Strategic Choices in Polygamous Households: Theory and Evidence from Senegal

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Abstract

This paper proposes a strategic framework to account for fertility choices in polygamous households. A theoretical model specifies the main drivers of fertility in the African context and describes how the fertility of one wife impacts the behavior of her co-wives. It generates predictions to test for the existence and type of strategic interactions. Exploiting original data from a household survey and the Demographic and Health Surveys in Senegal, empirical tests show that children are strategic complements : one wife raises her fertility in response to an increase by the other wife. This result is the first quantitative evidence of a reproductive rivalry between co-wives. By providing new insights into the demand for children, this paper has strong implications for population policies in Africa. It also contributes to the literature on household behavior as one of the few attempts to open the black box of non-nuclear households.

Keywords : Fertility, Polygamy, Africa, Noncooperative models, Duration models.

JEL Codes : C72, D13, J13, J16, O15, O55.

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Many parents in the developing world rely on offspring to secure their economic futures. Children mitigate the lack of insurance markets and social safety nets by taking care of parents in old age and during times of need. Banerjee and Duflo (2011) state that, for the poor, children represent "an insurance policy, a savings product as well as some lottery tickets". In the African context, fertility decisions are crucial ones. In fact, for women, they are literally a matter of life or death. On the one hand, women's security in case of widowhood or abandonment by the husband critically depends on their ability to have children (Bledsoe 1990). On the other hand, women put their lives in jeopardy each time they give birth. In Sub-Saharan Africa, the lifetime risk of maternal death is 1 in 38 against 1 in 3,700 in developed countries (WHO, UNICEF, UNFPA and The World Bank 2014). Maternal mortality is the second main cause of female excess mortality (Anderson and Ray 2010). Therefore, setting the right pace for births is probably one of the major challenge faced by African women.

Fertility choices are typically made by a couple, but in African countries, where polygamy[1] is widespread, the unit of decision is not clear. More than 10% of married men have several wives in 28 countries in Sub-Saharan Africa ; this proportion raises up to 40-50% in Burkina Faso, Cameroon and Senegal (Tertilt 2005). The term "polygamy belt" has been coined to define an area between Senegal in the west and Tanzania in the east, in which it is common to find that more than one third of married women are engaged in a polygamous union (Jacoby 1995).

Economists have devoted little effort to understanding the decision process over fertility in polygamous households, although evidence provided by other disciplines is rather puzzling. Anthropologists have documented that children are the main stake in polygamous unions and claimed that a strong reproductive rivalry exits between co-wives (Jankowiak, Sudakov, and Wilreker 2005). However, demographers have established a negative correlation between

1. Polygamy is a marriage that includes more than two partners. It encompasses both polygyny, in which a man has several wives, and polyandry, in which a woman has several husbands. Throughout this paper, I use the term polygamy to refer to the former situation, which is by far the most practiced form of polygamy.

fertility and polygamy : women engaged in polygamous unions tend to have fewer children than other women (Lesthaeghe 1989). This empirical result is at first sight in contradiction with the idea of co-wives competing for more children.

The goal of this paper is to propose a simple framework to test for strategic interactions in polygamous households. I build upon anthropologists' and demographers' findings to set up a model of fertility choices. The unitary view of the household is obviously inadequate when the crux of the matter is the interaction between members. As for collective models, they may be ruled out by various sources of inefficiencies such as social norms and information asymmetries. [2] I opted for a non-cooperative approach, in which each member maximizes her own utility taking the actions of others as given. To the best of my knowledge, there is no theoretical paper modeling strategic interactions in non-nuclear households. This paper aims at opening the way by focusing on a specific, but fundamental, decision : children.

The theoretical part aims at shedding light on the different forces driving fertility. It distinguishes three potential mechanisms through which polygamy might have an impact on fertility. First, an overbidding effect ; it reflects the idea that each wife cares about her relative number of children, which drives fertility upwards. Second, a substitution effect ; it comes from the side of the husband, who cares about his total number of children, and this imposes some limits on the fertility of each wife. Third, an exposure effect ; according to demographers, the intensity of exposure to pregnancy risk is lower in polygamous unions than in monogamous ones, which drives fertility downwards. The last effect exists independently of the fertility of the other wife, whereas the first two effects have a strategic component. Since these forces go in opposite directions, it is a priori not clear whether polygamy should increase or decrease fertility, nor whether children should be strategic complements or substitutes.

2. In Zambia, Ashraf, Field, and Lee (2014) show that even in monogamous couples, husbands and wives are not able to achieve efficient fertility outcomes. The few studies testing the efficiency of other outcomes in the African context have come to the same conclusion : Udry (1996) on agricultural production in Burkina Faso, Dercon and Krishnan (2000) on risk sharing in Ethiopia and Duflo and Udry (2004) on resource allocation in Cote d'Ivoire.

These questions have to be solved empirically, and the model provides some guidance to think about the identification strategy. First, to identify the total impact of polygamy on fertility, I look at the change in the first wife's birth spacing before and after the second marriage. Second, to identify the type of strategic interactions, I test if first wives react differently depending on who they face as a competitor. To do so, I exploit the variation in the timing of the second marriage, in particular in the husband's and wife's ages at marriage, which determine how much time ahead the second wife has to give birth to children. The idea is to use the length of this time period as a kind of instrument for her final number of children. Indeed, demographers have shown that it influences significantly completed fertility in the African context, where most couples have only access to traditional birth control methods. In the end, an order of magnitude of each effect can be estimated by comparing the birth rates of the same woman in the monogamous stage and in the polygamous stage. The main advantage of this strategy is to include individual fixed effects to deal properly with endogeneity issues related to time-invariant unobserved heterogeneity. Formally, I estimate a duration model of birth intervals with a baseline hazard specific to each woman. This specification allows fertility choices and the occurrence and timing of the second marriage to be jointly determined by some unobserved characteristics of the spouses. The key identification assumption is that these characteristics are fixed in time.

Implementing such a strategy requires information that is not available in standard surveys. In particular I need to observe all spouses in a given union, and to know the dates of successive marriages as well as the birth dates of all children. I exploit original data from a Senegalese household survey that provides information on a husband and all co-wives, even if they do not live in the same household, and detailed information on the timing of marriages and births. I find that first wives lengthen birth spacing in the polygamous stage. It means that the upward force (the overbidding effect) is dominated by a combination of the downward forces (the substitution effect and the exposure effect). The second result is that birth

spacing lengthens less when the second wife's reproductive period is longer. So a predictor of the second wife's number of children drives the first wife's fertility upwards. Last, there is suggestive evidence that second wives intensify birth spacing when they face a more fertile competitor, too. All this is consistent with a strategic complementarity, indicating that the overbidding effect dominates the substitution effect. Thus, my framework brings together the result of demographers that, on average, fertility is lower in polygamous unions, and the claim by anthropologists that the reproductive rivalry between co-wives is strong.

As far as I know, this paper provides the first quantitative evidence that co-wife rivalry has a detectable impact on fertility decisions. I further discuss how standard policy recommendations to curb fertility must be qualified to take into account reproductive externalities in polygamous households. I carry out the tests on Senegalese data, but my framework may fit with the reality of other countries as soon as co-wives have (i) competing reproductive interests, and (ii) some imperfect control over fertility. Both conditions seem likely to hold throughout Africa, but further research is needed to determine to what extent the behavior of polygamous households is culturally specific.

The outline of the paper is as follows. Section 1 provides background on fertility in polygamous unions. Section 2 presents the Senegalese data and some descriptive statistics. Section 3 sets up the model and derives testable predictions. Section 4 describes the empirical strategy, and Section 5 reports the results of the empirical tests. Section 6 deals with robustness tests and alternative models of fertility choices. Section 7 discusses the implications for population policies in Africa, and Section 8 concludes.

1 Polygamy and Fertility

1.1 Qualitative evidence : co-wife reproductive rivalry

Conflicts between co-wives are pervasive in polygamous societies : co-wife rivalry is a recurring theme in African novels [3] and it has been thoroughly studied by anthropologists and sociologists working on polygamous ethnic groups. Jankowiak, Sudakov, and Wilreker (2005) gathered information on co-wife interactions in 69 polygamous systems from all over the world (among which 39 are in Africa) to identify the determinants of co-wife conflict and cooperation. They conclude that conflict is widespread and primarily caused by competing reproductive interests. They note that "reproductive vitality, women's age in the marriage, and the presence or absence of children influence a woman's willingness to enter in or avoid forming some kind of pragmatic cooperative relationship with another co-wife." Thus, conflict is less prevalent when one wife cannot have children.

In the African context, Fainzang and Journet (2000) have documented that wives in polygamous unions overbid for children. Women commonly resort to marabouts to increase their own chances to get pregnant and to cause infertility or stillbirths for the co-wife. In most extreme cases, aggressions may jeopardize children's lives. Indeed, child mortality is found to be higher in polygamous households and co-wife rivalry is considered as one important risk factor (Strassman 1997, Areny 2002).

There are many reasons why wives care so much about their number of children, relative to the number of children of their co-wives. This difference defines social status, authority over co-wives and husband's respect (Fainzang and Journet 2000). It may also be interpreted as a sign of husband's sexual and emotional attention, which clearly matters for the wife's well-being when jealousy is rife (Jankowiak, Sudakov, and Wilreker 2005). In an

3. See for instance books by China Achebe, Sefi Atta, Mariama Ba, Fatou Diome, Buchi Emecheta, Aminata Saw Fall and Ousmane Sembene.

economic perspective, the relative number of children directly determines the wife's share in husband's resources. It is particularly important in case of widowhood because in most African countries, a man's bequest is to be shared among his children. The surviving wives generally have little control over inheritance other than through their own children (United Nations 2001, Lambert and Rossi 2014). In polygamous unions, each wife needs therefore to ensure that enough children are born to secure current and future access to husband's resources.

1.2 Quantitative evidence : lower fertility in polygamous unions

Although the reproductive rivarly has been qualitatively documented, quantitative evidence is very scarce. Demographers have been working on the relationship between polygamy and fertility for a long time, but their theoretical framework does not take into account strategic behaviors. They assume that a regime of natural fertility prevails in Sub-Saharan Africa. It implies that a woman's fertility is mostly determined by factors influencing the length and the intensity of her exposure to pregnancy risk : men's and women's ages at marriage, imperatives for widows or divorcees to remarry, breastfeeding practices, post-partum taboos etc. This framework emphasizes the importance of biological constraints and social norms, leaving little room for individual choices.

Demographers working on Sub-Saharan Africa have established that, at the woman level, fertility is lower in polygamous unions than in monogamous ones (see Chapter 7 by Pebley and Mbugua in Lesthaeghe (1989) or Garenne and van de Walle (1989) and Lardoux and van de Walle (2003) for specific studies on Senegal). This empirical regularity is first explained by infertile unions. On the one hand, women with low fecundity are over-represented in polygamous unions because infertility is a common motive for taking another wife. On the other hand, polygamous marriage may be a way for widowed or divorced women to fulfill the obligation to remarry. In such "safety nets" unions, spouses generally do not aim at

giving birth to children. This composition effect partly explains the difference in fertility between monogamous and polygamous unions. The second explanation is the difference in the timing of marriage : junior wives tend to get married older, and to an older husband. Consequently, these couples have shorter reproductive periods, which mechanically translates into fewer children in a natural fertility regime. The last explanation is about the frequency of intercourse, which is lower in polygamous unions. Indeed, a rule of rotation between the wives for marital duties is implemented and highly monitored. Wives have to share the husband's bed time, which lengthens the period of time before getting pregnant. This is especially true when spouses do not co-reside and the husband pays his wives regular visits. Also, the non-susceptible period following the birth of a child is longer in polygamous unions because the availability of alternative partners makes it easier for husbands to observe the post-partum abstinence.

Interestingly, some results in the study by Lardoux and van de Walle (2003) on Senegalese data cannot be rationalized in the natural fertility framework. First, the authors expected that post-partum abstinence would reduce the likelihood of simultaneous births by co-wives and foster alternate births. In fact, they find a strong positive association between each wife's probability of child-bearing during a given year. Strategic choices may well explain this pattern : if the wife targets a relative number of children, she gets pregnant as soon as her rival does to keep pace with her. The other unexpected result is that the presence of a wife who is past her fecund years impacts positively the fertility of the younger wives. The authors had hypothesized the opposite relationship assuming that the older wife would claim her share of bed time and enforce the compliance with intercourse taboos. In a strategic framework, this might be interpreted as a hint that junior wives are catching up with an older wife who already gave birth to many children.

To my knowledge, there is no empirical study providing evidence that fertility behavior of a given wife impacts the choices of her co-wife. This paper attempts to fill in this gap.

2 Data and Descriptive Statistics

2.1 Data

2.1.1 Poverty and Family Structure (PSF)

Most empirical tests are carried out on original data from a household survey entitled ”Poverty and Family Structure” (PSF) conducted in Senegal in 2006-2007. [4] It is a nationally representative survey conducted on 1,800 households spread over 150 primary sampling units drawn randomly among the census districts. About 1,750 records can be exploited.

The main advantage is that all spouses of the household head were surveyed, even if they do not co-reside. This is very important because approximately one fourth of women do not live with their husband. Standard household surveys only gather information on co-residing spouses. So typically, information is missing in incomplete unions, and the sample of complete unions is selected. Here, I have information on husband and *all* co-wives, whatever the residence status. There remain two selection biases : the sample is restricted (i) to household heads and their spouses, and (ii) to currently married people. [5]

The goal of the survey was to obtain, in addition to the usual information on individual characteristics, a comprehensive description of the family structure. In particular it registered the dates of birth for all living children below 25, even if they do not live with their parents. Children who died are also reported but there is no information on the timing of deaths. As a result, a woman's complete birth history for surviving children is available only if all her children are under 25 years old. Moreover, detailed information is collected on the marital history of all spouses : age at first marriage, date of current union, having or not broken unions, date of termination of latest union. Therefore, I am able to retrace the timing of

4. Detailed description in Vreyer, Lambert, Safir, and Sylla (2008). Momar Sylla and Matar Gueye of the Agence Nationale de la Statistique et de la Démographie of Senegal (ANSD), and Philippe De Vreyer (University of Paris-Dauphine and IRD-DIAL), Sylvie Lambert (Paris School of Economics-INRA) and Abla Safir (now with the World Bank) designed the survey. The data collection was conducted by the ANSD.

5. There is very limited information on co-widows.

marriages and to identify children from previous unions. The main weakness of PSF is that there is no information on fertility preferences.

The PSF sample consists of 1,317 unions : 906 monogamous unions and 411 polygamous unions, among which 321 with two wives, 66 with three wives and 24 with four wives. Roughly one half of women are engaged in a polygamous union, a proportion in line with demographers' estimations (see Chapter 27 by Antoine in Caselli, Vallin, and Wunsch (2002)).

2.1.2 Demographic and Health Surveys (DHS)

To get complementary information, I exploit Demographic and Health Surveys (DHS) collected in Senegal in 1992, 1997, 2005 and 2010. [6] DHS data contain stratified samples of households in which all mothers aged 15 to 49 are asked about their reproductive history, including children who left the household or who are dead at the time of the survey. Male questionnaires are further applied to all eligible men in a sub-sample of these households. [7]

DHS contain relevant data that is missing in PSF, in particular information on fertility preferences of husbands and wives. Respondents are asked how many children they would like to have, or would have liked to have, in their whole life, irrespective of the number they already have.

However, the main drawback is that the DHS sample of husbands and wives is not representative of all Senegalese unions. Indeed, a spouse is not surveyed if (i) she does not co-reside, or (ii) she is above the age limit, 50 for women, 60 for men in the last two waves. As a result, only one woman in five in the mother's record has a match with a male questionnaire. This is an issue especially in polygamous unions, because all wives of a given husband are not systematically observed. For instance, in bigamous unions, both wives are found in the

6. Three other surveys were conducted in Senegal in 1986, 1999 and 2012-13, but I do not consider them, because in 1986 and 2012-13, there is no information on men, and in 1999, the quality of data does not meet the criteria of standard DHS.

7. In 1992 and 1997, men should be older than 20 to be eligible, whereas in 2005 and 2010, they should be between 15 and 59 years old. The proportion of households selected to administer male questionnaires was 33%, except in 1997, when it was 75%.

sample in only half of the cases.

The DHS sample consists of 5,254 unions. The proportion of union types is similar to PSF : one half of women are in polygamous unions, and bigamous unions represent approximately 80% of polygamous unions.

2.2 Descriptive statistics on fertility in current union

I compute the descriptive statistics on completed fertility using PSF data on women over 45 years old. I consider surviving children born in the current union. Figure 1 shows the distribution for monogamous, senior and junior wives. Junior wives stand out with a large proportion (30%) having no child with their current husband. Those women are probably engaged in a kind of safety net union. The proportion of childless women is similar for monogamous and first wives. It contradicts the hypothesis that husbands would take another wife when the first one is infertile ; it rather supports the idea that husbands would repudiate an infertile wife, who would then end as a junior wife in another union.

Table 1 confirms and specifies these remarks. Women engaged in a polygamous union have, on average, one child fewer than in a monogamous union. The whole gap is driven by junior wives : they have, on average, 2.5 children fewer. As for senior wives, they have the same number as monogamous ones. In line with demographers' findings, roughly half of the gap between junior wives and the others is explained by infertile unions. When I restrict the sample to women having at least one child with their current husband, the difference decreases down to 1.4 children. Another explanation put forward in the literature review deals with the length of the couple's reproductive period. I construct a proxy T = min (45- wife's age at marriage ; 60-husband's age at marriage) reflecting the difference in age at fertility decline between men and women. As expected, it is an important driver of women's completed fertility.[8] After controlling for T, there remains a gap of approximately one child.

8. In Appendix A, Table A.1, I test whether husband's and wife's age at marriage are negatively correlated

This last figure is unchanged if I control for having children from previous unions.

If I disentangle the results by mother's rank in polygamous unions, I find that fertility decreases as the number of wives increases. The first column of Table A.2 in Appendix A shows that, controlling for T and children from previous unions, and excluding childless women, the number of children decreases with the mother's rank. Findings are similar when I consider birth spacing. In column 2, I report the estimates of a Cox duration model of birth intervals, and I find that the higher the rank of the mother, the longer the durations between births.

