The long-term impact of trade with firm heterogeneity

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Abstract

This paper develops a model of North-South trade with firm heterogeneity and growth through expanding varieties. Introducing simple dynamics in the heterogeneous firm model adds new static and dynamic effects to the well-known decrease in prices that increases welfare in the static model. The constant level of nominal expenditure is affected as firm selection changes the average value of firms which modifies consumers' resource constraint. The growth rate of real consumption is also affected by firm selection since greater average efficiency means a larger amount of resources are required to create a new variety. Country asymmetry yields differentiated results between countries. In all cases net welfare results depend on parameter values which highlights how much welfare evaluations depend on the initial setting imposed.

Keywords: firm heterogeneity, specialization, asymmetric countries, welfare.

JEL Classification numbers: F12, F15, H32, O40.

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1 Introduction

The rise and further development of the Heterogeneous Firm Trade (HFT) model introduced the possibility of analysing within-industry reallocation of resources caused by freer trade. In Melitz (2003) and Helpman et al. (2004) firms face costly discovery of their productivity and then make their producing and exporting decisions. A standard result in this model is that trade openness provides incentives for the most efficient firms in the industry to export while low-productivity firms are forced-out by tougher competition. As a result resources move from the latter firms to the former, average efficiency in the industry increases and welfare rises due to the subsequent reduction of the price index. The HFT model has been extensively used over the last decade to account for many previously unexplained facts in trade literature but a static approach has prevailed.

The dynamic effects of trade and especially the long term consequences of trade-induced resource allocation are traditionally tackled by growth theory. We can think about the most influential works on endogenous growth of the nineties as adding different types of growth mechanisms to a diversity of trade models. These works provided intuition on the long-term effects that trade could have on knowledge accumulation (Grossman and Helpman, 1990), human capital formation (Romer, 1990) or learning by doing (Young, 1991) among other factors that have an impact on growth and welfare.

The present paper contributes to this literature introducing very simple dynamics to a HFT model with two asymmetric countries (North-South). Growth is the result of expanding varieties as in Grossman and Helpman (1991). Introducing simple dynamics in the HFT model adds new static and dynamic effects to the well-known decrease in prices that increases welfare in Melitz (2003). Allowing agents to save resources in any given moment implies their consuming possibilities can be modified by freer trade if firm selection affects the activity towards which savings are allocated. In the present model savings go to the sector in charge of creating knowledge which is demanded by final producers so firm selection is able to affect returns on savings. Lower trade costs can produce dynamic effects if it has an impact on the rate at which new varieties are introduced in the economy which determines the rate at which prices decrease over time. In the present model, an increase in the average amount of knowledge required by firms determines that more resources must be devoted to the creation of a new variety which yields a lower variety growth rate.

The welfare effects of freer trade in the dynamic HFT model proposed here differ greatly from the standard static model, even with symmetric countries. While in Melitz (2003) both countries experience welfare gains through a price fall, in the present model this effect is enhanced by an increase in nominal expenditure due to higher returns from households' savings. However, firm selection increases the average requirement of knowledge thus increasing the cost of new varieties. The resulting negative dynamic effect turns long-term welfare outcomes uncertain.

Asymmetries in technological development between countries add an extra layer of heterogeneity that affects welfare outcomes. A small technological lag in the South still yields positive static effects in both countries but their magnitude differ being greater in the North. Moreover, even though growth rates converge to a lower value in

the long term they can differ in the short term. When the technological gap is large, the South may experience a reversed firm selection that makes prices rise and nominal expenditure fall. The negative static effect in the South contrasts with a positive effect in the North. Finally, the Ricardian aspect of the model may promote specialization even in a context of firm heterogeneity.

Among the papers introducing dynamics into the HFT model, the most relevant to my work is Baldwin and Robert-Nicoud (2008) where expanding varieties growth is inserted into a HFT model á la Helpman et al. (2004) in a context of country symmetry. Other works with similar proposals are Gustafsson and Segerstrom (2010) or Dinopoulos and Unel (2011). As opposed to these contributions the model in the present paper allows for country asymmetries and introduces a traditional sector that enables resource reallocation between sectors in addition to the usual within-sector reallocation of the HFT model.[1]

Previous works allowing for differences in size and technology between countries are Baldwin and Forslid (2010) and Demidova (2008) respectively. Models in those works are static so most of the welfare channels explored in the present paper are absent in them. Indeed, welfare results in both papers are completely driven by movements in the price index. Moreover Baldwin and Forslid (2010) there is no room for welfare losses, while this is possible in the present model.

The present work is also related to Sampson (2014) who also emphasises the potentiality that firm heterogeneity brings when dynamic effects are considered. That work explores welfare effects in the case the productivity distribution of firms is positively affected by knowledge spillovers. The resulting dynamic selection introduces a new source of gains from trade previously unexplored. My work can be seen as complementary to his in that, acknowledging the value of endogenizing the distribution of firms, I show that introducing simple dynamics, the HFT model presents richer welfare effects than the homogeneous firm model, even with an exogenous distribution of firms.

Finally, the present paper may also be seen as a contribution to the ongoing discussion regarding welfare effects in the HFT model. In a recent paper, Arkolakis et al. (2012) prove that in a class of models with certain characteristics, gains from trade can be expressed as a function of only two arguments: the import-penetration ratio and the trade elasticity. Armington model and the model in Krugman (1980) belong to this family of models along with the HFT model with Pareto distributed firms. This drives the conclusion that allowing for firm heterogeneity does not yield new welfare effects. On the other side of the debate, Melitz and Redding (2014) argue that allowing firms to endogenously decide whether to enter a market or not provides the HFT model with an extra adjustment margin that is efficient. Then, if both models are calibrated to yield the same welfare results for a certain level of trade costs, the extra margin in the HFT model results in greater welfare for all other parameter values. The conclusion is that welfare effects of trade can be said to be not only different but greater in the HFT model. The present paper contributes to this debate by showing that, when simple

[1]Another stream of dynamic HFT models have adopted a somehow different approach. Atkeson and Burstein (2010) allow differentiated-good producers to invest in innovations turning growth into the result of firm's strategic decisions. This choice, also followed by others like Burstein and Melitz (2013) or Alessandria and Choi (2014), yields rich firm dynamics that, although interesting, are unnecessary for the purposes of this work. So far this stream of the literature has not been expanded to asymmetric countries. This could be subject for future work.

dynamics are introduced in the HFT model, the new channels affecting welfare depend on firm selection and therefore are absent in models with homogeneous firms.

The rest of the paper is organized as follows. Section 2 presents the model identifying the main assumptions made. Section 3 defines the Balanced Growth Path and presents closed-form solutions for all endogenous variables under the assumption that firms' productivity are Pareto-distributed. Section 4 presents the long-term welfare results of increasing openness. First, it presents results in a context of firm heterogeneity within a given country but assuming country-symmetry to contrast these to those arising under firm homogeneity. Then, between-country firm heterogeneity is allowed for to show how dynamic effects can change welfare results when countries are asymmetric. Finally, section 5 concludes.

2 The model

The continuous time model is composed of two economies (North and South) indexed by $i = N, S$ difering only in size and degree of technological development. Each economy features two types of final good producers: one sector (denoted M) produces differentiated goods and sells them to consumers in a monopolistic competition market; the other one (sector C) produces a homogeneous good under perfect competition. The homogeneous good is freely traded in the world market. This imposes factor price equalization between countries as long as the C-good is produced in both. Economies also feature an R&D sector in charge of producing the knowledge (denoted K) that enables the production of new M-products. Each economy is endowed with a fixed amount of labour (we assume no population growth) that has to be distributed among the three productive activities $L_i = L_{K,i} + L_{M,i} + L_{C,i}$.

The timing of production in the M-sector is similar to that proposed in Melitz (2003). First, differentiated good producers buy blueprints for a new variety (and pay a sunk cost). Then they discover the productivity $(1/a)$ they could have if they engage monopolistic production of that new variety. With this information firms evaluate whether it is profitable for them to incur in the extra costs of serving the domestic market or even exporting at a greater cost. The fact that profits increase with productivity implies that there will always be a firm with marginal selling cost a_D (a_X) that is indifferent between producing for the domestic (foreign) market or not. This yields three types of firms: firms that exit and do not serve any market (i.e. those with $a_D < a$), those that only serve the local market $(a_X < a < a_D)$ and those serving both the domestic and the foreign markets $(a < a_X)$.

2.1 Consumers

Economy i is comprised of L_i homogeneous consumers at any t so I assume population does not change over time. Each worker inelastically supplies one unit of work obtaining a wage w_i in country i. Consumers in i have to make three choices. First, they have to choose how much to consume and save at each moment in time, i.e. they decide their optimal expenditure level $E_i(t)$. Then, they need to decide how much they are going to consume of each of the two types of final goods (i.e. chose $E_{M,i}(t)$ and $E_{C,i}(t)$ with $E_i(t) = E_{M,i}(t)+E_{C,i}(t)$) within each t. Finally, they need to establish how to split their consumption of M-goods among the different varieties available at each t.

Welfare at t is defined as present discounted value of future real consumption:

$$U_i(t) = \int_t^\infty e^{-\rho(s-t)} \ln(Q_i(s))\, \mathrm{d}s \tag{1}$$

where $\rho > 0$ is the rate of pure time preference, $Q_i(t) = E_i(t)/P_i(t)$ is real consumption and $P_i(t)$ is the aggregate price index at t.

At every moment in time, consumers from i maximize (1) subject to budget constraint $Y_i(t) = E_i(t) + \dot{W}_i(t)$. Here $W_i(t)$ is accumulated wealth and $Y_i(t)$ is current income which is composed by labour income and profits made by domestic firms in any market, i.e. $Y_i(t) = w_i(t)L_i + \Pi_i(t)$. I assume that what consumers get as revenue from their savings equals domestic firms' profits (in any market), i.e. $\Pi_i(t) = \iota(t)W_i(t)$ where $\iota(t)$ is the international interest rate. The underlying assumption I am making is that firms from country i can only be financed by households in i. The extent to which this is a realistic assumption depends strongly on the average size of firms in a country since small firms typically obtain their resources in local markets.

The Hamiltonian to this problem can be written as

$$H_i(t, \lambda) = e^{-\rho t} \ln(E_i(t)/P_i(t)) + \lambda(w_i(t)L_i + \iota(t)W_i(t) - E_i(t))$$

Remember that $P_i(t)$, $\iota(t)$ and $w_i(t)$ are exogenous to the consumer so her control variable is only $E_i(t)$. The optimality conditions that arise from this problem are a transversality condition and the following Euler equation

$$\frac{\dot{E}_i(t)}{E_i(t)} = \iota(t) - \rho \tag{2}$$

Consumers have two-tier utility functions with the upper-tier (Cobb-Douglas) dictating their consumption between the two goods and the lower-tier (CES) determining their demand for different varieties of the M-good. As consequence of the upper-tier utility function, at each moment in time t, each consumer in i devotes a fraction μ of their total expenditure to different varieties of the M-good and a fraction $1-\mu$ to purchases of the C-good. Then $E_{M,i}(t) = \mu E_i(t) = Q_{M,i}(t).P_{M,i}(t)$ and $E_{C,i}(t) = (1-\mu)E_i(t) = Q_{C,i}(t).P_{C,i}(t)$. Also, the aggregate price index in i is

$$P_i(t) = P_{C,i}(t)^{1-\mu} P_{M,i}(t)^{\mu} B \tag{3}$$

where $B = (1-\mu)^{\mu-1}\mu^{-\mu}$.

