Bounded Competition in Monetary Economies

Narayana R. Kocherlakota*

September 23, 2017

University of Rochester and NBER

Abstract

This paper has two parts. In the first part, I demonstrate that, in the absence of price and wage bounds, monetary models do not have *current* equilibria - and so lack predictive content - for a wide range of possible policy rules and/or *future* equilibrium outcomes. This non-existence problem disappears in models in which firms face (arbitrarily loose) finite upper bounds on prices or positive lower bonds on nominal wages. In the second part, I study the properties of a class of dynamic monetary models with these kinds of bounds on prices/wages. Among other results, I show that these models imply that the Phillips curve is L-shaped, are consistent with the existence of permanently inefficiently low output (secular stagnation), and do not imply that forward guidance is surprisingly effective. I show too that economies with lower nominal wage floors have even worse equilibrium outcomes in welfare terms. It follows that models with arbitrarily low but positive nominal wage floors are not well approximated by models without wage floors.

*I thank Eunmi Ko for excellent research assistance, I thank Mariacristina deNardi, Greg Kaplan and participants in a seminar at Cornell University for valuable comments, and I thank Ivan Werning, whose criticism of an earlier paper (Kocherlakota 2016a) spurred me to write this one. This is still a very preliminary draft. All comments are welcome and especially suggestions for other related literature that should be discussed.

1 Introduction

This paper has two parts. In the first part, I set forth a class of monetary models in which prices and wages adjust freely. I demonstrate that these models have an important defect: it is only possible to ensure existence of equilibrium in a given period by imposing (possibly tight) restrictions on the set of monetary policy rules and/or *future* equilibrium outcomes. It follows that the models are uninformative about what happens if governments and agents don't obey those restrictions. I demonstrate that this non-existence result disappears when firms face a finite upper bound on their price choices and a positive lower bound on their wage choices (regardless of how loose these bounds are). I conclude that useful monetary models require a finite price ceiling and a positive wage floor.

In the second part, I study the properties of a class of such monetary models. I show that they have a number of important properties that are distinct from the implications of conventional models with price-setting and wage-setting frictions. Specifically, I prove the following results about the models with price/wage bounds:[1]

- Whenever there is a negative output gap (output is inefficiently low), the inflation rate is equal to its lowest possible level. When the output gap is zero (output is efficient), the inflation rate varies. In this sense, the models predict an L-shaped Phillips curve that is horizontal when the output gap is negative and vertical when the output gap is zero. (The output gap cannot be positive.)

- The models are consistent with a form of secular stagnation in the sense that, under weak conditions, there is a set of equilibria in which the output gap is permanently negative.

- When output is efficient, the output multiplier on government purchases is zero. When output is inefficiently low, the output multiplier on government purchases is one.

[1]The models are finite horizon and have a set of equilibria that are indexed by final-period (possibly random) inflation. All of these results are conditional on a particular specification of final period inflation (which might be interpreted as "anchored" inflation expectations).

- There is no "forward guidance puzzle": if interest rate rules obey the Taylor Principle, the current impact of forward guidance about future interest rates declines exponentially with the horizon of the guidance.[2]

- Lowering the nominal wage floor makes inefficient equilibrium outcomes even worse in a welfare sense.[3]

In Appendix A, I describe highly accurate numerical solution methods for (the fully nonlinear) versions of this class of models in which exogenous shocks follow a Markov chain. I apply these methods in a numerical example that illustrates the (possibly surprising) power of slightly negative nominal interest rates.[4]

The theoretical argument in the first part of the paper justifies the imposition of *some* bounds on prices and wages. Of course, the quantitative implications of a model with such bounds necessarily depend on their magnitudes. But, as described above, the nature of this dependence is somewhat counter-intuitive. Reducing the nominal wage floor reduces inflation expectations, raises real interest rates, and (for a given interest rate rule) lowers output. This logic means that a world with very low nominal wage floors has highly inefficient equilibrium outcomes and so is not well-approximated by models without price/wage bounds or without monetary trade.

We can get some intuition for these apparently counterintuitive results about bounds through the lens of a simple two-person game, in which player i chooses a_i from the interval I. The players' payoffs are given by:

$$(a_1 a_2, 0) \text{ if } |a_1| > |a_2|$$

$$(0, a_1 a_2) \text{ if } |a_2| > |a_1|$$

$$(a_1 a_2/2, a_1 a_2/2) \text{ if } |a_1| = |a_2|$$

[2]See del Negro, Giannoni (2015) and MacKay, Nakamura, and Steinsson (2016) for a description of the "forward guidance puzzle" in New Keynesian models.

[3]The result resembles the "paradox of toil" described by Eggertsson (2010).

[4]The numerical methods mirror those described in Kocherlakota (2016b).

Suppose first that the interval is unbounded, so that $I = (-\infty, \infty)$. Then, player i has no best response when player j chooses $a_j \neq 0$. It follows that the unique Nash equilibrium in this game is that both players choose 0.

Now suppose instead that the interval is bounded from above and below, so that $I = [b, B]$ where $b < 0 < B$. It is still an equilibrium for both players to choose 0, but it is also an equilibrium for both players to choose b or for both players to choose B. (Indeed, the latter two extremal equilibria seem more robust, since the first requires the use of weakly dominated strategies.) As we increase the absolute value of the bounds, the set of equilibria in the latter bounded game diverges from the set of equilibria in the former unbounded game. A game without bounds on players' action sets need not be a useful approximation to a game with bounds on action sets, even when those bounds are very loose.

Why do the implications of monetary models with price/wage bounds differ from more conventional monetary models with nominal frictions (like models with Calvo pricing, Rotemberg pricing or menu costs)? The key difference lies in the nature of the inefficiency implied by the two kinds of models. In conventional models with (only) price-setting frictions, the allocation is inefficient because agents consume a lot of goods from some firms (with low prices) and they consume few goods from some firms (with high prices). In the class of models with price/wage bounds that I study in this paper, the allocation is inefficient because all firms are producing too little and all households are working too little.

Here's an example of why this matters. Suppose that we were to agree that output was inefficiently low in the US in late 2009 (when the unemployment rate was 10%). In conventional models with price-setting and/or wage-setting adjustment costs, such an inefficiency is wholly attributable to misallocations of production across otherwise identical firms and workers. In bounded competition models, the relevant inefficiency is attributable to all firms being unable to lower worker wages and product prices.

Throughout the paper, I'm agnostic about the source of the nominal wage floors or price ceilings. But I don't believe that either should be seen as emerging from legal restrictions of some kind. Rather, it seems clear that, at any point in time, businesses face non-statutory bounds on their price and wage decisions. The important empirical issue with bounds on price-setting and wage-setting is not whether they exist, but rather if and when they bind.

I defer a full discussion of the related literature until Section 6. However, for clarity, it is important to emphasize that the class of models that I study feature a lower bound on *nominal* wages. This bound has distinct implications from the more typical assumption that real wages are sticky (for prominent examples, see Blanchard and Gali (2005) or Christiano, Eichenbaum and Trabandt (2016)). In a model with sticky real wages, there is a gap between the marginal product of labor and the real wage. In a model with flexible prices and a lower bound on nominal wages, product market competition will eliminate any such gap. (There is a more related literature about downward nominal wage rigidity, which I will discuss in Section 6.)

I close with a final methodological comment. Throughout the paper, I abandon the recursivity/stationarity restrictions that macroeconomists usually impose on equilibria. These restrictions, as far as I can tell, have no economics behind them. Rather, they are ad hoc ways to ensure that macroeconomic models are "nice" from the point of view of computation (and, for some, estimation).

In contrast (and as in Kocherlakota (2016a)), I use finite horizon models. The upper-hemicontinuity of equilibrium correspondences with respect to horizon length implies that these models should have *fewer* equilibria than their infinite horizon analogs.[5] Nonetheless, the finite horizon models that I study actually exhibit an enormous amount of nominal and real indeterminacy - indeterminacy which, as will become clear in the next section, lies at the heart of the paper.

[5]Thus, in monetary models in which money is intrinsically valueless, there are only non-autarkic equilibria when the horizon is infinite. Similarly, infinitely repeated games may have many more equilibria than occurs when the same game is repeated a finite number of times.

2 Why Monetary Models Need Bounds on Prices and Wages

In this section, I illustrate, through a two-period example, why monetary models need to include bounds on prices and wages. I first consider a (standard) Walrasian two-period model of monetary exchange. In this model, the government imposes a lump-sum tax in period 2 equal to the average amount of money outstanding. As a result, essentially any price level is an equilibrium. However, the anticipation of many (possibly almost all) of these period 2 equilibria leads to non-existence of equilibrium in period 1. Put another way, we have to impose an otherwise artificial restriction on the set of period 2 equilibria to ensure that we get existence of equilibrium in period 1.

I switch to a model in which firms compete strategically by setting prices and wages. I show that, without bounds on the firms' choices, the same existence issue emerges as in the Walrasian case. I then add an upper bound to constrain the firms' choices of prices and a positive lower bound to constrain their choices of wages. Given these restrictions, there is an equilibrium in period 1 for any period 2 equilibrium.