Turning to men's total fertility, it is more complicated to avoid censoring issues because older men may still have the opportunity to take another fertile wife. When I restrict the sample to unions in which all wives are above 45, and the husband is above 60, figures are consistent with statistics computed on women's side. On average, monogamous men have 5 children, men with two wives 7.6 children, and men with three wives 9.7 children. [9]

2.3 Descriptive statistics on preferences

I compute the descriptive statistics on fertility preferences using DHS. Table 2 shows that preferences of men and women differ considerably. Women would like to have, on average, 5.7 children, whereas men want 9 children. Medians are respectively 5 and 7. Within couples, a husband wants on average 3.1 children more than his wife. Preferences are aligned in only 13% of the cases ; in almost two thirds of couples, the husband wants strictly more children than his wife. As pointed out by Field, Molitor, and Tertilt (2015), in polygamous unions,

to the number of children, controlling for the union status, and differentiating between spouses with a small and a large age difference. The idea of the test is that, if the age difference between spouses is small, the length of the reproductive period should be driven by the wife's age at marriage, not by the husband's age at marriage. Indeed, in this case, the wife is expected to reach the end of her reproductive life sooner than the husband. If the age difference between spouses is large, it should be the opposite. Signs are in line with predictions : when the age difference is small (resp. large), only the wife's (resp. husband's) age at marriage is significantly, negatively correlated to the number of children.

9. I do not report the average for unions with four wives because there are too few data points.

the discrepancy in preferences does not necessarily translate into a conflict, because men and women can realize their fertility individually.

I exploit DHS to identify the predictors of the ideal number of children of men and women (cf. Table A.3 in Appendix A). Since there is no information on preferences in PSF, I will use this set of predictors when empirical tests require controlling for preferences of the husband and the wife. Appendix A provides a detailed discussion of the predictors. Two results are worth highlighting here. On the wife's side, there is no difference between the preferences of women engaged in a monogamous or a polygamous union. On the husband's hand, the ideal number of children is much larger for polygamous men than for monogamous ones. If I consider the raw average, monogamous husbands want 7.6 children against 12.2 for polygamous husbands. The gap goes down to 2.7 children when I control for the whole set of predictors, but it remains significant. So there is a positive correlation between a man's taste for a large family and his likelihood to take a second wife.

2.4 Descriptive statistics on the timing of unions

The timing of unions plays a key role in my framework. Table 3 provides some descriptive statistics on ages at the husband's first and second marriages. Women tend to marry much older husbands : the median age at marriage is 17 for first wives against 28 for their husbands. The second marriage generally takes place around 12 years later : the husband is 40, the first wife 29, and the second wife 22. But the variation in the timing of the second marriage is large : it ranges from within 6 years in the first quartile, to after 16 years in the last quartile. First wives may be still very young when the rival arrives (below 24 years old in the first quartile) or already quite old (above 36 in the last quartile). The situation of second wives is even more diverse. Around one third have already been married, which explains why the age at marriage is so large in the last quartile (31 years old), while others are very young (below 17 in the first quartile). I will exploit this variation in the empirical tests.

I do not report statistics on the third and fourth marriages because there are too few observations. In the theoretical part, I consider a model with two players, and in the empirical part, I am interested in the impact of the second marriage. I could easily extend the model to three or four wives, but I would lack power to perform empirical tests on the third and fourth marriages.

3 The Model

3.1 A simple model of fertility choices

I propose a model of fertility choices in which the decision-maker is the woman, and the choice variable is the birth rate, λ. At date T, the couple reaches the end of the reproductive period with $n = \lambda.T$ children. [10] In concrete terms, it means that women choose to give birth every x years, where $x = \frac{1}{\lambda}$.

There is no uncertainty in this framework : λ is a frequency and $\lambda.T$ is the realized number of children. I do not explicitly model the intertemporal evolution of fertility choices and outcomes, although a dynamic stochastic model would better reflect the true decision process. There is a trade-off between realism and tractability. Most dynamic stochastic models do not generate closed-form solutions and the predictions derived from comparative dynamics vary from model to model (see the survey by Arroyo and Zhang (1997)). Since the aim of this paper is to evidence noncooperative behaviors, I chose the simplest framework that allowed me to model and test for the existence of strategic interactions.

Fertility choices are determined by three drivers. First, women face a standard economic trade-off modeled by a function $v(n, n_w^{id})$, that captures the net gain of having n children for the mother ; I assume that $v(.)$ has an inverted U-shape and reaches its maximum when

10. The number of children is therefore not necessarily an integer. Leung (1991) proposed to consider the number of children as a flow of child services in efficiency units when a continuous measure of family size is needed.

$n = n_w^{id}$, the mother's ideal number of children.

Second, even if the wife is the only decision-maker, husband's preferences over fertility influence the outcome by entering the wife's utility. The idea is that husbands are adversely affected if the birth rate is far from their expectations, and wives internalize such a loss, be it because of altruism, love or fear of a punishment. So women incur a cost to deviate from the husband's ideal number of children, n_h^{id}. I denote the cost function $H(n, n_h^{id})$ and I further assume that it has a U-shape and reaches its minimum when $n = n_h^{id}$. As shown in descriptive statistics and in line with the literature on Africa (Westoff 2010), n_h^{id} is generally greater than n_w^{id}.

Third, I build upon demographers' concept of natural fertility to introduce a natural birth rate, λ^{nat}. I further define a function $N(T.(\lambda - \lambda^{nat}))$ capturing the cost to deviate from the natural level. It has an U-shape in λ and reaches its minimum when $\lambda = \lambda^{nat}$. The nature of the cost is not exactly the same on each side of the threshold. If $\lambda > \lambda^{nat}$, the woman incurs a health cost generated by frequent pregnancies as well as a psychological cost for transgressing social norms.[11] If $\lambda < \lambda^{nat}$, the cost is related to acquiring and implementing birth control methods ; it might be an economic cost, an opportunity cost of time or a psychological cost to hide contraceptives from the husband or the community (Ashraf, Field, and Lee 2014). The argument in the cost function is $\int_0^T (\lambda - \lambda^{nat}) dt$; it corresponds to the accrual of instantaneous deviations during the whole reproductive period. One can think of n_i^{id} as the ideal number of children that individual i would like to have if the conception of children were free from any biological or social constraint.

3.1.1 When polygamy is banned

I start by considering societies in which polygamy is banned, in order to set up the key forces driving fertility choices. The next section will describe how this setting is mo-

11. Fainzang and Journet (2000) document that pregnancies in quick succession are frowned upon in West Africa. The mother is despised for giving in to her husband at the expense of the youngest child's health.

dified when polygamy is an option. In monogamous societies, a wife chooses λ maximizing $u(n_w^{id}, n_h^{id}, \lambda_m^{nat}, \lambda.T)$ where λ_m^{nat} is the natural rate for monogamous couples. I further assume that the wife's utility is separable in three components :

$$u(n_w^{id}, n_h^{id}, \lambda_m^{nat}, \lambda.T) = v(\lambda.T, n_w^{id}) - \theta^h H(\lambda.T, n_h^{id}) - \theta^n N(T.(\lambda - \lambda_m^{nat})) \tag{1}$$

$\theta^h \geq 0$ and $\theta^n \geq 0$ capture the intensity of marital and natural constraints, respectively. Payoffs are paid at the end of the reproductive period. This is important to ensure time consistency.[12] I further note $n^{nat} = T.\lambda_m^{nat}$.

A natural way to put more structure on this optimization problem is to assume that the wife minimizes a weighted sum of distances : distance to her ideal number (weight 1), distance to the husband's ideal number (weight θ^h), and distance to the natural number (weight θ^n). That is why I consider the following parametric forms :[13]

$$v(n, n_w^{id}) = -(n - n_w^{id})^2 \text{ and } H(n, n_h^{id}) = (n - n_h^{id})^2 \text{ and } N(n, n^{nat}) = (n - n^{nat})^2$$

The first order condition gives an optimal birth rate :[14]

$$\lambda^{NS} = \frac{n_w^{id} + \theta^h n_h^{id} + \theta^n n^{nat}}{(1 + \theta^h + \theta^n).T} \tag{2}$$

12. At date 0, the woman solves the maximization problem and chooses λ^*. Suppose that she can update her choice at date t. She already has $\lambda^*.t$ children, and maximizes over λ' :

$$v(\lambda^*.t + \lambda'.(T-t), n_w^{id}) - \theta^h H(\lambda^*.t + \lambda'.(T-t), n_h^{id}) - \theta^n N(t.(\lambda^* - \lambda_m^{nat}) + (T-t).(\lambda' - \lambda_m^{nat}))$$

It is equivalent to maximizing over $\mu = \frac{\lambda^*.t + \lambda'.(T-t)}{T}$:

$$v(\mu.T, n_w^{id}) - \theta^h H(\mu.T, n_h^{id}) - \theta^n N(T.(\mu - \lambda_m^{nat}))$$

Therefore $\mu = \lambda^* = \lambda'$. The woman is time-consistent.

13. In Appendix C1, I show that the main predictions of the model can be derived without specifying any parametric form for $v(.)$, $H(.)$ and $N(.)$.

14. I use the superscripts NS for Non-Strategic because there is no strategic interaction here.

Hence the optimal number of children, $n^{NS} = \lambda^{NS}.T$, is a weighted average of n_w^{id}, n_h^{id} and n^{nat}. The optimal birth rate and the optimal number of children increase with n_w^{id}, n_h^{id} and λ_m^{nat}. An increase in T raises the final number of children, but reduces the birth rate. Last, the impact of a variation in θ^h and θ^n depends on the relative size of n_w^{id}, n_h^{id} and n^{nat}.

From this simple model, I derive a testable implication. In monogamous societies, the final number of children should be a weighted average of the preferences of the wife, the preferences of the husband and a natural number proportional to marriage duration. I estimate the model using DHS data on monogamous couples, focusing on older ones. The first column of Table 4 suggests that my framework is relevant. The three drivers n_w^{id}, n_h^{id} and T are significantly correlated to total fertility, accounting for 36% of the variance. This is virtually the largest share in variance that can be explained by a linear probability model in this setting.[15] The constant is not significantly different from zero.[16] The estimation I get for the natural birth rate is sensible : one birth every three years during the whole reproductive period. Given that $T = 26$ years on average, it corresponds to approximately 8 children for the average couple. These numbers are consistent with estimates produced by demographers.[17] The natural number of children significantly constrains fertility choices : θ^n is estimated to be around three. Last, the order of magnitude of θ^h is one half, but I cannot reject the hypothesis that the preferences of the husband and those of the wife have the same weight.

However, this framework fails to account for behaviors in polygamous unions. Indeed, when I estimate the model on polygamous couples, in the second column, the fit is not as

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15. Indeed, the dependent variable is an integer. Assuming that the true data generating process is a Poisson model of parameter μ, then a LPM could explain at most $\mathbb{V}(\mu)/[\mathbb{E}(\mu) + \mathbb{V}(\mu)]$. In my sample, the empirical counterparts of $\mathbb{E}(\mu)$ and $\mathbb{V}(\mu)$ are respectively 6.44 and 2.65, leading to an upper bound of 0.30 for the R^2.

16. Yet, the constant is rather large in magnitude and negative. One explanation is that I consider women between 40 and 50 years old who may still have an additional child. There are not enough observations to restrict the sample to an older age bracket.

17. Using data from various populations in the world, the founding father of the natural fertility concept, Louis Henry, concludes that the completed fertility for a woman married at 20 years old is between 6 and 11 (Henry 1961). More specifically, in Africa, Hertrich (1996) reports that the most widespread norm advocates an interval of three years between two births.

good. The R^2 drops to 0.14 and the preferences of the wife are no longer significant, so that I can no longer get consistent estimates of the parameters. Something seems to be missing, and I claim in the next section that the missing element is the fertility of the other wife. As suggestive evidence, I added the preferences and the length of reproductive period of the co-wife as additional drivers.[18] Both have a positive and large effect on the completed fertility of the index wife, although not significant ; more importantly, the R^2 increases up to 0.30, which is close to the level observed in monogamous unions.

3.1.2 When polygamy is allowed

How does the eventuality of polygamy impact the framework described above ? The situation is best modeled as a game with two players, wife 1 and wife 2, characterized by their preferences $n^{id}_{w,i}$ and their reproductive period T_i. The husband is not a player but he has some preferences n^{id}_h and a type, monogamous (m) or polygamous (p). At $t = 0$, a monogamous couple is formed between the husband and wife 1. If the husband is of type m, the union remains monogamous and wife 2 never enters the game. If the husband is of type p, wife 2 enters at date S so that the timing is split into a monogamous stage until $t = S$ and a polygamous stage after $t = S$.

At $t = 0$, the first wife only knows $n^{id}_{w,1}$, n^{id}_h and T_1. She has some beliefs about the risk of polygamy, and in case of polygamy, about the date of the second marriage and about the characteristics of the second wife. In particular, she believes that her husband is of type p with probability π, and of type m with probability $(1 - \pi)$. As long as the second marriage has not taken place, the first wife does not know the type of her husband. She chooses λ_0 taking into account her expectations.

When the second marriage takes place, the type "polygamous" is revealed and all informa-

18. I observe the preferences and the length of reproductive period of the co-wife only in a very small sample : bigamous unions in which both wives are found and are in their first union (41 observations).

tion become public : S, T_2 and $n^{id}_{w,2}$.[19] The two wives play a simultaneous, non-cooperative game. The first wife chooses λ_1 and the second wife chooses λ_2. Payoffs are paid when both reproductive periods are over.

One important assumption in this setting is that the union type, the date of the second marriage and the characteristics of the second wife are given ex-ante. They may well be correlated with $n^{id}_{w,1}$, n^{id}_h and T_1. Therefore, they may be correlated with λ_0. But the model rules out any reverse causality : the occurrence and characteristics of the second marriage should not be caused by fertility choices made during the monogamous stage. I will test this assumption in the robustness section.[20]

When bigamy is allowed in a society, the utility function of wife i is given by :

$$u(n^{id}_w, n^{id}_h, n^{nat}_i, n_i, n_j) = v(n_i - \epsilon_i n_j, n^{id}_w) - \theta^h_i H(n_i + n_j, n^{id}_h) - \theta^n_i N(n_i - n^{nat}_i) \qquad (3)$$

Where n_j is the final number of children of the (potential) co-wife. Note that $n_j = 0$ when the husband is of type ”monogamous”. n_j enters directly the utility function of wife i along two dimensions.On the one hand, the wife cares about her *relative* number of children $(n_i - \epsilon_i n_j)$ for the reasons put forward in the literature review. The parameter ϵ is meant to capture the intensity of co-wife rivalry. On the other hand, the husband cares about his *total* number of children $(n_i + n_j)$. For him, the children of the first wife and the children of the second wife are perfect substitutes.[21]

I also define $n^{nat}_1 = \lambda^{nat}_m.S + \lambda^{nat}_p.(T_1 - S)$ and $n^{nat}_2 = \lambda^{nat}_p.T_2$, where λ^{nat}_p is the natural rate for polygamous couples. Building upon the demography literature, I assume that $\lambda^{nat}_m > \lambda^{nat}_p$.

19. In an extension of the model, I relax the assumption that $n^{id}_{w,2}$ is observed by the first wife (cf. Appendix C2).

20. An extreme case is when the first wife turns out to be infertile. Whatever the husband's type, he is likely to take another wife. This is one reason why, as mentioned above, I exclude infertile unions from the theoretical and empirical analysis.

21. In an extension of the model, I relax the assumption that the number of rivals enters the utility function in a linear way (cf. Appendix C1).

In the polygamous stage, the second wife chooses λ_2 maximizing :

$$u(\lambda_2.T_2, n_1) = v(\lambda_2.T_2 - \epsilon_2 n_1, n_w^{id}) - \theta_2^h H(\lambda_2.T_2 + n_1, n_h^{id}) - \theta_2^n N(T_2.(\lambda_2 - \lambda_p^{nat}))$$

Let me call λ_2^* the optimal birth rate of second wives, and $n_2^* = \lambda_2^*.T_2$, their optimal final number of children.

Turning to the first wife, λ_0 is the birth rate chosen in the monogamous stage. At date S, the first wife has $\lambda_0.S$ children, and she is able to update her choice. Her final number of children is given by $n_1 = \lambda_0.S + \lambda_1.(T_1 - S)$. So she maximizes over λ_1 :

$$u(\lambda_0.S + \lambda_1.(T_1 - S), n_2) = v(\lambda_0.S + \lambda_1.(T_1 - S) - \epsilon_1 n_2, n_w^{id}) -$$
$$\theta_1^h H(\lambda_0.S + \lambda_1.(T_1 - S) + n_2, n_h^{id}) - \theta_1^n N(S.(\lambda_0 - \lambda_m^{nat}) + (T_1 - S).(\lambda_1 - \lambda_p^{nat}))$$

Let me call $\lambda_1^*(\lambda_0)$ the optimal birth rate of first wives in the polygamous stage.

At $t = 0$, the first wife has to choose λ_0 that maximizes her expected utility given her beliefs. She maximizes over λ_0 :

$$(1 - \pi) \times u(\lambda_0.T_1, 0) + \pi \times \mathbb{E}[u(\lambda_0.S + \lambda_1^*(\lambda_0).(T_1 - S), n_2)]$$

The second term in the expected utility is unknown at $t = 0$ because it depends on S, $n_{w,2}^{id}$ and T_2.

3.2 Finding the equilibrium in bigamous unions

I solve the problem by backward induction : focusing on the polygamous stage, I determine the best response of each wife, in order to compute the equilibrium of the static game. Then I turn to the monogamous stage and determine the optimal initial birth rate.

3.2.1 First stage : best responses

I start by computing the functions of best response taking the number of rivals as exogenous. I replace $v(.), H(.), N(.)$ by the parametric forms and take the first-order condition for wife i when the other plays n_{-i}. I find :

$$n_i^* = n_i^{NS} + n_{-i}.B_i$$

Where $n_i^{NS} = \frac{n_w^{id} + \theta_i^h n_h^{id} + \theta_i^n n_i^{nat}}{1 + \theta_i^h + \theta_i^n}$ is the optimal choice in the absence of strategic interactions, and $B_i = \frac{\epsilon_i - \theta_i^h}{1 + \theta_i^h + \theta_i^n}$ is the strategic response.