A direct outcome of CES preferences between varieties of the M-sector, the aggregate demand in for the M-good is

$$Q_{M,i}(t) = \left[\int_{\theta \in \Theta_i(t)} q_i(\theta, t)^{1-1/\sigma} \mathrm{d}\theta\right]^{1/(1-1/\sigma)}$$

where $q_i(\theta, t)$ represents the demand in i for variety θ of the M-good, $\Theta_i(t)$ represents the mass of available varieties in the M-good market of this economy (both produced domestically and imported) at time t and $\sigma > 1$ is the constant elasticity of substitution between any two varieties of the M-good. As shown by Dixit and Stiglitz (1977) the perfect price index in the market with monopolistic competition can be written as

$$P_{M,i}(t) = \left[\int_{\theta \in \Theta_i(t)} p_i(\theta, t)^{1-\sigma} \mathrm{d}\theta\right]^{1/(1-\sigma)} \tag{4}$$

where $p_i(\theta, t)$ is the price of variety θ at time t in market i.

Using these expressions I can write the optimal consumption for each M-variety in i:

$$q_i(\theta, t) = Q_{M,i}(t) \left[\frac{p_i(\theta, t)}{P_{M,i}(t)} \right]^{-\sigma}$$

Then expenditure in each M-variety available $e_i(\theta, t)$ and aggregate expenditure in the M-good at country i are respectively:

$$e_i(\theta, t) = E_{M,i}(t) \left[\frac{p_i(\theta, t)}{P_{M,i}(t)} \right]^{1-\sigma} \quad ; \quad E_{M,i}(t) = \int_{\theta \in \Theta_i(t)} e_i(\theta, t) \mathrm{d}\theta \tag{5}$$

It must be emphasized that all aggregate variables represent the domestic market but include products produced both locally and imported. For example P_i is the price index in economy i and it aggregates prices of domestic and foreign products being sold in that country.

2.2 Final good producers

2.2.1 C-sector

This sector produces using labour as sole input. Technology of production is the same in both countries $Q_C = L_C$. As mentioned before, perfect competition imposes zero profits in this sector which implies that, at every t, marginal revenues must equal marginal costs, i.e. $P_{C,i}(t) = w_i(t)$. There are no trade costs for the C-good which imposes $P_{C,i}(t) = P_{C,j}(t)$. Then, as long as there is some production of the homogeneous good in both countries, factor price equalization holds and wages are the same in both economies. I can then normalize wages in both economies ($w = P_C = 1$).[2] By (3) a direct result from this is

$$P_i(t) = P_{M,i}(t)^{\mu} B \tag{6}$$

Revenues made by C-firms from country i are $R_{C,i} = L_{C,i}$. These revenues come from selling in the domestic and in the foreign markets. Unlike the autarkic or the symmetric country cases where revenues made by a sector equal domestic expenditure in the goods of that sector, in an asymmetric world with trade $R_{C,i}$ and $E_{C,i}$ are not necessarily equal. In our asymmetric country model $E_{C,i}$ equals to a fraction $1 - \mu_i$ of consumers current income at time t.

2.2.2 M-sector

At each moment in time there is a number of prospective entrants to the M-sector in country i. Prospective entrants pay the R&D sector a one-time sunk-entry cost $f_I > 0$ to buy a blueprint enabling the production of a new variety indexed by θ. After paying this sunk cost the firm learns the labour-unit marginal cost a at which it can produce. Marginal costs are drawn from a country-specific distribution $g_i(a)$ over the support $(0, a_{m,i})$ with cumulative distribution $G_i(a)$.

[2]Notice that when country symmetry is imposed, wages can change over time but they are always equalized to those in other countries. This allows symmetric-country models as those in Melitz (2003) or Baldwin and Robert-Nicoud (2008) to set wages as *numeraire*. In my asymmetric country setting wages can be set equal to 1 only as long as there is some production of the C-sector.

As explained in Krugman (1980, p.951) since the cost of differentiating products is zero and all varieties enter consumers' demand symmetrically, there are no incentives for firms to produce a variety that is already being produced by other firms. Then each variety is produced by only one firm. The fact that each firm also has a unique marginal cost a allows me to index goods and firms by θ or a indistinctly.

When the mass of varieties produced is large (as assumed here) the cross price elasticity of demand tends to zero between any pair of products. This means that each firm in the M-market acts independently of the others. Then I can treat each M-producer as enjoying monopolistic rights to sell its variety for as long as it stays in the market. I assume firms do not die, so they produce forever but it is easy to show that introducing an exogenous death probability for firms does not change the analysis in any relevant way.

Each M-firm in i uses labour as input and its production function is $q_i(a,t) = l_{M,i}(a,t)/a$. After paying the sunk cost f_I, and knowing the marginal cost at which it will produce, the firm must decide whether to serve the domestic market, for which it has to incur in an additional sunk cost of $f_D > 0$, or not. I assume $a_{m,i}$ is high enough to make production non-profitable for the higher values of a's support. Firms with too high marginal costs do not produce and make a profit of $-f_I$. Firms with a marginal cost below a certain cut-off $a_{D,i}$ chose to produce as future operating profits allows them to (at least) cover the sunk cost f_D. Notice that not all producing firms have non-negative profits as there are some for which the discounted value of operating profits are sufficient to cover f_D but not necessarily f_I.

To serve a foreign market, firms have to pay an additional sunk cost $f_X > 0$. International trade in the M-good also involves per-unit costs, modelled here as iceberg costs $\tau \geq 1$, i.e. τ units must be shipped for 1 unit to reach the final consumer in the foreign destination. Depending on their productivity level some of i's M-producers find profitable to export and some others do not. There will always be a firm with marginal cost level $a_{X,i}$ making zero profits from its foreign trade activities.

Each firm pays its corresponding sunk costs only once, at the beginning of their activities. All these costs are composed by units of knowledge so their value can be written as $f_{v,i} = \kappa_v.P_{K,i}$ for $v = I, D, X$, where κ_v is a fixed amount of units of knowledge and $P_{K,i}$ is the price of knowledge. I assume the κ's are the same worldwide while $P_{K,i}$ is country specific (it depends on the productivity of the local R&D sector). I also assume $\kappa_D < \kappa_X$ which reflects the fact that firms find it less costly to sell locally than to engage in business abroad.

Dixit-Stiglitz competition implies that successful firms price their products with a fixed mark-up of $1/(1-1/\sigma)$ over their marginal cost, which reflects their market power. A typical outcome of monopolistic competition with CES preferences, constant mark-ups imply that higher firm productivity goes entirely to lower prices and, given demands are elastic, to higher profits as well. I call m the marginal selling cost a firm with marginal cost a has in a given market. Then, since wages are equal to one in both countries m equals a for sales to the domestic market and τa for sales to the foreign market. This allows me to write the pricing rule in the M-sector as:

$$p(m,t) = \frac{m}{1-1/\sigma} \tag{7}$$

Notice we are assuming no differences in preference parameters ρ and σ between countries. The pricing rule in (7) implies each M-firm has a mark-up over its sales of $1/\sigma$ in each market so the operating profits a firm with selling cost m makes in market i is

$$\pi_i(m,t) = \frac{s_i(m,t)E_{M,i}(t)}{\sigma} \tag{8}$$

where $s_i(m,t)$ is the share that a firm with marginal selling cost of m has in market i.

For two different firms with marginal selling costs m_1 and m_2 respectively in market i, it is easy to show that $\frac{q_i(m_1)}{q_i(m_2)} = \left(\frac{m_2}{m_1}\right)^{\sigma}$, $\frac{p(m_1)}{p(m_2)} = \frac{m_1}{m_2}$ and $\frac{\pi_i(m_1)}{\pi_i(m_2)} = \left(\frac{m_2}{m_1}\right)^{\sigma-1}$. Firms with lower selling costs in market i enjoy larger demands and charge lower prices to consumers. The net result in this context is larger revenues and profits for the firm with lower marginal selling cost in that market.

Aggregate operating profits all firms make from serving i is

$$\Pi_i(t) = \frac{E_{M,i}(t)}{\sigma} \tag{9}$$

In the asymmetric-country model this latter concept differs to that denoted as $\mathsf{\Pi}_i$ in section 2.1 which refers to profits made by firms from country i in all markets.

Using (4), (5), (7) and (8) the market share a firm with marginal selling cost m has in market i can be written as:

$$s_i(m,t) = \left[\frac{p_i(m,t)}{P_{M,i}(t)}\right]^{1-\sigma} = m(\theta)^{1-\sigma}\left[\int_{\theta\in\Theta_i(t)} m(\theta)^{1-\sigma}\mathrm{d}\theta\right]^{-1} = \left[\frac{m}{\breve{m}_i}\right]^{1-\sigma} \tag{10}$$

with

$$\breve{m}_i(t)^{1-\sigma} = n_i(t)\hat{m}_i^{1-\sigma} + n_j(t)\nu_j\tilde{m}_i^{1-\sigma} \tag{11}$$

and

$$\begin{aligned} \nu_i &= G_i(a_{X,i})/G_i(a_{D,i}) \\ \hat{m}_i^{1-\sigma} &= \int_0^{a_{D,i}} a^{1-\sigma}\mathrm{d}G_i(a|a_{D,i}) \\ \tilde{m}_i^{1-\sigma} &= \phi\int_0^{a_{X,j}} a^{1-\sigma}\mathrm{d}G_j(a|a_{X,j}) \end{aligned} \tag{12}$$

Here $n_i(t)$ denotes the mass of varieties being produced in country i and $\phi = \tau^{1-\sigma}$ is a measure of the importance of variable trade costs in the industry ($0 \le \phi \le 1$, when $\tau \to \infty$ then $\phi \to 0$ and when $\tau \to 1$ then $\phi \to 1$). $\hat{m}_i$ is the marginal selling cost of the representative firm producing in i for the domestic market while $\tilde{m}_i$ is the marginal selling cost of the representative foreign firm selling in i. ν_i is the proportion of firms from i that serve the foreign market j. Then we can think of $\breve{m}_i$ as the aggregate marginal selling costs of firms from both countries serving market i. Equation (10) shows that the greater the marginal selling cost a firm has the lower its share in a given market. A firm's share also decreases the tougher the competition it faces in that market (i.e. the lower is $\breve{m}$).

Notice that (11) implicitly assumes that the set of produced varieties includes the set of exported varieties in both countries. This is formally stated in Assumption 2. Calling $N_i(t) = n_i(t) + \nu_j n_j(t)$ the total mass of firms that sell in country i, then $\breve{m}_i N_i^{1/\sigma-1}$

is the marginal selling cost of the representative firm serving that market.[3]

Finally using (4) and (7) I can write:

$$P_{M,i}(t) = \frac{\breve{m}_i(t)\sigma}{\sigma - 1} \tag{13}$$

This expression shows that the aggregate price of M-goods will be lower in the country where average selling costs are lower.

2.3 K-sector

This sector produces units of knowledge demanded by firms in the M-sector using labour as sole input. It has the following production function:

$$Q_{K,i}(t) = \frac{n_i(t)}{a_{K,i}} L_{K,i}(t) \tag{14}$$

where $L_{K,i}(t)$ is the amount of labour devoted to the sector in country i and $n_i(t)/a_{K,i}$ is the productivity of its workers. As in the standard expanding variety model, labour productivity in the K-sector is composed by an exogenous parameter $a_{K,i}$ and is affected by spillovers from existing blueprints $n_i(t)$ (see for example section 3.2 in Grossman and Helpman 1991).

Perfect competition in the market of knowledge imposes zero profits and therefore $a_{K,i}/n_i(t) = P_{K,i}(t)$ where $P_{K,i}(t)$ is the unitary price of knowledge expressed in units of labour at time t in country i.