2.1 Two Period Example Setup

There are two periods and a unit measure of agents who all live for two periods. The agents maximize the expectation of a cardinal utility function of the form:

$$u(c_1) + kc_2$$

where $k > 0$. Here, c_1 is consumption in period 1 and c_2 is consumption in period 2. The utility function u satisfies typical restrictions:

$$u', -u'' > 0$$

$$lim_{c \to 0} u'(c) = \infty$$

$$lim_{c \to \infty} u'(c) = 0$$

In period 1, the agents are each endowed with $\bar{N}$ units of time. In period 2, the agents are each endowed with Y units of consumption. Both consumption and leisure are required to be non-negative.

There are J firms who have identical constant returns to scale technologies that transform a measure n units of time in period 1, $n \geq 0$, into a measure n consumption goods. The agents have equal ownership of all firms.

The symmetric efficient allocation in this environment is easy to compute. Agents have no utility from leisure and their time can generate useful consumption goods in period 1. So, it is efficient for each agent to work 1 unit of time in period 1, consume $\bar{N}$ units of goods in period 1, and to consume Y units of goods in period 2.

2.2 Two-Period Walrasian Monetary Equilibrium

I now add money to this model. I treat money as an interest-bearing asset (akin to the interest-bearing reserves that banks hold with the Federal Reserve). Each person is endowed with M dollars of money in period 1. The government commits to an interest rate rule: in period 2, money pays a gross nominal interest rate $R(P_1)$, where R is a continuous function of the period 1 price level P_1. In terms of fiscal policy, all agents are required to pay a lump-sum tax of $MR(P_1)$ dollars in period 2.

In period 2, households trade money and goods. Given a period 2 price level P_2, the generic household's problem is:

$$max_{c_2,M_2} kc_2$$

$$s.t. P_2c_2 + M_2 \leq P_2Y + M_1'R(P_1^*)$$

$$M_2 \geq MR(P_1^*)$$

where P_1^* is the period 1 price level. Here, M_1' may differ across households. However, the average M_1' is equal to the initial per-capita money-holdings M. The last constraint is necessary to ensure that the household has enough money at the end of the period to pay its taxes.

It is straightforward to show that, for any P_2, it's optimal for households to set $M_2 = MR(P_1^*)$ and to set:

$$c_2 = Y + M_1'R(P_1)/P_2 - MR(P_1)/P_2$$

Given these choices, markets clear, because the average of M_1' across households equals the supply of money M. It follows that any positive real P_2 is an equilibrium.[6]

Now, we move back in time to period 1. Suppose the households rationally expect that the equilibrium period 2 price level P_2^* will equal the equilibrium period 1 price level P_1^* multiplied by (an endogenously determined variable) Π^*. Given these expectations, they trade in period 1. The households' problem in period 1 is then:

[6]Throughout the paper, I treat the final period price level as being indeterminate. There are certainly ways to use government policy to limit or eliminate this indeterminacy. The results in this section would then be viewed about the sensitivity of period 1 outcomes to period 2 government policy choices.

$$(c_1^*, n_1^*, c_2^*) \in argmax_{(c_1, n_1, c_2)} u(c_1) + kc_2$$

$$s.t. P_1^* c_1 + M_1' = W^* n_1 + M + J\Phi^*$$

$$\Pi^* P_1^* c_2 = M_1' R(P_1^*) - MR(P_1) + \Pi^* P_1^* Y$$

$$c_1, c_2, n_1, \bar{N} - n_1, M_1' \geq 0$$

where P_1^* is the period 1 price level, and W^* is the period 1 wage (in terms of dollars). The firms' problem is:

$$\Phi^* = max_{n_1 \geq 0} P_1^* n_1 - W^* n_1$$

Finally, the market-clearing conditions are:

$$c_1^* = n_1^*$$

$$c_2^* = Y$$

$$M_1' = M$$

It is straightforward to show that:

Proposition 1. *Given a monetary policy rule R and an anticipated period 2 gross inflation rate Π^*, there exists a period 1 Walrasian monetary equilibrium if and only if there exists some P_1^* such that:*

$$u'(\bar{N}) = k \frac{R(P_1^*)}{\Pi^*}$$

In any Walrasian monetary equilibrium, the equilibrium allocation is efficient.

Proof. We can show, as in the proof of Proposition 1, that in any Walrasian monetary equilibrium, $W^* = P_1^*$, $\Phi^* = 0$, and $n_1^* = \bar{N}$. It follows from market-clearing that:

$$c_1^* = \bar{N}$$

$$c_2^* = Y$$

which means that, in any Walrasian monetary equilibrium, the allocation is efficient.

However, people will be willing to hold M dollars of money from period 1 to period 2 if and only if the real interest rate paid by money is equal to the shadow real interest rate associated with the efficient allocation. Mathematically, we need:

$$u'(\bar{N}) = k\frac{R(P_1^*)}{\Pi^*}$$

which proves the proposition. □

Proposition 2 shows that there is a Walrasian monetary equilibrium if the real interest rate on money is equal to the efficient real interest rate. That Walrasian equilibrium is efficient.

However, there is an existence problem with Walrasian equilibrium. Suppose $R(P_1) = \bar{R}$ for all P_1, where:

$$u'(\bar{N})\Pi^*/k \neq \bar{R}$$

Then, a period 1 Walrasian equilibrium doesn't exist. The criterion of existence in period 1 imposes a constraint on the interest rate rule and/or what can happen in period 2. But this seems highly problematic. Why should we presume that the government will be informed enough or benevolent enough to choose an interest rate rule that is consistent with existence of equilibrium? Or that players in the future will necessarily make choices consistent with existence of equilibrium today?

2.3 Explicit Price and Wage Competition

How to fix this defect in Walrasian monetary equilibrium? To answer this question, I turn to a more explicitly strategic model of firm price and wage competition. I begin in this section by allowing the firms to choose from action sets that are unbounded from above or away from zero. As above, I assume that households and firms have rational beliefs about what will transpire in period 2.

The J firms play a game in period 1 in which they simultaneously choose a wage W, a price P, and a capacity constraint $\bar{Y}$. The capacity constraint simultaneously constrains the labor hired by the firm (to be no more than $\bar{Y}$) and output produced by the firm (to be no more than $\bar{Y}$). In this way, the capacity constraint decision ensures that the demand for the firm's product market and labor market activities cohere.

After observing firm choices, households choose, in a randomly determined sequence, how many goods to buy from each firm and how much time to supply to each firm. (The sequential nature of household choice limits the amount of inefficient queueing at any given firm.) In this second stage, a household's best labor market response is to supply all of its time to the capacity-unconstrained firm with the highest (positive) wage, given prior households' choices. In the product market, a household demands:

$$u'^{-1}(kR(P_1)P_1/P_2)$$

units of consumption in period 1, where P_1 is the equilibrium price level in period 1 and P_2 is the commonly known period 2 price level. The household buys that consumption from the capacity-unconstrained firm that offers the lowest prices, given the choices by prior households.

In this game, I focus on symmetric equilibria (in which all firms make identical choices). I further restrict attention to equilibria in which firms' production levels are positive and are equal to their chosen capacity constraints.[7] I refer to these equilibria as being *strategic competitive equilibria*. These equilibria have the following properties.

First, all firms choose $W = P_1$. If $W > P_1$, then all firms are making negative profits. A given firm can gain by setting its capacity constraint to zero. If $W < P_1$, then a given firm can gain by raising its capacity constraint a lot, cutting its price slightly (so as to generate more household demand), and raising its wage slightly (to attract more household labor supply).

Second, there is no equilibrium in which labor $n_1^* < \bar{N}$ **(these variables are per-household).** Suppose that $n_1^* < \bar{N}$, and a firm deviates by cutting its wage W by ϵ (ϵ small and positive). This deviation is profitable because, even at this lower wage, there are $(1 - n_1^*(J-1)/J)$ households who would like to supply labor to the firm.

Finally, there is no equilibrium in which:

$$u'(\bar{N}) \neq \frac{kR(P_1^*)}{\Pi^*}$$

where Π^* is the anticipated equilibrium inflation rate from period 1 to period 2. Suppose $u'(\bar{N}) < kR(P_1^*)/\Pi^*$. Then, households demand only $c_1 < \bar{N}$ units of consumption, which is contradicted by the earlier observation that $n_1^* = \bar{N}$ in equilibrium. Suppose that $u'(\bar{N}) > kR(P_1^*)/\Pi^*$. Then, a firm can profitably deviate by raising its price by ϵ. This deviation is profitable because all other firms are capacity-constrained, and (even at a slightly higher price) households demand more consumption than the putative equilibrium level $\bar{N}$.

[7] This last requirement is without loss of generality.

These three observations imply that the only possible equilibrium is one in which:

$$n_1^* = \bar{N}$$

$$c_1^* = \bar{N}$$

$$u'(\bar{N}) = kR(P_1^*)/\Pi^*$$

$$P_1^* = W_1^*$$

$$c_2^* = Y$$

It is straightforward to verify that this is, in fact, a strategic competitive equilibrium outcome. (No firm can reduce its wage without losing its supply of workers. No firm can raise its price without losing the demand for its products.)

Thus, the set of strategic competitive equilibria is in fact equivalent to the set of Walrasian monetary equilibria described in the prior subsection. This means that strategic competition gives rise to efficient outcomes. However, it also means that, just as was true for Walrasian equilibrium, there is no strategic competitive equilibrium for a wide range of rational beliefs about period 2 equilibrium and/or government policy rules.