B_i will play a key role in the analysis because its sign determines if children are strategic complements or substitutes. The sign of B_i is given by the difference between ϵ_i which captures the intensity of co-wife rivalry, and θ_i^h which is the weight given to husband's preferences. If $\epsilon_i > \theta_i^h$, then $B_i > 0$ and n_i is increasing in n_{-i}.

3.2.2 Second stage : Nash equilibrium

I solve the following system :

$$n_1^* = n_1^{NS} + n_2^*.B_1$$
$$n_2^* = n_2^{NS} + n_1^*.B_2$$

I get :

$$n_i^* = (n_i^{NS} + n_{-i}^{NS}.B_i) \times \frac{1}{1 - B_1.B_2} \text{ for } i = 1, 2 \tag{4}$$

I further impose that $\epsilon_i \in [0, 1]$ so that $B_i \in]-1, 1]$ and $(1 - B_1.B_2) \geq 0$. If $(1 - B_1.B_2) = 0$, there is no equilibrium. It happens when $\epsilon_i = 1, \theta_i^h = \theta_i^n = 0$ for $i = 1, 2$, meaning that the

rivalry effect is not offset by any kind of marital or biological constraint. The number of children of both wives is pushed to infinity. Another extreme case is when $B_i \to -1$ for $i = 1, 2$. It happens when $\epsilon_i = \theta_i^n = 0$ and θ_i^h is very large, meaning that only the deviation costs from husband's preferences are driving fertility choices. Here, there is an infinite number of equilibria : (n_1, n_2) s.t. $n_1 + n_2 = n_h^{id}$. Both cases are easily ruled out by the fact that fertility choices are not free from any biological constraints, so θ_i^n is never equal to zero.

Note that the equilibrium of the static game is fully determined by n_1^{NS}, n_2^{NS}, B_1 and B_2. Whatever λ_0, first wives adjust their birth rate after the second marriage ; they choose $\lambda_1^*(\lambda_0)$ such that $\lambda_0 \dot{} S + \lambda_1^*(\lambda_0).(T_1 - S) = n_1^* = (n_1^{NS} + n_2^{NS}.B_1) \times \frac{1}{1-B_1.B_2}$.

3.2.3 Third stage : back to $t = 0$

At $t = 0$, first wives maximize over λ_0 :

$$(1 - \pi) \times u(\lambda_0.T_1, 0) + \pi \times \mathbb{E}[u(n_1^*, n_2^*)]$$

Since $u(n_1^*, n_2^*)$ does not depend on λ_0, the maximization problem boils down to the problem in monogamous societies. The optimal initial birth rate is :

$$\lambda_0^* = \frac{n_w^{id} + \theta^h n_h^{id} + \theta^n n_0^{nat}}{(1 + \theta^h + \theta^n).T_1} = \frac{n_0^{NS}}{T_1}$$

Where $n_0^{nat} = \lambda_m^{nat}.T_1$. In Appendix B, I compare the equilibrium and the outcome maximizing total welfare. Consistently with most non-cooperative models, I find that household members are unable to reach an optimal allocation.

We can prove that an equilibrium exists as soon as $B_i \geq 0$ for $i = 1, 2$. If we come back to the Nash equilibrium described above, it exists iff $\lambda_i^* \geq 0$ for $i = 1, 2$. For the second wife, it is the case when $B_2 \geq 0$. For the first wife, it is the case when $n_1^*(S) \geq \lambda_0^*.S$. Let me note $f(S) = n_1^*(S) - \lambda_0^*.S$. $f(S)$ is monotonic, and $f(0)$ and $f(T_1)$ are both non-negative, so

$f(S) \geq 0$ for all S. In other words, whatever the length of the monogamous period, the first wife always wants more children than she currently has at the time of the second marriage.

One testable implication is that π should not influence the optimal initial birth rate. Holding everything else constant, women with different beliefs should take the same decision in the monogamous stage. One way to test the model is to come up with an estimate of π for each woman and to examine whether it influences birth spacing in the monogamous period. In Appendix E, Table E.1, I regress the union status on potential predictors of polygamy using the sample of monogamous and first wives older than 45 years old. I use this regression to predict the probability of a second marriage for women younger than 45 years old. Table E.2 shows that the predicted probability has no significant impact on birth spacing. However, the power of this test is limited because there is no strong predictor of polygamy ($R^2 = 0.16$ in the first stage).

The prediction that expectations about the second period do not influence choices in the first period is entirely driven by the assumption that the game is simultaneous. Indeed, in the basic model, I only consider the case when $S < T_1$, meaning that first wives have some time left to update their number of children. In an extension of the model, I include the eventuality that $S \geq T_1$ which corresponds to a sequential game. In this case, expectations play a key role (cf. Appendix C3).

3.3 Comparative statics

3.3.1 Main predictions

From this simple model, I derive a test for the existence of strategic interactions. The key quantity of interest is the difference in first wives' optimal birth rates before and after the second marriage. The advantage of looking at the change in birth rate is to get rid of time-invariant unobservable characteristics of spouses, as will be explained in the empirical

strategy. We have :

$$\lambda_1^* - \lambda_0^* = \frac{n_1^* - n_0^{NS}}{T_1 - S} = \frac{n_2^*.B_1}{T_1 - S} - \frac{\theta_1^n}{1 + \theta_1^h + \theta_1^n} \times (\lambda_m^{nat} - \lambda_p^{nat}) \tag{5}$$

The term can be decomposed into (i) an exposure effect, $-\frac{\theta_1^n}{1+\theta_1^h+\theta_1^n} \times (\lambda_m^{nat} - \lambda_p^{nat})$, that is always negative ; and (ii) a strategic effect, $\frac{n_2^*}{T_1 - S} \times B_1$, that might be positive or negative depending on the sign of B_1. Recalling that the sign of B_1 is given by $(\epsilon_i - \theta_i^h)$, the strategic effect is further split into a positive overbidding effect, driven by $(\epsilon \times n_2^*)$, and a negative substitution effect, driven by $(-\theta^h \times n_2^*)$.

First, I am interested in the sign of $\lambda_1^* - \lambda_0^*$. If it is negative, birth intervals lengthen after the second marriage. It happens when the positive force, the overbidding effect, is not large enough to compensate the negative forces, the sum of the substitution effect and the exposure effect. If $B_1 < 0$, birth spacing is unambiguously longer in the polygamous stage. If $B_1 > 0$, the second marriage causes a lengthening of birth spacing iff the global strategic response is weaker than the change in natural fertility.

Prediction 1 *First wives should lengthen birth spacing after the second marriage iff the exposure effect dominates the strategic effect.*

Second, I predict how $\lambda_1^* - \lambda_0^*$ should evolve with T_2. Using Equation 5, I find that $\frac{\partial \lambda_1^* - \lambda_0^*}{\partial T_2}$ has the same sign as B_1. In the case of a negative strategic effect, birth spacing should lengthen more when T_2 is longer. In the case of a positive effect, it should lengthen less (if $\lambda_1^* - \lambda_0^* < 0$) or shorten more (if $\lambda_1^* - \lambda_0^* > 0$) when T_2 is longer. The prediction is illustrated in Figure 2.

Prediction 2 *In the case that first wives shorten birth spacing after the second marriage : they should shorten more when the reproductive period of the second wife is longer.*
In the case that first wives lengthen birth spacing after the second marriage : they should

lengthen less when the reproductive period of the second wife is longer iff the strategic effect is positive.

3.3.2 Secondary predictions

The model also predicts how the second wife's birth spacing should be affected by the duration of the first wife's reproductive period split into the monogamous stage (S) and the polygamous one ($T_1 - S$). Substituting the value of n_2^* from Equation 4 into $\lambda_2^* = \frac{n_2^*}{T_2}$, I find that $\frac{\partial \lambda_2^*}{\partial (T_1 - S)}$ and $\frac{\partial \lambda_2^*}{\partial S}$ have the same sign as B_2.

Prediction 3 *The duration of the monogamous period and the time left before the end of the first wife's reproductive period should impact birth spacing of second wives in the same direction. They should shorten birth spacing iff the strategic effect is positive.*

Last, I am able to derive predictions on the equilibrium number of children. Using the closed form for n_1^* in Equation 4, I find that $\frac{\partial n_1^*}{\partial T_2}$ has the same sign as B_1 while $\frac{\partial n_1^*}{\partial T_1} > 0$ and $\frac{\partial n_1^*}{\partial S} > 0$. Similarly, for second wives, $\frac{\partial n_2^*}{\partial (T_1 - S)}$ and $\frac{\partial n_2^*}{\partial S}$ have the same sign as B_2, while $\frac{\partial n_2^*}{\partial T_2} > 0$.

Prediction 4 *The number of children of first wives should increase with their own reproductive period and with the duration of the monogamous period. It should also increase with the second wife's reproductive period iff the strategic effect is positive.*

Prediction 5 *The number of children of second wives should increase with the duration of their own reproductive period. The duration of the monogamous period and the time left before the end of the first wife's reproductive period should impact fertility in the same direction. They should raise the second wife's completed fertility iff the strategic effect is positive.*

The main weakness of this set of predictions is that I cannot account for unobserved heterogeneity in the corresponding empirical tests. I report the results as suggestive evidence of strategic interactions, but the empirical strategy focuses on the change in first wives' birth

spacing. The main notations of the model and the predictions are summarized in Tables 5 and 6.

4 Empirical Strategy

The empirical strategy aims at (i) identifying the total impact of polygamy on fertility and (ii) identifying the sign of the strategic effect. Tests are carried out on the sub-sample of first wives younger than 45 because I do not know the complete birth history of older women, as explained in Section 2.1. Moreover, to mitigate the issue of infertile unions, I exclude women having no child with the current husband in all tests.

4.1 Preliminary evidence on raw data

Before turning to the econometric specifications, Table 7 presents some descriptive statistics on the average birth intervals of first wives before and after the second marriage. The first column reports the statistics for the whole sample of first wives : birth intervals increase by six months, from 37.6 months in the monogamous stage up to 43.6 months in the polygamous stage. The sample is further split on the median T_2 into those facing a weak competitor (short T_2, column two) and those facing a strong competitor (long T_2, column three). Birth intervals rise by almost 10 months for the former, whereas the magnitude is halved and the increase is no longer significant for the latter. Note that both groups display very similar birth intervals in the monogamous period : 37.8 and 37.1 months, not significantly different from each other.

The analysis of raw data provides a first hint that polygamy reduces fertility, and differentially so according to the type of second wife. It also suggests that the timing of the second marriage is not correlated with the history of previous births.

These preliminary results will be confirmed and strengthened by the identification stra-

tegy presented below. I consider two specifications : a linear model with fixed effects and a duration model with individual baseline hazards. Through the explicit modeling of unobserved heterogeneity, they allow a causal interpretation under credible identification assumptions.

4.2 Econometric specifications

The main idea is to take advantage of the panel structure of the data. Indeed, I observe several birth intervals for a given woman i, some of them occurring before and others after the second marriage. [22]

The dependent variable is the duration between births j and $(j+1)$, denoted $t_{i,j}$, measured in months. The vector $x_{i,j}$ contains observed time-varying explanatory variables : woman's age and age squared at birth j. I also include a dummy for each birth rank $j \in [2, J]$ where J is the highest parity observed ; the reference category consists of intervals after the first birth. The term ν_i is meant to capture all the determinants of birth intervals that may vary across women, but not across birth ranks for a given woman.

In a first step, I estimate the total impact of a change in union status, from monogamous to polygamous. The covariate of interest is the dummy $After_{i,j}$, which is is equal to one if the second wife has arrived when the child $(j + 1)$ was conceived by woman i. This specification allows me to test Prediction 1. In a second step, I test if the impact is heterogenous by interacting $After_{i,j}$ with $T_{2,i}$, the reproductive period of the second wife faced by woman i. It provides an empirical test for Prediction 2.

I include monogamous wives in the sample to improve the precision of the estimates. They do not contribute to estimating the impact of polygamy, because $After$ is always equal to zero for them, but they help estimating the coefficients on $x_{i,j}$ and on the birth rank dummies. As a robustness test, I check that estimates are very similar if I exclude monogamous wives

22. The impact of the second marriage on the first wife's fertility is identified on women with at least two births before the second marriage and at least one birth after the second marriage. It is the case for roughly 40% of first wives. In the end, 318 birth intervals contribute to the identification.

(cf. Table E.3, columns 5 and 6).

I use robust standard errors clustered at the woman level to account for the correlation between the error terms related to the different birth intervals of the same woman.

4.2.1 Linear model with fixed effects

I start with a linear model to get a sense of magnitude of the effects. The specifications write :

$$t_{i,j} = \alpha_0.After_{i,j} + \beta.x_{i,j} + \sum_{k=2}^{k=J} \eta_k.\mathbb{1}_{k=j} + \nu_i + \epsilon_{i,j}$$

$$t_{i,j} = \alpha_1.After_{i,j} + \alpha_2.After_{i,j} \times T_{2,i} + \beta.x_{i,j} + \sum_{k=2}^{k=J} \eta_k.\mathbb{1}_{k=j} + \nu_i + \epsilon_{i,j}$$

Where $\epsilon_{i,j}$ is an idiosyncratic error term, and $\mathbb{1}_{k=j}$ is equal to one for birth rank j. In these specifications, I only consider non-censored durations (i.e. closed intervals). ν_i is treated as a woman fixed effect. I estimate α_0, α_1 and α_2 using the within estimation method.

Under the identification assumptions specified below, the linear model predicts that polygamy causes an average change in birth intervals by α_0 months. In the theoretical model, this quantity is given by $(1/\lambda_1^* - 1/\lambda_0^*)$. If $\alpha_0 > 0$, birth spacing lengthens in the polygamous stage. Moreover, the change in case of $T_2 = 0$ is predicted to be α_1 ; each additional year in T_2 translates into α_2 additional months. If $\alpha_2 < 0$, birth spacing lengthens less (if $\alpha_0 > 0$) or shorten more (if $\alpha_0 < 0$) when T_2 is longer.

4.2.2 A duration model of birth intervals with individual hazards

Now I turn to duration models, which are closer in spirit to the theoretical model and better suited to the nature of the dependent variable. They make it possible to exploit information from right-censored durations (i.e. time intervals between the birth of the latest child and the date of the survey). Another advantage of duration models is that $After_{i,j}$ may

vary within a spell j, and not only across spells. It captures more precisely the date of the change than in the linear model.[23] Formally, I consider a mixed proportional hazard model with multi-spell data. The hazard functions write :

$$\theta(t|x_{i,j}, v_i) = \theta_0(t, \nu_i) \times exp(\alpha'_0.After_{i,j} + \beta'.x_{i,j} + \sum_{k=2}^{k=J} \eta'_k.\mathbb{1}_{k=j})$$

$$\theta(t|x_{i,j}, v_i) = \theta_0(t, \nu_i) \times exp(\alpha'_1.After_{i,j} + \alpha'_2.After_{i,j} \times T_{2,i} + \beta'.x_{i,j} + \sum_{k=2}^{k=J} \eta'_k.\mathbb{1}_{k=j})$$

In these specifications, the baseline hazard θ_0 is specific to each woman. There is no restriction on the interaction of ν_i with the elapsed duration t in the hazard function. Moreover, no assumption on the tail of the distribution of the unobservables is needed. The main technical assumption is the proportional hazard assumption that I tested and failed to reject (cf. Figure E.1 in Appendix E). I estimate α'_0, α'_1 and α'_2 using a stratified partial likelihood.[24]

In proportional hazard models, coefficients are interpreted as hazard ratios. $exp(\alpha'_0)$ measures the hazard ratio between births occurring after and births occurring before the second marriage. In the theoretical model, it corresponds to λ^*_1/λ^*_0. If $exp(\alpha'_0) < 1$, birth spacing lengthens in the polygamous stage. The hazard ratio after-before also writes $exp(\alpha'_1) \times exp(\alpha'_2.T_2)$. If $exp(\alpha'_2) > 1$, the ratio increases with T_2. It means that birth spacing lengthens less (if $exp(\alpha'_0) < 1$) or shortens more (if $exp(\alpha'_0) > 1$) when T_2 is longer.

Note that the duration model predicts hazard rates while the linear model predicts durations. The higher the hazard rate, the shorter the duration. So the predictions of both models are consistent if their estimates have opposite signs. In particular we should have $\alpha > 0$ iff $exp(\alpha') < 1$.

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23. An alternative identification strategy would be to rely on the timing-of-events methodology developed by Abbring and den Berg (2003) that would exploit the variation in the union status occurring within a spell. Unfortunately, I only observe the year of the second marriage so there are large measurement errors when I split the spell into two intervals measured in months.

24. The method is described in more details in Appendix D. In the same way as in linear models, it is based on a suitable transformation that eliminates the ν.

4.3 Identification assumptions

Both specifications allow the explanatory variables and ν to be dependent. It means that fertility choices and the occurrence and timing of the second marriage may be jointly determined by some unobserved characteristics, provided that these characteristics are fixed in time (e.g. husband's taste for a large family). The key identification assumption is a strict exogeneity condition : the idiosyncratic error term ϵ should be uncorrelated with $After$ and T_2 of all past, current and future spells of the same individual. It corresponds in part to the assumption of the theoretical model that the second marriage is not caused by fertility choices in the monogamous stage.

It also entails a condition on the evolution of fertility during the woman's life. In the model, I abstract from such dynamics by assuming that natural birth rates were constant in time. In fact, birth rates tend to evolve with the life-cycle. Using a fixed effect specification, I find that the same woman (i) at a given age, has longer intervals if the birth rank is higher, and (ii) at a given rank, has shorter birth intervals if she is older, up to a certain age above which the relationship is reversed. Since $After$ is mechanically correlated with birth ranks and woman's age, I need to carefully address the issue to be sure that my test is not capturing spurious dynamics. I start by including woman's age and age squared at birth and a dummy for each birth rank as controls. [25] This is satisfactory iff the effect of the life-cycle does not differ between polygamous and monogamous wives, nor among polygamous wives depending on the timing of the second marriage. I call this assumption the common life-cycle assumption. It is close to the common trend assumption in a difference-in-difference framework. In the same way, it is testable on births that occurred *before* the second marriage.

