Preference for Housing Services and Rational House Price Bubbles

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Abstract

The recent financial crisis was triggered by the collapse of a housing bubble. This has raised interest among policy makers and researchers in understanding which economic environments are more prone to produce such bubbles. The literature focuses mainly on credit market conditions and their effect on housing bubbles. This paper instead explores the importance of the preference for housing services. In a companion paper, I provide a comprehensive empirical characterization of housing cycles using a large database covering 18 OECD countries over the period 1970:1-2013:4. Three novel stylized facts are identified across countries: the preference for housing services is highly negatively correlated with (1) homeownership rates, and (2) the *frequency* and (3) the *intensity* of independent housing booms and boom-bust cycles. This paper provides an explanation for the stylized facts discovered in the data. An overlapping generations model is used as a laboratory for the analysis of the impact of the preference of housing services on homeownership rates and housing bubble occurrence. Further, I will study the impact of different policies that affect agents' preferences for housing services and thereby influence economies tendencies to generate house price bubbles. Within this policy analysis, the impact of policy changes on the distribution of consumption across the population are considered, as are the general welfare implications.

Keywords: housing bubbles, housing booms and busts, homeownership, preference for housing services

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Contents

Definition of Variables

Y_t	Total Output of Final Consumption Good in period t
H_t	Total Housing Stock in period t
S_t	Total Rental Housing Stock in period t
C_t	Composite consumption good in period t
ω	Fraction of Households with high preference of housing services
$(1-\omega)$	Fraction of Households with low preference of housing services
ν^j	Fraction of type j Households
φ^j	Fraction of housing stock $H_{t\|t-k}$ bought by households of type j in period t.
R_t	Nominal Interest Rate
γ	Discount rate of Households
Q_t	Nominal Price of a unit of Housing Stock
$P_{c,t}$	Nominal Price of the final Consumption good
$P_{r,t}$	Nominal Rental Price
q_t	Real Price of Housing Stock in terms of final Consumption good
$p_{r,t}$	Real Rental Price of Housing Stock in terms of final Consumption good
W_t	Real Wage
D_t	Firm's profit in period t, received by Household as a lump-sum payment
ξ^k	Aggregate Preference for Housing Service relative to Consumption in country k
δ	Depreciation rate of Housing
Ψ_t	Nominal Cost Function of Monopolistic Firm in period t
ε	Elasticity of Substitution between differentiated intermediate inputs

1 Introduction

The bursting of the housing bubble in the United States (and other countries) is widely accepted to have played an important role in generating the great recession. Likewise, it is generally not questioned that housing market fluctuations have very strong impacts on overall economic performance. Several recent empirical studies show that recessions that are associated with house price busts are not only much longer but also much deeper than normal recessions or recessions associated with equity busts.[1]

These patterns reflect that the impact of house price crashes on the real economy tend to be larger than other types of bust. House purchases are funded by leverage, which can imply stability risks for banks (and the banking system) when the quality of mortgage books deteriorate during house price crashes. Additionally, there is a strong wealth effect associated with housing busts whereby homeowners feeling an equity loss will reduce their consumption to reflect the deterioration in their balance sheets. This tends to be stronger than the wealth effects associated with equity price crashes as equities are disproportionately held by wealthy economic agents, whose consumption does not change dramatically when faced with a short-term wealth setback.

In response to recent history, policy makers and researchers are now investing more analytical resources to understand how best to mitigate the risks associated with house price booms and busts. One key element of this work is to determine which economic environments are more prone to produce such bubbles. On this question, the current literature focuses mainly on credit market conditions and their effect on housing bubbles. This paper instead explores the relevance of preferences for housing services on house price fluctuations and homeownership rates. From this novel perspective the potential for alternative / non-traditional policy responses to housing bubbles are explored.

[1] Recessions that are associated with house price busts are much longer than recessions associated with equity busts, 4 years compared to 2.5 years. Following e.g. Claessens, Kose, and Terrones (2009) and IMF (2003), house price busts have larger effects on consumption and investment and therefore GDP. GDP drops by an average of 8% when house prices burst compared to 4% when equity prices burst.

In a companion paper, I have provided a comprehensive empirical characterization of housing cycles using a large database covering 18 OECD countries over the period 1970:1-2013:4. Three novel stylized facts are identified across countries: the preference for housing services is highly negatively correlated with (1) homeownership rates, and (2) the *frequency* and (3) the *intensity* of independent housing booms and boom-bust cycles. Countries with a lower preference for housing services are characterized with larger homeownership rates and experienced significantly more housing booms and boom-bust cycles over the time period 1970 to 2013.

This paper provides an explanation for the stylized facts discovered in the data. An overlapping generations model is used as a laboratory for the analysis of the impact of the preference for housing services on homeownership rates and housing bubble occurrence. Further, I study the impact of different policies that affect agents preferences for housing services and thereby investigate the potential for these policies to be used for mitigating house price bubbles. Within this policy analysis, the impact of policy changes on the distribution of consumption across the population is considered, as are the general welfare implications.

Within the model, the economy is populated by heterogenous households with differing preferences for housing services. The supply of rental housing and homeownership rates are determined endogenously. The bubble is modelled as part of the housing price, rather than as a shortage of assets in the economy as in Arce and López-Salido (2011). The model is the first to study the effects of rental market regulations on rational housing bubbles within such a framework.

Households differ in their preferences for housing services and these differences are assumed to be constant and exogenously given. The weighted average of the different preferences for housing services represents population's preferences for housing services of country k. This assumption is reasonable and based on empirical evidence, provided by Huber *et al.* (2015), who show that large cross-country differences in housing preferences exist and that these cross-country differences are persistent over time.[2]

[2]Huber *et al.* (2015) study the impact of culture on homeownership rates, using data on the tenure choice decision of second generation immigrants in the United States. They find that cultural preferences imported by the parents play a large role for the tenure choice decision.

This paper relates to the theorectical literature that studies why homeownership rates differ so greatly and persistently across countries. Further, it seeks to contribute to the literature investigating why some countries experience more (and or more extreme) housing bubbles than others and to the literature regarding the optimal policy design for the mitigation of such bubbles. In addition, the results of this project will help to understand how the preference for housing services and the fraction of homeownership within a country impact the effect of different macro-prudential policies in controlling housing bubbles - which is of particular importance for the ECB as homeownership rates and the preference for housing services vary significantly across European countries.[3]

The remainder of this paper is organized as follows: Section 2 provides a short summary of parts of the empirical evidence presented in the companion paper. Section 3 describes the overlapping generation model. Section 4 provides comparative statics and shows the impact of the preference for housing services on choice variables, prices and bubbles. Finally, section 5 concludes. Appendix A provides a detailed derivation of the model equations. Appendix B describes the methodology used in the companion paper to identify and measure house price booms and busts. A description of the data used for that analysis can be found in the appendix C, along with short descriptive statistics. For more details on the empirical part I refer to the companion paper *Housing Booms and Busts - Convergences and Divergences across OECD countries.*

[3]In my sample of 18 OECD countries, Switzerland has the lowest homeownership rate with around 35%, while Spain has the highest with around 80%. The preference for housing services measured by household spending on housing (% of disposable income) varies from 14% in Portugal to 30% in Denmark. The preference for housing services measured by National CPI weights (Housing, Water, Electricity, Gas and other Fuels) varies from around 10.3% in Portugal to around 28% in Denmark.

2 Empirical Evidence

This section provides a short summary of parts of the empirical evidence presented in the companion paper "*Housing Booms and Busts - Convergences and Divergences across OECD countries*". A description of the data used for that analysis can be found in the appendix along with short descriptive statistics. Further, the appendix provides a detailed explanation of how I measured (1) housing cycles, (2) independent housing booms, (3) boom-bust cycles for 18 OECD countries over the time period 1970 to 2014, and (4) the preference for housing services. In this paper, I want to highlight two novel stylized facts across countries:

First, across OECD countries, the preference for housing service is highly and negatively correlated with the number of independent booms and the number of boom-bust cycles. For instance, the number of completed independent housing booms (boom-bust cycles) that are associated with at least a 80% price increase, displays a correlation across countries with preference for housing services of -0.72 (-0.30).[4]

Second, across OECD countries, the preference for housing service is highly and negatively correlated with aggregate homeownership rates.[5]

Thus, countries with a lower preference for housing services are characterized with larger homeownership rates and experienced more independent housing booms as well as boom-bust cycles and therefore potentially more housing bubbles over the time period 1970 to 2014.

OLS regressions show that the preference for housing service inherits a high explanatory power to explain the frequency of independent housing booms and boom-bust cycles, see table (6) in the appendix. For example, regressing the preference for housing service on the frequency of independent booms that involve more than a 20% price increase (along with standard controls), shows that homeownership is significant at the 5% level with $R^2 = 0.59$.

But how important is the impact of preference for housing services quantitatively? An increase in the level of CPI by one standard deviation (across countries) is associated with a decrease in the average number of booms (associated with a

[4]Please refer to tables (7)-(8) in the appendix for more information.

[5]Refer to table (5) in the appendix C.

price increase of at least 20%) by 0.74 which accounts for 57% of the variation of the number of booms per country.[6] This is remarkable.[7] TO BE COMPLETED

3 Model

This paper provides a highly stylized overlapping generations model without capital and where labor is supplied inelastically. In equilibrium, aggregate employment and output are constant. However, this framework allows to study why countries with a *lower preference for housing services* experienced significantly more housing bubbles over the time period 1970-2013, and why these countries are characterized with larger homeownership rates. The model is used as a laboratory for the analysis of the impact of the preference of housing services on (1) housing bubble occurrence, and (2) homeownership rates. Further, I study the impact of different policies affecting the preference of housing services and hence house price bubbles, and as a result, the distribution of consumption across cohorts and welfare.

3.1 Households

I assume an overlapping generations structure where a continuum of households lives for two periods. The size of each generation (young and old) is normalized to unity. After dying, the old generation is replaced by a new, young one. Hence total population remains constant. Households have heterogeneous preferences for housing services. A continuum of measure ω has *high* preferences for housing services, while a continuum of measure $(1-\omega)$ has *low* preferences of housing services.

Households of type j born at time t maximize the expected lifetime utility

$$u(C^j_{1,t}) + \xi^j v(S^j_t) + \gamma E_t\{u(C^j_{2,t+1})\} \tag{3.1}$$

[6] An increase in the level of CPI by one standard deviation (across countries) is associated with a decrease in the average number of booms (associated with a price increase of at least 80%) by 0.63 which accounts for 109% of the variation of the number of booms per country.

[7] The regression results for independent housing booms and boom-bust cycles defined by a different threshold are very similar. The results are henceforth robust to various housing bubble identification rules. Please refer to the companion paper for more details.

where C_t denotes the non-durable composite consumption good.[8] Renting housing stock of size S_t yields utility $v(S_t)$. ξ^j denotes the preference for housing service relative to consumption of household type $j \in \{L, H\}$. Hence, aggregate preferences for *housing services* relative to consumption of country k are captured by

$$\xi_k = \omega\xi^H + (1-\omega)\xi^L \quad with \quad \xi^H > \xi^L$$

The young generation rents housing stock S_t from the old generation. I will use log utility as the functional form for what will follow, i.e. $u(\cdot) = v(\cdot) = log(\cdot)$.[9]

Young households are endowed with "know-how" to set up a new firm producing a differentiated consumption good. That firm only becomes productive after one and for one period only (i.e. when the founder is old), generating profits, D_t, for the owner when old.

When born, households are endowed with $\delta \in [0,1)$ units of housing stock whose price is $Q_{t|t} > 0$. Households can buy and trade houses.[10] Each period, the housing stock depreciates by the fraction δ, it follows that the total housing stock in the economy remains constant.