2.4 Equilibrium conditions

2.4.1 Cut-off conditions in the M-sector

Firms from country i discount future operating profits at a rate γ_i. The present value of operating profits that a firm from country i and selling cost m gets by serving its domestic market as $s_i(m,t)E_{M,i}/\sigma\gamma_i$. Operating profits from serving foreign market j equals $s_j(m,t)E_{M,j}/\sigma\gamma_i$. After paying the initial fixed cost f_I, firms take producing and exporting decisions comparing these values with the sunk cost required to engage each activity which must be paid pay at their country of origin. This allows me to define the cut-off conditions for serving market i as:

$$\frac{s_i(m_{D,i},t)E_{M,i}}{\sigma\gamma_i} = P_{K,i}\kappa_D \quad ; \quad \frac{s_i(m_{X,j},t)E_{M,i}}{\sigma\gamma_j} = P_{K,j}\kappa_X \tag{15}$$

where $m_{D,i} = a_{D,i}$ and $m_{X,j} = \tau a_{X,j}$. Notice that the second condition imposes that a firm $a_{X,j}$ from country j is indifferent between serving market i or not. There are four conditions for four indifferent firms since in each country there is one firm for which future profits exactly offset the costs for selling to the local market and another one for which the same is true regarding exports.

[3]Expression $\breve{m}_i^{1-\sigma}$ in the present model is the equivalent to $M_{ti}\tilde{\varphi}_{ti}^{\sigma-1}$ in Demidova (2008) and -if symmetry is imposed- to $n\bar{m}^{1-\sigma}$ in Baldwin and Robert-Nicoud (2008).

2.4.2 Free-entry conditions in the M-sector

At equilibrium, the ex-ante probability distribution of a's provided firms are successful (i.e. they are producing) is the same as the ex-post distribution of producing firm a's. Moreover, at any t the ex-ante expected value of operating profits of a firm from country i equals the ex-post average operating profits of all producing firms from that country. Using this property and expression (8) I can write:

$$V_i^e(t) = \frac{\bar{s}_{i,i}(t)E_{M,i}(t) + \nu_i \bar{s}_{i,j}(t)E_{M,j}(t)}{\sigma\gamma_i} \tag{16}$$

where $\bar{s}_{i,j}$ is the average share a firm from country i has on market j. The expected value at t of a producing firm from country i is composed by the discounted expected profits the firm would obtain selling to the domestic market plus the discounted expected profits the firm would obtain selling abroad times the probability that a firm from i accesses the foreign market.

Equation (16) shows a very interesting feature of the asymmetric country model. While under symmetry all firms face the same expected operating profits regardless of their country of origin the same is not true in the asymmetric country case. Indeed, it is possible that firms from one country, say N, enjoy larger average shares than firms from the other country S (e.g. it can happen that $\bar{s}_{N,N} > \bar{s}_{S,S}$ and $\bar{s}_{N,S} > \bar{s}_{S,N}$). In such a case, the present day average operating profits of firms from N is greater than that of firms from S. If this is the case, the average producing firm from N is more valuable than its counterpart form S.

Using (10) I can define average shares of firms from i in their domestic and foreign markets as:

$$\bar{s}_{i,i}(t) = \left[\frac{\hat{m}_i}{\breve{m}_i(t)}\right]^{1-\sigma} \quad ; \quad \bar{s}_{i,j}(t) = \left[\frac{\tilde{m}_j}{\breve{m}_j(t)}\right]^{1-\sigma} \tag{17}$$

The average share that firms from country i have in their domestic market depends negatively on their average marginal selling cost ($\hat{m}_i$) and positively on selling costs of all firms serving that market ($\breve{m}_i$). Similarly, the average share that firms from country i have in the foreign market depends also negatively on their average marginal selling cost ($\tilde{m}_j$) and positively on selling costs of all firms serving that market ($\breve{m}_j$).

Let me define $d_i = \bar{s}_{i,i}n_i$ as the total share of market i served by domestic firms and $x_i = \nu_i\bar{s}_{i,j}n_i$ as the total share of market j served by firms from i. Then I obtain:

$$d_i = \left[1 + \frac{n_j\nu_j}{n_i}\left(\frac{\tilde{m}_i}{\hat{m}_i}\right)^{1-\sigma}\right]^{-1} \text{ and } x_i = \left[1 + \frac{n_j}{n_i\nu_i}\left(\frac{\hat{m}_j}{\tilde{m}_j}\right)^{1-\sigma}\right]^{-1} \tag{18}$$

Using (18) it is possible to prove that $d_i \geq x_i$ and equality would only hold if $a_{D,i} = a_{X,i}$ in both countries and $\phi = 1$ (see proof 1 in the Appendix). Firms from country i always enjoy a greater aggregate share at their domestic market than abroad. By definition $d_i \leq 1$, $x_i \leq 1\forall i$ and $d_i + x_j = 1$. Then, as long as there is some degree of openness ($\phi > 0$), average shares are lower than $1/n_i$, i.e. the average share in autarky.

Free entry in the M-sector implies that at equilibrium there are no incentives for entry. The expected value of operating profits V_i^e must equal the expected sunk costs firms

pay. Using (16) I can write the free-entry condition (FEC) as:

$$\frac{\bar{s}_{i,i}E_{M,i} + \nu_i \bar{s}_{i,j}E_{M,j}}{\sigma\gamma_i} = P_{K,i}\bar{\kappa}_i \tag{19}$$

where

$$\bar{\kappa}_i = [\kappa_D G_i(a_{D,i}) + \kappa_X G_i(a_{X,i}) + \kappa_I]/G_i(a_{D,i}) \tag{20}$$

is the expected sunk cost a producing firm has to pay expressed in units of knowledge. The left-hand side in (19) can be viewed as the present-day expected stock value of a firm from country i while the right hand side is the expected replacement cost in that country (i.e. the cost associated to the creation of a new average producing firm in i). Then, as in Baldwin and Robert-Nicoud (2008), equation (19) resembles Tobin's $q = 1$ equation. Notice that the only source of country difference in $\bar{\kappa}_i$ comes from the ex-ante distribution of a's since I am assuming no country differences in the sunk costs expressed in units of knowledge (the κ's). The FEC can also be expressed in aggregate terms:

$$\frac{d_i E_{M,i} + x_i E_{M,j}}{\sigma\gamma_i} = a_{K,i}\bar{\kappa}_i \tag{21}$$

Notice that, by (16), $a_{K,i}\bar{\kappa}_i$ must equal $n_i V_i^e$ at equilibrium. Then, aggregate knowledge purchased by firms from i ($a_{K,i}\bar{\kappa}_i$) can be interpreted as the expected stock value of all firms from that country.

2.4.3 Static equilibrium

The clearing condition in the market for knowledge imposes that, at any t, the production of knowledge must be used in sunk costs of new entrants, which gives:

$$Q_{K,i}(t) = \dot{n}_i(t)\bar{\kappa}_i \tag{22}$$

The previous equation introduces a major departure from the standard expanding variety model with homogeneous firms. Indeed, in such a model all firms buy the same amount of blueprints and the value of that purchase can then be normalized to one which, in terms of our model imposes $\bar{\kappa}_i = 1$. The fact that the present model allows for an endogenous and country-specific value of $\bar{\kappa}_i$ implies introducing an externality in the K-sector that depends on firm selection apart from the usual spillovers in the standard expanding variety model stemming from the mass of existing firms. By definition, the rate at which new varieties are introduced in any economy i, must be positive (i.e. $\frac{\dot{n}_i}{n_i} > 0\ \forall i$). Then, using (14) I can express the growth rate of varieties in country i's M-sector as:

$$g_i = \frac{\dot{n}_i}{n_i} = \frac{L_{K,i}(t)}{a_{K,i}\bar{\kappa}_i} \tag{23}$$

To obtain closed-form expressions for the cut-off levels of marginal costs I use cut-off conditions in (15) and obtain:

$$\frac{a_{X,j}}{a_{D,i}} = \left(\frac{\phi}{T}\frac{P_{K,i}}{P_{K,j}}\right)^{\frac{1}{\sigma-1}} \tag{24}$$

where $T = \frac{\kappa_X}{\kappa_D} > 1$ measures how important the supplementary cost to export is relative to the cost of serving the domestic market. This expression shows that there exist a relationship between the range of varieties being exported to i and the range of varieties being produced in i. As in Demidova (2008), this relationship depends

on the degree of openness to trade (in variable and sunk costs), but equation (24) includes a new determinant of the ratio $a_{X,j}/a_{D,i}$, i.e. the relative price of knowledge between countries which introduces firm selection as a relevant factor here too. The range of exports from j to i, is enhanced by greater openness (i.e. larger ϕ and/or smaller T), lower relative price of knowledge in j (larger $P_{K,i}/P_{K,j}$) and greater average costs among local firms which follows from a larger $a_{D,i}$. Inversely, solving for $a_{D,i}$ the equation shows that greater openness, a larger relative price of knowledge in i and a lower range of varieties coming from abroad (smaller $a_{X,i}$) reduce average productivity of domestic firms in i.

To obtain simple closed-form solutions of cut-off values I impose the following assumption:

Assumption 1 *The distribution of firm's productivities in the M-sector follow a Pareto distribution in both countries.*

Power functions of a Pareto random variable are Pareto-distributed with a different shape parameter. In my model, this means that firm sizes and operating profits are Pareto too. The use of this distribution is customary in the literature although its empirical validation is not undebated.[4] Besides its analytical convenience, the use of the Pareto distribution serves one of the purposes of this paper. As shown in Arkolakis et al. (2012) a static HFT model like that in Melitz (2003) with Pareto-distributed firms yields the same welfare results than a similar model with homogeneous firms. But, as shown below, even with Pareto-distributed firms, the HFT model is able to account for new welfare channels when simple dynamics are set in place. Moreover, this paper shows that these channels are absent if firms are assumed homogeneous.

When firm productivity distribution $1 - G_i(a)$ is Pareto-shaped, the ex-ante distribution of marginal costs can be written as

$$G_i(a) = (a/a_{m,i})^{\alpha}$$

with $a \in (0, a_{m,i}]$ in country i. Here $a_{m,i} > 0$ is the scale parameter of the distribution and therefore it resumes information about the state of the technology in country i. Indeed, for any two countries, prospective entrants to the M-sector in the country with a lower a_m have a greater probability of obtaining a cost level below a certain value a^*, $\forall a^* \in (0, a_{m,i}]$. Parameter $\alpha > 0$ is the shape parameter for firm productivities and marginal costs. The shape parameter for firm sizes and operating profits is then $\beta = \frac{\alpha}{\sigma-1}$. As in Melitz and Redding (2012), I impose $\beta > 1$ as this is a necessary condition for integrals in (12) to converge. The average marginal cost, firm size and operating profit are guaranteed to be finite as a consequence. Then using (15) and (21) I get the cut-off value for domestic production:

$$a_{D,i} = a_{m,i} \left[\frac{\kappa_I}{\kappa_D} \frac{\beta - 1}{Z_i} \right]^{1/\alpha} \tag{25}$$

[4] An incomplete list of papers using this distribution may include Helpman et al. (2004), Melitz and Ottaviano (2008), Baldwin and Robert-Nicoud (2008), Falvey et al. (2011) or Melitz and Redding (2012). Nevertheless, empirical validation of the distribution is not settled. Axtell (2001) shows that the Pareto distribution is a good fit for sales and employment among US's firms. Using data for French firms, Combes et al. (2012) presents empirical evidence suggesting that the log-normal distribution fits firms' productivity better. That paper also finds that the main results in Melitz and Ottaviano (2008) hold unchanged without the need of any specific functional form for the distribution of productivities.

where

$$Z_i = \frac{1-\Omega^2}{1-\Omega R_i} \tag{26}$$

and $\Omega = \phi^\beta(1/T)^{\beta-1}$ is a measure of openness in the economy that combines the importance of variable (ϕ) and sunk (T) trade costs in a Cobb-Douglas function with increasing returns to scale. Remember that $0 < \phi < 1$ and $T > 1$, therefore $0 < \Omega < 1$. Then, Z_i shows how the Ricardian comparative advantage, given here by $R_i = [P_{K,j}/P_{K,i}]^\beta[a_{m,j}/a_{m,i}]^\alpha$, affects firm selection. Indeed, the relatively better the technology in country i (i.e. the greater $a_{m,j}/a_{m,j}$ or $P_{K,j}/P_{K,i}$) the higher Z_i and the lower $a_{D,i}$ which implies a greater average productivity for local M-firms. Expression (26) helps put forward that in the present model, Ricardian comparative advantage in the M-sector comes both from a better productivity distribution for M-firms but also from better productivity in the K-sector. It is important to notice that under country symmetry $R_i = 1$ and so $Z_i = Z_j = Z = 1 + \Omega$. Notice also that, when economies are closed ($\Omega \to 0$) then equation (25) reduces into its equivalent under autarky ($Z_i \to 1$) and technological asymmetries play no role in firm selection.

