Optimal Banking Regulation with Endogenous Liquidity Provision

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very, very preliminary, please do not circulate

Abstract

In a money-search model where deposits are used as means-of-payments, banks have expertise to obtain higher returns from assets with a cost and an economy of scale but are subject to limited commitment and moral hazard. They can pledge a proportion of asset holdings to issue deposits. Optimal regulation trades off efficiency in asset-management and liquidity service banks provide. An optimal charter system restricts banking licence to crate profits for banks to sustain a leverage ratio above the laissez-faire level to improve liquidity. As moral hazard becomes more serious, optimal regulation allows banks to be larger and have higher profits to compensate for stricter capital requirement due to moral hazard. When banks are heterogenous, it is optimal to allow higher leverage for larger banks. With uncertainty on bank returns, deposit insurance is optimal as it makes bank liabilities information insensitive. Finally, with moral hazard under deposit insurance, we show that it is not always optimal to exclude gambling behavior in equilibrium.

Keywords: Capital requirement, banking, moral hazard, deposit insurance

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1 Introduction

The serious interruption of the real economy from the Global Financial Crisis of 2008 has given rise to a renowned interest in understanding the role of financial intermediaries and how to regulate them. Two particular issues have surfaced both in the mass media and in the policy debate: first, bankers seem to make unjustified profits;[1] second, the banking sector seem to be too concentrated in few big banks. These issues surfaced to the public domain partly because the banking sector has been under two government protections: the deposit insurance that allows them to raise more deposits, and government bail-outs to many banking failures. These privileges seem even more unreasonable as it has been difficult to persecute any potential fraud in the sector.[2]

While some may take for granted that these protections and regulations that lead to big banks and high profit are undesirable, to have a meaningful debate we need to first understand the role of financial intermediaries in the working of the economy and to understand why regulations may be necessary. One particular aspect that may result in externality which requires regulation is the liquidity role banks' liabilities serve;[3] in most advanced economies, the majority of money supply consists of bank deposits. This role, which is mainly concerned with bank liabilities, motivates various regulations that promote stability, as bank failures would affect not only banks' shareholders but also the welfare of the general public who rely on banks' liquidity services. This liquidity service is provided by an asset transformation process: while banks issue deposits on the liability side, they also hold various assets to back those deposits. This process, as the financial crisis reveals, involves many credit mar-

[1]In a comment about the Dodd-Frank reform, New Yorker article ("Banking's New Normal," 2016 issue) has argued that "Bankers still make absurd amounts of money."

[2]For a popular view on difficulty in such persecutions, see New Yorker article, "Why Corrupt Bankers Avoid Jail," 2017 issue.

[3]See, for example, the Controversies section in *Economic Journal*, issue 106, May 1996, where all articles mentioned that banking is special because they produce "money," or assets that can be used as means-of-payments.

ket frictions from the banks, such as uncertainty to bank returns and limited commitment. The feature that banks supply liquidity and they are subject to frictions has important macroeconomic consequences, and thus, implications on policy and regulations.

In this paper, we propose a model of financial intermediaries with endogenous liquidity provision. We do this by introducing *banks* into a standard monetary model à la Lagos and Wright (2005) to maintain tractability. On the asset side, banks are the only agents with the necessary expertise to manage/monitor long-term loans (modeled as Lucas trees) to receive dividends. There is economy of scale in the sector by way of a fixed cost of operation. On the liability side, banks may issue deposits to finance their asset holdings, and, under the usual frictions that render means-of-payments essential (lack of commitment and monitoring) from the depositor side, this can generate a higher profits to banks by doing so. We consider two main frictions in the banking sector. First, banks cannot fully commit to honor their future obligations; instead, they could only credibly pledge a fraction of their assets that can be seized by the court upon bankruptcy. This friction constraints the amount of liquidity banks can provide and may prevent the first-best level of consumption for the depositors to be achieved. Second, the banks' efforts in managing the assets may not be observable and this moral hazard issue may hinder the liquidity role of the banks.

We first consider the limited commitment of the banks but no moral hazard. When banks can only make static contracts, the amount of deposits a bank can issue is constrained by limited pledgeability of assets through market discipline; no one would deposit in a bank unless it can credibly repay. Under free entry of banks, bank sizes are determined by a zero-profit condition that balances the variable cost of asset management and the fixed cost of entry, which coincide with the efficient level of asset holdings as far as the asset-management is concerned for the economy of scale. However, unless the pledgeability constraint is slack, depositors cannot achieve first-best level of consumption due to lack of liquidity and asset pricing exhibits liquidity premium. This pledgeability constraint also implies a capital requirement impose by the market: the bank will not repay any deposit beyond what is pledgeable because of limited commitment and hence the difference has to be financed by bank capital.

Against this free market arrangement, we show that a charter system with a banking regulator can improve social welfare. Under the charter system, the regulator can shut down a bank when it does not honor its obligation and hence allows for a dynamic incentive to relax the pledgeability constraint. For this dynamic incentive to be effective, however, it is necessary to limit the number of charters relative to the efficient number under free entry and to allow banks to earn economic profits. This scheme makes it incentive feasible for banks to issue unsecured deposits beyond the pledgable assets they own, and hence can increase the leverage ratio of banks. The optimal policy then trades off two inefficiencies: on the one hand, a smaller number of charters increases bank profits and hence helps increase liquidity, which improves depositors' welfare; on the other hand, a smaller number of charters increases the overall cost of banking operations as each one gets inefficiently large. Our main result demonstrate that, whenever liquidity is tight under static bank contracts, it is optimal to limit the number of charters relative to the number under efficient asset management, and to relax the pledgeability constraint through a lower overall leverage ratio requirement that its laissez faire level.

We then introduce the moral hazard issue with asset-management by banks, in which banks may shirk on their asset management and obtain a lower return, although doing so would be socially suboptimal. Precisely because of the two-sided nature of bank contracts, banks may have incentives to shirk, as that may increase their profits by lowering the cost while the depositors have to suffer the consequences. We show that, under static contract and free-market arrangement, market discipline would impose an additional proportional capital requirement to a bank's asset holding to ensure efforts. This, however, can be harmful to liquidity provision as it lowers the level of deposits banks can offer. In particular, as moral hazard becomes more serious, the liquidity service becomes poorer. In contrast, under the optimal charter system, although an additional proportional capital requirement is also necessary, the regulator would adjust the overall leverage ratio requirement as well as the number of licences to compensate for the liquidity loss. In particular, we show that as moral hazard becomes more serious, the optimal response is to allow for higher profits to sustain a higher unsecured deposit issuance, and to make each banks larger.

We extend our model to address two policy debates. The first is the optimal sizes of banks. In our model, bank sizes are endogenously determined by either free entry (in the absence of charter), or by the number of charters. We extend our model by allowing for heterogeneous management costs. In the absence of regulation, more efficient banks end up being larger in terms of asset holding, and it is efficient to do so. When liquidity is tight, we show that an optimal charter system would in fact make large banks even larger by allowing more generous unsecured borrowing. The intuition is simple: when the number of charters is limited, large banks make higher profits and hence it is more efficient to incentivise them to repay unsecured deposits. As a result, we obtain a positive correlation between bank size and leverage ratio under the optimal policy arrangement.

Second, we consider dividend uncertainty where the return to each bank's assets are subject to an idiosyncratic shock that affects all assets the bank holds. Moreover, we assume that depositors receive noisy signals regarding the shocks and hence the value of the deposits may be affected when used as means-of-payments. We show that as the noisy signal becomes more precise, the presence of shocks becomes a bigger hinderance to liquidity provision. A deposit insurance scheme that charge a premium on bank returns to bail out troubled banks, however, can make deposit contracts information insensitive again, and hence improve welfare. Nevertheless, as argued in the literature, deposit insurance may further intensify the moral hazard issue. To understand this issue, we introduce moral hazard in the following way: banks may secretly direct the asset to a more risky projects that have higher return to the bank that is not observable to the general public (and hence not subject to repayment to depositors) but have lower overall expected return. In the absence of a further capital requirement, there exists an equilibrium where all banks gamble. However, introducing a harsh capital requirement faces a new trade-off that is not in the literature: on the one hand, it discourages gambling and hence increase overall return; on the other hand, it directly decreases liquidity provision. We show that how the trade-off resolves would depend on the fundamentals.

Literature review

This paper is not the first one to point out that future bank profits play an essential role in banking regulations. On the empirical side, Keeley (1990) provides some evidence that charter value restricts banks' risk-taking behavior. On the theory side, Hellmann, Murdock, and Stiglitz (2000), in a model where banks have market powers and face moral hazard, show that it is optimal to use a combination of capital requirement and deposit-rate ceilings to create sufficient franchise value for banks to ameliorate the moral-hazard problem. Future bank profits are the main incentive device for prudent behavior, using deposit-rate ceiling to maintain profits. In contrast, profits are amintained by restricting entry and deposit-rate ceilings would be sub-optimal in our model. Two main modeling ingredients explain the difference: first, while deposit demand is exogenously given there, in our model it is driven by endogenous liquidity needs; second, asset prices (and hence returns to the loans) are endogenously determined in our model.

Our paper is also related to the literature on liquidity provision by banks. Using a means-of-payment-in-advance model with currency and deposits, Chari and Phelan (2014) show that if there is insufficient deflation, fractionally backed banks which offer interest-bearing deposits may be good, but such banks are subject to socially costly runs. Williamson (2016), shows that, when banks face limited commitment, and when short-maturity government debt has a greater degree of pledgeability than long-maturity government debt, quantitative easing can improve liquidity. These papers, however, do not address optimal financial regulations. Gorton and Winton (2017) also features a trade-off of raising capital requirement because bank debt is used for transactions purposes, while more bank capital can reduce the chance of bank failure; however, they assume exogenously given banks' charter value. Phelan (2016), in a model where deposits serve the liquidity function, shows that leverage also increases asset price volatility and so limiting leverage decreases the likelihood that the financial sector is undercapitalized. However, the model assumes that deposits exogenously generate utility to depositors, and hence it is then not clear how regulations may affect banks' function in providing means-of-payment, and through which, the economic activity.

Our rationale for deposit insurance differs from the usual Diamond-Dybvig (1983) motive but is related to Gorton, Holmstrom, Ordonez (2017), who show that, to produce money-like safe liquidity, banks keep detailed information about their loans secret, which provides a rationale for opaque banking examinations and capital requirements, and deposit insurance. With a similar argument, we show that deposit insurance makes deposit contract insensitive to information. This motive also implies a different policy recommendation; in contrast to most of the literature, we show that once we take endogenous liquidity provision into account, it is not necessary desirable to eliminate gambling behavior.

2 The Environment

The environment is borrowed from Rocheteau and Wright (2005). Time is discrete and has an infinite horizon, $t \in \mathbb{N}_0$. The economy is populated by three sets of agents; each set has a continuum of infinitely-lived agents with measure one. The first set consists of *buyers*, denoted by $\mathbb{B}$, and the second consists of *sellers*, denoted by $\mathbb{S}$. The third set consists of potential *banks.* Each date has two stages: the first has pairwise meetings of buyers and sellers in a decentralized market (called the DM), and the second has centralized meetings (called the CM) where all agents meet. In each DM, the probability that a buyer has a successful meeting with a seller is σ. There is a single perishable good produced in each stage, with the CM good taken as the numéraire. Agents' labels as buyers and sellers depend on their roles in the DM where only sellers are able to produce and only buyers wish to consume. While all agents can produce and consume in the CM, potential banks do not consume nor produce in the DM.

Buyers' preferences are represented by the following utility function

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[u(q_t)+x_t-h_t\right],$$

where $\beta \equiv (1+r)^{-1} \in (0,1)$ is the discount factor, q_t is DM consumption, x_t is CM consumption, and h_t is the supply of hours in the CM. Sellers' preferences are given by

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[-c(q_t)+x_t-h_t\right],$$

where $c(q)$ is the seller's disutility of producing q in the DM. The first-stage utility functions, $u(q)$ and $-c(q)$, are increasing and concave, with $u(0) = v(0) = 0$. The surplus function, $u(q)-c(q)$, is strictly concave, with $q^* = \arg\max\left[u(q)-c(q)\right]$. Moreover, $u'(0) = c'(\infty) = \infty$ and $c'(0) = u'(\infty) = 0$. All agents have access to a linear technology to produce the CM output from their own labor, $x = h$.

There is only one type of real assets, Lucas trees, which are long-lived. Each unit of the trees pays off dividend τ units of the CM goods at the beginning of CM. The average supply (per buyer) of the trees is $\bar{A}$. To receive dividends from the Lucas trees, however, it requires a potential bank to perform costly monitoring/management. This assumption is in line with the delegated monitoring model of financial intermediaries proposed by Williamson

(1986) or Diamond (1984).[4] In contrast to those papers, however, our main focus is on the role of banks in providing liquid deposits as means-of-payments. Specifically, buyers may use the bank's liability, or *deposits*, to finance their consumptions in the DM. Sellers have a technology that can access the records in the bank, and, upon buyers' agreement, may transfer deposits to the sellers' accounts in the bank. For instance, in a DM transaction the buyer gives to the seller a claim on deposits, which the seller may present in the CM to the bank to acquire funds.

There are two frictions associated with this financial intermediation. The first friction is the cost associated with managing/monitoring the asset. Only active banks can hold assets and issue deposits; to become active, a bank has to pay a fixed cost of γ each period. There is also a marginal cost of asset-management: for a banker to hold a units of assets, he needs to pay $\psi(a)$ (as a labor cost) to monitor/manage the assets. We assume that $\psi(0) = 0$, $\psi(a)$ is strictly increasing and strictly convex, and $\psi(\bar{A}) = \infty$.

Second, banks have limited liability and cannot commit to their future actions. However, we assume that if a bank files for bankruptcy, the court could seize ρ proportion of his assets. Thus, by holding a units of Lucas trees, a bank can credibly pledge ρ fraction of its value (plus dividends) but can take the rest away and sell them on the open market. Banks maximize their life-time profits.

3 Bank contracts

The course of events. In the CM, the course of events is as follows:

1. first, banks settle deposit obligations with depositors;

[4] Of course, in those models one needs to introduce asymmetric information between borrowers and lenders to give the financial intermediaries a role, while here the return of the Lucas trees is certain. One can interpret the return of Lucas trees here as the *diversified* return in those models where each bank represents a large number of depositors.

2. then, banks buys Lucas trees in competitive market at price ϕ (in terms of CM good);
3. finally, banks may issue deposit contract, promising a gross return R (in exchange for CM good).

We use d to denote the total amount of deposits that the bank promises to give out in the next CM (and hence it will receive d/R in the current CM). Note that there are two different markets in the CM—a spot market for deposits, and a spot market for assets. Because only banks can manage Lucas trees to receive dividends, with no loss of generality we assume that buyers and sellers do not participate in the asset market.[5] We also assume that only buyers enter the deposit contracts in the CM but not sellers.

In the DM, upon a successful meeting with a seller, the buyer makes a take-it-or-leave-it offer, (q, z), where q is the DM consumption and z is the amount of deposit (in terms of the coming CM goods) transfer. This is feasible because, as mentioned earlier, there is record-keeping technology under which the accounts of the buyer can be transferred to the seller.

3.1 Static bank contracts

Here we consider the case where the free entry of banks implies a zero-profit condition, which in turn implies that banks cannot credibly promise any amount beyond what could be seized by the court.