Young households supply their labor service inelastically for a real wage W_t, and allocate their net wealth between consuming the bundle $C_{1,t}$, renting housing stock of size S_t, save/invest in a one period bond of value Z_t and purchasing housing stock of size H_t. The return to saving Z_t is given by the nominal interest rate $(1+i_t)$. For

[8] $C_{1,t} \equiv \left(\int_0^1 C_{1,t}^{1-\frac{1}{\varepsilon}}(i)di\right)^{\frac{\varepsilon}{\varepsilon-1}}$ and $C_{2,t+1} \equiv \left(\int_0^1 C_{2,t+1}^{1-\frac{1}{\varepsilon}}(i)di\right)^{\frac{\varepsilon}{\varepsilon-1}}$ are the bundles consumed when young and old, respectively. In each period, there exists a continuum of differentiated goods, each produced by a different firm, and with a constant elasticity of substitution denoted by ε. Henceforth I assume $\varepsilon > 1$. Goods (and the firms producing them) are indexed by $i \in [0,1]$.

[9] As in Iacoviello (2005), I assume that housing service and consumption are separable. The decision of choosing a log specification over housing service and consumption is based e.g. on Davis and Ortalo-Magne (2011), who find that the expenditure share on housing is constant (over time and across U.S. cities). Bernanke (1984) studies the joint behavior of the consumption of durable and non-durable goods, and finds that a separable log specification is a good approximation. Note the functions $u(\cdot), v(\cdot)$ are continuous and twice differentiable, with $\lim_{C^j \to 0} u(C^j) = -\infty$ and $\lim_{C^j \to 0} u'(C^j) = \infty$, $\lim_{S^j \to 0} v(S^j) = -\infty$ and $\lim_{S^j \to 0} v'(S^j) = \infty$.

[10] Assuming that housing is a partially bubbly asset, it follows that households are endowed with a partially bubbly asset as in Galí (2014). With the difference that in Galí (2014) households are endowed with a pure bubbly asset, that is intrinsically useless.

future reference, I define the real interest rate as

$$R_t \equiv (1+i_t)E_t\left\{\frac{P_t}{P_{t+1}}\right\} \tag{3.2}$$

Accordingly, the budget constraint of the young of type j at time t is given by

$$C^j_{1,t} + \frac{Z^j_t}{P_t} + \sum_{k=0}^{\infty} q_{t|t-k}H^j_{t|t-k} + p^r_t S^j_t \leq W^j_t + \nu^j \delta q_{t|t} \tag{3.3}$$

where P_t is the price of the composite consumption good in period t. The rental and purchasing price of one unit of housing stock is denoted by P^r_t and Q_t, respectively. With prices written in lowercase letters, I define prices relative to the consumption bundle, so $q_t = \frac{Q_t}{P_t}$ and $p^r_t = \frac{P^r_t}{P_t}$. Further, $H_{t|t-k}$ denotes the quantity of the housing stock purchased in t, introduced by the cohort born in period $t-k$, and whose current price is $q_{t|t-k}$ for $k = 0, 1, 2, ...$ φ^j denotes the fraction of type j households.

The budget constraint when old is given by equation (3.4). By purchasing the consumption bundle $C_{2,t+1}$, the household consumes all it's wealth. The household's wealth consists of (1) the rental income from renting his housing stock to the young generation, which is given by $p^r_{t+1}H_t$, (2) the re-selling value of his housing stock[11], (3) the payoff from his maturing bond holding and (4) real profits generated by his intermediate firm, D_{t+1}. Formally, for each household type we have

$$C^j_{2,t+1} \leq \frac{(1+i_t)Z^j_t}{P_{t+1}} + \sum_{k=0}^{\infty} p^r_{t+1}H^j_{t|t-k} + (1-\delta)\sum_{k=0}^{\infty} q_{t+1|t-k}H^j_{t|t-k} + D^j_{t+1} \tag{3.4}$$

where $H^j_t = \sum_{k=0}^{\infty} H^j_{t|t-k}$.

The inter-temporal budget constraint (IBC) is thus given by

$$\begin{aligned} & C^j_{1,t} + \sum_{k=0}^{\infty} q_{t|t-k}H^j_{t|t-k} + p^r_t S^j_t + \frac{P_{t+1}}{P_t(1+i_t)}\left(C^j_{2,t+1} - \sum_{k=0}^{\infty}\left(p^r_{t+1} + (1-\delta)q_{t+1|t-k}\right)H^j_{t|t-k}\right) \\ \leq \quad & W_t + \delta q_{t|t} + \frac{P_{t+1}}{P_t(1+i_t)}D^j_{t+1} \end{aligned} \tag{3.5}$$

[11] At the end of the period the old household sells his remaining housing stock, $(1-\delta)q_{t+1}H_t$, to the young generation.

Lagrangian and First Order Conditions (FOCs)[12]

$$\max_{C_{1,t},C_{2,t+1}H_t,S_t}\left\{\begin{array}{l} u(C_{1,t})+\xi v(S_t)+\gamma E_t\{log(C_{2,t+1})\} \\ -\lambda_t\left(C_{1,t}+\frac{Z_t}{P_t}+\sum_{k=0}^{\infty}q_{t|t-k}H_{t|t-k}+p_t^rS_t-W_t-\delta q_{t|t}\right) \\ -\phi_t\left(E_t\{C_{2,t+1}\}-\frac{(1+i_t)Z_t}{E_t\{P_{t+1}\}}-\sum_{k=0}^{\infty}E_t\left\{\left(p_{t+1}^r+(1-\delta)q_{t+1|t-k}\right)H_{t|t-k}-D_{t+1}\right\}\right) \\ +\mu_tC_{1,t} \\ +\gamma_tC_{2,t+1} \\ +\kappa_tH_t \\ +\varphi_tS_t \\ +\psi_tZ_t \end{array}\right\}$$

The households first order conditions (FOCs) and complementary slackness conditions (CSCs) are given by

$$C_{1,t}:\quad u'(C_{1,t})-\lambda_t+\mu_t=0 \tag{3.6}$$

$$\text{with}\quad \mu_t,C_{1,t}\geq 0\quad\text{and}\quad \mu_tC_{1,t}=0$$

$$C_{2,t+1}:\quad \gamma E_t\{u'(C_{2,t+1})\}-\phi_t+\gamma_t=0 \tag{3.7}$$

$$\text{with}\quad \gamma_t,C_{2,t+1}\geq 0\quad\text{and}\quad \gamma_tC_{2,t+1}=0$$

$$H_{t|t-k}:\quad -\lambda_tq_{t|t-k}+\phi_t(1-\delta)E_t\{q_{t+1|t-k}\}+\phi_tE_t\{p_{t+1}^r\}+\kappa_t=0 \tag{3.8}$$

$$\text{with}\quad \kappa_t,H_{t|t-k}\geq 0\quad\text{and}\quad \kappa_tH_{t|t-k}=0$$

$$S_t:\quad \xi v'(S_t)-\lambda_tp_t^r+\varphi_t=0 \tag{3.9}$$

$$\text{with}\quad \varphi_t,S_t\geq 0\quad\text{and}\quad \varphi_tS_t=0$$

$$Z_t:\quad -\frac{\lambda_t}{P_t}+\phi_t\frac{(1+i_t)}{E_t\{P_{t+1}\}}+\psi_t=0$$

$$\Leftrightarrow \lambda_t=\phi_t(1+i_t)\frac{P_t}{E_t\{P_{t+1}\}}+\psi_t \tag{3.10}$$

$$\text{with}\quad \psi_t,Z_t\geq 0\quad\text{and}\quad \psi_tZ_t=0$$

Note: I focus on the case where consumption is positive in both periods of life, i.e. $C_{1,t},C_{2,t+1}>0$, this is a realistic assumption as it is empirically motivated. One time period corresponds to around 35 years. Hence, $\mu_t=\gamma_t=0$. Further I assume that the constraints on H_t,S_t,Z_t are not binding, i.e. $\kappa=\varphi_t=\psi_t=0$.

[12]For the purpose of notation clarity, I omit the index j.

3.1.1 Household Optimality Conditions

The Euler Equation for households of type j is derived using FOCs (3.6), (3.7) and (3.10):

$$1 = \gamma(1+i_t)E_t\left\{\left(\frac{C^j_{1,t}}{C^j_{2,t+1}}\right)\left(\frac{P_t}{P_{t+1}}\right)\right\} \tag{3.11}$$

The intra-temporal Optimality Condition for households of type j is derived using FOCs (3.6) and (3.7)

$$\xi^j\frac{C^j_{1,t}}{S^j_t} = p^r_t \tag{3.12}$$

The optimal saving/investment decision for households of type j is derived using FOCs (3.8) and (3.10)

$$q_{t|t-k} = \frac{1}{(1+i_t)}\frac{E_t\left\{P_{t+1}\right\}}{P_t}E_t\left\{p^r_{t+1} + (1-\delta)q_{t+1|t-k}\right\} \tag{3.13}$$

using the Euler Equation for households of type j, the previous equation can be rewritten as

$$q_{t|t-k} = \gamma E_t\left\{\left(\frac{C^j_{1,t}}{C^j_{2,t+1}}\right)\left(p^r_{t+1} + (1-\delta)q_{t+1|t-k}\right)\right\} \tag{3.14}$$

3.2 Homeownership: Definitions and Assumptions

The good housing is a consumption good, providing housing services $v(S_t)$, *and* an investment good, H_t. Household of type j becomes a

- homeowner if $S^j_t = H^j_t$,
- homeowner and landlord if $S^j_t < H^j_t$,
- renter and landlord if $S^j_t > H^j_t$

It follows that, all else equal, households with a low preference for housing services will be homeowners, while households with a high preference for housing services become renters. The higher the fraction $(1-\omega)$,the continuum of households with a low preference for housing services, the larger the aggregate homeownership rate in country k.

3.3 The Price of Housing: Definitions and Assumptions

I define the **Housing price** as

$$q_t \equiv q_t^F + q_t^B \tag{3.15}$$

where the **fundamental component** is defined as the present discounted value of future rental income and is hence given by

$$q_t^F \equiv E_t \left\{ \sum_{k=1}^{\infty} \prod_{j=0}^{k-1} \frac{1}{R_{t+j}} (1-\delta)^{k-1} p_{r,t+k} \right\} \tag{3.16}$$

The **bubble component** is defined as

$$q_t^B \equiv b_t + u_t^b \tag{3.17}$$

with $b_t \equiv \delta \sum_{k=1}^{\infty} (1-\delta)^k q_{t|t-k}^B$ and $u_t^b \equiv \delta q_{t|t}^B$, where b_t denotes the value of pre-existing bubbles in the economy and u_t^b the value of the newly introduced bubbles in t. I assume that u_t^b will follow an exogenous i.i.d. process with mean u^b.

It can be shown that (3.16) satisfies

$$q_{t|t-k}^F = E_t \left\{ \frac{1}{R_t} \left(p_{r,t+1} + (1-\delta) q_{t+1|t-k}^F \right) \right\} \tag{3.18}$$

Using (3.15), (3.18) and (3.14), it follows that the bubble component must satisfy

$$q_t^B \equiv b_t + u_t^b = E_t \left\{ \frac{1}{R_t} b_{t+1} \right\} \tag{3.19}$$

Hence, an increase in the interest rate will raise the expected growth of the bubble (as long as $u^b > 0$), while the fundamental component of the housing price will be affected negatively by a rise in the interest rate, refer to equation (3.16) .

