Nonparametric Estimation for Regulation Models *

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Abstract

Regulation models are a special class of contract models that have received a lot of attention from economists in the last few decades. This continuous interest has been motivated by an increasing need in designing regulatory policies in a world where decentralization and delegation of public services play an important role. A more recent and less rich strand of economic literature is devoted to the structural analysis of contract theory models. The economic setting underlying these models gives rise to complex nonlinear inverse problems and hence the difficulties in uniquely recovering the primitives of the model from the data. The novelty of our paper comes from the fact that we globally identify the static version of a classical adverse selection model and we also provide a quantile estimation procedure for the parameter of interest along with a discussion of the asymptotic properties of our estimator. We also present two extensions where we allow for semiparametric forms of the cost function.

Keywords: L-functionals, regulation models, principal-agent model, adverse selection, nonparametric statistics, structural econometrics

JEL Classification: $C40, D86, L51$

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1 Introduction

Contracts are part of our daily life as they are present in any economic activity we might think about: financial markets (the relationship between a lender and a borrower), labor market (employer-employee), selling mechanisms (owner-buyer), public utilities (regulatory agency-supplier), insurance market (brokerage company-customers), corporate organisation (shareholders-managers) and the list is far from being exhaustive. All these instances previously enumerated have in common one main feature: they all can be represented as a principal-agent relation with incomplete information.

In practice, we encounter very often the situation where a local municipality delegates the task of providing a so-called "public service" (water supply, water management, waste management, public transportation, energy) to another entity which is usually a natural monopoly [1]. We face here a principal-agent relationship [2] characterised by the presence of informational asymmetries under the form of adverse selection or moral hazard or very often a mix of the two (for an extensive description of these issues see Laffont and Martimort (2001)). The adverse selection arises from the fact that the local municipality doesn't have a perfect information about the demand or the costs conditions faced by the firm. Hence, in order to increase its revenues, the natural monopolist may try to distort the reality when reporting to the principal. A low cost firm may try to pass for a high cost supplier in order to be able to fix a higher price for its product or service. Another informational disadvantage faced by the principal comes from the fact she cannot verify if the managerial team made enough efforts to reduce the costs of operating in the industry. This creates the so-called moral hazard problem or "hidden action". For example, the firm may find it in its own interest not to invest into a new technology, if the revenues received from the principal depend on the observed costs.

We focus our attention on the public-utilities problem, but our econometric approach

[1] Usually in these sectors of activity, the production features make optimal the provision of the good by a small number of firms or even one.

[2] Throughout this paper, we will refer to the principal as "she" and to the agent as "he".

can be extended to other examples. In this paper we take into account only cases of pure adverse selection (i.e. we assume that the principal doesn't face an effort shirking problem). Moreover she is not able to observe the ex-post realisations of the cost (no possibility of auditing). This could be the case of development countries where the regulatory agency has a limited regulatory power that leads to a weak, and hence useless, auditing system (for an in-debt discussion about the consequences of institutional failures on the design of regulatory policies in development countries, see Estache and Wren-Lewis (2009)). In order to account for the informational asymmetries concerning the cost opportunities, the local municipality must design a regulation policy which will limit the rents the monopolistic firm can extract thanks to its informational advantage. The main instruments at the regulator's disposal for inducing truth telling is the price for the regulated product and the monetary transfers (taxes or subsidies). Thus, the principal will offer a menu of contracts that specifies the prices for the good or service at different cost realisations and also a payment scheme. These options are designed in such a manner that the firm will have incentives to report its actual cost. For a survey of the early literature on regulation see Baron (1989), Armstrong and Sappington (2007) and for more recent developments in the field see Joskow (2007).

Although the economic theory behind different forms of informational asymmetries is very well developed, the econometric literature that treats these models is still scarce. The early empirical applications in the field of economic regulation were conducted using a reduced-form approach. Moreover, these papers were mainly concerned with the implications of regulatory policies on the level and structure of prices or on the innovation and production growth, but they were not incorporating the effects of informational asymmetries on the agent's behaviour. The first paper that used a structural approach to account for the presence of private information in a regulator-utility relation is Wolak (1994). The author used a parametric setting to estimate the parameters of the agent's production and cost function for the water supplier in California. Other empirical ap-

plications are in particular conducted by Ivaldi and Martimort (1994), Bontemps et al. (2010) or Chiappori and Salanié (2002). Perrigne and Vuong (2011) analyse the non-parametric point identification issues in a static mixed model of asymmetric information and d'Haultfoeuille and Février (2010) provide a partial identification result for the distribution of agents types and the cost function. Although the identification issue in the static problem of contracting has been studied previously in Perrigne and Vuong (2011), our contribution comes from the fact we present also the estimation issues and compute the speed of convergence for our estimator.