2.4 Price and Wage Bounds

In this subsection, I discuss *why* we obtained the non-existence result in the prior subsection. As I did in the introduction, I argue that it is attributable to the openness of the firms' action sets. I describe and characterize *bounded strategic competitive equilibria.*

Suppose that the period 2 gross inflation rate Π^* is such that:

$$u'(\bar{N}) < \frac{kR(P_1)}{\Pi^*}$$

for all values of P_1. Then, for any period 1 price level, the households demand less consumption than can be produced if they all work full-time. But we saw in the last subsection it is impossible for $n_1 < \bar{N}$ in a strategic competitive equilibrium because, regardless of how low P_1 is, firms can increase their profits by *cutting wages*.

Similarly, suppose that Π^* is such that:

$$u'(\bar{N}) > \frac{kR(P_1)}{\Pi^*}$$

for all P_1. Then, for any price level, the households demand more consumption than can be produced if they all work full-time. But we saw in the last subsection that it is impossible for this situation to occur in a strategic competitive equilibrium, because, regardless of how high P_1 is, firms can always increase their profits by *raising prices*.

Thus, the non-existence of period 1 equilibrium, conditional on beliefs about what will happen in period 2, is attributable to the non-compact nature of the firms' action sets. The issue is similar to what occurs in a game between two players who are asked to simultaneously name two natural numbers, with a prize being awarded to the player who names the higher number. There is, of course, no equilibrium to this game because a player can always increase her chance of winning the prize by choosing a (possibly mixed) strategy that stochastically dominates her initial one. In contrast, suppose we compactify the players' action sets from above from B, so that they are allowed to name a natural number that is less than or equal to B. Then, the unique equilibrium is one in which both players choose B and split the prize.

This analogy suggests that we can resolve the non-existence problem by imposing bounds on the firms' action sets. With that in mind, I now consider the same game as in the prior subsection, except that a firm's choice of its price is bounded from above by P_{UB} and its choice of a wage is bounded from below by $W_{LB} > 0$. As before, I focus on symmetric equilibria in which the firm production equals their chosen positive capacity. I term such equilibria *bounded strategic competitive equilibria.*

Proposition 2. *Given a monetary policy rule R, and a period 2 gross inflation rate Π^*, an outcome (c_1^*, n_1^*, W, P_1^*) is part of a bounded strategic competitive equilibrium outcome if and* *only if* $c_1^* = n_1^*, W^* = P_1^*$, *and one of the three following sets of conditions are satisfied:*

1. $P_1^* = W^{LB}$; $n_1^* < \bar{N}$; *and* $u'(c_1^*) = kR(W^{LB})/\Pi^*$
2. $P_1^* = P^{UB}$; $n_1^* = \bar{N}$; *and* $u'(\bar{N}) > kR(P^{UB})/\Pi^*$
3. $W^{LB} \leq P_1^* \leq P^{UB}$; $n_1^* = \bar{N}$; *and* $u'(\bar{N}) = kR(P_1^*)/\Pi^*$

Proof. I first show that these cases are, in fact, equilibria. In case 1: $P_1^* = W^{LB}$, and so no firm can deviate by cutting its wage. Raising its price means that no customers will demand its goods. It follows that firms can't find a profitable deviation.

In case 2: $P_1^* = P^{UB}$. A firm can't deviate by increasing its price. Lowering its wage means that it won't be able to hire any labor. It follows that firms can't find a profitable deviation.

In case 3: There is no profitable deviation for any firm, because a firm loses all labor supply if it lowers its wage and loses all product demand if raises its price.

Are there other equilibria? It is straightforward to show that, in any equilibrium, $c_1^* = n_1^*$ and $P_1^* = W^*$. Given the gross real return on money ($R(P_1^*)/\Pi^*$), households demand:

$$c_1^d(P_1^*) = u'^{-1}(kR(P_1^*)/\Pi^*)$$

units of consumption from each firm. If $c_1^d(P_1^*) < \bar{N}$, then firms will set their capacity constraints at $c_1^d(P_1^*)/J$ and hire $n_1^* = c_1^d(P_1^*) < \bar{N}$ units of time. That can only be an equilibrium if $W^* = W_{LB}$ (case 1).

If $c_1^d(P_1^*) > \bar{N}$, firms are unable to hire enough workers to meet the households' demand for goods. The households end up consuming only $\bar{N}$ units of goods each. This can only be an equilibrium if $P_1^* = P_{UB}$ (case 2).

If $c_1^d(P_1^*) = \bar{N}$, firms have no incentive to adjust their prices or wages, so this can be an equilibrium for any price level (case 3). □

Proposition 2 describes three kinds of bounded strategic competitive equilibria. In all of them, while they can substitute between consumption and interest-bearing money, the households end up spending their wage income ($W^*n_1^*$) to buy $P_1^*c_1^*$ period 1 goods. Money plays no substantive role in the economy: households simply hold their initial money-holdings M into period 2 and then use that money to pay their taxes.

The first kind of equilibria (case 1) are *inefficien*t, because households consume $c_1^* < \bar{N}$ and work $n_1^* < \bar{N}$. The inefficiency cannot be alleviated because firms can't lower their wages. The second kind of equilibria (case 2) are efficient (because households consume $c_1^* = \bar{N}$ and work $n_1^* = \bar{N}$). However, they are *efficient with rationing*: households systematically would like to sell their money for more goods from firms than the firms can produce (given their capacity constraints). The rationing cannot be alleviated because the firms are unable to adjust their prices upward. The final kind of equilibria (case 3) are *efficient without rationing* and correspond to the Walrasian monetary equilibria.

2.5 Existence of Bounded Strategic Competitive Equilibria

In this subsection, I prove that for any (continuous) interest rate rule R, any period 2 gross inflation rate Π^*, for any price upper bound P^{UB}, and for any W^{LB}, there exists a bounded strategic competitive equilibrium.

Proposition 3. *For any continuous interest rate rule R, any period 2 gross inflation rate Π^*, price upper bound P^{UB}, and nominal wage floor W^{LB}, there exists a bounded strategic competitive equilibrium.*

Proof. If there isn't an inefficient equilibrium (case 1), then:

$$u'(\bar{N}) \geq \frac{kR(W^{LB})}{\Pi^*}$$

If there isn't an efficient equilibrium with rationing (case 2), then:

$$u'(\bar{N}) \leq \frac{kR(P^{UB})}{\Pi^*}$$

Since R is continuous, these two inequalities imply via the intermediate value theorem that:

$$u'(\bar{N}) = \frac{kR(P_1^*)}{\Pi^*}$$

so that there exists at least one efficient equilibrium without rationing. □

There is a bounded strategic competitive equilibrium for all (continuous) interest rate rules and all Π^*. Note that the proof of existence is valid regardless of how large P^{UB} is or how small W^{LB} is.

2.6 Summary

In this section, I illustrated a problem with the concept of Walrasian equilibrium: to obtain existence in a given period, we need to restrict the set of future equilibrium outcomes. I showed how to fix this problem by using *bounded strategic competitive equilibrium* - a notion of equilibrium which incorporates bounds on firm price-setting and wage-setting.

Bounded strategic competitive equilibrium outcomes may be inefficient. The essence of the inefficiency is that households would like to work and consume more. However, a given household can only trade its labor for consumption via firms that own the means of production. Those firms can't profitably expand their scale of operation because they can't cut nominal wages.

It shouldn't be surprising that the relevant inefficiency involves underproduction, not overproduction. Underproduction represents mutually beneficial trades not being consummated. This kind of inefficiency is simply a form of incompleteness of markets. In contrast, overproduction would (somehow) require forcing firms and workers to engage in trades that are not mutually beneficial.

3 Dynamic Equilibrium

In this section, I extend the above definition of equilibrium to a finite horizon economies. (Henceforth, I use the short-hand term "equilibrium" to refer to a bounded strategic competitive equilibrium.)

3.1 Description of the Economy

Consider a $(T+1)$ period stochastic version of the above economy, where T is finite but arbitrarily large. In each period, firms have a technology in which n units of time translates into n units of output. Agents have no disutility from labor, and are endowed with $\bar{N}_t$ units of time in period $t = 1, , ..., T$, where $\{\bar{N}_t\}_{t=1}^{T}$ is a stochastic process. They are endowed with a random amount Y_{T+1} units of consumption in the final period $(T+1)$. Agents have expected utility, with a time separable cardinal utility function over consumption processes:

$$\sum_{s=1}^{T+1} \beta^s u(c_s; \lambda_s)$$

where $\{\lambda_s\}_{s=1}^{T+1}$ is a stochastic process.

In terms of monetary policy, money is again an interest-bearing asset. The one-period nominal interest rate from period $(t-1)$ to period t is given by $R(\pi_t; \varepsilon_t)$, where π_t is the gross inflation rate from period $(t-1)$ to period t and $(\varepsilon_s)_{s=1}^{T}$ is an exogenous stochastic process of monetary policy shocks. (Without loss of generality, I fix $P_0 = 1$.) I assume that the interest rate rule R is strictly increasing and continuous in its first argument. In the final period, the government levies a lump-sum tax equal to the per-capita money supply on each agent.