In equation (25), the cut-off cost to produce for the domestic market depends negatively on the ratio $\frac{\kappa_D}{\kappa_I}$ reflecting that the greater the cost of domestic production (relative to the initial sunk cost) the narrower the range of varieties the economy is going to produce. The threshold $a_{D,i}$ also depends positively on $a_{m,i}$ reflecting that a better technology (a lower $a_{m,i}$) necessarily yields greater average efficiency in the economy (lower $a_{D,i}$). These results are quite intuitive and resemble what has been obtained under symmetry. What the asymmetric-country expression helps put forward is the effect that openness and technological differences have on this threshold. Indeed, unlike the symmetric country case in which greater openness leads to a monotonic reduction in a_D this is not necessarily the case when countries are technologically asymmetric. This is analysed in detail in section 4.2.

Merging equations (24) and (25) yields two useful expressions for $a_{X,i}$:

$$a_{X,i} = a_{m,j}\left[\Omega\frac{\kappa_I}{\kappa_X}(\beta-1)\frac{1}{Z_j}\left(\frac{P_{K,j}}{P_{K,i}}\right)^\beta\right]^{1/\alpha} = a_{m,i}\left[\frac{\kappa_I}{\kappa_X}(\beta-1)\left(1-\frac{1}{Z_i}\right)\right]^{1/\alpha} \tag{27}$$

Although a detailed analysis of comparative statics for $a_{X,i}$ is provided below, direct inspection of these expressions allows to see that the range of varieties that are being exported by country i depends negatively on the sunk cost to export κ_X relative to the sunk entry-cost κ_I in units of knowledge. Following intuition, if trade costs are at their highest ($\Omega \to 0$) there are no firms efficient enough to export ($a_{X,i} \to 0$).

Abundant evidence supports the intuitive idea that only a fraction of producing firms export to foreign markets (see for example Bernard and Jensen 1999 or Eaton et al. 2004). The present model should reflect this and therefore condition $a_{X,i} < a_{D,i}$ is imposed:

Assumption 2 *To ensure that only a subset of producing firms export, assume $a_{X,i} < a_{D,i}$ $\forall i = N, S$.*

As shown in Proof this imposes $Z_i \in [\frac{T+\Omega^2}{T}, 1+T]$ $\forall i = N, S$. Notice the lower bound for Z_i is greater than 1 which means that $Z_i > 1$ $\forall i$ and guarantees $a_{D,i} > 0$ and

$a_{X,i} > 0 \ \forall i = N, S$ without need of further assumptions.

Finally, there is no capital account in this model so trade must be balanced at every t. This means that trade imbalances in the M-sector, which amount to $x_i E_{M,j} - x_j E_{M,i}$ are exactly compensated by an imbalance in the C-sector.

3 Balanced Growth Path

3.1 Definition and direct implications

To proceed to the BGP of the model let me introduce the following definition:[5]

Definition 1 *A Balanced Growth Path (BGP) in this model is characterized by a fixed allocation of labour among sectors (i.e. constant $L_{K,i}$, $L_{M,i}$ and $L_{C,i}$), constant cut-off values ($a_{D,i}$ and $a_{X,i}$) and aggregate shares (d_i and x_i), and endogenous variables n_i and E_i growing at a constant rate $\forall i = N, S$.*

The fact that the cut-off thresholds must be constant at the BGP implies that $\bar{\kappa}_i$, $\hat{m}_i$ and $\tilde{m}_i$ are constant too. Then by (17) I can obtain equations describing the evolution of average shares:

$$\frac{\dot{\bar{s}}_{i,i}}{\bar{s}_{i,i}} = (\sigma - 1)\frac{\dot{\breve{m}}_i}{\breve{m}_i} \quad ; \quad \frac{\dot{\bar{s}}_{i,j}}{\bar{s}_{i,j}} = (\sigma - 1)\frac{\dot{\breve{m}}_j}{\breve{m}_j} \tag{28}$$

where

$$\frac{\dot{\breve{m}}_i}{\breve{m}_i} = \frac{\frac{\dot{n}_i}{n_i} d_i + \frac{\dot{n}_j}{n_j} x_j}{1 - \sigma} \tag{29}$$

The first expression in (28) shows that the average market share for domestic firms in market i evolves proportionally to the aggregate marginal cost of firms in that market. When the aggregate marginal cost of firms serving market i is decreasing then the share of the domestic average firm decreases as well. The second expression shows a similar relationship for firms from i serving the foreign market: their share decreases when the average marginal cost of firms serving market j decreases. It is also straightforward to show that $\frac{\dot{\bar{s}}_{i,i}}{\bar{s}_{i,i}} = \frac{\dot{\bar{s}}_{j,i}}{\bar{s}_{j,i}}$ and $\frac{\dot{\bar{s}}_{j,j}}{\bar{s}_{j,j}} = \frac{\dot{\bar{s}}_{i,j}}{\bar{s}_{i,j}}$.

Equation (29) shows that the aggregate marginal selling cost decreases in time since $\sigma > 1$ and every term in the numerator is positive. Indeed, the rate at which aggregate marginal costs of firms serving country i is reduced is a weighted average of the rates at which countries introduce new varieties, where weights are the importance that firms from each country have on market i.

Constant aggregate shares impose that the BGP is characterized by $\frac{\dot{n}_i}{n_i} = -\frac{\dot{\bar{s}}_{i,i}}{\bar{s}_{i,i}} = -\frac{\dot{\bar{s}}_{i,j}}{\bar{s}_{i,j}}$. This means that average shares of firms from country i in both markets decrease at the rate at which new varieties are introduced in country i. Using this result I can also get $\frac{\dot{n}_i}{n_i} = \frac{\dot{n}_j}{n_j} = \frac{\dot{n}}{n}$. This is a necessary condition for aggregate shares to remain constant over time. Then I can write the following lemma:

[5] Using a different definition for the BGP yields a different result. In particular, removing the requirement of constant d_i and x_i at BGP, it is possible to prove both variables would asymptotically converge to 1 for one country while they converge to zero in the other (see proof 2). Complete specialization in the M-sector arises and the model reduces to a standard Ricardian model of trade where firm heterogeneity plays no role.

Lemma 1 *At BGP, the M-sector introduces new varieties at the same rate in both countries (i.e. $g_i = g_j = g$).*

By (29) a direct implication of this result is:

$$\frac{\dot{\breve{m}}_i}{\breve{m}_i} = \frac{\dot{\breve{m}}_j}{\breve{m}_j} \tag{30}$$

The aggregate marginal selling cost in both economies decreases at the same rate. This is due to the fact that new varieties are introduced at the same rate in both countries.

Since $a_{K,i}$ is an exogenous parameter then the price of knowledge $P_{K,i} = \frac{a_{K,i}}{n_i}$ decreases at the same rate the mass of varieties increases over time (i.e. g_i). Nevertheless notice that, as a second implication from Lemma 1, the ratio n_i/n_j is constant over time which, by (26), implies that Z_i is a constant at BGP.

Finally I can obtain the expressions for constant $\bar{\kappa}_i$, d_i and x_i:

$$\bar{\kappa}_i = \frac{\beta\kappa_D Z_i}{(\beta-1)}; \quad d_i = \left[1 + \frac{a_{K,j}}{a_{K,i}}(Z_j - 1)\right]^{-1}; \quad x_i = \left[1 + \frac{a_{K,j}}{a_{K,i}}(Z_i - 1)^{-1}\right]^{-1} \tag{31}$$

3.2 Discount factor

Let me find now the expression of the discount factor γ_i. This is the rate firms use to evaluate future cash flows at present time. The value at t of future operating profits for a firm from country i selling in its domestic market and producing with marginal cost a is

$$V_i(a,t) = \int_t^\infty e^{-\rho(s-t)}\pi_i(a,s)\mathrm{d}s = \frac{E_{M,i}(t)}{\sigma}\left(\frac{a}{\breve{m}_i(t)}\right)^{1-\sigma}\int_t^\infty e^{-(\rho+g_i)(s-t)}\mathrm{d}s$$

where last equality follows from (8). Besides selling to the domestic market, a fraction ν_i of firms from i export to j. For them, the present value of operating profits from exporting, which amounts to $\phi V_j(a,t)$, must also be added. Using this and (30), the discount factor for firms from i is

$$\gamma = \rho + g_i \tag{32}$$

The discount factor that firms consider to evaluate future cash flows is the sum of the time preference of consumers (ρ) and the rate at which new varieties are introduced in their economy (g_i). Since both values are equal between economies it follows that firms in both countries discount the future at the same rate. As a consequence, by equation (19), the ratio between firms' market value and their replacement cost is the same in both countries at BGP.

Notice that using the previous expressions I can also express the expected value of a producing firm from economy i as:

$$\begin{aligned} V_i^e(t) &= \frac{1}{n_i}\int_{\theta\in\Theta_i^i(t)} \left[V_i(\theta,t) + \nu_i\phi V_j(\theta,t)\right]\mathrm{d}\theta \\ &= \frac{1}{n_i}\int_0^{a_{D,i}} \left[V_i(a,t) + \nu_i\phi V_j(a,t)\right] n_i \mathrm{d}G_i(a|a_{D,i}) = \frac{\bar{s}_{i,i}E_{M,i} + \nu_i\bar{s}_{i,j}E_{M,j}}{\sigma\gamma} \end{aligned}$$

where $\Theta_i^i(t)$ is subset of $\Theta_i(t)$ composed of varieties produced locally.

3.3 Expenditure and Savings

In this model the only sector with non-zero ex-post operating profits is the M-sector. Total operating profits earned in any market by firms from country i at a given moment in time are:

$$\Pi_i = \frac{1}{\sigma}\left[d_i E_{M,i} + x_i E_{M,j}\right] \tag{33}$$

Notice that by (16) the following equality holds: $V_i^e = \frac{\Pi_i}{n_i \gamma}$. The expected value of operating profits for a producing firm equals the discounted average operating profit firms make in the M-sector.