As a benchmark, we first begin with the situation where banks cannot issue deposits at all. In this case, the price of the Lucas trees can be easily pinned down by a no arbitrage condition (i.e., banks' profit-maximizing condition) and the number of banks pinned down

[5] We implicitly assume that there is no friction within the two spot markets in the sense that all agents (especially buyers and bankers) can make promises to deliver the CM goods within the same-date CM stage when making the portfolio decisions, and hence, as usual in Lagos and Wright (2005) frameworks, the timing of the trades within CM does not matter and we can work with the *net* consumption in the CM for various agents.

by free entry. It is convenient to define

$$\Pi(A) = \psi'(A)A - \psi(A). \tag{1}$$

We assume that

$$\Pi(\bar{A}) < \gamma < \Pi\left[(\psi')^{-1}\left(\frac{\tau}{1+r}\right)\right]. \tag{2}$$

Assumption (2) ensures that there is sufficient entry to the banking sector. Indeed, as will be clear below, $\Pi(A) - \gamma$ will be the profit for a bank with A units of trees. Free entry then requires banks to hold $A = \Pi^{-1}(\gamma)$ and hence only a measure $m^* = \bar{A}/\Pi^{-1}(\gamma)$ of banks will enter. (2) ensures that $m^* < 1$ and hence a unit measure of banks is sufficient to provide free entry. Note that for a social planner who wants to minimize the cost of asset management will solve

$$\min_{m \geq 0} m\gamma + m\psi(\bar{A}/m). \tag{3}$$

The measure m^* also solves this problem. We may define the fundamental value of the trees as

$$\phi^* = \frac{\tau - (1+r)\psi'\left(\frac{\bar{A}}{m^*}\right)}{r}, \tag{4}$$

which will be the price for the asset if the banks cannot issue any deposits.

For expositional purposes, we define a variable,

$$\iota \equiv \frac{1+r}{R} - 1.$$

Given R (and hence ι) and ϕ, and for a given asset holding, a, and a given deposit issuance, d, (in terms of next CM promised value), a bank's profit is given by

$$\begin{aligned}\pi(a,d;\phi,R) &= \frac{d}{R} - \phi a - \gamma - \psi(a) + \beta\{(\phi+\tau)a - d\} \qquad (5)\\ &= \beta\left\{\iota d - (r\phi - \tau)a - (1+r)[\psi(a)+\gamma]\right\},\end{aligned}$$

and is subject to the pledgeability constraint,

$$d \le \rho(\phi+\tau)a. \qquad (6)$$

As mentioned, under static contracts, banks can only pledge what could be seized by the court, namely, ρ fraction of the value of their assets; (6) captures this constraint.

Let $A(\phi,\iota)$ be the optimal asset holding that maximizes (5) subject to (6). Note that whenever $\iota > 0$, the constraint (6) is binding and $A(\phi,\iota)$ is determined by the following FOC:

$$-(r-\iota\rho)\phi + (1+\iota\rho)\tau = (1+r)\psi'(a). \qquad (7)$$

When the pledgeability constraint is binding, the bank needs to own capital, $\phi a - \frac{d}{R}$, to finance some of its asset holdings.

Now we turn to depositors' behavior. Given R, a depositor's problem is given by

$$\begin{aligned}&\max_{d\ge 0} -\frac{d}{R} + \beta\left\{\sigma[u(q(d)) - c(q(d))] + d\right\}, \qquad (8)\\ \text{where}\quad & c(q(d)) = d \text{ if } d < c(q^*) \text{ and } q(d) = q^* \text{ otherwise.}\end{aligned}$$

Note that d is the *promised value* of the deposit in the coming CM. The FOC to (8) is

$$\iota = \frac{\sigma[u'(q(d)) - c'(q(d))]}{c'(q(d))}. \qquad (9)$$

Let $D(\iota)$, the deposit demand per depositor, be the solution to (9). Note that for any $\iota > 0$, $D(\iota)$ is uniquely determined; when $\iota = 0$, $D(\iota)$ is not pinned down but $D(\iota) \geq c(q^*)$. Without loss of generality we may take $D(0)$ as its minimum. Then, $D(\iota)$ is continuous and strictly decreasing in ι.

Equilibrium then requires market clearing conditions for deposits and assets:

$$D(\iota) = \rho(\tau + \phi)\bar{A}; \tag{10}$$

$$mA(\iota, \phi) = \bar{A}. \tag{11}$$

Finally, free-entry implies that all active banks have to have zero profits.

Lemma 3.1. *There is a unique equilibrium allocation, (m, ϕ, ι, q, d), in which $m = m^*$, and (ϕ, ι, q, d) is characterized as follows.*

(a) Suppose that

$$\rho\frac{1+r}{r}\left[\tau - \psi'\left(\frac{\bar{A}}{m^*}\right)\right]\bar{A} \geq c(q^*). \tag{12}$$

Then, $\phi = \phi^*$, $q = q^*$, and $\iota = 0$.

(b) Suppose that (12) does not hold. Then,

$$\phi = \frac{(\iota\rho + 1)\tau - \psi'\left(\frac{\bar{A}}{m^*}\right)(1+r)}{r - \iota\rho}, \tag{13}$$

with $q = c^{-1}(D(\iota)) < q^*$ and with $\iota \in (0, \frac{r}{\rho})$ as the unique solution to

$$D(\iota) = \rho(1+r)\frac{\tau - \psi'\left(\frac{\bar{A}}{m^*}\right)}{r - \iota\rho}\bar{A}. \tag{14}$$

Condition (12) gives a precise condition for the first-best trades to occur in DM in equilibrium. It is obtained by plugging the fundamental value ϕ^* given by (4) into (6), which implies that the amount of deposits banks can credibly issue when assets are valued at their fundamental values is more than enough to buy q^* for each buyer. In this case the net return to depositors is r. When (12) fails, Lemma 3.1 (b) shows that ϕ will be higher than the fundamental price, and hence the asset price exhibits *liquidity premium*, and R will be lower than $1+r$. Finally, note that since m always equals m^* under free entry, there is distortion in asset-management; all the potential inefficiency comes from insufficient liquidity provision.

Here we show that, whenever the first-best is not implementable, higher pledgeability leads to higher welfare. Given the measure of banks, m, and an allocation, (ϕ, ι, q, d), the associated welfare is given by

$$W = \sigma[u(q) - c(q)] - m\psi(\bar{A}/m) - m\gamma. \tag{15}$$

From (8), if the first-best is not obtained, $c(q(d)) = d$. Using (14), $d = D(\iota)$, and given $\tau \geq \psi'\left(\frac{\bar{A}}{m}\right)(1+r)$, we have

$$\frac{\partial W}{\partial \rho} = \frac{r(1+r)\sigma\left[\frac{u'(q(d))}{c'(q(d))} - 1\right]\left[\tau - \psi'\left(\frac{\bar{A}}{m}\right)\right]\bar{A}}{(r - \iota\rho)^2} > 0.$$

Indeed, welfare is increased by the pledgeability.

For our discussion below, it is convenient to define a threshold of ρ, $\tilde{\rho}$, such that $R \geq 1$, or, equivalently, $\iota \leq r$. If $R < 1$, then the government could make buyers better off by introducing fiat money. Let $y(d) = u'(q(d))/c'(q(d))$. Then, (9) implies that

$$D(\iota) = y^{-1}\left(\frac{\sigma + \iota}{\sigma}\right).$$

Thus, the solution to (14) satisfies $\iota \leq r$ if and only if

$$y^{-1}\left(\frac{r+\sigma}{\sigma}\right) \leq \rho(1+r)\frac{\left[\tau - \psi'\left(\frac{\bar{A}}{m^*}\right)\right]\bar{A}}{r - r\rho},$$

which is equivalent to

$$\rho \geq \tilde{\rho} \equiv \frac{y^{-1}\left(\frac{r+\sigma}{\sigma}\right)}{\frac{(1+r)\left[\tau-\psi'\left(\frac{\bar{A}}{m^*}\right)\right]}{r}\bar{A} + y^{-1}\left(\frac{r+\sigma}{\sigma}\right)}. \tag{16}$$

3.2 The charter system and dynamic bank contracts

Here we introduce the charter system with a banking authority or regulator. Under the charter system, there are two policy parameters for the regulator. The first parameter is the number of banking licences, denoted by m. The second is the amount of deposit issuance beyond what is allowed by the pledgeability constraint under static contract, denoted by κ. As mentioned, since the pledgeability constraint, (6), effectively implies that banks need to hold some capital to finance their asset holdings, the parameter κ may also be interpreted as a requirement on the *overall* leverage ratio for a bank. Given κ, the pledgeability constraint (6) is thus modified to

$$d \leq \rho(\phi + \tau)a + \kappa. \tag{17}$$

The two policy parameters, m and κ, are intimately connected in the incentive provision for banks. As we shall demonstrate later, when $m < m^*$, chartered banks make positive profits that will be lost if the charter is terminated. Hence, by terminating chartered banks who fail to repay their deposit obligations, the regulator can use future profits as a discipline device to ensure repayments beyond what can be seized by the court. Thus, to be incentive compatible, the amount κ has to be consistent with the equilibrium bank profits, which is determined by m.

Each chartered bank still maximizes the profit given by (5), but subject to (17). Note that the asset demand $A(\phi, \iota)$ is independent of κ. A depositor's problem is still given by (8), and deposit demand, $D(\iota)$, remains the same. For given κ and m, the market-clearing conditions are given by

$$\begin{aligned} D(\iota) &= \rho(\tau + \phi)\bar{A} + m\kappa; \qquad (18) \\ mA(\phi, \iota) &= \bar{A}. \qquad (19) \end{aligned}$$

Moreover, we only consider m's that satisfy

$$\tau \geq \psi'\left(\frac{\bar{A}}{m}\right)(1+r). \qquad (20)$$

By (2) and convexity of ψ, there exists a unique $\bar{m} < m^*$ such that (20) is satisfied for all $m \in [\bar{m}, m^*]$. It can be verified that it is never optimal to have $m < \bar{m}$.

Lemma 3.2. *Let $m \in [\bar{m}, m^*]$ and let $\kappa \geq 0$ be given. There is a unique allocation (ϕ, ι, q, d) that satisfies the market-clearing conditions that can be characterized as follows:*

$$\phi = \frac{(\iota\rho + 1)\tau - \psi'\left(\frac{\bar{A}}{m}\right)(1+r)}{r - \iota\rho}, \qquad (21)$$

and $q = c^{-1}(D(\iota))$ with $\iota = \iota(m, \kappa) \in [0, \frac{r}{\rho})$ as the unique ι such that $\iota \geq 0$ and

$$D(\iota) \leq \rho(1+r)\frac{\tau - \psi'\left(\frac{\bar{A}}{m}\right)}{r - \iota\rho}\bar{A} + m\kappa, \qquad (22)$$

with equality whenever $\iota > 0$. The profit for each bank is given by $\Pi\left(\frac{\bar{A}}{m}\right) - \gamma + \frac{\iota \cdot \kappa}{1+r}$.

It is straightforward to verify that $\iota(m, \kappa) = 0$ if and only if

$$\rho\frac{1+r}{r}\left[\tau - \psi'\left(\frac{\bar{A}}{m}\right)\right]\bar{A} + m\kappa \geq c(q^*). \qquad (23)$$

Lemma 3.2 then generalizes Lemma 3.1, the latter being a special case of the former with $\kappa = 0$. When (23) holds, we also have $q = q^*$ and $\phi = \phi^*$; otherwise, we have $q < q^*$ and $\phi > \phi^*$. The issuance of unsecured debt, κ, affects the equilibrium deposit holdings, as well as bank's profits. This implies that κ has effects on the efficiency of both DM trades and asset managements.

Now we turn to the incentive compatibility of κ. Since the court can only seize ρ proportion of a bank's asset, the bank has temptation not to repay the κ component of his liability in (17). To deter this temptation, the regulator can remove the bank charter and stop the bank from future business if the bank fails to honor his deposit obligations. Thus, if a bank defaults, he loses the pledged assets, $\rho(\phi+\tau)\bar{A}/m$, as well as the charter to run the business, beginning from the period when he fails to repay. As a result, a bank is willing to repay deposits if and only if

$$-\kappa - \rho(\phi+\tau)\bar{A}/m + \sum_{t=0}^{\infty} \beta^t \left[\Pi\left(\frac{\bar{A}}{m}\right) - \gamma + \frac{\iota \cdot \kappa}{1+r} \right] \geq -\rho(\phi+\tau)\bar{A}/m.$$

This constraint can be simplified as

$$-(r-\iota)\kappa + (1+r)\left[\Pi\left(\frac{\bar{A}}{m}\right) - \gamma \right] \geq 0. \tag{24}$$

We have the following definition.

Definition 3.1. A policy, $(m, \kappa) \in [\bar{m}, m^*] \times \mathbb{R}_+$, is *implementable* if (24) holds for $\iota = \iota(m, \kappa)$.

We have the following theorem.

Theorem 3.1. *Let $m \in [\bar{m}, m^*]$ be given. Then, there exists a greatest κ, denoted $\bar{\kappa}(m)$, such that $(m, \bar{\kappa}(m))$ is implementable and whose allocation has the highest welfare among all implementable policies for the given m.*

Theorem 3.1 gives a full characterization of the best implementable equilibrium outcome for a given number of charters, m. The regulator's goal, however, is to design (m, κ) such that the social welfare is maximized. Note that by Lemma 3.2, for any given (m, κ), there is a unique allocation (ϕ, ι, q, d) that satisfies the market-clearing conditions (18)- (19). However, the regulator is constrained by banks' repayment incentive constraint, (24).

Theorem 3.2. *Assume (A0). There exists an optimal policy* (m, κ) *that maximizes (15) subject to implementability.*
(a) If (12) holds, then $(m, \kappa) = (m^*, 0)$ *is an optimal policy.*
(b) Suppose that (12) does not hold and that $\rho \in (\tilde{\rho}, 1)$*. Then, any optimal policy has* $m < m^*$ *and* $\kappa > 0$*.*

Theorem 3.2 shows that, when designing an optimal charter system, the regulator has to balance efficiency in asset management and efficiency in liquidity provision. When there is abundant pledgable assets so that (12) holds, full efficiency can be achieved on both aspects, according to Theorem 3.2 (a). Otherwise, according to Theorem 3.2 (b), the constrained efficient arrangement has to sacrifice full efficiency on both aspects. Restricting the number of charters reduces competition and increases banks' profits; this is suboptimal regarding efficiency in asset-management. However, higher profits make it easier for banks' incentive constraint, (24), to hold and allow for a positive κ without having the banks defaulting on their debts. Thus, this financial stability in our framework is possible because of positive profits banks enjoy, and it is useful to enhance social welfare because banks provides liquidity services as their liabilities are used as means-of-payments.

Since the optimal κ regulates the amount of deposits a bank can issue through the pledgeability constraint (17), one can interpret the policy parameter κ as an *overall* leverage ratio requirement. In contrast to the common capital requirements that depend only on the asset characteristics a bank holds, optimal κ also depends on other bank characteristics such as ρ (proportion of asset that can be secured for repayment) and ψ (marginal cost of asset-management). Our framework then implies a holistic approach to capital requirement that would take both idiosyncratic feature of a specific bank as well as the global environment into account when designing the optimal capital regulations.

3.3 Moral hazard

We have shown that when designing an optimal policy in the presence of limited commitment of banks, the regulator needs to trade off efficiency and stability. Here we introduce another friction that is more akin the conventional moral hazard issue discussed in the literature. Our main focus is to what extent the competitive market can correct this issue and how this issue would interact with the optimal overall leverage ratio requirements we obtained in the last section.

Suppose that the dividends of Lucas trees that a bank holds are subject to moral hazard. By shirking the cost of managing a units of assets is $\psi(a) - ea + \gamma$, but the return is lower; it will be $\tau_0 < \tau$. The decisions to shirk are not observable, but the realized returns are. We assume that

$$(1 + r)e < \tau - \tau_0. \tag{25}$$

Condition (25) ensures that putting effort is socially beneficial.

