} \\
&= \frac{1}{(\sigma^2_{\theta_i}+\sigma^2_{\epsilon_i})\sigma^2_p - \sigma^2_{\theta_i,p}} \cdot [\sigma^2_{\theta_i} \quad \sigma_{\theta_i,p}] \begin{bmatrix} \sigma^2_p & -\sigma_{\theta_i,p} \\ -\sigma_{\theta_i,p} & \sigma^2_{\theta_i}+\sigma^2_{\epsilon_i} \end{bmatrix} \begin{bmatrix} s_{in} \\ p \end{bmatrix} \\
&= \frac{1}{(\sigma^2_{\theta_i}+\sigma^2_{\epsilon_i})\sigma^2_p - \sigma^2_{\theta_i,p}} \cdot \left[(\sigma^2_{\theta_i}\sigma^2_p - \sigma^2_{\theta_i,p})s_{in} + \sigma_{\theta_i,p}\sigma^2_{\epsilon_i}\, p\right] \\
&= \frac{1}{\sigma^2_{\theta_i}+\sigma^2_{\epsilon_i} - \beta_i\sigma_{\theta_i,p}} \cdot \left[(\sigma^2_{\theta_i} - \beta_i\sigma_{\theta_i,p})s_{in} + \beta_i\sigma^2_{\epsilon_i} p\right] \\
&= \frac{1}{\sigma^2_{\theta_i|p}+\sigma^2_{\epsilon_i}} \cdot \left(\sigma^2_{\theta_i|p}s_{in} + \beta_i\sigma^2_{\epsilon_i}p\right),
\end{aligned}$$

and

$$E(v_i|p) = \frac{\sigma_{\theta_i,p}}{\sigma^2_p}p = \beta_i p.$$

The conditional variances are:

$$\begin{aligned}
\mathrm{Var}(v_i|s_{in},p) &= \sigma^2_{v_i} - [\sigma^2_{\theta_i} \quad \sigma_{\theta_i,p}] \begin{bmatrix} \sigma^2_{\theta_i}+\sigma^2_{\epsilon_i} & \sigma_{\theta_i,p} \\ \sigma_{\theta_i,p} & \sigma^2_p \end{bmatrix}^{-1} \begin{bmatrix} \sigma^2_{\theta_i} \\ \sigma_{\theta_i,p} \end{bmatrix} \\
&= \sigma^2_{v_i} - \frac{1}{(\sigma^2_{\theta_i}+\sigma^2_{\epsilon_i})\sigma^2_p - \sigma^2_{\theta_i,p}} \cdot [\sigma^2_{\theta_i} \quad \sigma_{\theta_i,p}] \begin{bmatrix} \sigma^2_p & -\sigma_{\theta_i,p} \\ -\sigma_{\theta_i,p} & \sigma^2_{\theta_i}+\sigma^2_{\epsilon_i} \end{bmatrix} \begin{bmatrix} \sigma^2_{\theta_i} \\ \sigma_{\theta_i,p} \end{bmatrix} \\
&= \sigma^2_{v_i} - \frac{1}{(\sigma^2_{\theta_i}+\sigma^2_{\epsilon_i})\sigma^2_p - \sigma^2_{\theta_i,p}} \cdot \left[(\sigma^2_{\theta_i}\sigma^2_p - \sigma^2_{\theta_i,p})\sigma^2_{\theta_i} + \sigma^2_{\theta_i,p}\sigma^2_{\epsilon_i}\right] \\
&= \sigma^2_{v_i} - \frac{1}{\sigma^2_{\theta_i}+\sigma^2_{\epsilon_i} - \beta_i\sigma_{\theta_i,p}} \cdot \left[(\sigma^2_{\theta_i} - \beta_i\sigma_{\theta_i,p})\sigma^2_{\theta_i} + \beta_i\sigma_{\theta_i,p}\sigma^2_{\epsilon_i}\right] \\
&= \sigma^2_{\eta_i} + \frac{(\sigma^2_{\theta_i} - \beta_i\sigma_{\theta_i,p})\sigma^2_{\epsilon_i}}{\sigma^2_{\theta_i}+\sigma^2_{\epsilon_i} - \beta_i\sigma_{\theta_i,p}} \\
&= \sigma^2_{\eta_i} + \frac{\sigma^2_{\theta_i|p}\sigma^2_{\epsilon_i}}{\sigma^2_{\theta_i|p}+\sigma^2_{\epsilon_i}},
\end{aligned}$$

and

$$\mathrm{Var}(v_i|p) = \sigma^2_{\theta_i|p} + \sigma^2_{\eta_i}.$$

Plugging these conditional moments into (2), we get the desired result. $\square$

Proof of Proposition 3.2 We proceed under the provisional assumption that $\sigma^2_{\theta_i|p} > 0$ for all i, i.e. the price function does not (fully) reveal θ_i for any i. We will verify later that this assumption does in fact hold. From (4),

$$q_i = \gamma_i \theta_i - k_i p, \tag{18}$$

where

$$\gamma_i := \frac{\lambda_i}{r_i} \cdot \frac{\sigma^2_{\theta_i|p}}{(\sigma^2_{\theta_i|p} + \sigma^2_{\epsilon_i})\sigma^2_{\eta_i} + \sigma^2_{\theta_i|p}\sigma^2_{\epsilon_i}}, \tag{19}$$

and

$$k_i := \frac{\lambda_i}{r_i} \cdot \frac{\sigma^2_{\theta_i|p} + (1-\beta_i)\sigma^2_{\epsilon_i}}{(\sigma^2_{\theta_i|p} + \sigma^2_{\epsilon_i})\sigma^2_{\eta_i} + \sigma^2_{\theta_i|p}\sigma^2_{\epsilon_i}} + \frac{1-\lambda_i}{r_i} \cdot \frac{1-\beta_i}{\sigma^2_{\theta_i|p} + \sigma^2_{\eta_i}}. \tag{20}$$

Using the market-clearing condition, $\sum_i q_i = 0$, we obtain:

$$\left(\sum_i k_i\right) p = \sum_i \gamma_i \theta_i. \tag{21}$$

Suppose first that $\sum_i k_i = 0$. Then, $\sum_i \gamma_i \theta_i = 0$. Due to the positive definiteness of the covariance matrix of $\boldsymbol{\theta}$, we must have $\gamma_i = 0$ for all i. Since $\sigma^2_{\theta_i|p} > 0$ by assumption, it follows from (19) that $\lambda_i = 0$ for all i. But this contradicts Assumption A1, which says that $\lambda_i > 0$ for at least two types. We conclude that $\sum_i k_i \neq 0$.

