Guided Search

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Preliminary and incomplete *

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Abstract

This paper studies a truly fundamental problem. A monopolist that offers various differentiated products sets a price and search costs for each product. A consumer searches products, sequentially and in his preferred search order, in order to learn his valuations for the monopolist's products until he purchases a product or leaves the store. In equilibrium the monopolists sets strictly positive search costs in order to guide the consumer's search with the outcome that the consumer searches more expensive products first. The monopolist obtains the second best profits, under which the consumer purchases the most expensive product that supplies weakly positive utility. There always exist equilibria in which the consumer's expected utility is zero such that his expected information rent is equal to his expected search costs. It is discussed how the differentiated products determine the equilibrium search order.

1 Introduction

In many economic situations a consumer is initially uninformed about the various products that a firm offers, but has the opportunity to search these products in order

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learn whether he likes a particular variant. As the consumer, typically, does not search all product available, the consumer's search order affects his demand for the firm's products. Firms' have acknowledged the important role of the consumer's search order and, as a consequence, seek to guide the consumer in their favor. Many examples ....

This paper considers this truly fundamental problem from a theoretical perspective. A monopolist offers several differentiated products, who supply a random utility to the consumer that is ex ante unknown to the consumer and the monopolist. The consumer can search any product available and learn about his valuation for the product, in which case this information is private. The novel aspect is that the search costs that the consumer incurs are a choice variable of the monopolist. Thus, for each product the monopolist sets a price and search costs. The consumer observes the monopolist's choice, and decides which product to search, whether to buy a searched product or whether to leave. Thus, in his strategic considerations the monopolist not only has to take into account how prices affect the profits due to the purchase of a product, but how prices jointly with search costs affect the consumer's search order.

This paper shows that the monopolists sets strictly positive search costs (obfuscates), so that the consumer can not find his preferred product for free. There are two main motives that lead the monopolist to obfuscate. First, obfuscation allows the monopolists to guide the consumer's search such that the consumer prefers to search more expensive products first. Second, the monopolist can deteriorate the consumer's benefits due to continued search, which induces the consumer to purchase the expensive variants that he searched earlier. As a result, the firm obfuscates such that in equilibrium the consumers finds it optimal to search products in a decreasing price order, and the monopolist is able to achieve second best profits, under which the consumer purchases the most expensive product that supplies weakly positive utility. The equilibrium outcome is disastrous for the consumer. There always exists an equilibrium in which the consumer's expected information rent is equal to his expected total search costs, so that his expected utility is zero.

I discuss which products will be searched first. I identify a relation among the distribution functions $F_k(\varepsilon)$ from which the consumer's valuations for product

$k \in K$ is drawn that allows me to characterize the consumer's search order. If the distribution function are ordered by means of rotations [Johnson and Myatt (2006)], I obtain a partial characterization. For a full characterization stronger conditions are needed and provided – in particular, log-supermodularity of the distribution functions $F_k(\varepsilon)$ and log-submodularity of the corresponding reliability functions $1 - F_k(\varepsilon)$.

The paper contributes to large literature that explores techniques (mechanism) how a monopolist can extract parts of the private information rent that the consumer obtains, as his valuations for the firm's products are private. Mc Afee (1983), ...

Directed search, Zhou (2012), Armstrong, Vickers, Zhou (2009).

Very few other studies have considered search problems with endogenous search costs before. Ellison and Wolitzky (2012) argue that in a competitive market, with non-directed consumer search, firms have incentives to obfuscate in order to fatigue consumers. Formally, they assume that search costs are convex, instead of linear, such that sunk search costs affect the costs of continued search and discourage consumers from continuing their search. Gamp (2015) shows ... . Most related, with regard to this aspect, is Wilson (2010), who shows in a directed search model that in a duopoly firms differentiate in search costs in order to relax the competitive pressure and to split the market.

2 Model

The market consists of a representative consumer with unit demand and a multi-product, profit-maximizing monopolist. The monopolist offers K-many differentiated products. Her marginal costs of production are normalized to zero for each product. If the consumer buys product $k \in \{1, \ldots, K\}$ his quasi-linear utility absent any search costs is

$$u(\varepsilon_k, p_k) = \varepsilon_k - p_k, \tag{1}$$

where $p_k \in \mathbb{R}_0^+$ is the price of product k, and the match-value ε_k is the consumer's valuation for the monopolist's k-th product. Let ε_k be an independent draw from

the cumulative density function F_k. The support of F_k is $Supp(F_k) = (\underline{\varepsilon}_k, \overline{\varepsilon}_k)$; assume furthermore that the supports of any two distributions overlap such that any product, absent prices, may be preferred to any other. Finally, let $F_k \in C^2$ be twice continuously differentiable.

Ex ante the consumer only knows the products' prices, but does not know the realizations of match-values. However, he can sequentially search products in order to learn these, in which case this is his private information. Each such search is time-consuming and hence costly. More precisely, let c_k be the product dependent search costs that the consumer incurs in order to learn ε_k. I interpret search costs $c_k > 0$ as a measure of the complexity of information acquisition. The key departure of my model is that $c_k \in \mathbb{R}_0^+$ is a choice variable of the monopolist such that the monopolist can aggravate or simplify the complexity of information acquisition with his obfuscation strategy.

The timing of the model is as follows. First, the monopolist chooses a price and search costs for each product – a sequence $[p_k, c_k] \in \{\mathbb{R}_0^+ \times \mathbb{R}_0^+\}^K$, which the consumer observes. Then, the consumer searches until he leaves or purchases a product. The consumer's search proceeds as follows. The consumer can (*i*) search any product k: learn ε_k for c_k, (ii) purchase any previously searched product k, (*iii*) leave: leave the store, and obtain a utility of zero absent search costs. Thus, the consumer's search is directed and his search order is consequently not random. Assume that the consumer has perfect and costless recall. The equilibrium concept that I apply is Sequential Equilibrium in pure strategies.

