How competition determines the success of an eco-label

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Abstract

The article presents a model of vertical differentiation with more than two firms. Goods are distinguished according to the environmental impact of their production process. An eco-label is introduced that certifies those firms whose environmental effort respects some given requirements. The model explores the hypothesis that the number of firms obtaining the eco-label depends not only on the cost of respecting those requirements, but also on the competition level both in the labeled and in the non-labeled segment of the industry. This approach offers new insights on the welfare implications of eco-labels. If the certification body imposes very mild requirements, many firms will afford to obtain the label, but their environmental effort will hardly be noticeable; if, instead, requirements are very strict, labeled firms will engage in significant environmental practices, but there will be too few of them. The model is able to define this trade-off and to endogenize the choice of the requirements by the certification body, whose goal is to maximize the *success* of the eco-label, defined as the total environmental effort that is put in place within the industry.

Keywords: Ecolabel, quality, differentiation, oligopolistic competition, asymmetric information

JEL Classification: L15, L13, D82

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1 Introduction

The economic motivation for the existence of quality labels is well-known: they signal the quality of a product and thereby address the problem of asymmetric information.

A large literature studies quality labels along different dimensions (for recent reviews of the literature, see Roe et al. (2014) and Bonroy and Constantatos (2015)). The present model focuses on labels with the following characteristics:

- Adoption is **voluntary**;
- They are **binary** (either the product has a certain characteristic, or it doesn't), as opposed to continuous labels (e.g. those that certify energy efficiency for electronic appliances along a scale: A++, A+...);
- They are promoted by the **public sector** or by an NGO;
- They convey information that is valuable for consumers: consumers trust them and understand them;
- They certify that some requirements have been respected during the **production process**, instead of signalling a characteristic of the good itself.

The last point deserves particular attention: the present article models a specific kind of "credence attributes", that is, characteristics that a consumer is not able to distinguish even after consumption. As an example, take some coffee produced according to Fair Trade standards: without the label, a consumer would never be able to distinguish between a cup of Fair Trade coffee and standard coffee, because the dimension along which the two goods are different is not something that the consumer can perceive. The firm does *not* invest in the quality of the production process so that the final good is safer, more reliable, more efficient, and so on: in these examples, the investment of the firm would result in characteristics that consumers could observe without the need for a label. The focus of this article is instead on those intangible characteristics that are typically the object of Corporate Social Responsibility (CSR) strategies: firms aim at strengthening the quality of their products and production

processes to reduce their environmental impact. Taking the cynical perspective that the only reason firms would choose these strategies is to preserve the reputation of their brands in the eyes of the relevant stakeholders (consumers, NGOs...), it can be safely assumed that labels are necessary to give firms incentives to invest in CSR practices: without such a signalling mechanism, stakeholders would never learn about a firm's commitment to better practices, therefore such a commitment would not entail any positive returns.

Finally, the model adopts a vertical differentiation approach: it is assumed that, if a labeled good and a non-labeled good are sold at the same price, all consumers would agree that the labeled good is better and would buy it (Gabszewicz and Thisse (1979), Shaked and Sutton (1982)). In other words, we abstract from any other characteristic that could make the non-labeled good preferred to the labeled one (e.g. we ignore the fact that a consumer might prefer the standard coffee to the Fair Trade coffee because she likes its taste more). In this kind of models, the labeled good is sold at a higher price (to compensate for the higher production costs); therefore, only consumers who attach enough importance to the characteristic certified by the label will be willing to pay a price premium to have the certified good.

Most of the so-called *eco-labels*, that is, labels that certify that a firm has made an effort to lower the environmental footprint of its product, fit into this line of reasoning. Table 1 lists some of the most well-known eco-labels of this kind and shows their adoption rate, ranging from 5% to 14% of the industry[1].

The first question explored in this article is: What are the factors that determine the adoption rate of label? It is reasonable to expect that a label will be adopted by more firms if consumers attach a lot of importance to the characteristic that is being certified; therefore, consumers' tastes should affect adoption rates positively. On the other hand, a label requiring drastic changes in the mode of production will entail higher costs for firms, thus affecting

[1]Sources: http://www.rainforest-alliance.org/about/business-practices, http://www.msc.org/business-support/key-facts-about-msc, http://www.pefc.org/about-pefc/who-we-are/facts-a-figures, "Facts and figures on organic agriculture in the European Union", October 2013, European Commission

Programme	**Aim**	**Adoption Rate**
Rainforest Alliance	Biodiversity, Sustainable livelyhoods	14%
Marine Stewardship Council	Sustainable fishing practices	10%
Programme for the Endorsement of Forest Certification	Sustainable Forest Management	9%
EU organic label	Organic food	5%

Table 1: Eco-labels and adoption rate

adoption rates negatively.

While these two factors have been extensively taken into account in the literature on labels, the present approach adds an additional ingredient: competition forces in the labeled and in the non-labeled segment. The hypothesis is that a firm observing that few firms are certified would expect a low level of competition should it decide to obtain the certification; certification would therefore be seen as relatively more profitable than if many firms were already certified. As competition in the certified segment of the market increases, though, certification might not be worthwhile any longer, given the cost it entails and the limited demand for labeled goods (most consumers are not willing to pay a premium price for this kind of "warm-glow" characteristics, because of income constraints or low environmental awareness). Therefore, some firms might find it more profitable to save on production costs and sell to "low-preference" consumers. The level of competition in both the certified and the non-certified segments of the market is explicitly taken into account in this model.

The second question is: How should we define the success of an eco-label, and how can a certification body maximize it? It will be shown that a very high adoption rate is only possible if the costs of certification are very low, or, in other words, if the requirements to

obtain the certification are very mild. This line of reasoning lead us to question the idea that *full adoption* in itself is the desirable outcome of a label policy: a label adopted by fewer firms, but that requires a more considerable effort, could achieve its sustainability goals more efficiently. On the other hand, if certification requirements are too high, the number of firms adopting the label would be too small. There is therefore a trade-off between the environmental requirements of a label and its adoption rate. The success of an eco-label should be ultimately defined as the overall environmental effort put in place within the industry as a whole. The present model allows to endogenize the choice of the stringency of certification requirements in order to maximize the success of the label.