We can now solve (21) for the price function p, and we see that it is indeed given by (5), with $k = (\sum_i k_i)^{-1}$. From (19), it is immediate that $\gamma_i = \lambda_i (r_i \sigma^2_{\eta_i})^{-1}$ if $\sigma^2_{\epsilon_i} = 0$ (type i is asymmetrically informed), while $\gamma_i = \lambda_i (r_i \sigma^2_{\epsilon_i})^{-1}$ if $\sigma^2_{\eta_i} = 0$ (type i is differentially informed).

Finally, we verify that a price function of the form (1) does not reveal θ_i for any i ($\sigma^2_{\theta_i|p} > 0$ for all i). Suppose not, say p reveals θ_j. Then, since the covariance matrix of $\boldsymbol{\theta}$ is positive definite, so that θ_j is not perfectly correlated with any linear combination of the remaining θ_i's, we must have $p = a_j \theta_j, a_j \neq 0$. Since p does not reveal θ_i for $i \neq j$, equations (18)–(20) still hold for $i \neq j$. For type j, Lemma 3.1 does not apply, but q_j can be calculated directly from (2). Assuming for the moment that $\sigma^2_{\eta_j} > 0$, $q_j = (\theta_j - p)(r_j \sigma^2_{\eta_j})^{-1}$. Thus q_j is given by (18), with $\gamma_j = k_j = (r_j \sigma^2_{\eta_j})^{-1}$. From (21), we see that $\sum_i k_i \neq 0$, for otherwise $\gamma_i = 0$ for all i, a contradiction. Hence the price function is given by (5). But since $p = a_j \theta_j$, $\gamma_i = 0$ for all $i \neq j$, which in turn implies that $\lambda_i = 0$ for all $i \neq j$. This contradicts Assumption A1. For the case where $\sigma^2_{\eta_j} = 0$, we must have $p = \theta_j$, and the optimal trade of a type j agent is indeterminate. However, the aggregate trade of type j is pinned down by market clearing, i.e. $q_j = -\sum_{i \neq j} q_i = \sum_{i \neq j}(k_i p - \gamma_i \theta_i) = \sum_{i \neq j}(k_i \theta_j - \gamma_i \theta_i)$. Invoking Assumption A3, q_j is measurable with respect to the information of type j agents. Hence, we must have $\gamma_i = 0$ for all $i \neq j$, leading to the same contradiction that we

arrived at above. $\square$

Proof of Lemma 5.1 We have

$$\begin{aligned}
\mathcal{V}_i^I &= \frac{\left(\sigma_{\theta_i}^2 - \frac{\sigma_{\theta_i}^4}{\sigma_{\theta_i}^2+\sigma_{\epsilon_i}^2}\right) - \frac{(\sigma_{\theta_i}^2-\beta_i\sigma_{\theta_i,p})\sigma_{\epsilon_i}^2}{\sigma_{\theta_i}^2+\sigma_{\epsilon_i}^2-\beta_i\sigma_{\theta_i,p}}}{\sigma_{\theta_i}^2 - \frac{\sigma_{\theta_i}^4}{\sigma_{\theta_i}^2+\sigma_{\epsilon_i}^2}} \\
&= \frac{\beta_i\sigma_{\theta_i,p}\sigma_{\epsilon_i}^2}{\sigma_{\theta_i}^2(\sigma_{\theta_i}^2+\sigma_{\epsilon_i}^2-\beta_i\sigma_{\theta_i,p})} \\
&= \frac{\mathcal{V}_i\sigma_{\epsilon_i}^2}{(1-\mathcal{V}_i)\sigma_{\theta_i}^2+\sigma_{\epsilon_i}^2}.
\end{aligned}$$

The result follows. $\square$

Proof of Proposition 5.2 The expression for $\mathcal{V}_i$ is immediate from (8). Differentiating this expression, we obtain

$$\frac{\partial\mathcal{V}_i}{\partial\lambda_j} = \frac{2\rho_{ij}(\boldsymbol{\gamma}^\top\mathbf{R}\boldsymbol{\gamma})(\mathbf{R}_i^\top\boldsymbol{\gamma}) - 2(\mathbf{R}_i^\top\boldsymbol{\gamma})^2(\mathbf{R}_j^\top\boldsymbol{\gamma})}{(\boldsymbol{\gamma}^\top\mathbf{R}\boldsymbol{\gamma})^2}\cdot\frac{\partial\gamma_j}{\partial\lambda_j}.$$

Therefore,

$$\frac{\partial\mathcal{V}_i}{\partial\lambda_j} \propto \rho_{ij}(\mathbf{R}_i^\top\boldsymbol{\gamma}) - \mathcal{V}_i(\mathbf{R}_j^\top\boldsymbol{\gamma}).$$

In particular,

$$\frac{\partial\mathcal{V}_i}{\partial\lambda_i} \propto (1-\mathcal{V}_i)(\mathbf{R}_i^\top\boldsymbol{\gamma}) \propto \mathbf{R}_i^\top\boldsymbol{\gamma},$$

where we have used the fact that $\mathcal{V}_i \in [0,1)$.