Our paper is essentially devoted to the analysis of the adverse selection problem (the "hidden information") in a static interaction between the regulated supplier and his regulator. For this purpose, we will use the economic settings exposed in Baron and Myerson (1982) and Laffont and Tirole (1993) as follows.

2 Economic model

Let us first present the features of the static model of hidden information *à* la Baron and Myerson (1982).

We consider the situation where the private information of the agent consists of the knowledge of his marginal cost denoted by θ and in order to simplify notations suppose that $\theta \in [0,1]$. The principal cannot observe the θ but knows that its distribution is given by $F : [0,1] \to [0,1]$ and $f(\theta) \equiv F'(\theta)$. The principal also knows the demand function denoted by $X(p)$, where p is the transaction price. The supplier's production cost is a function of his efficiency and the quantity provided and is denoted by $C(x,\theta)$.

The following assumptions are the standard ones and come from the economic literature (see Baron and Myerson (1982)):

Assumption 1. $\lambda(\theta) = \dfrac{F(\theta)}{f(\theta)}$ *is a well-defined function on* $[0,1]$ *(*f *is continuously differentiable with strictly positive support on* $[0,1]$*) and is nondecreasing as a function of*

θ.

Assumption 2. $C_\theta > 0, C_{x,\theta} > 0, C_{xx} \geq 0, C_{xx\theta} \geq 0, C_{x\theta\theta} \geq 0$, $\forall x, \theta$.

Using the Revelation Principle [3], the regulator offers the firm the possibility to self-select from a menu of contracts specifying pairs of price and transfers $\{p(\theta), T(\theta)\}$. One can think about the transfer $T(\theta)$ as being a subsidy (in this case, the pricing decision belongs to the regulator, but she has to pay a subsidy to the firm when the price doesn't fully cover the cost) or as tax (the agent decides the price, but he has to pay a tax corresponding to different price levels). The benevolent regulator solves a maximisation problem of the weighted sum of consumer surplus and the profit obtained by the firm. If we denote by S the surplus and by Π the profit of the firm, the principal will maximise $S + \alpha\Pi$, where α is the importance that the principal gives to the rent left to the agent. The usual assumption is that $\alpha \in [0,1)$ as the principal is more concerned with the taxpayers' welfare. This hypothesis is quite important as without it the solution to the principal's problem will be to simply allow the firm to be the decision maker and to maximise its profit.

We write the decision maker's problem as follows:

$$\int_0^1 \{S(p(\theta)) - T(\theta) + \alpha[\Pi(p(\theta), T(\theta); \theta)]\} f(\theta)\, \mathrm{d}\theta$$

subject to the participation constraint and, respectively, the incentive compatibility constraint:

$$\Pi(p(\theta), T(\theta); \theta) \geq 0^{4}$$

$$\Pi(p(\theta), T(\theta); \theta) \geq \Pi(p(\hat{\theta}), T(\hat{\theta}); \theta), \ \ \forall \theta, \hat{\theta} \in [0,1]$$

After some computations (which can be found in Baron and Myerson (1982)) and

[3] The Revelation Principle is a powerful tool that tells us the principal can focus without loss of generality on a direct revelation mechanism that is individual rational and incentive compatible.

[4] We set the agent's reservation profit to 0.

denoting by $V(x) = \int_0^x P(u)\,\mathrm{d}u$ and $P(x)$ the inverse demand function [5], one can write the previous problem in an equivalent way:

$$\int_0^1 \left\{ V(x(\theta)) - C(x(\theta),\theta) - (1-\alpha)\frac{F(\theta)}{f(\theta)} C_\theta(x(\theta),\theta) \right\} f(\theta)\,\mathrm{d}\theta$$

s.t.:

$$T(\theta) = P(x(\theta))x(\theta) - C(x(\theta),\theta) - \int_\theta^1 C_\theta(x(u),u)\,\mathrm{d}u$$

$$p(\theta) = P(x(\theta))$$

and

$$x(\theta) = X(p(\theta)) \text{ is weakly decreasing}^{6}.$$

The optimal contract under the presence of asymmetric information is characterised by the following First Order Condition:

$$P(x(\theta)) = C_x(x(\theta),\theta) + (1-\alpha)\frac{F(\theta)}{f(\theta)} C_{x\theta}(x(\theta),\theta) \tag{1}$$

One can see from (1) that a higher inefficiency is associated with a higher price (the benchmark being price equals marginal cost).