3.4 Firms

3.4.1 Final Production Sector

The final consumption good production is perfectly competitive, hence final consumption good producers make zero profits. Each final consumption good producer has the following production function:

$$y_t \equiv \left(\int_0^1 y_t(i)^{\left(\frac{\varepsilon-1}{\varepsilon}\right)} \quad di \right)^{\left(\frac{\varepsilon}{\varepsilon-1}\right)} \qquad with \qquad \varepsilon > 1 \tag{3.20}$$

where $y_t(i)$ is the quantity of the intermediate good i with the demand function:

$$y_t(i) = \left(\frac{P_t\,(i)}{P_t} \right)^{-\varepsilon} y_t \qquad \forall i \in [0,1] \tag{3.21}$$

And it follows that the price of the final consumption good is given by

$$P_t \equiv \left(\int_0^1 P_t(i)^{(1-\varepsilon)} \quad di \right)^{\left(\frac{1}{1-\varepsilon}\right)} \tag{3.22}$$

The optimization problem of the representative final producer is therefore

$$\begin{aligned} \max \qquad & P_t y_t - \left(\int_0^1 P_t(i) y_t(i) \quad di \right) \\ s.t. \qquad & y_t = \left(\int_0^1 y_t(i)^{\left(\frac{\varepsilon-1}{\varepsilon}\right)} \quad di \right)^{\left(\frac{\varepsilon}{\varepsilon-1}\right)} \end{aligned}$$

3.4.2 Intermediate Production Sector

The production function uses labor as the only input and is given by

$$y_t(i) \equiv L_t(i) \qquad \forall i \in [0,1] \tag{3.23}$$

Every firm has monopolistic power in the production of his own variety. The monopolist sets his price $P_t(i)$ in order to maximize his profits subject to the demand

constraint (3.21). The optimization problem of the monopolistic firm is given by

$$\max_{P_t^*} \quad E_{t-1}\left\{\Lambda_{t-1,t}\left(\underbrace{P_t^* y_t(i) - \Psi_t\left(y_t(i)\right)}_{profit}\right)\right\} \tag{3.24}$$

$$s.t. \quad y_t(i) = \left(\frac{P^*}{P_t}\right)^{-\varepsilon} y_{c,t} \tag{3.25}$$

where the price P_t^* is set at the end of $t-1$ (price set in advance), which introduces nominal rigidities in the model. $\Psi_t\left(y_t(i)\right)$ denotes the nominal cost function of firm i. $\Lambda_{t-1,t}$ denotes the discount factor. As households own the intermediate production firms, they will get the profits as a lump-sum payment when old.[13]

The first order condition (FOC) is given by:

$$E_{t-1}\left\{\Lambda_{t-1,t}\left(y_t(i) + P_t^*(-\varepsilon)\left(\frac{P_t^*}{P_t}\right)^{-\varepsilon-1}\frac{y_t}{P_t} - \Psi_t^{'}\left(y_t(i)\right)(-\varepsilon)\left(\frac{P_t^*}{P_t}\right)^{-\varepsilon-1}\frac{y_t}{P_t}\right)\right\} = 0$$

After some manipulations, we get the optimal pricing condition:

$$E_{t-1}\left\{\Lambda_{t-1,t}y_t(i)\left(P_t^* - \frac{\varepsilon}{\varepsilon-1}\Psi_t^{'}\left(y_t(i)\right)\right)\right\} = 0 \tag{3.26}$$

Each firm chooses its new price equal to a fixed markup over its current nominal marginal cost, i.e. $\mathcal{M} = \frac{\varepsilon}{\varepsilon-1}$.

In the <u>*case of flexible prices*</u> *and or no uncertainty*, the FOC (3.26) is satisfied with

$$P_t^* = \underbrace{\frac{\varepsilon}{\varepsilon-1}}_{\text{markup}}\underbrace{\Psi_t^{'}\left(y_t(i)\right)}_{\text{nom Marg Costs}} \qquad \Leftrightarrow \qquad P_t^* = \mathcal{M}W_tP_t \tag{3.27}$$

From this it follows that the real wage is given by $W = \frac{1}{\mathcal{M}} \quad \forall t$

[13] Note that households take prices as given, therefore the discount factor used in the firm maximization problem must be slightly different then the one of the household. But as the difference just has to be infinitesimally small, the discount factor can be approximated by the discount factor of the household. So, the relevant discount factor will be derived from the Euler Equation and will be given by $\Lambda_{t-1,t} \approx \gamma\frac{E_t\left\{u'(C_{2,t})\right\}}{u'(C_{1,t-1})}$.

And aggregate (real) profits are given by

$$\begin{aligned} D =& \frac{1}{P_t}\left(P_t^* \int_0^1 y_t(i)di - \int_0^1 \Psi_t\left(y_t(i)\right)di\right) = \frac{1}{P_t}\left(\mathcal{M}W_tP_t - W_tP_t\right) \\ =& \left(1-\frac{1}{\mathcal{M}}\right) \qquad \forall t \end{aligned}$$

3.5 Equilibrium

Final consumption good market clearing

$$Y_t = \omega\left(C_{1,t}^H + C_{2,t}^H\right) + (1-\omega)\left(C_{1,t}^L + C_{2,t}^L\right) \tag{3.28}$$

From the income side, I can write

$$Y_t = D_t + W_t \tag{3.29}$$

Labor Market Clearing

Under the assumption that only young households supply inelastically one unit of labor it follows that each period, total labor employed in the economy is given by[14]

$$L_t = \int_0^1 L_t(i) \quad di = 1 \tag{3.30}$$

Labor market clearing implies

$$L_t = \int_0^1 Y_t(i) \quad di = Y_t = 1 \tag{3.31}$$

the second equality follows because I assume that all firms set the same price and produce the same amount in a symmetric equilibrium.

[14]Given that $Y_t(i) = L_t(i)$, it follows that $\int_0^1 Y_t(i)di = \int_0^1 L_t(i)di$, hence $Y_t = L_t = 1$.

Housing Market Clearing

Houses exist in fixed supply. The aggregate supply of housing stock is given by

$$\bar{H}_t = \delta + \delta(1-\delta) + \delta(1-\delta)^2 + ... = \delta \sum_{k=0}^{\infty}(1-\delta)^k = 1 \tag{3.32}$$

with $\bar{H}_{t|t-k} = \delta(1-\delta)^k$. The supply of houses has to equal demand each period. Hence,

$$H_t \equiv \omega H_t^H + (1-\omega)H_t^L = 1 \qquad \text{and} \qquad H_{t|t-k} = \delta(1-\delta)^k \quad \forall t \tag{3.33}$$

I denote $H_{t|t-k}^j = \varphi^j \bar{H}_{t|t-k}$. Hence,

$$H_{t|t-k}^H \equiv \varphi^H H_{t|t-k} = \varphi^H \delta(1-\delta)^k \tag{3.34}$$

$$H_{t|t-k}^L \equiv \varphi^L H_{t|t-k} = \varphi^L \delta(1-\delta)^k \tag{3.35}$$

$$H_t^H \equiv \sum_{k=0}^{\infty} H_{t|t-k}^H = \varphi^H \tag{3.36}$$

$$H_t^L \equiv \sum_{k=0}^{\infty} H_{t|t-k}^L = \varphi^L \tag{3.37}$$

with $\varphi^H + \varphi^L = 1$.

Rental Market Clearing

The supply of houses is constant and normalized to one. The aggregate supply of the housing stock that is available for rent is given by the aggregate housing stock itself and is denoted by $\bar{S}_t$. Formally,

$$\bar{S}_t = \delta + \delta(1-\delta) + \delta(1-\delta)^2 + ... = \delta \sum_{k=0}^{\infty}(1-\delta)^k = 1 \tag{3.38}$$

The supply of rental-homes has to equal it's demand each period. It follows that

$$S_t \equiv \omega S_t^H + (1-\omega)S_t^L = 1 \tag{3.39}$$

Bond market Clearing

Market clearing implies that the aggregate value of the bond market must equal to zero (the bond is in zero net supply).[15]

$$Z_t \equiv \omega Z_t^H + (1-\omega) Z_t^L = 0 \tag{3.40}$$

Market clearing conditions (3.28), (3.31), (3.33), (3.39), (3.40) and the optimal price setting equation (3.26) together with the optimality conditions of the household (3.11)-(3.14) and the definition of the housing price (3.15) with (3.18) describe the equilibrium of the economy.

Equilibrium Equations

$$L_t = \int_0^1 L_t(i)di = \int_0^1 Y_t(i)di = Y_t = 1$$
$$Y_t = \omega\left(C_{1,t}^H + C_{2,t}^H\right) + (1-\omega)\left(C_{1,t}^L + C_{2,t}^L\right)$$
$$Y_t = D_t + W_t = 1$$
$$S_t = H_t = 1$$
$$H_t = \omega H_t^H + (1-\omega) H_t^L$$
$$S_t = \omega S_t^H + (1-\omega) S_t^L$$
$$C_{l,t} = \omega C_{l,t}^H + (1-\omega) C_{l,t}^L \quad for \quad l \in \{1,2\}$$

$$C_{1,t}^j = C_{1,t}^H = \tfrac{1}{1+\xi^j}\left[\nu^j\left(W_t + u_t\right) - \varphi^j\left(f_t + b_t + u_t\right)\right] \quad for \quad j \in \{H, L\}$$
$$p_t^r = \Sigma_j \tfrac{\xi^j \nu^j}{1+\xi^j}\left[\nu^j\left(W_t + u_t\right) - \varphi^j\left(f_t + b_t + u_t\right)\right] \quad for \quad j \in \{H, L\}$$
$$C_{2,t}^j = \varphi^j p_t^r + \varphi^j\left(f_t + b_t\right) + \nu^j(1 - W_t) \quad for \quad j \in \{H, L\}$$
$$S^j = \frac{\frac{\xi^j}{1+\xi^j}\left[\nu^j(W_t+u_t) - \varphi^j(f_t+b_t+u_t)\right]}{\Sigma_j \frac{\xi^j \nu^j}{1+\xi^j}\left[\nu^j(W_t+u_t) - \varphi^j(f_t+b_t+u_t)\right]} \quad for \quad j \in \{H, L\}$$

$$q_t^b \equiv b_t + u_t = \gamma E_t\left\{\left(\frac{C_{1,t}^j}{C_{2,t+1}^j}\right)\left(b_{t+1}\right)\right\}$$
$$q_t^f = \gamma E_t\left\{\left(\frac{C_{1,t}^j}{C_{2,t+1}^j}\right)\left(p_{t+1}^r + (1-\delta) q_{t+1}^f\right)\right\}$$
$$1 = \gamma(1+i_t) E_t\left\{\left(\frac{C_{1,t}^j}{C_{2,t+1}^j}\right)\left(\frac{P_t}{P_{t+1}}\right)\right\}$$

[15] The dividend is simply a return paid by one agent to some other agent. The asset is not productive in the sense that it adds to the total resources in the economy.

Next, I will characterize the deterministic equilibrium for which an exact solution exists. Emphasis is placed on the existency conditions for a bubbly equilibrium with positive fundamental. Subsequently, I will discuss comparative statics of the two steady states.

3.6 Conditions for the Existence of Bubbles

3.6.1 Affordability Constraint

The investement good house has to be affordable. Given that the young are the only agents that buy houses, the affordability constraint is derived from the budget constraint of the young. In a bubbly equilibrium it must hold that

$$b_t \in \left[0; W - q_t^f\right] \quad \textit{for all } t \tag{3.41}$$

Lemma 3.1. *The larger the fundamental value of the house today, the smaller the maximum pre-existing aggregate bubble value today. Proof: See Appendix.*

3.6.2 No-Arbitrage Condition and its implication on the bubble size

The bubbly asset has to be attractive. In equilibrium the expected return of investing in the one-period bond (i.e. saving) and the expected return of investing in houses has to be equal. This condition is given by the household optimality condition (3.13), and it's *deterministic version* can be rewritten as

$$R_t = \frac{p_{t+1}^r}{q_t} + \frac{f_{t+1} + b_{t+1}}{q_t} \tag{3.42}$$

Deterministic Steady State Interest Rate

Case 1: No Bubble Word $(f(u^f) > 0, b(u^b) = 0)$

In a deterministic steady state with $f(u^f = 0) > 0$, the steady state interest rate equals $R(b = 0, f(0)) = 1 + \frac{p^r}{q}$, hence larger than one, as p^r is always positive, given the log-utility function assumed.[16]

[16]With $u^f > 0$, the interest rate $R(b = 0, f(u^f))$ is larger than one if $W > f(u^f) + \frac{(1+\xi)}{\xi}u^f$.

Case 2: Pure Bubble Word ($\xi = f = 0, b(u^b) > 0$)
In a pure bubble world without heterogeneous households, the model collapses to the economy in Gali (2014). If $u^b = 0$ and $\xi = f(0) = 0$, the deterministic steady state interest rate is given by $R\,(0, b(0)) = 1$, the interest rate corresponding to the upper bound of the steady state bubble size. Note if $u^b > 0$ and $\xi = f(u^f) = 0$, it follows from (7.26) that $R(b(u^b)) < 1$.