Since $\mathcal{V}_i$ is homogeneous of degree zero in $\boldsymbol{\gamma}$, by Euler's theorem,

$$\sum_j \gamma_j\frac{\partial\mathcal{V}_i}{\partial\gamma_j} = 0.$$

If $\lambda_i > 0$ and $\partial\mathcal{V}_i/\partial\lambda_i > 0$, then $\gamma_i(\partial\mathcal{V}_i/\partial\gamma_i) > 0$. It follows that $\gamma_j(\partial\mathcal{V}_i/\partial\gamma_j) < 0$ for some $j \neq i$, which in turn implies that $\partial\mathcal{V}_i/\partial\lambda_j < 0$. $\square$

Proof of Lemma 5.4 Let $\mathbf{1} := (1,\ldots,1)^\top$ and $\mathbf{v}_j := (-1,0,\ldots,0,1,0,\ldots,0)^\top$, where the 1 is in the j'th place. Then $\mathbf{R1} = [1+\rho(N-1)]\mathbf{1}$, and $\mathbf{R}\mathbf{v}_j = (1-\rho)\mathbf{v}_j$, for $j = 2,3,\ldots,N$. Thus the eigenvalues of $\mathbf{R}$ are $[1+\rho(N-1)]$ and $(1-\rho)$, the latter with multiplicity $N-1$. Since $\mathbf{R}$ is a symmetric matrix, it is positive definite if and only if all its eigenvalues are positive, i.e. if and only if $1+\rho(N-1) > 0$. $\square$

Proof of Proposition 5.6 Since $\lambda_i = 1$ for $i \neq j$, we have (from (11)):

$$\mathcal{V}_i = \frac{\big[(1-\rho) + \rho(N-1+\lambda_j)\big]^2}{(1-\rho)(N-1+\lambda_j^2) + \rho(N-1+\lambda_j)^2}, \qquad i \neq j,$$
$$\mathcal{V}_j = \frac{\big[(1-\rho)\lambda_j + \rho(N-1+\lambda_j)\big]^2}{(1-\rho)(N-1+\lambda_j^2) + \rho(N-1+\lambda_j)^2}.$$

Using the fact that $\rho > \rho_{\min}$, we can directly verify statement (i) of the proposition. Statement (iii) is straightforward to check as well, and statement (ii) is immediate from (12). $\Box$

Proof of Lemma 6.1 With the understanding that the cost c_i is paid by agent in only if he is informed, his ex ante expected utility is

$$\begin{aligned} E[-\exp(r_i c_i - r_i W_{in})] &:= -e^{r_i c_i} E\big[E(\exp(-r_i W_{in})|\mathcal{I}_{in})\big] \\ &= -e^{r_i c_i} E\big[\exp(-r_i \mathcal{E}_{in})\big], \end{aligned} \tag{22}$$

where

$$\begin{aligned} \mathcal{E}_{in} &:= E(W_{in}|\mathcal{I}_{in}) - \frac{r_i}{2}\mathrm{Var}(W_{in}|\mathcal{I}_{in}) \\ &= \big[E(v_i|\mathcal{I}_{in}) - p\big] q_{in} - \frac{r_i}{2} q_{in}^2 \mathrm{Var}(v_i|\mathcal{I}_{in}). \end{aligned}$$

From (2), $E(v_i|\mathcal{I}_{in}) - p = r_i q_{in} \mathrm{Var}(v_i|\mathcal{I}_{in})$. Therefore,

$$\mathcal{E}_{in} = \frac{r_i}{2} q_{in}^2 \mathrm{Var}(v_i|\mathcal{I}_{in}).$$

Substituting for q_{in} from Lemma 3.1,

$$-r_i \mathcal{E}_{in}^I = -\frac{1}{2} \cdot \frac{\Big[\sigma_{\theta_i|p}^2 s_{in} - \big[\sigma_{\theta_i|p}^2 + (1-\beta_i)\sigma_{\epsilon_i}^2\big] p\Big]^2}{(\sigma_{\theta_i|p}^2 + \sigma_{\epsilon_i}^2)\big[(\sigma_{\theta_i|p}^2 + \sigma_{\epsilon_i}^2)\sigma_{\eta_i}^2 + \sigma_{\theta_i|p}^2 \sigma_{\epsilon_i}^2\big]},$$
$$-r_i \mathcal{E}_{in}^U = -\frac{1}{2} \cdot \frac{(1-\beta_i)^2}{\sigma_{\theta_i|p}^2 + \sigma_{\eta_i}^2} p^2.$$

In order to evaluate (22), we use the fact that if $y \sim N(0, \sigma^2)$, then $E[e^{-\frac{1}{2}y^2}] = (1+\sigma^2)^{-\frac{1}{2}}$. We obtain

$$\mathcal{U}_{in}^I = e^{-2r_i c_i} \left[1 + \frac{(\sigma_{\theta_i}^2 + \sigma_{\epsilon_i}^2)\sigma_{\theta_i|p}^4 + \big[\sigma_{\theta_i|p}^2 + (1-\beta_i)\sigma_{\epsilon_i}^2\big]^2 \sigma_p^2 - 2\big[\sigma_{\theta_i|p}^2 + (1-\beta_i)\sigma_{\epsilon_i}^2\big]\sigma_{\theta_i|p}^2 \sigma_{\theta_i,p}}{(\sigma_{\theta_i|p}^2 + \sigma_{\epsilon_i}^2)\big[(\sigma_{\theta_i|p}^2 + \sigma_{\epsilon_i}^2)\sigma_{\eta_i}^2 + \sigma_{\theta_i|p}^2 \sigma_{\epsilon_i}^2\big]}\right],$$

which, after some algebraic manipulation, gives us the expression for $\mathcal{U}_{in}^I$ in the statement of the lemma. Also,

$$\mathcal{U}_{in}^U = 1 + \frac{(1-\beta_i)^2 \sigma_p^2}{\sigma_{\theta_i|p}^2 + \sigma_{\eta_i}^2},$$

which yields the desired expression for $\mathcal{U}_{in}^U$. $\square$

Proof of Lemma 6.2 From Lemma 6.1:

$$\frac{\mathcal{U}_i^I}{\mathcal{U}_i^U} = e^{-2r_i c_i} \cdot \frac{(\sigma_{\theta_i|p}^2 + \sigma_{\epsilon_i}^2)(\sigma_{\theta_i|p}^2 + \sigma_{\eta_i}^2)}{(\sigma_{\theta_i|p}^2 + \sigma_{\epsilon_i}^2)\sigma_{\eta_i}^2 + \sigma_{\theta_i|p}^2 \sigma_{\epsilon_i}^2}$$
$$= e^{-2r_i c_i}\left[1 + \frac{\sigma_{\theta_i|p}^4}{\sigma_{\theta_i|p}^2(\sigma_{\epsilon_i}^2 + \sigma_{\eta_i}^2) + \sigma_{\epsilon_i}^2\sigma_{\eta_i}^2}\right].$$