3 Analysis

Henceforth, whenever I refer to some sequence I indicate so by the use of squared brackets – i.e. p_k is the k-th element of $[p_k]$.

3.1 The consumer's search rule

The consumer's behavior follows Weitzman's famous rule. Therefor, I define the reservation utility, as is standard.

Definition 1 (Reservation utility) *For every* (p_k, c_k) *define the reservation utility* $U_k^{\text{res}}(p_k, c_k)$ *implicitly by*

$$c_k \stackrel{!}{=} \int_{U_k^{\text{res}}(p_k,c_k)+p_k}^{\bar{\varepsilon}_k} \Big\{\Big(\varepsilon_k - p_k\Big) - U_k^{\text{res}}(p_k, c_k)\Big\} dF_k(\varepsilon_k). \tag{2}$$

This means that $U_k^{\text{res}}(p_k, c_k)$ is the highest utility of the consumer's outside option such that the consumer would weakly prefer to search product k if he could at most search one product. Thus, $U_k^{\text{res}}(p_k, c_k)$ equates the benefits of evaluating product k, the RHS of equation (2), with its costs, the LHS of equation (2). Weitzman's rule states that the consumer searches those products with the highest reservation values first, and ends his search once the utility that his most preferred product supplies exceeds the highest reservation value of the remaining products. Denote the set of products that the consumer has searched by $E \subseteq K$; denote the remaining products by $R = K \backslash E$. Denote the utility of the consumer's hitherto preferred product by $U_E^{\text{best}}\Big([p_k, \varepsilon_k]\Big) = max_{k \in E}\Big\{\varepsilon_k - p_k, 0\Big\}$; denote the highest reservation utility among the remaining products by $U_R^{\text{res}}\Big([p_k, c_k]\Big) = max_{k \in R}\Big\{U_k^{\text{res}}(p_k, c_k), -1\Big\}$.[1]

Lemma 1 (Weitzman (1979)) *The optimal consumer search rule is:*

i) If $U_E^{\text{best}}\Big([p_k, \varepsilon_k]\Big) < U_R^{\text{res}}\Big([p_k, c_k]\Big)$*, then search a product in* $argmax_{k \in R}\Big\{U_k^{\text{res}}[p_k, c_k]\Big\}$.

ii) If $U_E^{\text{best}}\Big([p_k, \varepsilon_k]\Big) = U_R^{\text{res}}\Big([p_k, c_k]\Big)$*, then purchase/search a product in* $argmax_{k \in R}\Big\{U_k^{\text{res}}[p_k, c_k]\Big\} \cup argmax_{k \in I}\Big\{\varepsilon_k - p_k\Big\}$.

iii) If $U_E^{\text{best}}\Big([p_k, \varepsilon_k]\Big) > U_R^{\text{res}}\Big([p_k, c_k]\Big)$*, then purchase a product in* $argmax_{k \in I}\Big\{\varepsilon_k - p_k\Big\}$ *if* $max_{k \in I}\Big\{\varepsilon_k - p_k\Big\} \geq 0$*, and leave otherwise.*

The consumer's behavior is unique up to how indifferences are resolved. In the following, I assume that in case of indifference, whenever the set products among the consumer chooses is not a singleton, the consumer chooses the most expensive product with the lowest k. Thus, I resolve all indifferences in favor of the monopolist in order to obtain a unique consumer behavior. Moreover, this ensures that the

[1] The consumer's option to leave is treated as a produt that supplies zero utility. Consequently, the utility of the consumer's so far preferred product is bounded from below by zero. The reservation utility of the consumer's preferred product to search is bounded from below by -1 only so that $U_R^{\text{res}}\Big([p_k, c_k]\Big)$ is well-defined.

monopolist's profits $\pi([p_k, c_k])$ are upper semi-continuous in $[p_k, c_k]$, and guarantees the existence of an equilibrium, which otherwise might fail to exist.

The consumer's search rule induces a search order, in which he searches products, where products are ranked by their reservation values. I denote this search order with ϕ: if the consumer searches k-many products, then k-th product that the consumer searches is the product $\phi(k)$.[2]

3.2 The monopolist's pricing and obfuscation strategy

As the consumer's valuation for products are his private information, he obtains an information rent and the monopolist can't obtain the full surplus due to trade. I define the second best profits as follows.

Definition 2 (second best profits) *For any prices $[p_k] \in \{\mathbb{R}_0^+\}^K$ define the second best profits $\overline{\pi}\big([p_k]\big)$ as those profits that the monopolists obtains if the consumer purchases a most expensive product that supplies weakly positive utility.*

A formal definition of $\bar{\pi}$ is delegated to the appendix. Intuitively, the best scenario for the monopolist is when the consumer purchases the most expensive product that he weakly prefers to his outside option. The second best profits $\overline{\pi}\big([p_k]\big)$ constitute hence an upper bound on the monopolist's profits generated by the prices $[p_k]$. Now, the astonishing result that I establish in theorem 1 is that the monopolist can guide the consumer's search and deteriorate the consumer's utility of continued search in a manner that for any prices he can achieve the second best profits $\overline{\pi}\big([p_k]\big)$.

Theorem 1 *For any prices $[p_k]$ there exists an obfuscation strategy such that the monopolist's obtains the second best profits $\overline{\pi}\left([p_k]\right)$.*

Proof. Consider the obfuscation strategy yolo defined as follows.