Case 3: Bubble Word with positive Fundamental ($f(u^f) > 0, b(u^b) > 0$)
To sustain a bubbly equilibrium, the interest rate has to be $R(b(u^b), 0) \leq 1$. In the deterministic steady state, the definition of the fundamental price (3.16) becomes $q_t^F \equiv f + u^f = \frac{p_r}{R-(1-\delta)}$ and hence it must hold that $R(f(u^f), b(u^b)) > (1-\delta)$ for the price of the fundamental component to be positive.

Consequently, in a bubbly deterministic steady state with a positive fundamental value, the real interest rate lies between $(1-\delta) < R(f(u^f), b(u^b)) \leq 1$.

NOTE: The remaining Part of this Chapter will be solved for the Representative Agent Economy. (to be completed for the heterogeneous household model)

The size of the Fundamental and of the Bubble
Using the *deterministic version* of the Euler equation (3.11) and the definition of the real interest rate (3.2), I can write:

$$R_t = \frac{(1-W) + \xi^k + f_{t+1} + b_{t+1}}{\gamma(W - f_t - b_t)} \tag{3.43}$$

Using (3.43), the affordability constraint (3.41), and the conditions on the real interest rate derived above (*cases 1-3*), it can be shown that the size of the fundamental and the bubble is given by

Case 1: No Bubble Word ($f(u^f) > 0, b(u^b) = 0$)

$$f \in \left(W - \frac{1+\xi^k}{1+(1-\delta)\gamma}, W \right) \quad \textit{where} \quad R > (1-\delta) \tag{3.44}$$

Case 2: Pure Bubble Word ($\xi = f = 0, b(u^b) > 0$)

$$b \in \left(0, W - \frac{1}{(1+\gamma)}\right) \qquad where \quad R \leq 1 \tag{3.45}$$

Case 3: Bubble Word with positive Fundamental ($f(u^f) > 0, b(u^b) > 0$)

$$b \in \left(W - f - \frac{(1+\xi^k)}{1+(1-\delta)\gamma}, W - f - \frac{(1+\xi^k)}{1+\gamma}\right) \quad where \quad (1-\delta) < R \leq 1 \tag{3.46}$$

3.6.3 Bubbly Equilibrium: Existency Condition

Proposition 3.2. *A necessary condition for the existence of a deterministic bubbly steady state with a positive fundamental and bubble value is given by*

$$\left(\frac{1}{\mathcal{M}}\right) > f(\xi^k, \gamma, \delta) + \left(\frac{1+\xi^k}{1+(1-\delta)\gamma}\right)$$

Proof: See Appendix

Corollary 3.3. *The higher ξ^k, i.e. the larger the population's preference for housing services relative to consumption in country k, the lower the probability that a positive bubble exists (that Proposition 3.2 holds).*

The higher ξ^k, the larger the fraction of income spent on housing services and the lower the fraction of income spent on other consumption goods. Hence according to Proposition 3.2, countries with a higher income share spent on housing services relative to other consumption goods, are less prone to experience housing bubbles.[17]

Corollary 3.4. *The higher the fundamental component of the housing price, the less likely that Proposition 3.2 holds, i.e. that a positive bubble exists.*

Corollary 3.5. *The lower the depreciation rate δ, the more likely that Proposition 3.2 holds, i.e. that a positive bubble exists.*

[17] In a companion paper, I provide empirical evidence and show that countries with a larger preference for housing services experienced significantly less housing bubbles over 1970-2013.

Proposition 3.6. *A necessary and sufficient condition for the existence of a deterministic pure bubbly steady state without fundamental value is given by*

$$\left(\frac{1}{\mathcal{M}}\right) > \left(\frac{1}{1+\gamma}\right)$$

Proof: See Appendix

Proposition 3.7. *A necessary condition for the existence of a deterministic non-bubbly steady state with a positive fundamental value is given by*

$$\left(\frac{1}{\mathcal{M}}\right) > \left(\frac{1+\xi^k}{1+(1-\delta)\gamma}\right)$$

Proof: See Appendix

3.7 Equilibrium Dynamics: The Deterministic Case

3.7.1 Steady State

The steady state interest rate solves the equation[18]

$$R^2 + \underbrace{\frac{(1-\gamma)W + U - (1+\xi^k)}{\gamma(W+U)}}_{\equiv Z} R - \underbrace{\frac{(1-W)}{\gamma(W+U)}}_{\equiv F} = 0 \tag{3.47}$$

where W is a constant. Solving for R gives

$$R_{1,2}(U) = \frac{(1+\xi^k) - U - W(1-\gamma) + \mp\sqrt{-4\gamma(1-W)(U+W) + [U + (1-\gamma)W - (1+\xi^k)]^2}}{2\gamma(U+W)}$$

Solving if $Z^2 - 4F = 0$ for u gives

$$\tilde{u}_{1,2} \equiv (\gamma + \xi^k) + (1+\gamma)(1-W) \mp 2\sqrt{\gamma(1-W)(1+\gamma+\xi^k)} \tag{3.48}$$

Resulting in two real solutions $R_1(\tilde{u_1}) = R_2(\tilde{u_1})$ and $R_1(\tilde{u_2}) = R_2(\tilde{u_2})$.[19] In the following we focus on the range $U \in (0, \tilde{u_1})$.

Bubbly deterministic steady state with a positivie fundamental value:

$$\begin{cases} \exists \quad \text{two sets of steady states with} \quad R_1(U) \neq R_2(U) & \text{for } U \in]\underline{u}_{R_1}, \tilde{u_1}). \\ \exists \quad \text{one set of steady states with} \quad R_2(U) & \text{for } U \in (\underline{u}_{R_2}, \tilde{u_1}). \end{cases}$$

where

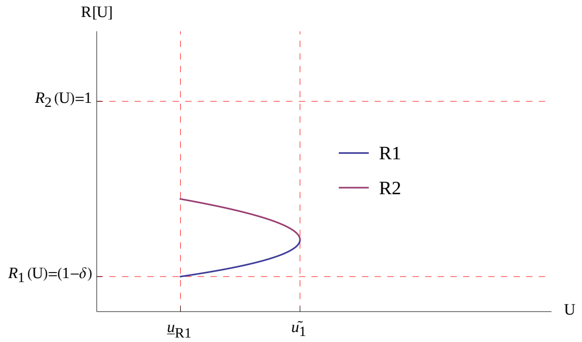
$$\underline{u}_{R_1} = \left(\frac{\xi^k + \delta\left[W(1+\gamma) - (1+\xi^k)\right] - W\gamma\delta^2}{[1+\gamma(1-\delta)]\,(1-\delta)} \right) \qquad \underline{u}_{R_2} = \left(\frac{\xi^k}{1+\gamma} \right)$$

Proof 1: $\exists$ two sets of steady states with $R_1(U) \neq R_2(U)$ for $U \in]\underline{u}_{R_1}, \tilde{u_1})$
$R_2(U) > R_1(U)$ and $\dfrac{\partial R_1}{\partial U} > 0$ and $\dfrac{\partial R_2}{\partial U} < 0$ for $U < \tilde{u_1}$. Given the restriction that $(1-\delta) < R(U) \leq 1$, recall (3.46), the lower bound on U for both real interest rates can be derived and is given by $R_1(\underline{u}_{R_1}) = (1-\delta)$ and $R_2(\underline{u}_{R_2}) = 1$. Hence,

[18]Solving the version of the Euler Equation (**??**) for R yields this quadratic equation.

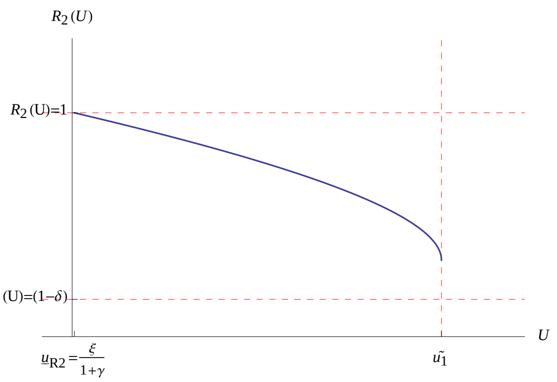
[19]For $U > \tilde{u_2}$, two real, non-positive solutions $R_1(U) \neq R_2(U)$. For $\tilde{u_1} < U < \tilde{u_2}$, two complex solutions $R_1(U) \neq R_2(U)$. For $U < \tilde{u_1}$, two real, positive solutions $R_1(U) \neq R_2(U)$.

$\underline{u} = max\{\underline{u}_{R_1}, \underline{u}_{R_2}, 0\}$ where $\underline{u}_{R_1} = \left(\frac{\xi^k + \delta[W(1+\gamma) - (1+\xi^k)] - W\gamma\delta^2}{[1+\gamma(1-\delta)](1-\delta)}\right)$, $\quad \underline{u}_{R_2} = \left(\frac{\xi^k}{1+\gamma}\right)$.
Using the necessary condition (Proposition 1.2.) for the existence of a deterministic bubbly steady state with positive fundamental, it can be shown that $\underline{u}_{R_1} > \underline{u}_{R_2}$ and hence $\underline{u} = \underline{u}_{R_1}$.[20] Note that the set U $\in]\underline{u}_{R_1}, \tilde{u_1})$ is non-empty for combinations of parameter restrictions displayed on page 20.



Notes: $\gamma = 0.99$, $\xi = 0.03$, $\delta = 0.25$, $W = 0.6$.

Figure 1: Set of Steady State interest rates $R_1(U), R_2(U)$ over full solution space



Notes: $\gamma = 0.99$, $\xi = 0.03$, $\delta = 0.25$, $W = 0.6$.

Figure 2: Set of Steady State interest rates $R_2(U)$ over full solution space

Proof 2: $\exists$ one set of steady states with$R_2(U)$ for $U \in (\underline{u}_{R_2}, \tilde{u_1})$
$R_2(U)$ is decreasing in U, hence a sufficient condition is $R_2(\tilde{u_1}) - (1-\delta) > 0$. The the solution set, where all parameter restrictions and the existency condition (Proposition 2.1) hold is given by the explicit representation of the following region:

$0 < \delta < 1 \quad \wedge \quad 0 < \gamma < 1 \quad \wedge \quad 0 < \xi < -\frac{\gamma(\delta-1)\delta}{\gamma(\delta^2-3\delta+2)+1}$
$\wedge \quad \frac{\xi+1}{\gamma(-\delta)+\gamma+1} < W < \frac{1-\gamma(\delta-1)(\delta\xi+\delta-\xi+1)}{(\gamma(\delta-1)-1)^2}$

[20] This follows from Proposition 1.2. and the fact that $\frac{\partial \underline{u}_{R_1}}{\partial W} = \frac{\delta}{1-\delta} > 0$.