2 The label model

The traditional model of vertical differentiation, where two firms strategically choose their quality level and then compete in prices, is not able to incorporate the oligopolistic competition forces that, according to the hypothesis stated above, play a major role in firms' decisions.
Therefore, we present a model of vertical quality differentiation in an oligopoly with more than two firms that compete in quantities (Cournot competition)[2] and are faced with a discrete choice between two exogenously given quality levels (label *vs* no label).

In the case of eco-labels, some consumers are willing to pay more for a labeled product out of a sense of altruism or compliance to some social norm, but would be unable to distinguish the labeled good from the unlabeled one if they were to consume the two goods without any knowledge about their origin. These industries (food, natural resources extraction) are more realistically described by competition *à la* Cournot (the most relevant choice variable is production volumes, whereas prices are strongly influenced by global market conditions), which reinforces our modeling choice.

[2]The model of vertical differentiation with Cournot competition follows Motta (1993).

In the presence of quality certification, the industry is segmented into two groups of firms, one that bears the costs of the label and targets consumers willing to pay a higher price for the certified product, and another that does not make any quality effort and targets those consumers that prioritize low prices over quality. This novel modeling approach implies that each firm's reaction function depends not only on the level of intra-segment competition (number of firms that belong to the same segment of the industry), but also on the level of inter-segment competition (number of firms that belong to the other segment).

All consumers get the same utility from consumption, $u = \theta s_i - p_\ell$ $(i = L, H)$.

Parameter $\theta \in [0, 1]$ represents the heterogeneity of consumers' taste for quality and is uniformly distributed. Consumers buy at most one unit. The market is not fully covered: some consumers do not buy the product and get a utility equal to zero. An explicit condition to allow for a partially covered market will be provided below.

Certified firms produce "High" quality s_h, and non-certified firms produce "Low" quality s_l (such that $s_h > s_l$). Quality levels are assumed to be exogenous from the point of view of the firm: s_h is defined by the certification body, and s_l can be seen as the "Business As Usual" quality level that costs nothing to the firm and does not change once the possibility to get certification is available.

The cost of obtaining the label depends on the extent of the additional effort made by the labeled firms, that is, the difference $(s_h - s_l)$. We assume that the cost of certification is an increasing and convex function of environmental effort, as follows:

$$c(s_i) = \frac{(s_i - s_l)^2}{\Delta}, \quad i = (L, H), \quad \Delta > 0, \tag{1}$$

where $1/\Delta$ measures the convexity of the cost function (the lower Δ, the faster the cost of quality increases as the environmental effort required becomes more significant).

Therefore:

$$c(s_l) = 0 \qquad c(s_h) = \frac{(s_h - s_l)^2}{2}$$

The number of firms in the industry is denoted by I; all firms are symmetric and their constant marginal cost is equal to zero. Among these firms, n invest in the higher quality and m do not, such that $n + m = I$. Unlike previous models of vertical differentiation, that described a duopoly situation, in this model we allow I to be any number larger than or equal to 2.

The setting of the game is as follows. In the first stage of the game, the quality requirement necessary to obtain the label is endogenously chosen by the certification body and announced to the industry. In the second stage, firms choose whether to obtain the label. In the third stage, firms compete *à la* Cournot.
As usual, the model is solved by backward induction.

3 Stage 3: intra-segment and inter-segment Nash Equilibrium

The consumer indifferent between buying s_h and s_l is denoted by $\theta_{hl} = (p_h - p_l)/(s_h - s_l)$, whereas the consumer indifferent between buying nothing and s_l is denoted by $\theta_{\emptyset l} = p_l/s_l$. All consumers characterized by $\theta < \theta_{\emptyset l} = p_l/s_l$ don't buy anything.

Demand functions for the two goods are therefore:

$$Q_h = 1 - \frac{p_h - p_l}{s_h - s_l}, \tag{2}$$

$$Q_l = \frac{p_h - p_l}{s_h - s_l} - \frac{p_l}{s_l}. \tag{3}$$

It is helpful to express demand functions in their indirect form:

$$p_h = s_h - Q_h s_h - Q_l s_l, \tag{4}$$

$$p_l = s_l - Q_h s_l - Q_l s_l. \tag{5}$$

The high quality s_h is supplied by n symmetric firms denoted by the index h ($h = \{1, 2, ..., n\}$), while the low quality s_l is supplied by m symmetric firms denoted by the index

l ($l = \{1, 2, ..., m\}$). Defining $q_{-h} = Q_h - q_h$ (respectively, $q_{-l} = Q_l - q_l$) as the sum of the quantities produced by all firms but firm h (resp., firm l), the demand functions can be re-written as

$$p_h = s_h - (q_h s_h + q_{-h} s_h) - Q_l s_l, \tag{6}$$

$$p_l = s_l - Q_h s_l - (q_l s_l + q_{-l} s_l). \tag{7}$$

Firm h chooses q_h in order to maximize $\pi_h = q_h(p_h - c)$. The cost function c (see (1)) represents the marginal cost of producing a certified good. It is constant with respect to the quantity of output produced, but it is assumed to be increasing and convex with respect to the environmental effort borne by the certified firms.