Remember that in each economy $E_i = w_i(L_i - L_{K,i}) + \Pi_i$, then since $w_i = 1$ and using (21), (23) and (32) it is possible to reach $E_i(t) = L_i + \rho a_{K,i}\bar{\kappa}_i$ which is a similar expression to that obtained by Baldwin and Robert-Nicoud (2008) under country symmetry. Notice this means that E_i is constant at BGP since everything in the right-hand side of the expression is a constant at BGP or a parameter of the model. Constant expenditure means, by (2), that $\rho = \iota$, so the interest rate is constant at BGP. This gives the following expression:

$$E_i(t) = L_i + \iota a_{K,i}\bar{\kappa}_i \tag{34}$$

Equation (34) means that aggregate expenditure in country i equals income that domestic consumers get from labour (L_i) plus the interests (ι) they get on their savings. Remember that in this model savings are directed towards the production of knowledge so the value of the stock of savings equals aggregate knowledge purchased by firms ($a_{K,i}\bar{\kappa}_i$). Then aggregate expenditure equals *permanent income* in a given country. This contrast with static models like Melitz (2003) where, since time is set not to play a role in the model ($\rho = \iota = 0$) , then $E_i = L_i$ which means permanent expenditure is unaffected by the static reallocation of resources. More importantly, (34) shows that permanent expenditure is affected by firm selection as that process affects the expected value of firms through $\bar{\kappa}_i$. In a model where externalities in the creation of knowledge are independent of firm selection (i.e. $\bar{\kappa}_i = 1$ as in the standard expanding variety model) aggregate expenditure would be greater than L_i but still exogenous. On the contrary, in the present model externalities coming from firm selection and affecting the expected stock value of firms in country i can have an impact on expenditure levels and therefore welfare in that country.

Having a path for aggregate expenditure means that the path of savings which determines resources in the R&D sector and therefore the path of $L_{K,i}$ is fully determined: $L_{K,i} = (\gamma - \rho)a_{K,i}\bar{\kappa}_i$. Allocation of labour in the M-sector is

$$L_{M,i} = \int_{\theta \in \Theta_i^i(t)} l_{M,i}(\theta)\mathrm{d}\theta = (\sigma - 1)\gamma a_{K,i}\bar{\kappa}_i \tag{35}$$

Then, I can determine the amount of labour used in the C-sector which is: $L_{C,i} = L_i - (\gamma\sigma - \rho)a_{K,i}\bar{\kappa}_i$. As is clear from the previous equations the allocation of the productive resource in this world is constant at BGP but depends on firm selection.

Finally notice that $E_{M,i} = \mu E_i$ and $E_{C,i} = (1 - \mu)E_i$ and so aggregate expenditure in both sector C and M are a constant fraction of aggregate expenditure at BGP. Due to Cobb-Douglas preferences between goods, firm selection affecting aggregate

expenditure affects equally expenditure in both final products.

A constant $E_{C,i}$ at BGP yields constant real expenditure in the C sector since the price of the C-good has been set to one. Given that $E_{M,i}$ is constant, the growth rate of real consumption of the M-good is $g_{C_M,i} = -\frac{\dot{\breve{m}}_i}{\breve{m}_i} = \frac{g_i}{\sigma-1}$ which is the same expression that arises under country symmetry. The term in the right-hand side is the rate at which the aggregate price of the M-good decreases over time due to decreasing marginal costs.

The economy described here has constant cut-off values $a_{D,i}$ and $a_{X,i}$ as described by (25) and (27) at any moment in time. By Definition 1, $\bar{\kappa}_i$, d_i, x_i, g_i, γ, $L_{K,i}$ and $L_{M,i}$ are also constant at BGP. As previously stated, $P_{K,i}$ and n_i are not constant at BGP but grow at a constant rate which means $\breve{m}_i^{1-\sigma}$ and $P_{M,i}$ also do.

Time-path of dynamic variables at BGP are given by:

$$n_i(t) = n_i(s)e^{(t-s)g_i}; \quad \breve{m}_i^{1-\sigma}(t) = \frac{\beta a_{D,i}^{1-\sigma} n_i(t)}{(\beta-1)d_i} \tag{36}$$

$$P_{M,i}(t) = \frac{\sigma \breve{m}_i(t)}{\sigma-1}; \quad P_{K,i}(t) = \frac{a_{K,i}}{n_i(t)}; \quad P_i(t) = \left[\frac{\sigma \breve{m}_i(t)}{\sigma-1}\right]^{\mu} B \tag{37}$$

where $n_i(s)$ is the mass of producing firms from country i at time s and the initial value $n_i(0)$ is a parameter of the model. In this expression the growth rate in i is given by:

$$g_i = \frac{\mu d_i Z_i}{\sigma}\left[\frac{L_i}{a_{K,i}\bar{\kappa}_i} + \rho\right] - \rho \tag{38}$$

All the expressions can be reduced to their counterparts under autarky just by imposing $\Omega = 0$. They can also be transformed into their symmetric-country version by setting $i = j$ (as shown in section 4.1).

3.4 Welfare

Welfare at time t is defined here by equation (1), as a function of the present value of real consumption. Using that expression and our solutions at BGP I can express welfare in country i at any t in the long run as:

$$U_i(t) = \frac{1}{\rho}\left[\ln\left(\frac{E_i(t)}{P_i(t)}\right) + \frac{\mu g_i}{\rho(\sigma-1)}\right] \tag{39}$$

According to (39), aggregate welfare depends on static reallocation of resources which determine real expenditure $\frac{E_t}{P_t}$ and its growth rate over time. Changes affecting real expenditure at a given moment are referred here as *static effects* while changes modifying its growth rate are *dynamic effects*.

This expression helps put forward that we can have three types of welfare effects of trade in this model. First, we can have static effects on nominal expenditure when a change in the degree of openness alters the value of E_i which is constant at BGP. Second, there can also be a static effect on prices as freer trade changes the value of $P_i(t)$ at t. Finally, due to our choice for the numeraire, the only dynamic effect in this model comes from changes in the rate at which prices in the M-sector fall over time which is determined by the rate at which new varieties are introduced in that sector.

The previous expression also highlights that dynamic effects are less important when consumers assign greater value to the consumption of the traditional C-good (i.e. $\mu \to 0$). Intuitively, while the price of the M-good decreases as new varieties are introduced in the market period after period, the price of the C-good in terms of units of labour remains constant. Therefore, the more consumers value their consumption of the traditional product the lower the impact that decreasing prices in the M-sector have on welfare.

4 Results

This section analyses the effect freer trade has on welfare. In the present model, freer trade can be a consequence of lower iceberg costs ($\mathrm{d}\phi > 0$), lower sunk-costs for exporting ($\mathrm{d}\kappa_X < 0$) or a combination of both. All these possibilities can be synthesised in $\mathrm{d}\Omega > 0$. Therefore, welfare effects of trade result from total differentiation of the previous equation with respect to trade openness Ω:

$$\frac{\mathrm{d}U_i(t)}{\mathrm{d}\Omega} = \frac{1}{\rho}\left[\frac{\mathrm{d}E_i(t)}{\mathrm{d}\Omega}\frac{1}{E_i(t)} - \frac{\mathrm{d}P_i(t)}{\mathrm{d}\Omega}\frac{1}{P_i(t)} + \frac{\mu}{\rho(\sigma-1)}\frac{\mathrm{d}g_i(t)}{\mathrm{d}\Omega}\right] \tag{40}$$

As shown in Proof 5, comparative static results stemming from an increase in openness ($\mathrm{d}\Omega > 0$) are qualitatively similar to those that arise if only changes in variable costs of international trade are considered ($\mathrm{d}\phi > 0$). This makes our results comparable to the literature that mostly considers a reduction of iceberg costs. Analysing the effects of a reduction in the exporting sunk costs is an additional feature of this work that does not change the main conclusions of the paper. Whenever the effects of a reduction in κ_X yields different results to an increase in ϕ, these differences are explicitly underlined.

In what follows, I split the analysis in two cases, one with symmetric countries and another one where country asymmetry is allowed. This allows me to better highlight the role that dynamic effects play on welfare when firms are different between countries.

4.1 Symmetric countries

A symmetric country setting simplifies greatly the analysis and provides a benchmark to better understand results under country asymmetry. It also suffices to show some of the key welfare features in the heterogeneous firm model that are absent when homogeneous firms is assumed. By country symmetry $\frac{a_{K,i}}{a_{K,j}} = 1$, $\frac{a_{m,i}}{a_{m,j}} = 1$, $R = 1$, and so $Z = 1 + \Omega$. The fact that the degree of openness is related to Z in such a simple way simplifies greatly the effects trade has on welfare. Previous analytic solutions get reduced to:

$$a_D = a_m\left[\frac{\kappa_I(\beta-1)}{\kappa_D(1+\Omega)}\right]^{1/\alpha}; \quad a_X = a_m\left[\Omega\frac{\kappa_I(\beta-1)}{\kappa_X(1+\Omega)}\right]^{1/\alpha} \tag{41}$$

$$\bar{\kappa} = \frac{\beta\kappa_D(1+\Omega)}{\beta-1}; \quad d = \frac{1}{1+\Omega}; \quad x = \frac{\Omega}{1+\Omega} \tag{42}$$

$$\breve{m}^{1-\sigma} = \frac{\beta a_D^{1-\sigma} n}{(\beta-1)d}; \quad P = \left[\frac{\sigma\breve{m}}{\sigma-1}\right]^{\mu} B; \quad g = \frac{\mu}{\sigma}\left[\frac{L}{a_K\bar{\kappa}} + \rho\right] - \rho \tag{43}$$

Then, using our previous expressions for prices, nominal expenditure and the growth rate of varieties I obtain:

$$\begin{aligned}\frac{\mathrm{d}E}{\mathrm{d}\Omega} =& \rho a_K \frac{\mathrm{d}\bar{\kappa}}{\mathrm{d}\Omega} \\ \frac{\mathrm{d}P(t)}{\mathrm{d}\Omega} =& \frac{\mu P(t)}{a_D}\left[\frac{a_D}{(\sigma-1)d}\frac{\mathrm{d}d}{\mathrm{d}\Omega} + \frac{\mathrm{d}a_D}{\mathrm{d}\Omega}\right] \\ \frac{\mathrm{d}g}{\mathrm{d}\Omega} =& -\frac{\mu L}{\sigma a_K \bar{\kappa}^2}\frac{\mathrm{d}\bar{\kappa}}{\mathrm{d}\Omega}\end{aligned} \tag{44}$$

To reach the second of these expressions I have imposed $\frac{\mathrm{d}n}{\mathrm{d}\Omega} = 0$ which follows from (36). Indeed, since the mass of firms in one economy (n) is determined by history, it does not experience an static change when the level of openness is modified. This does not mean n is unaffected by openness. The fact that greater openness increases the average amount of knowledge required to generate a producing firm ($\bar{\kappa}$) implies the rate at which new varieties are introduced into de economy (g) is lower. Therefore the new BGP path for n is always below the one it had before the fall in trade costs. This dynamic effect captured in (40) by the fall in g.

In order to highlight the importance of firm heterogeneity, what follows contrasts welfare results in the symmetric country version of the current model to those that arise in a context where firm are homogeneous as in Krugman (1980) or its dynamic version in Grossman and Helpman (1991). If firms are homogeneous within the M-sector of each country, i.e. all M-firms have the same exogenous marginal cost a^* then they all make the same producing and exporting decisions ($a_D = a^*$). To keep international trade flows, let me assume they all produce and serve the domestic and foreign market. Having the same marginal costs, all firms in the M-sector of country i enjoy the same share in their home market $s_{i,i} = \bar{s}_{i,i} = (a^*/\breve{m})^{1-\sigma}$ and foreign market $s_{i,j} = \bar{s}_{i,j} = \phi(a^*/\breve{m})^{1-\sigma}$, they all hire the same amount of labour, make the same operating profits and pay the same sunk costs, so $\bar{\kappa} = \kappa_I + \kappa_D + \kappa_X$. A version of the cut-off conditions still operate since, although all firms are the cut-off firm for domestic production and exporting, it is still true that the value of both activities must equal the sunk cost to engage in it. This yields the condition $\Omega = \phi = T$. Finally, free-entry condition (21) still holds but now its right-hand side is only composed by exogenous parameters $\bar{\kappa}$ and a_K.