Given the following combinations of paramter restrictions, there exists a range for the exogenous process $U \in]\underline{u}, \tilde{u}_1)$ where two steady states exist and where $(1-\delta) < R(U) \leq 1$, $0 < \gamma < 1$, $0 < \delta < 1$, $\xi > 0$, $0 < W < 1$ and the necessairy condition holds: $W > f + \left(\frac{1+\xi}{1+(1-\delta)\gamma}\right)$.

$$0 < \xi < \frac{1}{3}\left(2\sqrt{3}-3\right) \quad \wedge \quad \frac{\xi+1}{2} < W \leq \frac{\xi^2+4\xi+3}{2(\xi+2)} - \frac{1}{2}\sqrt{\frac{-3\xi^4-12\xi^3-14\xi^2-4\xi+1}{(\xi+2)^2}} \wedge \frac{\xi-W+1}{W} < \gamma < 1 \wedge 0 < \delta < \frac{-\xi+\gamma W+W-1}{\gamma W};$$

$$\vee 0 < \xi < \frac{1}{3}\left(2\sqrt{3}-3\right) \quad \wedge \quad \frac{\xi^2+4\xi+3}{2(\xi+2)} - \frac{1}{2}\sqrt{\frac{-3\xi^4-12\xi^3-14\xi^2-4\xi+1}{(\xi+2)^2}} < W < \frac{\xi^2+4\xi+3}{2(\xi+2)} + \frac{1}{2}\sqrt{\frac{-3\xi^4-12\xi^3-14\xi^2-4\xi+1}{(\xi+2)^2}}$$

$$\wedge \frac{\xi-W+1}{W} < \gamma \leq \frac{-\xi^3-3\xi^2-3\xi-\xi W^2-W^2+2\xi^2W+4\xi W+2W-1}{W^2+\xi^2W-W} \quad \wedge \quad 0 < \delta < \frac{-\xi+\gamma W+W-1}{\gamma W};$$

$$\vee \frac{-\xi^3-3\xi^2-3\xi-\xi W^2-W^2+2\xi^2W+4\xi W+2W-1}{W^2+\xi^2W-W} < \gamma < 1 \quad \wedge \quad 0 < \delta < \frac{\xi+\gamma W+W}{\xi+\gamma W+1} - \sqrt{\frac{\gamma+\xi-\gamma W-\xi W-W+1}{\gamma(\xi+\gamma W+1)^2}};$$

$$\vee \frac{-\xi^3-3\xi^2-3\xi-\xi W^2-W^2+2\xi^2W+4\xi W+2W-1}{W^2+\xi^2W-W} < \gamma < 1 \wedge \frac{\xi+\gamma W+W}{\xi+\gamma W+1} - \sqrt{\frac{\gamma+\xi-\gamma W-\xi W-W+1}{\gamma(\xi+\gamma W+1)^2}} < \delta < \frac{-\xi+\gamma W+W-1}{\gamma W};$$

$$\vee 0 < \xi < \frac{1}{3}\left(2\sqrt{3}-3\right) \quad \wedge \quad \frac{\xi^2+4\xi+3}{2(\xi+2)} + \frac{1}{2}\sqrt{\frac{-3\xi^4-12\xi^3-14\xi^2-4\xi+1}{(\xi+2)^2}} \leq W < 1 \quad \wedge \frac{\xi-W+1}{W} < \gamma < 1 \quad \wedge 0 < \delta < \frac{-\xi+\gamma W+W-1}{\gamma W};$$

$$\vee \frac{1}{3}\left(2\sqrt{3}-3\right) \leq \xi < 1 \quad \wedge \quad \frac{\xi+1}{2} < W < 1 \wedge \frac{\xi-W+1}{W} < \gamma < 1 \quad \wedge \quad 0 < \delta < \frac{-\xi+\gamma W+W-1}{\gamma W}$$

Given the steady state interest rates, the corresponding steady state allocations can be calculated. Figures (3) and (4) show the steady state realizations over the full set of steday states with interest rate R_1.

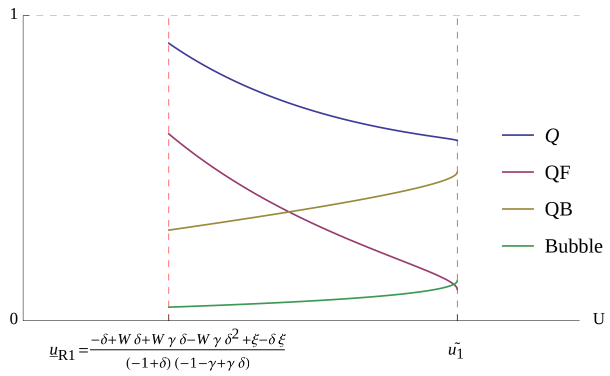


Figure 3: Steady State realizations for full set of Steady States with R_1.

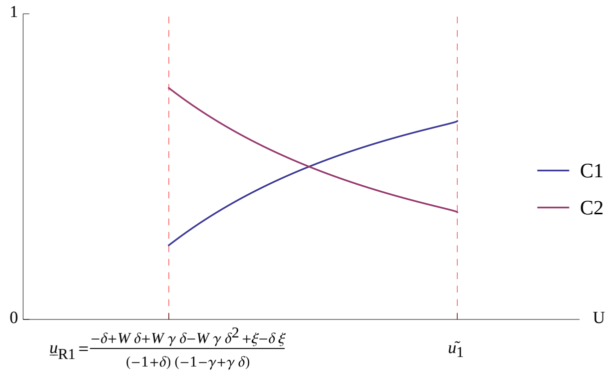


Figure 4: Steady State realizations for full set of Steady States with R_1.

TO BE COMPLETED: PROOF THAT THIS IS THE SET OF STABLE STEADY STATES

TO BE COMPLETED: DO THIS PART FOR HETEROGENEOUS HOUSEHOLD MODEL

4 Comparative Statics

4.1 Impact of the Preference of Housing Services on Steady State Allocations

The impact of ξ^k, the aggregate preference for housing services of country k, on the set of Steady States with R_1 is depicted in figures (3) and (4). Refer also to table (1). Note: ξ^k increases when either ξ^H is larger or when the fraction ω of households with a high preference for housing services is larger.

An increase in ξ captures an increase in household's preference for housing services relative to consumption. The 50% increase induces a decrease the fraction of income spend on consumption when young (hence C_1 decreases) and an increase in the fraction of income spend on renting, hence the rental price increases. The fundamental price q_F, is the discounted stream of rental prices, and q^F increases.[21] The price of houses q increases, while the price-rent-ratio decreases. The bubble component decreases. Consumption when old increases.

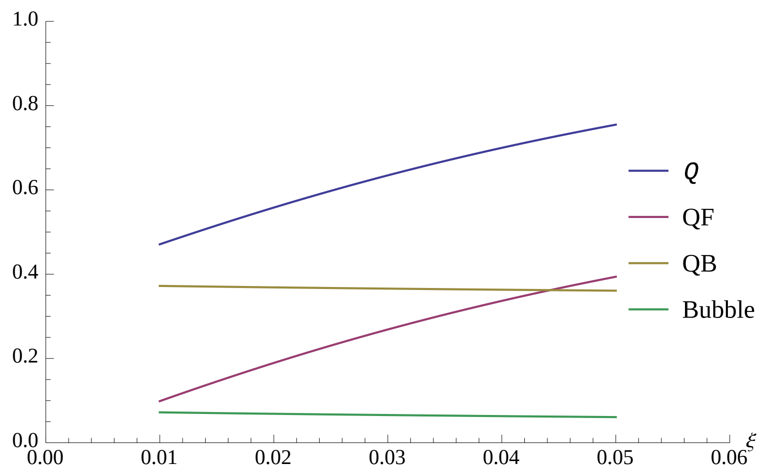


Figure 5: Comparative Statics - Steady State with R_1: Increase in ξ.

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[21] Note that there are two effects on the fundamental price: A positive effect steaming from the increase in the rental price and the effect, coming from the decrease in the interest rate, which also increases the fundamental component.

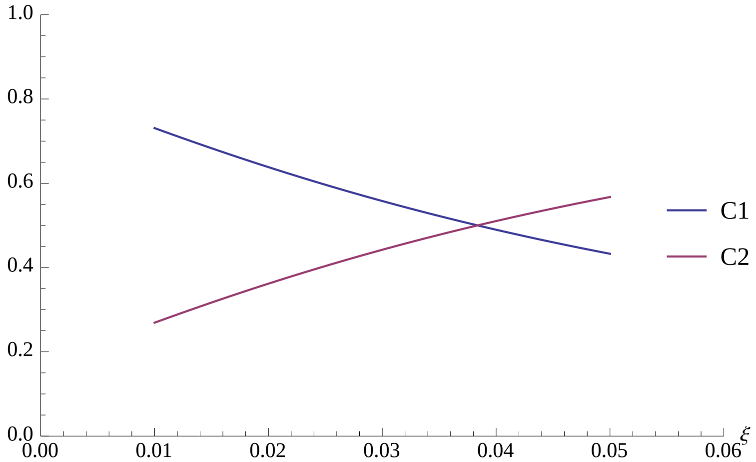

Figure 6: Comparative Statics - Steady State with R_1: Increase in ξ.

	$\xi \uparrow$	$\gamma \uparrow$	$\delta \uparrow$
R_1^*	decr.	decr.	no effect
C_1^*	decr.	decr.	incr.
C_2^*	incr.	incr.	decr.
q^*	incr.	incr.	decr.
p_r^*	incr.	decr.	incr
q^*/p_r^*	decr.	incr.	decr.
q_F^*	incr.	incr.	decr.
q_B^*	decr.	decr.	no effect
b^*	decr.	decr.	no effect

Table 1: Comparative Statics for Case: Set of steady states with R_1

	$\xi \uparrow$	$\gamma \uparrow$	$\delta \uparrow$
R_2^*	incr.	incr.	no effect
C_1^*	decr.	decr.	incr.
C_2^*	incr.	incr.	decr.
q^*	incr.	incr.	decr.
p_r^*	incr.	decr.	incr
q^*/p_r^*	decr.	incr.	decr.
q_F^*	incr.	decr.	decr.
q_B^*	incr.	incr.	no effect
b^*	incr.	incr.	no effect

Table 2: Comparative Statics for Case: Set of steady states with R_2

5 Policy Analysis

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6 Conclusion

The recent financial crisis was triggered by the collapse of a housing bubble. This has raised interest among policy makers and researchers in understanding which economic environments are more prone to produce such bubbles. The literature focuses mainly on credit market conditions and their effect on housing bubbles. The companion paper instead explores the importance of the preference of housing services and homeownership rates empirically. First, it provides a comprehensive empirical characterization of housing cycles using a large database covering 18 industrialized countries over the period 1970:1-2013:4. Second, the interaction between homeownership, preferences for housing services, real estate specific credit variables and house price fluctuations is investigated. This paper focuses on two novel stylized facts that are identified across countries: the preference for housing services is highly negatively correlated with (1) homeownership rates, and (2) with the frequency of independent housing booms and boom-bust cycles.

Thus, countries with a lower preference for housing services are characterized with larger homeownership rates and experienced more independent housing booms as well as boom-bust cycles and therefore potentially more housing bubbles over the time period 1970 to 2014. These results call for a better understanding of the interaction between home ownership rates, preferences for housing services and house price fluctuations.

This paper provides a theoretical explanation for the two novel stylized facts discovered in the data. An overlapping generations model is used as a laboratory for the analysis of the impact of the preference of housing services on homeownership rates and housing bubble occurrence. The model features heterogeneous households that differ in their preference for housing services relative to other consumption goods. These differences are assumed to be constant and exogenously given. The weighted average of the different preferences for housing services represents population's preferences for housing services of country k. This assumption is reasonable

and based on empirical evidence, provided by Huber *et al.* (2015), who show that large cross-country differences in housing preferences exist and that these cross-country differences are persistent over time.[22]

In the model, it can be shown that countries featuring a larger aggregate preference for housing services are characterized with (1) larger homeownership rates and, (2) with a larger fundamental housing price. The bubble component of the housing price is smaller. Further, in the model, countries with a larger aggregate preference for housing services are less prone to experience housing bubbles.

TO BE COMPLETED

[22]Huber *et al.* (2015) study the impact of culture on homeownership rates, using data on the tenure choice decision of second generation immigrants in the United States. They find that cultural preferences imported by the parents play a large role for the tenure choice decision.

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7 Appendix A: Model

7.1 Affordability Constraint

The good House has to stay affordable, as the young are the only agents that buy houses the affordability constraint is derived from the budget constraints of the young households:[23]

$$\sum_{k=0}^{\infty} q_{t|t-k} H^{j}_{t|t-k} \leq W^{j}_{t} + \nu^{j} \delta q_{t|t} \tag{7.1}$$

Hence,

$$\varphi^{H} \sum_{k=0}^{\infty} q_{t|t-k} H_{t|t-k} \leq \omega (W_t + \delta q_{t|t}) \tag{7.2}$$

$$\varphi^{L} \sum_{k=0}^{\infty} q_{t|t-k} H_{t|t-k} \leq (1-\omega)(W_t + \delta q_{t|t}) \tag{7.3}$$

As all houses need to be owned in equilbrium, $\varphi^{H} = (1 - \varphi^{L})$, it follows

$$\sum_{k=0}^{\infty} q_{t|t-k} H_{t|t-k} \leq W_t + \delta q_{t|t}$$

$$q_t \leq W_t + \delta q_{t|t}$$

$$b_t \leq W - f_t \tag{7.4}$$

in a bubbly equilibrium it must hold that

$$b_t \in [0; W - f_t] \quad \textit{for all } t \tag{7.5}$$

7.2 Derviation of Choice Variables

We can derive p^{r}_{t+1} in terms of f_t, b_t, u_t as follows: Starting with the intra-temporal optimality condition (3.12) and using the fact in equilibrium
$S_t = \omega S^{H}_{t} + (1-\omega) S^{L}_{t} = 1 \quad \forall t$, I can write

$$\xi^{H} C^{H}_{1,t} = p^{r}_{t} S^{H}_{t} \tag{7.6}$$

$$\xi^{L} C^{L}_{1,t} = p^{r}_{t} S^{L}_{t} \tag{7.7}$$

$$\Leftrightarrow \quad p^{r}_{t} = \omega \xi^{H} C^{H}_{1,t} + (1-\omega) \xi^{L} C^{L}_{1,t} \tag{7.8}$$

[23] Setting consumption $C^{j}_{1,t}$, bond holding Z^{j}_{t} and renting S^{j}_{t} to zero, I can derive the space for b_t that it has to belong to in equilibrium.