If type i is asymmetrically informed ($\sigma_{\epsilon_i}^2 = 0$), we get

$$\frac{\mathcal{U}_i^I}{\mathcal{U}_i^U} = e^{-2r_i c_i}\left[1 + \frac{\sigma_{\theta_i|p}^2}{\sigma_{\eta_i}^2}\right].$$

Substituting for $\sigma_{\theta_i|p}^2$, using (7), gives us the desired expression for the utility ratio. In the differential information case, we get the same expression with $\sigma_{\eta_i}^2$ replaced by $\sigma_{\epsilon_i}^2$. $\square$

Proof of Lemma 6.5 If $\mathcal{V}_i = \alpha_i$, then in particular $\mathcal{V}_i > 0$, so that $\mathbf{R}_i^\top \boldsymbol{\lambda} \neq 0$ (from (9)). Moreover, from (10), $\partial \mathcal{V}_i / \partial \lambda_i \propto \mathbf{R}_i^\top \boldsymbol{\lambda}$. The result follows from the definition of stability. $\square$

Proof of Proposition 6.6 Price informativeness for type i is given by (11). By Lemma 6.5,

$$(1-\rho)\lambda_\ell + \rho \sum_k \lambda_k > 0, \tag{23}$$

for all ℓ satisfying $\mathcal{V}_\ell = \alpha_\ell$. In particular, this is the case if $\lambda_\ell \in (0,1)$. Moreover, if $\lambda_\ell = 1$, the LHS of (23) is equal to $1 + \rho\sum_{k\neq\ell}\lambda_k$, which is positive since $\rho > \rho_{\min}$. Thus (23) applies as long as $\lambda_\ell \in (0,1]$, and in particular for $\ell = i, j$ in the statement of the proposition. It follows that $\lambda_i > \lambda_j$ if and only if $\mathcal{V}_i > \mathcal{V}_j$, and $\lambda_i = \lambda_j$ if and only if $\mathcal{V}_i = \mathcal{V}_j$. It remains to show that $c_i < c_j$ if and only if $\mathcal{V}_i > \mathcal{V}_j$ (since we can reverse the indices i and j, this in turn implies that $c_i = c_j$ if and only if $\mathcal{V}_i = \mathcal{V}_j$).

Suppose first that $\mathcal{V}_i > \mathcal{V}_j$. Then $\lambda_i > \lambda_j$ and hence $\lambda_j \in (0,1)$. Furthermore, $\alpha_i \geq \mathcal{V}_i > \mathcal{V}_j = \alpha_j$, so that $c_i < c_j$. Next suppose that $c_i < c_j$, or equivalently $\alpha_i > \alpha_j$. If $\mathcal{V}_i \leq \mathcal{V}_j$, we have $\lambda_i \leq \lambda_j$, implying that $\lambda_i \in (0,1)$, and $\alpha_i > \alpha_j \geq \mathcal{V}_j \geq \mathcal{V}_i = \alpha_i$, a contradiction. Therefore $\mathcal{V}_i > \mathcal{V}_j$. $\square$

Proof of Proposition 6.7 Let

$$\hat{\lambda}_i := \frac{\lambda_i}{\sum_{k\neq i}\lambda_k}, \qquad \text{and} \qquad \delta_i := \frac{\sum_{k\neq i}\lambda_k^2}{\left(\sum_{k\neq i}\lambda_k\right)^2},$$

which are well-defined for any i since $\sum_{k\neq i} \lambda_k > 0$ by Lemma 6.3. Using (11), we can write $\mathcal{V}_i$ as follows:

$$\begin{aligned}
\mathcal{V}_i &= \frac{\left[\lambda_i + \rho \sum_{k\neq i} \lambda_k\right]^2}{(1-\rho)\left[\lambda_i^2 + \sum_{k\neq i} \lambda_k^2\right] + \rho\left[\lambda_i + \sum_{k\neq i} \lambda_k\right]^2} \\
&= \frac{(\hat{\lambda}_i + \rho)^2}{(1-\rho)(\hat{\lambda}_i^2 + \delta_i) + \rho(\hat{\lambda}_i + 1)^2} \\
&= \frac{(\hat{\lambda}_i + \rho)^2}{(\hat{\lambda}_i + \rho)^2 + (1-\rho)(\delta_i + \rho)}. \qquad (24)
\end{aligned}$$

Notice that $\mathcal{V}_i$ is strictly decreasing in δ_i and, if $\rho \geq 0$, strictly increasing in $\hat{\lambda}_i$. Hence, provided $\rho \geq 0$, a lower bound for $\mathcal{V}_i$ is obtained from (24) by setting $\hat{\lambda}_i$ equal to its lowest possible value, which is 0, and δ_i equal to its highest possible value, which is 1 ($\delta_i = 1$ if and only if there is only one type $k, k \neq i$, for which $\lambda_k > 0$). This gives us $\mathcal{V}_i \geq \rho^2$; if $\lambda_i > 0$, we have $\mathcal{V}_i > \rho^2$.

By Lemmas 6.3 and 6.4, there are at least two types for which $\lambda_i > 0$ and consequently $\mathcal{V}_i \leq \alpha_i$. If $\rho \geq 0$, we must therefore have $\rho^2 < \mathcal{V}_i \leq \alpha_i$ for these two types, implying that $\rho < \sqrt{\alpha_2}$ (recall that we have ranked the α_k's in descending order). Thus there is no equilibrium if $\rho \geq \sqrt{\alpha_2}$.

Now suppose $\lambda_i = 1$ for all i. Then $\mathcal{V}_i = \bar{\mathcal{V}}$, where

$$\bar{\mathcal{V}} = \frac{1 + \rho(N-1)}{N}.$$

This is an equilibrium provided all agents have a (weak) incentive to acquire information, i.e. if $\bar{\mathcal{V}} \leq \alpha_i$ for all i, or $\bar{\mathcal{V}} \leq \alpha_N$. From this we obtain the upper bound on ρ in the proposition. $\square$

Proof of Proposition 7.1 For $i \neq N$, we can choose c_i sufficiently low so that $\lambda_i = 1$. Then, from Proposition 5.6 (part (ii)), $\mathcal{V}_N(\lambda_N)$ is minimized at λ^*; $\lambda^* \in (0, 1)$ since $\rho \in (\rho_{\min}, 0)$. If $\alpha_N < \mathcal{V}_N(0)$, we obtain the two equilibria as stated in the proposition. See Figure 1 for the case of $N = 3$ and the discussion in the text.