In the robustness section, I will present empirical tests to provide some support to these assumptions.

25. The impact of birth ranks and the impact of $After$ are separately identified because the second marriage takes place at different parities for different women.

5 Results

5.1 Main results

Testing Predictions 1 and 2 Table 8 reports the estimation of the linear model with fixed effects. Polygamy causes a significant increase in birth intervals, by seven months. The impact is highly heterogeneous : 14 months for first wives facing a weak competitor against zero for those facing a strong competitor. These estimates are in line with the raw data. The econometric model further predicts that intervals lengthen by approximately two years when $T_2 = 0$, and that an increase by one year in T_2 reduces this change by one month. The coefficient on the interaction term is significant at 10%. It remains the same when I control for $After$ interacted with S and with T_1.

The duration model with individual hazard gives similar results. Table 9 reports $exp(\alpha')$. The hazard ratio after-before is lower than one, although not significant. [26] It suggests that birth rates are lower after the second marriage, i.e. intervals are longer. Next, the hazard ratio on the interaction term is larger than one, and significant at 5%, meaning that first wives lengthen less when T_2 is longer. Again, results are robust to the introduction of interaction terms with S and with T_1.

Going back to Equation 5, these empirical results imply that $\lambda_1^* < \lambda_0^*$ while $B_1 > 0$. The overall impact of the second marriage is negative because the exposure effect is large enough to dominate the positive strategic effect. Since the strategic effect is proportional to n_2^*, it is only possible when n_2^* is not too large, more precisely when $n_2^* < \frac{\theta^n.(\lambda_m^{nat} - \lambda_p^{nat}).(T_1 - S)}{\epsilon_1 - \theta_h^1}$. I need to make further assumptions on the values of the parameters to assess whether this condition is likely to hold in the majority of households. I use estimations on monogamous unions for $\theta^h \approx 1/2$, $\theta^n \approx 3$ and $\lambda_m^{nat} \approx 1/3$. Under the additional assumption that $\lambda_p^{nat} = \frac{\lambda_m^{nat}}{2}$ and

26. The p-value is 0.16. The ratio becomes significant at conventional levels when the specification is slightly modified (cf. Table E.3 discussed in the robustness checks).

$\epsilon = 1$, the condition rewrites $n_2^* < (T_1 - S)$. This is verified in 90% of households.[27]

5.2 Secondary results

5.2.1 Second wives

What about second wives? In this section, I provide suggestive evidence that they do behave strategically too. However, the test is less conclusive because I only observe second wives in the polygamous stage. So it is not possible to account for unobserved determinants of fertility. One might be worried that some of those are correlated to the type of marriage the woman ends up in. The best I can do is controlling for a wide range of characteristics : the predictors of the ideal number of children identified in Table A.3, characteristics of husband's occupation (income and a dummy for the public sector) which were not available in DHS but are likely to influence preferences, co-residence status, having children from previous unions and having dead children from current union. I estimate a Cox model of birth intervals with a baseline hazard common to all women. I include durations between marriage and first birth, because they convey useful information on second wives' reactions.

Testing Prediction 3 The idea, again, is to test if the second wife responds to the length of the first wife's reproductive period. If her strategic response is also positive, I should observe that birth intervals are shorter when S and $(T_1 - S)$ are longer. When I know the complete birth history of the first wife, I can deduce n_{ini}, the number of children she had at the time of the second marriage. Prediction 3 would be modified to state that birth intervals of second wives should be shorter when n_{ini} is higher, holding $(T_1 - S)$ constant. n_{ini} is much more informative than S because it captures the optimal birth rate of the first wife.[28] Table 10 summarizes the test using S in column 1 and n_{ini} in column 2. Signs are

27. The distribution of the second wife's final number of children (excluding infertile unions) is $Q_1 = 2$, $Q_2 = 4$ and $Q_3 = 6$ while the distribution of $T_1 - S$ in the same sample is $Q_1 = 9$, $Q_2 = 16$ and $Q_3 = 22$.

28. Empirically, the duration of the monogamous period is a strong predictor of the number of children already born at the time of the second marriage. If I regress n_{ini} on S and no constant, I find a coefficient

in line with expectations. The hazard rate is positively correlated with all predictors of the first wife's completed fertility : $(T_1 - S)$, S and n_{ini}. If I break down the effects by birth ranks, they are particularly strong on the duration between marriage and first birth. All this suggests that second wives also intensify their fertility when they face a more fertile rival.

5.2.2 Completed fertility

To test the predictions on the total number of children, I consider the sub-sample of women over 45 years old, who have reached the end of their reproductive life, in order to avoid censoring issues. Since there is only one observation per woman, sample sizes are reduced. As described above, I control for many determinants of fertility to mitigate potential omitted variable issues, but this is not as satisfactory as panel data analysis. Under this caveat, looking at completed fertility provides circumstantial evidence that the impact of strategic interactions on birth spacing patterns translates into sizeable differences in terms of total number of children.

Testing Prediction 4 Starting with first wives, Table 11 shows that the number of children increases with the duration of their own reproductive period, with the duration of the monogamous period, and with the duration of the co-wife's reproductive period. First wives are predicted to eventually have two children fewer when the second wife has a short time left before fertility decline (less than 10 years).

Testing Prediction 5 Turning to second wives, although the sample of women over 45 years old is small, Table 12 suggests that their own reproductive period has a positive impact on completed fertility. Moreover, second wives have three children fewer when the monogamous period was short (below the median), which left little time for the first wife to have many children. And they have two children fewer when the first wife has no time left to react after the second marriage. Both effects are consistent with a positive strategic

of 0.24 highly significant. It corresponds to one birth every four years on average, or five children in 20 years of marriage.

response of second wives.

To sum up, all empirical tests point to children being strategic complements in polygamous unions : characteristics raising the fertility of one wife intensify the fertility of her co-wife.

5.3 More empirical evidence based on the gender of children

So far, I have considered only the quantity of children, putting quality aside. Nonetheless, it might be argued that co-wife rivalry is also about children's characteristics such as, for instance, educational achievement, social success, commitment to norms or responsibility taken in the family welfare. According to the literature on Africa, one characteristic plays a key role : gender. Having sons substantially improves women's status and security (Lesthaeghe 1989). It is particularly true in patrilineal ethnic groups, and where the influence of the Islamic law is strong, like in Senegal where 95% of the population is Muslim. In a previous work on Senegal, I show that women have a stronger preference for sons when the current husband already has children from ex-wives, either divorced or deceased. The explanation rests on the rivalry for inheritance between the husband's children. In presence of children from ex-wives, current wives need a son to secure access to their late husbands' resources in case of widowhood (Lambert and Rossi 2014). The same rationale might be at play in polygamous households, and even exacerbated by the rivalry for *current* resources, be it material or emotional.

The hypothesis I want to test in this section is that the gender of children matters in polygamous households. Ideally, I would like to predict how the birth of a boy vs. a girl impacts the subsequent optimal birth rate of both wives, and whether this effect depends on the gender composition of the other wife. However, my model of fertility choices lacks a true time dimension to adequately account for the uncertainty related to a child's gender. Therefore, I build on the above-mentioned work on the rivalry with ex-wives to derive the

following predictions regarding co-wives. On the one hand, the arrival of a second wife should exacerbate the preference for sons of first wives. Indeed, they move from a situation in which no other child can compete with their own offspring, to a situation with rivals. So the necessity to have a son should be stronger in the polygamous stage. On the other hand, the behavior of the second wife should depend on the gender composition of the first wife's children, boys representing a more serious threat than girls. The second wife's fertility should therefore increase more with the number of boys than with the number of girls already born to the first wife.

To test the prediction on the change in son preference of first wives, I rely on a duration model of birth intervals with individual baseline hazards. I introduce a dummy $No\ son$ equal to one if the woman had no son at the time of the index birth. This variable varies across births and captures the impact of having only daughters vs. at least one son on the next interval, holding birth rank and age at birth constant. If the hazard ratio is larger than one, meaning that having only daughters decreases the expected interval, then one can infer the existence of son preference.[29] Then I interact the gender composition with the dummy $After$ to test whether the arrival of the second wife has an impact on son preference. Results for bigamous unions are reported in Table 13. Son preference exists in the monogamous stage : the hazard ratio on $No\ son$ is significantly greater than one. But it is substantially exacerbated by the second marriage : the hazard ratio on the interaction term, capturing the difference in son preference before and after, is equal to two. Controlling for life-cycle dynamics, *the same woman* is predicted to display much larger son preference once her rival has arrived than she used to do.

Regarding the prediction on second wives, I assume that the baseline hazard is common to all women, and that all unobserved heterogeneity is accounted for by the controls. I split

29. Rossi and Rouanet (2015) discuss in more details how to infer the existence of gender preferences using duration models of birth intervals.

n_{ini}, the first wife's number of children at the time of the second marriage, into $Boys_{ini}$, her number of boys and $Girls_{ini}$, her number of girls. I restrict the analysis to bigamous unions. Table 14 shows that second wives react more to the number of boys than to the number of girls. Hazard ratios on $Boys_{ini}$ and $Girls_{ini}$ are both larger than one, but the former is 60% larger than the latter and only $Boys_{ini}$ has a significant impact.

Both tests suggest that co-wife rivalry raises the relative value of sons against daughters. It is an additional piece of evidence that potential and realized fertility outcomes of one wife influence the behavior of the other wife.

6 Robustness and Placebo Tests

6.1 Robustness checks

I start by checking that the main results are robust to changes in the duration model specification. Results are reported in Table E.3 in Appendix E. In columns 1 and 2, I estimate the impact of polygamy on the very next birth interval rather than the impact on all subsequent intervals. Coefficients are larger in magnitude and more significant than in the baseline specification. One interpretation is that the difference between λ_m^{nat} and λ_p^{nat} would tend to die out as spouses get older. The exposure effect would be strong at the beginning of the polygamous stage, and would gradually disappear over time.[30]

In columns 3 and 4, I exclude first wives with more than one co-wife, and in columns 5 and 6, I exclude monogamous wives. In columns 7 and 8, I impose that $After = 1$ for the whole spell during which the second marriage takes place, which is the approximation made in the linear model. Estimates are very similar to the baseline results in terms of magnitude.

30. Another explanation is based on learning effects. In a more complex game, the preferences of each wife could be private information and both wives would update their beliefs as and when the other wife gives birth. Such a setting is beyond the scope of this paper, but it could help explaining the dynamic reaction of first wives.

The hazard ratio after-before, which is close to significance in the baseline specification, is significant at 10% when I focus on bigamous unions, and significant at 1% when $After$ does not vary within a spell.

The last concern is that I do not observe the birthdate of deceased children, which might lead me to overestimate the true duration between successive births. I check in columns 9 and 10 that the main findings still hold when I restrict the sample to women who lost no child. Estimates are even slightly larger in magnitude and more significant.

6.2 Testing identification assumptions

First I test the assumption that the occurrence and characteristics of the second marriage are not caused by fertility choices in the monogamous stage. The idea is to examine the empirical relationship between λ_0 on the one hand, and the union type, S and T_2 on the other hand. [31] According to the model, these characteristics are unknown in the monogamous stage so they cannot directly influence λ_0. However, they might be correlated with $n^{id}_{w,1}$, n^{id}_h and T_1 and hence with λ_0. For instance, the descriptive statistics have shown that polygamous husbands report wanting more children. One way to test the model is to regress λ_0 on the union type, S and T_2, controlling for $n^{id}_{w,1}$, n^{id}_h and T_1. If there is no reverse causality, the correlation should be nil.

Table E.4 in Appendix E reports the estimates of a Cox proportional hazard model. The first column deals with the occurrence of polygamy. I compare the birth rates of first wives before the second marriage to the birth rates of monogamous wives. I construct a dummy "Future first wives" indicating if the mother is in a polygamous union at the time of the survey. I restrict the sample to women over 40 to ensure that most women in the reference category "monogamous wives" will remain the sole wife in this union. Controlling specifically for marriage duration and predictors of preferences, I find that first wives do behave like

31. Ideally I would implement the same test with $n^{id}_{w,2}$ but I do not observe the preferences in PSF.

monogamous wives as long as the second wife has not arrived. So there is no evidence that some specific fertility patterns would drive the likelihood of polygamy. The second column is about the timing of polygamy. I consider first wives before the second marriage and I test if T_2 and S are systematically correlated with birth rates. Again, holding T_1 and predictors of preferences constant, I find that choices in the monogamous stage do not predict the timing of the second marriage.

Second, I turn to dynamic effects. The identification assumption is that intervals may depend on birth ranks, but the effect of birth ranks should be uncorrelated with the occurrence and timing of polygamy. It is possible to test if this holds during the monogamous stage using a duration model with individual hazards. Results are reported in Table E.5 in Appendix E. In the first column, I interact birth rank dummies with "Future first wives" to compare the effect of the life-cycle between monogamous wives and first wives before the second marriage. I find that hazard ratios are all the more lower than one as the birth rank rises, meaning that the higher the parity, the longer the birth interval. But not differentially so according to the union type. Coefficients on the interaction terms are close to one, and never significantly different from one. In the second column, I restrict the sample to future first wives in the monogamous stage and I interact birth rank dummies with T_2. Again, there is no evidence of an heterogeneous birth rank effect.

Another way to ensure that the ratio after-before is not capturing spurious dynamics is to run a placebo test using a duration model with individual hazards. The idea is to replace the true date of the second marriage S by alternative cut-offs : the mean, the first quartile and the last quartile taken from the distribution of S in the sample. I consider two specifications : the baseline with all intervals, and the specification using only the very next birth interval. If the coefficient on $After$ was purely driven by a decline in fertility over time, it should remain stable whatever the cut-off. Table E.6 in Appendix E reports the new ratios : they are much closer to one than the baseline ratio, and never significantly different from one.

Therefore, the placebo test suggests that the change in first wives' birth spacing precisely coincides with the second marriage.

6.3 Alternative models without strategic interactions

There are other frameworks describing fertility choices in polygamous unions that do not rely on strategic interactions. Could they provide an alternative explanation for my results ?

For instance, in Lardoux and van de Walle (2003), the natural birth rate is the only driver of fertility, and it may differ across wives depending on sexual favoritism. The authors hypothesize that husbands may play favorites, hence generating differential birth rates between co-wives. They further explain that husbands are more likely to favor the youngest wife so that the natural rate of a given wife would decrease with her age and increase with the age of her co-wife. One testable implication is that, controlling for her own age, the birth rate of first wives should be lower when they face a younger rival. It implies that λ_1 should decrease with T_2, which is at odds with empirical findings.

In the same vein, birth rates might simply reflect living arrangements. In the data, husbands are less likely to live with their second wives when T_2 is shorter. It makes sense because older second wives might be living in the house of a deceased husband or with an adult son. Moreover, T_2 has no significant impact on the probability to live with the first wife. All in all, husbands tend to spend more time with the first wife when T_2 is shorter, which should raise the exposure to pregnancy risk. This story predicts, again, a negative correlation between λ_1 and T_2.

Overall, the empirical results cannot be explained by a potential unobserved heterogeneity in the exposure effect ; if they were indeed at play, these mechanisms would only attenuate my prediction.

7 Discussion

For a long time, controlling population growth has been a concern for policy makers. The general view is that population becomes a problem when it starts growing faster than resources, because such a dynamic leads to general impoverishment.[32] In recent years, the rising environmental awareness has brought the issue back up (Sachs 2008). Since costs and benefits to children are partly passed on society, there is a role for public policies to influence fertility decisions made by parents. Health concerns are another rationale for state intervention, insofar as frequent pregnancies raise the risk of maternal and child mortality. In Africa, more and more effort has been devoted to reducing the rate of population growth. In 1976, policies to lower the level of fertility were implemented in only 25% of African countries, while that proportion increased up to 83% in 2013 (United Nations 2013). Measures are generally taken on the supply-side to increase access to family planning services and to birth control methods. However, identifying the determinants of the demand for children is crucial to design adequate policies. By providing a simple framework to analyze which forces drive fertility, this paper shows that policy makers should definitely take into account the structure of households.

Policy recommendations to curb fertility have generally been designed for monogamous societies. They build on the fact that, on average, wives want fewer children than their husbands, and fewer children than they would have in a natural fertility regime. For instance, Sen (1999) argues that the main drivers of fertility transition are improving women's bargaining power and facilitating access to birth control methods. In my framework, it means alleviating the marital and natural constraints weighing on monogamous women. Indeed, the lower θ^h and θ^n, the closer n^{NS} to n_w^{id}, and therefore, in general, the lower n^{NS}.

However, these standard recommendations might be counterproductive in polygamous

32. For a review of the literature over the so-called "population problem", see Dasgupta (1995).

societies, because they overlook reproductive externalities. Empirical tests carried out in this paper show that, in Senegal, polygamy is associated with lower birth rates at the micro level.[33] This is because the overbidding effect is dominated by the sum of substitution effect and exposure effect. But this might change if women's choices are less and less constrained by husbands and social norms. Indeed, the negative correlation between polygamy and fertility is driven by the marital and natural constraints. If θ^h and θ^n decrease, the relative magnitude of the overbidding effect rises. It makes it more likely to observe an overall positive impact of polygamy on fertility. The correlation might also become positive if couples comply less with intercourse taboos regulating marital duties. In this case, the gap between λ_m^{nat} and λ_p^{nat} might be closed and the exposure effect would disappear.

A key lever to curb fertility in polygamous societies is acting on the causes of co-wife reproductive rivalry, that is decreasing ϵ. Tackling the emotional dimension is probably a long-term endeavor, but policy makers could start by reducing women's economic reliance on children. One way ahead is to give more opportunities for self-support to women. Concretely, it means easing labor market restrictions and constraints stemming from social norms to improve female labor force participation. Another recommendation would be to reform family law in order to improve women's status in terms of property rights and inheritance rights ; for instance by entitling wives to a significant share of the husband's bequest irrespective of the number of children.

8 Conclusion

This paper proposes a strategic framework to account for fertility choices in polygamous unions. Using data from Senegal, it shows that children of co-wives are strategic complements.