Definition 3 (Yolo) *For every k and $p_k \leq \overline{\varepsilon}_k$ define yolo $\tilde{c}_k(p_k)$ by*

$$\tilde{c}_k(p_k) := \int_{p_k}^{\overline{\varepsilon}^k} \Big(\varepsilon_k - p_k\Big) dF_k(\varepsilon_k). \tag{3}$$

[2]Formally, let Φ denote the set of permutations of K. Thus, Φ is set of bijections that map from K to K. Then, a search order is an element of Φ.

Equivalently, $\tilde{c}_k(p_k)$ solves $U_k^{\text{res}}\big(p_k, \tilde{c}_k(p_k)\big) \overset{!}{=} 0$. Thus, the yolo strategy sets for a given price the reservation utility of a product to zero. Now, consider an arbitrary pricing strategy $[p_k]$, and suppose the monopolist uses yolo for each product k: $c_k = \tilde{c}_k(p_k)$ for every k. Then, the reservation utility for each product is zero: $U_k^{\text{res}}(p_k, c_k) = 0$ for all k, and the consumer is indifferent about in which order to search products or whether to leave. In fact, if the monopolist could resolve indifferences as he wanted to, then the monopolist could implement any search order for any price sequence! However, I assumed that indifferences are resolved in favor of the monopolist such that in the case of indifference the consumer searches more expensive products firsts. Thus, the consumer searches products in a decreasing order with respect prices. Then, if the consumer finds a product that supplies a utility that weakly exceeds zero, he ends his search and purchases the product, as it supplies a utility that exceeds the highest reservation utility of the remaining products, since $U_R^{\text{res}}\big([p_k, c_k]\big) = 0$ if R is non-empty, and $U_R^{\text{res}}\big([p_k, c_k]\big) = -1$ otherwise. Therefore, the consumer purchases the most expensive product that supplies positive utility! Thus, the monopolist's profits are $\overline{\pi}\big([p_k]\big)$. ■

Now, the approach taken here to derive equilibrium becomes evident. I identified an upper bound on the monopolist's profits for any prices, and theorem 1 then tells me that there exists an obfuscation strategy that generates these profits. Consequently, a monopolist's strategy is an equilibrium strategy if and only if it generates the profits $\tilde{\pi}$ that are generated by those prices $[p_k]$ that maximize $\overline{\pi}\big([p_k]\big)$.

Corollary 1 *The set of the monopolist's equilibrium strategies Σ is non-empty. A strategy $[p_k, c_k] \in \Sigma$ if and only if $\pi\big([p_k, c_k]\big) = \tilde{\pi}$ such that the monopolist obtains the second best profits and the consumer purchases a most expensive product that supplies weakly positive utility.*

One equilibrium strategy of the monopolist is to choose prices that maximize $\overline{\pi}\big([p_k]\big)$ and the yolo strategy for each product. This equilibrium has further noteworthy characteristica. Not only purchases the consumer the most expensive products, but he searches products in a decreasing price order, never returns to a previously searched product and obtains a utility of zero. The latter holds, since $U_k^{\text{res}} = 0$ for all k such that the consumer is indifferent about whether to enter the market or not.

The next proposition shows to what extent these properties hold in any equilibrium.

Proposition 1 *Let $[p_k^*, c_k^*] \in \Sigma$ and let ϕ^* be the induced consumer's search order. Then, the following holds:*

i) Each product is bought with strictly positive probability.

ii) The consumer searches more expensive products first: $[p_{\phi^*(k)}^*]$ is a weakly decreasing sequence.

iii) For any product that is cheaper than the most expensive product, the consumer is indifferent between evaluating the product and leaving the store: if $p_k^* < p_l^*$ for some l, then $U^{res}(p_k^*, c_k^*) = 0$.

iv) The consumer never returns in order to purchase a more expensive, previously searched product: If the consumer has searched $\phi^*(k)$, then he does not purchase $\phi^*(l)$ for $l < k$ if $p_{\phi^*(k)}^* < p_{\phi^*(l)}^*$.

v) For any prices that are part of an equilibrium strategy there exists an equilibrium with the same prices such that the consumer obtains zero utility; if $[p_k^*, c_k^*] \in \Sigma$, then $[p_k^*, \tilde{c}_k(p_k)] \in \Sigma$ such that $U_k^{res} = 0$ for all k.

The meaning of proposition 1 is self-explaining. What remains to be answered is whether the equilibrium is unique. Proposition 1 implies that the market equilibrium is unique up to the search costs for the most expensive product if $argmax_{[p_k] \in \{\mathbb{R}_0^+\}^K} \overline{\pi}\left([p_k]\right)$ is a singleton and the most expensive product is strictly more expensive than all other products. More precisely, in that case the search costs for each product, but the most expensive product, are chosen such that each reservation utility is zero, while the search costs for the most expensive product are arbitrary to the extent that they only have to satisfy that the reservation utility is weakly positive. This suffices to ensure that the most expensive product is searched first. Note, however, that the search costs for the most expensive product pin down the consumer's expected utility. Only, if the consumer uses yolo for all products, then the consumer's expected utility is zero.

The remainder of the paper is devoted to determine $argmax_{[p_k] \in \{\mathbb{R}_0^+\}^K} \overline{\pi}\left([p_k]\right)$. Less we are interested in a 'formular' to determine the optimal prices, which nev-

ertheless will be given in proposition 2, than in the induced search order of products. In particular, we would like to know which products are offered first to the consumer by the monopolist – i.e. whether the monopolist offers products with higher variance in the consumer's valuation first. However, recall that the question is equivalent to which products are more expensive, since the consumer searches products in a decreasing price sequence.