What are the implications of imposing firm homogeneity for welfare effects of trade? With homogeneous firms the average amount of knowledge bought by firms (or what is equivalent, the value of the stock of savings) is independent of openness, i.e. $\frac{\mathrm{d}\bar{\kappa}}{\mathrm{d}\Omega} = 0$. By (44), this means there can be no static effect from trade on the permanent nominal expenditure level. Similarly, trade policy does not have any impact on the growth rates of varieties ($\frac{\mathrm{d}g}{\mathrm{d}\Omega} = 0$) and therefore on the rate at which prices on the M-sector decrease over time. Notice also that in this setting labour allocation between sectors is unaffected by freer trade.

The sole welfare effect of freer trade with homogeneous firms is a static effect on prices. A one-time decrease in trade costs permanently reduces the prices of imported goods. Everything else constant this lowers the average price consumers pay and rises welfare. In (44) this shows as a decrease in d which is the only thing changing in P. Indeed, even when all firms compete with the same number of local and exporting firms as before, the reduction in trade costs reduces the price at which foreign firms sell and this

increases their market share in detriment of local firms. In other words, the price index is reduced by both a lower price of imports and a greater weight of those imports in domestic purchases. It is possible to conclude that the introduction of simple dynamics does not qualitatively change welfare effects from trade in the homogeneous firm model since no new effects come from the dynamic aspects of the model. The fact that the only welfare effect of freer trade is a decrease in the price index is a point in which the homogeneous firm model resembles the static HFT. Indeed, in Melitz (2003) welfare gains from trade are also channelled exclusively through a fall of the price index.

Result 1 *Introducing simple dynamics into the homogeneous firm model does not qualitatively change the welfare effects of reducing trade costs as gains come exclusively from the static decrease in the price index. This resembles what is obtained in the static versions of the homogeneous and the heterogeneous firm models.*

When firm heterogeneity is allowed many new effects come to life. A one-time increase in the degree of openness between two economies generates a well-known firm selection effect in both. The reduction of international trade costs reduce a_D meaning that the least efficient domestic firms cannot compete with foreign exporters and are forced-out of the market. This also yields a reduction in the share of the domestic market served by local firms (d). Lower costs to send products abroad increases export opportunities and therefore the threshold marginal cost to do so (a_X) is larger. It is easy to see that the share of a market served by foreign firms (x) increases. Average productivity in the M-sector rises as a direct consequence of firm selection. As a combination of the increase in average productivity and the enhanced export possibilities, firms' expected profitability increases which rises in the average sunk cost paid by firms $\bar{\kappa}$.

The reduction of foreign prices present with homogeneous firms is also there when firms are heterogeneous but its effect on the price index is enhanced by the increase in average efficiency in the domestic M-sector ($\frac{\mathrm{d}a_D}{\mathrm{d}\Omega} < 0$). Both forces drive the reduction of the price index. Moreover, a static positive effect appears on the permanent level of aggregate expenditure. The rise of the stock of knowledge purchased by M-firms $a_K\bar{\kappa}$ increases households disposable income and therefore their expenditure level. Finally, the positive static effect on real expenditure is accompanied by a negative dynamic effect. The fact that a greater amount of knowledge is required by the average firm means that it is more costly to create a new variety. This results in a lower rate of creation of new varieties which yields a lower rate of price reduction over time. I can summarize the previous findings as follows:

Result 2 *When simple dynamics are introduced into the HFT model the sign of the net welfare effect in the long run depends on the magnitude of opposing forces even considering symmetric countries. The well-known price decrease driving welfare results in the static model is enhanced by a positive static effect on expenditure but opposed by a negative dynamic effect on growth.*

4.2 Asymmetric countries

Country asymmetry adds another layer of heterogeneity among firms and allows for welfare results to differ among countries. In a static context Demidova (2008) finds that, if the technological gap between countries is small both countries experience welfare gains from trade, but if the gap is too large then the laggard economy can experience losses. Her static model features no externalities and all welfare effects are channelled through the price index. This effect is also present in the current model

but it is accompanied by other effects that can revert or enhance its impact on both economies.

Without loss of generality let me assume the North is the technologically advanced country and therefore has a natural comparative advantage in the production of M-goods. In the present model this is imposed by setting $R_N > R_S$. Comparative advantage in the M-good can come from a better technology in the M-sector itself (i.e. $a_{m,N} < a_{m,S}$), a better technology in the K-sector ($a_{K,N} < a_{K,S}$), greater spillovers from learning ($n_N > n_S$) or some combination of them.[6] Joining this with Assumption 2 and expressions (25) and (27) immediately yields $a_{X,S} < a_{X,N} < a_{D,N} < a_{D,S}$ for every level of openness Ω.

The effect freer trade has on welfare in i depends on how openness (Ω) affects firm selection in that country, which according to (25), depends to a great extent on $\frac{\mathrm{d}Z_i}{\mathrm{d}\Omega}$. Then it is important to consider the following lemma (see proof 4 in the Appendix):

Lemma 2 *The sign of* $\frac{\mathrm{d}Z_i}{\mathrm{d}\Omega}$ *depends on the original level of openness and the size of the technological gap between countries. While for the North* $\frac{\mathrm{d}Z_N}{\mathrm{d}\Omega} > 0$ *holds, for the South* $\frac{\mathrm{d}Z_S}{\mathrm{d}\Omega} > 0$ *if and only if* $R_S > R^* = \frac{2\Omega}{1+\Omega^2}$ *and* $\frac{\mathrm{d}Z_S}{\mathrm{d}\Omega} < 0$ *otherwise.*

Since the threshold R^* depends positively on Ω then for a certain gap level R_S an infinitesimal increase in Ω from an initially small value may increase Z_S while a similar change from an initially larger value may reduce Z_S.

A case in which technological asymmetries are not strong (i.e. $R_S > R^*$) is then characterized by $\frac{\mathrm{d}Z_N}{\mathrm{d}\Omega} > 0$ and $\frac{\mathrm{d}Z_S}{\mathrm{d}\Omega} > 0$. This means greater trade openness reduces threshold $a_{D,i}$ in both countries increasing average efficiency. Firm selection operates in both economies as in the symmetric country case. New export opportunities make it easier for firms from both countries to engage in foreign trade which translates into a reduction of $a_{X,i}$ $\forall i = N, S$. In aggregate terms d_i decreases and x_i increases for both regions as can be deduced from (31). The average sunk cost paid by firms from country i ($\bar{\kappa}_i$) increases due to increased profitability in the M-sector in both countries. As in the previous section, a greater value of the stock of knowledge implies that consumers get greater income from their savings which generates a positive static effect on expenditure. The increased efficiency due to firm selection plus the lower costs to imports make the price index fall. Therefore, as in the symmetric country case, greater openness has a positive static effect on real consumption in both countries. What distinguishes the asymmetric country case is the fact that the positive effect is relatively greater in the North. This is easy to prove noting the fact that $\frac{\mathrm{d}Z_N}{\mathrm{d}\Omega} > \frac{\mathrm{d}Z_S}{\mathrm{d}\Omega}$.

In what respects to the effect that freer trade has on variety growth, it must be pointed out that, as in the symmetric country case the greater requirement of knowledge by M-firms in both countries is a detrimental force to variety-growth. This makes the growth rate at the new BGP lower than the value before the shock. In the short-term however, growth rates in each country may jump in different directions. Proof 6 shows that if firms from the North are able to expand in the foreign market relatively more

[6] Remember that setting $a_{m,N} < a_{m,S}$ implies that the support of the distribution of marginal costs in the North excludes the highest values compared to that of the South. M-firms from the North have a greater probability of getting a marginal cost below any threshold than those from the South.

than their counterparts from the South, a positive growth rate of northern varieties is possible.[7]

Result 3 *A small technological gap can yield asymmetric welfare results. Both countries experience a positive static effect on real expenditure but this effect is relatively larger in the North. For both countries the long-term dynamic effect is negative, but g_N and g_S can take divergent paths in the short term before reaching the new BGP. In particular a rising g_N in the short-term is possible.*

When the technological gap is large enough to make $\frac{\mathrm{d}Z_S}{\mathrm{d}\Omega} < 0$ (i.e. $R_S < R^*$), trade induced movements of most endogenous variables go in different directions for the North and the South. An increase in openness pushes down $a_{D,N}$ increasing average efficiency in the North but rises $a_{D,S}$ so average efficiency in the laggard economy decreases. The possibility for this inverse firm selection in the South was first documented in Demidova (2008). Reducing barriers for trade of the M-good in such a context promotes the expansion of firms from the North in both markets. By Proof 5, $a_{X,N}$, d_N and x_N rise while $a_{X,S}$, d_S and x_S fall.[8] Moreover, by (35), the case in which the technological asymmetry is large is the only one that yields an unambiguous reallocation of labour towards the M-sector in the North while the opposite happens in the South. This gives the following result:

Result 4 *Unlike the case where countries are technologically close, when technological differences are large enough, the North increases its specialization in the M-good while the opposite happens in the South.*

Increasing specialization in a context of firm heterogeneity in not a new result. In a Heckscher-Ohlin context, Bernard et al. (2007) finds that when countries differ in their relative factor abundance, trade openness reinforces each country's comparative advantage. Since in my model comparative advantages come from differences in relative productivities, the previous result can be considered the Ricardian version of that conclusion and shows that for comparative advantages to be reinforced by freer trade in a Ricardian model, technological asymmetries must be larger than certain threshold.[9]

In what respects to welfare outcomes, the case of large technological gap is characterized by an unambiguously divergent static effect on nominal expenditure. As established in Proof 5, having $\frac{\mathrm{d}Z_S}{\mathrm{d}\Omega} < 0$ and $\frac{\mathrm{d}Z_N}{\mathrm{d}\Omega} > 0$ implies that the average amount of knowledge used by producing M-firms increases in the North while decreases in the South. This means that returns on savings increase in the North rising the constant consumption level while the opposite happens in the South.

[7] The fact that short-term growth rates of both economies are affected differently by a one-time shock in openness highlights the importance of the study of transitional dynamics towards the BGP in order to quantify net welfare effects in both countries. This should be a subject of future work.

[8] By Proof 5 a case in which $a_{X,S}$ rises is possible even in the case of a large technological gap. This is the case if the increase in Ω is to a large extent an outcome of reduction in κ_X. However, this does not affect the results that follow.

[9] This result is independent to the dynamic feature of my model and arises if growth is assumed away. Okubo (2009) reaches a similar conclusion in a static Ricardian model with a somewhat different setting. In his model with multiple homogeneous-good sectors, some sectors have comparative advantages in one country and some other sectors feature advantages in the other. The result of freer trade is that sectors with Ricardian advantages expand relatively more than the others, but no sector shrinks.