33. Note that, at the macro level, polygamy has always been associated with higher fertility (Lesthaeghe 1989). This institution is indeed closely related to early marriages and quick remarriages of women. Although the natural birth rate is lower, the length of women's exposure to marriage is maximized, so that, overall, fertility is higher in polygamous societies.

Exposing such reproductive externalities may have strong implications for population policies in Africa. The general consensus is that giving women a greater say in fertility decisions and more efficient means to implement them are key drivers of the fertility transition. I claim that these standard recommendations might be inefficient in polygamous societies, as long as co-wife rivalry generates incentives for women to want many children. Women's empowerment must be understood in a much broader sense : disassociating women's status and economic security from their offspring is a prerequisite for curbing the demand for children.

More generally, policy makers should take into account strategic interactions in the household when designing population policy instruments. As already noted by Alderman, Chiappori, Haddad, Hoddinott, and Kanbur (1995), the unitary view of the household may ignore or obscure important policy issues that are especially relevant in the context of developing economies. This paper is one of the few attempts to open the black box of non-nuclear households, but further research is needed to understand decisions related to most topics on the development agenda such as poverty, migration, labor supply, home production, land tenure, health and education.

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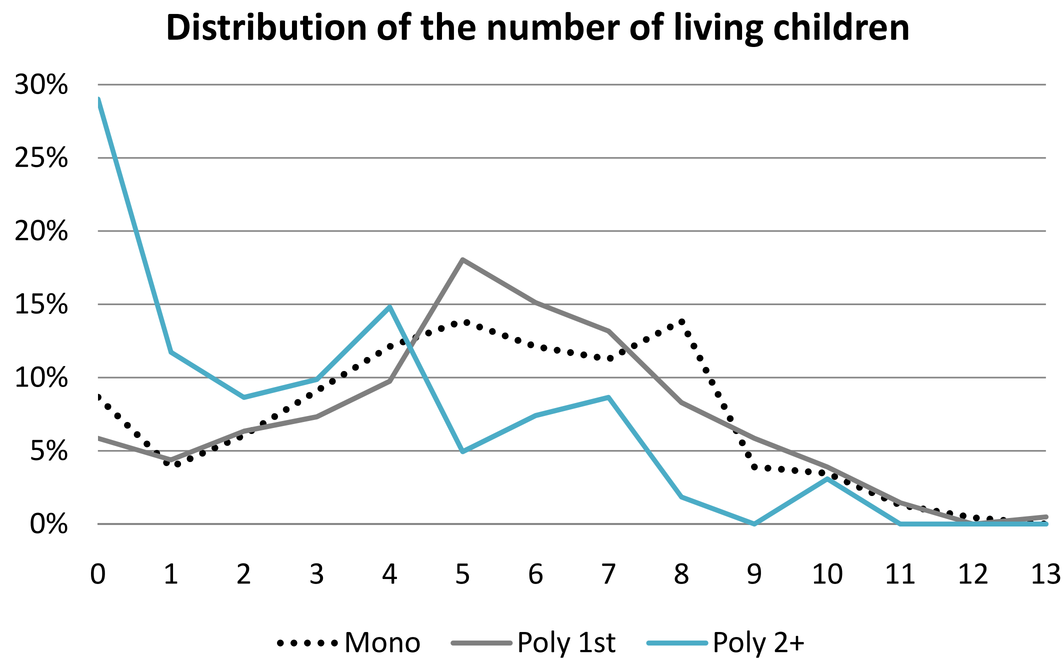
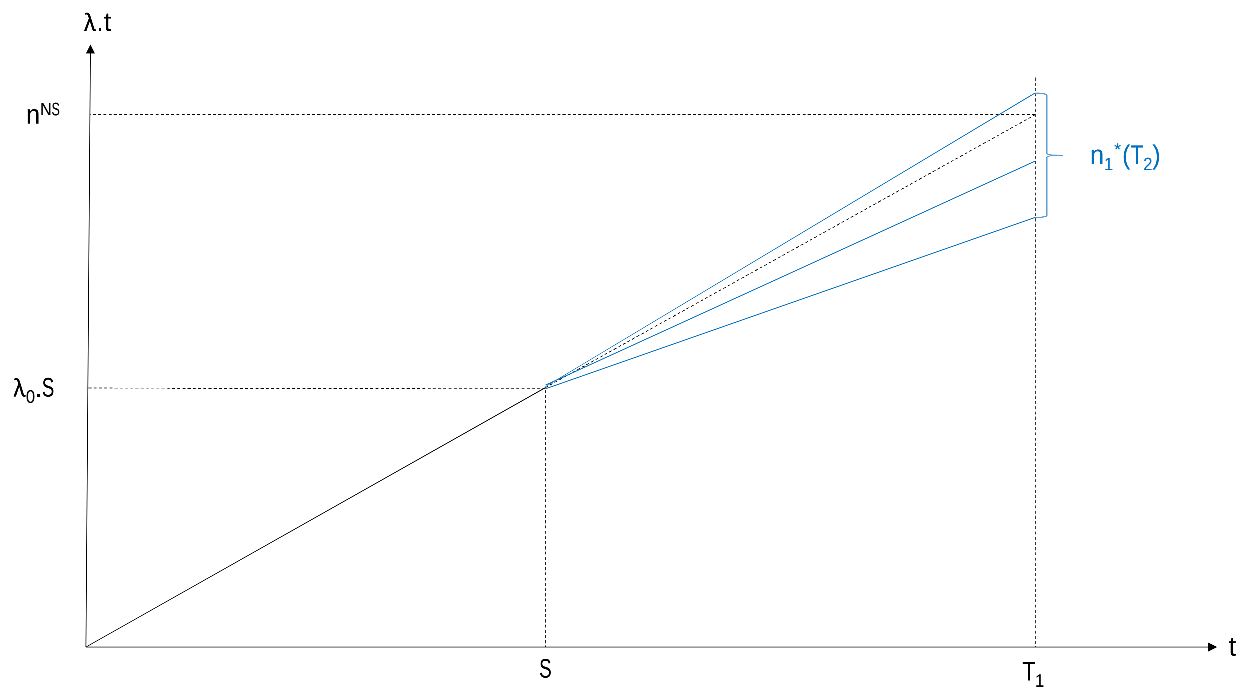
Figures and Tables

Figure 1: Number of children in current union, by union type



Data : PSF. Sample : married women over 45 years old. Mono= monogamous wives (231 obs), Poly 1^{st}= senior wives (205 obs) and Poly 2+= junior wives (162 obs).

Figure 2: Theoretical change in first wives' birth rate



The graph plots the first wife's realized number of children $\lambda.t$ as a function of time. In the monogamous stage, between $t = 0$ and $t = S$, the wife targets n^{NS} and hence chooses λ_0. At date S, the polygamous stage starts. The wife updates her choice and chooses λ_1 in order to reach n_1^* at the end of her reproductive period T_1. n_1^* can be above or below n^{NS} depending on the sign and magnitude of the strategic response. According to the model, the change in birth rate should depend on the reproductive period of the other wife T_2. When the strategic response is positive, λ_1 should be higher when T_2 is longer; indeed, an increase in T_2 means more time for the second wife to have children, which raises the fertility of the both wives at equilibrium.

Table 1: Number of children in current union, by union type

Sample	All		At least one child		
Constant (Monogamous union)	5.074***	5.074***	5.477***	3.687***	4.364***
	(0.183)	(0.170)	(0.157)	(0.338)	(0.373)
Polygamous union	-1.071***				
	(0.232)				
Senior wife		0.188	0.138	-0.130	-0.139
		(0.250)	(0.230)	(0.228)	(0.225)
Junior wife		-2.512***	-1.370***	-0.933***	-0.948***
		(0.260)	(0.270)	(0.279)	(0.276)
T				0.081***	0.059***
				(0.013)	(0.014)
Children from previous unions					-1.236***
					(0.312)
Observations	674	674	568	551	550

Data : PSF. Sample : women over 45 years old. In the last three columns, I restrict the sample to women having at least one child with their current husband.
Dep. Var. : number of children in current union. T = min (45- wife's age at marriage ; 60-husband's age at marriage) : length of couple's reproductive period.
Significance levels : * p<0.10, ** p<0.05, *** p<0.01.

Table 2: Ideal number of children

Dep. var.	n_w^{id}	n_h^{id}	$n_h^{id} - n_w^{id}$
Sample	Wives	Husbands	Couples
Mean	5.7	9.0	3.1
Std. Dev.	2.3	5.7	5.6
Q_1	4	5	0
Q_2	5	7	2
Q_3	7	10	5
Observations	2535	2291	1824

Data : DHS, waves 2005 and 2010. Weights.
n_w^{id} and n_h^{id} are the ideal number of children reported by women and men, respectively. The last column shows the difference between these numbers within a couple.
The number of observations is different in each column because statistics are computed on the sample of individuals who gave a numerical answer for the first two columns, and on the sample of couples in which *both* spouses gave a numerical answer for the last column. "Non-numerical answer" means that the respondent answered "I don't know" or any non-numerical statement (e.g. "it is up to God") to the question "How many children would you like to have, or would you have liked to have, in your whole life ?".

Table 3: Timing of unions

Median age	Husband	1^{st} wife	2^{nd} wife
First marriage	28 (Q1=24 ; Q3=32)	17 (Q1=15 ; Q3=21)	na
Second marriage	40 (Q1=34 ; Q3=46)	29 (Q1=24 ; Q3=36)	22 (Q1=17 ; Q3=31)

Data : PSF. Sample : polygamous unions (411 unions).

If I restrict the sample to bigamous unions (321 unions), all median ages increase by one year.

Table 4: Estimating the model on monogamous and polygamous unions

Sample	Monogamous	Polygamous
n_w^{id}	0.232**	-0.024
	(0.092)	(0.096)
n_h^{id}	0.128**	0.067*
	(0.064)	(0.039)
T	0.205***	0.188***
	(0.046)	(0.047)
Constant	-1.083	1.785
	(0.985)	(1.302)
R^2	0.36	0.14
pval $n_w^{id} = n_h^{id}$	0.36	0.39
Observations	109	151
Structural parameters		
θ^h	0.55	na
θ^n	2.77	na
λ^{nat}	0.32	na
n^{nat} (mean)	8.30	na

Data : DHS. Sample : women over 40, having at least one child. Women with broken unions are excluded because I only know the date of first marriage, so I am able to deduce the timing of successive marriages only when all wives are in their first marriage. For monogamous unions, I also restrict the sample to husbands in their first union. Weights.

Dep. Var. : total number of births. T = min (45- wife's age at marriage ; 60-husband's age at marriage). $n^{nat} = \lambda^{nat} \times$ mean(T) with mean$(T) = 26$.

Significance levels : * p<0.10, ** p<0.05, *** p<0.01.

For polygamous unions, the model specification is not right. Since the coefficient on n_w^{id} is not significantly different from zero, I cannot compute the estimates of the structural parameters.

Table 5: Summary of main notations

Dependent variables	
λ_0^*	First wife's birth rate in the monogamous stage
λ_1^*	First wife's birth rate in the polygamous stage
λ_2^*	Second wife's birth rate in the polygamous stage
n_1^*	First wife's completed fertility
n_2^*	Second wife's completed fertility
Explanatory variables	
T_1	Length of first wife's reproductive period
T_2	Length of second wife's reproductive period
S	Length of the monogamous stage
Parameters	
θ_i^h	Weight on marital constraint in the utility of wife i
θ_i^n	Weight on natural constraint in the utility of wife i
ϵ_i	Intensity of co-wife rivalry for wife i
B_i	Strategic response of wife i ($B_i = \frac{\epsilon_i - \theta_i^h}{1+\theta_i^h+\theta_i^n}$)

Table 6: Summary of predictions

Label	Statement	Test
	Main predictions	
Prediction 1	$\lambda_1^* - \lambda_0^* < 0$ iff the exposure effect dominates the strategic effect.	Tables 8 and 9
Prediction 2	$\frac{\partial \lambda_1^* - \lambda_0^*}{\partial T_2} > 0$ iff the strategic effect is positive.	Tables 8 and 9
	Secondary predictions	
Prediction 3	$\frac{\partial \lambda_2^*}{\partial S} > 0$ and $\frac{\partial \lambda_2^*}{\partial (T_1 - S)} > 0$ iff the strategic effect is positive.	Table 10
Prediction 4	$\frac{\partial n_1^*}{\partial T_2} > 0$ iff the strategic effect is positive. $\frac{\partial n_1^*}{\partial T_1} > 0$, $\frac{\partial n_1^*}{\partial S} > 0$.	Table 11
Prediction 5	$\frac{\partial n_2^*}{\partial S} > 0$ and $\frac{\partial n_2^*}{\partial (T_1 - S)} > 0$ iff the strategic effect is positive. $\frac{\partial n_2^*}{\partial T_2} > 0$.	Table 12

TABLE 7: Birth intervals before and after the second marriage, by type of competitor

Birth intervals in months	Whole sample	Weak competitor	Strong competitor
Before	37.6	37.8	37.1
	(21.0)	(21.9)	(19.0)
nb obs.	216	147	69
After	43.6	47.4	41.8
	(26.2)	(31.1)	(23.6)
nb obs.	184	58	126
Difference after-before	6.0**	9.6***	4.7
	(2.4)	(3.6)	(3.5)

Data : PSF. Average birth intervals in months. Standard deviations are in parentheses. Non-censored durations.
Sample : first wives below 45 years old, for whom the complete birth history is known, having at least one child from current union ; the sample is further split on the median T_2 (20 years) into those facing a weak competitor in the second column (T_2 below the median) and those facing a strong competitor in the third column (T_2 above the median).
The last line reports the difference after-before. Standard errors are in parentheses. Significance levels : * p<0.10, ** p<0.05, *** p<0.01.

TABLE 8: Change in first wives' birth spacing : linear model

Birth intervals in months	Whole sample (1)	Weak compet. (2)	Strong compet. (3)	Interaction (4)	Control S and T_1 (5)
After	6.988**	14.000***	0.698	25.613**	-6.465
	(3.491)	(4.975)	(4.260)	(10.280)	(20.758)
After * T_2				-0.994*	-1.164**
				(0.513)	(0.586)
Controls	Birth rank dummies, mother's age and age squared at birth j				
Woman FE	Yes				
Observations	1715	1509	1508	1715	1711
Clusters	597	525	532	597	596

Data : PSF. Dep. var. : duration between births j and $(j+1)$. Non-censored durations. Extreme values are excluded (larger than seven years, top 5%). Results are qualitatively similar when I include them.
Sample : monogamous and first wives below 45 years old, for whom the complete birth history is known, having at least one child from current union ; first wives are further split on the median T_2 (20 years) into those facing a weak competitor in the second column (T_2 below the median) and those facing a strong competitor in the third column (T_2 above the median).
$After$ is a time-varying variable indicating if the second wife has arrived when the child $(j+1)$ was conceived.
T_2 = min (45-second wife's age at marriage ; 60-husband's age at second marriage). In the last column, I control for $After$ interacted with S and with T_1.
Linear estimation with woman fixed effects ; robust standard errors of the coefficients are in parentheses (clustered at the woman level). Significance levels : * p<0.10, ** p<0.05, *** p<0.01.

TABLE 9: Change in first wives' birth spacing : duration model

Hazard ratios	Total impact	Heterogenous impact	Control S and T_1
After	0.770	0.201**	0.321
	(0.143)	(0.140)	(0.360)
After * T_2		1.077**	1.081*
		(0.036)	(0.047)
Controls	Birth rank dummies, mother's age and age squared at birth j		
Baseline hazard	Woman-specific		
Observations	2483	2483	2478
Clusters	716	716	715

Data : PSF. Dep. var. : duration between births j and $(j+1)$. Sample : monogamous and first wives below 45 years old, for whom the complete birth history is known, having at least one child from current union. $After$ is a time-varying variable indicating if the second wife has arrived. $T_2 =$ min (45-second wife's age at marriage ; 60-husband's age at second marriage). In the last column, I control for $After$ interacted with S and with T_1.

Stratified partial likelihood estimation with baseline hazards specific to each woman ; Breslow method to handle ties among non-censored durations. Robust standard errors of the coefficients are in parentheses (clustered at the woman level). Significance levels (for hazard ratio = 1) : * p<0.10, ** p<0.05, *** p<0.01.

Table 10: Second wives' birth spacing : λ_2^*

Hazard ratios	Using S	Using n_{ini}
T_2	0.961	0.874*
	(0.032)	(0.063)
$T_1 - S$	1.011	1.166***
	(0.015)	(0.063)
S	1.043***	
	(0.015)	
n_{ini}		1.258**
		(0.119)
Specific controls	Predictors of preferences	
Additional controls	Yes	
Baseline hazard	Common to all women	
Observations	446	213
Clusters	119	64

Data : PSF. Dep. var. : duration between births j and $(j+1)$. Duration between marriage and first birth is also included.

Sample : second wives, below 45 years old, for whom the complete birth history is known, having at least one child from current union. In column 2, I focus on unions in which the complete birth history of the first wife is known to be able to compute n_{ini}, the first wife's number of children as the second wife arrived.

T_2 = min (45-second wife's age at marriage ; 60-husband's age at second marriage) ; S = (husband's age at second marriage - husband's age at first marriage) ; $(T_1 - S)$ = min (45-first wife's age at marriage ; 60-husband's age at first marriage)-S.

Predictors of preferences : religion (husband and wife), ethnic group (husband and wife), education (husband and wife), rural dummy (husband and wife), income (husband), employment status (wife), birth cohort (husband and wife), region of residence, age at marriage (husband and wife), be in first marriage (husband and wife). Additional controls : co-residence status, work in public sector (husband), at least one child from previous union (husband and wife), having at least one dead child from current union, mother's age and age squared at birth j, a dummy for each j.

Cox estimation with a baseline hazard common to all women. Breslow method to handle ties among non-censored durations. Robust standard errors of the coefficients are in parentheses (clustered at the woman level). Significance levels (for hazard ratio = 1) : * p<0.10, ** p<0.05, *** p<0.01.