At the equilibrium with positive λ_N, we have $\mathcal{V}_N \leq \alpha_N < \mathcal{V}_N(0)$, i.e. price informativeness is strictly lower for type N. From Proposition 5.6 (part (i)), this is the case for the other types as well. $\square$

Proof of Proposition 7.2 We consider a situation in which m types are informed and the remaining $(N-m)$ types are uninformed. By symmetry, all informed types have the same price informativeness, which we denote by $\mathcal{V}_{(1)}$. All uninformed types also have the same price informativeness, $\mathcal{V}_{(0)}$. From (11):

$$\mathcal{V}_{(1)} = \frac{1 + \rho(m-1)}{m}, \qquad \text{and} \qquad \mathcal{V}_{(0)} = \frac{\rho^2 m}{1 + \rho(m-1)}. \qquad (25)$$

This is an equilibrium provided $\mathcal{V}_{(0)} > \alpha_1 \geq \alpha_2 \geq \ldots \geq \alpha_N > \mathcal{V}_{(1)}$, by Lemma 6.4. Thus we require $\mathcal{V}_{(0)} > \mathcal{V}_{(1)}$. Using (25), and noting that $1 + \rho(m-1) > 0$ due to the fact that $\rho > \rho_{\min}$, we get the following condition

$$\rho_{\min} = -\frac{1}{N-1} < \rho < -\frac{1}{2m-1}.$$

In particular, we require that m be an integer strictly greater than $N/2$, or $m \geq (N+1)/2$. The condition $\rho < -N^{-1}$ ensures that $\rho < -(2m-1)^{-1}$ for all such values of m. The open subset of C for which the proposition holds corresponds to a choice of $\{\alpha_i\}$ in the interval $(\mathcal{V}_{(1)}, \mathcal{V}_{(0)})$. The same choice must apply for all $m \geq (N+1)/2$. Since $\mathcal{V}_{(1)}$ is decreasing in m and $\mathcal{V}_{(0)}$ is increasing in m, the appropriate interval is the one for $m = (N+1)/2$.

Now let us compare an equilibrium with $m = N$ to one in which $m < N$. Since $\mathcal{V}_{(1)}$ is strictly decreasing in m, price informativeness for the types who remain informed is higher for $m < N$ than at $m = N$. For any type i that switches from being informed to being uninformed, price informativeness must go up, since $\mathcal{V}_i < \alpha_i$ in the first case and $\mathcal{V}_i > \alpha_i$ in the second. $\square$

Proof of Lemma 8.1 From Proposition 3.2,

$$p = \frac{k}{r\sigma_\eta^2} \cdot \boldsymbol{\lambda}^\top \boldsymbol{\theta}.$$

Therefore,

$$\sigma_p^2 = \sigma_\theta^2 \left(\frac{k}{r\sigma_\eta^2}\right)^2 \boldsymbol{\lambda}^\top \mathbf{R} \boldsymbol{\lambda},$$

$$\sigma_{\theta_i,p} = \sigma_\theta^2 \left(\frac{k}{r\sigma_\eta^2}\right) \mathbf{R}_i^\top \boldsymbol{\lambda},$$

so that

$$\beta_i = \frac{\sigma_{\theta_i,p}}{\sigma_p^2} = r\sigma_\eta^2 k^{-1} \cdot \frac{\mathbf{R}_i^\top \boldsymbol{\lambda}}{\boldsymbol{\lambda}^\top \mathbf{R} \boldsymbol{\lambda}}, \tag{26}$$

$$\sigma_{\theta_i|p}^2 = \sigma_\theta^2 - \beta_i \sigma_{\theta_i,p} = \sigma_\theta^2 (1 - \mathcal{V}_i). \tag{27}$$

From (20):

$$\begin{aligned} k_i &= \frac{\lambda_i}{r\sigma_\eta^2} + \frac{1-\lambda_i}{r} \cdot \frac{1-\beta_i}{\sigma_{\theta_i|p}^2 + \sigma_\eta^2} \\ &= \frac{1}{r\sigma_\eta^2}\left[\lambda_i + (1-\lambda_i) \cdot \frac{1 - r\sigma_\eta^2 k^{-1} \cdot \frac{\mathbf{R}_i^\top \boldsymbol{\lambda}}{\boldsymbol{\lambda}^\top \mathbf{R} \boldsymbol{\lambda}}}{1 + \xi(1-\mathcal{V}_i)}\right], \end{aligned}$$

where $\xi := \sigma_\theta^2/\sigma_\eta^2$. Summing over i, and recalling that $k = (\sum_i k_i)^{-1}$, we obtain

$$k^{-1} = \frac{1}{r\sigma_\eta^2}\left[\sum_i \lambda_i + \sum_i (1-\lambda_i)\cdot \frac{1 - r\sigma_\eta^2 k^{-1}\cdot \frac{\mathbf{R}_i^\top \boldsymbol{\lambda}}{\boldsymbol{\lambda}^\top \mathbf{R}\boldsymbol{\lambda}}}{1+\xi(1-\mathcal{V}_i)}\right].$$

Now we can solve for k:

$$k = r\sigma_\eta^2 \cdot \frac{1 + \sum_i \frac{1-\lambda_i}{1+\xi(1-\mathcal{V}_i)}\cdot \frac{\mathbf{R}_i^\top \boldsymbol{\lambda}}{\boldsymbol{\lambda}^\top \mathbf{R}\boldsymbol{\lambda}}}{\sum_i \frac{1+\lambda_i\xi(1-\mathcal{V}_i)}{1+\xi(1-\mathcal{V}_i)}}.$$

We have

$$\begin{aligned}
\sigma_{v_i-p}^2 &= \sigma_\theta^2 + \sigma_\eta^2 + \sigma_p^2 - 2\sigma_{\theta_i,p} \\
&= \sigma_\theta^2 + \sigma_\eta^2 + \sigma_\theta^2\left(\frac{k}{r\sigma_\eta^2}\right)^2 \boldsymbol{\lambda}^\top \mathbf{R}\boldsymbol{\lambda} - 2\sigma_\theta^2\left(\frac{k}{r\sigma_\eta^2}\right)\mathbf{R}_i^\top \boldsymbol{\lambda},
\end{aligned}$$

so that

$$\frac{\sigma_{v_i-p}^2}{\sigma_\eta^2} = 1 + \xi + \xi\cdot\frac{k}{r\sigma_\eta^2}\left[\frac{k}{r\sigma_\eta^2}\boldsymbol{\lambda}^\top \mathbf{R}\boldsymbol{\lambda} - 2\mathbf{R}_i^\top \boldsymbol{\lambda}\right].$$