3 Econometric model

The econometric model is mainly based on a analysis of the First Order Condition presented in (1). The data available to the econometrician consists of prices, traded quantities and transfers. Therefore one can observe the left-hand side variable of the (1) (i.e. the prices specified in the contract). The unobservables are the cost function and the

[5] $S(p) = V(x) - P(x)x$.

[6] The monotonicity of the hazard ratio imposed in Assumption 1 is crucial for the monotonicity of the quantity with respect to the cost parameter (and thus to avoid pooling)

distribution of the efficiency parameters.

Let us adopt a simple framework where we suppose that the cost function is known and has the following shape: $C(x,\theta) = \theta x$ [7]. Later we will study more complex specifications of this model.

Thus we observe an i.i.d. sample of the random element p defined by:

$$p = \theta + \frac{F(\theta)}{f(\theta)} \tag{2}$$

Our goal is to estimate the structural parameter F from the observations $(p_i)_{i=1,\dots,n}$. Remember that the principal knows the distribution from which the costs are drawn, but this is unobservable for the econometrician.

We concentrate our analysis on equation (2) which is a particular case of a general class of game-theoretic models characterized by a relation between an observable variable p and an inobservable variable θ of the form $p = \sigma_F(\theta)$. The σ_F is the structural element of the model and is assumed to be a Bayesian Nash Equilibrium. The strategic component of the game is formalized by the fact that σ depends on θ and on its distribution, F. As remarked by Florens et al. (1998) and Florens and Sbaï (2010), the dependence of σ on F modifies the structural properties of the model (identification and estimation). This paper illustrates the importance of this dependence relation.

3.1 Global identification

The first step in any econometric analysis is to check if the model is identified, i.e., if there is a one-to-one mapping between the distribution of the latent variables and the distribution of observables.

The distribution of primitives and the distribution of observables in our simple model of adverse selection are related by the following functional relation:

[7] We also suppose that the weight on the firm's profit, α, is 0.

$$G(p) = P(P \leq p) = P(\sigma_F(\theta) \leq p) = F(\sigma_F^{-1}(p)) \tag{3}$$

where G denotes the cdf of p and $\sigma_F(\cdot)$ is an increasing function of its argument. One can note that $G(\cdot)$ is identified by the data. From (3) we obtain the implicit relationship between F and G:

$$F \circ \sigma_F^{-1} = G \text{ or } F = G \circ \sigma_F \tag{4}$$

G is identified and can be estimated by a natural estimator $\widehat{G}$. The equation of interest is thus the following

$$\widehat{G} = F \circ \sigma_F^{-1} + U$$

where U is the estimation noise.

The previous equation can be written in a quantile version:

$$G^{-1} = \sigma_F \circ F^{-1} \Rightarrow \widehat{G^{-1}} = \sigma_F \circ F^{-1} + V$$

where again V is the estimation noise. This paper is based on the analysis of this quantile equation. G^{-1} is estimated by the quantile function of the observables and F^{-1} is the solution of an inverse problem.

The identification of the model can be easily assessed by solving the quantile equation $G^{-1} = \sigma_F \circ F^{-1}$ (as we are in the identification stage, the population distribution of the observable is supposed to be known and therefore there is no noise). The model is identified if the equation $G^{-1}(\alpha) = F^{-1}(\alpha) + \alpha F^{-1'}(\alpha)$ has a unique solution in F^{-1}. This is a first order differential equation whose solution is

$$F^{-1}(\alpha) = \frac{1}{\alpha} \int_0^{\alpha} G^{-1}(u)\, \mathrm{d}u \tag{5}$$

Therefore the model is non parametrically globally identified and without any over iden-

tifying restriction. Proposition (1) below summarises this finding:

Proposition 1. *In a pure classical model of adverse selection, the distribution of cost parameters $F(\theta)$, is globally nonparametrically identified without any restrictions and the quantile function corresponding to it, $F^{-1}(\alpha)$, is described by the following relation:*

$$F^{-1}(\alpha) = \frac{1}{\alpha}\int_0^{\alpha} G^{-1}(u)\,\mathrm{d}u \tag{6}$$