Firm h's reaction function is:

$$q_h^*(q_{-h}^*) = \frac{(1 - q_{-h}^*)s_h - Q_l s_l - c}{2s_h}. \tag{8}$$

Exploiting the symmetry of the model[3], we can rewrite the n FOCs of the high segment (respectively, the m FOCs of the low segment) into a single expression:

$$q_h^* = \frac{s_h - Q_l s_l - c}{(n+1)s_h}, \tag{9}$$

$$q_l^* = \frac{1 - Q_h}{(m+1)}. \tag{10}$$

Expressions 9 and 10 can be re-written as reaction functions that express how much each firm decides to produce given the quantity produced in the other segment of the market:

$$q_h^*(q_l^*) = \frac{s_h - (mq_l^*)s_l - c}{(n+1)s_h}, \tag{11}$$

$$q_l^*(q_h^*) = \frac{1 - (nq_h^*)}{(m+1)}. \tag{12}$$

The Nash equilibrium is given by:

$$q_h^* = \frac{(s_h - c - s_l)m + (s_h - c)}{(n+1)(m+1)s_h - nms_l}, \tag{13}$$

$$q_l^* = \frac{cn + s_h}{(n+1)(m+1)s_h - nms_l}. \tag{14}$$

[3]Since firms are symmetric, $Q_h^* = nq_i^*$. This, together with the definition of q_{-h}^*, gives $q_{-h}^* = (n-1)q_h^*$.

The ratio $s = s_h/s_l > 1$ represents the degree of differentiation: it is a pure number (the unit of measure which is specific to the kind of quality considered cancels out) and can be interpreted as how many times a high-quality good is “better” than a good produced according to the BAU standard. Without loss of generality and for easiness of notation, we can assume $s_l \equiv 1$. Therefore, high quality level s_h coincides with the degree of differentiation s, and equilibrium output, prices and profits can be written as:

$$q_h^* = \frac{(s-c-1)m + (s-c)}{(n+1)(m+1)s - nm} \qquad q_l^* = \frac{cn+s}{(n+1)(m+1)s - nm}$$

$$p_h^* = \frac{[(s-1)m + s](cn+s)}{(n+1)(m+1)s - nm} \qquad p_l^* = \frac{cn+s}{(n+1)(m+1)s - nm}$$

$$\pi_h^* = \left[\frac{(s-c-1)m + (s-c)}{(n+1)(m+1)s - nm}\right]^2 \cdot s \qquad \pi_l^* = \left[\frac{cn+s}{(n+1)(m+1)s - nm}\right]^2$$

The market is partially covered if there exist some consumers characterized by a value of parameter θ between $\underline{\theta} = 0$ and $\theta_{\emptyset l} = p_l$. It is straightforward to show that $p_L^* > 0$ (since $s > 1$).

The output and the mark-up of a certified firm are positive provided the following inequality is satisfied: $s - c \geq \frac{m}{m+1}$.

The functions q_h and q_l share the element $1/[(n+1)(m+1)s - nm]$. This U-shaped function is symmetric in n and m: the symmetry represents the fact that, although the market is divided into a high quality and a low quality segment, there is some *inter-segment competition* (e.g. firms in the high segment are affected by the level of competition in the low segment as much as they are affected by the level of competition in the high segment, and vice-versa), due to the fact that the goods sold in the two segments are highly substitutable. This function reaches its minimum when $n = m = I/2$: the overall level of competition is higher when the industry is such that half of the firms are in the high segment and half in the low segment.

There is no other segmentation such that the level of competition in the "other" segment is weaker without the level of competition in the "own" segment being tougher (and viceversa). The role of *intra-segment competition* (e.g. firms in the high segment suffer from competition from direct competitors in the high segment, and likewise for the low-segment), instead, is evident in the numerator of q_h and q_l. We observe that q_h increases in m:[4] this is equivalent to saying that it decreases in n, the level of competition in the high segment. Similarly, q_l increases in n, which is equivalent to saying that it decreases in m, the level of competition in the low segment.

Therefore, while both q_h and q_l are U-shaped due to the symmetry given by their common denominator, q_l reaches its minimum for $m > I/2$ and q_h reaches its minimum for $n > I/2$ due to the impact of the intra-segment competition represented by their numerator.

4 Stage 2: the label decision

The number of firms adopting the label is stable when, *at the same time*, $\pi_l^*(n^*) \geq \pi_h^*(n^*+1)$ and $\pi_h^*(n^*) \geq \pi_l^*(n^*-1)$. Intuitively, each firm in the non-labeled segment would not be better off if it unilaterally decided to obtain the label, and each labeled firm would not be better off if it unilaterally abandoned the labeled segment. In addition, this number should be an integer number between 0 and I.

In the Appendix we show that, given any level of quality differentiation (s), there exists at most one equilibrium number of firms that adopt the quality label, denoted by n^*. This

[4] The derivative of q_h with respect to m, $(s-c-1)$, is not positive for any value of s; however, it is positive over the domain of s where an equilibrium adoption rate can be analytically found. This is further explained in the next section. For an analysis of the sign of $(s-c-1)$, see equation (36) in the Appendix.

value satisfies the following system:

$$
\begin{cases}
\pi_l^*(n) \geq \pi_h^*(n+1) & (15)\\
\pi_h^*(n) \geq \pi_l^*(n-1) & (16)\\
0 \leq n \leq I & (17)\\
n \in \mathbb{Z} & (18)
\end{cases}
$$

To illustrate these four conditions, consider Figure 1. The value of n at which $\pi_l(n)$ and $\pi_h(n+1)$ cross is denoted by n_1, while the one at which $\pi_h(n)$ and $\pi_l(n-1)$ cross is denoted by n_2. Condition (15) is satisfied for all values of n larger than n_1; condition (16) is satisfied for all values of n smaller than n_2.