3.3 The monopolist's pricing strategy

In this paragraph, I derive a recursive formula for the optimal pricing strategy of the monopolist given the consumer's search order ϕ^*.

Proposition 2 *Let $[p_k^*, c_k^*] \in \Sigma$ be a monopolist's equilibrium strategy that induces the search order ϕ^*. Then, the price sequence $[p_k^*]$ is given by the recursive formula that is defined by the equations (4), (5) and (6).*

$$p^*_{\phi^*(k)} \in argmax_{p \in \mathbb{R}_0^+}\ p\tilde{F}_{\phi^*(k)}(p) + F_{\phi^*(k)}(p)\pi_k^* \tag{4}$$

$$\pi_K^* = 0 \tag{5}$$

$$\pi_{k-1}^* = p^*_{\phi^*(k)}\tilde{F}_{\phi^*(k)}(p^*_{\phi^*(k)}) + F_{\phi^*(k)}(p^*_{\phi^*(k)})\pi_k^* \tag{6}$$

The monopolist's profits are π_0^*.

The profits π_k^* denote the firm's continuation profits if the consumer does not purchase any of the first k-many searched products. If the consumer purchases no product, then the firm's profits are zero: $\pi_K^* = 0$ as stated in equation (5). Since the consumer never returns, the price of the k-th searched product does not affect the monopolist's continuation profits π_k^*, and does not affect the firm's demand for the first $k-1$-many searched products. Therefore, the monopolist chooses the price of the k-th searched product so that it maximizes the continuation profits π_{k-1}^* – which is context of equation (4). Finally, equation (6) is the missing recursive definition of continuation profits that completes the 'formula'.

Proposition 2 allows us to determine the optimal pricing strategy, once we have determined the optimal permuation ϕ^*. In the following I identify a relation among the distribution functions that allows me to determine this order.

3.4 The equilibrium search order

Definition 4 states a property of the family of distribution functions $[F_k]$ that allows for a partial characterization of the equilibrium search order. This notion has already appeared before in Johnson and Myatt (2006) to describe demand rotations.[3]

Definition 4 (Rotation, Johnson and Myatt (2006)) *A family of cumulative density functions $[F_k]$ is ordered by a sequence of rotations if there exist a rotation point $(\varepsilon_{\text{rot}}, F_{\text{rot}})$ such that for every $k < l$*

$$\varepsilon \gtreqless \varepsilon_{\text{rot}} \Leftrightarrow F_k(\varepsilon) \lesseqgtr F_l(\varepsilon)$$

The definition menas that, loosely speaking, the distribution F_l can be 'generated' by a counter clockwise rotation of F_k around the point $(\varepsilon_{\text{rot}}, F_{\text{rot}})$, which is illustrated in figure 1. In that case I say that F_l is a (counter clockwise) rotation of F_k. Therefore, for $\varepsilon \gtreqless \varepsilon_{\text{rot}}$ the family $[F_k]$ is completely ordered by means of first order stochastic dominance. The definition encompasses first order stochastic dominance, as the rotation point per se has not to be in the support of the distribution functions. The value of the definition lies in the fact that it is sufficient to partially characterize the equilibrium search order. For a full characterization I provide sufficient conditions in the following proposition.

Proposition 3 *Let $[F_k]$ be ordered by a sequence of rotations. Let $[p_k^*, c_k^*] \in \Sigma$ and let ϕ^* be the induced search order. Let $k^\dagger = max\left\{0, k \in K | p^*_{\phi^*(k)} \geq \varepsilon_{\text{rot}}\right\}$. Then, the equilibrium search order ϕ^* satisfies the following:*

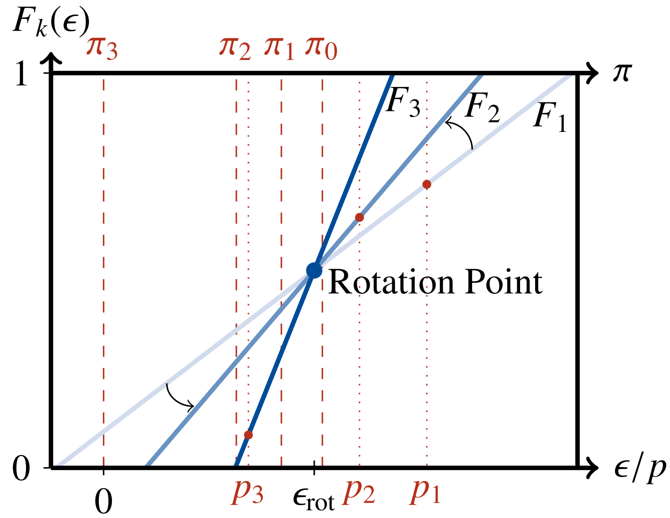
i) The consumer searches a set of more diverse products first: $\phi^*(k) \in \{1, \ldots, k^\dagger\}$ for $k \leq k^\dagger$.

ii) If $F_k(\varepsilon)$ is log-supermodular for $\varepsilon \geq \varepsilon_{\text{rot}}$, then for $k \leq k^\dagger$ the order induced by rotations and the equilibrium search order coincide: $\phi^*(k) = k$.

iii) If $\tilde{F}_k(\varepsilon)$ is log-submodular for $\varepsilon \leq \varepsilon_{\text{rot}}$, then for $k > k^\dagger$ the order induced by rotations and the equilibrium search order coincide: $\phi^*(k) = k$.

[3] For a discussion, and the relation to related properties as mean preserving spreads and second order stochastic dominance the interested reader is referred to Johnson and Myatt (2006).