We now set $\lambda_i = 1$ for $i \neq j$, and write all variables as functions of λ_j. For $\lambda_j = 1$, we have $k(1) = r\sigma_\eta^2/N$, and hence

$$\begin{aligned}
\frac{\sigma_{v_i-p}^2(1)}{\sigma_\eta^2} &= 1 + \xi + \xi N^{-1}\Big[N^{-1}N[1+\rho(N-1)] - 2[1+\rho(N-1)]\Big] \\
&= 1 + \xi - \xi N^{-1}[1+\rho(N-1)] \qquad (28)\\
&= 1 + \xi[1-\mathcal{V}_i(1)], \qquad (29)
\end{aligned}$$

for all i. If $\lambda_j = 0$, we have:

$$\begin{aligned}
k(0) &= r\sigma_\eta^2 \cdot \frac{1 + \xi[1-\mathcal{V}_j(0)] + \frac{\mathbf{R}_j^\top \boldsymbol{\lambda}}{\boldsymbol{\lambda}^\top \mathbf{R}\boldsymbol{\lambda}}}{N + \xi(N-1)[1-\mathcal{V}_j(0)]} \\
&= r\sigma_\eta^2 \cdot \frac{[1+\rho(N-1)][1+\xi(1-\rho)]}{(1-\rho) + (N-1)[1+\rho(N-1)][1+\xi(1-\rho)]}.
\end{aligned}$$

For type $i \neq j$,

$$\begin{aligned}
\frac{\sigma_{v_i-p}^2(0)}{\sigma_\eta^2} &= 1 + \xi + \xi\cdot\frac{k(0)}{r\sigma_\eta^2}\left[\frac{k(0)}{r\sigma_\eta^2}(N-1)[1+\rho(N-2)] - 2[1+\rho(N-2)]\right] \\
&= 1 + \xi - \xi\cdot\frac{[1+\rho(N-1)][1+\rho(N-2)][1+\xi(1-\rho)]}{\Big[(1-\rho) + (N-1)[1+\rho(N-1)][1+\xi(1-\rho)]\Big]^2} \\
&\quad \cdot \Big[2(1-\rho) + (N-1)[1+\rho(N-1)][1+\xi(1-\rho)]\Big].
\end{aligned}$$

It is easy to check that the above expression is lower than (28). For type j,

$$\begin{aligned}\frac{\sigma^2_{v_j-p}(0)}{\sigma^2_\eta} &= 1+\xi+\xi\cdot\frac{k(0)}{r\sigma^2_\eta}\left[\frac{k(0)}{r\sigma^2_\eta}(N-1)[1+\rho(N-2)]-2\rho(N-1)\right] \\ &= 1+\xi-\xi\cdot\frac{(N-1)[1+\rho(N-1)][1+\xi(1-\rho)]}{\Big[(1-\rho)+(N-1)[1+\rho(N-1)][1+\xi(1-\rho)]\Big]^2} \\ &\qquad\cdot\Big[2\rho(1-\rho)+(\rho N-1)[1+\rho(N-1)][1+\xi(1-\rho)]\Big], \end{aligned} \tag{30}$$

which is greater than (28). $\square$

Proof of Proposition 8.3 An equilibrium with the properties (a) and (b) stated in the proposition exists if, for any type j, $\mathcal{U}^U_j(1) < \mathcal{U}^I_j(1) < \mathcal{U}^U_j(0)$ (we write all variables as functions of λ_j, which is either 0 or 1). The first inequality says that at the proposed equilibrium, with $\lambda_i = 1$ for all i, agents of type j have an incentive to acquire information, while the second inequality says that they are worse off compared to the case where none of them do. Using (14) and (15), and recalling that $\xi := \sigma^2_\theta/\sigma^2_\eta$, this system of inequalities can be written as follows:

$$\frac{1}{1+\xi[1-\mathcal{V}_j(1)]}\cdot\frac{\sigma^2_{v_j-p}(1)}{\sigma^2_\eta} < e^{-2rc_j}\cdot\frac{\sigma^2_{v_j-p}(1)}{\sigma^2_\eta} < \frac{1}{1+\xi[1-\mathcal{V}_j(0)]}\cdot\frac{\sigma^2_{v_j-p}(0)}{\sigma^2_\eta}.$$

Substituting for $\sigma^2_{v_j-p}(1)/\sigma^2_\eta$ from (29), the condition becomes

$$1 < e^{-2rc_j}\Big[1+\xi[1-\mathcal{V}_j(1)]\Big] < \frac{1}{1+\xi[1-\mathcal{V}_j(0)]}\cdot\frac{\sigma^2_{v_j-p}(0)}{\sigma^2_\eta}. \tag{31}$$

The first inequality in (31) holds for all $c_j \in (0, c^*_j)$, where[19]

$$c^*_j := \frac{1}{2r}\log\Big[1+\xi[1-\mathcal{V}_j(1)]\Big],$$

and, moreover, becomes arbitrarily close to holding as an equality as c_j converges to c^*_j. We can also verify by direct computation, using (30), that the RHS of the second inequality in (31) is greater than 1. Therefore, (31) holds for $c_j \in (c^*_j - \varepsilon_j, c^*_j)$ for sufficiently small $\varepsilon_j > 0$.

The above argument applies for any choice of j. The proposition is thus established for a vector of costs $(c_j)_{j=1}^N$ that lies in the Cartesian product of the open intervals $(c^*_j - \varepsilon_j, c^*_j)$, $j = 1, \ldots, N$, an open subset of C. $\square$

[19] Since we have a symmetric economy, c^*_j is actually the same for all j, but this is not needed for our argument.