The fact that there exists one and only one integer value of n in the interval $[n_1, n_2]$ (condition (18)) is ensured by the fact that the function $\pi_l(n-1)$ is the horizontal translation of the function $\pi_l(n)$ by a lenght of 1 to the right, while the function $\pi_h(n)$ is the horizontal translation of the function $\pi_h(n+1)$ by a lenght of 1 to the right. Therefore, the distance between n_1 and n_2 is exactly one, and there can be only one integer number belonging to this interval.[5]

Finally, for n^* to be included between 0 and I (condition (17)) we need to impose some constraints on the values of s, which are defined in the Appendix. Indeed, the value of s such that $n_1 = -1$ and $n_2 = 0$ is the highest possible value of s (denoted by $\bar{\bar{s}}$ in the Appendix) above which requirements are so strict that less than 0 firms would obtain the label; for $s > \bar{\bar{s}}$, condition (17) is violated. Similarly, the value of s such that $n_1 = I$ and $n_2 = I+1$ is the lowest possible value of s (denoted by $\underline{\underline{s}}$ in the Appendix) below which requirements are so mild that more than I firms would obtain the label; for $s < \underline{\underline{s}}$, condition (17) is again violated. However, it can be said that, even if values of s outside these thresholds would not allow an analytical solution to the system of equilibrium conditions, we can intuitively assume corner solutions such that, for $s < \underline{\underline{s}}$, all firms obtain the label and, for $s > \bar{\bar{s}}$, no firm obtains the label.

[5]Of course, the exception is when both n_1 and n_2 are integer numbers; however, this is a special case that does not invalidate the results. To rule it out, we could set a cut-off rule by which either condition (15) or (16) is a strict inequality.

Approximation of $n^*(s)$

The function $n^*(s)$ is discontinuous. Indeed, as long as s is lower than the threshold at which $n_1 = x$ and $n_2 = x+1$, n^* is equal to $x+1$; as soon as s goes above this threshold, n^* jumps to x. This discontinuity would create issues in the remainder of the analysis; we therefore define a function $n_a^*(s)$, where a stands for "approximate", which corresponds to the value of n at which $\pi_l(n)$ and $\pi_h(n)$ (see Figure 1). This value being always between n_1 and n_2, it will always be at a distance of less than 1 from the equilibrium value n^*. It is defined as:

$$n_a^*(s) = \frac{I(s-c-1)\sqrt{s} + (s-c)\sqrt{s} - s}{-[(s-c-1)\sqrt{s} + c]} \tag{19}$$

In the Appendix, it is shown that $n_a^*(s)$ is decreasing in s.

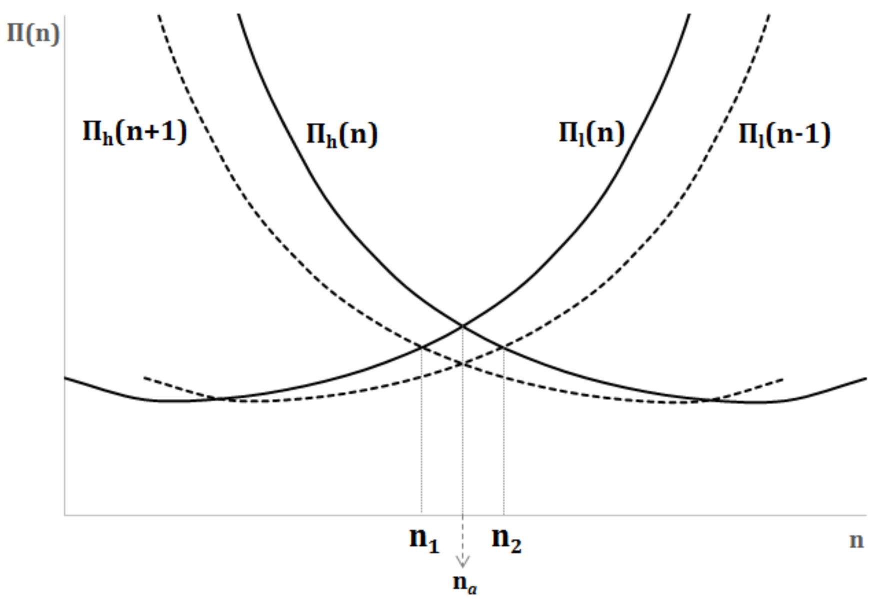

Figure 1: The equilibrium conditions

5 Stage 1: endogenous quality requirements

At the first stage, the government promotes the introduction of a label and sets its quality requirements, anticipating the impact of this choice on environmental emissions. The objective of the government is to maximize the success of the eco-label, defined as the minimization of the environmental emissions from the industry. This, as it will be shown, does not coin-

cide with the objective of having the whole industry following the voluntary label programme.

We assume that each unit of a good produced with the Business As Usual technology causes one unit of polluting emissions (denoted by E). Therefore, the emissions produced by the Low-quality segment is simply equal to $E_l = (I - n^*)q_l^*(n) = Q_l(n)$. However, a good produced according to the high technology pollutes s times less: the total emission produced by the High-segment is therefore equal to $E_h = n^* q_h^*(n)/s = Q_h(n)/s$. Total emissions of the industry are then defined by the following function:

$$E(n(s)) = \frac{Q_h(n(s))}{s} + Q_l(n(s)) \tag{20}$$

If we replace the discontinuous function $n(s)$ by the continuous function $n_a(s)$, we can take the derivative of total emissions with respect to s:

$$\begin{aligned} \frac{\partial E(n_a(s))}{\partial s} &= \frac{\partial Q(n_a)}{\partial n_a}\frac{\partial n_a(s)}{\partial s}\frac{1}{s} + Q_h(n_a(s))\left(-\frac{1}{s^2}\right) + \frac{\partial Q_l(n_a)}{\partial n_a}\frac{\partial n_a(s)}{\partial s} = \\ &= \underset{(-)}{\frac{\partial E(n_a)}{\partial n_a}}\underset{(-)}{\frac{\partial n_a(s)}{\partial s}} - \frac{Q_h(n_a(s))}{s^2} \end{aligned} \tag{21}$$

The second element in (21) ($-Q_h/s^2$) is negative and represents the fact that, when requirements become stricter, everything else being equal, emissions decrease thanks to the improved environmental performance of the labeled firms.