Firms compete in each period $t = 1, ..., T$ by setting prices and wages. Their price choice in period t is constrained by an upper bound given by $\pi^{UB} P_{t-1}^*$, where P_{t-1}^* is the period $(t-1)$ price level. Their wage choice is constrained by a lower bound given by $\pi^{LB} W_{t-1}^*$. Since labor productivity is constant over time, this lower bound on nominal wages translates into an equivalent lower bound on inflation.[8]

3.2 Definition of Equilibrium

As in the 2-period economy, the gross inflation rate π_{T+1} in the final period $(T+1)$ is indeterminate. So, the set of equilibria should be seen as being indexed by the (random) inflation rate π_{T+1}.

Given the exogenous processes $\{\varepsilon_s, \lambda_s, , \bar{N}_s\}_{s=1}^{T}$ and the exogenous period $(T+1)$ shocks $(\lambda_{T+1},\ \pi_{T+1}, Y_{T+1})$, an equilibrium is a joint consumption-inflation process $(c_t, \pi_t)_{t=1}^{T}$ that satisfies two sets of restrictions. The first is a set of Euler equations:

$$u'(c_t; \lambda_t) = max(\beta R(\pi^{LB}; \varepsilon_t) E_t(\frac{u'(c_{t+1}; \lambda_{t+1})}{\pi_{t+1}}), u'(\bar{N}_t; \lambda_t)), t = 1, .., T \tag{1}$$

where $c_{T+1} = Y_{T+1}$. The period t level of consumption is equal to its efficient level $\bar{N}_t$ if inflation is above its lower bound. Otherwise, the period t level of consumption is shaped by the current marginal utility of future dollars given that period t inflation is at its lower bound.

The second set of restrictions describes how inflation is determined subject to its upper and lower bounds. If inflation is unconstrained by its bounds, then the level of consumption is efficient. The inflation rate in period t is determined by the interest rate rule so as to satisfy the Euler equation:

$$u'(\bar{N}_t; \lambda_t) = \beta R(\pi_t; \varepsilon_t) E_t(\frac{u'(c_{t+1}; \lambda_{t+1})}{\pi_{t+1}})$$

However, there may be no π_t in $[\pi^{LB}, \pi^{UB}]$ such that the Euler equation is satisfied. More

[8]If labor productivity varied over time, or (in a richer model) mark-ups varied over time, then the lower bound on inflation would be a stochastic process.

generally, for $t = 1, ..., T$, the second set of equilibrium restrictions take the form:

$$\pi_t = \pi^{UB} \text{ if } \beta R(\pi^{UB}; \varepsilon_t) E_t(\frac{u'(c_{t+1}; \lambda_{t+1})}{\pi_{t+1} u'(\bar{N}_t; \lambda_t)}) \leq 1 \tag{2}$$

$$= \pi^{LB} \text{ if } \beta R(\pi^{LB}; \varepsilon_t) E_t(\frac{u'(c_{t+1}; \lambda_{t+1})}{\pi_{t+1} u'(\bar{N}_t; \lambda_t)}) \geq 1 \tag{3}$$

$$= R^{-1}(\beta^{-1}\{E_t \frac{u'(c_{t+1}; \lambda_{t+1})}{\pi_{t+1} u'(\bar{N}_t; \lambda_t)}\}^{-1}; \varepsilon_t) \text{ otherwise} \tag{4}$$

where $c_{T+1} = Y_{T+1}$. Note that, because R is strictly increasing, these restrictions characterize a unique π_t.

The three cases can be classified much as in the two-period economy discussed in the prior section. If:

$$\beta R(\pi^{UB}; \varepsilon_t) E_t(\frac{u'(c_{t+1}; \lambda_{t+1})}{\pi_{t+1} u'(\bar{N}_t; \lambda_t)}) < 1$$

the equilibrium exhibits *rationing* in period t because agents demand more than the maximal amount that can be produced. In contrast, if:

$$\beta R(\pi^{LB}; \varepsilon_t) E_t(\frac{u'(c_{t+1}; \lambda_{t+1})}{\pi_{t+1} u'(\bar{N}_t; \lambda_t)}) > 1,$$

then equilibrium output is inefficiently low in period t.

3.3 Constructing Equilibrium

It is straightforward to apply backward induction to these two sets of restrictions (1) and (2) to construct the set of equilibria. Fix an arbitrary random period $(T+1)$ gross inflation rate π_{T+1} and set $c_{T+1} = Y_{T+1}$. Then, we can use the period T restrictions (1) to solve for period T consumption:

$$u'(c_T; \lambda_T) = max(\beta R(\pi^{LB}; \varepsilon_T) E_T(\frac{u'(Y_{T+1}; \lambda_{T+1})}{\pi_{T+1}}), u'(\bar{N}_T; \lambda_T))$$

And we can use the period T restriction (2) to solve for inflation:

$$\begin{aligned}\pi_T &= \pi^{UB} \text{ if } \beta R(\pi^{UB};\varepsilon_T)E_T(\frac{u'(Y_{T+1};\lambda_{T+1})}{u'(\bar{N}_T;\lambda_T)\pi_{T+1}}) \leq 1 \\ &= \pi^{LB} \text{ if } \beta R(\pi^{LB};\varepsilon_T)E_T(\frac{u'(Y_{T+1};\lambda_{T+1})}{u'(\bar{N}_T;\lambda_T)\pi_{T+1}}) \geq 1 \\ &= R^{-1}(\beta^{-1}\{E_T\frac{u'(Y_{T+1};\lambda_{T+1})}{\pi_{T+1}u'(\bar{N}_T;\lambda_T)}\}^{-1};\varepsilon_T) \text{ otherwise}\end{aligned}$$

There is a unique solution for π_T because the interest rate rule R is strictly increasing and continuous.

We can then continue using backward induction to construct the full equilibrium $(c_T, \pi_T)_{t=1}^{T}$. In this way, given any (random) terminal inflation π_{T+1}, there is a unique equilibrium (c, π).

Note that in any equilibrium of this form, the (identical) households choose never to trade money and goods. As in the two-period model, money is simply a store of value used to pay their taxes in the final period. Nonetheless, the opportunity to hold interest-bearing money can in fact create inefficiencies.

3.4 The Role of the Price/Wage Bounds

What if there were no bounds on prices or wages? In that case, the equilibrium would necessarily be efficient and inflation would satisfy the restrictions:

$$\pi_t = R^{-1}(\frac{\beta^{-1}u'(\bar{N}_t;\lambda_t)}{E_t(\frac{u'(\bar{N}_{t+1};\lambda_{t+1})}{\pi_{t+1}})};\varepsilon_t), t = 1, ...(T-1)$$

But these restrictions imply that too that the terminal inflation rate π_{T+1} must be such that for all $t = 1, ..., T$:

$$\frac{\beta^{-1}u'(\bar{N}_t;\lambda_t)}{E_t(\frac{u'(\bar{N}_{t+1};\lambda_{t+1})}{\pi_{t+1}})}$$

lies in the range of the interest rate rule R. This (potentially complex) restriction on future equilibria to ensure period t existence is exactly what I criticized in the two-period example.

4 Results

In this section, I derive a number of properties of the (bounded strategic competitive) equilibria of the models set forth in section 3. I focus on implications that are distinct from models with more conventional pricing frictions. As in Kocherlakota (2016a), the proofs rely on backward induction.

4.1 Secular Stagnation

In this section, I show that under weak uniform boundedness conditions, there are equilibria in which output is permanently lower than would be efficient. In my view, this kind of outcome[9] corresponds to what Summers (2013) terms "secular stagnation". The key to these equilibria is that money is viewed as a highly valuable asset because long-run inflation π_{T+1} is expected to be low.

Proposition 4. *Suppose that the interest rate rule R satisfies the condition:*

$$R(\pi^{LB};\varepsilon) \geq R_{LB}$$

for all ε. Suppose too that for some constants L_Y and L_N

$$u'(Y_{T+1};\lambda_{T+1}) \geq L_Y > 0 \text{ w.p. } 1$$
$$\text{For } t = 1,...,T,\ u'(\bar{N}_t;\lambda_t) < L_N \text{ w.p. } 1$$

Then there is a set of equilibria (with sufficiently low inflation π_{T+1}) in which consumption $c_t < \bar{N}_t$ with probability one for all $t \leq T$.

Proof. In Appendix B. □

The following example is a simple illustration of Proposition 4. The example is based on the premise that the efficient allocation is constant. It then shows that if $\beta/\pi^{LB} = 1$,

[9] See Eggertsson and Mehrotra (2014) for an overlapping generations model of secular stagnation.

and the nominal interest rate lower bound equals one, there is a set of *constant* inefficient equilibria.[10]

Example 1. Suppose $Y_{T+1} = 1$, $\bar{N}_t = 1, t = 1, ..., T$, and that there are no marginal utility shocks. Suppose

$$R(\pi^{LB}; \varepsilon) = 1$$

for all ε and $\pi^{LB} = \beta$. Set π_{T+1} to be any constant so that:

$$\beta > \pi_{T+1}$$

Then, using backward induction, we can construct an inefficient equilibrium in which period $(T+1)$ inflation equals π_{T+1}, period t inflation $\pi_t = \pi_{LB}$, and period t consumption satisfies:

$$u'(c_t) = \frac{\beta u'(1)}{\pi_{T+1}} > u'(1)$$

Begin with period T. We know that:

$$u'(c_T) = max(\frac{\beta u'(1)}{\pi_{T+1}}, u'(1))$$

Since $\beta/\pi_{T+1} > 1$, it follows that:

$$u'(c_T) > u'(1)$$

and $\pi_T = \pi_{LB}$. Now suppose inductively that:

$$u'(c_{t+1}) = \frac{\beta u'(1)}{\pi_{T+1}}$$

[10]Suppose that there (also) exists $\bar{\pi} > \pi_{LB}$ such that $\beta R(\bar{\pi})/\bar{\pi} = 1$. Then, there is also a set of constant efficient equilibria.

and $\pi_{t+1} = \pi_{LB}$. Then:

$$u'(c_t) = max(\frac{\beta u'(c_{t+1})}{\pi_{LB}}, u'(1))$$
$$= \frac{\beta u'(1)}{\pi_{T+1}}$$

Since

$$\beta/(\pi_{T+1}) > 1,$$

it follows that $u'(c_t) > u'(1)$, and that inflation must be at its lower bound π^{LB}. The equilibrium construction follows by induction.