Unlike previous cases, the direction of price movements is not straightforward when technological differences are large. Given the expression for prices in (37), inspection of (11) reveals that the effect freer trade has on the price index in country i is a combination of the effects it has on (a) the average selling costs of domestic firms, (b) the average selling costs of foreign firms exporting to i, and (c) the proportion of foreign firms that reach market i. In the case where countries are symmetric, lower trade costs make a_D rise and a_X fall in both countries which yields a reduction of prices from local firms and an increase in the proportion of firms reaching the foreign market. The effect on the average price exporters charge is ambiguous since on the one hand firms with higher marginal costs reach the foreign market but on the other hand all exporters face lower trading costs. This means that (a) and (c) tend to reduce prices while (b) is indeterminate. However it is easy to show that, since $\beta > 1$, the net effect (b)+(c) makes prices fall. This drives the well-known result that, under country symmetry, trade liberalization unambiguously reduces the price index. As it is easy to see, the same result is obtained if there is a small technological gap as all endogenous variables move in the same direction. But when the technological asymmetry is large, the effect freer trade has on the price index is not determined in both countries. In the North, domestic producers sell at a lower average price, but the effect stemming from foreign firms is not straightforward. While the average selling price of foreign firms is lower (both because of greater efficiency and lower tariff costs) the proportion of exporters among foreign firms gets reduced. In the South the picture is the exact opposite but the effect is still ambiguous. Unlike the case of a small gap, analytical solutions do not show one effect prevailing over the others when the technological gap is large. Proof 7 shows that price movements in both countries depend on parameter values.

Finally, a one-time increase in openness in a case of large technological asymmetry undoubtedly pushes growth rates in different directions in the short-term. The immediate response of g_S is to fall while g_N rises (see Proof 6). As in the previous cases both rates converge in the long-term to a common value, smaller than its value before the shock.

The following result summarizes welfare conclusions for the case with a large technological gap.

Result 5 *Country asymmetry characterized by a large technological gap yields very different welfare results between countries. The movement on prices is not determined by the model but while the North experiences a positive static effect on nominal expenditure and a positive dynamic effect in the short-term, the South faces the exact opposite. In the long-term however, the dynamic effect is negative in both countries.*

5 Conclusions

The basic HFT model with symmetric countries yields very straightforward welfare conclusions, i.e. a reduction of trade costs makes prices fall and welfare increases. This paper shows that introducing very simple dynamics into the model can change this conclusion. The model presented here features expanding varieties and allows consumers to save a part of their income. A symmetric-country version of the model shows that besides the reduction of the price index an increase in average productivity due to tougher competition by foreign firms pushes up the average amount of knowledge required by firms. Since all savings are devoted to the production of knowledge,

firm selection increases the revenues from savings that households get. The static increase in welfare in the present model is therefore driven both by a reduction of prices and an increase in nominal expenditure. However, the positive static welfare effect is accompanied by a negative dynamic effect stemming from the fact that, because of the greater need for knowledge, more resources are needed in the creation of new varieties and therefore openness may slow down variety growth. The long-term net welfare effect of freer trade is therefore ambiguous and depends on which of these effects prevails.

Allowing countries to differ in their degree of technological development yields different welfare results between them. When the technological gap is small both countries experience a positive static effect on real expenditure as in the symmetric case. Nevertheless, I show that the static welfare gain is relatively larger in the North. For both countries the long-term dynamic effect is negative, but growth rates of varieties can take divergent paths in the short term. The short-term dynamic effect in the North can even be positive.

When technological asymmetries are large, welfare results differ greatly between countries. Firm selection increases efficiency in the North but an inverted process causes the opposite in the South. This causes a positive static effect on nominal expenditure in the former country and a negative effect in the latter. The net static effect on real expenditure is not straightforward since the combination of greater efficiency in the North and lower efficiency in the South along with lower trade costs between markets and changes in market shares induce an indeterminate price movement in both economies. The technological advantage of the North pushes its economy to increase its specialization in the production of the differentiated good while it is reduced in the South. Finally, the North experiences a jump in its growth rate due to the activity increase in its M-sector. This is not permanent as both growth rates are equal in the long term.

Previous conclusions can be seen as an argument for the view of HFT models as a step forward in the development of tools to evaluate welfare effects of freer trade. As shown in Arkolakis (2013), a family of models that includes a version of Melitz (2003) with Pareto distributed firms, yields the same welfare outcomes as Krugman (1980) when the two main parameters are equally calibrated. My conclusions show that, when simple dynamics are introduced, firm selection can create externalities affecting welfare that are overlooked assuming homogeneous firms.

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6 Appendix

Proof 1 $d_i \geq x_i \; \forall i$

By their definitions in (18), $d_i \geq x_i \Leftrightarrow \nu_j \left[\frac{\tilde{m}_i}{\hat{m}_j}\right]^{1-\sigma} \geq \frac{1}{\nu_i}\left[\frac{\hat{m}_i}{\tilde{m}_j}\right]^{1-\sigma}$.

By (12), $\left[\frac{\hat{m}_i}{\tilde{m}_j}\right]^{1-\sigma} = \frac{\int_0^{a_{D,i}} a^{1-\sigma} \mathrm{d}G_i(a|a_{D,i})}{\phi \int_0^{a_{X,i}} a^{1-\sigma} \mathrm{d}G_i(a|a_{X,i})} = \frac{\nu_i}{\phi} \frac{\int_0^{a_{D,i}} a^{1-\sigma} \mathrm{d}G_i(a|a_{D,i})}{\int_0^{a_{X,i}} a^{1-\sigma} \mathrm{d}G_i(a|a_{D,i})}$.

Notice that, since $a^{1-\sigma} > 0 \; \forall \; a > 0$ and assuming $a_{X,i} < a_{D,i}$ (see Assumption 2), then $\left[\frac{\hat{m}_i}{\tilde{m}_j}\right]^{1-\sigma} = \frac{\nu_i}{\phi} M$ with $M > 1$.

Following a symmetric reasoning the condition for $d_i \geq x_i$ can be written as

$$\phi \frac{\int_0^{a_{X,j}} a^{1-\sigma} \mathrm{d}G_j(a|a_{D,j})}{\int_0^{a_{D,j}} a^{1-\sigma} \mathrm{d}G_j(a|a_{D,j})} \leq \frac{1}{\phi} \frac{\int_0^{a_{D,i}} a^{1-\sigma} \mathrm{d}G_i(a|a_{D,i})}{\int_0^{a_{X,i}} a^{1-\sigma} \mathrm{d}G_i(a|a_{D,i})}$$

This condition always holds as the left-hand side is lower or equal than one while the right-hand side is greater or equal than unity. The previous condition shows that strict equality holds only if $a_{D,i} = a_{X,i} \; \forall i$ and $\phi = 1$. ■

Proof 2 *A BGP where $\dot{d} \neq 0$ yields a degenerate equilibrium that converges to a Ricardian model of trade*

Assume the BGP follows the next definition instead of Definition 1:

Definition 2 *A Balanced Growth Path (BGP) in this model is characterized by a fixed allocation of labour among sectors (i.e. constant $L_{K,i}$, $L_{M,i}$ and $L_{C,i}$), constant cut-off values ($a_{D,i}$ and $a_{X,i}$) and endogenous variables n_i and E_i growing at a constant rate $\forall i = N, S$.*

Notice that the only difference between the BGP in Definition 1 and that in Definition 2 is that the latter allows for $\dot{d} \neq 0$. It is possible to prove that a BGP characterized by Definition 2 yields a situation where firms from one country take over the entire global M-market (i.e. if $\dot{d}_i > 0 \Rightarrow d_i \to 1$, $x_i \to 1$, $d_j \to 0$ and $x_j \to 0$).

First let me prove that $\dot{d}_i > 0 \Leftrightarrow \dot{x}_i > 0 \Leftrightarrow \frac{\dot{n}_i}{n_i} > \frac{\dot{n}_j}{n_j}$. By definition $d_i = 1 - x_j$ $\Rightarrow \dot{d}_i = -\dot{x}_j$. Also by definition $\dot{d}_i = d_i \left(\frac{\dot{n}_i}{n_i} + \frac{\dot{\bar{s}}_{i,i}}{\bar{s}_{i,i}}\right)$ and $\dot{x}_i = x_i \left(\frac{\dot{n}_i}{n_i} + \frac{\dot{\bar{s}}_{i,j}}{\bar{s}_{i,j}}\right)$ where $\frac{\dot{\bar{s}}_{i,i}}{\bar{s}_{i,i}}$, $\frac{\dot{\bar{s}}_{i,i}}{\bar{s}_{i,i}}$ and $\frac{\dot{n}_i}{n_i}$ are constants $\forall i, j$.

Without loss of generality assume $\dot{d}_N > 0$. This implies $\frac{\dot{n}_N}{n_N} > |\frac{\dot{\bar{s}}_{N,N}}{\bar{s}_{N,N}}|$. Under an alternative definition of BGP where constant a_D and a_X are included, then $\frac{\dot{\bar{s}}_{N,N}}{\bar{s}_{N,N}} = \frac{\dot{\bar{s}}_{S,N}}{\bar{s}_{S,N}}$ and $\frac{\dot{\bar{s}}_{S,S}}{\bar{s}_{S,S}} = \frac{\dot{\bar{s}}_{N,S}}{\bar{s}_{N,S}}$. Therefore $\dot{d}_N > 0 \Leftrightarrow \frac{\dot{n}_N}{n_N} > |\frac{\dot{\bar{s}}_{S,N}}{\bar{s}_{S,N}}|$

The assumption also implies $\dot{x}_S < 0 \Leftrightarrow \frac{\dot{n}_S}{n_S} < |\frac{\dot{\bar{s}}_{S,N}}{\bar{s}_{S,N}}| \Leftrightarrow \frac{\dot{n}_S}{n_S} < |\frac{\dot{\bar{s}}_{N,N}}{\bar{s}_{N,N}}|$. I can then establish that $\frac{\dot{n}_S}{n_S} < |\frac{\dot{\bar{s}}_{N,N}}{\bar{s}_{N,N}}| = |\frac{\dot{\bar{s}}_{S,N}}{\bar{s}_{S,N}}| < \frac{\dot{n}_N}{n_N}$. Since $\frac{\dot{n}_S}{n_S} < \frac{\dot{n}_N}{n_N}$, then $\frac{\dot{n}_S}{n_S} < d_S \frac{\dot{n}_S}{n_S} + x_N \frac{\dot{n}_N}{n_N} = |\frac{\dot{\bar{s}}_{S,S}}{\bar{s}_{S,S}}| \Leftrightarrow$ $\dot{d}_S < 0 \Leftrightarrow \dot{x}_N > 0$.

Second, let me prove that when $\dot{d}_i > 0$, both d_i and x_i converge to 1 asymptotically (and therefore d_j and x_j converge to 0).

Using (28) and (29), I can write $\dot{d}_i(t) = \left(\frac{\dot{n}_i}{n_i} - \frac{\dot{n}_j}{n_j}\right)(d_i(t) - d_i(t)^2)$ which is a Bernoulli differential equation with solution:

$$d_i(t) = \frac{1}{1 + me^{-\left(\frac{\dot{n}_i}{n_i} - \frac{\dot{n}_j}{n_j}\right)t}}$$

where $m \in \mathbb{R}$. According to this expression, if $\frac{\dot{n}_i}{n_i} < \frac{\dot{n}_j}{n_j}$, d_i converges to 0, while if $\frac{\dot{n}_i}{n_i} > \frac{\dot{n}_j}{n_j}$, d_i smoothly converges to 1. A situation where $\dot{d}_N > 0$ $\forall t$ must correspond to $\frac{\dot{n}_N}{n_N} > \frac{\dot{n}_S}{n_S}$ and therefore d_N converges to 1. Replicating the reasoning for a similar Bernoulli equation for x_N, it is possible to conclude that $\dot{d}_N > 0$ implies d_N and x_N converging to 1 and d_S and x_S converging to 0.$\blacksquare$

Proof 3 $a_{D,i} > a_{X,i} \Leftrightarrow Z_i \in [\frac{T+\Omega^2}{T}, 1+T]$ $\forall i = N, S$

Using (25) and (27) it is easy to see that $a_{D,i} > a_{X,i} \Leftrightarrow Z_i < 1 + T$ $\forall i = N, S$. Without loss of generality let us assume it is the North that reaches a maximum level of Ricardian comparative advantage R_N^{max} such that $Z_N^{max} = 1 + T$. Solving for such value I obtain $R_N^{max} = \frac{T+\Omega^2}{\Omega(T+1)}$ which by definition yields $R_S^{min} = \frac{\Omega(T+1)}{T+\Omega^2}$. By definition, this value for R_S^{min} yields $Z_S^{min} = \frac{T+\Omega^2}{T}$.