The first element relates to the fact that, everything else being equal, emissions only depend on the total output produced, and whether this output his produced by a labeled or a non-labeled firm. The first term represents how emissions in the labeled and in the non labeled segment change when the number of labeled firms changes; overall, total emissions decrease when more firms obtain the certification (the analysis is left to the Appendix). Since higher level of requirements decrease the number of labeled firms (the second term in the first element), the sign of the first element is always positive, representing the fact that total emissions increase when s increases due to the fact that fewer firms adopt the label.

For low values of s, the second element prevails on the first and the sign of the total derivative is negative: a marginal increase in s improves the environmental performance of labeled costs, and this more-than-offsets the fact that requirements become costlier and, thereby, the number of labeled firms decreases.

As s increases, the second element becomes smaller and, eventually, the sign of the total derivative is reversed: the fact that fewer firms obtain the label when s increases offsets the beneficial impact on the environmental performance of labeled firms, and total emissions increase.

We can therefore state that total emissions initially decrease with stricter certification requirements because labeled firms increase their environmental effort; however, when s is above the emission-minimizing level, emissions increase with stricter requirements because less firms decide to adopt the label (which is becoming increasingly costly) and the share of output produced by non-labeled firms in the industry increases.

To conclude, a social planner willing to prioritize environmental protection should set an intermediate level of quality requirements s that minimizes emissions. This is because there is a trade-off between the adoption rate of an eco-label and its impact on the environment: in order to be adopted by many, possibly by all firms in the industry, a label would need to impose very mild requirements, that do not impose a significant environmental commitment; on the other hand, a demanding eco-label would make the certified firms make an important environmental effort, but there would be too few of them.

6 Conclusion

The model presented in this article explores the hypothesis that the number of firms obtaining the eco-label depends not only on the cost of respecting those requirements, but also on the competition level both in the labeled and in the non-labeled segment of the industry. This approach offers new insights on the welfare implications of eco-labels. In particular, it is possible to endogenize the level of stringency that the certification body requires to labeled firms, in such a way to maximize the success of the eco-label, defined as the total environmental effort that is put in place within the industry. The emission-minizing level of requirements is shown to be an intermediate one: indeed, if requirements are too mild, many firms will afford to obtain the label, but their environmental effort will hardly be noticeable; if, instead,

requirements are too strict, labeled firms will engage in significant environmental practices, but there will be too few of them. The model is able to define the trade-off beween the stringency of the requirements of a label and its adoption rate.

Further extensions of the current model could include heterogeneous firms (assuming that more efficient firms would be the first to adopt the label, since they can better absorb the additional quality cost) or explore different distribution of environmental taste among consumers (instead of the uniform distribution assumed in this model).

7 Appendix

To simplify notation, and without loss of generality, the discussion in the Appendix is done for an economy with three firms ($I = 3$).

7.1 Proof of the uniqueness of the equilibrium

The proof that there is one and only one value of n^* that satisfies the system of equations (15)-(18) is done in three steps:

1. Define the range of values of s such that (17) is respected ($0 \leq n^* \leq I$).
2. Express conditions (15) and (16) as cubic functions of n, and define the conditions under which (18) is respected ($n^* \in \mathbb{Z}$).
3. Show that, for all the values of s belonging to the range defined at point 1, the equilibrium is unique.

Step 1: Maximum and minimum values of s

After isolating c, the first and second conditions at equilibrium (inequalities (15) and (16)) can be written, respectively, as:

$$c \geq \frac{P(n+1)Q(n)\sqrt{s} - Q(n+1)s}{nQ(n+1) + (3-n)Q(n)\sqrt{s}} \equiv g(n+1) \tag{22}$$

$$c \leq \frac{P(n)Q(n-1)\sqrt{s} - Q(n)s}{(n-1)Q(n) + (4-n)Q(n-1)\sqrt{s}} \equiv g(n), \tag{23}$$

where

$$P(n) = s + (3-n)(s-1) \tag{24}$$

$$Q(n) = (n+1)(4-n)s - n(3-n). \tag{25}$$

When the cost of quality is comprised between $g(n+1)$ and $g(n)$ evaluated at $n = x$, the number of firms at equilibrium is $n^* = x$.

For instance, when $g(1) \leq c \leq g(0)$, the equilibrium number of firms at equilibrium is 0 (*no adoption scenario*). The functions $g(0)$ and $g(1)$ can be expressed as a function of s only:

$$c \leq g(0) \Leftrightarrow c \leq \frac{-s^{\frac{3}{2}} + 4s - 3}{-s^{\frac{1}{2}} + 4} \tag{26}$$

$$c \geq g(1) \Leftrightarrow c \geq \frac{6s^{\frac{3}{2}} - 3s - 4s^{\frac{1}{2}} + 1}{6s^{\frac{1}{2}}} \tag{27}$$

Similarly, when $g(4) \leq c \leq g(3)$, the equilibrium number of firms at equilibrium is 3 (*full adoption scenario*).

$$c \leq g(3) \Leftrightarrow c \leq \frac{3s^2 - 2s^{\frac{3}{2}} - s}{3s + 4s^{\frac{1}{2}} - 1} \tag{28}$$

$$c \geq g(4) \Leftrightarrow c \geq \frac{s^{\frac{3}{2}} - s}{3} \tag{29}$$

These inequalities should be interpreted as follows:

- In order to obtain an equilibrium that respects conditions (15)-(17), the cost of quality should not exceed $g(0)$ as defined in (26), otherwise the label would be so expensive that less than zero firms would obtain it - which contradicts condition (17);

- Moreover, the cost of quality should not be lower than $g(4)$ as defined in (29) (in the general case with I firms, c should not be lower than $g(I+1)$), otherwise the label would be so cheap that four firms or more would obtain it - which again contradicts condition (17);

- In order to obtain a *partial adoption* scenario, the cost of quality should be lower than $g(1)$ as defined in (27) and higher than $g(3)$ as defined in (28).