Figure 1: Uniform distributions
The figure shows in different shades of blue the distributions functions for three products, where the consumer's valuations are uniformly distributed, $F_k(\varepsilon) = \frac{1}{2} + k(\varepsilon - \varepsilon_{\text{rot}})$. $[F_k]$ is ordered by a sequence of rotations around $(\varepsilon_{\text{rot}}, \frac{1}{2})$, which is indicated by arrows. Equilibrium prices and continuation profits are shown on the abscissas. The search order of products coincides with their enumeration.



Proof. Here, I proof proposition 3 *i*), which moreover will be helpful to provide intuition for the proof of the remainder of proposition 3, that is delegated to the appendix.

i): Consider the equilibrium prices $[p_k^*]$, which generate the profits $\overline{\pi}([p_k^*])$. Then, there may no exist prices $[p_k]$ for which $\overline{\pi}([p_k]) > \overline{\pi}([p_k^*])$. In particular, it may not be profitable to switch the prices of two distinct products k, l. Such a switch is definitely profitable if the probability that the consumer's valuation exceeds p_k^* is higher for product l than for product k, and vice versa. Thus, if $\tilde{F}_k(p_l^*) \geq \tilde{F}_l(p_l^*)$ and $\tilde{F}_l(p_k^*) \geq \tilde{F}_k(p_k^*)$, where at least one inequality must be strict.

Now, if some product $k \leq k^\dagger$ does not belong to the $k^\dagger$-many products that are searched first: $\phi^{-1}(k) > k^\dagger$, then there exists as well a product $l > k^\dagger$ that satisfies $\phi^{-1}(l) \leq k^\dagger$. By the definition of $k^\dagger$, $p_l^* \geq \varepsilon_{\text{rot}}$ and $p_k^* < \varepsilon_{\text{rot}}$. Furthermore, by assumption, F_l is a rotation of F_k, so that F_k first order dominates F_l for $\varepsilon > \varepsilon_{\text{rot}}$ and vice versa. This implies $\tilde{F}_k(p_l^*) \geq \tilde{F}_l(p_l^*)$ and $\tilde{F}_l(p_k^*) > \tilde{F}_k(p_k^*)$. ■

If F_l is a rotation of F_k, then product k is superior to product l for prices that exceed ε_{rot}, and the other way around for prices below ε_{rot}. As a consequence, the price of product l only exceeds ε_{rot} if the price of product k exceeds ε_{rot}, since otherwise it would be profitable to switch prices for the two products. This is the idea of the

proof presented above and an alternative interpretation of proposition 3 *i*).

If the price of product k and product l exceed ε_{rot}, this argument does not prevail anymore, as product k is superior for both prices. For a full characterization of the equilibrium search stronger conditions are hence needed. Intuitively, whether a switch of prices is profitable depends in a complex way on the differences $\tilde{F}_k(p_l^*) - \tilde{F}_l(p_l^*)$ and $\tilde{F}_k(p_k^*) - \tilde{F}_l(p_k^*)$. In the proof of proposition 3 *ii*), I show that product k, recall $k < l$, will be searched first if $F_k(\varepsilon)$ is log-supermodular with respect to ε and k for $\varepsilon > \varepsilon^{\text{rot}}$, as otherwise a switch of prices is profitable. Loosely speaking, log-supermodularity $F_k(\varepsilon)$ can be interpreted in this context as a sufficient condition that implies that $\tilde{F}_k(\varepsilon) - \tilde{F}_l(\varepsilon)$ is sufficiently increasing with respect to ε for $\varepsilon > \varepsilon^{rot}$. More precisely, supermodularity states that the relative increase of F_k that is given by $f_k(\varepsilon)/F_k(\varepsilon)$ is increasing in k.[4]

Similarily, if F_l is a rotation of F_k and the price of product k and product l are below ε_{rot}, then the product l is superior to product k. Then, log-submodularity of the reliability function $\tilde{F}_k(\varepsilon)$ can be interpreted as a sufficient condition that implies that $\tilde{F}_l(\varepsilon) - \tilde{F}_k(\varepsilon)$ is sufficiently decreasing with respect to ε for $\varepsilon < \varepsilon^{\text{rot}}$ such that product k is searched first [proposition 3 *iii*)]. More precisely, log-submodularity of the reliability function means that the hazard rate $f_k(\varepsilon)/\tilde{F}_k(\varepsilon)$ is increasing with respect k.

I end with an example that illustrates the usefulness of proposition 3.

Example cont': A family of uniform distribution

Consider, again, the family of uniform distribution parametrized by k that is given by $F_k(\varepsilon) = \frac{1}{2} + k(\varepsilon - \varepsilon_{\text{rot}})$. *The special case for three products is illustrated in figure 1. This family is ordered by a sequence of rotation around the rotation point* $(\varepsilon_{\text{rot}}, \frac{1}{2})$. *Furthermore, simple algebra yields* $\frac{d^2}{dk\,d\varepsilon} log F_k(\varepsilon) = \frac{1}{2(F_k(\varepsilon))^2} > 0$, *which implies that* $F_k(\varepsilon)$ *is log-supermodular. Analagously,* $\frac{d^2}{dk\,d\varepsilon} log \tilde{F}_\mu(\varepsilon) = \frac{-1}{2(\tilde{F}_\mu(\varepsilon))^2} < 0$ *implies that* $\tilde{F}_k(\varepsilon)$ *is log-submodular. Consequently, proposition 3 applies and the order induced by rotations and the equilibrium search order coincide:* $\phi^*(k) = k$. *Here, this can be interpreted as that those products with higher variance are searched first.*

[4]Log-supermodularity is a well-known property – i.e. from the search and matching literature, where it is identified as a sufficient condition for positive assortative matching [Smith (2006)].