Proof of Proposition 8.4 In order to calculate $\mathcal{U}_i^I$ and $\mathcal{U}_i^U$, we need to determine $\sigma^2_{v_i-p}$. We proceed as in the proof of Lemma 8.1. The expressions for β_i and $\sigma^2_{\theta_i|p}$ are the same as in (26) and (27), but with σ^2_η replaced by σ^2_ϵ. From (20):

$$\begin{aligned} k_i &= \frac{\lambda_i}{r\sigma^2_\epsilon} + \frac{1}{r}\cdot\frac{1-\beta_i}{\sigma^2_{\theta_i|p}} \\ &= \frac{1}{r\sigma^2_\epsilon}\left[\lambda_i + \frac{1 - r\sigma^2_\epsilon k^{-1}\cdot\frac{\mathbf{R}_i^\top\boldsymbol{\lambda}}{\boldsymbol{\lambda}^\top\mathbf{R}\boldsymbol{\lambda}}}{\zeta(1-\mathcal{V}_i)}\right], \end{aligned}$$

where $\zeta := \sigma^2_\theta/\sigma^2_\epsilon$. Summing over i, and recalling that $k = (\sum_i k_i)^{-1}$, we obtain

$$k^{-1} = \frac{1}{r\sigma^2_\epsilon}\left[\sum_i \lambda_i + \sum_i \frac{1 - r\sigma^2_\epsilon k^{-1}\cdot\frac{\mathbf{R}_i^\top\boldsymbol{\lambda}}{\boldsymbol{\lambda}^\top\mathbf{R}\boldsymbol{\lambda}}}{\zeta(1-\mathcal{V}_i)}\right].$$

Now we can solve for k:

$$k = r\sigma^2_\epsilon\cdot\frac{\zeta + \sum_i(1-\mathcal{V}_i)^{-1}\frac{\mathbf{R}_i^\top\boldsymbol{\lambda}}{\boldsymbol{\lambda}^\top\mathbf{R}\boldsymbol{\lambda}}}{\zeta\sum_i\lambda_i + \sum_i(1-\mathcal{V}_i)^{-1}}.$$

We have

$$\begin{aligned} \sigma^2_{v_i-p} &= \sigma^2_\theta + \sigma^2_p - 2\sigma_{\theta_i,p} \\ &= \sigma^2_\theta + \sigma^2_\theta\left(\frac{k}{r\sigma^2_\epsilon}\right)^2\boldsymbol{\lambda}^\top\mathbf{R}\boldsymbol{\lambda} - 2\sigma^2_\theta\left(\frac{k}{r\sigma^2_\epsilon}\right)\mathbf{R}_i^\top\boldsymbol{\lambda}, \end{aligned}$$

so that

$$\frac{\sigma^2_{v_i-p}}{\sigma^2_\theta} = 1 + \frac{k}{r\sigma^2_\epsilon}\left[\frac{k}{r\sigma^2_\epsilon}\boldsymbol{\lambda}^\top\mathbf{R}\boldsymbol{\lambda} - 2\mathbf{R}_i^\top\boldsymbol{\lambda}\right]. \tag{32}$$

We now set $\lambda_i = 1$ for $i \neq j$, and write all variables as functions of λ_j. For $\lambda_j = 1$, we have $k(1) = r\sigma^2_\epsilon/N$, and hence

$$\begin{aligned} \frac{\sigma^2_{v_i-p}(1)}{\sigma^2_\theta} &= 1 + N^{-1}\Big[N^{-1}N[1+\rho(N-1)] - 2[1+\rho(N-1)]\Big] \\ &= 1 - N^{-1}[1+\rho(N-1)], \\ &= 1 - \mathcal{V}_i(1), \end{aligned} \tag{33}$$

for all i. If $\lambda_j = 0$, we have:

$$\begin{aligned} k(0) &= r\sigma^2_\epsilon\cdot\frac{\zeta + [1-\mathcal{V}_i(0)]^{-1} + \frac{\rho}{1+\rho(N-2)}\cdot[1-\mathcal{V}_j(0)]^{-1}}{\big[\zeta + [1-\mathcal{V}_i(0)]^{-1}\big](N-1) + [1-\mathcal{V}_j(0)]^{-1}} \\ &= r\sigma^2_\epsilon D^{-1}\Big(\rho(N-2) + \big[1+\rho(N-1)\big]\big[(N-1) + \zeta(N-2)(1-\rho)\big]\Big), \end{aligned}$$

where

$$D := (N-2)\big[1+\rho(N-2)\big] + (N-1)\big[1+\rho(N-1)\big]\big[(N-1)+\zeta(N-2)(1-\rho)\big].$$

For type $i \neq j$,

$$\begin{aligned}
\frac{\sigma^2_{v_i-p}(0)}{\sigma^2_\theta} &= 1 + \big[1+\rho(N-2)\big]\cdot\frac{k(0)}{r\sigma^2_\eta}\left[\frac{k(0)}{r\sigma^2_\eta}(N-1)-2\right] \\
&= (N-2)(1-\rho)D^{-2} \\
&\quad \cdot\Big((N-1)\big[1+\rho(N-1)\big]^2\big[(N-1)+\zeta(N-2)(1-\rho)\big]^2 \\
&\qquad + 2(N-2)\big[1+\rho(N-1)\big]\big[1+\rho(N-2)\big]\big[(N-1)+\zeta(N-2)(1-\rho)\big] \\
&\qquad + (N-2)\big[1+\rho(N-2)\big]\big[1+\rho(N-2)-\rho\big]\Big).
\end{aligned}$$

Using (16), some tedious but straightforward algebra shows that $\mathcal{U}_i^I(1) > \mathcal{U}_i^I(0)$ for $i \neq j$ if and only if the following condition is satisfied:

$$f(\zeta) > (N-2)(1-\rho)^{-1}\big[1+\rho(N-2)\big]\big[N+\rho(N^2-2N-1)\big], \tag{34}$$

where

$$\begin{aligned}
f(\zeta) := {}& \zeta^3(N-1)(N-2)^2[1+\rho(N-1)]^2(1-\rho)^2 \\
&+ 2\zeta^2(N-2)[1+\rho(N-1)](1-\rho)\Big[(N-1)^2[1+\rho(N-1)] + (N-2)[1+\rho(N-2)]\Big] \\
&+ \zeta\Big[(N-1)^3[1+\rho(N-1)]^2 + 2(N-1)(N-2)[1+\rho(N-1)][1+\rho(N-2)] \\
&\quad - (N-2)^2[1+\rho(N-2)]^2\Big].
\end{aligned}$$