Using the budget constraint of the young (3.3) and the intra-temporal optimality condition (3.12) and using the fact that market clearing implies $S_t = S_t^H + S_t^L = 1, Z_t = 0$, we can derive the rental price $p_t^r(f_t, b_t, \varphi^H)$ and consumption level when young $C_{1,t}(f_t, b_t, \varphi^H)$ as follows[24]

$$
\begin{aligned}
C_{1,t}^j &= -\frac{Z_t^j}{P_t} - \sum_{k=0}^{\infty} q_{t|t-k} H_{t|t-k}^j - p_t^r S_t^j + W_t^j + \nu^j \delta q_{t|t} \\
\Leftrightarrow \quad C_{1,t}^H &= \frac{1}{1+\xi^H} \left[\omega \left(W_t + u_t \right) - \varphi^H \left(f_t + b_t + u_t \right) \right] \qquad (7.9) \\
\Leftrightarrow \quad C_{1,t}^L &= \frac{1}{1+\xi^L} \left[\left(1-\omega \right) \left(W_t + u_t \right) - \varphi^L \left(f_t + b_t + u_t \right) \right] \qquad (7.10)
\end{aligned}
$$

Plugging (7.9)-(7.10) into (7.8) gives the equation for the rental price. Using the fact that $C_{1,t} = C_{1,t}^H + C_{1,t}^L$, I can write[25]

$$
\begin{aligned}
p_t^r(f_t, b_t, \varphi^H) &= \frac{\xi^H \omega}{1+\xi^H} \left[\omega \left(W_t + u_t \right) - \varphi^H \left(f_t + b_t + u_t \right) \right] \qquad (7.11) \\
&+ \frac{\xi^L (1-\omega)}{1+\xi^L} \left[\left(1-\omega \right) \left(W_t + u_t \right) - \varphi^L \left(f_t + b_t + u_t \right) \right] \\
C_{1,t}(f_t, b_t, \varphi^H) &= \frac{\omega}{1+\xi^H} \left[\omega \left(W_t + u_t \right) - \varphi^H \left(f_t + b_t + u_t \right) \right] \qquad (7.12) \\
&+ \frac{(1-\omega)}{1+\xi^L} \left[\left(1-\omega \right) \left(W_t + u_t \right) - \varphi^L \left(f_t + b_t + u_t \right) \right]
\end{aligned}
$$

We derive the choice variable housing services, S_t^j using (7.6) - (7.10)

$$
S_t^H(f_t, b_t) = \frac{\frac{\xi^H}{1+\xi^H} \left[\omega \left(W_t + u_t \right) - \varphi^H \left(f_t + b_t + u_t \right) \right]}{\frac{\xi^H \omega}{1+\xi^H} \left[\omega \left(W_t + u_t \right) - \varphi^H \left(f_t + b_t + u_t \right) \right] + \frac{\xi^L (1-\omega)}{1+\xi^L} \left[\left(1-\omega \right) \left(W_t + u_t \right) - \varphi^L \left(f_t + b_t + u_t \right) \right]} \qquad (7.13)
$$

$$
S_t^L(f_t, b_t) = \frac{\frac{\xi^L}{1+\xi^L} \left[\left(1-\omega \right) \left(W_t + u_t \right) - \varphi^L \left(f_t + b_t + u_t \right) \right]}{\frac{\xi^H \omega}{1+\xi^H} \left[\omega \left(W_t + u_t \right) - \varphi^H \left(f_t + b_t + u_t \right) \right] + \frac{\xi^L (1-\omega)}{1+\xi^L} \left[\left(1-\omega \right) \left(W_t + u_t \right) - \varphi^L \left(f_t + b_t + u_t \right) \right]} \qquad (7.14)
$$

---

[24]Recall ν^j is the fraction of type j households in the economy, $\nu^H = \omega, \nu^L = (1-\omega)$. φ^j denotes the fraction of the housing stock bought by type j agents, with $\varphi^H = 1 - \varphi^L$.

[25]Note if $\xi^j > 1$ for $\forall j$ (i.e. the household receives relatively more utility from renting than from consumption when young), ceteris paribus, the equations (7.9), (7.10) and (7.11) imply that $p_t^r(f_t, b_t) > C_{1,t}(f_t, b_t)$, i.e. in equilibrium, the total spending on renting exceeds total spending on consumption when young ($P_t^r S_t > P_t C_{1,t}$) as $P_t = S_t = 1$ and $p_t^r(f_t, b_t, \varphi^H) > C_{1,t}(f_t, b_t, \varphi^H)$. It is intuitive that $p_t^r(f_t, b_t, \varphi^H)$ and $C_{1,t}(f_t, b_t, \varphi^H)$ are both increasing in the wage rate W and decreasing in both components of the housing price f_t, b_t. Further, we find that $p_t^r(f_t, b_t, \varphi^H)$ is increasing and total consumption of the young generation $C_{1,t}(f_t, b_t, \varphi^H)$ is decreasing in ξ^H, ξ^L.

Derivation of φ^H, the fraction of houses bought by the H-Type:
Given the fact, that $\bar{H}_t = 1$, and markets have to clear, we can derive the fraction of houses bought by the H-Type, i.e. φ^H, using equation (3.33):

$$H_t \equiv \omega\varphi^H + (1-\omega)(1-\varphi^H) = 1 \qquad \Rightarrow \qquad \varphi^H = \frac{1-\omega}{2\omega+1} \quad \forall t \tag{7.15}$$

Derivation of consumption levels of the old generation: Using the budget constraint of the old (3.4) and (7.11) and the fact that market clearing implies $Z_t = 0$, yields $C^j_{2,t+1}(f_{t+1}, b_{t+1})$:[26]

$$C^j_{2,t+1} = \sum_{k=0}^{\infty} p^r_{t+1} H^j_{t|t-k} + (1-\delta)\sum_{k=0}^{\infty} q_{t+1|t-k} H^j_{t|t-k} + D^j_{t+1}$$

$$\Leftrightarrow \quad C^j_{2,t+1} = \varphi^j p^r_{t+1} \sum_{k=0}^{\infty} q_{t+1|t-k} H_{t|t-k} + \varphi^j (1-\delta) \sum_{k=0}^{\infty} q_{t+1|t-k} H_{t|t-k} + (1 - W^j_{t+1})$$

Hence,

$$\begin{aligned} C^H_{2,t+1}(f_{t+1}, b_{t+1}) &= \varphi^H p^r_{t+1} + \varphi^H (f_{t+1} + b_{t+1}) + \omega(1 - W_{t+1}) \\ C^L_{2,t+1}(f_{t+1}, b_{t+1}) &= \varphi^L p^r_{t+1} + \varphi^L (f_{t+1} + b_{t+1}) + (1-\omega)(1 - W_{t+1}) \end{aligned}$$

Plugging in p^r_{t+1}, gives

$$\begin{aligned} C^H_{2,t+1}(f,b) = {} & \frac{\varphi^H \xi^H \omega}{1+\xi^H}\left[\omega\left(W_{t+1} + u_{t+1}\right) - \varphi^H\left(f_{t+1} + b_{t+1} + u_{t+1}\right)\right] \\ & + \frac{\varphi^H \xi^L (1-\omega)}{1+\xi^L}\left[(1-\omega)\left(W_{t+1} + u_{t+1}\right) - \varphi^L\left(f_{t+1} + b_{t+1} + u_{t+1}\right)\right] \\ & + \varphi^H\left(f_{t+1} + b_{t+1}\right) + \omega(1 - W_{t+1}) \end{aligned} \tag{7.16}$$

$$\begin{aligned} C^L_{2,t+1}(f,b) = {} & \frac{\varphi^L \xi^H \omega}{1+\xi^H}\left[\omega\left(W_{t+1} + u_{t+1}\right) - \varphi^H\left(f_{t+1} + b_{t+1} + u_{t+1}\right)\right] \\ & + \frac{\varphi^L \xi^L (1-\omega)}{1+\xi^L}\left[(1-\omega)\left(W_{t+1} + u_{t+1}\right) - \varphi^L\left(f_{t+1} + b_{t+1} + u_{t+1}\right)\right] \\ & + \varphi^L\left(f_{t+1} + b_{t+1}\right) + (1-\omega)(1 - W_{t+1}) \end{aligned} \tag{7.17}$$

[26] The lower the wage rate W and the higher the components of the housing price when old, ceteris paribus the higher consumption when old. The impact of ξ is not that clear cut. If the wage rate is large enough, $W > f_{t+1} + b_{t+1}$, the consumption level when old is increasing in ξ.

7.3 Deriviation of the Existency Condition

Using the expressions for the consumption levels (7.9), (7.10), (7.16), (7.17) and the deterministic version of the Euler equations for each household type j (3.11) and the definition of the real interest rate (3.2), I can write:

$$R_t = \gamma^{-1} \frac{C^H_{2,t+1}(f, b, \varphi^H)}{C^H_{1,t}(f, b, \varphi^H)} \tag{7.18}$$

$$R_t = \gamma^{-1} \frac{C^L_{2,t+1}(f, b, \varphi^H)}{C^L_{1,t}(f, b, \varphi^H)} \tag{7.19}$$

Note: *In representative household economy*, this equation has the simple expression:

$$E_t \{R_t\} = \gamma^{-1} E_t \left\{ \frac{(1-W) + \xi + f_{t+1} + b_{t+1}}{W - f_t - b_t} \right\} \tag{7.20}$$

Start with (7.18), we know that as long as $f \geq 0, b \geq 0$, it must be that $R_t \leq 1$. Set $R_t \leq 1$ and solve for b:

$$\begin{aligned} b \leq & \left(\frac{(1+\xi^L)(1+\gamma+\xi^H(1-\varphi^H))\omega - \xi^L \varphi^H (1-\omega)}{\varphi^H \left[1 + \gamma(1+\xi^L) + \xi^H(1-\varphi^H) + \xi^L \left(2 + \xi^H(2-\varphi^H) - \varphi^H(1+\xi^H)\right)\right]} \right) W \\ & - f + \left(\frac{\xi^L \varphi^H (1+\xi^H)(2-\varphi^H-\omega) + (1+\xi^L)(\gamma - \varphi^H \xi^H)(\varphi^H - \omega)}{(1+\xi^L)(1+\gamma+\xi^H(1-\varphi^H))\omega - \xi^L \varphi^H (1+\xi^H)(1-\omega)} \right) u \\ & - \left(\frac{(1+\xi^L)(1+\xi^H)\omega}{\varphi^H \left[1 + \gamma(1+\xi^L) + \xi^H(1-\varphi^H) + \xi^H \left(2 + \xi^H(2-\varphi^H) - \varphi^H(1+\xi^H)\right)\right]} \right) \end{aligned} \tag{7.21}$$

Start with (7.18), we know that as long as $f \geq 0, b \geq 0$, it must be that $R_t \geq (1-\delta)$. Set $R_t \geq (1-\delta)$ and solve for b, for it's lower bound:

$$\begin{aligned} & \left[-\varphi^H(1+\xi^L)(\varphi^H\xi^H - \gamma(1-\delta)) - \varphi^H\xi^L(1+\xi^H)(1-\varphi^H) + \varphi^H(1+\xi^H)(1+\xi^L)\right] b \geq \\ & - \left[-\varphi^H(1+\xi^L)(\varphi^H\xi^H - \gamma(1-\delta)) - \varphi^H\xi^L(1+\xi^H)(1-\varphi^H) + \varphi^H(1+\xi^H)(1+\xi^L)\right] f \\ & + \left[\omega(1+\xi^H)(1+\xi^L) - \omega\left(\varphi^H\xi^H(1+\xi^L) - \gamma(1-\delta)(1+\xi^L)\right) - \varphi^H\xi^L(1+\xi^H)(1-\omega)\right] W \\ & - \left[\varphi^H\xi^H(1+\xi^L) - \gamma(1-\delta)(1+\xi^L) - \varphi^H\xi^L(1+\xi^H)\right](\omega - \varphi^H)u \\ & - \omega(1+\xi^H)(1+\xi^L) \end{aligned} \tag{7.22}$$

It follows

$$b \geq x_{wb} W - f - x_{ub} u - x_{cb} \tag{7.23}$$

where

$$x_{wb} \equiv \tag{7.24}$$

which is equation (42) in the text. Using (7.11), the "no-arbitrage condition" (7.26) can be written as:

$$E_t \{R_t\} = \frac{1}{(1+\xi)} E_t \left\{ \frac{\xi W + f_{t+1} + b_{t+1}}{f_t + b_t + u_t} \right\} \tag{7.25}$$

which is equation (41) in the text.