We now replace c by the cost function $c(s) = (s-1)^2/\Delta$ (see (1)). In this way, the inequalities (26)-(29) will be expressed only as a function of the variable s and parameter Δ, and we can obtain four critical values of s:

- $\underline{\underline{s}}$, the threshold value between a scenario with no equilibrium (below this level, labels requirements are so mild that more than I firms would obtain the label) and the *full adoption* scenario;

- $\underline{s}$, the threshold value between the *full adoption* and the *partial adoption* scenario;

- $\bar{s}$, the threshold value between the *partial adoption* and the *no adoption* scenario;

- $\bar{\bar{s}}$,the threshold value between the *no adoption* and a scenario with no equilibrium (above this level, labels requirements are so strict that less than zero firms obtain the label).

These four values are defined by the following inequalities:

$$s \geq \underline{\underline{s}} \text{ whenever } \quad 3s^2 - \Delta s^{\frac{3}{2}} + (\Delta - 6)s + 3 \geq 0 \tag{30}$$

$$s \geq \underline{s} \text{ whenever } \quad 3s^3 + 4s^{\frac{5}{2}} - (3\Delta + 7)s^2 + (2\Delta - 8)s^{\frac{3}{2}} + (\Delta + 5)s + 4s^{\frac{1}{2}} - 1 \geq 0 \tag{31}$$

$$s \leq \bar{s} \text{ whenever } \quad 6s^{\frac{5}{2}} - (6\Delta + 12)s^{\frac{3}{2}} + 3\Delta s + (4\Delta + 6)s^{\frac{1}{2}} - \Delta \leq 0 \tag{32}$$

$$s \leq \bar{\bar{s}} \text{ whenever } \quad - s^{\frac{5}{2}} + 4s^2 + (\Delta + 2)s^{\frac{3}{2}} - (4\Delta + 8)s - s^{\frac{1}{2}} + (3\Delta + 4) \leq 0 \tag{33}$$

Table 2 summarizes the threshold levels of s for some given values of Δ.

We observe that the more convex the cost function (the lower Δ), the smaller the values of s that sustain an equilibrium number of certified firms between 0 and 3, and the narrower their range. This is obvious: if costs increase faster, then label adoption will only be possible

Table 2: Threshold levels of s for given values of Δ $(I = 3)$.

Δ	$\underline{\underline{s}}$	$\underline{s}$	$\bar{s}$	$\bar{\bar{s}}$
0.5	1.09	1.17	1.35	1.42
1	1.19	1.37	1.73	1.84
1.5	1.30	1.57	2.13	2.26
2	1.44	1.79	2.53	2.68
3	1.75	2.27	3.38	3.51
4	2.17	2.79	4.24	4.32

for milder requirements, and for a narrower set of requirements.

Notice that, for any $s < \underline{\underline{s}}$ (resp. any $s > \bar{\bar{s}}$), *full adoption* (resp. *no adoption*) is a corner solution although it does not characterize an equilibrium as defined in (15)-(17).

Step 2: Equilibrium conditions expressed as a cubic functions of n

We manipulate again conditions (15) and (16), but this time, instead of isolating c, we want to obtain a cubic function of n. The first condition becomes:

$$f_1(n) = a_1 n^3 + b_1 n^2 + c_1 n + d_1 \leq 0, \tag{34}$$

where

$$a_1 = (s-1)[(s-c-1)\sqrt{s} + c],$$

$$b_1 = (s-1)[-3(2s-2c-1)\sqrt{s} + 2\sqrt{s} + s - c],$$

$$c_1 = -4(s-c-1)s^{\frac{3}{2}} - (s-1)s + 3(3s-3c-2)(s-1)\sqrt{s} - 2(3s-1)c, \text{and}$$

$$d_1 = 4(3s-3c-2)s^{\frac{3}{2}} - 2(3s-1)s.$$

The second condition becomes:

$$f_2(n) = a_2 n^3 + b_2 n^2 + c_2 n + d_2 \geq 0, \tag{35}$$

where

$$
\begin{aligned}
a_2 &= (s-1)[(s-c-1)\sqrt{s}+c], \\
b_2 &= (s-1)[-9(s-c-1)\sqrt{s}-\sqrt{s}+s-4c], \\
c_2 &= 4(s-c-1)(5s-6)\sqrt{s}+5\sqrt{s}(s-1)-4cs-3(s-c)(s-1), \text{and} \\
d_2 &= 16(s-c-1)\sqrt{s}+4\sqrt{s}-4(s-c)s.
\end{aligned}
$$

Notice that $a_1 = a_2 = a$.

The functions $f_1(n)$ and $f_2(n)$ can have one, two or three roots. These roots are the values of n at which, respectively, $\pi_l(n)$ crosses $\pi_h(n+1)$ and $\pi_h(n)$ crosses $\pi_l(n-1)$.

In the next step, we carry out some analysis to show that at most one root of each function can belong to the range $n \in [0, I]$; in the case of equation (34), we denote this root by n_1, and, in the case of equation (35), we denote it by n_2 (see Figure 1).

Notice that there can only be one integer number between n_1 and n_2. Indeed, it is possible to show that $f_1(n)$ and $f_2(n)$ are the horizontal translation of each other, by a lenght of 1 ($f_2(n) = f_1(n-1)$).[6] Therefore, $n_2 - n_1 = 1$, and there can be only one integer number between n_1 and n_2. It remains now to show that the adoption level at equilibrium is indeed an integer number belonging to the interval $[n_1, n_2]$. For this to be the case, $f_1'(n_1)$ and $f_2'(n_2)$ must be negative, so that the first condition is satisfied for $n \geq n_1$ (where $f_1(n) \leq 0$) and the second is satisfied for $n \leq n_2$ (where $f_2(n) \geq 0$).