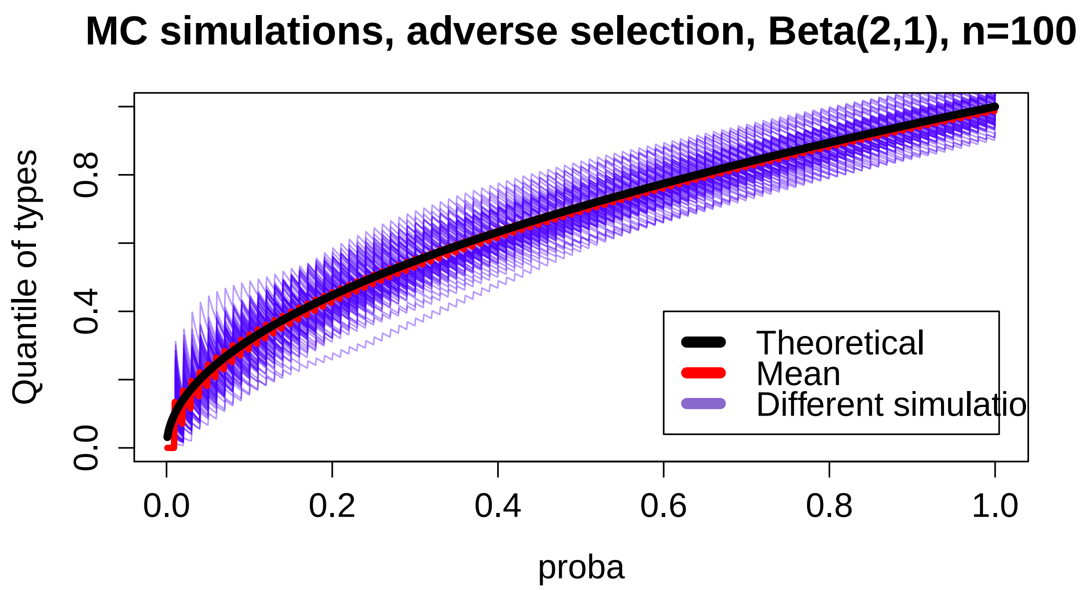
where $\alpha \in [0,1]$, $F^{-1}(\alpha)$ is the quantile function of the productivity/cost parameters and $G^{-1}(\alpha)$ is the quantile function of prices.

3.2 Quantile estimation

The estimation of the quantile function of types is conducted by using the order statistics of the data, in our case the prices. Thus if $X_{(i:n)}$ denotes the i^{th} order statistic of the sample of size n, we know that $\widehat{G^{-1}}(u) = \sum_{i=1}^{n} X_{(i:n)} \mathbb{1}\left\{\frac{i-1}{n} < u \leq \frac{i}{n}\right\}$. If we plug this expression into (6), we get that:

$$\widehat{F^{-1}}(\alpha) = \frac{1}{\alpha}\int_0^{\alpha} \widehat{G^{-1}}(u)\,\mathrm{d}u = \frac{1}{\alpha n}\sum_{i=1}^{n} \mathbb{1}\{\frac{i}{n} \leq \alpha\} X_{(i:n)} \tag{7}$$

We conduct different configurations of simulations to check that our estimator performs well in small samples (n=100 and the types are drawn from Beta distributions with different parameters).