First we begin with the case where there is no regulation and hence only static contracts are feasible. We shall impose free entry, but for now assume that the number of active banks is given by a fixed m. Even in the absence of regulation, depositors can potentially discipline the bank to exert efforts by not depositing in a bank without sufficient capital in place. Of course, there could potentially be an equilibrium where all banks shirk and depositors, by rational expectation, understand this and demand a more stringent capital requirement. Indeed, if a bank with a units of trees shirks, the return will then be τ_0 and hence would only pay out the bank will only pay out

$$d_0 = \rho(\phi + \tau_0)a \tag{26}$$

to the depositors. As a result, in such an equilibrium, the pledgeability constraint would then be given by

$$d \leq \rho(\phi + \tau_0)a. \tag{27}$$

The following lemma, however, shows that such an equilibrium does not exist under assumption (25).

Lemma 3.3. *Consider the static contract. There is no equilibrium with all banks shirking.*

Lemma 3.3 is shown by a simple contrapositive argument. If an equilibrium exists where all banks shirk, then depositors would demand the pledgeability constraint (27). However, under such a constraint, a bank receives all the additional return by exerting efforts, and hence (25) implies that all banks are willing to exert efforts. This leads to a contradiction.

By Lemma 3.3, we can focus only on equilibria where all banks exert efforts. In this case, the constraint (6) may no longer be appropriate as it may not induce efforts. Instead, a more general pledgeability constraint is needed:

$$d \leq \rho(\phi + \tau_0)a + \omega(\tau - \tau_0)a, \tag{28}$$

for some $\omega \in [0, \rho]$. Note that when $\omega = 0$, (28) coincides with (27); when $\omega = \rho$, (28) coincides with (6). The parameter ω also has a simple interpretation: $1 - \omega$ stands for the share of the additional return that goes to the bank by exerting efforts. As argued earlier, when $\omega = 0$ all banks are willing to exert efforts.

Here we give a remark about what we mean by equilibrium under moral hazard. As mentioned, here the pledgeability constraint is endogenous in the sense that a bank who reaches the constraint cannot credibly issue more deposits. Thus, in equilibrium, the pledgeability constraint (28) must satisfy two conditions: first, it has to ensure that the banks are willing to exert efforts; second, it cannot be the case that any bank can credibly issue more deposits than what the constraint requires. Equivalently, equilibrium requires the *highest* ω that is consistent with banks exerting effort under the constraint.

To do this, we can modify our previous analysis and obtain market clearing conditions. Recall that we assume a fixed number of active banks, m. By exerting efforts, the bank profit is obtained by substituting (28) at equality into (5):

$$\begin{aligned} \pi(a, d; \phi, R) &= \frac{d}{R} - \phi a - [\psi(a) + \gamma] + \beta\{(\phi + \tau)a - d\} \\ &= \beta\left\{-(r - \iota\rho)\phi a + [(\tau + \iota\rho\tau_0) + \iota\omega(\tau - \tau_0)]a - (1 + r)[\psi(a) + \gamma]\right\}. \end{aligned} \tag{29}$$

The FOC for (29) is thus

$$-(r - \iota\rho)\phi + (\tau + \iota\rho\tau_0) + \iota\omega(\tau - \tau_0) = (1 + r)\psi'(a).$$

Thus, the equilibrium price for trees is pinned down by market-clearing, $a = \bar{A}/m$:

$$\phi = \frac{(1 + \iota\omega)\tau + \iota(\rho - \omega)\tau_0 - \psi'\left(\frac{\bar{A}}{m}\right)(1 + r)}{r - \iota\rho}; \tag{30}$$

Note that, as before, the bank profit is then given by $\Pi(\bar{A}/m) - \gamma$. Given ϕ, the equilibrium ι, denoted by $\iota \in (0, \frac{r}{\rho})$, is then the unique solution to (with equality whenever $\iota > 0$)

$$D(\iota) \leq \frac{\rho\tau + r\rho\tau_0 + \omega r(\tau - \tau_0) - \rho(1 + r)\psi'\left(\frac{\bar{A}}{m}\right)}{r - \iota\rho}\bar{A}. \tag{31}$$

Finally, we also need to consider the profit to a bank if he shirks, taking ϕ as given. Recall that a shirking bank only pays $d_0 = \rho(\phi + \tau_0)a$ to depositors under return τ_0; hence, the bank profit is given by

$$\begin{aligned} \pi^s(a, d; \phi, R) &= \frac{d}{R} - \phi a - [\psi(a) - ea + \gamma] + \beta\{(\phi + \tau_0)a - d_0\} \\ &= \beta \left\{ \begin{array}{c} -(r - \iota\rho)\phi a + [(\tau_0 + \iota\rho\tau_0) + \iota\omega(\tau - \tau_0) + \omega(\tau - \tau_0)]a \\ -(1 + r)[\psi(a) - ea + \gamma] \end{array} \right\}. \end{aligned} \tag{32}$$

The FOC implies that the asset holding for a shirking bank is given by A^s that solves

$$-(r - \iota\rho)\phi + (\tau + \iota\rho\tau_0) + (\iota + 1)\omega(\tau - \tau_0) = (1 + r)\psi'(A^s) - e.$$

Hence, the bank profit under shirking is given by $\Pi(A^s) - \gamma$. Thus, to ensure that banks have no incentive to shirk, we the following condition:

$$\Pi(\bar{A}/m) - \Pi(A^s) \geq 0. \tag{33}$$

To summarize, equilibrium conditions the consist (30), (31), and (33). We have the following lemma.

Lemma 3.4. *Consider the static contract and let m be given. The highest ω under which all banks exert efforts in equilibrium is given by*

$$\omega_1 \equiv \min\left\{1 - \frac{(1 + r)e}{\tau - \tau_0}, \rho\right\}. \tag{34}$$

Lemma 3.4 shows that the market can discipline banks to exert effort by demanding additional capital requirement parameterized by ω_1. When ρ is relatively small, i.e., when $\rho \leq 1 - \frac{(1+r)e}{\tau - \tau_0}$, $\omega_1 = \rho$ and the presence of moral hazard does not affect the equilibrium allocation. In contrast, when ρ is relatively small and hence $\omega_1 < \rho$, the presence of moral hazard does limit the ability of the banks to provide liquidity. Under fee entry, we can use the same arguments before and derive the equilibrium m would be m^*.

Charter system with moral hazard

Now we turn to the charter system with moral hazard. Relative to the literature, the novelty here is to study the two capital regulations together, one parameterized by ω and the other by κ. Under the charter system with moral hazard, the general pledgeability constraint is given by:

$$d \leq \rho(\phi + \tau_0)a + \omega(\tau - \tau_0)a + \kappa. \tag{35}$$

We remark here that Lemma 3.3 can be generalized in this dynamic setting, and hence it is without loss of generality to consider only equilibria with all banks exerting efforts. Thus, the policy parameter now becomes (m, κ, ω).

Now we move to equilibrium analysis for a given policy parameter. By exerting efforts, the bank profit is given by:

$$\begin{aligned} \pi(a, d; \phi, R) &= \frac{d}{R} - \phi a - [\psi(a) + \gamma] + \beta\{(\phi + \tau)a - d\} \qquad (36) \\ &= \beta\left\{-(r - \iota\rho)\phi a + [(\tau + \iota\rho\tau_0) + \iota\omega(\tau - \tau_0)]a + \iota\kappa - (1 + r)[\psi(a) + \gamma]\right\}. \end{aligned}$$

Note that the only difference between (29) and (36) is the term $\beta\iota\kappa$, they share the same FOC's and hence the equilibrium ϕ is still given by (30). The profit to each bank in equilibrium is then $\Pi(\bar{A}/m) - \gamma + \beta\iota\kappa$. Given ϕ, the equilibrium ι, denoted by $\iota(m, \kappa, \omega) \in (0, \frac{r}{\rho})$, is then the unique solution to (with equality whenever $\iota > 0$)

$$D(\iota) \leq \frac{\rho\tau + r\rho\tau_0 + \omega r(\tau - \tau_0) - \rho(1 + r)\psi'\left(\frac{\bar{A}}{m}\right)}{r - \iota\rho}\bar{A} + m\kappa. \tag{37}$$

Again, note that the only difference between (31) and (37) is the term $m\kappa$.

Now we turn the incentive compatibility of banks to exert efforts and to repay κ. We assume that banks with return τ_0 will have their charters terminated and it is easy to see that this is the optimal punishment. Thus, a shirking bank only pays d_0 given by (26) to depositors under return τ_0. Thus, the profit to a shirking bank is given by

$$\begin{aligned} \pi^s(a, d; \phi, R) &= \frac{d}{R} - \phi a - [\psi(a) - ea + \gamma] + \beta\{(\phi + \tau_0)a - d_0\} \\ &= \beta \left\{ \begin{array}{c} -(r - \iota\rho)\phi a + [(\tau_0 + \iota\rho\tau_0) + \iota\omega(\tau - \tau_0) + \omega(\tau - \tau_0)]a \\ +(1+\iota)\kappa - (1+r)[\psi(a) - ea + \gamma] \end{array} \right\}. \end{aligned} \tag{38}$$

Again, note that the only difference between (32) and (38) is the term $\beta(1+\iota)\kappa$ and hence has no bearings on FOC; so the optimal asset holding is still given by A^s and the profit is $\Pi(A^s) - \gamma + \beta(1+\iota)\kappa$.

To ensure that banks follow equilibrium behavior, we have two incentive compatibility conditions, one for repaying κ, the other for exerting efforts. Since we assume that in equilibrium all banks exert effort, the first one is the same as before, (24); note that, however, equilibrium ι is affected by ω through (37). The second condition is new and is given by

$$-\left[\Pi(A^s) - \Pi(\bar{A}/m) + \beta\kappa\right] + \frac{\beta}{1-\beta}\left[\Pi\left(\frac{\bar{A}}{m}\right) - \gamma + \frac{\iota \cdot \kappa}{1+r}\right] \geq 0. \tag{39}$$

Theorem 3.3. *Let $m < m^*$ be given. Suppose that $\rho \in (0,1)$. The optimal capital requirement is such that $\omega = \omega_1$ given by (34), and κ is the highest κ that satisfies (24) with ι determined by (37) and with $\omega = \omega_1$.*

Since Theorem 3.3 holds for any given m, it follows that we can extend Theorem 3.2 to the case with moral hazard. In particular, Theorem 3.3 states that the highest κ exists for which (24) holds with ι determined by (37) and with $\omega = \omega_1$. One can then solve for the optimal m and, as in Theorem 3.2, we will have $m < m^*$ and $\kappa > 0$ unless the first-best is implementable under $m = m^*$ and $\kappa = 0$, as well as $\omega = \omega_1$.

Compared against Lemma 3.4, Theorem 3.3 shows that under the charter system the optimal ω is the same as that under market-discipline, and that it is optimal to use the dynamic incentive to increase κ and κ only. Note that, however, optimal κ is indeed affected by moral hazard, since the choice of ω does affect the amount banks can provide through asset prices and returns on deposits. The following theorem shows that when moral hazard affects welfare, it is in fact to allow bank profits to increase and hence to allow for higher unsecured lending to increase liquidity.

Theorem 3.4. *Suppose that $\omega_1 < \rho$ and that the first-best is not implementable. Fix some $m < m^*$. Then, when e increases, optimal κ increases and banks make higher profits under the optimal arrangement.*

As e increases and hence the moral hazard issue becomes more serious, ω_1 increases as well by Theorem 3.3. This directly decreases the amount of liquidity banks can provide. However, Theorem 3.4 shows that the optimal response to such change is to increase κ by allowing for higher profits to the banks. This implies a nontrivial interaction between the conventional capital requirement designed to counter the moral hazard issue and the overall leverage ratio requirement in our charter system that aims to balance stability and liquidity. Crucially, this result follows from our explicit treatment of liquidity provision from banks.

4 Heterogeneity, Leverage, and Deposit Insurance

In this section we introduce heterogeneity across banks. We consider two types of heterogeneity. The first is concerned with efficiency of different banks. The main focus is on the distribution of bank sizes and optimal charter system when banks differ in sizes. The second is concerned with idiosyncratic shocks to individual bank returns. In this case, our main focus is on how information leakage about bank returns affect liquidity provision from banks.

4.1 Heterogenous bank sizes and profits

Here we consider heterogenous banks in terms of their efficiency in asset management. Specifically, for each $n \in \{1, ..., N\}$, the economy has measure μ_n of type-n banks with $\sum_{n=1}^{N} \mu_n = 1$, and the cost function for a bank of type-n is $\lambda_n \psi(a) + \gamma$. The parameter λ_n is then a measurement of how efficient type-n banks are in terms of asset management. We assume that $\lambda_n \in [1, \bar{\lambda}]$ is strictly increasing in n, and hence type-1 banks are the most efficient ones while type-n are the least efficient ones.

First we consider efficient asset management in this environment. Without deposit issuance, efficient asset management requires the measures of type-n active banks, denoted by m_n, to solve

$$\min_{m_n \in [0, \mu_n], A_n \geq 0, =1, ..., N} \sum_{n=1}^{N} [m_n \gamma + \lambda_n \psi(A_n)] \tag{40}$$
$$\text{s.t. } \sum_{n=1}^{N} m_n A_n = \bar{A}.$$

Parallel to (2), to ensure that there is sufficient entry we assume that

$$\sum_{n=1}^{N} \mu_n \Pi^{-1}(\gamma/\lambda_n) > \bar{A}. \tag{41}$$

To characterize the solution to (40), first for each $\mathbf{m} = (m_1, ..., m_N)$ with $m_1 > 0$, we define $\{A_n(\mathbf{m})\}_{n=1}^{N}$ as the solution to

$$\sum_{n=1}^{N} m_n A_n = \bar{A}, \;\; \lambda_1 \psi'(A_1) = \lambda_n \psi'(A_n) \text{ if } m_n > 0, \;\; A_n = 0 \text{ otherwise.} \tag{42}$$

We have the following claim.

Claim 4.1. *Assume (41). The solution to (40) is unique, denoted by* $\mathbf{m}^*$, *is characterized by* $\bar{n} \in \{1, ..., N\}$ *and* $0 < m^*_{\bar{n}} \leq \mu_{\bar{n}}$ *such that*

$$\mathbf{m}^* = (\mu_1, .., \mu_{\bar{n}-1}, m^*_{\bar{n}}, 0..., 0), \tag{43}$$

$$\lambda_n \Pi(A_n(\mathbf{m}^*)) \geq \gamma, \text{ for all } n = 1, ..., \bar{n}, \tag{44}$$

$$\lambda_n \Pi(A_{\bar{n}}(\mathbf{m}^*)) = \gamma \text{ if } m_{\bar{n}} < \mu_{\bar{n}}, \tag{45}$$

$$\lambda_n \Pi(A_n(\mathbf{m}^*)) < \gamma, \text{ for all } n = \bar{n} + 1, ..., N. \tag{46}$$

Now we consider static contracts with free entry. Given R (and hence ι) and ϕ, and for a given asset holding, a, and deposits giving out, d, (in terms of next CM promised value), the profit of a type-n bank is given by

$$\pi_n(a, d; \phi, R) = \beta \left\{ \iota d - (r\phi - \tau)a - (1 + r)[\lambda_n \psi(a) + \gamma] \right\},$$

and is subject to the pledgeability constraint,

$$d \leq \rho(\phi + \tau)a. \tag{47}$$

This gives rise to a well-defined asset demand $A_n(\phi, \iota)$ determined by the following FOC:

$$-(r - \iota\rho)\phi + (1 + \iota\rho)\tau = (1 + r)\lambda_n \psi'(a). \tag{48}$$

That is,

$$A_n(\phi, \iota) = (\psi')^{-1} \left(\frac{-(r - \iota\rho)\phi + (1 + \iota\rho)\tau}{(1 + r)\lambda_n} \right). \tag{49}$$

Let $\phi^*_{\mathbf{m}}$ be the unique solution to

$$\int_{n=1}^{N} m_n A_n(\phi, 0) = \bar{A}.$$

As before, we may call $\phi^*_{\mathbf{m}^*}$ the fundamental value of the asset, the price for the trees if banks were not allowed to issue deposits; in that situation the measures of active banks would be given by $\mathbf{m}^*$.