7.3.1 No-Arbitrage Condition

The bubbly asset has to be attractive. In equilibrium the return of investing in the one-period bond (i.e. saving) and the return of investing in homes has to be equal. This condition is given by the optimality condition (3.13), which is the same for both household types:

$$\begin{aligned} q_t \equiv f_t + b_t + u_t = E_t \left\{ \frac{1}{R_t} \left(p^r_{t+1} + f_{t+1} + b_{t+1} \right) \right\} \\ \Rightarrow \quad E_t \{R_t\} = E_t \left\{ \frac{p^r_{t+1} + f_{t+1} + b_{t+1}}{f_t + b_t + u_t} \right\} \end{aligned} \tag{7.26}$$

In the deterministic case, I can write (7.26) as

$$\begin{aligned} R_t(f_t, b_t, \varphi^H) = & \frac{1}{(1+\xi^H)(1+\xi^L)(f_t + b_t + u_t)} \left[\left((1+\xi^L)\xi^H \omega + (1+\xi^H)\xi^L(1-\omega) \right) (W_{t+1} + u_{t+1}) \right. \\ & - \left((1+\xi^L)\xi^H \varphi^H + (1+\xi^H)\xi^L \varphi^L \right) u_{t+1} \\ & \left. + \left(1 + \xi^H(1 - \varphi^H + \xi^H) + \varphi^H \xi^H \right) (f_{t+1} + b_{t+1}) \right] \end{aligned}$$

8 Appendix B: Methodology

A detailed explanation of how housing cycles are measured is given in section 5.1. Defining and thus measuring a housing bubble proves to be more challenging than identifying house price cycles. Section 5.2 discusses this issue and explains in detail how housing bubbles are measured in this study.

8.1 Identifying House Price Cycles

Housing cycles are identified with a method that falls into the category of "classical" approaches. Cycles are identified in the level of the reference variable.[27] The alternative concept of measuring cycles is that of growth cycles, fluctuations in economic activity around a long-run trend. The so-called classical approach has the following advantages: (1) up- and downturns in the level of housing prices are more relevant for policy makers[28]; (2) detrending involves an arbitrary distinction between trend and cycle, where there is no clear consensus on the best method for this distinction[29]; and (3) turning points are robust to the inclusion of newly available data, in contrast to methods that require detrending (where the inclusion of new data can affect the estimated trend and hence the identification of a cycle).

Harding and Pagan (2002)'s BBQ algorithm is used to detect turning points in quarterly house price data.[30] This algorithm belongs to the strand of pattern-recognition methods pioneered by Burns and Mitchell (1946) in their work on business cycles for the National Bureau of Economic Research (NBER), and later formalized by Bry and Boschan (1971).[31] The dating procedure consists in finding a series of local maxima and minima that allow a segmentation of the series into expansions and contractions.

The algorithm requires the implementation of the following three steps on a quarterly series:

1. Identification rule. Identification of points which are higher or lower than a window of surrounding observations. Using a window of j quarters on each

[27] The described dating procedure was employed among others by Bracke (2013), Igan and Loungani (2012), Claessens *et al.* (2011), Andre (2010), Girouard *et al.* (2006) and Borio and McGuire (2004).

[28] The change in the level of house prices induces e.g. wealth effects and hence the impact on real economic outcomes.

[29] See King *et al.* (1991) among others. The identification of cycles does clearly depend on the detrending method chosen. Moreover, key growth cycle characteristics depend on the detrending method employed, see Canova (1998).

[30] The algorithm is denominated BBQ because it is a quarterly (Q) application of the Bry and Boschan (1971) algorithm designed to find business cycles in monthly data.

[31] To date housing cycles, this dating procedure was employed among others by Igan and Loungani (2012), Claessens *et al.* (2011), Bracke (2013), Andre (2010), Girouard *et al.* (2006) and Borio and McGuire (2004).

side, a local maximum q_t^{max} is defined as an observation of the house price series such that $(q_{t-j}, ..., q_{t-1}) < q_t^{max} > (q_{t+1}, ..., q_{t+j})$. Symmetrically, a local minimum q_t^{min} satisfies $(q_{t-j}, ..., q_{t-1}) > q_t^{min} < (q_{t+1}, ..., q_{t+j})$.

2. Alternation rule. A local maximum must be followed by a local minimum, and vice versa. In the case of two consecutive maxima (minima), the highest (lowest) q_t is chosen.

3. Censoring rule. The distance between two turning points has to be at least x quarters, where x is chosen by the analyst in order to retrieve only the significant series movements and avoid some of the series noise.[32]

As housing cycles are longer than GDP cycles, the threshold parameter for the identification and censoring rule should be set higher. For housing cycles, Borio and McGuire (2004) suggest a rolling window of 13 quarters to be appropriate, which implies $j = 6$. For the censoring rule I follow Girouard *et al.* (2006). The distance between two turning points has to be at least 6 quarters, i.e. $x = 6$.[33] The decisions over the length of the rolling window (j) and the minimum phase duration (q) correspond to the choices made by Bracke (2013).

8.2 Identifying Housing Bubbles

In the literature there is no clean or generally accepted definition for the term asset price bubble. Researchers often focus on a single specific aspect of a generally vague concept: rapid and large price increases[34], unrealistic expectations of future price increases[35], the departure of prices from fundamentals[36], or large drops in prices after the bubble pops[37].

The empirical literature measuring housing bubbles can be decomposed into two main strands. The first, the ”fundamental analysis”, tries to explicitly measure the departure of the housing price from fundamental values, inferred from the residual of an error-correction framework with real house prices regressed on fundamental variables.[38] The number and selection of variables that are seen as fundamental to housing prices is subjective and varies significantly across studies.[39] The selection

[32]Harding and Pagan (2002) choose $x = 2$ for U.S. GDP.

[33]It follows that a housing cycle has a minimum duration of 3 years.

[34]Baker (2002).

[35]Case and Shiller (2003).

[36]Garber (2000) and Lansing (2006).

[37]Siegel (2003), p.3.

[38]Theoretically, researchers would need to quantify the unobserved expected future values of fundamentals on which the fundamental asset price depends.

[39]Examples for fundamental variables included in empirical studies are (1) short run factors like current real GDP per capita, construction costs, the real interest rate, investment demand, (2) long run factors like population and economic growth and (3) institutional factors as supply of land, taxes, financial deregulation (...).

of fundamental variables is crucial when measuring housing bubbles with this approach. This is very problematic and therefore I will not follow this route.[40]

The second strand of literature that identifies housing bubbles empirically uses the "technical analysis", that has a strong descriptive character. This method is intuitive and has the big advantage that fundamental factors do not need to be known. Researchers simply need to know the development of housing prices to identify housing bubbles. According to this method, a necessary feature of a housing bubble is a "dramatic price increase", the literature calls this phenomenon an asset price boom.[41] An obvious criticism follows from the fact that a rapid price increase could also result from a pure change in fundamentals.[42] Given this criticism, researchers extended the concept to boom-bust cycles, i.e. a rapid price increase has to be directly followed by a dramatic bust.[43] However, for the identification of a housing bubble, many researchers do not require booms to be followed by busts. Allowing booms being disconnected from busts is appropriate from a theoretical perspective as well, as bubbles do not need to burst.[44] This study will use both concepts to identify housing bubbles; (1) independent booms and (2) boom-bust cycles.

In summary, the technical analysis can only provide indications for housing bubbles. Nevertheless, advantages of this method, namely (1) it is clearly defined and economically intuitive concept, (2) it has a low requirement of information, and (3) it allows to exactly date housing bubbles - outweigh the disadvantages.

The next section explains how housing booms and busts - as a measure for housing bubbles - are identified.

[40] The selection of fundamental factors will determine the unexplained residual of the regression and hence the bubble size.

[41] Detken and Smets (2004), pp.9. However, it should be noted that from a theoretical perspective, bubbles do not have to involve past rise in prices.

[42] Case and Shiller (2003): "The mere fact of rapid price increases is not itself conclusive evidence of a bubble." Helbling (2005): "However, large price increases - which will be referred to as booms - are only a sufficient but not a necessary condition for bubbles."

[43] Following Garber (2000) the general criticism also applies to the boom-bust cycle, it is still just "an empirical statement about the pattern of prices." This aspect is also highlighted by Haines and Rosen (2007) : "Thus, what appears to be a bubble in some markets might just be a reflection of normally high volatility in those markets".

[44] Despite the debates concerning the measurement of housing bubbles, there is a widespread consensus that many boom-bust cycles in housing prices were accompanied by financial instabilities and recessions.

8.2.1 Measurement: Housing Price Booms and Busts

The identification of housing booms and busts requires two steps. The first step, the determination of housing price cycles, was described in detail in section 3.1.[45] The second step, the identification of housing price booms and busts, involves the choice of a cut-off value for a house price increase (decrease) to be considered as large enough to denote a boom (bust). Such a threshold for the identification of booms and busts is clearly rather arbitrary and varies across studies.[46] This analysis will therefore consider four different cut-off values, leading to four different bubble identification methods. A housing price boom (bust) is defined as a upturn (downturn) that is accompanied with at least a 10%, 15%, or 20% price increase (decrease). In addition, booms (busts) are defined by those episodes involving price increases (decreases) that fall into the top (bottom) quartile of all recorded trough-peak (peak-bottom) price increases (decreases) in the sample.

The stylized facts presented in this study remain robust, when the threshold is altered from e.g. 20% to 15% or 10%, and when booms (busts) are defined as those price increases (decreases) that fall into the top (bottom) quartile of all recorded trough-peak (peak-bottom) price increases (decreases) in the sample.

This two-step procedure does not require booms to be followed by busts, as these two events are determined independently.[47]

Recall that independent booms are the first measure for housing bubbles. The second approach (boom-bust cycles) considers only those booms that are followed by busts.

9 Appendix C: Empricial Work

9.1 Data Sources and short Descriptive Statistics

This section outlines the data sources and provides a short descriptive statistics of the data used in the forthcoming analysis.

[45]The described dating procedure was employed among others by Bracke (2013), Igan and Loungani (2012), Claessens *et al.* (2011), Andre (2010), Girouard *et al.* (2006) and Borio and McGuire (2004).

[46]Girouard *et al.* (2006) identifies booms and busts episodes when a real price change exceeds 15%. Claessens *et al.* (2011), Helbling (2005), IMF (2003) chose the quartile as cutoff value. IMF (2009) choses a methodology similar to Bordo and Jeanne (2002) where turning points are not determined. Busts (booms) are defined as periods when the four-quarter trailing moving average of the annual growth rate of the housing price, in real terms, falls below (above) 5%, equivalent to an accumulated (decrease) increase of 20%.

[47]Bordo and Jeanne (2002) and Helbling (2005) among others also use a procedure whereby booms and busts are determined independently.