MC simulations, adverse selection, Beta(2,1), n=100
Quantile of types
0.0
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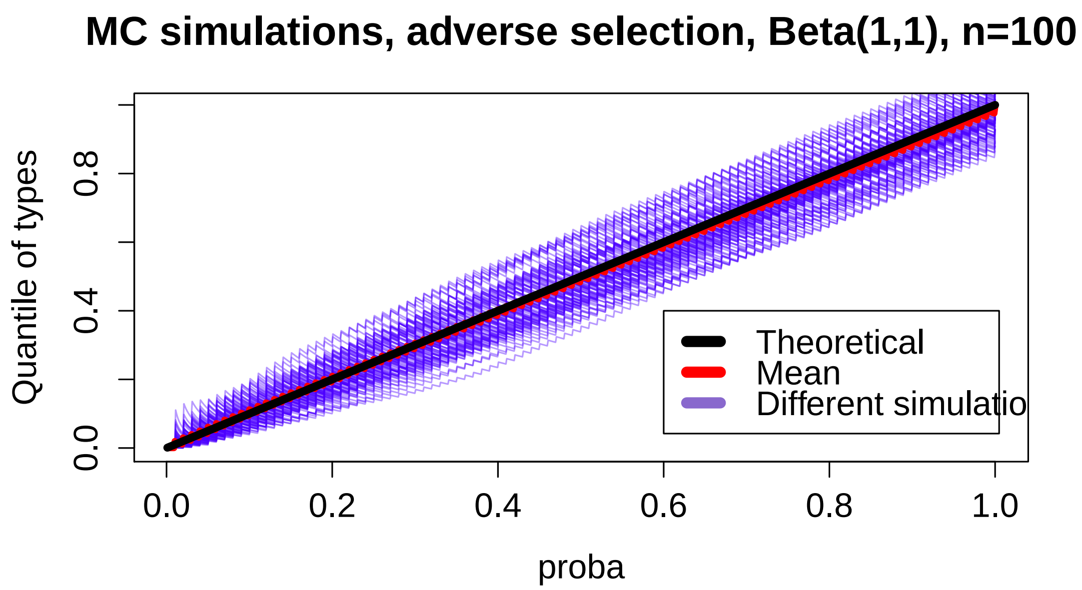
MC simulations, adverse selection, Beta(1,1), n=100
Quantile of types
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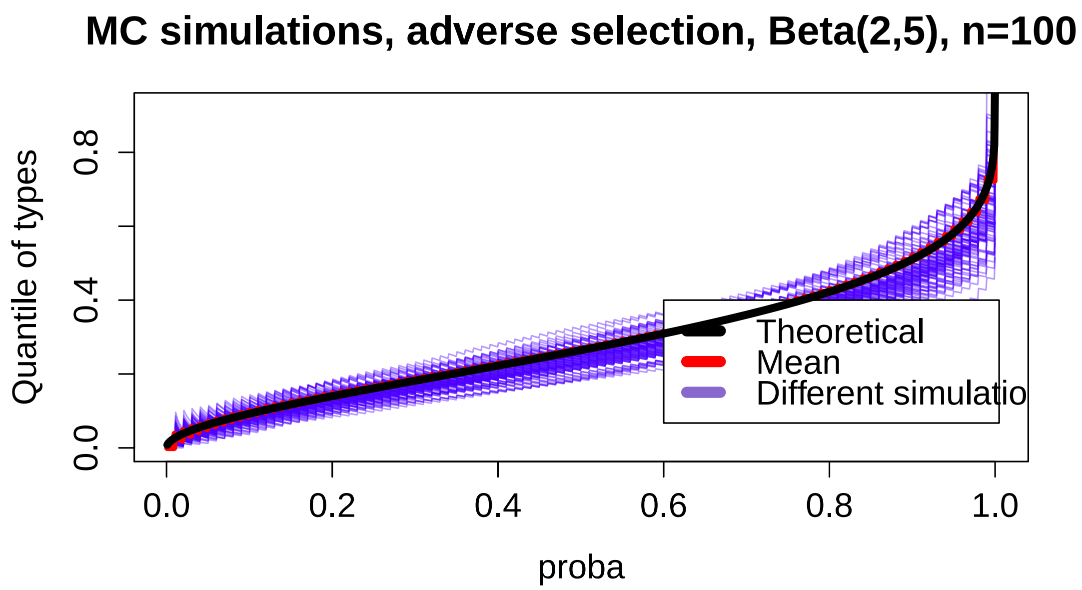

3.3 Asymptotics properties

Our main tool is a general theorem on the asymptotic behaviour of linear functions of order statistics (see Moore (1968), Stigler (1969), Stigler (1974)) [8]. We will first define the "L-estimators". Thus if $P_{(i:n)}$ denotes the i^{th} order statistic of an iid sample of prices of size n and drawn from the cdf G , with $P_{(1:n)} \leq ... \leq P_{(n:n)}$, then a L-estimator is a linear function of order statistics with the following shape:

$$T_n = \frac{1}{n}\sum_{i}^{n} J\left(\frac{i}{n}\right) P_{(i:n)}$$

where J is a well-behaved function defined on $[0,1]$.

The idea is to write the $\widehat{F^{-1}}$ as a L-estimator and to show its asymptotic normality. In

[8]The so-called "L-estimators".

our particular case $J\left(\frac{i}{n}\right) = \frac{1}{\alpha}\mathbb{1}\left(\frac{i}{n} \leq \alpha\right)$ and therefore we have:

$$
\begin{aligned}
\widehat{F^{-1}}(\alpha) =& \frac{1}{n}\sum_{i=1}^{n}\frac{1}{\alpha}\mathbb{1}\left(\frac{i}{n} \leq \alpha\right)P_{(i:n)} = \frac{1}{n}\sum_{i=1}^{n}J\left(\frac{i}{n}\right)P_{(i:n)} \\
=& \int_0^1 J(\alpha)\widehat{G^{-1}}(\alpha)\,\mathrm{d}\alpha = \int_{-\infty}^{\infty} pJ(\widehat{G}(p))\,\mathrm{d}\widehat{G}(p)
\end{aligned}
$$