Let m_n be the measure of active type-n banks, $n = 1, ..., N$. Then, equilibrium objects include asset price ϕ, returns to deposits ι, and the measure of active type-n banks, m_n for each $n = 1, ..., N$ ($m_n = 0$ means that no type-n bank is active). The market-clearing conditions and free entry condition are given by (note that $D(\iota)$ is still given by (9))

$$\begin{aligned} D(\iota) &= \rho(\tau + \phi)\bar{A}; && (50) \\ \sum_{n=1}^{N} m_n A_n(\phi, \iota) &= \bar{A}; && (51) \\ \lambda_n \Pi[A_n(\phi, \iota)] \geq \gamma \text{ if } m_n > 0, \qquad & \lambda_n \Pi[A_n(\phi, \iota)] \leq \gamma \text{ if } m_n < \mu_n. && (52) \end{aligned}$$

We have the following lemma.

Lemma 4.1. *Assume (41). There exists a unique equilibrium. The equilibrium measures of active banks are given by* $\mathbf{m}^*$ *and equilibrium asset holding is given by* $A_n(\mathbf{m}^*)$ *according to (42) for type-n banks. If the equilibrium DM production is* q^**, then equilibrium* $\phi = \phi^*_{\mathbf{m}^*}$*; otherwise, equilibrium* $\phi > \phi^*_{\mathbf{m}^*}$*.*

Lemma 4.1 shows that even with heterogenous banks, the result that without regulation efficiency of asset-management is achieved still holds. However, here we obtain an endogenous distribution of bank balance sheets. Specifically, (42) implies that $A_n(\mathbf{m}^*) > A_{n+1}(\mathbf{m}^*)$ for all $n = 1, ..., \bar{n}-1$, and hence, under free entry, more efficient banks are also larger in terms of asset holdings. Moreover, the FOC also implies that the profit for bank of type-n is given by

$\lambda_n \Pi[A_n(\phi,\iota)] - \gamma$, and hence Claim 4.1 implies that even under the efficient arrangement for asset management, some banks may make positive profits. Strict convexity also implies that $\lambda_n \Pi(A_n(\mathbf{m}^*)) > \lambda_{n+1}\Pi(A_{n+1}(\mathbf{m}^*))$ for all $n = 1, ..., \bar{n} - 1$, and hence, more efficient banks also make higher profits. In what follows, we assume that the solution satisfies $m^*_{\bar{n}} < \mu_{\bar{n}}$.

Heterogenous bank leverages

Here we consider the charter system. We assume that bank efficiency, λ_n, is observable. Given this assumption, the policy parameters are now a measure of banks for each type, $\mathbf{m} = (m_1, ..., m_N)$, and a unsecured deposit limit κ_n for each type n with $m_n > 0$. Note that the demand for assets from banks of type-n is still given by (49) (since κ_n does not affect the FOC). For given $\mathbf{m}$ and $\{\kappa_n\}$, the market-clearing conditions are given by (note that $D(\iota)$ is still given by (9))

$$\begin{aligned} D(\iota) &= \rho(\tau+\phi)\bar{A} + \sum_{n=1}^{N} m_n \kappa_n; & (53) \\ \int_{n=1}^{N} m_n A_n(\phi,\iota) &= \bar{A}. & (54) \end{aligned}$$

We have the following lemma.

Lemma 4.2. *Let $\mathbf{m} \le \mathbf{m}^*$ with $m_1 > 0$ and $\{\kappa_n\}$ be given. There is a unique allocation (ϕ, ι, q, d) that satisfies the market-clearing conditions, and can be characterized as follows:*

$$A_n = A_n(\mathbf{m}),$$

$$\phi = \frac{(1+\iota\rho)\tau - (1+r)\lambda_1\psi'(A_1)}{r-\iota\rho}, \tag{55}$$

$$D(\iota) \le \rho(1+r)\frac{\tau - \lambda_1\psi'(A_1)}{r-\iota\rho}\bar{A} + \sum_{n=1}^{N} m_n\kappa_n, \text{ with equality if } \iota > 0. \tag{56}$$

Let the unique ι that satisfies (56) be denoted by $\iota(\mathbf{m}, \{\kappa_n\})$. Moreover, the profit for bank *of type n is given by*

$$\lambda_n \Pi(A_n(\mathbf{m})) - \gamma + \frac{\iota(\mathbf{m}, \{\kappa_n\})\kappa_n}{(1+r)}. \tag{57}$$

The assumption that $m_1 > 0$ is with no loss of generality; if, instead, $m_1 = 0$ but $m_n > 0$ for some other n, then we can simply replace 1 by n in (55) and (56). Note also that since we are only concerned with market clearing and not entry, banks may make negative profits (because of the fixed cost γ). However, a full equilibrium analysis also requires incentive compatibility for repayment of κ, which would require nonnegative profits. As before, banks fail to repay depositors will be closed and hence lose their future profits. Thus, given a policy, $\mathbf{m}$ and $\{\kappa_n\}$, a bank of type-n is willing to repay deposits if and only if

$$-\kappa_n - \rho(\phi+\tau)A_n(\mathbf{m}) + \sum_{t=0}^{\infty} \beta^t [\lambda_n \Pi(A_n(\mathbf{m})) - \gamma + \iota(\mathbf{m}, \{\kappa_n\})\kappa_n/(1+r)] \geq -\rho(\phi+\tau)A_n(\mathbf{m}).$$

This constraint can be simplified as

$$-r\kappa_n + (1+r)[\lambda_n \Pi(A_n(\mathbf{m})) - \gamma + \iota(\mathbf{m}, \{\kappa_n\})\kappa_n/(1+r)] \geq 0. \tag{58}$$

The regulator then chooses policy parameters to maximize the social welfare. For a given policy $\mathbf{m}$ and $\{\kappa_n\}$ and the DM trade q, the regulator maximizes the welfare given by

$$\sigma[u(q) - c(q)] - \sum_{n=1}^{N} m_n \left[\lambda_n \psi(A_n(\mathbf{m})) - \gamma\right], \tag{59}$$

subject to equilibrium implementation $c(q) = D(\iota(\mathbf{m}, \{\kappa_n\}))$ and incentive compatibility condition (58). The following lemma characterize optimal $\{\kappa_n\}$ for a given $\mathbf{m}$.

Lemma 4.3. *Let* $\mathbf{m}$ *be given such that*

$$\lambda_n \Pi(A_n(\mathbf{m})) \geq \gamma \textit{ for all } n \textit{ with } m_n > 0.$$

(a) Let $\hat{\kappa}_n(\mathbf{m}) = \frac{1+r}{r}[\lambda_n \Pi(A_n(\mathbf{m})) - \gamma]$ *for each* $n = 1, ..., N$. *If*

$$c(q^*) \leq \rho(1+r)\frac{\tau - \lambda_1 \psi'(A_1(\mathbf{m}))}{r}\bar{A} + \sum_{n=1}^{N} m_n \hat{\kappa}_n(\mathbf{m}), \tag{60}$$

then $\{\hat{\kappa}_n(\mathbf{m})\}$ *is optimal under* $\mathbf{m}$. *In this case,* $\iota = 0$ *and* $q = q^*$ *in equilibrium.*

(b) Suppose that (60) does not hold. Then, there exists an optimal $\{\kappa_n\}$ *under* $\mathbf{m}$*, denoted by* $\{\bar{\kappa}_n(\mathbf{m})\}$*, such that the constraint (58) is binding for all* n *with* $m_n > 0$.

Now we are ready to characterize optimal policy.

Theorem 4.1. *There exists an optimal policy* $\mathbf{m}$ *and* $\{\kappa_n\}$*; in any optimal policy, we have that* $\mathbf{m} \leq \mathbf{m}^*$*, and that* $m_n = \mu_n$ *or* $m_n = 0$ *except for at most one* n.

(a) Suppose that (60) holds for $\mathbf{m} = \mathbf{m}^*$*, then* $(\mathbf{m}^*, \{\hat{\kappa}_n(\mathbf{m}^*)\})$ *is an optimal policy.*

(b) Suppose that (60) does not hold for $\mathbf{m} = \mathbf{m}^*$.

(b.1) Any optimal policy $(\mathbf{m}, \{\bar{\kappa}_n(\mathbf{m})\})$ *have* $m_{\bar{n}} < m^*_{\bar{n}}$.

(b.2) Suppose that $\psi(A) = A^x$ *for some* $x > 1$. *Then, for any optimal policy* $(\mathbf{m}, \{\bar{\kappa}_n(\mathbf{m})\})$,

$$\mathcal{L}_n = \frac{\rho(\phi+\tau)A_n(\mathbf{m}) + \bar{\kappa}_n(\mathbf{m})}{A_n(\mathbf{m})}$$

is strictly decreasing in n.

Theorem 4.1 (b.1) shows that unless the first-best is implementable, restriction in banking licence is optimal. This generalizes Theorem 3.2. Moreover, (b.2) shows that under the optimal arrangement, not only the regulator would allow higher unsecured deposit issuance for larger banks, the ration between total debt and total asset also increases with the bank size. If we assume that all banks issue less debts than their assets (for example, by having ρ not too large), this also implies that it is optimal to allow for a higher leverage ratio requirement for larger banks.

4.2 Dividend uncertainty and deposit insurance

Here we introduce another dimension of heterogeneity, namely, fluctuation over each bank's return on their assets. Specifically, the dividends of Lucas trees that a bank holds are subject to bank-specific shocks: by holding a units of the trees, the return, denoted by τ_s, is determined by the state of the bank, s, which can be either h or ℓ and $\tau_h > \tau_\ell$. State s fully realizes in the CM and occurs with probability p_s. We assume that the state is observable to all in the CM.

Moreover, this fluctuation in dividends can cause disturbances to transactions, by way of noisy signals transmitted to agents in the DM before it realizes in the following CM. Specifically, when the state for a bank will be s in the coming CM, a buyer who has made deposits with the bank and the seller who met the buyer in the DM receive the same signal, which can be either g or b, and the conditional probability is given by $p(g|h) = p(b|\ell) = \nu > 1/2$. The parameter ν then measures the informativeness of the signal. Because uncertainty in asset returns may affect the value of deposits, the DM consumption may depend on the noisy signals and hence ν.

Because of uncertainty in the realization of dividends, the fundamental price, ϕ^*, of Lucas trees changes as well. Suppose there is a measure m of banks. Let $\mathbb{E}(\tau) \equiv p_h\tau_h + p_\ell\tau_\ell$. Then

$$\phi^* = \frac{\mathbb{E}(\tau) - (1+r)\psi'\left(\frac{\bar{A}}{m}\right)}{r}, \tag{61}$$

which would be the price for the asset if banks cannot issue any deposits.

We focus only on the charter system here. For now we only consider policy parameters m and κ. The pledgeability also requires some modification. When the realized state is s, the regulator can seize $\rho(\phi + \tau_s)a$ from each bank and require κ from each bank (both in terms of CM goods). This in turn affects the bank contract with the depositors. We focus on a contract of the following form: one unit of deposits is sold at price $1/R$; for a buyer that deposits d (in terms of promised CM goods next period) with the bank, if $s = h$, then the bank pays $d(1+\delta_h)$; otherwise, the bank pays $d(1+\delta_\ell)$. Given this contract (parameterised by R and δ), the buyer's problem is then given by

$$\max_{d\geq 0} -\frac{d}{R} + \beta\left\{\sigma \sum_s (p_h p(s|h) + p_\ell p(s|\ell))[u(q(d,s)) - c(q(d,s))] + d + d(p_h\delta_h + p_\ell\delta_\ell)\right\},$$

$$\text{where } c(q(d,s)) = \begin{cases} d + d[p(h|s)\delta_h + p(\ell|s)\delta_\ell] & \text{if } d + d[p(h|s)\delta_h + p(\ell|s)\delta_\ell] < c(q^*), \\ c(q^*) & \text{otherwise.} \end{cases} \tag{62}$$

This gives a well-defined demand for deposits, $D(\iota;\delta_h,\delta_\ell)$. Note that the function $D(\iota;\delta_h,\delta_\ell)$ is implicitly a function of ν, precision of the noisy signal, as well. We have the following claim.

Claim 4.2. *Fix some ι and δ_h, δ_ℓ such that $p_h\delta_h + p_\ell\delta_\ell = 0$ with $\delta_h > 0$. Then, $D(\iota;\delta_h,\delta_\ell)$ is strictly decreasing in ν.*

For the bank, we assume that the pledgeability takes the form

$$d \leq \rho[\phi + \mathbb{E}(\tau)]a + \kappa. \tag{63}$$

Now, we consider a bank's problem, whose profit is given by

$$\begin{aligned} \pi(a,d;\phi,R) &= \frac{d}{R} - \phi a - \psi(a) - \gamma + \beta\{(\phi + \mathbb{E}(\tau)a - (p_h d_h + p_\ell d_\ell)\} \quad (64) \\ &= \beta\left\{\iota d - (r\phi - \mathbb{E}(\tau))a - (1+r)[\psi(a)+\gamma]\right\}, \\ &= \beta\left\{-(r-\iota\rho)\phi a + (1+\iota\rho)\mathbb{E}(\tau)a + \iota\kappa - (1+r)[\psi(a)+\gamma]\right\}, \end{aligned}$$

where we use the fact that

$$d_h = \rho(\phi + \tau_h)a + \kappa \text{ and } d_\ell = \rho(\phi + \tau_\ell)a + \kappa. \tag{65}$$

The FOC for the bank is

$$-(r - \iota\rho)\phi + (1 + \iota\rho)\mathbb{E}(\tau) = (1 + r)\psi'(a), \tag{66}$$

which implies a well-defined asset demand, $A(\phi, \iota)$.

Now we can formulate the market clearing conditions:

$$\delta_s = \frac{\rho[\tau_s - \mathbb{E}(\tau)]\bar{A}}{\rho[\phi + \mathbb{E}(\tau)]\bar{A} + m\kappa} \text{ for } s = h, \ell, \tag{67}$$

$$D(\iota; \delta_h, \delta_\ell) = \rho[\mathbb{E}(\tau) + \phi]\bar{A} + m\kappa, \tag{68}$$

$$mA(\phi, \iota) = \bar{A}. \tag{69}$$

The following lemma characterizes the allocation that satisfies the above conditions.

Lemma 4.4. *Let $m \geq m^*$ and let $\kappa \geq 0$ be given. There exists an upper bound $\bar{\nu} > 1/2$ such that for all $\nu \leq \bar{\nu}$, there exists a unique pair $\phi(m, \kappa, \nu)$ and $\iota(m, \kappa, \nu)$ that clears the market, in which ϕ is still given by (21) and ι is characterized by*

$$D(\iota; \delta_h, \delta_\ell) \leq \rho(1 + r)\frac{\mathbb{E}(\tau) - \psi'\left(\frac{\bar{A}}{m}\right)}{r - \iota\rho}\bar{A} + m\kappa, \tag{70}$$

with equality whenever $\iota > 0$. The profit for each bank is still given by $\left[\Pi\left(\frac{\bar{A}}{m}\right) - \gamma\right] + \frac{\iota(m,\kappa)\cdot\kappa}{1+r}$.