4.2 Reducing the Nominal Wage Floor

The main result in this subsection is that reducing the nominal wage floor makes inefficient equilibrium outcomes even worse in welfare terms. I first use example 1 to illustrate this claim, and then provide a more general proof.

Example 2. Consider the parameter setting in example 1, except that the wage lower bound π^{LB} is set to be less than β. (Note that $R(\pi^{LB}; \varepsilon)$ still equals one for this lower value of π^{LB}.) Then, we can use reverse induction as before to show that, given long-run inflation π_{T+1}, the equilibrium marginal utility in period t is given by:

$$u'(c_t) = \frac{\beta^{T-t+1} u'(1)}{(\pi^{LB})^{T-t} \pi_{T+1}}.$$

This equilibrium marginal utility is an exponentially increasing function of π^{LB}, meaning that the equilibrium becomes arbitrarily less efficient as the wage lower bound π^{LB} is made smaller.

The intuition behind Example 2 is simple. Reducing the wage lower bound in period $(t + 1)$ also reduces expected inflation in period t. Because the lower bound on the interest rate rule has been left unchanged, the fall in expected inflation results in a higher real interest rate, lower demand, and lower output.

The following proposition generalizes Example 2.

Proposition 5. *Consider two economies that are identical except that they have distinct inflation lower bounds* $\pi_H^{LB} > \pi_L^{LB}$ *and interest rate rules* R, R' *such that:*

$$R'(\pi_L^{LB}; \varepsilon) = R(\pi_H^{LB}; \varepsilon)$$

for all ε. *Suppose* $(c_t, \pi_t)_{t=1}^T$ *is an equilibrium in the former economy given random period* $(T+1)$ *inflation* π_{T+1} *such that* $c_t < \bar{N}_t$ *with probability one for all* $t \leq T$. *Then, given the same random period* $(T+1)$ *inflation* π_{T+1}, *there is an equilibrium* (c'_t, π'_t) *in the latter economy such that:*

$$u'(c_t; \lambda_t)(\frac{\pi_H^{LB}}{\pi_L^{LB}})^{T-t-1} \leq u'(c'_t; \lambda_t)$$

with probability one for all $t \leq T$.

Proof. In Appendix B. □

Proposition 4 established that, for a wide class of economies, there is a "secular stagnation" equilibrium in which output is inefficiently low in all periods. Proposition 5 shows that reducing the nominal wage floor, without changing the minimum nominal interest rate, makes this secular stagnation equilibrium even worse. The intuition is the same as in Example 2: The smaller value of the nominal wage floor translates into lower realized inflation and lower inflation expectations. With a fixed lower bound on the nominal interest rate, the lower inflation expectations translate into higher real interest rates, lower demand and lower output.

4.3 L-Shaped Phillips Curve

In this class of economies, the Phillips curve relating the output gap to inflation is L-shaped.

Proposition 6. *Consider an equilibrium* (c, π)*. In this equilibrium,* $\pi_t = \pi^{LB}$ *in any date and state in which* $c_t < \bar{N}_t$*. In these dates and states:*

$$u'(c_t; \lambda_t) = \beta R(\pi^{LB}; \varepsilon_t) E_t\{\frac{u'(c_{t+1}; \lambda_{t+1})}{\pi_{t+1}}\}$$

As well, $c_t = \bar{N}_t$ *in any date and state in which* $\pi_t > \pi^{LB}$*.*

Proof. Straightforward implication of the definition of equilibrium. □

In any equilibrium, the Phillips curve is horizontal with inflation equal to π^{LB} when there is a negative output gap (so that output is inefficiently low). In these dates and states, firms bid down wages and prices to the common lower bound. In contrast, when the output gap is zero (output is efficient), current inflation is determined by expectations about future inflation and future consumption:

$$\pi_t = min(R^{-1}(\beta^{-1} \frac{1}{E_t\{\frac{u'(c_{t+1};\lambda_{t+1})}{u'(\bar{N}_t;\lambda_t)\pi_{t+1}}\}}; \varepsilon_t), \pi^{UB})$$

In this vertical portion of the curve, higher future expected inflation and higher future expected consumption is associated with higher current inflation. This L-shaped Phillips curve is quite different from the log-linear Phillips curve that emerges from the New Keynesian paradigm.

Friedman (1968) famously argued that, in the long-run, the Phillips curve is necessarily vertical. But in these models, there is no force that ensures that the economy converges to the vertical portion of the Phillips curve. Instead, as Proposition 4 and Example 1 illustrate, the economy may remain stuck permanently on the horizontal portion of the Phillips curve. More generally, we should expect the economy to fluctuate between the two branches of the L.

4.4 Fiscal Multipliers

In this subsection, I discuss how fiscal multipliers work in this class of models.

Suppose that the government buys an exogenously specified process $g = \{g_t\}_{t=1}^{T}$ of consumption goods. (The purchases could be financed in a number of ways. To be explicit, suppose that the government issues debt that pays off only in period $(T + 1)$ and pays off that debt using lump-taxes.) Then, the efficient level of private consumption in any period t becomes $(\bar{N}_t - g_t)$, where g_t is the amount of public consumption in that period. The equilibrium conditions for private consumption become:

$$u'(c_t; \lambda_t) = max(\beta R(\pi^{LB}; \varepsilon_t) E_t(\frac{u'(c_{t+1}; \lambda_{t+1})}{\pi_{t+1}}), u'(\bar{N}_t - g_t; \lambda_t)), t = 1, .., T \tag{5}$$

Now suppose that we perturb the government purchases process by increasing period t purchases g_t by a small positive Δ, while keeping final period inflation π_{T+1} remaining unchanged. This increase in government purchases has no effect on private consumption if $c_t < (\bar{N}_t - g_t)$. In this case, the output multiplier is one. The impact on welfare depends on how government purchases enter into agents' utility functions. There is no effect on period t inflation (if Δ is small).

In contrast, if $c_t + g_t = \bar{N}_t$, then raising g_t by Δ will lower c_t by Δ. The output multiplier is zero, as we get completely crowding out. If there is no rationing, then the period t inflation rate is given by:

$$\pi_t = R^{-1}(\beta^{-1} \frac{1}{E_t\{\frac{u'(c_{t+1}; \lambda_{t+1})}{u'(\bar{N}_t - g_t - \Delta; \lambda_t)\pi_{t+1}}\}}; \varepsilon_t)$$

and so it is an increasing function of Δ.

4.5 Failure of Neo-Fisherian Logic

In recent papers, Cochrane (2016) and Schmitt-Grohe and Uribe (2014) have argued that increasing the nominal interest rate rule will result in higher inflation. In this subsection, I consider this claim in the context of the models with bounded competition studied in this paper. I consider two policy rules (R, R') such that $R' > R$ for all π, ε. The following proposition shows that, given a random variable π_{T+1}, the implied equilibrium (c', π') under R' is no larger than the implied equilibrium under R.

Proposition 7. *Consider two interest rate rules R, R' such that $R'(\pi;\varepsilon) > R(\pi;\varepsilon)$ for all (π, ε). If (c^*, π^*) is an equilibrium given R with (random) period $(T+1)$ inflation π_{T+1}, and (c', π') is an equilibrium given R' with the same (random) period $(T+1)$ inflation, then $c'_t \leq c^*_t$ and $\pi'_t \leq \pi^*_t$ for all t with probability one.*

Proof. In Appendix B. □

The key neo-Fisherian premise is that, for any interest rate rule, the long run real interest rate is necessarily efficient. Given this premise, the Fisher equation then implies that the long-run inflation rate has to move one-for-one with the long-run nominal interest rate. But, as Proposition 4 demonstrates, this presumption of a policy-invariant long-run real interest rate is not valid in models with bounded competition. In these models, there is a set of equilibria indexed by the long-run inflation rate, and the long-run real interest rate can vary both across and within these equilibria.

4.6 The Forward Guidance Puzzle

Del Negro, et. al. (2015) and MacKay, et. al. (2016) demonstrate that forward guidance about future monetary policy is puzzlingly powerful in the New Keynesian modeling paradigm. In this subsection, I analyze the effect of forward guidance on (inefficiently low) current output within the class of models studied in this paper. I make two main points:

- The effect of forward guidance is completely summarized through its impact on the inflation rate during the period in which output returns to an efficient level.

- If the (logged) interest rate rule obeys the Taylor Principle, forward guidance becomes exponentially less effective with respect to the horizon.