Table 11: First wives' completed fertility

Dep. var.	n_1^*	
T_1	0.220	0.245
	(0.198)	(0.189)
S	0.056	0.114**
	(0.069)	(0.048)
T_2	0.023	
	(0.073)	
T_2^{low}		-2.098***
		(0.761)
T_2^{high}		0.580
		(0.684)
Specific controls	Predictors of preferences	
Additional controls	Yes	
Woman FE	No	
Observations	101	

Data : PSF. Dep. var. : number of children in current union. Sample : first wives over 45 years old who were younger than 45 years old as the second wife arrived, having at least one child from current union.
S = (husband's age at second marriage - husband's age at first marriage) ; T_1 = min (45-first wife's age at marriage ; 60-husband's age at first marriage) ; T_2 = min (45-second wife's age at marriage ; 60-husband's age at second marriage) ; T_2^{low} : T_2 is below Q_1 (10 years) and T_2^{high} : T_2 is above Q_3 (22 years).
Predictors of preferences : religion (husband and wife), ethnic group (husband and wife), education (husband and wife), rural dummy (husband and wife), income (husband), employment status (wife), birth cohort (husband and wife), region of residence, age at marriage (husband and wife), be in first marriage (husband and wife). Additional controls : co-residence status, work in public sector (husband), at least one child from previous union (husband and wife), having at least one dead child from current union.
OLS estimation. Significance levels : * p<0.10, ** p<0.05, *** p<0.01.

Table 12: Second wives' completed fertility : n_2^*

Dep. var.	n_2^*	
T_2	0.074	0.334*
	(0.244)	(0.160)
S	0.040	
	(0.107)	
$T_1 - S$	0.013	
	(0.157)	
S^{low}		-3.354**
		(1.013)
$(T_1 - S)^{low}$		-2.307
		(1.753)
Specific controls	Predictors of preferences	
Additional controls	Yes	
Woman FE	No	
Observations	48	

Data : PSF. Dep. var. : number of children in current union. Sample : second wives over 45 years old, having at least one child from current union.

T_2 = min (45-second wife's age at marriage ; 60-husband's age at second marriage) ; S = (husband's age at second marriage - husband's age at first marriage) ; S^{low} : S is below the median (9 years). $(T_1 - S)$ = min (45-first wife's age at marriage ; 60-husband's age at first marriage)-S ; $(T_1 - S)^{low}$: $T_1 - S = 0$.

Predictors of preferences : religion (husband and wife), ethnic group (husband and wife), education (husband and wife), rural dummy (husband and wife), income (husband), employment status (wife), birth cohort (husband and wife), region of residence, age at marriage (husband and wife), be in first marriage (husband and wife). Additional controls : co-residence status, work in public sector (husband), at least one child from previous union (husband and wife), having at least one dead child from current union.

OLS estimation. Significance levels : * p<0.10, ** p<0.05, *** p<0.01.

Table 13: Testing changes in son preference of first wives

Hazard ratios	Bigamous unions
After	0.693*
	(0.143)
No son	1.383***
	(0.164)
After * No son	2.135*
	(0.846)
Controls	Birth rank dummies, mother's age and age squared at birth j
Baseline hazard	Woman-specific
Observations	2397
Clusters	695

Data : PSF. Dep. var. : duration between births j and $(j+1)$. Sample : monogamous and first wives below 45 years old, for whom the complete birth history is known, having at least one child from current union ; I focus on bigamous unions.
$After$ is a time-varying variable indicating if the second wife has arrived. *No son* is a dummy equal to one if the woman had no son among her first j births.
Stratified partial likelihood estimation with baseline hazards specific to each woman ; Breslow method to handle ties among non-censored durations. Robust standard errors of the coefficients are in parentheses (clustered at the woman level). Significance levels (for hazard ratio = 1) : * p<0.10, ** p<0.05, *** p<0.01.

TABLE 14: Testing if second wives react to the gender composition of first wives' children

Hazard ratios	Bigamous unions
$Boys_{ini}$	2.266***
	(0.406)
$Girls_{ini}$	1.430
	(0.452)
Specific controls	$T_2, (T_1 - S)$ and predictors of preferences
Additional controls	Yes
Baseline hazard	Common to all women
Observations	151
Clusters	46

Data : PSF. Dep. var. : duration between births j and $(j+1)$. Duration between marriage and first birth is also included. Sample : second wives, below 45 years old, for whom the complete birth history is known, having at least one child from current union. I focus on bigamous unions in which the complete birth history of the first wife is known to be able to observe the gender composition of the first wife's children at the time of the second marriage.

$Boys_{ini}$ and $Girls_{ini}$ measure the number of boys and girls born to the first wife as the second wife arrived. T_2 = min (45-second wife's age at marriage ; 60-husband's age at second marriage) ; S = (husband's age at second marriage - husband's age at first marriage) ; $(T_1 - S)$ = min (45-first wife's age at marriage ; 60-husband's age at first marriage)-S. Predictors of preferences : religion (husband and wife), ethnic group (husband and wife), education (husband and wife), rural dummy (husband and wife), income (husband), employment status (wife), birth cohort (husband and wife), region of residence, age at marriage (husband and wife), be in first marriage (husband and wife). Additional controls : co-residence status, work in public sector (husband), at least one child from previous union (husband and wife), having at least one dead child from current union, mother's age and age squared at birth j, a dummy for each j.

Cox estimation with a baseline hazard common to all women. Breslow method to handle ties among non-censored durations. Robust standard errors of the coefficients are in parentheses (clustered at the woman level). Significance levels (for hazard ratio = 1) : * p<0.10, ** p<0.05, *** p<0.01.

Appendix : For online publication

Appendix A : Additional descriptive statistics

A1. Realized fertility

Table A.1: Impact of ages at marriage on total fertility

Threshold for large age difference	15 years	18 years
Age wife	-0.078***	-0.087***
	(0.027)	(0.022)
Age husband	-0.011	0.000
	(0.027)	(0.021)
Age wife * Large age difference	0.021	0.040
	(0.048)	(0.059)
Age husband * Large age difference	-0.049	-0.104*
	(0.044)	(0.055)
Specific controls	Union type	
Additional controls	Yes	
Nb obs	564	
F-test (p-val)		
Age wife + Age wife * Large=0	0.165	0.413
Age husband+ Age husband * Large=0	0.093*	0.045**

Data : PSF. Sample : women over 45 years old. Dep. var. : number of children in current union.

Age means age at marriage ; Union type : monogamous, senior wife, junior wife ; Large age difference is dummy equal to 1 if the age difference between spouses is larger than 15 years in column 1 or 18 years in column 2. These thresholds correspond roughly to the difference in ages at fertility decline between men and women.

Additional controls : co-residing with husband, education, area of residence, at least one child from previous union, being in first marriage, employment status, ethnic group.

OLS estimation. Significance levels : * p<0.10, ** p<0.05, *** p<0.01.

TABLE A.2: Fertility in current union, by mother's rank

Dep. var.	Number of children	Birth intervals
Estimation	OLS	Cox (hazard ratios)
rank1	-0.134	0.889**
	(0.226)	(0.046)
rank2	-0.870***	0.783***
	(0.294)	(0.041)
rank3	-1.222**	0.696***
	(0.533)	(0.075)
rank4	-1.467	0.561**
	(1.065)	(0.149)
Controls	T and children from previous unions	
Observations	550	3717

Data : PSF. Sample : in column 1, women over 45 years old having at least one child with current husband. In column 2, women below 45 years old, having at least one child with current husband, and for whom the complete birth history is known. Reference category : monogamous wives.

$T = \min$ (45- wife's age at marriage ; 60-husband's age at marriage).

In column 1 : OLS estimation ; the unit of observation is the woman. In column 2 : Cox estimation on pooled durations between births j and $(j+1)$; the unit of observation is the birth ; Breslow method to handle ties among non-censored durations ; robust standard errors clustered at the woman level.

Significance levels (for hazard ratio= 1 in the Cox estimation) : * p<0.10, ** p<0.05, *** p<0.01.

In both regressions, the coefficient on $rank1$ is significantly different from the coefficients on all other ranks, but the coefficients on $rank2$, $rank3$, and $rank4$ are not significantly different from one another (pair-wise F-tests). Since there are few women of rank 3 and 4, the estimates are very imprecise.

A2. Fertility preferences

I compute the descriptive statistics on fertility preferences using the most recent DHS waves (2005 and 2010), which were conducted in the same years as the PSF survey (2006-2007). Table A.3 below reports the predictors of the ideal number of children, for men and women separately. First, socio-economic status is clearly negatively correlated with the ideal family size : educated and wealthier men and women, as well as urban men, want fewer children. This is also the case for younger cohorts. Then, marital history matters : men and women who got married younger, and men who got married more than once, want more children. Household heads and their wives display on average the same preferences as other members.[34] Last, there is some variation across religions – the ideal family size is smaller for Christians – ethnic groups and regions.

Note that a sizeable proportion of the respondents gave a non-numerical answer to the question "How many children would you like to have, or would you have liked to have ?". 23% of women and 31% of men answered "I don't know" or any non-numerical statement such as "It is up to God". As a result, I know the ideal number of children for both spouses in a bit more than one half of the couples (55%). The selection is not random : those couples are more "westernized" than couples in which at least one spouse has a non-numerical ideal family size. Table A.4 below shows that couples in which both spouses report their ideal number of children are more urban, richer and more educated. The proportion of Christians is larger, as well as the proportion of monogamous unions. They belong to younger cohorts and got married older. There is also some variation across regions - they are more likely to live in Dakar, Saint-Louis, Fatick, Kolda or Kaffrine, and less likely to live in Diourbel, Kaolack, Thies or Louga - and across ethnic groups - the proportion of Serer and Jola is

34. Since the PSF sample is restricted to household heads, I ran the same regression on the sub-sample of household heads in DHS. Predictors of the ideal number children are the same as in the whole sample, except that age at first marriage and being in first marriage are no longer significant for men, while being employed becomes significant for women.

larger while that of Fula is smaller.

One may wonder whether respondents rationalize ex-post their fertility behavior, and report that they would have wanted exactly the same number of children as they actually have had. This would be an issue when I use the reported ideal number as a proxy for innate preferences, because it would be driven by the outcome of the whole decision process. To assess the validity of such a concern, I restrict the sample to older couples (all wives above 40, husbands above 50) and I compare the ideal family size to the realized one. Ideal and realized numbers coincide for only 10% of husbands and 17% of wives. Over one third of men and one half of women declare that they would have wanted fewer children than they actually have had. Such figures provide some level of assurance that the scope for ex-post rationalization is limited.

TABLE A.3: Predictors of the ideal number of children

Sample	Husbands	Wives
Christian	-1.731***	-0.935***
	(0.481)	(0.209)
Other religion	-0.437	0.351
	(1.311)	(0.650)
Serere	-0.259	0.434**
	(0.402)	(0.198)
Poular	-0.891**	0.224*
	(0.352)	(0.131)
Mandingue	-0.337	-0.000
	(0.417)	(0.240)
Sarakole	-1.908*	0.290
	(1.024)	(0.509)
Diola	-0.728	-0.146
	(0.550)	(0.235)
Other ethnic group	-0.356	0.138
	(0.512)	(0.202)
No education	0.821***	0.421***
	(0.295)	(0.119)
Rural	0.937***	0.187
	(0.341)	(0.138)
Wealth index	-0.555***	-0.246***
	(0.138)	(0.052)
Employed	-0.265	0.158
	(0.419)	(0.110)
Head or head's spouse	0.295	-0.187
	(0.289)	(0.114)
Age at first marriage	-0.073***	-0.041***
	(0.022)	(0.014)
Being in first marriage	-0.727**	-0.047
	(0.352)	(0.146)
Monogamous union	-2.709***	-0.123
	(0.479)	(0.117)
Constant	14.739***	7.055***
	(1.286)	(0.419)
Cohort Fixed Effect	Yes	Yes
Region Fixed Effect	Yes	Yes
Observations	1928	2523
R2	0.26	0.15

Data : DHS, waves 2005 and 2010. Weights. Dep. var : ideal number of children. Sample : respondents who gave a numerical answer to the question "How many children would you like to have, or would you have liked to have, in your whole life ?" Reference categories are "Muslims" for the religion and "Wolof" for the ethnic group.
OLS estimation. Significance levels : * $p<0.10$, ** $p<0.05$, *** $p<0.01$.

Table A.4: Balancing tests between couples declaring a numerical vs. non-numerical ideal family size

Sample	Numerical ideal family size	Non-numerical ideal family size	p-value
Rural	0.540	0.658	0.000
Wealth index	3.010	2.756	0.000
Monogamous union	0.708	0.622	0.000
Head or head's spouse	0.622	0.675	0.001
No education (husband)	0.538	0.699	0.000
No education (wife)	0.637	0.791	0.000
Employed (husband)	0.884	0.925	0.000
Employed (wife)	0.409	0.408	0.969
Age (husband)	41.055	43.061	0.000
Age (wife)	30.451	31.664	0.000
Age at first marriage (husband)	27.105	26.354	0.000
Age at first marriage (wife)	18.481	17.450	0.000
Being in first marriage (husband)	0.655	0.566	0.000
Being in first marriage (wife)	0.858	0.878	0.102
Christian (husband)	0.046	0.018	0.000
Christian (wife)	0.054	0.022	0.000
Observations	1824	1484	

Data : DHS, waves 2005 and 2010. Weights.

The first column present descriptive statistics of couples in which both spouses report her ideal number of children. The second column present the same statistics for couples in which at least one spouse gave a non-numerical answer to the question "How many children would you like to have, or would you have liked to have, in your whole life ?" The third column reports the p-value of the t-tests comparing the means in both sub-samples : a low p-value indicates that they are statistically different with respect to the corresponding covariate.

Appendix B : Welfare analysis

In this section, I compare the Nash equilibrium and the outcome maximizing total welfare. I focus on two quantities of interest : the total number of children ($N = n_1 + n_2$) and the relative number of children ($\Delta = n_1 - n_2$). I consider the simple case when parameters are the same for both wives : $\theta_1^h = \theta_2^h = \theta^h$, $\theta_1^n = \theta_2^n = \theta^n$ and $\epsilon_1 = \epsilon_2 = \epsilon$.

From equation 4, I derive the quantities at equilibrium :

$$N^* = n_1^* + n_2^* = \frac{N^{id} + 2\theta^h n_h^{id} + \theta^n N^{nat}}{1 - \epsilon + 2\theta^h + \theta^n}$$

$$\Delta^* = n_1^* - n_2^* = \frac{\Delta^{id} + \theta^n \Delta^{nat}}{1 + \epsilon + \theta^n}$$

Where $N^{id} = n_1^{id} + n_2^{id}$, $N^{nat} = n_1^{nat} + n_2^{nat}$, $\Delta^{id} = n_1^{id} - n_2^{id}$ and $\Delta^{nat} = n_1^{nat} - n_2^{nat}$.

I further define the total welfare function as $W(n_1, n_2) = u_1(n_1, n_2) + u_2(n_2, n_1)$, where $u_i(n_i, n_{-i})$ is the utility of wife i. By maximizing $W(n_1, n_2)$ over n_1 and n_2, I find :

$$N^{Opt} = n_1^{Opt} + n_2^{Opt} = \frac{(1-\epsilon)N^{id} + 4\theta^h n_h^{id} + \theta^n N^{nat}}{(1-\epsilon)^2 + 4\theta^h + \theta^n}$$

$$\Delta^{Opt} = n_1^{Opt} - n_2^{Opt} = \frac{(1+\epsilon)\Delta^{id} + \theta^n \Delta^{nat}}{(1+\epsilon)^2 + \theta^n}$$

The same elements drive N and Δ when I maximize total welfare and when I compute the Nash equilibrium. But the weights given to each element differ. The table below summarizes the drivers and their weights in both cases.

Weight on each driver	Nash equilibrium	Welfare-maximizing outcome
Drivers of N		
$N^{id}/(1-\epsilon)$	$\frac{(1-\epsilon)}{(1-\epsilon)+2\theta^h+\theta^n}$	$\frac{(1-\epsilon)^2}{(1-\epsilon)^2+4\theta^h+\theta^n}$
n_h^{id}	$\frac{2\theta^h}{(1-\epsilon)+2\theta^h+\theta^n}$	$\frac{4\theta^h}{(1-\epsilon)^2+4\theta^h+\theta^n}$
N^{nat}	$\frac{\theta^n}{(1-\epsilon)+2\theta^h+\theta^n}$	$\frac{\theta^n}{(1-\epsilon)^2+4\theta^h+\theta^n}$
Drivers of Δ		
$\Delta^{id}/(1+\epsilon)$	$\frac{(1+\epsilon)}{(1+\epsilon)+\theta^n}$	$\frac{(1+\epsilon)^2}{(1+\epsilon)^2+\theta^n}$
Δ^{nat}	$\frac{\theta^n}{(1+\epsilon)+\theta^n}$	$\frac{\theta^n}{(1+\epsilon)^2+\theta^n}$

Compared to the welfare-maximizing outcome, the total number of children at equilibrium depends too much on the preferences of the wives, $N^{id}/(1-\epsilon)$, and too little on the preferences of the husband, n_h^{id}. The relative number of children depends too much on the difference in wives' natural fertility, Δ^{nat}, and too little on the difference in wives' preferences, Δ^{id}.

To sum up, if wives were to agree on maximizing total welfare instead of maximizing their own utility, the total number of children would be closer to the husband's preferences. The relative number of children would reflect more the difference in wives' preferences and less the difference in natural fertility.