Then $R_i \in [\frac{\Omega(T+1)}{T+\Omega^2}, \frac{T+\Omega^2}{\Omega(T+1)}]$ and $Z_i \in [\frac{T+\Omega^2}{T}, 1+T]$ $\forall i = N, S$.$\blacksquare$

Proof 4 *Proof of Lemma 2*

The effect that trade openness (Ω) has on Z_i can be expressed by $\frac{\mathrm{d}Z_i}{\mathrm{d}\Omega} = \frac{1}{1-\Omega R_i}(R_i Z_i - 2\Omega)$. Since $0 < \Omega < 1$ and $Z_i > 0$, then:

$$\begin{aligned}\frac{\mathrm{d}Z_i}{\mathrm{d}\Omega} &> 0 \Leftrightarrow R_iZ_i > 2\Omega \\ \frac{\mathrm{d}Z_i}{\mathrm{d}\Omega} &< 0 \Leftrightarrow R_iZ_i < 2\Omega \\ \frac{\mathrm{d}Z_i}{\mathrm{d}\Omega} &= 0 \Leftrightarrow R_iZ_i = 2\Omega \Leftrightarrow R_i = R^* = \frac{2\Omega}{1+\Omega^2}\end{aligned}$$

Notice that under country symmetry, $R = 1$ and $Z = 1 + \Omega$ so the condition for $\frac{\mathrm{d}Z_i}{\mathrm{d}\Omega} > 0$ is reduced to $1 + \Omega > 2\Omega$ and holds for both countries. Under country asymmetry $R_N > 1$ and $R_S < 1$. Since the left-hand side in the condition is monotonically increasing in R_i it holds for N and therefore $\frac{\mathrm{d}Z_N}{\mathrm{d}\Omega} > 0$.

As shown in to Proof 3, Assumption 2 imposes a lower limit to R_S given by $R_S^{min} = \frac{\Omega(T+1)}{T+\Omega^2}$. It is easy to show that $T > 1$ implies $R_S^{min} < R^*$. Then for values of the technological gap that set R_S to the range $[R_S^{min}, R^*)$ we are going to have $\frac{\mathrm{d}Z_S}{\mathrm{d}\Omega} < 0$, while when R_S is in $(R^*, 1]$ we are going to have $\frac{\mathrm{d}Z_S}{\mathrm{d}\Omega} > 0$.■

Proof 5 *Comparative statics of trade effect*

Using (25)-(36) it is possible to derive the effect of greater openness upon each of the main variables. In what follows I consider rises in the degree of openness ($\mathrm{d}\Omega > 0$) stemming from both a reduction of the variable cost ($\mathrm{d}\phi > 0$) or in the fixed cost ($\mathrm{d}\kappa_X < 0$) indistinctly.

- $\frac{\mathrm{d}a_{D,i}}{\mathrm{d}\Omega} = \frac{-a_{D,i}}{\alpha Z_i}\frac{\mathrm{d}Z_i}{\mathrm{d}\Omega}$
- $\frac{\mathrm{d}\bar{\kappa}_i}{\mathrm{d}\Omega} = \frac{\bar{\kappa}_i}{Z_i}\frac{\mathrm{d}Z_i}{\mathrm{d}\Omega}$
- $\frac{\mathrm{d}d_i}{\mathrm{d}\Omega} = -d_i^2\frac{a_{K,j}}{a_{K,i}}\frac{\mathrm{d}Z_j}{\mathrm{d}\Omega}$
- $\frac{\mathrm{d}x_i}{\mathrm{d}\Omega} = \frac{x_i^2}{(Z_i-1)^2}\frac{a_{K,j}}{a_{K,i}}\frac{\mathrm{d}Z_i}{\mathrm{d}\Omega}$

Changes in the degree of openness affect $a_{X,i}$ differently whether they stem from a reduction of the variable cost ($\mathrm{d}\phi > 0$) or in the fixed cost ($\mathrm{d}\kappa_X < 0$). Indeed,

- $\frac{\mathrm{d}a_{X,i}}{\mathrm{d}\phi} = \frac{a_{X,i}\beta\Omega}{\alpha Z_i(Z_i-1)\phi}\frac{\mathrm{d}Z_i}{\mathrm{d}\Omega}$
- $\frac{\mathrm{d}a_{X,i}}{\mathrm{d}\kappa_X} = \frac{-a_{X,i}}{\alpha\kappa_X}\left[\frac{(\beta-1)\Omega}{Z_i(Z_i-1)}\frac{\mathrm{d}Z_i}{\mathrm{d}\Omega} + 1\right]$

From these expressions it is clear that $\frac{\mathrm{d}Z_i}{\mathrm{d}\Omega} > 0$ is a necessary and sufficient condition for $\frac{\mathrm{d}a_{D,i}}{\mathrm{d}\Omega} < 0$, $\frac{\mathrm{d}\bar{\kappa}_i}{\mathrm{d}\Omega} > 0$, $\frac{\mathrm{d}d_j}{\mathrm{d}\Omega} < 0$, $\frac{\mathrm{d}x_i}{\mathrm{d}\Omega} > 0$ and $\frac{\mathrm{d}a_{X,i}}{\mathrm{d}\phi} > 0$. The condition for $\frac{\mathrm{d}a_{X,i}}{\mathrm{d}\kappa_X} < 0$ is $\frac{\mathrm{d}Z_i}{\mathrm{d}\Omega} > \frac{Z_i(1-Z_i)}{(\beta-1)\Omega}$ with $\frac{Z_i(1-Z_i)}{(\beta-1)\Omega} < 0$, so $\frac{\mathrm{d}Z_i}{\mathrm{d}\Omega} > 0$ is sufficient but not necessary.■

Proof 6 *Comparative statics of trade effect on the growth rate of varieties*

The effect that a one-time increase in openness (Ω) immediately has on the growth rate of country i (g_i) is given by the following expression:

$$\frac{\mathrm{d}g_i}{\mathrm{d}\Omega} = \frac{\mu d_i E_i}{\sigma a_{K,i}\bar{\kappa}_i}\left[\left(1 - \frac{L_i}{E_i}\right)\frac{\mathrm{d}Z_i}{\mathrm{d}\Omega} - d_iZ_i\frac{a_{K,j}}{a_{K,i}}\frac{\mathrm{d}Z_j}{\mathrm{d}\Omega}\right]$$

The growth rate of local varieties in country i depends on the signs of $\frac{\mathrm{d}Z_i}{\mathrm{d}\Omega}$ and $\frac{\mathrm{d}Z_j}{\mathrm{d}\Omega}$. Tracing back the origins of these two effects it is possible to notice that $\frac{\mathrm{d}Z_i}{\mathrm{d}\Omega}$ is there for two reasons. One (a) is that the growth rate of varieties produced in i is affected by the market share that firms from i cover in the foreign market (represented here by $1\frac{\mathrm{d}Z_i}{\mathrm{d}\Omega}$). The other (b) is that, by (31), the average knowledge required by new firms $\bar{\kappa}_i$ changes with freer trade (represented here by $\frac{L_i}{E_i}\frac{\mathrm{d}Z_i}{\mathrm{d}\Omega}$). By (34), $1-\frac{L_i}{E_i} > 0$ which means that the first of these effects is always greater than the second. The fact that $\frac{\mathrm{d}Z_j}{\mathrm{d}\Omega}$ is in the previous expression too is related to the fact that (c) freer trade can hinder growth in i if foreign firms increase their market share in i in detriment of local firms. When countries are symmetric $\frac{\mathrm{d}Z_i}{\mathrm{d}\Omega} = \frac{\mathrm{d}Z_j}{\mathrm{d}\Omega}$ and $d_i Z_i \frac{a_{K,j}}{a_{K,i}} = 1$ so the previous expression reduces to (44) and g decreases with Ω due to the increase in $\bar{\kappa}$ in both countries. By symmetry, (a) and (c) perfectly offset each other. But when countries are asymmetric that might not be the case and that enables $g_S \neq g_N$. Since by Lemma 1 both rates must converge to the same value at the new BGP any divergence must happen only in the short-term. By Lemma 2 we can separate two cases:

- The technological gap is relatively small and therefore $\frac{\mathrm{d}Z_i}{\mathrm{d}\Omega} > 0\ \forall i = N, S$. It can be shown that the net effect on growth is negative for the South while for the North its sign depends on the size of the technological gap.
- The technological gap is relatively large and therefore $\frac{\mathrm{d}Z_N}{\mathrm{d}\Omega} > 0$ and $\frac{\mathrm{d}Z_S}{\mathrm{d}\Omega} < 0$. In such a case it is easy to see that the growth rate in the South is negatively affected by openness while the growth rate in the North unambiguously increases with freer trade.■

Proof 7 *Comparative statics of trade effect on the price index*

By (37) and (36), the effect that a one-time increase in openness (Ω) immediately has on the price index of country i (P_i) is given by the following expression:

$$\frac{\mathrm{d}P_i}{\mathrm{d}\Omega} = -\frac{\mu P_i}{\alpha Z_i}\left[\frac{\mathrm{d}Z_i}{\mathrm{d}\Omega} + \beta Z_i d_i \frac{a_{K,j}}{a_{K,i}}\frac{\mathrm{d}Z_j}{\mathrm{d}\Omega}\right]$$

This shows how in cases where technological asymmetries are not large (which by Lemma 2 implies $\frac{\mathrm{d}Z_i}{\mathrm{d}\Omega} > 0\ \forall i = N, S$) the price index unequivocally decreases with freer trade. However, when the technological gap between countries is large (which means $\frac{\mathrm{d}Z_N}{\mathrm{d}\Omega} > 0$ and $\frac{\mathrm{d}Z_S}{\mathrm{d}\Omega} < 0$) the effect freer trade has on prices is determined by the following conditions:

- $\frac{\mathrm{d}P_N}{\mathrm{d}\Omega} < 0 \Leftrightarrow \frac{\mathrm{d}Z_N}{\mathrm{d}\Omega} > \beta Z_N d_N \frac{a_{K,S}}{a_{K,N}}\left|\frac{\mathrm{d}Z_S}{\mathrm{d}\Omega}\right|$ and $\frac{\mathrm{d}P_N}{\mathrm{d}\Omega} > 0$ otherwise,
- $\frac{\mathrm{d}P_S}{\mathrm{d}\Omega} < 0 \Leftrightarrow \left|\frac{\mathrm{d}Z_S}{\mathrm{d}\Omega}\right| < \beta Z_S d_S \frac{a_{K,N}}{a_{K,S}}\frac{\mathrm{d}Z_N}{\mathrm{d}\Omega}$ and $\frac{\mathrm{d}P_S}{\mathrm{d}\Omega} > 0$ otherwise.

By Proof 4, a case of a large technological is one where $R_S \in [R_S^{min}, R^*)$ and when $R_S \to R^*$ then $\frac{\mathrm{d}Z_S}{\mathrm{d}\Omega} \to 0$. By the previous conditions, this implies it is possible to have falling prices even in a situation where the technological gap is large.■