Transition Inflation as a Summary Statistic

Suppose that the interest rate rule satisfies $R(\pi^{LB};\varepsilon) = R_{LB}$ for all ε. Consider an equilibrium such that, in some event Ξ_t, consumption is known to be inefficiently low in periods $(t+s)$, $s = 0, ..., \tau$, and known to be efficient in all periods after $(t+s)$. This is a description of a deterministic liquidity trap, in which the nominal interest rate is known to be pinned at its lowest level for the next τ periods.

In any equilibrium of this kind, we know that:

$$\begin{aligned} u'(c_t;\lambda_t) &= \beta R_{LB} E_t\{\frac{u'(c_{t+1};\lambda_{t+1})}{\pi^{LB}}\} \\ &= (\beta R_{LB}/\pi^{LB})^{\tau}\beta R_{LB} E_t\{\frac{u'(\bar{N}_{t+\tau+1};\lambda_{t+\tau+1})}{\pi_{t+\tau+1}}\} \end{aligned} \tag{6}$$

This restriction implies that the impact of any form of post-trap forward guidance is completely summarized through its impact on the inflation rate $\pi_{t+\tau+1}$, during the single period in which the economy exits the trap. Note too that the effect of changes in this transition inflation rate $\pi_{t+\tau+1}$ on prior consumption is independent of the anticipated duration τ of the liquidity trap.

Decaying Effect of Forward Guidance

We've seen that post-liquidity trap forward guidance affects outcomes in the trap only through the inflation rate during the period in which the economy transits from the trap. How is this transition inflation rate affected by the level of future (that is, post-trap) interest rates? The answer to this question depends on the interest rate rule that maps realized inflation into interest rates.

By way of example, return to the liquidity trap described in the prior subsection. Suppose that, after the liquidity trap ends, the marginal $u'(\bar{N}_{t+s};\lambda_{t+s})$ is equal to a constant MU^{EFF} (for all $s > \tau$). Suppose too that, after the liquidity trap ends, the interest rate rule takes the form:

$$R(\pi; \varepsilon) = B\pi^{\gamma}, \gamma > 1$$

The logged version of this interest rate rule obeys the Taylor Principle (so that the nominal interest rate adjusts more than one-for-one with the inflation rate).

Now consider a form of forward guidance in which the central bank lowers B to $B' = \lambda B$, $0 < \lambda < 1$, in a single period $(t + k)$, where $k > (\tau + 1)$. This change in policy in a future period increases the inflation rate in that period:

$$\pi'_{t+k} = (\frac{B'^{-1}\beta^{-1}}{E_{t+k}(1/\pi_{t+k+1})})^{1/\gamma} = \pi_{t+k}\lambda^{-1/\gamma}$$

This increase in period $(t + k)$ inflation feeds back into prior inflation rates, so that:

$$\pi'_{t+k-r} = \lambda^{-1/\gamma^r}\pi_{t+k-r}, r > 1$$

But, since $\gamma > 1$, λ^{-1/γ^r} converges to 1 as r converges to infinity. Unlike in the New Keynesian model, the impact of forward guidance declines exponentially with the relevant horizon.

5 Literature

In this section, I discuss some antecedents for this paper in the existing literature.[11]

5.1 Flat Phillips Curve

The basic New Keynesian model implies that there is a positive log-linear relationship between current inflation and current output (conditional on short-term expected inflation). But the data from the past nine years suggest that there is little connection between resource underutilization and inflation. Thus, most measures of labor market slack rose sharply from 2008 to 2009 and inflation fell relatively little over that same time period. Similarly, inflation

[11] I welcome suggestions for further references that seem relevant.

has remained essentially unchanged while most measures of labor market slack have fallen considerably over the past four years (2013-17).

These observations about inflation don't seem all that surprising when viewed through the lens of the bounded competition models analyzed in this paper. As long as there is a negative output gap, the Phillips curve is flat: there is no connection between the magnitude of the gap and inflation.[12] The Phillips curve becomes vertical only when the output gap rises back to zero. And, when the Phillips curve is vertical, inflation is determined by the interaction of the nominal interest rate rule, the efficient real interest rate, and expected inflation.

5.2 Sticky Wages and Prices

I treat prices and wages as completely flexible, except for extremal bounds. There is considerable evidence that prices and wages are not completely flexible, although the degree of inflexibility remains a subject of much empirical study (see Nakamura and Steinsson (2013) for a recent survey of the relevant evidence). As discussed in the introduction, the misallocation inefficiency that emerges in conventional nominal frictions models is quite different from the underproduction inefficiency that emerges in models with bounds. I also focus on models with perfect competition. However, I believe that the main results would generalize to models with 1) Dixit-Stiglitz monopolistic competition 2) output subsidies to correct the baseline market power distortion and 3) flexible prices/wages except price-setting firms face a ceiling and wage-setting households face a floor.

In a recent paper, Benigno and Ricci (2011) study a class of Dixit-Stiglitz models in which prices and wages are flexible, except forward-looking wage-setting households are not able to lower wages. In these models, the *endogeneity* of lower bounds on wage choices introduces dynamic incentives for wage-setters that are absent from the models studied in this paper, in which wage lower bounds are exogenous. Benigno and Ricci abstract from the equilibrium

[12] See Daly and Hobijn (2014) for a similar justification for the flatness of the Phillips curve.

indeterminacy that lies at the heart of my analysis.

5.3 Indeterminacy and Finite Horizon

In the class of finite horizon models studied in this paper, the final period inflation rate is not pinned down. By construction, the wage floor and price ceiling guarantee that there is a dynamic equilibrium associated with each of these possible final period outcomes. Equilibrium indeterminacy is intrinsic to this class of economies.

In some sense, this indeterminacy shouldn't be viewed as all that unusual. For example, Cochrane (2011) describes how, even under active Taylor Rules, there is a set of equilibrium outcomes in New Keynesian models. It is typical practice to discard all but one of these equilibria because they lead to explosive inflationary paths. But, as Cochrane rightly emphasizes, there is no economics to justify that practice.

However, there is a key difference between the finite horizon indeterminacy highlighted in this paper and the infinite horizon indeterminacy that Cochrane discusses. The set of equilibria in this paper is indexed by the final period random inflation. This is a large set, because it consists of all random variables that are measurable with respect to past realizations of the exogenous processes in the economy. In contrast, the infinite horizon indeterminacy is indexed by a one dimensional variable: initial inflation.

How can it be that the indeterminacy in these finite horizon models is so much larger than the indeterminacy in the infinite horizon models? The main reason is that users of the infinite horizon models typically impose an auxiliary restriction that equilibria be time homogeneous. This restriction also has no economics behind it.

6 Conclusions and Extensions

This paper demonstrates that, for a wide set of policy rules, monetary models without price and wage bounds have no equilibria and so are unable to make any predictions about the implications of those policy rules. Motivated by this observation, I study the properties of a class of dynamic monetary models with (arbitrarily loose) price ceilings and wage floors. Among other results, I show that these models imply that the Phillips curve is L-shaped, are consistent with the existence of permanent secular stagnation, and do not imply that forward guidance is surprisingly effective. Perhaps most importantly, I prove that lowering the wage floor toward zero leads to less efficient outcomes emerging as equilibria. It follows that models with very low wage floors have materially different implications from non-monetary models or monetary models without wage floors.[13]

I've deliberately kept the class of models simple in many respects. It would be useful to extend the analysis in a number of directions such as:

- exploring the consequences of adding dimensions of heterogeneity, like different price/wage bounds across firms and different, imperfectly substitutable, forms of labor.
- adding some kind of inflation cost, like a Friedmanian tax on transactions or a New Keynesian relative price distortion
- incorporating social costs of rationing (associated with the price ceiling).
- including worker-firm matching impediments in the labor market.

Perhaps most importantly, I treat the price and wage bounds as exogenous. Future work should investigate how these bounds are determined.

References

[1] Benigno, G., and Ricci, "The Inflation-Output Trade-Off and Downward Wage Rigidities," *American Economic Review* 101, 1436-66.

[2] Blanchard, O., and Gali, J., "Real Wage Rigidities and the New Keynesian Model," *Journal of Money, Credit, and Banking* 39, 35-65.

[13] This "discontinuity" result (which of course is no such thing) echoes the findings of Kocherlakota (2016a).

[3] Christiano, L., Eichenbaum, M., and Trabandt, M., "Unemployment and Business Cycles," *Econometrica* 84, 1523-1569.

[4] Cochrane, J., "Determinacy and Identification with Taylor Rules," *Journal of Political Economy* 119, 565-615, 2011.

[5] Cochrane, J., "Do Higher Interest Rates Raise or Lower Inflation?" The University of Chicago working paper, 2016.

[6] Daly, M., and Hobijn, B., "Downward Nominal Wage Rigidities Bend the Phillips Curve," Federal Reserve Bank of San Francisco working paper, 2014.

[7] del Negro, M., Giannoni, M., and Patterson, C., "The Forward Guidance Puzzle," Federal Reserve Bank of New York working paper, 2015.

[8] Eggertsson, G., "The Paradox of Toil," Federal Reserve Bank of New York working paper.

[9] Eggertsson, G., and Mehrotra, N., "A Model of Secular Stagnation," NBER working paper 20574, 2014.

[10] Friedman, M., "The Role of Monetary Policy," *American Economic Review* 58, 1-17, 1968.

[11] Kocherlakota, N., "Fragility of Purely Real Macroeconomic Models," National Bureau of Economic Research working paper 21866, 2016a.