Are there too many or too few children at equilibrium ? We have :

$$N^* - N^{Opt} = \frac{(\frac{N^{id}}{1-\epsilon} - n_h^{id}).2\theta^h(1-\epsilon^2) + (\frac{N^{id}}{1-\epsilon} - N^{nat}).\theta^N\epsilon(1-\epsilon) + (N^{nat} - n_h^{id}).2\theta^h\theta^n}{((1-\epsilon)^2 + 4\theta^h + \theta^n)(1-\epsilon+2\theta^h+\theta^n)}$$

The comparison between N^* and N^{Opt} depends on the relative values of $\frac{N^{id}}{1-\epsilon}$, n_h^{id} and N^{nat}. In particular, if $\frac{N^{id}}{1-\epsilon} \geq N^{nat} \geq n_h^{id}$, then $N^* \geq N^{Opt}$; whereas if $\frac{N^{id}}{1-\epsilon} \leq N^{nat} \leq n_h^{id}$, then $N^* \leq N^{Opt}$. The first statement is all the more likely to hold as ϵ rises. In the empirical tests, I found that ϵ is large enough to induce a positive strategic reaction. One plausible value for

ϵ is therefore its upper bound. When $\epsilon = 1$, we have :

$$N^* - N^{Opt} = \frac{N^{id}.(4\theta^h + \theta_n) + (N^{nat} - n_h^{id}).2\theta^h\theta^n}{(2\theta^h + \theta^n)(4\theta^h + \theta^n)}$$

Using the values of the parameters estimated on monogamous unions ($\theta^h \approx 1/2$ and $\theta^n \approx 3$), it means that $N^* \geq N^{Opt}$ iff $N^{id} \geq \frac{3}{5}(n_h^{id} - N^{nat})$. This condition is likely to hold in the vast majority of cases given that preferences in polygamous unions are on average 5 or 6 children for each wife, and 12 children for the husband. To sum up, a deficit of children would be observed at equilibrium only when n_h^{id} reaches uncommonly high values. In general, the non-cooperative model leads to a surplus of children with respect to the outcome maximizing total welfare.

Appendix C : Extensions of the model

C1. Relaxing the assumptions on functional forms

In this section, I investigate whether the predictions of the model remain valid with more general functional forms. Starting from equation 1 and using the fact that $v(.)$ is concave while $H(.)$ and $N(.)$ are convex, one can show that $\frac{\partial n^*}{\partial T} > 0$. The implication that an increase in the reproductive period should raise the final number of children still holds. It is not the case for the implication on birth spacing, because the sign of $\frac{\partial \lambda^*}{\partial T}$ is now ambiguous.

Turning to predictions on bigamous societies, I use Equation 3 to compute the functions of best response. I find a new condition for the existence of strategic complements. n_i^* increases with n_j iff :

$$b_i = -\epsilon_i v''(n_i - \epsilon_i n_j, n_w^{id}) - \theta_i^h H''(n_i + n_j, n_h^{id}) > 0$$

In the specification minimizing the sum of distances, $v''(.) = -2$ and $H''(.) = 2$ so the

condition boils down to $\epsilon_i - \theta_i^h > 0$. When considering more general forms, one needs to take into account the concavity of $v(.)$ and the convexity of $H(.)$. For instance, if $v(.)$ is very concave while $H(.)$ is almost flat, the overbidding effect could dominate the substitution effect, even if ϵ is smaller than θ^h. The predictions on how the reproductive period of one wife should impact fertility choices of her co-wife are still valid if we replace the condition $B_i > 0$ by the more general one stated above. In particular, I find that $\frac{\partial n_1^*}{\partial T_2}$ and $\frac{\partial \lambda_1^* - \lambda_0^*}{\partial T_2}$ have the same sign as b_1 while $\frac{\partial n_2^*}{\partial S}$, $\frac{\partial n_2^*}{\partial (T_1 - S)}$, $\frac{\partial \lambda_2^*}{\partial S}$ and $\frac{\partial \lambda_2^*}{\partial (T_1 - S)}$ have the same sign as b_2.

Another parametric assumption is related to the introduction of the number of rivals in the utility function. By considering the relative number of children and the total number of children, I modeled strategic interactions in a linear way. To be more general, I could write $v(n_i, n_j, n_w^{id})$ and $H(n_i, n_j, n_h^{id})$, and further impose that the cross-derivatives of $v(.)$ and $H(.)$ with respect to n_i and n_j are both positive. Here, children are predicted to be strategic complements iff :

$$\frac{\partial^2 v}{\partial n_i \partial n_j} - \theta_i^h \frac{\partial^2 H}{\partial n_i \partial n_j} > 0$$

We compare the impact of an additional rival on the marginal net gain from children to the mother and the impact on the marginal net cost related to the husband's constraint.

Overall, the empirical tests for the existence of strategic interactions are not driven by specific functional forms. However, what is not clear when I leave the linear framework is whether a Nash equilibrium always exists and is unique. The analysis of multiple equilibria is beyond the scope of this paper but understanding what kind of households coordinate on a low or on a high fertility equilibrium could be a promising line of research.

C2. Relaxing the assumption that information on preferences is complete

When I model the interaction between both wives as a non-cooperative, simultaneous game, I implicitly assume that each player knows the payoff of the other player. In my context, it means that each wife knows the ideal number of children of the other. From the second wife's perspective, it might seem reasonable to assume that she infers $n_{w,1}^{id}$ from the behavior of the first wife in the monogamous stage. She observes S and the number of children as she enters the household, so she is able to deduce the optimal initial birth rate of the first wife, and hence her preferences. Things are not as straightforward from the first wife's perspective, who only observes T_2, but has no piece of evidence to infer $n_{w,2}^{id}$.

In this extension, I consider that $n_{w,2}^{id}$ is private information of the second wife. To simplify the notations, let me call $l = n_{w,2}^{id}$ the type of the second wife, and $f(l)$ the density, defined on an interval $I \in \mathbb{R}^+$. The first wife knows the distribution of types in the population of second wives.[35] I denote $n_2(l)$ the strategy played by a second wife of type l. From section 3.2.2, the best response of a second wife of type l when the first wife plays n_1 is :

$$n_2(t) = n_2^{NS}(l) + B_2.n_1 \tag{6}$$

Where $n_2^{NS}(l) = \frac{l+\theta_2^h n_h^{id} + \theta_2^n n_2^{nat}}{1+\theta_2^h+\theta_2^n}$.

What is the best response of first wives when second wives of type l play $n_2(l)$? First wives maximize their expected utility :

$$\mathbb{E}[u(n_1, n_2(l)] = \int_I u(n_1, n_2(l))f(l)dl$$

35. Note that the first wife does not update her beliefs during the game.

When $n_2(l)$ is considered as given, $\frac{\partial u}{\partial n_1}$ is linear in n_1 and in $n_2(l)$, so the FOC gives :

$$n_1 = n_1^{NS} + B_1 \int_I n_2(l) f(l) dl \tag{7}$$

The Nash equilibrium is the intersection of all best responses. Plugging the expression of $n_2(l)$ from Equation 6 into Equation 7, I get :

$$n_1^* = (n_1^{NS} + B_1 \mathbb{E}[n_2^{NS}]) \times \frac{1}{1 - B_1.B_2}$$

$$n_2^*(l) = (n_2^{NS}(l) + B_2.n_1^{NS} + B_1.B_2(\mathbb{E}[n_2^{NS}] - n_2^{NS}(l))) \times \frac{1}{1 - B_1.B_2}$$

Where $\mathbb{E}[n_2^{NS}] = \frac{\int_I l f(l) dl + \theta_2^h n_h^{id} + \theta_2^n n_2^{nat}}{1 + \theta_2^h + \theta_2^n}$.

Under incomplete information, the comparative statics described above are still valid. The first wife responds to the preferences of the average second wife.[36] This framework can be easily extended to relax the assumption that $n_{w,1}^{id}$ is known by the second wife.

C3. Relaxing the assumption that the game is simultaneous

So far, I have considered that either the first wife always remains in a monogamous union, or a second wife arrives at date S and both wives play a simultaneous game. In fact, when the first wife is relatively old as the second marriage takes place, the game is not simultaneous, but sequential. Indeed, the first wife has already given birth to $n_1 = \lambda_0.T_1$ children, and she can no longer update this quantity. Then the second wife chooses her best response to n_1, and payoffs are paid when the reproductive period of the second wife is over. Therefore, if $T_1 \leq S$, the interaction between both wives is best described by a Stackelberg leadership

36. In theory, I could investigate whether the assumption of complete information is likely to hold by testing if n_1^* depends on $n_{w,2}^{id}$. In practice, the DHS sample of complete bigamous unions is too small to perform a credible test.

model.[37]

To account for the possibility of a late second marriage, I create a new husband's type, late polygamous. The first wife's belief about the probability of polygamy, π, is split into π_a the probability of late polygamy, and π_b the probability of early polygamy. At $t = 0$, first wives consider three scenarios : no strategic interaction, sequential game, and simultaneous game. Keeping the notations of section 3.2.2, first wives maximize their expected utility over λ_0 :

$$(1-\pi) \times u(\lambda_0.T_1, 0) + \pi_a \times \mathbb{E}[u(\lambda_0.T_1, n_2(\lambda_0.T_1))] + \pi_b \times \mathbb{E}[u(n_1^*, n_2^*)]$$

Where $n_2(\lambda_0.T_1)$ is the best response of the second wife when she faces a first wife with $\lambda_0.T_1$ children. As already noted above, $u(n_1^*, n_2^*)$ does not depend on λ_0.

To find the subgame perfect Nash equilibrium, I solve the game by backward induction. I start by considering the last stage of the sequential game. Building on section 3.2.2, I know that $n_2(n_1) = n_2^{NS} + B_2.n_1$. First wives anticipate the reaction of second wives. Let me compute the best strategy of first wives for a given T_2 (which determines n_2^{NS}). They maximize $u(n_1, n_2^{NS}(T_2) + B_2.n_1)$. The FOC gives :

$$n_1^{st}(T_2) = \frac{n_w^{id}(1 - B_2\epsilon_1) + \theta_1^h n_h^{id}(1 + B_2) + \theta_1^n n_0^{nat} + n_2^{NS}(T_2)(\epsilon_1(1 - B_2\epsilon_1) - \theta_1^h(1 + B_2))}{(1 - B_2\epsilon_1)^2 + \theta_1^h(1 + B_2)^2 + \theta_1^n} \quad (8)$$

Note that n_1^{st} increases with T_2 iff $B_1^{st} = \epsilon_1(1 - B_2\epsilon_1) - \theta_1^h(1 + B_2) \geq 0$. In the simultaneous game, I found that n_1^* increases with T_2 iff $\epsilon_1 - \theta_1^h \geq 0$. To understand the difference, one needs to consider the cross-derivative of $u_1(.)$ with respect to n_1 and n_2. When n_2 is taken

37. In the rest of the section, I use the superscripts st to denote the equilibrium quantities when I introduce the Stackelberg interaction in the basic model.

as exogenous, I have :

$$\frac{\partial^2 u_1}{\partial n_1 \partial n_2} = 2(\epsilon_1 - \theta_1^h)$$

When n_2 depends on n_1 in such a way that $\frac{\partial n_2}{\partial n_1} = B_2$, the expression is :

$$\frac{\partial^2 u_1}{\partial n_1 \partial n_2} = 2(\epsilon_1 \times (1 - \epsilon_1 B_2) - \theta_1^h \times (1 + B_2))$$

When n_2 is fixed, having one more child for the first wife means that her relative number of children increases by one unit, and that the total number of children of the husband increases by one unit. Whereas when n_2 depends on n_1, having one more child for the first wife means that the second wife will have B_2 additional children. So her relative number of children increases by $(1 - \epsilon_1 B_2)$, and the total number of children of the husband increases by $(1 + B_2)$.[38]

Now, first wives do not know T_2 before the second marriage takes place. But they have some information about the timing of events that can be exploited to refine the expectation about T_2. Indeed, it can be shown that $\mathbb{E}[T_2|T_1, T_h]$ is increasing in $(T_h - T_1)$, where T_h is the length of the husband's reproductive period at $t = 0$. The intuition is that the expected value of T_2 depends on S : the later the second marriage takes place in the husband's life, the shorter the time left to the second wife to have children. T_2 cannot be larger than $(T_h - S)$. Moreover, $S \geq T_1$ in the sequential scenario so that T_2 is bounded above by $(T_h - T_1)$.[39]

38. I rewrite $B_1^{st} = B_1(1 + \theta_1^h + \theta_1^n) - B_2(\theta_1^h + \epsilon_1^2)$. It may be the case that $B_1^{st} < 0$ even if $B_1 \geq 0$, for instance when B_2 is much larger than B_1. The intuition is that, when the second wife reacts very strongly to an increase in n_1, the increase in the relative number is small compared to the increase in the total number of children. On the other hand, when $B_1 \geq B_2 \geq 0$, then $B_1^{st} \geq 0$. In other words, when the strategic response of the second wife is not stronger than the one of the first wife, the first wife is always better off raising her number of children when she faces a more fertile rival.

39. The formal proof is as follows. Denote $L = T_h - T_1$ the length of the time period between the end of the first wife's reproductive life and the end of the husband's reproductive life. In the sequential game, this is the length of the second stage, when the second arrives and has children. Denote μ and ν two random variables representing, respectively, the entry date and the exit date of the second wife. The only assumption on their distributions $f(\mu)$ and $g(\nu)$ is that both have a positive support. Using these notations, I can rewrite

First wives maximize on n_1 their expected utility, $\mathbb{E}[u(n_1, n_2^{NS}(T_2) + B_2.n_1)]$. The derivative of $u(.)$ is linear in T_2 so I can write the equilibrium number of children using Equation 8 :

$$n_1^{st} = n_1^{st}(\mathbb{E}[T_2|T_1, T_h])$$

Since $\mathbb{E}[T_2|T_1, T_h]$ is increasing in $(T_h - T_1)$, n_1^{st} is also increasing in $(T_h - T_1)$ as long as $B_1^{st} \geq 0$.

Let me come back to $t = 0$ and consider the optimal initial birth rate under the three scenarios that I mentioned above : (i) no strategic interaction : $\lambda_0 = \frac{n_0^{NS}}{T_1}$; (ii) sequential game : $\lambda_0 = \frac{n_1^{st}}{T_1}$; and (iii) simultaneous game : indifferent between any $\lambda_0 \geq 0$. The first-order condition is a weighted average of the first-order condition under no strategic interaction (weight $(1-\pi)$) and the first-order condition of the sequential game (weight π_a). As result, the optimal initial birth rate λ_0^{st} lies between $\frac{n_0^{NS}}{T_1}$ and $\frac{n_1^{st}}{T_1}$.

How does n_1^{st} compare to n_0^{NS} ? It depends on the sign of B_1^{st} and on the relative magnitude of $n_{w,1}^{id}$ and n_h^{id}. The case that seems the most consistent with empirical evidence is $B_1 \approx B_2 \geq 0$ (implying that $B_1^{st} \geq 0$) and $n_{w,1}^{id} \leq n_h^{id}$. In this case, $n_1^{st} \geq n_0^{NS}$. When the strategic reaction is similar for both wives, and the husband wants more children than the first wife, the likelihood of a sequential game raises the initial birth rate. The first wife intensifies her fertility to improve her position in the event of a late second marriage.

It is possible to test for such a strategic overshooting by comparing the choices of women more or less exposed to the risk of a late second marriage. The idea is to exploit the variation

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$T_2 = min(\nu, max(L - \mu, 0))$. T_2 can never be larger than L. When L increases, it enlarges the widow of opportunity for a second wife to enter and exit after a substantial period of time. Intuitively, an increase in L can only raise the expected value of T_2. Formally, we have :

$$\mathbb{E}[T_2|L] = \int_0^{+\infty} \int_0^{+\infty} min(\nu, max(L - \mu, 0)) g(\nu) f(\mu) d\nu d\mu$$

Let $L' \geq L$, then $min(\nu, max(L' - \mu, 0)) \geq min(\nu, max(L - \mu, 0)) \ \forall \ (\mu, \nu)$. So $\mathbb{E}[T_2|L'] \geq \mathbb{E}[T_2|L]$.

in $(T_h - T_1)$, which is driven by the age difference between the first wife and the husband. When they have a very large age difference, then $(T_h - T_1) = 0$ because the length of the reproductive period of the couple (T_1) is determined by the length of the husband's period (T_h). It is very unlikely that the husband starts having children with another wife when the first one is already in her late forties because he will either be dead or infertile. On the contrary, when the age difference is low, then $(T_h - T_1)$ may be as large as 15 years, which leaves time for a potential rival to have many children.

In Table C.1 below, I test whether the fertility of monogamous wives is indeed higher if the age difference with the husband is lower. It is crucial in this test to control for the preferences of each spouse because their age difference is correlated to their ideal family size. [40] On the other hand, I do not need information on the timing of unions since I consider anticipations. This is why I perform the test on DHS instead of PSF.

Regarding completed fertility, the prediction is verified. The first column shows that, controlling for T_1 and the preferences of each spouse, $(T_h - T_1)$ has a positive and significant impact on the final number of children. In column 2, I investigate whether the effect is truly linear or driven by the difference between women not exposed at all and the others. I create three categories of women depending on their exposure to the risk of a late second marriage : not exposed if $(T_h - T_1) = 0$, weakly exposed if $(T_h - T_1)$ is below the median ; strongly exposed if $(T_h - T_1)$ is above the median. I find that women not exposed have significantly fewer children than the others. Also, among exposed women, the degree of exposure matters : strongly exposed women have more children than weakly exposed ones.

Regarding birth spacing, the effect of $(T_h - T_1)$ is of predicted sign, although not significant. In the last column, I interact $(T_h - T_1)$ with different ranks of birth ; the impact is all

40. Since I consider only the monogamous stage, I can not rely on a specification with fixed effects. So there could be other omitted variables such as the wife's bargaining power. Using a cross-section of nations, Cain (1984) shows that the median age difference between spouses is positively correlated to total fertility rate. If age difference is a proxy for women empowerment at the household level, then a low $(T_h - T_1)$ would be correlated with a high θ_1^h, and hence with a large n_1^{NS}. This would create an attenuation bias.

the larger as the rank is high, and it is significant after birth 5. One interpretation is that wives update upwards the relative likelihood of a late marriage as time passes by.[41]

41. In the model, to keep things simple, I assume that the ratio $\frac{\pi_a}{1-\pi}$ remains constant over the monogamous stage. The idea is that, even if the probability of an early second marriage (π_b) decreases as time goes by, it does not change the relative likelihood of a late marriage compared to the likelihood of no second marriage. In fact, first wives seem to consider that $(1-\pi)$ is fixed, and that the decreasing risk of an early second marriage is fully converted into a rising risk of late marriage.