4 Conclusion

This study has shown that a monopolist obfuscates in order to guide the consumer's search and discourage to intense consumer search. Obfuscation allows the monopolist to achieve second best profits, while for the consumer there always exists an equilibrium in which the consumer's information rent equals his expected costs of search such that his utility is zero.

I conclude with a discussion of the robustness of the presented results.

Robustness of the results

i) If the consumer had imperfect recall, so that he could not return to any previously searched product, all results remain valid. This is the case as the consumer has no incentive to return to previously searched products.

ii) The consumer's outside option is assumed to supply a utility of zero.[5] If the consumers outside option supplies a non-zero utilitiy, similar corresponding results are obtained. In particular, the monopolist could set the reservation utility of all products equal to the utility that the outside option supplies.

iii) If prices or search costs are unobservable to the consumer, each equilibrium would remain to be an equilibrium of the corresponding new game, since the monopolist has no incentives to deviate from his pricing strategy or obfuscation strategy. There would be, however, additional equilibria as the monopolist would not be able to affect the consumer's search order, as any deviation would be unobserved ex ante. But, as the consumer only purchases products that supply weakly positive utility, none of these equilibria generate profits that exceed $\tilde{\pi}$ and would be preferred by the monopolist.

[5] Note that this is not the outcome of a normalisation, since the single degree of freedom available is used to set production costs to zero.

5 Appendix

Proof of lemma 1: A proof can be found in Weitzman (1979). ■

Definition 2: Let $\phi^{\mathrm{D}} \in \Phi$ be a permutation such that $\left[p_{\phi^{\mathrm{D}}(l)}\right]$ is a weakly decreasing price sequence. Then, the second best profits are define by

$$\overline{\pi}\Big([p_k]\Big) := \sum_{k \in K} \left\{ \prod_{l<k} F_{\phi^{\mathrm{D}}(l)}\Big(p_{\phi^{\mathrm{D}}(l)}\Big) \right\} \tilde{F}_{\phi^{\mathrm{D}}(k)}\Big(p_{\phi^{\mathrm{D}}(k)}\Big) p_{\phi^{\mathrm{D}}(k)}, \tag{7}$$

where $\tilde{F}_k := 1 - F_k$ denotes the reliability function of F_k. Equation (7) is the sum over the profits generated by each product, where the term in brackets ¨$\{\ldots\}$¨ denotes the probability that all more expensive products generate negative utility and the term on the RHS of the brackets denotes the probability that the considered product generates positive utility multiplied with its price.[6] ■

Proof of corollary 1: A necessary condition for the purchase of any product is that it generates positive utility to the consumer. Therefore, the second best, that the monopolist can achieve, is that the consumer purchases the most expensive product that generates weakly positive utility. Then, the claim is obvious, except that it remains to be shown that $\tilde{\pi}$ is well-defined. Let (0_k) be a sequence of zeros. Let $\overline{\varepsilon} := max_{k \in K}\{\overline{\varepsilon}^k\}$. Then, $P^{\overline{\varepsilon}} := \{[p_k] \in \{\mathbb{R}_0^+\}^K | p_k \leq \overline{\varepsilon}$ for every $k\}$ is a compact set. Furthermore, $\overline{\pi}([p_k]) = 0$ for every $[p_k] \in \{\mathbb{R}_0^+\}^K \backslash P^{\overline{\varepsilon}}$. Then, $\tilde{\pi}$ is well-defined, as a maximum exists by the Bolzano-Weierstrass theorem. ■

Proof of proposition 1: Let $[p_k^*, c_k^*] \in \Sigma$. Recall that, by corollary 1, a most expensive product that yields positive utility is bought in equilibrium.

i): Assume the contrary – i.e. that for some $[p_k^*, c_k^*]$ product l is not bought with strictly positive probability. Set $p_k = p_k^*$ for $k \neq l$. Set $p_l = \Big(min_{\{k \in K\}}(\overline{\varepsilon}_k) + max_{\{k \in K\}}(\underline{\varepsilon}_k)\Big)/2$. Recall that by assumption $Supp(F_m) \cap Supp(F_n)$ has positive measure for any m, n, which implies $min_{\{k \in K\}}\overline{\varepsilon}_k < max_{\{k \in K\}}\underline{\varepsilon}_k$. Recall that it suffices to show that l is the strictly most expensive product that yields positive utility for

[6]Let me add two technical remarks for the careful reader. First, $\overline{\pi}([p_k])$ is well-defined, since it is irrelevant which permutation ϕ^{D} that generates a weakly decreasing price sequence is considered.

$[p_k]$ with positive probability, as this implies $\hat{\pi}([p_k]) > \tilde{\pi}$. First, $\tilde{F}_l(p_l) > 0$ which means that product l yields positive utility with strictly positive probability. Second, if $p_k \geq p_l$, then $\tilde{F}_k(p_k) < 1$ by the definition of p_l. Thus, all more expensive products yield strictly negative utility with positive probability. $\square$.