[12] Kocherlakota, N., "Sluggish Inflation Expectations," National Bureau of Economic Research working paper 22009, 2016b.

[13] MacKay, A., Nakamura, E., and Steinsson, J., "The Power of Forward Guidance Revisited," *American Economic Review* 106, 2016.

[14] Nakamura, E., and Steinsson, J., "Price Rigidity: Microeconomic Evidence and Macroeconomic Implications," *Annual Review of Economics* 5, 133-63.

[15] Summers, L, "IMF Fourteenth Annual Research Conference in Honor of Stanley Fischer," 2013, http://larrysummers.com/imf-fourteenth-annual-research-conference-in-honor-of-stanley-fischer/.

Appendix A: Exact Numerical Solution Methods in the Markovian Case

In this appendix, I consider the properties of equilibria in a numerical example in which the driving processes are Markov chains. As above, the horizon is finite and there is a large set of equilibria indexed by the final period outcomes. However, I restrict attention to settings in which the dependence of period t equilibrium outcomes on final period $(T+1)$ outcomes is small when $(T-t)$ is large. The main purpose of the example is to demonstrate the (surprising) power of negative nominal interest rates.

A. 1 Markov Chain Setup

I define

$$g_{t+1}^{EFF} = \frac{\lambda_{t+1} u'(\bar{N}_{t+1})}{\lambda_t u'(\bar{N}_t)}$$

to be the growth rate of marginal utility from period t to period $(t+1)$ in an efficient allocation. I denote the marginal utility "gap" in period t by:

$$\hat{m}_t = \frac{u'(c_t)}{u'(\bar{N}_t)}$$

Note that, in any equilibrium, $\hat{m}_t$ is bounded below by 1. In an equilibrium, we know from (1):

$$\hat{m}_t = max(\beta R(\pi_t, \varepsilon_t) E_t \hat{m}_{t+1} g_{t+1}^{EFF} / \pi_{t+1}, 1), t = 1, ..., T-1$$

In this economy, there are two relevant exogenous processes: $\{g_t^{EFF}, \varepsilon_t\}_{t=1}^{T}$. I assume that they are governed by a Markov chain s_t, which is a Markov chain with state space $\{1, 2, .., J\}$ and transition matrix P. I define a state space $\{(g_j, \varepsilon_j)\}_{j=1}^{J}$ and assume that $(g_t^{EFF}, \varepsilon_t) = (g_{s_t}^{EFF}, \varepsilon_{s_t})$ for $t = 1, ..., T$.

In what follows, it will be useful to define $Q(\pi)$ to be a $J \times J$ matrix:

$$Q_{ij}(\pi) = \frac{\beta R(\pi^{LB}, \varepsilon_i) P_{ij} g_j^{EFF}}{\pi_j}$$

I require that:

$$||Q(\pi^{LB})|| < 1 \quad (7)$$

where $||.||$ represents the Euclidean norm of the matrix.[14] This restriction implies that:

$$||Q(\pi)|| < 1$$

for all values of π.

A. 2 Solving for Markov Equilibrium

In what follows, I assume that in period $(T+1)$:

$$(\frac{u'(Y_{T+1})}{u'(\bar{N}_T)}, \pi_{T+1})$$

is a function of the state s_{T+1} in period $(T+1)$. We can then apply backward induction to these terminal conditions. Given the restriction (7), the resulting equilibrium is approximately

[14] The Euclidean norm $||P||$ of a matrix P is the square root of the maximal eigenvalue of $P'P$.

Markov in periods t such that $(T-t)$ is large.

To be more specific, construct a sequence of marginal utility gap vectors and inflation vectors by setting $\hat{m}^0 = \vec{1}$ and $\pi^0 = \pi^{UB}\vec{1}$ and iterating as follows:

$$
\begin{aligned}
\hat{m}_i^{n+1} &= max(1, Q_i(\pi^n)\hat{m}^n) \\
\pi_i^{n+1} &= \pi^{LB} \text{ if } Q_i(\pi^n)\hat{m}^n \geq 1 \\
&= R^{-1}\left(\frac{1}{\sum_{j=1}^{J} \frac{\beta P_{ij} g_j^{EFF}}{\pi_j^n} \hat{m}_j^n}; \varepsilon_i\right) \text{ otherwise}
\end{aligned}
$$

For any i, the sequence $\{\pi_i^n\}_{n=1}^{\infty}$ is a decreasing sequence in n that is bounded from below by π^{LB}. So, it converges to π_i^*. Given π^*, and since $||Q(\pi^*)|| < 1$, $\{\hat{m}^n\}_{n=1}^{\infty}$ converges to the unique solution to:

$$\hat{m}_i^* = max(1, Q_i(\pi^*)\hat{m}^*), i = 1, ..., J$$

This is the best Markov equilibrium.

We can solve for the worst Markov equilibrium in a similar fashion. Let $\pi^0 = \pi^{LB}\vec{1}$. Because $||Q(\pi^0)|| < 1$, we can define $\hat{m}^{max}$ to be the unique solution to:

$$\hat{m}_i^{max} = max(1, Q_i(\pi^0)\hat{m}^{max}), i = 1, ..., J$$

We begin the reverse iteration process by leting $\hat{m}^0 = \hat{m}^{max}$. We can then iterate as above:

$$
\begin{aligned}
\hat{m}_i^{n+1} &= max(1, Q_i(\pi^n)\hat{m}^n) \\
\pi_i^{n+1} &= \pi^{LB} \text{ if } Q_i(\pi^n)\hat{m}^n \geq 1 \\
&= R^{-1}\left(\frac{1}{\sum_{j=1}^{J} \frac{\beta P_{ij} g_j^{EFF}}{\pi_j^n} \hat{m}_j^n}; \varepsilon_i\right) \text{ otherwise}
\end{aligned}
$$

For any i, the sequence $\{\hat{m}_i^n\}_{n=1}^{\infty}$ is decreasing in n and bounded from below. At the same time, the sequence $\{\pi_i^n\}_{n=1}^{\infty}$ is increasing in n and is bounded from above (by π^{UB}). So, both sequences converge (to what is the worse Markov equilibrium).

In the numerical examples that follow, the best and worst Markov equilibria always coincide.

A. 3 Numerical Example

In this subsection, I describe the properties of a numerical example. The main point of the example is to illustrate how the risk of an economic downturn can create inefficiencies in apparently "normal" states.[15] I discuss to what extent these inefficiencies can be ameliorated with negative interest rates or a higher inflation target.

In the example, there are three states. I define the state space for the growth rate of efficient marginal utility to be:

$$g^{EFF} = (0.97, 0.97, 1.11)$$

This parameterization is meant to suggest that the growth rate of marginal utility in an efficient allocation is "normal" in the first two states. In the last state, the growth rate of marginal utility in an efficient allocation is very large, meaning that this should be viewed as a "bad" state. The interest rate rule is defined to be:

$$R(\pi_i, \varepsilon_i) = \varepsilon_i \pi_i^{1.5}$$

I vary ε in the example.

I set the transition matrix as follows:

$$T = \begin{matrix} 0.975 & 0 & 0.025 \\ 0.06 & 0.55 & 0.39 \\ 0 & 0.5 & 0.5 \end{matrix}$$

[15] I thank Eunmi Ko for great research assistance with these simulations.

This parameterization ensures that in state 1, there is only a low risk of entering the bad state 3. Once in state 3, it is impossible to transit to state 1 directly. However, in state 2, there is a large risk of entering the bad state 3.

This specification of the transition matrix implies that the average stay in (the good) state 1 is 40 years. Once the economy transits into (the bad) state 3, it stays (only) two years on average in that state. But it can only return to state 1 after going through the (risky) state 2. As a result, after exiting state 1, the economy takes over thirty years on average to return to that state..

Finally, I set the lower bound π^{LB} on the gross inflation rate to equal one and the upper bound to equal 1.1 (the latter specification is irrelevant, as long as it is sufficiently high).

Baseline

In the baseline specification, I set $\varepsilon = (1.03425, 1, 1)$. This setting results in a Markov equilibrium in which:

$$\hat{m} = (1, 1.24, 1.28)$$

$$\hat{\pi} = (1.02, 1, 1)$$

I chose the parameter ε_1 so as to ensure that the inflation in period 2 is 2%.

The equilibrium allocation is efficient in state 1 but (highly) inefficient in state 3. (Note that this inefficiency is in addition to the low rate of efficient growth in this state.) More interestingly, the risk of falling into this inefficient state 3 also creates a similarly sized inefficiency in state 2.

Negative Interest Rates

In this subsection, I set $\varepsilon = (1.0413, 0.995, 0.995)$. The resulting Markov equilibrium is:

$$\hat{m} = (1, 1.02, 1.04)$$

$$\hat{\pi} = (1.02, 1, 1)$$

I've chosen $\varepsilon_2 = \varepsilon_3 = 0.995$ to be 50 basis points below the zero lower bound. This means that the nominal interest rate is now (slightly) negative in states 2 and 3. As before, I've chosen ε_1 so as to ensure that the inflation rate in the good state 1 is equal to 2%.

The main point of this simulation is that lowering the nominal interest rate by only half a percent in states two and three greatly reduces the inefficiencies in those states. Intuitively, what matters for stimulus is not the decrease in the annual real interest rate, but the decline in the cumulated real interest rate before the allocation of consumption is once more efficient (that is, the economy returns to state 1). The structure of the transition matrix implies that if the economy is in states 2 or 3, it is expected to remain out of state 1 for a long period of time (nearly 30 years on average in state 2 and a couple years longer on average in state 3). Over such a long time period, the cumulative impact of a 50 basis point reduction in the annual real interest rate is very large.