Step 3: Uniqueness of the equilibrium

Firstly, we analyse the sign of a to understand the shape of $f_1(n)$ and $f_2(n)$. We find that, as long as $s \in [\underline{\underline{s}}, \bar{\bar{s}}]$, a is positive: this implies that $f_i(-\infty) = -\infty$ and $f_i(\infty) = \infty$ $(i = 1, 2)$.

[6] When $n = n_1$, the lines $\pi_l(n)$ and $\pi_h(n+1)$ cross. These two lines are the horizontal translation of the lines $\pi_l(n+1)$ and $\pi_h(n)$ to the left, and the length of the horizontal movement is one. Therefore, the value of n where the two latter lines cross (which is $n = n_2$) must be equal to $n_1 + 1$.

In fact, the first component of a, $(s-1)$, is always positive by the definition of s. The second component is positive if $(s-c-1)$ is positive. Indeed,

$$s - c - 1 \geq 0 \quad \Leftrightarrow \quad -s^2 + (\Delta+2)s - (\Delta+1) \geq 0 \tag{36}$$

is true when s belongs to the interval $[1, \Delta+1]$. A quick look at table 2 shows that the range of values that is relevant to the analysis ($[\underline{\underline{s}}, \bar{\bar{s}}]$) is always comprised in the interval $[1, \Delta+1]$, for any value of Δ.

Now we analyse function $f_1(n)$ and we observe that:

$$\begin{aligned}
f_1(0) = d_1 \geq 0 \quad &\Leftrightarrow \quad -6s^{\frac{5}{2}} + (6\Delta+12)s^{\frac{3}{2}} - 3\Delta s - (4\Delta+6)s^{\frac{1}{2}} + \Delta \geq 0 \\
f_1(3) = 27a + 9b_1 + 3c_1 + d_1 \leq 0 \quad &\Leftrightarrow \quad -3s^2 + \Delta s^{\frac{3}{2}} - (\Delta-6)s - 3 \leq 0
\end{aligned}$$

The right-hand part of these two expressions are exactly equivalent to the condition for $s \leq \bar{s}$ ((32)) and to the one for $s \geq \underline{\underline{s}}$ ((30)); this is not surprising since they are derived in the same way (by replacing n by 0 and 3 in the first equilibrium condition).

What is important, however, is to observe that as long as $s \in [\underline{s}, \bar{s}]$ the function $f_1(n)$ is positive when $n = 0$ and negative when $n = 3$; because of the shape of the function and of the Intermediate Value Theorem, this shows that, for this range of s, there can be at most one value of $n \in [0, 3]$ such that $f(n) = 0$; this value is $n = n_1$ and the function $f_1(n)$ is decreasing at that point ($f_1'(n_1) < 0$).

Instead, when $s \in (\bar{s}, \bar{\bar{s}}]$, $f_1(0) = d_1 < 0$. It is possible to show that, for that range of s, $f_1(-1) = f_2(0) > 0$ (see next paragraph). Therefore, it has to be that $-1 < n_1 < 0$.

Similarly, for $f_2(n)$:

$$\begin{aligned}
f_2(0) = d_2 \geq 0 \quad &\Leftrightarrow \quad s^{\frac{5}{2}} - 4s^2 - (\Delta+2)s^{\frac{3}{2}} + (4\Delta+8)s + s^{\frac{1}{2}} \\
&\qquad -(3\Delta+4) \geq 0 \\
f_2(3) = 27a + 9b_2 + 3c_2 + d_2 \leq 0 \quad &\Leftrightarrow \quad -3s^3 - 4s^{\frac{5}{2}} + (3\Delta+7)s^2 - (2\Delta-8)s^{\frac{3}{2}} \\
&\qquad -(\Delta+5)s - 4s^{\frac{1}{2}} + 1 \leq 0
\end{aligned}$$

Again, the right-hand expressions correspond, respectively, to $s \leq \bar{\bar{s}}$ ((33)) and $s \geq \underline{s}$ ((31)); therefore, as long as $s \in [\underline{s}, \bar{\bar{s}}]$, function $f_2(n)$ is positive when $n = 0$ and negative

when $n = 3$; thanks to the fact that $a > 0$ and to the Intermediate Value Theorem, we can say that there can be at most one value of $n \in [0, 3]$ such that $f_2(n) = 0$; this value is $n = n_2$ and $f_2'(n_2) < 0$.

Similarly to the previous case, when $s \in [\underline{\underline{s}}, \underline{s})$, $f_2(3) > 0$. It was shown earlier that, for that range of s, $f_1(3) = f_2(4) < 0$. Therefore, it has to be that $3 < n_2 < 4$.

The following graph summarizes the values of n_1, n_2 and n^* at different values of s, for a general number of firms I:

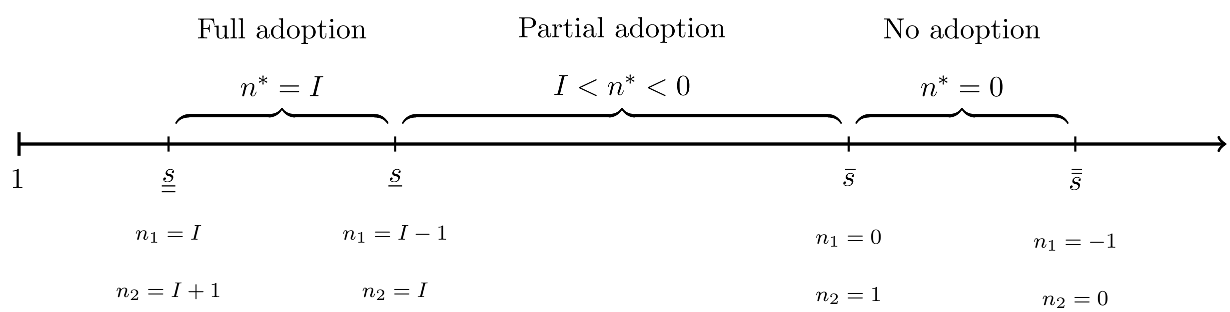

Figure 2: Equilibrium at different values of s.