Table C.1: Testing strategic overshooting

Dep. var.	Total number of births		Birth intervals	
Estimation	OLS		Cox (hazard ratios)	
n_w^{id}	0.195**	0.229**	1.023*	1.024*
	(0.095)	(0.092)	(0.013)	(0.013)
n_h^{id}	0.119**	0.145***	1.031***	1.030***
	(0.059)	(0.052)	(0.005)	(0.005)
T_1	0.281***	0.612***	0.994	0.996
	(0.054)	(0.114)	(0.017)	(0.017)
$T_h - T_1$	0.131**		1.009	
	(0.056)		(0.007)	
$T_h - T_1$ below median		1.585**		
		(0.710)		
$T_h - T_1$ above median		2.783**		
		(1.073)		
$(T_h - T_1) \times \{j = 1, 2\}$				1.003
				(0.008)
$(T_h - T_1) \times \{j = 3, 4\}$				1.010
				(0.011)
$(T_h - T_1) \times \{j \geq 5\}$				1.028**
				(0.014)
pval test below=above		0.141		
Controls	No	Yes	Yes	Yes
Observations	109	109	2768	2768
Clusters	na	na	768	768

Data : DHS. Sample : women in a monogamous union, in first union, at least one child. In column 1, I restrict to women over 40. Weights.

In column 1 : OLS estimation; the unit of observation is the woman. In column 2 : Cox estimation on durations between births j and $(j+1)$; the unit of observation is the birth; baseline hazard common to all women; Breslow method to handle ties among non-censored durations; robust standard errors clustered at the woman level.

T_1= min (45-first wife's age at marriage; 60-husband's age at marriage). $T_h - T_1$ = (60 - husband's age at marriage - T_1). The median is 7 years. In column 2, the reference category is $T_h - T_1 = 0$ (16 obs).

Controls : in column 2, husband's and wife's age at marriage; in columns 3 and 4, wife's age at marriage and a dummy for each j.

Significance levels (for hazard ratio = 1 in the Cox estimation) : * p<0.10, ** p<0.05, *** p<0.01.

Appendix D : Stratified partial likelihood estimation

This section draws on Ridder and Tunali (1999), and on the chapter "Duration Models : Specification, Identification, and Multiple Durations" by Gerard van den Berg in Heckman and Leamer (2001).

I consider a mixed proportional hazard model with multi-spell data. It means that several durations, indexed by j, are generated by a single individual i, which is characterized by a vector of observed explanatory variables x and an unobserved heterogeneity term ν. It is possible to identify the impact of x on the hazard function under very weak conditions (in addition to the proportional hazard assumption) if x varies between spells for a given individual while ν does not. Formally, the hazard function writes :

$$\theta(t|x_{i,j}, v_i) = \theta_0(t, \nu_i).exp(\beta.x_{i,j})$$

In this specification, the baseline hazard θ_0 is allowed to differ across individuals. There is no restriction on the interaction of ν with the elapsed duration t in the hazard function. Moreover, x and ν may be dependent. We do not need any assumption on the tail of the distribution of the unobservables.

The intuition underlying the estimation method is to construct a Cox partial likelihood within individuals (or strata) by ordering the uncensored durations. For each duration $t_{i,k}$, we can compute the probability that item k fails at $t_{i,k}$ given that exactly one item in i fails at $t_{i,k}$. It writes :

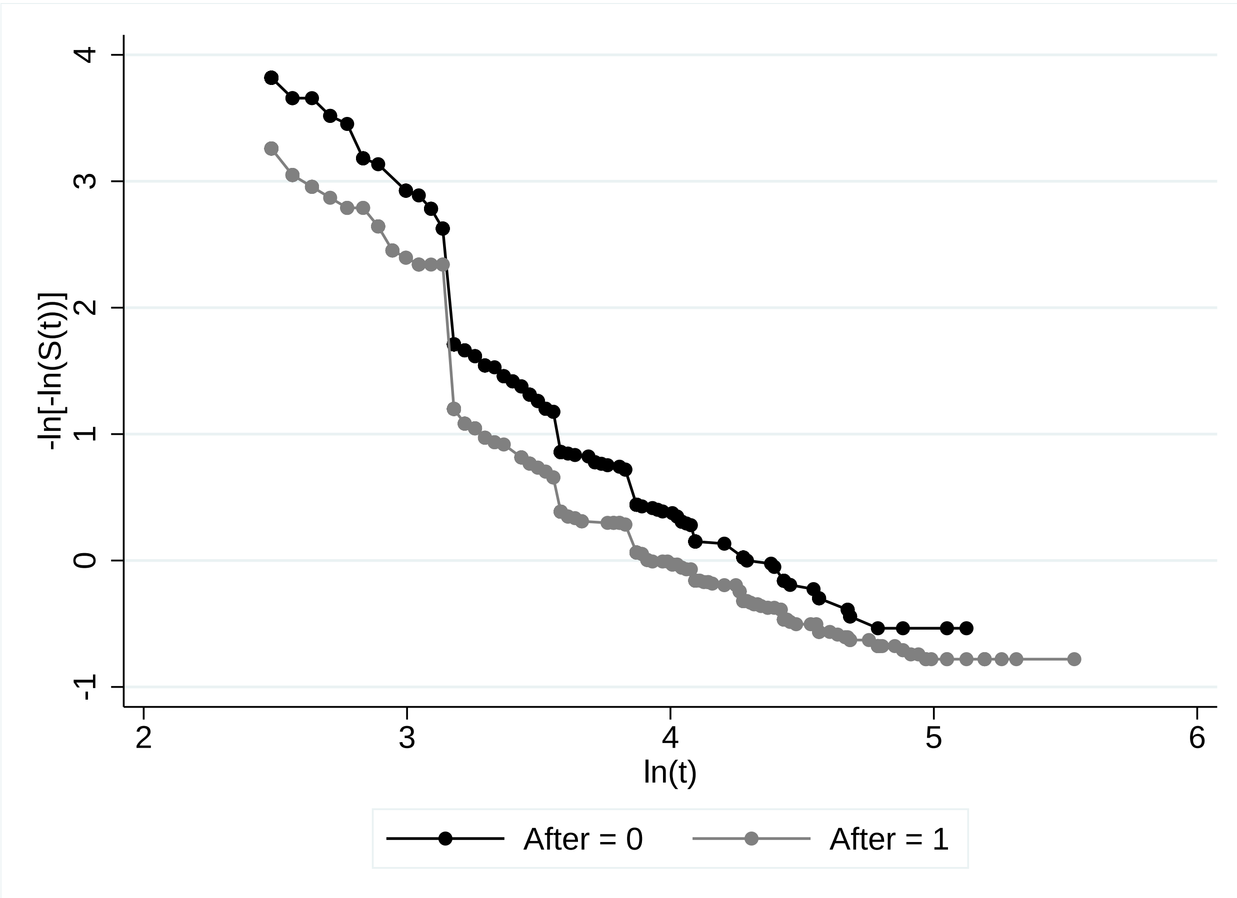
$$\frac{\theta(t_{i,k}|x_{i,k}, v_i)}{\Sigma_{j\in R_i(k)}\theta(t_{i,k}|x_{i,j}, v_i)} = \frac{exp(x_{i,k}\beta)}{\Sigma_{j\in R_i(k)}exp(x_{i,j}\beta)}$$

Where $R_i(k)$ is the set of observations in i at risk when k fails. This expression can be written for each spell of each individual ; the product gives the stratified partial likelihood.

The main caveat with multi-spell data is the issue of censoring. If all individuals are observed for the same period of time, then the right-censoring variable is not independent from previous durations, and therefore not independent from the current duration (because durations are jointly determined by the unobserved heterogeneity). This violates a standard assumption in duration analysis. It is not the case here : women are subject to censoring at the date of the survey, but they have different starting points corresponding to the date of first birth. So I do not follow them for a fixed time.

Appendix E : Robustness Tests

Figure E.1: Testing the proportional hazard assumption (log-log plot)



Data : PSF. Sample : first wives below 45 years old, for whom the complete birth history is known, having at least one child from current union.
The log-log plot graphs $-ln(-ln(S(t)))$ against $ln(t)$ for the category $After = 0$ (birth intervals occurring before the second marriage) and for the category $After = 1$ (birth intervals occurring after the second marriage). Estimates are adjusted for covariates : a dummy for each j, mother's age and age squared at birth j. As shown by the graph, the curves are parallel, meaning that the proportional hazard assumption is not violated.
Another way to test the proportional hazard assumption is to follow the procedure developed by Grambsch and Therneau (1994) and based on the Schoenfeld (1980) partial residuals. I fail to reject the assumption, both in the regression estimating the impact of $After$ (p-value=0.30) and in the regression with the interaction term $After \times T_2$ (p-value=0.51).

Table E.1: Predictors of polygamy

Sample	Polygamous union
Husband's father is polygamous	0.079
	(0.065)
Fostered before age 15 (wife)	-0.029
	(0.087)
Fostered before age 15 (husband)	0.092
	(0.078)
Age at first marriage (wife)	0.002
	(0.007)
Age at first marriage (husband)	-0.009
	(0.006)
Income (husband)	0.006
	(0.010)
Work in public sector (husband)	0.091
	(0.096)
No education (wife)	-0.055
	(0.090)
No education (husband)	0.143*
	(0.080)
Rural household	-0.006
	(0.089)
Children from previous unions (wife)	-0.308
	(0.247)
Children from previous unions (husband)	-0.144
	(0.135)
Being in first marriage (wife)	-0.128
	(0.235)
Being in first marriage (husband)	-0.242*
	(0.130)
Never worked (wife)	0.013
	(0.076)
Serere (wife)	0.028
	(0.179)
Poular (wife)	-0.115
	(0.200)
Diola	-0.438
	(0.271)
Mandingue (wife)	0.079
	(0.241)
Sarakole (wife)	-0.129
	(0.447)
Serere (husband)	-0.093
	(0.182)
Poular (husband)	0.017
	(0.201)
Mandingue (husband)	-0.057
	(0.244)
Sarakole (husband)	0.145
	(0.391)
Christian (wife)	-0.004
	(0.342)
Christian (husband)	0.005
	(0.347)
Cohort dummies	Yes
Region dummies	Yes
Observations	1928
Pseudo R2	0.16

Data : PSF. Dep. var : being in a polygamous union at the time of the survey. Sample : monogamous and first wives, older than 45 years old, having at least one child from current union.
Probit estimation. Marginal effects are reported. The average predicted probability of polygamy is 45%. Significance levels : * p<0.10, ** p<0.05, *** p<0.01.

Table E.2: Testing if the risk of polygamy influences birth spacing

Hazard ratios	Monogamous stage
$\hat{\pi}$	0.858
	(0.623)
Specific controls	T and predictors of preferences
Additional controls	Yes
Baseline hazard	Common to all women
Observations	986
Clusters	299

Data : PSF. Dep. var. : duration between births j and $(j+1)$. Sample : monogamous and senior wives *before the second marriage*, below 45 years old, for whom the complete birth history is known, having at least one child from current union.
$\hat{\pi}$ is the predicted probability of a second marriage (cf. Probit estimation in Table E.1).
Predictors of preferences : religion (husband and wife), ethnic group (husband and wife), education (husband and wife), rural dummy (husband and wife), income (husband), employment status (wife), birth cohort (husband and wife), region of residence, age at marriage (husband and wife), be in first marriage (husband and wife). Additional controls : co-residence status, work in public sector (husband), at least one child from previous union (husband and wife), having at least one dead child from current union, mother's age and age squared at birth j, a dummy for each j.
Cox estimation with a baseline hazard common to all women. Breslow method to handle ties among non-censored durations. Robust standard errors of the coefficients are in parentheses (clustered at the woman level). Significance levels (for hazard ratio = 1) : * p<0.10, ** p<0.05, *** p<0.01.

TABLE E.3: Change in first wives' birth spacing : robustness

Hazard ratios	Next interval		Bigamous		Excl. monogamous		Rough measure		No dead child	
After	0.557**	0.050***	0.694*	0.226**	0.844	0.176**	0.507***	0.166**	0.666	0.036***
	(0.142)	(0.042)	(0.145)	(0.163)	(0.182)	(0.130)	(0.099)	(0.116)	(0.180)	(0.038)
After * T_2		1.142***		1.067*		1.089**		1.064*		1.168***
		(0.044)		(0.039)		(0.038)		(0.035)		(0.060)
Controls	Birth rank dummies, mother's age and age squared at birth j									
Baseline hazard	Woman-specific									
Observations	2286		2399		553		2483		1563	
Clusters	699		695		144		716		468	

Data : PSF. Dep. var. : duration between births j and $(j+1)$. Sample : monogamous and first wives below 45 years old, for whom the complete birth history is known, having at least one child from current union. $After$ is a time-varying variable indicating if the second wife has arrived. T_2 = min (45-second wife's age at marriage ; 60-husband's age at second marriage).

Stratified partial likelihood estimation with baseline hazards specific to each woman ; Breslow method to handle ties among non-censored durations. Robust standard errors of the coefficients are in parentheses (clustered at the woman level). Significance levels (for hazard ratio = 1) : * p<0.10, ** p<0.05, *** p<0.01.

In columns 1 and 2, I consider all intervals in the monogamous stage, and only the very next interval after the second marriage. I exclude first wives with more than one co-wife in columns 3 and 4, and I exclude monogamous wives in columns 5 and 6. In columns 7 and 8, I impose that $After = 1$ for the whole spell during which the second marriage takes place, which is the approximation made in the linear model, whereas in the baseline duration model, $After$ varies within a spell. In columns 9 and 10, I exclude women who have lost at least one child from current union.

Table E.4: Testing if polygamy is caused by choices in the monogamous stage

Hazard ratios	Occurrence	Timing
Future first wives	0.953	
	(0.154)	
T_2		0.909
		(0.067)
S		0.910
		(0.065)
Specific controls	T_1 and predictors of preferences	
Additional controls	Yes	
Baseline hazard	Common to all women	
Observations	477	222
Clusters	106	83

Data : PSF. Dep. var. : duration between births j and $(j+1)$.

Sample : in column 1, monogamous and senior wives *before the second marriage*, between 40 and 45 years old, for whom the complete birth history is known, having at least one child from current union. In column 2, senior wives *before the second marriage*, below 45 years old, for whom the complete birth history is known, having at least one child from current union ; I restrict the analysis to unions with exactly two competitors. "Future first wives" is equal to 1 if the woman is in a polygamous union at the time of the survey. T_i= min (45- age at marriage of wife i ; 60-husband's age at marriage with wife i). S = (husband's age at second marriage - husband's age at first marriage).

Predictors of preferences : religion (husband and wife), ethnic group (husband and wife), education (husband and wife), rural dummy (husband and wife), income (husband), employment status (wife), birth cohort (husband and wife), region of residence, age at marriage (husband and wife), be in first marriage (husband and wife). Additional controls : co-residence status, work in public sector (husband), at least one child from previous union (husband and wife), having at least one dead child from current union, mother's age and age squared at birth j, a dummy for each j.

Cox estimation with a baseline hazard common to all women. Breslow method to handle ties among non-censored durations. Robust standard errors of the coefficients are in parentheses (clustered at the woman level). Significance levels (for hazard ratio = 1) : * p<0.10, ** p<0.05, *** p<0.01.

Table E.5: Testing the common life-cycle assumption

Hazard ratios	Between polygamous and monogamous wives	Among polygamous wives
$\{j = 2\}$	0.237***	0.128***
	(0.034)	(0.071)
$\{j = 3\}$	0.066***	0.018***
	(0.016)	(0.017)
$\{j = 4\}$	0.015***	0.005**
	(0.006)	(0.011)
Future first wives * $\{j = 2\}$	0.842	
	(0.168)	
Future first wives * $\{j = 3\}$	0.844	
	(0.227)	
Future first wives * $\{j = 4\}$	1.121	
	(0.414)	
$T_2 \times \{j = 2\}$		0.995
		(0.028)
$T_2 \times \{j = 3\}$		1.004
		(0.034)
$T_2 \times \{j = 4\}$		0.946
		(0.091)
Controls	Mother's age and age squared at birth j	
Baseline hazard	Woman-specific	
Observations	1411	207
Clusters	571	94

Data : PSF. Dep. var. : duration between births j and $(j+1)$. Birth ranks higher than 4 are excluded because there is not enough observations in each cell.

Sample : in column 1, monogamous and senior wives *before the second marriage*, below 45 years old, for whom the complete birth history is known, having at least one child from current union. In column 2, senior wives *before the second marriage*, below 45 years old, for whom the complete birth history is known, having at least one child from current union.

"Future first wives" is equal to 1 if the woman is in a polygamous union at the time of the survey. $T_2 = \min$ (45-second wife's age at marriage ; 60-husband's age at second marriage).

Stratified partial likelihood estimation with baseline hazards specific to each woman ; Breslow method to handle ties among non-censored durations. Robust standard errors of the coefficients are in parentheses (clustered at the woman level). Significance levels (for hazard ratio = 1) : * p<0.10, ** p<0.05, *** p<0.01.

TABLE E.6: Placebo test : alternative cut-offs S

Hazard ratios	Baseline : true S		$S = 10$ years (mean)		$S = 6$ years (Q_1)		$S = 13$ years (Q_3)	
	Next int.	All int.	Next int.	All int.	Next int.	All int.	Next int.	All int.
After	0.557**	0.770	0.668	0.848	0.791	0.992	0.863	0.920
	(0.142)	(0.143)	(0.170)	(0.140)	(0.191)	(0.162)	(0.211)	(0.199)
Controls	Birth rank dummies, mother's age and age squared at birth j							
Baseline hazard	Woman-specific							
Observations	2286	2483	2285	2483	2165	2483	2353	2483
Clusters	699	716	703	716	688	716	712	716

Data : PSF. Dep. var. : duration between births j and $(j+1)$. In odd-numbered columns, I consider all intervals in the monogamous stage, and the very next interval after the second marriage. In even-numbered columns, I consider all intervals.

Sample : monogamous and first wives below 45 years old, for whom the complete birth history is known, having at least one child from current union.

$After$ is a time-varying variable indicating if the index birth occurred after S. In column 1, S is the observed length of the monogamous period. In the last three columns, I run Placebo tests using alternative cut-offs, respectively the mean, the first quartile and the last quartile taken from the distribution of S in the sample. Stratified partial likelihood estimation with baseline hazards specific to each woman ; Breslow method to handle ties among non-censored durations. Robust standard errors of the coefficients are in parentheses (clustered at the woman level). Significance levels (for hazard ratio = 1) : * p<0.10, ** p<0.05, *** p<0.01.