Moreover, $D(\iota(m, \kappa, \nu))$ is strictly decreasing in ν and equilibrium welfare is strictly decreasing in ν for all $\nu \leq \bar{\nu}$.

Lemma 4.4 shows that precision in the signal actually hurts efficiency. Moreover, since the incentive compatibility for implementing κ under m is still given by (24), and since $\iota(m, \kappa, \nu)$ is decreasing in ν, this conclusion is not affected even if we consider optimal policy. In the next subsection we introduce deposit insurance that can restore information-insensitivity of the deposit contract.

Deposit insurance

Here we consider a deposit insurance scheme. Under this scheme, the regulator can use funds collected from banks at state h to pay for depositors with banks under state ℓ. This reduce the depositors' problem to the original problem with the demand for deposits given by $D(\iota)$. The market clearing conditions are then:

$$D(\iota) = \rho[\mathbb{E}(\tau) + \phi]\bar{A} + m\kappa, \tag{71}$$

$$mA(\phi, \iota) = \bar{A}. \tag{72}$$

Theorem 4.2. *For a given m and κ, there exists a unique equilibrium $\iota(m, \kappa)$ that satisfies (71)-(72). The allocation under deposit insurance is the same as the one without it but with $\nu = 1/2$ and hence is better than any equilibrium allocation without it.*

Moral hazard in deposit insurance

Here we consider moral hazard issue with dividend uncertainty. Suppose that each bank can shirk. By shirking, the probability of state h is $q_h < p_h$ and that of state ℓ is $q_\ell > p_\ell$. The return at state h is $\tau_h' \geq \tau_h$, while at state ℓ is still τ_ℓ. At state h, however, the difference $\tau_h' - \tau_h$ is not observable. Moreover, by shirking the cost of managing a units of assets will be $\psi(a) - ea$. We use $\mathbb{E}_1(\tau) = q_h\tau_h' + q_\ell\tau_\ell$ to denote the average return for a shirking bank. We assume that

$$(1 + r)e \leq \mathbb{E}(\tau) - \mathbb{E}_1(\tau). \tag{73}$$

Under (73), it is socially optimal to have banks exert efforts.

Note that the set of possible observable returns to a shirking bank is identical to that of a hard-working bank. To punish a shirking bank, then, the regulator has to (randomly) shut down some banks in state ℓ (but probably not all of them). Alternatively, the regulator can place a capital requirement. Before considering such punishments and requirements, we first consider the conditions under which no further regulation is necessary.

Lemma 4.5. *Consider the same DI scheme as in the previous section. An equilibrium with banks exerting efforts exists if and only if*

$$\rho \leq \omega_2 \equiv 1 - \frac{q_h(\tau_h' - \tau_h) + (1+r)e}{(\tau_h - \tau_\ell)(p_h - q_h)}. \tag{74}$$

When it exists, it is also unique.

When (74) does not hold, there are two alternatives. The first is to keep the original regulation but take into account that banks may shirk (and hence change the deposit insurance premium and pledgeability constraints). Under this alternative, the pledgeability constraint is given by

$$d \leq \rho[\phi + \mathbb{E}_0(\tau)]a + \kappa, \tag{75}$$

where $\mathbb{E}_0(\tau) = q_h\tau_h + q_\ell\tau_\ell$, but (65) remains the same. Given these constraints, the profit of a shirking bank is given by

$$\begin{aligned} & \frac{d}{R} - \phi a - [\psi(a) - ea + \gamma] + \beta\{[\phi + \mathbb{E}_1(\tau)]a - [q_h d_h + q_\ell d_\ell]\} \\ = \;& \beta\left\{\iota d - [r\phi - \mathbb{E}_1(\tau)]a - (1+r)[\psi(a) - ea + \gamma]\right\} \\ = \;& \beta\left\{-(r - \iota\rho)\phi a + [\mathbb{E}_1(\tau) + \iota\rho\mathbb{E}_0(\tau)]a + \iota\kappa - (1+r)[\psi(a) - ea + \gamma]\right\}. \end{aligned} \tag{76}$$

Thus, if all banks shirk, the equilibrium ϕ is the given by

$$\phi = \frac{\mathbb{E}_1(\tau) + \iota\rho\mathbb{E}_0(\tau) - (1+r)[\psi'(\bar{A}/m) - e]}{r - \iota\rho}, \tag{77}$$

and the equilibrium condition for ι is given by

$$D(\iota) \le \rho \frac{\mathbb{E}_1(\tau) + r\mathbb{E}_0(\tau) - (1+r)[\psi'(\bar{A}/m) - e]}{r - \iota\rho} \bar{A} + m\kappa. \tag{78}$$

We have the following lemma.

Lemma 4.6. *Consider the DI scheme with pledgeability constraint (75). An equilibrium with banks shirking exists if and only if $\rho \ge \omega_2$. When it exists, it is also unique.*

Now, the other alternative is to incentivize banks to exert efforts, even when (74) does not hold. To do so, the regulator can place a capital requirement similar to (35) and/or to shut down a bank in state ℓ. Let the probability of closing a bank at state ℓ be $1 - \pi$. The pledgeability constraints are given as follows.

$$\begin{aligned} d &\le \rho(\phi + \tau_\ell)a + p_h\omega(\tau_h - \tau_\ell)a + \kappa(p_h + p_\ell\pi), && (79) \\ d_h &= \rho(\phi + \tau_\ell)a + \omega(\tau_h - \tau_\ell)a + \kappa && (80) \\ d_\ell &= \rho(\phi + \tau_\ell)a + \kappa, && (81) \\ d'_\ell &= \rho(\phi + \tau_\ell)a, && (82) \end{aligned}$$

where d_h is the funds collected from a bank in state h, d_ℓ from a bank in state ℓ but allowed to survive, d'_ℓ from a bank in state ℓ but shut down.

By exerting efforts in asset management, a bank's profit is given by

$$\begin{aligned}
& \frac{d}{R} - \phi a - [\psi(a) + \gamma] + \beta\{[\phi + \mathbb{E}(\tau)]a - [p_h d_h + p_\ell(\pi d_\ell + (1-\pi)d'_\ell)]\} \\
= \;& \beta\left\{\iota d - [r\phi - \mathbb{E}(\tau)]a - (1+r)[\psi(a)+\gamma]\right\} \\
= \;& \beta\left\{\begin{array}{c} -(r-\iota\rho)\phi a + [\mathbb{E}(\tau) + \iota\rho\tau_\ell + \iota\omega p_h(\tau_h - \tau_\ell)]a + \iota(p_h + p_\ell\pi)\kappa \\ -(1+r)[\psi(a)+\gamma] \end{array}\right\}
\end{aligned} \tag{83}$$

where the third equality is obtained by using (79) at equality. By shirking, the bank profit is given by

$$\frac{d}{R} - \phi a - [\psi(a) - ea + \gamma] + \beta\{[\phi + \mathbb{E}_1(\tau)]a - [q_h d_h + q_\ell(\pi d_\ell + (1-\pi)d'_\ell)]\} \tag{84}$$

$$= \beta\left\{\begin{array}{c} \iota d - [r\phi - \mathbb{E}(\tau)]a - (1+r)[\psi(a)+\gamma] \\ +[-(p_h - q_h)(1-\omega)(\tau_h - \tau_\ell) + q_h(\tau'_h - \tau_h) + (1+r)e]a + (p_h - q_h)(1-\pi)\kappa \end{array}\right\}.$$

Thus, assuming that the bank holds a units of assets regardless of shirking or not, the gain from shirking is given by

$$\Phi(a) = \beta\left\{[-(p_h - q_h)(1-\omega)(\tau_h - \tau_\ell) + q_h(\tau'_h - \tau_h) + (1+r)e]a + (p_h - q_h)(1-\pi)\kappa\right\}. \tag{85}$$

Now, let V be the value for a surviving bank, and it satisfies

$$V = [\Pi(\bar{A}/m) - \gamma] + \frac{\iota(p_h + p_\ell\pi)}{1+r}\kappa + \beta(p_h + p_\ell\pi)V.$$

Thus, exerting effort is incentive compatible after the choice of asset as the equilibrium amount, $\bar{A}/m$, only if

$$-\Phi(\bar{A}/m) + \beta(p_h + p_\ell\pi)V \geq \beta(q_h + q_\ell\pi)V.$$

Note that, however, this is only a necessary condition, as a bank may choose a different amount of asset holding when planning to shirk.

Simple algebra yields the following condition for a bank to exert effort:

$$\left[(1-\omega)(\tau_h-\tau_\ell)-\frac{q_h}{p_h-q_h}(\tau_h'-\tau_h)-\frac{(1+r)e}{p_h-q_h}\right]\frac{\bar{A}}{m}+(1-\pi)\left\{-\kappa+V\right\}\geq 0. \tag{86}$$

The incentive for a surviving bank to repay κ requires

$$-\kappa+V\geq 0. \tag{87}$$

Finally, assuming that all banks exert efforts, the equilibrium ι is determined by

$$D(\iota)\leq\frac{\rho\mathbb{E}(\tau)+r[\rho\tau_\ell+p_h\omega(\tau_h-\tau_\ell)]-\rho(1+r)\psi'\left(\frac{\bar{A}}{m}\right)}{r-\iota\rho}\bar{A}+m(p_h+p_\ell\pi)\kappa. \tag{88}$$

Theorem 4.3. *Suppose that (74) does not hold, that* $\rho\in(0,1)$*, and that*

$$p_h\geq\frac{(1+r)\rho}{r+\rho}. \tag{89}$$

Then, among implementable outcomes with banks exerting efforts, it is optimal to set $\pi=1$ *and to set capital requirement as* $\omega=\omega_2$*.*

Remark 4.1. The condition (89) for optimal ω to be ω_2 is a sufficient condition. We conjecture that we can use implicit function theorem to extend this result for a range of lower p_h as well.

Theorem 4.3 shows that for high p_h's, it is optimal to place a minimum capital requirement, $\omega=\omega_2$. However, since this is a rather tight requirement, it may be the case that it is better to place no requirement other than that implied by ρ, and, when (74) does not hold, to expect that in equilibrium all banks shirk. The following theorem gives a precise condition when it is better to induce banks' efforts and when it is better not to.

Theorem 4.4. *Suppose that (74) does not hold and that the best equilibrium with banks exerting effort requires $\omega = \omega_2$. Then, for any given m and κ, such an equilibrium is better (from the depositors' perspective) than the one with all banks shirking if and only if*

$$(1+r)e \leq \frac{(\rho+r)p_h - \rho(1+r)q_h}{(\rho+r)p_h - \rho q_h}(p_h - q_h)(\tau_h - \tau_\ell) - q_h(\tau_h' - \tau_h) \tag{90}$$

5 Concluding remarks

In this paper we take the liquidity role of banks seriously and derive optimal banking regulations. We have shown that when banks are subject to limited commitment, an overall leverage ratio requirement with restricted banking licence can be optimal for welfare in a charter system. In particular, we have shown that under such arrangement, banks have higher profits and higher leverage ratio relative to the laissez-faire economy without banking regulations. This is broadly consistent with the contrast in these two dimensions for the US banking industry entering the Great Depression (an era where no serious regulations) and the industry entering the recent Financial Crisis (an era when more regulations are in place).

Compared to most of the literature, we have shown that considerations for liquidity provision can change many conventional wisdom about banking regulation. First, we show that when moral hazard issue becomes more serious, while it is optimal to increase asset-specific capital requirement, the overall leverage ratio requirement should not be proportionally increased, and it is in fact optimal to allow higher profits for banks to make them more trustworthy. Second, while it is true that in our model under deposit insurance moral hazard would require capital requirement from the regulator, it is not always the case that the regulator should discourage gambling.

Appendix

Proof of Lemma 3.1 (a) In equilibrium $A(\iota, \phi) = \bar{A}/n$. Taking $\iota = 0$ and $a = \bar{A}/n$ into (7), we obtain ϕ given by (4). Finally, (12) ensures that (10) is satisfied with $D(0) = q^*$.
(b) Again, in equilibrium $A(\iota, \phi) = \bar{A}/n$, and substituting $a = \bar{A}/n$ into (7) we obtain ϕ given by (13). We will do guess and verify. Take ϕ given by (13), we have

$$\phi + \tau = (1+r)\frac{\tau - \psi'\left(\frac{\bar{A}}{n}\right)}{r - \iota\rho}, \tag{91}$$

and hence (10) is satisfied iff ι is given by (14). When $\iota = 0$, since (12) does not hold, the left-side of (14) is strictly greater than the right-side. Given that $\tau > \psi'\left(\frac{\bar{A}}{m^*}\right)$ as $\iota \to \frac{r}{\rho}$ the right-side goes to infinity and the left-side remains finite. Since $D(\iota)$ is strictly decreasing and the right-side of (14) is strictly increasing in ι for $\iota \in [0, \frac{r}{\rho}]$, there is a unique $\iota \in (0, \frac{r}{\rho})$ that solves (14).

Proof of Theorem 3.1

$$\tilde{\kappa}(m) = \frac{1+r}{r}\left[\Pi\left(\frac{\bar{A}}{m}\right) - \gamma\right]. \tag{92}$$

(b) Let

$$\hat{\kappa} = \frac{1}{m}\left\{c(q^*) - \left(\rho\frac{1+r}{r}\left[\tau - \psi'\left(\frac{\bar{A}}{m}\right)\right]\bar{A}\right)\right\}. \tag{93}$$

The fact that $\tilde{\kappa}(m)$ does not satisfy (23) implies that $\tilde{\kappa}(m) < \hat{\kappa}$, and, by Lemma 3.2 (b), $\iota(m, \tilde{\kappa}(m)) > 0$ and hence $\tilde{\kappa}(m)$ satisfies (24) with a strict inequality. By Lemma 3.2 (a), $\iota(m, \hat{\kappa}) = 0$ and hence $\hat{\kappa}$ fails (24). The intermediate value theorem implies that there exists a greatest $\bar{\kappa}(m) \in (\tilde{\kappa}(m), \hat{\kappa})$ that satisfies (24) exactly.

Proof of Theorem 3.2 (b) Since (12) does not hold, Lemma 3.1 (b) implies that under $(m,\kappa)=(m^*,0)$ the equilibrium allocation has $q<q^*$. We show that any optimal policy has $m>m^*$.

Now, define

$$\mathcal{S}(\iota,m)=\max_{\kappa\geq 0}\rho(1+r)\frac{\tau-\psi'\left(\frac{\bar{A}}{m}\right)}{r-\iota\rho}\bar{A}+m\kappa,$$

subject to (24). This implies that

$$\mathcal{S}(\iota,m)=\max_{\kappa\geq 0}\rho(1+r)\frac{\tau-\psi'\left(\frac{\bar{A}}{m}\right)}{r-\iota\rho}\bar{A}+m\frac{(1+r)\left[\Pi\left(\frac{\bar{A}}{m}\right)-\gamma\right]}{r-\iota}.$$

It is easy to verify that for any m, the optimal κ and ι is determined by $D(\iota)\leq\mathcal{S}(\iota,m)$ and with equality whenever $\iota>0$. Let $\iota(m)$ be the unique solution. Now, since $\rho>\tilde{\rho}$, $\iota(m^*)<r$.