It has been showed (see Moore (1968), Stigler (1969), Stigler (1974)) the following result:

Theorem 1. *The linear functions of order statistics, $T_n = \frac{1}{n}\sum_{i}^{n}J\left(\frac{i}{n}\right)P_{(i:n)}$ is asymptotically normal*

$$
\sqrt{n}\left[T_n - \int_0^1 J(\alpha)G^{-1}(\alpha)\,\mathrm{d}\alpha\right] \rightsquigarrow N(0,\sigma^2)
$$

where

$$
\begin{aligned}
\sigma^2 =& 2\int\int J(G(p))J(G(r))(G(p\wedge r) - G(p)G(r))\,\mathrm{d}p\mathrm{d}r \\
=& \int_0^1\int_0^1 J(\alpha)J(\beta)(\alpha\wedge\beta - \alpha\beta)\,G^{-1}(\mathrm{d}\alpha)G^{-1}(\mathrm{d}\beta)
\end{aligned}
$$

if P_i have finite mean, $\sigma^2 < \infty$ and J is continuously differentiable s.t. J' has bounded variation on $[0,1]$ (possibly except at a finite number of points).

Using Moore (1968)'s theorem we obtain the following proposition for our model:

Proposition 2. *In a static adverse selection model, the empirical process of the quantile function of types converge in distribution to a gaussian process in $L^2_{[0,1]}$ at the parametric speed of convergence, $\sqrt{n}$:*

$$
\sqrt{n}\left(\widehat{F^{-1}}(t) - F^{-1}(t)\right) \rightsquigarrow N(0,\Sigma) \tag{8}
$$

where Σ denotes the asymptotic variance and has the following form:

$$\sigma(s,t) = \int_0^1 \int_0^1 \mathbb{1}(u \le s)\mathbb{1}(v \le t)(u \wedge v - uv)\, G^{-1}(\mathrm{d}u) G^{-1}(\mathrm{d}v)$$

For the estimation of the cdf function, the approach is quite straightforward:

$$\widehat{F}(\theta) = \frac{1}{n} \sup \left\{ i | \widehat{F^{-1}} \left(\frac{i}{n} \right) \le \theta \right\}$$

Another way of estimating the cdf would be to construct a smooth version of $\widehat{F^{-1}}$ and then compute the usual inverse function.

Next if we apply the delta method, we can easily see that the empirical process of the cdf of the types converges to a Gaussian process in $L^2_{[0,1]}$. Before checking the convergence of the pdf, we need to compute the derivative of the quantile function:

$$F^{-1'}(\alpha) = -\frac{1}{\alpha^2} \int_0^\alpha G^{-1}(u)\, \mathrm{d}u + \frac{1}{\alpha} G^{-1}(\alpha)$$

One can easily show that $\sqrt{n}\left(\widehat{F^{-1'}} - F^{-1'}\right)$ also converges to a Gaussian process (as $F^{-1'}$ is a function of G^{-1} and of an integral of G^{-1}). Moreover, the probability distribution function can be written as:

$$f(\theta) = \frac{1}{F^{-1'}(F(\theta))} \Rightarrow \widehat{f}(\theta) = \frac{1}{\widehat{F^{-1'}}(\widehat{F}(\theta))}$$

Using again delta theorem, we get that $\sqrt{n}(\widehat{f} - f) \rightsquigarrow$ to a Gaussian process.

Note that the estimation of f at a $\sqrt{n}$ speed is not in contradiction with the minimax speed of convergence of a density. Actually this result applies if f is the density of the observed data, which is not the case in our example. The density f is not the density of the data. The speed of convergence of $\widehat{f}$ is due to the strategic feature of the game model which may improve or diminish the speed of convergence.

4 Extensions

In this section we will present two extensions of the previously described model. Both extensions will relax the assumption about the linearity of the cost function by allowing for two semi-parametric specifications of the agent's cost functions.

We assume the cost function as being given, but we just adopt a more flexible form of it.