What happens if I instead set $\varepsilon = (1.0413, 1, 1)$, so that the nominal interest rate was raised in the good state 1 relative to the benchmark but the nominal interest rate remained zero in states 2 and 3? The equilibrium allocation would be (slightly) worse in states 2 and 3. However, the inflation rate would be much lower in state 1.

$$\hat{m} = (1, 1.26, 1.29)$$

$$\pi = (1.006, 1, 1)$$

Contrary to the neo-Fisherian view, raising interest rates lowers inflation and lowers output.

Higher Inflation Target

In this subsection, I set $\varepsilon = (1.02425, 1, 1)$. The resulting Markov equilibrium is:

$$\hat{m} = (1, 1.22, 1.25)$$
$$\hat{\pi} = (1.04, 1, 1)$$

I've chosen a lower value of ε_1 so as to increase the inflation rate in state 1 to 4%, as opposed to 2%.

Raising the state 1 inflation rate has no effect on the (already efficient) level of economic activity in that state. However, it does provide a partial mitigant against the inefficiently low levels of output in the other states: the marginal utility gap is about 2% lower in state 2 and is about 3% lower in state 3. Doubling the inflation target helps, but not by much.

We can understand these simulation results using equation (6). The effect of reducing the nominal interest rate lower bound R^{LB} grows exponentially with the expected duration of the liquidity trap. In contrast, the effect of raising the post-trap inflation target is independent of the duration of the trap.

Appendix B: Remaining Proofs

Proof of Proposition 4

Given the restrictions on the efficient marginal utility process and on the interest rate rule, there exists L such that for all $t = 1, ..., T$:

$$\frac{\beta^{T-t+1} R_{LB}^{T-t+1}}{(\pi^{LB})^{T-t}} \frac{E_t u'(Y_{T+1}; \lambda_{T+1})}{u'(\bar{N}_t; \lambda_t)} > L$$

with probability one. Pick any positive constant π_{T+1} that is less than L. I proceed by reverse induction to show that, with probability one:

$$u'(c_t) > \frac{\beta^{T-t+1} R_{LB}^{T-t+1}}{(\pi^{LB})^{T-t}\pi_{T+1}} E_t u'(Y_{T+1};\lambda_{T+1}) > u'(\bar{N}_t;\lambda_t)$$

and $\pi_t = \pi^{LB}$.

Note first that:

$$\begin{aligned} u'(c_T) &\geq \beta R(\pi^{LB};\varepsilon_T) E_T\{u'(Y_{T+1};\lambda_{T+1})/\pi_{T+1}\} \\ &> L u'(\bar{N}_T;\lambda_T)/\pi_{T+1} \\ &> u'(\bar{N}_T;\lambda_T) \end{aligned}$$

with probability one, which implies that $\pi_T = \pi^{LB}$ with probability one.

Now, inductively assume that:

$$u'(c_{t+1};\lambda_{t+1}) > \frac{\beta^{T-t} R_{LB}^{T-t}}{(\pi^{LB})^{T-t-1}\pi_{T+1}} E_{t+1} u'(Y_{T+1};\lambda_{T+1})$$

with probability one and $\pi_{t+1} = \pi^{LB}$ with probability one. Then, if we roll back one period, we can show that:

$$\begin{aligned} u'(c_t;\lambda_t) &\geq \beta R(\pi^{LB};\varepsilon_t) E_t \frac{u'(c_{t+1};\lambda_{t+1})}{\pi^{LB}} \\ &> (\beta R_{LB}) \frac{\beta^{T-t} R_{LB}^{T-t}}{(\pi^{LB})^{T-t}\pi_{T+1}} E_t u'(Y_{T+1};\lambda_{T+1}) \\ &= \frac{\beta^{T-t+1} R_{LB}^{T-t+1}}{(\pi^{LB})^{T-t}\pi_{T+1}} E_t u'(Y_{T+1};\lambda_{T+1}) \end{aligned}$$

with probability one. It follows that:

$$u'(c_t;\lambda_t) > L u'(\bar{N}_t;\lambda_t)/\pi_{T+1} > u'(\bar{N}_t;\lambda_t)$$

with probability one, which in turn shows that $\pi_t = \pi^{LB}$ with probability one.

Proof of Proposition 6

We proceed by reverse induction. Note first that:

$$\begin{aligned} u'(c'_T; \lambda_T) &= R'(\pi_L^{LB}; \varepsilon_T) E_T(u'(Y_{T+1}; \lambda_{T+1})/\pi_{T+1}) \\ &= R(\pi_H^{LB}; \varepsilon_T) E_T(u'(Y_{T+1}; \lambda_{T+1})/\pi_{T+1}) \\ &= u'(c_T; \lambda_T) \end{aligned}$$

Now suppose inductively that:

$$u'(c_{t+1}; \lambda_{t+1})(\frac{\pi_H^{LB}}{\pi_L^{LB}})^{T-t-1} \leq u'(c'_{t+1}; \lambda_{t+1})$$

with probability one for some $t \leq (T-1)$. Then, $u'(c'_{t+1}; \lambda_{t+1}) > u'(\bar{N}_{t+1}; \lambda_{t+1})$ with probability one and $\pi'_{t+1} = \pi_L^{LB}$ with probability one. Similarly, since $c_{t+1} < \bar{N}_{t+1}$ with probability one, $\pi_{t+1} = \pi_H^{LB}$ with probability one.

Next move backwards in time to period t. We can show that with probability one:

$$\begin{aligned} u'(c_t; \lambda_t) &= max(\beta R(\pi_H^{LB}; \varepsilon_t) E_t\{u'(c_{t+1}; \lambda_{t+1})/\pi_{t+1}\}, u'(\bar{N}_t; \lambda_t)) \\ &= \beta R(\pi_H^{LB}; \varepsilon_t) E_t\{u'(c_{t+1}; \lambda_{t+1})/\pi_H^{LB}\} \\ &\leq \beta R'(\pi_L^{LB}; \varepsilon_t) E_t\{u'(c'_{t+1}; \lambda_{t+1})(\pi_L^{LB}/\pi_H^{LB})^{T-t-1}/\pi_L^{LB}\}(\pi_L^{LB}/\pi_H^{LB}) \\ &\leq u'(c'_t; \lambda_t)(\pi_L^{LB}/\pi_H^{LB})^{T-t} \end{aligned}$$

which implies that:

$$u'(c_t; \lambda_t)(\pi_H^{LB}/\pi_L^{LB})^{T-t} \leq u'(c'_t; \lambda_t)$$

with probability one.

We have established that $u'(c'_T; \lambda_T) = u'(c_T; \lambda_T)$ with probability one, and that if $c'_{t+1} \leq$

c_{t+1} with probability one for $t \leq (T-1)$, then $c'_t < c_t$ with probability one. The proposition is proved.

Proof of Proposition 7

We can prove the proposition via reverse induction. Suppose inductively that $c'_{t+1} \leq c^*_{t+1}$ and $\pi'_{t+1} \leq \pi^*_{t+1}$ with probability one. Then:

$$\begin{aligned} u'(c'_t;\lambda_t) &= max(u'(\bar{N}_t;\lambda_t), \beta R'(\pi^{LB};\varepsilon_t)E_t\{u'(c'_{t+1};\lambda_{t+1})/\pi'_{t+1}\}) \\ &\geq max(u'(\bar{N}_t;\lambda_t), \beta R(\pi^{LB};\varepsilon_t)E_t\{u'(c^*_{t+1};\lambda_{t+1})/\pi^*_{t+1}\}) \\ &= u'(c^*_t;\lambda_t) \end{aligned}$$

which implies that $c'_t \leq c^*_t$. In terms of inflation, consider any event in which $\pi^*_t = \pi^{LB}$. In that event:

$$\beta R(\pi^{LB};\varepsilon_t)E_t\{u'(c^*_{t+1};\lambda_{t+1})/\pi^*_{t+1}\} \geq 1,$$

and it follows that:

$$\beta R'(\pi^{LB};\varepsilon_t)E_t\{u'(c'_{t+1};\lambda_{t+1})/\pi'_{t+1}\} > 1$$

which implies that $\pi'_t = \pi^{LB}$ in that event.

Next, consider any event in which $\pi^*_t = \pi^{UB}$. In that event, $\pi'_t \leq \pi^{UB} = \pi^*_t$.

Finally, consider any event in which $\pi^{LB} < \pi^*_t < \pi^{UB}$ with probability one. Then:

$$\beta R(\pi^*_t;\varepsilon_t)E_t\{u'(c^*_{t+1};\lambda_{t+1})/\pi^*_{t+1}\} = 1$$

In that event:

$$\beta R'(\pi^*_t;\varepsilon_t)E_t\{u'(c'_{t+1};\lambda_{t+1})/\pi'_{t+1}\} > 1$$

which implies that $\pi'_t \leq \pi_t$.

Note that π_{T+1} is the same in the two equilibria, and $c_{T+1} = Y_{T+1}$ in the two equilibria.

Hence, the reverse induction above implies that:

$$c'_t \leq c^*_t$$

$$\pi'_t \leq \pi^*_t$$

for all t and with probability one.