Now, for all $\iota<r$,

$$\begin{aligned}
\frac{\partial}{\partial m}\mathcal{S}(\iota,m^*) &= \frac{\rho(1+r)\psi''\left(\frac{\bar{A}}{m^*}\right)}{r-\iota\rho}\frac{\bar{A}^2}{(m^*)^2}+\frac{1+r}{r-\iota}\left[\Pi\left(\frac{\bar{A}}{m^*}\right)-\Pi'\left(\frac{\bar{A}}{m^*}\right)\left(\frac{\bar{A}}{m^*}\right)-\gamma\right]\\
&= \frac{\rho(1+r)\psi''\left(\frac{\bar{A}}{m^*}\right)}{r-\iota\rho}\frac{\bar{A}^2}{(m^*)^2}-\frac{1+r}{r-\iota}\left[\psi''\left(\frac{\bar{A}}{m^*}\right)\frac{\bar{A}^2}{(m^*)^2}\right]\\
&= (1+r)\psi''\left(\frac{\bar{A}}{m^*}\right)\frac{\bar{A}^2}{(m^*)^2}\left[\frac{\rho}{r-\iota\rho}-\frac{1}{r-\iota}\right]\\
&= -(1+r)\psi''\left(\frac{\bar{A}}{m^*}\right)\frac{\bar{A}^2}{(m^*)^2}\frac{-(1-\rho)r}{(r-\iota\rho)(r-\iota)}<0.
\end{aligned}$$

Thus, when $\iota(m^*)>0$, for $m<m^*$ but sufficiently close to m^*, we have $\iota(m)<\iota(m^*)$ and $D(\iota(m))>D(\iota(m^*))$.

Proof of Lemma 3.4 Suppose that $\omega\leq\omega_1$. We show that no bank has incentive to shirk. Thus, a shirking bank only pays $d_0=\rho(\phi+\tau_0)a$ to depositors under return τ_0, and hence,

(**??**) implies that the bank profit is

$$
\begin{aligned}
\pi^s(a,d;\phi,R) &= \beta\left\{\iota d + (d-d_0) - (r\phi-\tau_0)a - (1+r)[\psi(a)-ea+\gamma]\right\}, \\
&= \beta\left\{-(r-\iota\rho)\phi a + [(\tau_0+\iota\rho\tau+\rho(\tau-\tau_0)]a - (1+r)[\psi(a)-ea+\gamma]\right\}.
\end{aligned}
$$

The FOC is then given by

$$
-(r-\iota\rho)\phi + [\tau_0+\iota\rho\tau+\rho(\tau-\tau_0)] = (1+r)[\psi'(a)-e]. \tag{94}
$$

If a bank exerts effort, its profit is given by (5), where $d = \rho(\phi+\tau)a$, and the FOC is (7).Let A^s be the a that satisfies (94), and $\bar{A}/m$, the equilibrium amount of asset holding, satisfies (7), for ϕ and ι given by (21) and (22). Since $\tau_0 < \tau$ and since (34) holds, it follows that $A^s \leq \bar{A}/m$; moreover, the profit is also lower, as the profit is given by

$$
\begin{aligned}
& \beta\left\{\{-(r-\iota\rho)\phi + [\tau_0+\iota\rho\tau+\rho(\tau-\tau_0)]\}A^s - (1+r)[\psi(A^s)-eA^s+\gamma]\right\} \\
= \; & \beta\left\{(1+r)[\psi'^s) - e]A^s - (1+r)[\psi(A^s)-eA^s+\gamma]\right\} \\
= \; & [\Pi(A^s)-\gamma] \leq [\Pi(\bar{A}/m)-\gamma].
\end{aligned}
$$

This shows that the profit for shirking is weakly lower than exerting effort.

Proof of Lemma 3.3 Suppose that such an equilibrium exists. The bank profit, under shirking, is then given by (note that here $d \leq \rho(\phi+\tau_0)a$)

$$
\begin{aligned}
\pi^s(a,d;\phi,R) &= \frac{d}{R} - \phi a - [\psi(a)-ea+\gamma] + \beta\{(\phi+\tau_0)a - d\} \qquad (95) \\
&= \beta\left\{\iota d - (r\phi-\tau_0)a - (1+r)[\psi(a)-ea+\gamma]\right\}, \\
&= \beta\left\{-(r-\iota\rho)\phi a + (\tau_0+\iota\rho\tau_0)a - (1+r)[\psi(a)-ea+\gamma]\right\}.
\end{aligned}
$$

On the other hand, by exerting effort, the bank profit is given by

$$\beta\left\{-(r-\iota\rho)\phi a+(\tau+\iota\rho\tau_0)a-(1+r)[\psi(a)+\gamma]\right\}.$$

Using the same reasoning as in the proof of Lemma 3.4, one can show that, given $(1+r)e<\tau-\tau_0$, a bank that exerts efforts holds more assets than shirking banks. Hence, it is profitable to exert efforts. This leads to a contradiction.

Proof of Theorem 3.3 First we show that for any given m, it is optimal to set $\omega=\omega_1$. Note that since m determines asset-management efficiency, the regulator's goal is only to increase liquidity, or, equivalently, to have the lowest equilibrium ι among all (κ,ω) that are incentive compatible. First we show that To do this, we consider a relaxed problem. Instead of working with the constraint (39), we consider a relaxed constraint: we assume that the shirking bank also chooses $\bar{A}/m$. In this case, the gain from shirking is the difference between two expressions (36) and (38) with $a=\bar{A}/m$, which is given by

$$\Phi\equiv\beta\{[\omega(\tau-\tau_0)-(\tau-\tau_0)]\bar{A}/m+(1+r)e\bar{A}/m+\kappa\}. \tag{96}$$

Thus, for banks holding $\bar{A}/m$ units of trees not to shirk it requires

$$-\Phi+\frac{\beta}{1-\beta}\left[\Pi\left(\frac{\bar{A}}{m}\right)-\gamma+\frac{\iota\cdot\kappa}{1+r}\right]\geq 0,$$

which can be simplified to

$$-r\left[\omega(\tau-\tau_0)+(1+r)e-(\tau-\tau_0)\right]\frac{\bar{A}}{m}+\left\{-(r-\iota)\kappa+(1+r)\left[\Pi\left(\frac{\bar{A}}{m}\right)-\gamma\right]\right\}\geq 0. \tag{97}$$

Note that we could rewrite (39) as

$$-r(1+r)\left[\Pi(A^s)-\Pi\left(\frac{\bar{A}}{m}\right)\right]+\left\{-(r-\iota)\kappa+(1+r)\left[\Pi\left(\frac{\bar{A}}{m}\right)-\gamma\right]\right\}\geq 0.$$

Moreover, $\Pi(A^s)-\Pi\left(\frac{\bar{A}}{m}\right)$ and $[\omega(\tau-\tau_0)+(1+r)e-(\tau-\tau_0)]$ have the same sign for all ω (it is positive for $\omega>\omega_1$, negative for $\omega<\omega-1$, zero for $\omega=\omega_1$), and

$$\Pi(A^s)-\Pi\left(\frac{\bar{A}}{m}\right)>[\omega(\tau-\tau_0)+(1+r)e-(\tau-\tau_0)]$$

for all $\omega>\omega_1$. Since when both terms are negative the corresponding constraints are weaker than (24), (97), combined with (24), is indeed weaker than (39) combined with (24). When $\omega=\omega_1$, they are equivalent.

Now, define

$$\mathcal{S}(\iota) \quad = \quad \max_{\omega,\kappa}\frac{\rho\tau+r\rho\tau_0+\omega r(\tau-\tau_0)-\rho(1+r)\psi'\left(\frac{\bar{A}}{m}\right)}{r-\iota\rho}\bar{A}+m\kappa$$

subject to (97) and (24). We claim that the minimum equilibrium ι subject to (97) and (24) is determined by $D(\iota)\leq\mathcal{S}(\iota)$ (at equality whenever $\iota>0$). Note that $\mathcal{S}(\iota)$ is strictly increasing in ι: as ι increases both constraints (97) and (24) are more relaxed, and the objective function is strictly increasing in ι.

For any fixed ι, the maximization problem in $\mathcal{S}(\iota)$ is a linear programming problem in (κ,ω) and can be reduced to

$$\begin{aligned}
&\max_{\kappa,\omega}\frac{\omega r(\tau-\tau_0)}{r-\iota\rho}\bar{A}+m\kappa,\\
\text{s.t.}\quad &-r\omega(\tau-\tau_0)\frac{\bar{A}}{m}-(r-\iota)\kappa+C\geq 0,\\
&-(r-\iota)\kappa+D\geq 0.
\end{aligned}$$

where

$$C = r\left[(\tau-\tau_0)\frac{\bar{A}}{m} - (1+r)e\frac{\bar{A}}{m}\right] + (1+r)\left[\Pi\left(\frac{\bar{A}}{m}\right) - \gamma\right] > D = (1+r)\left[\Pi\left(\frac{\bar{A}}{m}\right) - \gamma\right].$$

Since $\rho < 1$, the optimal choice is given by

$$\kappa = \frac{1+r}{r-\iota}D,\ \omega_1 = \frac{C-D}{r(\tau-\tau_0)\frac{\bar{A}}{m}} = \omega_1.$$

□

Before we prove Lemma 4.1, we first prove Claim 4.1.

Proof of Claim 4.1 It is easy to verify that for any given $\mathbf{m}$, $[A_1(\mathbf{m}), ..., A_N(\mathbf{m})]$ given by (42) uniquely solves

$$\min_{(A_1,...,A_N)} \sum_{n=1}^{N} [m_n \lambda_n \psi(A_n) + m_n \gamma]$$

s.t. $\sum_{n=1}^{N} m_n A_n = \bar{A}$. Moreover, these solutions can be characterized as follows: for any $\mathbf{m}$, define $C(\mathbf{m})$ as the solution to

$$\sum_{n=1}^{N} m_n (\psi')^{-1}\left(\frac{C}{\lambda_n}\right) = \bar{A}. \tag{98}$$

$C(\mathbf{m})$ is well-defined by strict convexity of ψ. Then,

$$A_n(\mathbf{m}) = (\psi')^{-1}\left(\frac{C(\mathbf{m})}{\lambda_n}\right) \text{ if } m_n > 0,\ A_n(\mathbf{m}) = 0 \text{ otherwise.}$$

Now, we can compute the derivatives:

$$\begin{aligned}
\frac{\partial}{\partial m_n} C &= -\frac{A_n(\mathbf{m})}{\sum_{j=1}^N \frac{m_j}{\lambda_j \psi''[A_j(\mathbf{m})]}}, &(99)\\
\frac{\partial}{\partial m_n} A_{n'} &= -\frac{A_n(\mathbf{m})}{\sum_{j=1}^N \frac{m_j}{\lambda_j \psi''[A_j(\mathbf{m})]}} \frac{1}{\lambda_{n'} \psi''[A_{n'}(\mathbf{m})]}. &(100)
\end{aligned}$$

Now, define

$$\Psi(\mathbf{m}) \equiv \sum_{n=1}^N \left[m_n \lambda_n \psi(A_n(\mathbf{m})) + m_n \gamma \right], \tag{101}$$

and we can rewrite the original problem, (40), as

$$\min_{\mathbf{m}} \Psi(\mathbf{m}) \text{ s.t. } m_n \leq \mu_n, \ n = 1, ..., N.$$

By (100), we have

$$\begin{aligned}
\frac{\partial}{\partial m_n} \Psi(\mathbf{m}) &= \lambda_n \psi(A_n(\mathbf{m})) + \gamma - \sum_{k=1}^N m_k \lambda_k \psi'(A_k(\mathbf{m})) \frac{A_n(\mathbf{m})}{\sum_{j=1}^N \frac{m_j}{\lambda_j \psi''[A_j(\mathbf{m})]}} \frac{1}{\lambda_k \psi''[A_k(\mathbf{m})]} \\
&= \lambda_n \psi(A_n(\mathbf{m})) + \gamma - \lambda_n \psi'(A_n(\mathbf{m})) A_n(\mathbf{m}) \frac{\sum_{k=1}^N m_k \frac{1}{\lambda_k \psi''[A_k(\mathbf{m})]}}{\sum_{j=1}^N \frac{m_j}{\lambda_j \psi''[A_j(\mathbf{m})]}} \\
&= -\lambda_n [\psi'(A_n(\mathbf{m})) A_n(\mathbf{m}) - \psi(A_n(\mathbf{m}))] + \gamma \\
&= -\lambda_n \Pi(A_n(\mathbf{m})) + \gamma, &(102)
\end{aligned}$$

where the second equality follows from (42). Since for any $\mathbf{m}$, $\lambda_n \Pi(A_n(\mathbf{m}))$ is strictly decreasing in n among which $m_n > 0$. This implies the optiomal solution has the form given by (43)-(46). Note that (41) guarantees that $\bar{n} \leq N$.

Proof of Lemma 4.1 We show that the unique equilibrium is given by $\mathbf{m} = \mathbf{m}^*$, and

$$\phi = \frac{(1 + \iota\rho)\tau - (1 + r)C(\mathbf{m}^*)}{r - \iota\rho},$$

where $C(\mathbf{m})$ is given by (98), and $A_n = A_n(\mathbf{m}^*)$. Moreover, ι is determined by

$$D(\iota) \leq \rho(1+r)\frac{\tau - C(\mathbf{m})}{r - \iota\rho}\bar{A}, \text{ with equality if } \iota > 0.$$

It is straightforward to verify that these satisfy the market clearing conditions and free entry. Now, uniqueness follows from the fact that market-clearing for asset market and the FOC for asset holdings imply (42), and monotonicity of $\lambda_n \Pi(A_n(\mathbf{m}))$.

Proof of Lemma 4.2 Suppose that $x < y$. Then, equilibrium requires

$$\lambda_x \psi'(A_x) = \lambda_y \psi'(A_y).$$

Hence,

$$\lambda_x \left[\psi'\left(A_x\right) A_x - \psi\left(A_x\right)\right] > \lambda_y \left[\psi'\left(A_y\right) A_y - \psi\left(A_y\right)\right]$$

iff

$$A_x - \frac{\psi\left(A_x\right)}{\psi'(A_x)} > A_y - \frac{\psi\left(A_y\right)}{\psi'(A_y)}.$$

Now,

$$\frac{\mathrm{d}}{\mathrm{d}A}\left[A - \frac{\psi(A)}{\psi'(A)}\right] = 1 - \frac{[\psi'(A)]^2 - \psi(A)\psi''(A)}{[\psi'(A)]^2} = \frac{\psi(A)\psi''(A)}{[\psi'(A)]^2} > 0.$$

Proof of Lemma 4.3 Let C be given by (98). Define

$$\mathcal{S}(\iota, \mathbf{m}) = \max_{\kappa \geq 0} \rho(1+r)\frac{\tau - C(\mathbf{m})}{r - \iota\rho}\bar{A} + \bar{\kappa},$$

subject to

$$-[r - \iota]\bar{\kappa} + (1+r)\sum_{n=1}^{N} m_n \left[\lambda_n \Pi(A_n(\mathbf{m})) - \gamma\right] \geq 0. \tag{103}$$

This implies that

$$\begin{aligned}
\mathcal{S}(\iota,\mathbf{m}) &= \rho(1+r)\frac{\tau - C(\mathbf{m})}{r-\iota\rho}\bar{A} + \frac{(1+r)\sum_{n=1}^{N} m_n\left[\lambda_n \Pi(A_n(\mathbf{m})) - \gamma\right]}{r-\iota} \\
&= \rho(1+r)\frac{\tau - C(\mathbf{m})}{r-\iota\rho}\bar{A} + \frac{(1+r)[-\Psi(\mathbf{m}) + \sum_{n=1}^{N} m_n\lambda_n\psi'(A_n(\mathbf{m}))A_n(\mathbf{m})]}{r-\iota},
\end{aligned}$$

where Ψ is given by (101). Note that, for any fixed $\mathbf{m}$, $\mathcal{S}(\iota,\mathbf{m})$ is strictly increasing in ι.