7.2 The function $n_a^*(s)$ decreases in s

The function $n_a^*(s)$ defined in (19) can be rewritten as a function of parameter Δ and variable s:

$$n_a^*(s) = \frac{4s^{\frac{5}{2}} - (4\Delta + 8)s^{\frac{3}{2}} + \Delta s + (3\Delta + 4)s^{\frac{1}{2}}}{s^{\frac{5}{2}} - s^2 - (\Delta + 2)s^{\frac{3}{2}} + 2s + (\Delta + 1)s^{\frac{1}{2}} - 1} \tag{37}$$

This function is discontinuous since the denominator is equal to zero for three values of s: the first root is always lower than 1 so we can ignore it; the second root is $s = 1$, value at which also the numerator is equal to 0 (the function has a hole); we denote the third root by $s = \hat{s}_a$, value at which the numerator is positive and the function approaches a vertical asymptote.[7]

[7] The vertical asymptote is such that $\lim_{s \to \hat{s}_a^-} n_a^*(s) = -\infty$ and $\lim_{s \to \hat{s}_a^+} n_a^*(s) = +\infty$.

However, the interval of s within which $n_a^*(s) \in [0,3]$ (which, to be consistent with the previous notation, is denoted by $s \in [\underline{\underline{s}}_a, \bar{\bar{s}}_a])$ is always included in the interval $(1, \hat{s}_a)$ (see Table 3 for numerical results): therefore, over the relevant domain of s, the function $n_a^*(s)$ shows no discontinuity.

Table 3: Threshold levels of s_a for given values of Δ $(I = 3)$.

Δ	$\underline{\underline{s}}_a$	$\bar{\bar{s}}_a$	$\hat{s}_a$
0.5	1.06	1.44	2.40
1	1.13	1.89	3.24
1.5	1.21	2.35	4.00
2	1.29	2.81	4.70
3	1.47	3.74	6.05
4	1.68	4.68	7.34

The range $s \in [\underline{\underline{s}}_a, \bar{\bar{s}}_a]$ is defined by the following inequalities:

$$s \geq \underline{\underline{s}}_a \text{ whenever } \quad s^{\frac{5}{2}} + 3s^2 - (\Delta + 2)s^{\frac{3}{2}} + (\Delta - 6)s + s^{\frac{1}{2}} + 3 \geq 0 \tag{38}$$

$$s \leq \bar{\bar{s}}_a \text{ whenever } \quad 4s^2 - (4\Delta + 8)s + \Delta s^{\frac{1}{2}} + (3\Delta + 4) \leq 0 \tag{39}$$

The first derivative of (37) is given by

$$\begin{aligned} \frac{\partial n_a^*(s)}{\partial s} = & \frac{-2s^{\frac{7}{2}} - \left[\frac{7}{2}\Delta - 8\right]s^{\frac{5}{2}} - \left[12 - \frac{1}{2}\Delta(\Delta + 3)\right]s^{\frac{3}{2}} + \left[\frac{1}{2}\Delta(\Delta + 7) + 8\right]s^{\frac{1}{2}} - \left[\frac{3}{2}\Delta + 2\right]s^{-\frac{1}{2}}}{\left[s^{\frac{5}{2}} - s^2 - (\Delta + 2)s^{\frac{3}{2}} + 2s + (\Delta + 1)s^{\frac{1}{2}} - 1\right]^2} \\ & + \frac{3\Delta s^2 - \Delta(\Delta^2 + 2)s - \Delta}{\left[s^{\frac{5}{2}} - s^2 - (\Delta + 2)s^{\frac{3}{2}} + 2s + (\Delta + 1)s^{\frac{1}{2}} - 1\right]^2} \end{aligned} \tag{40}$$

For any value of Δ, the derivative is never positive on its domain (which is $s \in \mathbb{R} : s > 0$); its global maximum is at $s = 1$, where it is equal to 0.

7.3 Derivative of total emissions with respect to the number of certified firms

First, we write $E_h = Q_h/s$ and $E_l = Q_l$ as functions of n by replacing m by $I - n = 3 - n$. We get:

$$E_h(n) = \frac{-(s-c-1)n^2 + (4s-4c-3)n}{[-(s-1)n^2 + 3(s-1)n + 4s]s} \qquad E_l(n) = \frac{-cn^2 - (s-3c)n + 3s}{-(s-1)n^2 + 3(s-1)n + 4s} \tag{41}$$

The derivatives are:

$$\frac{\partial E_h(n)}{\partial n} = \frac{(s-1)(s-c)n^2 - 8(s-c-1)sn + 4(4s-4c-3)s}{s[-(s-1)n^2 + 3(s-1)n + 4s]^2} \tag{42}$$

$$\frac{\partial E_l(n)}{\partial n} = \frac{-(s-1)sn^2 + 2(3s-4c-3)sn - (13s-12c-9)s}{[-(s-1)n^2 + 3(s-1)n + 4s]^2} \tag{43}$$

The derivative of total emissions with respect to the number of certified firms is therefore:

$$\frac{\partial E(n)}{\partial n} = \frac{\partial E_h(n)}{\partial n} + \frac{\partial E_l(n)}{\partial n} = \frac{an^2 + bn + c}{s[-(s-1)n^2 + 3(s-1)n + 4s]^2} \tag{44}$$

where

$$a = -(s-1)(c + s(s-1))/s$$

$$b = 2(s-1)(3s-4c-4)$$

$$c = -[3(s-1)(3s-4c-4) + 4(c + s(s-1))]$$

The numerator in (44) is a quadratic bell-shaped function that has no real roots (its discriminant is always negative)[8]. Therefore, it is always negative.

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