Also, $\mathcal{U}_i^I(1) < \mathcal{U}_i^I(0)$ for $i \neq j$ if and only if $f(\zeta)$ is strictly less than the RHS of (34). Using the fact that $\rho > -(N-1)^{-1}$, it is easy to check that the function f is strictly convex on $(0,\infty)$, with $\lim_{\zeta\downarrow 0} f(\zeta) = 0$ and $\lim_{\zeta\uparrow\infty} f(\zeta) = \infty$. Hence there is a unique $\zeta^* > 0$ such that $f(\zeta^*)$ is equal to the RHS of (34) (which is positive). Clearly (34) is satisfied if and only if $\zeta > \zeta^*$. $\square$

Proof of Proposition 8.5 Using (32), we obtain

$$\begin{aligned}
\frac{\sigma^2_{v_j-p}(0)}{\sigma^2_\theta} &= 1 + (N-1)\frac{k(0)}{r\sigma^2_\epsilon}\left[\frac{k(0)}{r\sigma^2_\epsilon}[1+\rho(N-2)] - 2\rho\right] \\
&= 1 - (N-1)\Big(\rho(N-2) + \big[1+\rho(N-1)\big]\big[(N-1)+\zeta(N-2)(1-\rho)\big]\Big)D^{-2} \\
&\quad \cdot\Big(\rho(N-2)\big[1+\rho(N-2)\big] + (\rho N-1)\big[1+\rho(N-1)\big]\big[(N-1)+\zeta(N-2)(1-\rho)\big]\Big).
\end{aligned} \tag{35}$$

We now proceed as in the proof of Proposition 8.3. We require that, for any type j, $\mathcal{U}_j^U(1) < \mathcal{U}_j^I(1) < \mathcal{U}_j^U(0)$. Using (16) and (17), and recalling that $\zeta := \sigma_\theta^2/\sigma_\epsilon^2$, this system of inequalities can be written as follows:

$$\frac{1}{1-\mathcal{V}_j(1)} \cdot \frac{\sigma^2_{v_j-p}(1)}{\sigma_\theta^2} < e^{-2rc_j}\Big[\zeta + [1-\mathcal{V}_j(1)]^{-1}\Big]\frac{\sigma^2_{v_j-p}(1)}{\sigma_\theta^2} < \frac{1}{1-\mathcal{V}_j(0)} \cdot \frac{\sigma^2_{v_j-p}(0)}{\sigma_\theta^2}.$$

Substituting for $\sigma^2_{v_j-p}(1)/\sigma^2_\theta$ from (33), the condition becomes

$$1 < e^{-2rc_j}\Big[1 + \zeta[1-\mathcal{V}_j(1)]\Big] < \frac{1}{1-\mathcal{V}_j(0)} \cdot \frac{\sigma^2_{v_j-p}(0)}{\sigma_\theta^2}. \tag{36}$$

The first inequality in (36) holds for all $c_j \in (0, c_j^*)$, where

$$c_j^* := \frac{1}{2r}\log\Big[1 + \zeta[1-\mathcal{V}_j(1)]\Big],$$

and, moreover, becomes arbitrarily close to holding as an equality as c_j converges to c_j^*. We can also verify by direct computation, using (35), that the RHS of the second inequality in (36) is greater than 1. The rest of the proof is the same as that of Proposition 8.3. $\square$

References

Banerjee, S. and Green, B. (2015). Signal or noise? Uncertainty and learning about whether other traders are informed. *Journal of Financial Economics,* forthcoming.

Barlevy, G. and Veronesi, P. (2008). Information acquisition in financial markets: A correction. Working Paper, University of Chicago.

Breon-Drish, B. (2012). Asymmetric information in financial markets: Anything goes. Working Paper, Stanford University.

Diamond, D. and Verrecchia, R. (1981). Information aggregation in a noisy rational expectations economy. *Journal of Financial Economics*, 9(3):221–235.

Ganguli, J. and Yang, L. (2009). Complementarities, multiplicity, and supply information. *Journal of the European Economic Association*, 7:90–115.

García, D. and Strobl, G. (2011). Relative wealth concerns and complementarities in information acquisition. *Review of Financial Studies*, 24:169–207.

Goldstein, I., Li, Y., and Yang, L. (2014). Speculation and hedging in segmented markets. *Review of Financial Studies*, 27(3):881–922.

Goldstein, I. and Yang, L. (2014). Information diversity and complementarities in trading and information acquisition. *Journal of Finance,* forthcoming.

Grossman, S. and Stiglitz, J. (1980). On the impossibility of informationally efficient markets. *American Economic Review*, 70:393–408.

Hellwig, M. (1980). On the aggregation of information in competitive markets. *Journal of Economic Theory*, 22:477–498.

Manzano, C. and Vives, X. (2011). Public and private learning from prices, strategic substitutability and complementarity, and equilibrium multiplicity. *Journal of Mathematical Economics*, 47:346–369.

Mele, A. and Sangiorgi, F. (2015). Uncertainty, information acquisition and price swings in asset markets. *Review of Economic Studies,* forthcoming.

Mendel, B. and Shleifer, A. (2012). Chasing noise. *Journal of Financial Economics*, 104:303–320.

Peress, J. (2014). Learning from stock prices and economic growth. *Review of Financial Studies*, 27(10):2998–3059.

Rahi, R. (1996). Adverse selection and security design. *Review of Economic Studies*, 63:287–300.

Rostek, M. and Weretka, M. (2012). Price inference in small markets. *Econometrica*, 80(2):687–711.

Stein, J. C. (1987). Informational externalities and welfare-reducing speculation. *Journal of Political Economy*, 95(6):1123–1145.

Veldkamp, L. L. (2006a). Information markets and the comovement of asset prices. *Review of Economic Studies*, 73:823–845.

Veldkamp, L. L. (2006b). Media frenzies in markets for financial information. *American Economic Review*, 96(3):577–601.

Vives, X. (2008). *Information and Learning in Markets*. Princeton University Press.

Vives, X. (2014). On the possibility of informationally efficient markets. *Journal of the European Economic Association*, 12(5):1200–1239.