For each n, let

$$\bar{\kappa}_n(\mathbf{m},\iota) = \frac{(1+r)\left[\lambda_n\Pi(A_n(\mathbf{m})) - \gamma\right]}{r-\iota}.$$

Fixed some $\mathbf{m}$, we consider two cases.

(i) If $D(0) \le \mathcal{S}(0,\mathbf{m})$, then q^* is implementable with $\kappa_n = \bar{\kappa}_n(\mathbf{m},0)$, which is optimal under $\mathbf{m}$.

(ii) Otherwise, let $\iota(\mathbf{m})$ be the unique solution to

$$D(\iota) \le \mathcal{S}(\iota,\mathbf{m}). \tag{104}$$

Then, $\kappa_n = \bar{\kappa}_n(\mathbf{m},\iota(\mathbf{m}))$ is optimal under $\mathbf{m}$.

Proof of Theorem 4.1 (b) Let $\iota(\mathbf{m})$ be defined by (104). Then, $\iota(\mathbf{m}^*) > 0$. Now,

$$\begin{aligned}
\frac{\partial}{\partial m_{\bar{n}}}\mathcal{S}(\iota,\mathbf{m}^*) &= -\frac{\rho(1+r)\bar{A}}{r-\iota\rho}\frac{\partial}{\partial m_{\bar{n}}}C(\mathbf{m}^*) + \frac{1+r}{r-\iota}\lambda_{\bar{n}}\psi'(A_{\bar{n}}(\mathbf{m}^*))A_{\bar{n}}(\mathbf{m}^*) \\
&+ \frac{(1+r)\left[-\frac{\partial\Psi(\mathbf{m}^*)}{\partial m_{\bar{n}}} + \sum_{n=1}^{N} m_n\lambda_n[\psi''(A_n(\mathbf{m}^*))A_n(\mathbf{m}^*) + \psi'(A_n(\mathbf{m}^*))]\frac{\partial}{\partial m_{\bar{n}}}A_n(\mathbf{m}^*)\right]}{r-\iota} \\
&= -\frac{\rho(1+r)\bar{A}}{r-\iota\rho}\frac{\partial}{\partial m_{\bar{n}}}C(\mathbf{m}^*) + \frac{1+r}{r-\iota}\lambda_{\bar{n}}\psi'(A_{\bar{n}}(\mathbf{m}^*))A_{\bar{n}}(\mathbf{m}^*) \\
&+ \frac{(1+r)\left[\sum_{n=1}^{N} m_n\lambda_n[\psi''(A_n(\mathbf{m}^*))A_n(\mathbf{m}^*) + \psi'(A_n(\mathbf{m}^*))]\frac{\partial}{\partial m_{\bar{n}}}A_n(\mathbf{m}^*)\right]}{r-\iota},
\end{aligned}$$

since by (102) and by definition of $\bar{n}$,

$$\frac{\partial}{\partial m_{\bar{n}}}\Psi(\mathbf{m}^*) = -\left[\lambda_{\bar{n}}\Pi(A_{\bar{n}}(\mathbf{m}^*)) - \gamma\right] = 0.$$

Now, by (100),

$$\begin{aligned}
& \sum_{n=1}^{N} m_n\lambda_n\psi''(A_n(\mathbf{m}^*))A_n(\mathbf{m}^*)\frac{\partial}{\partial m_{\bar{n}}}A_n(\mathbf{m}^*) \\
= \quad & -\sum_{n=1}^{N} m_n\lambda_n\psi''(A_n(\mathbf{m}^*))A_n(\mathbf{m}^*)\frac{A_{\bar{n}}(\mathbf{m}^*)}{\sum_{j=1}^{N}\frac{m_j}{\lambda_j\psi''[A_j(\mathbf{m})]}}\frac{1}{\lambda_n\psi''[A_n(\mathbf{m}^*)]} \\
= \quad & -\left(\sum_{n=1}^{N} m_n A_n(\mathbf{m})\right)\frac{A_{\bar{n}}(\mathbf{m}^*)}{\sum_{j=1}^{N}\frac{m_j}{\lambda_j\psi''[A_j(\mathbf{m})]}} \\
= \quad & -\frac{\bar{A}A_{\bar{n}}(\mathbf{m}^*)}{\sum_{j=1}^{N}\frac{m_j}{\lambda_j\psi''[A_j(\mathbf{m})]}},
\end{aligned}$$

and

$$\begin{aligned}
& \sum_{n=1}^{N} m_n\lambda_n\psi'(A_n(\mathbf{m}^*))\frac{\partial}{\partial m_{\bar{n}}}A_n(\mathbf{m}^*) \\
= \quad & -\sum_{n=1}^{N} m_n\lambda_n\psi'(A_n(\mathbf{m}^*))\frac{A_{\bar{n}}(\mathbf{m}^*)}{\sum_{j=1}^{N}\frac{m_j}{\lambda_j\psi''[A_j(\mathbf{m})]}}\frac{1}{\lambda_n\psi''[A_n(\mathbf{m}^*)]} \\
= \quad & -\lambda_{\bar{n}}\psi'(A_{\bar{n}}(\mathbf{m}^*))A_{\bar{n}}(\mathbf{m}^*)\frac{\sum_{n=1}^{N}\frac{m_n}{\lambda_n\psi''[A_n(\mathbf{m}^*)]}}{\sum_{j=1}^{N}\frac{m_j}{\lambda_j\psi''[A_j(\mathbf{m})]}} \\
= \quad & -\lambda_{\bar{n}}\psi'(A_{\bar{n}}(\mathbf{m}^*))A_{\bar{n}}(\mathbf{m}^*),
\end{aligned}$$

where the second last equality follows from the fact that $\lambda_{\bar{n}}\psi'(A_{\bar{n}}(\mathbf{m}^*)) = \lambda_n\psi'(A_n(\mathbf{m}^*))$ for all n with $\mathbf{m}_n^* > 0$.

Now, combining the terms and use (99), we obtain

$$\begin{aligned}
\frac{\partial}{\partial m_{\bar{n}}}\mathcal{S}(\iota,\mathbf{m}^*) &= \frac{\rho(1+r)\bar{A}}{r-\iota\rho}\frac{A_{\bar{n}}(\mathbf{m}^*)}{\sum_{j=1}^{N}\frac{m_j}{\lambda_j\psi''[A_j(\mathbf{m})]}} + \frac{1+r}{r-\iota}\lambda_{\bar{n}}\psi'(A_{\bar{n}}(\mathbf{m}))A_{\bar{n}}(\mathbf{m}) \\
&- \frac{1+r}{r-\iota}\left[\lambda_{\bar{n}}\psi'(A_{\bar{n}}(\mathbf{m}^*))A_{\bar{n}}(\mathbf{m}^*) + \frac{\bar{A}A_{\bar{n}}(\mathbf{m}^*)}{\sum_{j=1}^{N}\frac{m_j}{\lambda_j\psi''[A_j(\mathbf{m})]}}\right] \\
&= \left[\frac{\rho(1+r)}{r-\iota\rho} - \frac{(1+r)}{r-\iota}\right]\left[\frac{\bar{A}A_{\bar{n}}(\mathbf{m}^*)}{\sum_{j=1}^{N}\frac{m_j}{\lambda_j\psi''[A_j(\mathbf{m})]}}\right] < 0.
\end{aligned}$$

Proof of Claim 4.2 Define

$$\begin{aligned}
f(d,\nu) &= \frac{[u'(q(d,g)) - c'(q(d,g))]}{c'(q(d,g))}\{[p_h\nu + p_\ell(1-\nu)] + [p_h\nu\delta_h + p_\ell(1-\nu)\delta_\ell]\} \\
&+ \frac{[u'(q(d,b)) - c'(q(d,b))]}{c'(q(d,b))}\{[p_\ell\nu + p_h(1-\nu)] + [p_\ell\nu\delta_\ell + p_h(1-\nu)\delta_h]\}.
\end{aligned}$$

Then, $D(\iota;\delta_h,\delta_\ell)$ is determined by $-\iota + \sigma f(d,\nu) = 0$. Now,

$$\begin{aligned}
\frac{\partial}{\partial\nu}f(d,\nu) &= \frac{[u''(q(d,g))c'(q(d,g)) - u'(q(d,g))c''(q(d,g))]}{c'(q(d,g))^2}\frac{\{[p_h\nu(1+\delta_h) + p_\ell(1-\nu)\delta_\ell]\}^2}{[p_h\nu + p_\ell(1-\nu)]}\mathbf{1}_{q(d,g)<q^*} \\
&+ \frac{[u'(q(d,g)) - c'(q(d,g))]}{c'(q(d,g))}\{[p_h - p_\ell] + [p_h\delta_h - p_\ell\delta_\ell]\} \\
&+ \frac{[u''(q(d,b))c'(q(d,b)) - u'(q(d,b))c''(q(d,b))]}{c'(q(d,b))^2}\frac{\{[p_\ell\nu(1+\delta_\ell) + p_h(1-\nu)\delta_h]\}^2}{[p_\ell\nu + p_h(1-\nu)]} \\
&- \frac{[u'(q(d,b)) - c'(q(d,b))]}{c'(q(d,b))}\{[p_h - p_\ell] + [p_h\delta_h - p_\ell\delta_\ell]\}.
\end{aligned}$$

Since

$$\frac{[u''(q(d,g))c'(q(d,g)) - u'(q(d,g))c''(q(d,g))]}{c'(q(d,g))^2} < 0,\ \frac{[u''(q(d,b))c'(q(d,b)) - u'(q(d,b))c''(q(d,b))]}{c'(q(d,b))^2} < 0,$$

and since

$$\frac{[u'(q(d,g)) - c'(q(d,g))]}{c'(q(d,g))} < \frac{[u'(q(d,b)) - c'(q(d,b))]}{c'(q(d,b))},$$

it follows that $\frac{\partial}{\partial \nu} f(d,\nu) < 0$.

Proof of Theorem 4.2 Consider the case where the first best is not achievable. Without deposit insurance, for a buyer that deposits d, the bank pays $d_h = D(\iota;\delta_h,\delta_\ell) + \delta_h$ if state $s = h$; otherwise, the bank pays $d_\ell = D(\iota;\delta_h,\delta_\ell) + \delta_\ell$, where $D(\iota;\delta_h,\delta_\ell) = \rho[\mathbb{E}(\tau) + \phi]\bar{A} + m\kappa$ and δ_h and δ_ℓ are defined in (67). Then, given m and κ, welfare is given by

$$\sum_s \sigma[u(q_s) - c(q_s)] = \sigma\left\{p_h\left[u(q(d_h)) - c(q(d_h))\right] + p_\ell[u(q(d_\ell)) - c(q(d_\ell))]\right\}.$$

Under deposit insurance, welfare is

$$\sum_s \sigma[u(q_s) - c(q_s)] = \sigma\left[u(q(d)) - c(q(d))\right],$$

where $d = D(\iota) = \rho[\mathbb{E}(\tau) + \phi]\bar{A} + m\kappa$. Note that $p_h d_h + p_\ell d_\ell = D(\iota;\delta_h,\delta_\ell) = D(\iota)$. Because $c(q(d)) = d$, $q = c^{-1}(d)$, and therefore, $u(c^{-1}(d))$ is strictly concave. Hence, deposit insurance improves welfare.

Proof of Lemma 4.5 The profit with efforts is still given by (64). The profit with shirking, however, is given by

$$\begin{aligned}
& \frac{d}{R} - \phi a - [\psi(a) - ea - \gamma] - \gamma + \beta\{[\phi + \mathbb{E}_1(\tau)]a - [q_h d_h + q_\ell d_\ell]\} \\
= \;& \beta\left\{\iota d + \rho(p_h - q_h)(\tau_h - \tau_\ell) - [r\phi - \mathbb{E}_1(\tau)]a - (1+r)[\psi(a) - ea + \gamma]\right\} \\
= \;& \beta\left\{\begin{array}{c} -(r-\iota\rho)\phi a + (1+\iota\rho)\mathbb{E}(\tau)a + \iota\kappa - (1+r)[\psi(a)+\gamma] \\ +[(\mathbb{E}_1(\tau) - \mathbb{E}(\tau)) + \rho(p_h - q_h)(\tau_h - \tau_\ell) + (1+r)e]a \end{array}\right\}.
\end{aligned} \tag{105}$$

Comparing (105) to (64), it is straightforward ro verify that shirking is not profitable if and only if

$$(\mathbb{E}_1(\tau) - \mathbb{E}(\tau)) + \rho(p_h - q_h)(\tau_h - \tau_\ell) + (1+r)e \le 0,$$

which is equivalent to (74).

Proof of Lemma 4.6 The profit with shirking is still given by (76). The profit with efforts, however, is given by

$$
\begin{aligned}
& \frac{d}{R} - \phi a - [\psi(a) - \gamma] - \gamma + \beta\{[\phi + \mathbb{E}(\tau)]a - [p_h d_h + p_\ell d_\ell]\} \\
= \;& \beta\left\{\iota d - \rho(p_h - q_h)(\tau_h - \tau_\ell) - [r\phi - \mathbb{E}_1(\tau)]a - (1+r)[\psi(a) + \gamma]\right\} \\
= \;& \beta\left\{\begin{array}{c} -(r - \iota\rho)\phi a + [\mathbb{E}_1(\tau) + \iota\rho\mathbb{E}_0(\tau)]a + \iota\kappa - (1+r)[\psi(a) - ea + \gamma] \\ +[(\mathbb{E}(\tau) - \mathbb{E}_1(\tau)) - \rho(p_h - q_h)(\tau_h - \tau_\ell) - (1+r)e]a \end{array}\right\}. \quad (106)
\end{aligned}
$$

Comparing (76) to (106), it is straightforward to verify that exerting effort is not profitable if and only if

$$
(\mathbb{E}_1(\tau) - \mathbb{E}(\tau)) + \rho(p_h - q_h)(\tau_h - \tau_\ell) + (1+r)e \geq 0.
$$

Proof of Theorem 4.3 Let ι be given. The IC for repaying κ is given by

$$
-\kappa + \frac{1}{1 - \beta(p_h + p_\ell\pi)}\left[\Pi(\bar{A}/m) - \gamma + \frac{\iota(p_h + p_\ell\pi)}{1+r}\kappa\right] \geq 0. \quad (107)
$$

Hence, we may rewrite (86) as

$$
\begin{aligned}
& \bar{r}\left[(1-\omega)(\tau_h - \tau_\ell) + \frac{q(\tau_h' - \tau_h) + (1+r)e}{p_h - q_h}\right]\frac{\bar{A}}{m} \quad (108) \\
+ \;\; & (1-\pi)\left[-(\bar{r} - \iota)\kappa + (1+\bar{r})[\Pi(\bar{A}/m) - \gamma]\right] \geq 0,
\end{aligned}
$$

where

$$
\bar{r} = \frac{1 - \beta(p_h + p_\ell\pi)}{\beta(p_h + p_\ell\pi)}.
$$

Now, consider the following problem:

$$\begin{aligned}
&\max_{\kappa,\omega} \frac{rp_h\omega(\tau_h-\tau_\ell)}{r-\iota\rho}\bar{A}+m(p_h+p_\ell\pi)\kappa,\\
\text{s.t.}\quad &-\omega(\tau_h-\tau_\ell)\frac{\bar{A}}{m}-(1-\pi)\frac{\bar{r}-\iota}{\bar{r}}\kappa+C\geq 0.
\end{aligned}$$

Note that this is maximizing the right-side of (88) subject to the constraint (108) and is a linear problem. The solution has $\kappa\cdot\omega=0$. Note that since the ratio

$$\frac{(p_h+p_\ell\pi)\bar{r}}{(1-\pi)(\bar{r}-\iota)}$$

is increasing in π, the problem has a positive solution for ω only if it has one under $\pi=0$. Now, when $\pi=0$, (89) ensures that the solution is $\omega=0$.