4.1 First functional form

Let us first consider the following semiparametric specification of the cost function $C(x,\theta) = \frac{x^2}{2} + (1-\varphi(\theta))x$. Then the equation (1) becomes:

$$x - p + 1 = \varphi(\theta) + \frac{F(\theta)}{f(\theta)}\varphi'(\theta)$$

We denote $x - p + 1 = y$ and then our observations become y. We suppose that y is distributed with H and H^{-1} is the quantile function associated to the data. We have that $G^{-1} = \sigma \circ F^{-1}$, which in this case leads to:

$$H^{-1}(\alpha) = \varphi\left(F^{-1}(\alpha)\right) + \frac{\alpha}{f \circ F^{-1}(\alpha)}\varphi'\left(F^{-1}(\alpha)\right)$$

equivalently to:

$$H^{-1}(\alpha) = \varphi\left(F^{-1}(\alpha)\right) + \alpha F^{-1'}(\alpha)\varphi'\left(F^{-1}(\alpha)\right)$$

or

$$H^{-1}(\alpha) = \left(\alpha\varphi\left(F^{-1}(\alpha)\right)\right)'$$

The solution to this equation is:

$$F^{-1}(\alpha) = \varphi^{-1}\left[\frac{1}{\alpha}\int_0^{\alpha} H^{-1}(u)\,\mathrm{d}u\right]$$

Let us suppose that one can have from the data an estimation for φ^{-1}, therefore a straightforward estimator for the quantile of latent variables is:

$$\widehat{F^{-1}}(\alpha) = \widehat{\varphi^{-1}}\left[\frac{1}{\alpha}\int_0^{\alpha} \widehat{H^{-1}}(u)\,\mathrm{d}u\right]$$

Using the Fréchet derivatives we have that:

$$\widehat{F^{-1}}(\alpha) - F^{-1}(\alpha) = \varphi^{-1'}\left[\frac{1}{\alpha}\int_0^{\alpha}\left(\widehat{G^{-1}} - G^{-1}\right) u\,\mathrm{d}u\right] + \left(\widehat{\varphi^{-1}} - \varphi^{-1}\right)\left[\frac{1}{\alpha}\int_0^{\alpha} G^{-1}(u)\,\mathrm{d}u\right] + o(1)$$

The first term converges at speed $\sqrt{n}$ and therefore the speed of convergence for $\widehat{F^{-1}}(\alpha) - F^{-1}(\alpha)$ depends on the speed of convergence of the second term (i.e. whether we use nonparametric or parametric methods of estimation for $\widehat{\varphi^{-1}}$).

4.1.1 A second functional form

A second approach assumes that $C(x,\theta) = \theta C(x)$ and in this case the FOC (1) becomes:

$$P(x(\theta)) = \theta C'(x) + \frac{F(\theta)}{f(\theta)} C'(x)$$

or equivalently

$$P(x(\theta)) = C'(x)\left(\theta + \frac{F(\theta)}{f(\theta)}\right)$$

This can be written under the quantile form as:

$$G^{-1}(\alpha) = C'(x)\left(\alpha F^{-1}(\alpha)\right)'$$

The solution to this equation is simply:

$$F^{-1}(\alpha) = \frac{1}{C'(x)} \frac{1}{\alpha} \int_{0}^{\alpha} G^{-1}(u)\,\mathrm{d}u$$

If we denote $\frac{1}{C'(x)} = a(x)$ and we suppose that there is available an estimation of a denoted with $\widehat{a}$ we have the following result:

$$\widehat{F^{-1}} - F^{-1} = (\widehat{a} - a)\left(\frac{1}{\alpha}\int_{0}^{\alpha} G^{-1}(u)\,\mathrm{d}u\right) + a\left(\frac{1}{\alpha}\int_{0}^{\alpha}\left(\widehat{G^{-1}} - G^{-1}\right)(u)\,\mathrm{d}u\right) + o(1)$$

This formula shows that if a is estimated at a parametric speed, this will also be the case for $\widehat{F^{-1}} - F^{-1}$. Therefore the first term is the one that dictates the speed of convergence of our estimator, as $\widehat{G^{-1}} - G^{-1}$ converges at a parametric speed.

5 Conclusions

The presence of asymmetric information between the regulator and the agency leads to the implementation of a less efficient outcome (the so-called "second best") than the benchmark outcome (the "first best") achieved in case of symmetric information. Providing estimates for the primitives of supplier's production process is crucial as it guides the regulator in the rate-setting process. Besides this, one can test the optimality of the contracts being implemented and also conduct counterfactuals to see under which payment schemes the consumer surplus would be increased. Once one has estimated the distribution of supplier's costs and the cost function, another policy simulation that can be envisioned is to check what are the effects on the level of prices and quantities of letting another supplier enter the market for the public service.