House Price Data
The dataset consists of 22 OECD countries and contains real and nominal prices for housing markets and are reported from national statistical sources. It includes: Australia, Belgium, Canada, Switzerland, Germany, Denmark, Spain, Finland, France, the United Kingdom, Greece, Ireland, Israel, Italy, Japan, Korea, Portugal, the Netherlands, Norway, New Zealand, Sweden, and the United States. The series are provided on a quarterly basis, are seasonally adjusted, and the average of the observations in 2010 is indexed to 100. Most of the series contain observations from 1970Q1 to 2013Q4 except for 5 countries that have later starting points.[48]

Due to the much shorter sample sizes I discard Greece, Israel, Korea and Portugal from the analysis. Spain is included, thereby leaving a total of 18 OECD countries.

Data on Homeownership Rates
Aggregate homeownership rates have been collected from different sources. Andrews and Sanchez (2011) provide an average for the 1990s and a point estimate for the year 2004. Aggregate homeownership rates for the year 2009, were collected from the OECD Economic Survey, Luxembourg, 2012, chapter 2. Homeownership rates differ significantly accross countries, an overview is given in table (6). Switzerland has the lowest homeownership rate in the sample with around 35%, while Spain has the highest with around 80%.

Preference for Housing Services measured with two proxies:

- National CPI weights (Housing, Water, Electricity, Gas and other Fuels)
  Per thousand of the National CPI Total. Annual frequency over the time period 1992 to 2013 (if available) for 17 countries, data for Australia is missing. Source: OECD.stat
- Household spending on housing (% of disposable income)
  Point estimate for the years 1995 and 2005 for 18 OECD countries. Source: OECD Outlook No 86 and OECD National Accounts.
  (varies from 14% in Portugal to 30% in Denmark)

[48] These are Spain (1971Q1), Greece (1997Q1), Israel (1994Q1), Portugal (1988Q1) and Korea (1986Q1). I thank Natalie Girouard (OECD) for providing me with the house price data.

	Homeownership Rates						
	1970	1990	2004	2009	2010	average (1990-2009)	Difference (1990-2009)
Australia		71.40	69.5	69.8		70.23	-1.6
Belgium		67.7	71.73	68		69.14	0.3
Canada	60	61.3	68.91	68	69	66.07	6.7
Denmark		51	51.6	49		50.53	-2.0
Finland		65.41	66	59.2		63.53	-6.2
France	45	55.3	54.83	57	58	55.71	1.7
Germany	36	36.29	41	43	45	40.1	6.7
Irland		75	81.41	79.5		78.63	4.5
Italy		64.2	67.90	71.40		67.83	7.2
Japan		n.a.	n.a.	35.79		35.79	n.a.
Netherlands		47.5	55.43	57		53.31	9.5
New Zealand		n.a.	n.a.	67		67	n.a.
Norway		n.a.	n.a.	63		63	n.a.
Spain		77.8	83.2	80.60		80.53	2.8
Sweden		n.a.	n.a.	56		56	n.a.
Switzerland	29	33.1	38.4	34.6	37	35.37	1.5
United Kingdom	50	67.5	70.7	70	64	69.40	2.5
United States	63	66.2	68.7	68	65	67.63	1.8

Source: OECD Economic Surveys: Luxembourg, 2012, Ch. 2, and Andrews and Sanchez (2011). scar Jord *et al.* (2014) provides the data for 1970 and 2010. The last column measures the % point difference in homeownership rates between 1990 and 2009.

Table 3: Aggregate Homeownership Rates in %

	Homeownership Rates				
	1970	1990	2004	2009	2010
1970	1.00				
1990	0.90	1.00			
2004	0.92	0.98	1.00		
2009	0.93	0.95	0.98	1.00	
2010	0.95	0.95	0.97	0.98	1.00

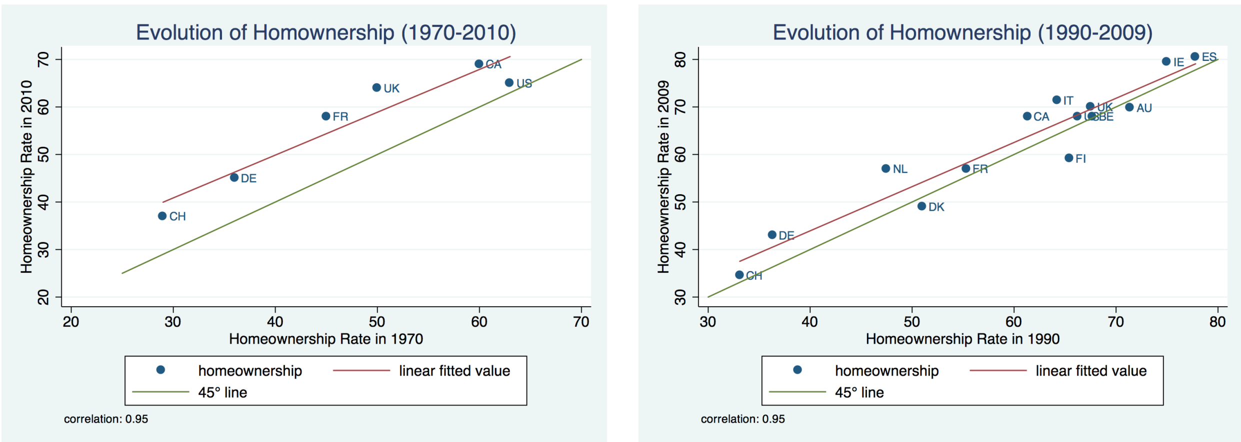
Table 4: Correlations of aggregate Homeownership rates for selected year pairs

Descriptive Statistics of Homeownership Rates

Homeownership rates rose in many OECD countries over time, but important country differences remained.[49] Table (4) shows the correlations of homeownership rates for selected year pairs. The correlations are large and positive.

Figure (3a) plots for a sample of six OECD countries the inital observation of homeownership (year 1970) against the last observation of homeownership available (year 2010). The fitted line is close to be parallel to the 45 degree line. Hence, homeownership rates rose proportionally in these OECD countries.

Figure (3b) plots for 18 OECD countries the inital observation of homeownership (year 1990) against the last observation of homeownership available (year 2009). The fitted line is nearly parallel to the 45 degree line. Hence, homeownership rates rose proportionally in the OECD countries. Cross-country differences in homeownership are very persistent over time.



(a) Evolution of Homeownership for 6 countries

(b) Evolution of Homeownership for 18 countries

Figure 7: Evolution of Homeownership rates

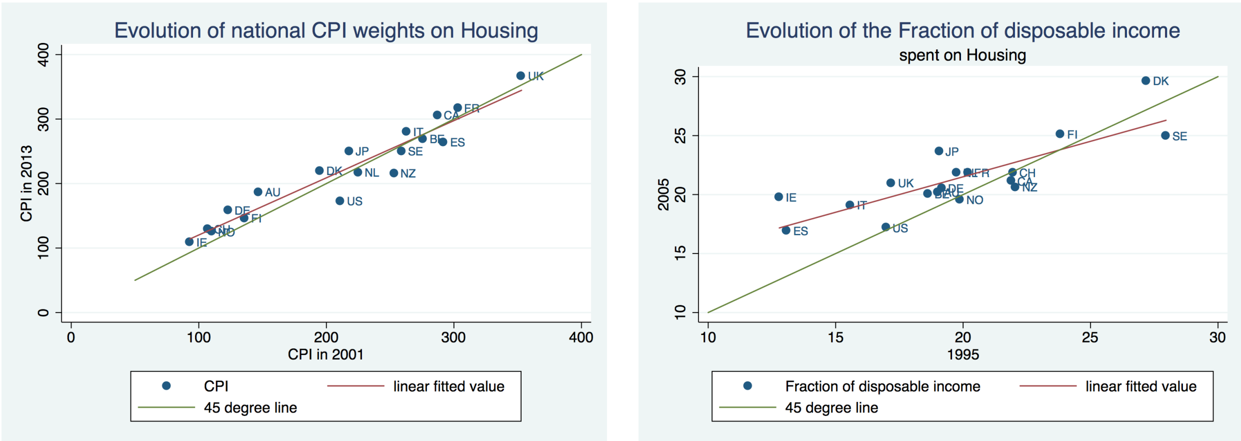
Descriptive Statistics of Preference for Housing Services

Preference of housing services differs significantly across OECD countries.

Figure (4a) plots for a sample of 17 OECD countries the inital observation of CPI weight for housing (year 2001) against the last observation of homeownership available (year 2013). The fitted line is close and parallel to the 45 degree line. Hence, CPI weights remained constant in these OECD countries.

Figure (4b) plots for 18 OECD countries the inital observation of fraction of income spent on housing services (year 1995) against the last observation of fraction of income spent on housing services available (year 2005). Cross-country differences in the preference of housing services are very persistent over time.

[49] On average, homeownership rates rose by 2.53 % points from 1990 to 2009. Please refer for details to table (6). For 1970, homeownership rates are available for six countries. In this sample homeownership rates rose by 9.2% points from 1970 to 2010.



(a) Evolution of CPI weights

(b) Evolution of fraction of disposable income

Figure 8: Evolution of the Preference for Housing Services

9.2 Stylized Fact: Homeonwership rates and Preference for Housing Service

HO	**National CPI weights** (Housing, Water, Elec, Gas & other Fuels)				**Spending on Housing** (% of disposable income)		
	1997	2004	2009	avg	1995	2005	avg
1990	-0.51	-0.53	-0.50	-0.54	-0.55*	-0.39	-0.50
2004	-0.49	-0.52	-0.48	-0.52	-0.63*	-0.47	-0.57*
2009	-0.50	-0.56*	-0.48	-0.56*	-0.54*	-0.56*	-0.58*
avg	-0.49	-0.55*	-0.47	-0.56*	-0.49*	-0.51*	-0.52*

The average of the National CPI weight is an average over the years 1992-2013 (if available). *: correlation coef sign at 5%

Spending on housing (% of disposable income) available for the years 1995 and 2005. Imputed rents included in both measures.

Table 5: Correlations: CPI weights on housing and aggregate Homeownership Rates

9.3 Stylized Fact: Number of Housing Booms (Boom-Bust Cycles) and Preference for Housing Service

	Booms				Boom-Busts	
	(1)	(2)	(3)	(4)	(5)	(6)
	> 80%	> 80%	> 20%	> 20%	> 20%	> 20%
CPI	-0.0636***	-0.0791*	-0.0854**	-0.0924*	-0.0450	-0.0702*
	(-4.30)	(-3.23)	(-3.43)	(-2.44)	(-1.60)	(-2.71)
IMF Mortgage Index		1.684		4.382*		4.099**
		(2.07)		(2.96)		(3.68)
typical LTV		0.0170		-0.0310		-0.0645
		(1.30)		(-1.10)		(-1.89)
max LTV		-0.00690		0.00151		0.0239
		(-0.45)		(0.08)		(0.88)
unemployment rate		-0.0207		-0.0429		-0.0967
		(-0.29)		(-0.42)		(-1.54)
GDP per head (PPP)		-0.0000802		-0.000175		-0.0000927
		(-1.30)		(-1.44)		(-1.26)
constant	2.111***	3.178	4.006***	8.989*	2.465***	6.904*
	(5.98)	(1.59)	(7.33)	(2.86)	(4.21)	(2.55)
N	17	15	17	15	17	15
R^2	0.515	0.691	0.272	0.586	0.137	0.633
adj. R^2	0.483	0.460	0.223	0.275	0.079	0.357

t statistics in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Estimation with robust standard errors. *unemployment rate* measured by rate of unemployment as % of civilian labor force. *GDP per head* measured per head, constant PPPs, OECD base year. *Maximum LTVs* from Heitor *et al.* (2006). *Typical LTVs* from Catte *et al.* (2004) and Calza *et al.* (2013).

Table 6: OLS: Preference for Housing Services and Number of indep. Booms and Boom-Busts

Booms	**CPI**
price rise larger	
$> 80\%$	-0.72
$> 20\%$	-0.52
$> 15\%$	-0.52
$> 10\%$	-0.26

Table 7: Correlation of CPI weights with Frequency of independent Booms

Boom-Busters	**CPI**
price rise larger	
$> 80\%$	-0.30
$> 20\%$	-0.37
$> 15\%$	-0.31
$> 10\%$	-0.06

Table 8: Correlation of CPI weights with Frequency of Boom-Bust Cycles