Thus, the optimal policy has ω as small as possible. Since (87) must hold, this gives the lowest $\omega=\omega_2$ as in (74). Finally, given that we set $\omega=\omega_2$, it is optimal to set $\pi=1$.

Proof of Theorem 4.4 For depositors' welfare, the only thing that matters is the amount of equilibrium deposits. Now, by (78) and by (88), for any given ι, the amounts of deposits in real terms are given by

$$D^s(\iota) = \rho\frac{\mathbb{E}_1(\tau)+r\mathbb{E}_0(\tau)-(1+r)[\psi'(\bar{A}/m)-e]}{r-\iota\rho}\bar{A}+m\kappa, \tag{109}$$

$$D^e(\iota) = \frac{\rho\mathbb{E}(\tau)+r[\rho\tau_\ell+p_h\omega_2(\tau_h-\tau_\ell)]-\rho(1+r)\psi'\left(\frac{\bar{A}}{m}\right)}{r-\iota\rho}\bar{A}+m\kappa, \tag{110}$$

where D^s is the amount under the shirking equilibrium and D^e is the amount under exerting efforts (with $\omega = \omega_2$ and $\pi = 1$). Now, by simple algebra, $D^e(\iota) \geq D^s(\iota)$ if and only if

$$
\begin{aligned}
& \rho\left[\mathbb{E}(\tau) - \mathbb{E}_1(\tau) - (1+r)e\right] + \rho r\left[\tau_\ell - \mathbb{E}_0(\tau)\right] + rp_h\omega_2(\tau_h - \tau_\ell) \geq 0, \\
\iff \quad & (\tau_h - \tau_\ell)\left\{[\rho(p_h - q_h) + rp_h]\omega_2 - \rho r q_h\right\} \geq 0, \\
\iff \quad & \omega_2 \geq \frac{\rho r q_h}{(\rho + r)p_h - \rho q_h}, \\
\iff \quad & \frac{(\rho+r)p_h - \rho(1+r)q_h}{(\rho+r)p_h - \rho q_h}(p_h - q_h)(\tau_h - \tau_\ell) - q_h(\tau_h' - \tau_h) \geq (1+r)e.
\end{aligned}
$$

Appendix B.

Deriving comparative statics in Section 3.2 When liquidity is not sufficient to buy the efficient quantity of consumption, the asset bears a liquidity premium, and $\iota > 0$. The effects of parameters on ι and the asset price are as follows:

$$
\frac{\partial \iota}{\partial \kappa} < 0\,,\ \frac{\partial \iota}{\partial m} < 0,\ \frac{\partial \iota}{\partial \rho} < 0,\ \frac{\partial \iota}{\partial \sigma} > 0,\ \frac{\partial \phi}{\partial \kappa} < 0,\ \frac{\partial \phi}{\partial \sigma} > 0, \tag{111}
$$

and

$$
-D'(\iota)(r - \iota\rho)(1+r)\frac{\bar{A}}{m^2}\psi''\left(\frac{\bar{A}}{m}\right) \;>\; \kappa\rho(1+r)\left[\tau - \psi''\left(\frac{\bar{A}}{m}\right)\right] \Leftrightarrow \frac{\partial \phi}{\partial m} > 0, \tag{112}
$$

$$
-D'(\iota)(r - \iota\rho)\iota \;>\; (1+r)\rho\bar{A}\left[\tau - \psi'\left(\frac{\bar{A}}{m}\right)\right] \Leftrightarrow \frac{\partial \phi}{\partial \rho} > 0. \tag{113}
$$

(See the Appendix for details of derivation.) When banks are allowed to issue more unsecured debt, demand for deposits must rise to clear the market, and thus, ι must fall because $D'(\iota) < 0$. (A similar mechanism underlies the effects of an increase in banking licenses and pledgeability; $\frac{\partial \iota}{\partial m} < 0$ and $\frac{\partial \iota}{\partial \rho} < 0$.) Also, when κ increases, assets are not as important to back deposits, so ϕ decreases. When trade frictions in the DM is reduced, demand for deposits rise, resulting a higher ι. The asset price also increases, because banks need more assets in order to match higher demand for deposits. However, the effects of a change in m and ρ on the asset price depend on two opposing forces. We have seen that, when the regulator licences more banks, ι falls, which increases the demand for deposits and, therefore, banks' demand for assets. This effect pushes up ϕ. On the other hand, more severe competition erodes banks' profits, decreasing the demand for assets. From (112) and (113), when $-D'(\iota)$ is big enough; i.e., the force via a rise in deposit demand is sufficiently large, the effects of m and ρ on ϕ are positive. In addition, as κ is close to zero, $\frac{\partial \phi}{\partial m} > 0$. Alternatively, we express the effects as:

$$\rho < \frac{-D'(\iota)r(1+r)\frac{\bar{A}}{m^2}\psi''(\frac{\bar{A}}{m})}{-\left\{\kappa(1+r)[\tau - \psi'(\frac{\bar{A}}{m})] - D'(\iota)\iota(1+r)\frac{\bar{A}}{m^2}\psi''(\frac{\bar{A}}{m})\right\}} \qquad \Leftrightarrow \frac{\partial \phi}{\partial m} > 0,$$

$$\rho < \frac{-D'(\iota)r\iota}{-D'(\iota)\iota^2 + (1+r)\bar{A}[\tau - \psi'(\frac{\bar{A}}{m})]} \qquad \Leftrightarrow \frac{\partial \phi}{\partial \rho} > 0.$$

An increase in m and ρ has a positive effect on ϕ when pledgeability is not sufficiently large.

Using (21) and (22), let Δ_0 denote the determinant of the following matrix:

$$\begin{bmatrix} -\rho(\phi+\tau) & r - \iota\rho \\ D'(\iota)(r-\iota\rho) - \rho[D(\iota) - m\kappa] & 0 \end{bmatrix},$$

where $D'(\iota) = \frac{\partial D(\iota)}{\partial \iota} < 0$. Given that $r - \iota\rho > 0$, and $D(\iota) - m\kappa > 0$ from (22), we know

$\Delta_0 > 0$, and from (9), $\frac{\partial D(\iota)}{\partial \sigma} > 0$. Then, we have

$$
\begin{aligned}
\frac{\partial \iota}{\partial \kappa} &= \frac{-m(r-\iota\rho)^2}{\Delta_0} < 0, \\
\frac{\partial \iota}{\partial m} &= \frac{-(r-\iota\rho)[\kappa(r-\iota\rho) + \frac{(1+r)\rho\bar{A}^2\psi''(\frac{\bar{A}}{m})}{m^2}]}{\Delta_0} < 0, \\
\frac{\partial \iota}{\partial \rho} &= \frac{-(r-\iota\rho)\{\iota[D(\iota) - m\kappa] + (1+r)\bar{A}[\tau - \psi'(\frac{\bar{A}}{m})]\}}{\Delta_0} < 0, \\
\frac{\partial \iota}{\partial \sigma} &= \frac{\frac{\partial D(\iota)}{\partial \sigma}(r-\iota\rho)^2}{\Delta_0} > 0, \\
\frac{\partial \phi}{\partial \kappa} &= \frac{-m(r-\iota\rho)\rho(\tau+\phi)}{\Delta_0} < 0, \\
\frac{\partial \phi}{\partial m} &= \frac{-\kappa\rho(1+r)[\tau - \psi'(\frac{\bar{A}}{m})] - D'(\iota)(r-\iota\rho)(1+r)\frac{\bar{A}}{m^2}\psi''(\frac{\bar{A}}{m})}{\Delta_0}, \\
\frac{\partial \phi}{\partial \rho} &= \frac{-(\tau+\phi)\left\{D'(\iota)(r-\iota\rho)\iota + (1+r)\rho\bar{A}[\tau - \psi'(\frac{\bar{A}}{m})]\right\}}{\Delta_0}, \\
\frac{\partial \phi}{\partial \sigma} &= \frac{\frac{\partial D(\iota)}{\partial \sigma}(r-\iota\rho)\rho(\tau+\phi)}{\Delta_0} > 0.
\end{aligned}
$$

Moreover, we express the conditions on the signs of $\frac{\partial \phi}{\partial m}$ and $\frac{\partial \phi}{\partial \rho}$ as follows.

$$
\begin{aligned}
-D'(\iota)(r-\iota\rho)(1+r)\frac{\bar{A}}{m^2}\psi''(\frac{\bar{A}}{m}) > \kappa\rho(1+r)[\tau - \psi''(\frac{\bar{A}}{m})] \quad &\Leftrightarrow \frac{\partial \phi}{\partial m} > 0, \\
-D'(\iota)(r-\iota\rho)\iota > (1+r)\rho\bar{A}[\tau - \psi'(\frac{\bar{A}}{m})] \quad &\Leftrightarrow \frac{\partial \phi}{\partial \rho} > 0.
\end{aligned}
$$

Or, equivalently,

$$
\begin{aligned}
\rho < \frac{-D'(\iota)r(1+r)\frac{\bar{A}}{m^2}\psi''(\frac{\bar{A}}{m})}{-\left\{\kappa(1+r)[\tau - \psi'(\frac{\bar{A}}{m})] - D'(\iota)\iota(1+r)\frac{\bar{A}}{m^2}\psi''(\frac{\bar{A}}{m})\right\}} \quad &\Leftrightarrow \frac{\partial \phi}{\partial m} > 0, \\
\rho < \frac{-D'(\iota)r\iota}{-D'(\iota)\iota^2 + (1+r)\bar{A}[\tau - \psi'(\frac{\bar{A}}{m})]} \quad &\Leftrightarrow \frac{\partial \phi}{\partial \rho} > 0.
\end{aligned}
$$

Deriving comparative statics in Section 3.3 Using (30) and (31) with equality, we let Δ_1 denote the determinant of the following matrix:

$$\begin{bmatrix} -\rho(\phi+\tau_0)-\omega(\tau-\tau_0) & r-\iota\rho \\ D'(\iota)(r-\iota\rho)-\rho[D(\iota)-m\kappa] & 0 \end{bmatrix}.$$

Given that $r-\iota\rho>0$, and $D(\iota)-m\kappa>0$ from (31), we know $\Delta_0>0$, and from (9), $\frac{\partial D(\iota)}{\partial\sigma}>0$. Then, we have

$$\begin{aligned}
\frac{\partial\iota}{\partial\kappa} &= \frac{-m(r-\iota\rho)^2}{\Delta_1}<0,\\
\frac{\partial\iota}{\partial m} &= \frac{-(r-\iota\rho)[\kappa(r-\iota\rho)+\frac{(1+r)\rho\bar{A}^2\psi''(\frac{\bar{A}}{m})}{m^2}]}{\Delta_1}<0,\\
\frac{\partial\iota}{\partial\omega} &= \frac{-r(r-\iota\rho)\bar{A}(\tau-\tau_0)}{\Delta_1}<0,\\
\frac{\partial\iota}{\partial\rho} &= \frac{(r-\iota\rho)\{-\iota[D(\iota)-m\kappa]+\bar{A}[(1+r)\psi'(\frac{\bar{A}}{m})-\tau]-\bar{A}r\tau_0\}}{\Delta_1}<0,\\
\frac{\partial\iota}{\partial\sigma} &= \frac{\frac{\partial D(\iota)}{\partial\sigma}(r-\iota\rho)^2}{\Delta_1}>0,\\
\frac{\partial\phi}{\partial\kappa} &= \frac{-m(r-\iota\rho)[\rho(\phi+\tau_0)+\omega(\tau-\tau_0)]}{\Delta_1}<0,\\
\frac{\partial\phi}{\partial m} &= \frac{-1}{m^2\Delta_1}\left\{\begin{array}{c} [D'(\iota)(r-\iota\rho)-\rho(D(\iota)-m\kappa)](1+r)\bar{A}\psi''(\frac{\bar{A}}{m}) \\ +[\kappa m^2(r-\iota\rho)+(1+r)\rho\bar{A}^2\psi''(\frac{\bar{A}}{m})][\rho(\phi+\tau_0)+\omega(\tau-\tau_0)] \end{array}\right\},\\
\frac{\partial\phi}{\partial\omega} &= \frac{(\tau-\tau_0)}{\Delta_1}\left\{-D'(\iota)(r-\iota\rho)\iota+\iota\rho[D(\iota)-m\kappa]-r\bar{A}[\rho(\phi+\tau_0)+\omega(\tau-\tau_0)]\right\},\\
\frac{\partial\phi}{\partial\rho} &= \frac{\iota(\phi+\tau_0)\{-D'(\iota)(r-\iota\rho)+\rho[D(\iota)-m\kappa]\}}{\Delta_1}\\
&\quad -\frac{[\rho(\phi+\tau_0)+\omega(\tau-\tau_0)]\{\iota[D(\iota)-m\kappa]+\bar{A}[\tau-(1+r)\psi'(\frac{\bar{A}}{m})+r\tau_0]\}}{\Delta_1},\\
\frac{\partial\phi}{\partial\sigma} &= \frac{\frac{\partial D(\iota)}{\partial\sigma}(r-\iota\rho)[\rho(\phi+\tau_0)+\omega(\tau-\tau_0)]}{\Delta_1}>0.
\end{aligned}$$

Moreover,

$$
\begin{aligned}
-D'(\iota)(r-\iota\rho)\iota + \iota\rho[D(\iota) - m\kappa] > r\bar{A}[\rho(\phi+\tau_0) + \omega(\tau-\tau_0)] \qquad &\Leftrightarrow \frac{\partial\phi}{\partial\omega} > 0,\\
-D'(\iota)(r-\iota\rho)\iota > -\{r\bar{A}[(1+r)e + \tau_0 - \tau] + \rho\bar{A}[(1+r)\psi'(\frac{\bar{A}}{m}) - \tau] - r\rho\tau_0\bar{A}\} \qquad &\Leftrightarrow \frac{\partial\phi}{\partial\omega} > 0,\\
-D'(\iota)(r-\iota\rho)\iota + \iota\rho[D(\iota) - m\kappa] > r\bar{A}[\rho(\phi+\tau_0) + \omega(\tau-\tau_0)] \qquad &\Leftrightarrow \frac{\partial\phi}{\partial\omega} > 0.
\end{aligned}
$$

When the optimal policy is such that $\omega = \rho$, we have

$$
\begin{aligned}
\rho < \frac{-D'(\iota)r\iota}{\{r\bar{A}(\phi+\tau_0) - \iota[D(\iota) - m\kappa]\} - D'(\iota)\iota^2 + r\bar{A}(\tau-\tau_0)} \qquad &\Leftrightarrow \frac{\partial\phi}{\partial\omega} > 0,\\
\rho < \frac{-D'(\iota)r\iota(\phi+\tau_0)}{(\phi+\tau)[\tau - (1+r)\psi'(\frac{\bar{A}}{m}) + r\tau_0]\bar{A} - (\phi+\tau_0)D'(\iota)\iota^2 + (\tau-\tau_0)\iota[D(\iota) - m\kappa]} \qquad &\Leftrightarrow \frac{\partial\phi}{\partial\omega} > 0.
\end{aligned}
$$

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