ii): Assume the contrary – i.e. that there exist k and l that satisfy $p^*_k > p^*_l$ such that the consumer searches product l first. Hence, $U^{\text{res}}_k(p^*_k, c^*_k) < U^{\text{res}}_k(p^*_l, c^*_l)$ holds. Suppose that product k is the most expensive product that supplies positive utility. It holds that $\overline{\varepsilon}_l - p^*_l \geq U^{\text{res}}_k(p^*_l, c^*_l)$ by the definition of the reservation utility in equation (2). Consequently, $\varepsilon_l - p^*_l > U^{\text{res}}_k(p^*_k, c^*_k)$ with strictly positive probability, in which case consumer ends his search and purchases product l – a contradiction. $\square$

iii): Assume the contrary – i.e. that there exists $p^*_k < p^*_{k'}$ such that $c^*_k \neq \tilde{c}^k(p^k)$. Since product k is bought with strictly positive utility, $U^{\text{res}}_k(p^*_k, c^*_k) > 0$ must hold, as otherwise the consumer would never search the product, where the strictness of the inequality follows from $c^*_k \neq \tilde{c}^k(p^k)$.Now, with strictly positive probability k is the second most expensive product that supplies positive utility, product k' is the most expensive product that supplies positive utility, and product k supplies a greater utility. Then, there are two cases to consider. First, if product k is searched before k', then the consumer purchases product k – a contradiction. Second, product k' is searched first. Then, with strictly positive probability the utility that k' supplies is strictly lower than $U^{\text{res}}_k(p^*_k, c^*_k)$, in which case the consumer searches k and purchases k – a contradiction. $\square$

iv): A corollary to claim *iii)*, since $p^*_{\phi^*(l)} < p^*_{\phi^*(l')}$ implies $U^{\text{res}}_{\phi^*(l)} = 0$. $\square$

v): The claim follows from $\pi\Big([p^*_k, \tilde{c}_k(p_k)]\Big) = \tilde{\pi}$ so that $[p^*_k, \tilde{c}_k(p_k)] \in \Sigma$, in which case $U^{\text{res}}_k = 0$ for each k. $\blacksquare$

Proof of proposition 2: The monopolist's equilibrium profits are $\pi^* = max_{[p_k]\in\{\mathbb{R}^+_0\}^K}\overline{\pi}\Big([p_k]\Big)$. Since $[p^*_{\phi^*(l)}]$ is a decreasing sequence this implies

$$[p^*_k] \in argmax_{[p_k]\in\{\mathbb{R}^+_0\}^K} \sum_{l\in K} \left\{\prod_{m<l} F_{\phi^*(l)}\Big(p_{\phi^*(l)}\Big)\right\} \tilde{F}_{\phi^*(l)}\Big(p_{\phi^*(l)}\Big)p_{\phi^*(l)}.$$

This means that $p^*_{\phi^*(l)}$ maximizes the firm's profits if all product that are insepcted

before and hence satisfy$(\phi^*)^{-1}(k) < l$ yield negative utility. Denote the monopolist's expected profits if all product that satisfy $(\phi^*)^{-1}(k) \leq l$ yield negative utility by π_l^*. Hence, π_l^* denotes the continuation profits if the first l-many previously searched products yield negative utility. Then, it must hold that

$$p^*_{\phi^*(l)} \in argmax_{p \in \mathbb{R}_0^+} \; p\tilde{F}_{\phi^*(l)}(p) + F_{\phi^*(l)}(p)\pi_l^*.$$

If the maximization problem has an interior solution, then the following first order condition holds:

$$\frac{\tilde{F}_{\phi^*(l)}(p^*_{\phi^*(l)})}{f_{\phi^*(l)}(p^*_{\phi^*(l)})} = p^*_{\phi^*(l)} - \pi_l^*. \tag{8}$$

As is standard, the price markup (here with respect to continuation profits) is equal to the inverse of the hazard rate.

If all products yield negative utility the monopolist's profits are zero. Hence, $\pi_K = 0$. Furthermore, the monopolist's continuation profits are given by the recursive formular

$$\pi^*_{l-1} = p^*_{\phi^*(l)}\tilde{F}_{\phi^*(l)}(p^*_{\phi^*(l)}) + F_{\phi^*(l)}(p^*_{\phi^*(l)})\pi_l^*.$$

Finally, the monopolist's equilibrium profits are the continuation profits if no product has been searched. Hence, $\pi^* = \pi_0^*$. ■

Proof of proposition 3: The proof of claim *i*) is given in the main part of the paper. *ii)*: Proof by contradiction. Assume the contrary – i.e. that there exist $l' \leq r^\dagger$ such that $\phi^*(l') \neq l'$. Then, claim *i*) implies that there exist $l < r^\dagger$ such that $\phi^*(l) > \phi^*(l+1)$. Now, set $p_{\phi^*(l)} = p^*_{\phi^*(l+1)}$, set $p_{\phi^*(l+1)} = p^*_{\phi^*(l)}$ and $p_k = p_k^*$ for each $k \notin \{\phi^*(l), \phi^*(l+1)\}$. Analagously, this means that the resulting decreasing price sequence remains unaffected, however the products $\phi^*(l)$ and $\phi^*(l+1)$ switch their position in the consumer's search order. This is strictly profitable if the monopolist's profits generated by the deviation exceed the equilibrium profits: $\bar{\pi}([p_k^*]) < \bar{\pi}([p_k])$. A sufficient and necessary condition is that the equilibrium continuation profits after

the $l-1$-th searched product

$$\pi^*_{l-1} = \tilde{F}_{\phi^*(l)}(p^*_{\phi^*(l)})p^*_{\phi^*(l)} + F_{\phi^*(l)}(p^*_{\phi^*(l)})\Big[\tilde{F}_{\phi^*(l+1)}(p^*_{\phi^*(l+1)})p^*_{\phi^*(l+1)} + F_{\phi^*(l+1)}(p^*_{\phi^*(l+1)})\pi^*_{l+1}\Big]$$

are lower than the corresponding continuation profits for the deviation. These are after substituting

$$\pi_{l-1} = \tilde{F}_{\phi^*(l+1)}(p^*_{\phi^*(l)})p^*_{\phi^*(l)} + F_{\phi^*(l+1)}(p^*_{\phi^*(l)})\Big[\tilde{F}_{\phi^*(l)}(p^*_{\phi^*(l+1)})p^*_{\phi^*(l+1)} + F_{\phi^*(l)}(p^*_{\phi^*(l+1)})\pi^*_{l+1}\Big].$$