In this paper we provide a framework for the identification and estimation of the distribution of agent's types in a static case of pure adverse selection where the cost

function is known. We find that the distribution of costs is globally identified without any restrictions (except those underlying the economic model). Next we provide a quantile estimation for the quantile of types and using the order statistic theory we show that the estimation of the density function of the types is done at a root-n speed of convergence. We also briefly present two extensions where we allow for semi-parametric forms of the cost function. We show that in this latter case, the speed of convergence of the density of latent variables is not necessary root-n, but it depends on the speed at which the cost function is estimated.

References

Mark Armstrong and David EM Sappington. Recent developments in the theory of regulation. *Handbook of industrial organization*, 3:1557–1700, 2007.

David P Baron. Design of regulatory mechanisms and institutions. 1989.

David P. Baron and David Besanko. Regulation and information in a continuing relationship. *Information Economics and Policy*, 1(3):267–302, 1984.

David P Baron and Roger B Myerson. Regulating a monopolist with unknown costs. *Econometrica: Journal of the Econometric Society*, pages 911–930, 1982.

Marco Battaglini and Rohit Lamba. Optimal dynamic contracting. *Economic Theory Center Working Paper*, (46-2012), 2012.

David Besanko. Multi-period contracts between principal and agent with adverse selection. *Economics Letters*, 17(1):33–37, 1985.

C. Bontemps, D. Martimort, and A. Thomas. Estimating optimal contracts under delegated management in the french water sector. 2010.

Pierre-Andre Chiappori and Bernard Salanié. Testing contract theory: A survey of some recent work. 2002.

X. d'Haultfoeuille and P. Février. Identification of a class of adverse selection models with contracts variation. 2010.

Andreea Enache and Jean-Pierre Florens. Identification and estimation in a third-price auction model. 2012.

Heinz Werner Engl, Martin Hanke, and Andreas Neubauer. *Regularization of inverse problems*, volume 375. Springer, 1996.

Antonio Estache and Liam Wren-Lewis. Toward a theory of regulation for developing countries: Following jean-jacques laffont's lead. *Journal of Economic Literature*, pages 729–770, 2009.

J.P. Florens and E. Sbaï. Local identification in empirical games of incomplete information. *Econometric Theory*, 26(06):1638–1662, 2010.

J.P. Florens, C. Protopopescu, and J.F. Richard. *Identification and estimation of a class of game theoretic models*. Institut d'economie industrielle, Université des Sciences sociales de Toulouse, 1998.

E. Guerre, I. Perrigne, and Q. Vuong. Optimal nonparametric estimation of first-price auctions. *Econometrica*, 68(3):525–574, 2000.

Cheng Hsiao. Identification. 1983.

Leonid Hurwicz. Generalization of the concept of identification. *Statistical Inference in Dynamic Economic Models (T. Koopmans, ed.). Cowles Commission, Monograph*, 10: 245–257, 1950.

Marc Ivaldi and David Martimort. Competition under nonlinear pricing. *Annales d'Economie et de Statistique*, pages 71–114, 1994.

Paul L Joskow. Regulation of natural monopoly. *Handbook of law and economics*, 2: 1227–1348, 2007.

Jan Jung. On linear estimates defined by a continuous weight function. *Arkiv för matematik*, 3(3):199–209, 1956.

Tjalling C Koopmans and Olav Reiersol. The identification of structural characteristics. *The Annals of Mathematical Statistics*, pages 165–181, 1950.

J.J. Laffont and D. Martimort. *The theory of incentives: the principal-agent model*. Princeton University Press, 2001.

J.J. Laffont and J. Tirole. *A theory of incentives in procurement and regulation*. MIT press, 1993.

Rosa L Matzkin. Nonparametric identification. *Handbook of Econometrics*, 6:5307–5368, 2007.

DS Moore. An elementary proof of asymptotic normality of linear functions of order statistics. *The Annals of Mathematical Statistics*, 39(1):263–265, 1968.

Isabelle Perrigne and Quang Vuong. Nonparametric identification of a contract model with adverse selection and moral hazard. *Econometrica*, 79(5):1499–1539, 09 2011.

Thomas J Rothenberg. Identification in parametric models. *Econometrica: Journal of the Econometric Society*, pages 577–591, 1971.

R.J. Serfling. Approximation theorems of mathematical statistics. *New York*, 1980.

Stephen M Stigler. Linear functions of order statistics with smooth weight functions. *The Annals of Statistics*, pages 676–693, 1974.

Stephen Mack Stigler. Linear functions of order statistics. *The Annals of Mathematical Statistics*, pages 770–788, 1969.

Frank A Wolak. An econometric analysis of the asymmetric information, regulator-utility interaction. *Annales d'Economie et de Statistique*, pages 13–69, 1994.