Simple algebra yields that the difference is:

$$\begin{aligned}\pi^*_{l-1} - \pi_{l-1} = &\underbrace{\Big(F_{\phi^*(l+1)}(p^*_{\phi^*(l)}) - F_{\phi^*(l)}(p^*_{\phi^*(l)})\Big)}_{\Xi_1<0}\underbrace{\Big[p^*_{\phi^*(l)} - p^*_{\phi^*(l+1)}\Big]}_{\Xi_2\geq 0} \\ &+ \underbrace{\Big(F_{\phi^*(l+1)}(p^*_{\phi^*(l)})F_{\phi^*(l)}(p^*_{\phi^*(l+1)}) - F_{\phi^*(l)}(p^*_{\phi^*(l)})F_{\phi^*(l+1)}(p^*_{\phi^*(l+1)})\Big)}_{\Xi_3}\underbrace{\Big[p^*_{\phi^*(l+1)} - \pi^*_{l+1}\Big]}_{\Xi_4>0},\end{aligned} \tag{9}$$

The inequality Ξ_1 follows from the fact that $[F_k]$ is an ordered sequence of rotations. The price sequence that is induced by the search order is weakly decreasing, which implies Ξ_2 and moreover Ξ_3, since then the continuation payoff after the m-th searched product cannot exceed $p^*_{\phi^*(m)}$. Hence, the difference is strictly negative if Ξ_4 is negative. This is the case if and only if $F_k(\varepsilon)$ is strictly log-supermodular with respect to k and ε in the regime under consideration, which is $\varepsilon > \varepsilon_{\text{rot}}$. □

iii): Proof by contradiction. Assume the contrary – i.e. that there exist $l' > r^\dagger$ such that $\phi^*(l') \neq l'$. Then, claim *i*) implies that there exist $l > r^\dagger$ such that $\phi^*(l) > \phi^*(l+1)$. Now, set $p_{\phi^*(l)} = p^*_{\phi^*(l+1)}$, set $p_{\phi^*(l+1)} = p^*_{\phi^*(l)}$ and $p_k = p^*_k$ for each $k \notin \{\phi^*(l), \phi^*(l+1)\}$. Analagously to the proof of claim *ii*), I find that this deviation is profitable if the difference in continuation payoffs as given by equation (9) is strictly negative.

Now, substitute a low price p_L for $p^*_{\phi^*(l+1)}$ and a high price p_H for $p^*_{\phi^*(l)}$, since $p^*_{\phi^*(l)}$ exceeds $p^*_{\phi^*(l+1)}$, as the price sequence induced by the search order is decreasing. Equivalenty, substitute F_L for $F_{\phi^*(l+1)}$ and F_H for $F_{\phi^*(l)}$, as by assumption

$\phi^*(l+1) < \phi^*(l)$. The resulting function Δ is

$$\begin{aligned}\Delta(p_L, p_H) &= \Big(F_L(p_H) - F_H(p_H)\Big)\Big[p_H - p_L\Big] \\ &\quad + \Big(F_L(p_H)F_H(p_L) - F_H(p_H)F_L(p_L)\Big)\Big[p_L - \pi^*_{l+1}\Big].\end{aligned} \tag{10}$$

Note that for $p_L = p_H$ the function Δ takes the value of zero. Hence, a sufficient condition for the deviation to profitable is that the derivate of Δ with respect to p_L is strictly positive for $p_H = p^*_{\phi^*(l)}$ and $p_L \in (p^*_{\phi^*(l+1)}, p^*_{\phi^*(l)})$. Simple algebra yields

$$\begin{aligned}\frac{d\Delta}{dp_L}(p_L, p_H) &= F_H(p_H)\tilde{F}_L(p_L) - F_L(p_H)\tilde{F}_H(p_L) \\ &\quad + \Big(F_L(p_H)f_H(p_L) - F_H(p_H)f_L(p_L)\Big)\Big[p_L - \pi^*_{l+1}\Big].\end{aligned}$$

It holds that $p_L - \pi^*_{l+1} > p^*_{\phi^*(l+1)} - \pi^*_{l+1} = \frac{\tilde{F}_L(p_L)}{f_L(p_L)}$, where the second equality follows from the first order condition that has to hold here, since . Therefore, if the second line is positive, then we obtain the following lower bound:

$$\begin{aligned}\frac{d\Delta}{dp_L}(p_L, p_H) &> F_H(p_H)\tilde{F}_L(p_L) - F_L(p_H)\tilde{F}_H(p_L) \\ &\quad + \Big(F_L(p_H)f_H(p_L) - F_H(p_H)f_L(p_L)\Big)\frac{\tilde{F}_L(p_L)}{f_L(p_L)}.\end{aligned}$$

This is justified, as we will show that above value exceeds zero. Then, p_H and p_L below ε_{rot} imply that the first line is strictly negative, since $[F_k]$ is an ordered sequence of rotations. This in turn implies that the second line is positive. Finally, simple algebra yields

$$\frac{d\Delta}{dp_L}(p_L, p_H) > \frac{F_L(p_H)\tilde{F}_H(p_L)\tilde{F}_L(p_L)}{f_1(p_L)}\Big(\phi_H(p_L) - \phi_L(p_L)\Big) > 0,$$

where ϕ_k denotes the hazard rate of F_k and the last inequality holds of the hazard rate is increasing with respect to k. This is however equivalentl to $\tilde{F}_k(\varepsilon)$ being log-submodular with respect to k and ε. ■