Propaganda, Patriotic Protest, and Diplomatic Persuasion

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Abstract

When diplomatic disputes loom, governments often instil hostility in their own citizens to encourage protests against other countries. Promoting protests, however, may cause unrest and escalation of disputes. This paper provides a theory explaining why governments nevertheless have an incentive to promote protests. The foreign government will be more likely to concede if the general public in the home country protests, but when observing a protest, it cannot distinguish whether the general public or only nationalists are protesting. The home government chooses its propaganda balancing the benefit of more concessions versus the cost of more unrest from increased protests. I show that because generating hostility is costly, a government may prefer its people to have 'restrained patriotism'—intermediate responsiveness to international controversies. I also show that governments benefit most from inciting protests when: 1) there is an intermediate level of media freedom; 2) the government is to some degree fragile to political protests.

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"During the first day and a half of the crisis, many of our colleagues, especially those in the Chancery and at some of the Consulates, were in significant danger. Though U.S. Marines protected the Chancery from direct assault, officers on the spot engaged in a full-scale destruction of classified materials that might fall into the hands of demonstrators should the Embassy be overrun. In hindsight, it appears the danger was never that close, but several Chinese did jump the compound wall and had to be confronted by Marines in full battle gear before they were persuaded to jump back over the wall. Except for Shanghai, with its own Marine guard contingent, the other Consulates were protected only by Chinese security guards. In Chengdu those guards were of virtually no help. Demonstrators climbed the compound wall, set fire to the Consul's residence, and smashed their way through the outer door of the Consulate. They were using a bike rack to try to crash into the interior - while screaming that they were going to exact vengeance - when city security forces finally arrived and routed them. Our colleagues were understandably terrified through this ordeal. They were frantically calling the Embassy and local contacts, and getting increasingly agitated by the slow, almost grudging response of the Chengdu authorities.

—Paul Blackburn, Foreign Service Officer, The Association for Diplomatic Studies and Training "

[1]

1 Introduction

When diplomatic disputes loom, governments often instil hostility in their own citizens to encourage protests against other countries. In May 1999, the US bombed the Chinese embassy in Yugoslavia, leading to the death of three journalists and twenty injuries.[2] After internal discussions following the attack, the Chinese leadership decided to take measures against the US, including encouraging protests: the Chinese leadership gave a speech on national TV to condemn the US,[3] while the official Chinese media claimed that the Chinese people's anger was justified. This was seen as an effort to steer citizens into protesting, in

[1] **Blackburn, Paul.** "Dealing with a PR Disaster-The U.S. Bombing of the Chinese Embassy in Belgrade". The Association for Diplomatic Studies and Training: Foreign Affairs Oral History Project. Retrieved 8 May 2013. https://adst.org/2013/05/dealing-with-a-pr-disaster-the-u-s-bombing-of-the-chinese-embassy-in-belgrade/

[2] Whether this was the intentional behaviour of the US is subject to heavy debate. The US side claims this was not intentional and due to outdated information over the position of the Chinese embassy. However, this did not convince the Chinese side, and it is widely believed in China that there was some conspiracy or intentional provocation. **Mizokami, Kyle.** "In 1999, America Destroyed China's Embassy in Belgrade (And Many Chinese Think It Was on Purpose)" The National Interest, January 21, 2017. https://nationalinterest.org/blog/the-buzz/1999-america-destroyed-chinas-embassy-belgrade-many-chinese-19124

[3] Then Vice-President Hu said in a speech that "Chinese government firmly supports and protects any demonstration that is held according to law". "Vice-President Hu gave speech over the bombing of China embassy on 9th May (Chinese)",Sina News, 25th May, 2003. http://news.sina.com.cn/c/2003-05-25/14421097103.shtml

marked contrast to the Chinese government's policy of quenching any public protest.

The US officials were perfectly aware of the Chinese government's efforts to incite protests. However, they also realized that the bombing did generate popular anger in the Chinese population,[4] and this may disrupt the US-China relationship. Eventually, US president Bill Clinton made a public apology and various commitments to investigate the event and improve the Sino-US relationship. After these public concessions, the Chinese government moved to subdue the protests successfully.[5]

This is far from an isolated example: in many other diplomatic disputes, there is intense hostility among the public of one or both sides[6] which leads to people protesting against the foreign opponent.[7] This paper tries to understand the logic of governments using their citizens' activism in international disputes by answering the following questions. First, why does a government encourage protests against a foreign opponent, given that protests can be very costly?[8] Second, through which mechanisms can a government benefit from encouraging protests? Last, which kind of regimes can benefit most from these mechanisms?

To study these questions, I consider the following model: Two countries—Country 1 (Home country) and Country 2 (Foreign country)—seek to resolve an international dispute. Country 1 is the 'aggrieved' country, and its people may go to the streets to protest against Country 2. Country 1's government can choose the level of propaganda that will affect Country 1's people's utility of participating in a protest. In Country 2 there is no internal politics, and its government can only choose to either concede or not.

In Country 1, its people is composed of two factions: the General Public (G) and the Nationalist (N). Both will be more likely to protest if there is a higher level of hostility. The opponent will only concede if the general public is protesting. However, the opponent cannot observe the identities of the protesters. Thus, the opponent is willing to concede as long as it is sufficiently likely that the general public will participate in that protest. This

[4]In Weiss (2013), the author interviewed some senior level US diplomat. That diplomat claims "This thing got out of control. The government and the Foreign Ministry did not realize how determined and angry these people were ... at the United States, but also, as it went on, partially directed at the Chinese government".

[5]See Weiss (2013) and Weiss (2014) for more details.

[6]To name a few, the recent dispute over the name of Macedonia; Japan-South Korea's dispute over the issue of "comfort woman"; and various territorial disputes.

[7]Again, in the previous example of Macedonia naming dispute, large-scale protests broke out on both sides. In South-Korea, widespread protests over the issue of "comfort women" persist years after both governments reaching binding-agreement over this issue

[8]Protests can lead to significant disruptions to society, reducing investors' confidence and sometimes creating threats to the regime.

implies that a protest has to be sufficiently *informative* for the opponent to concede when seeing one. In the case of the US bombing the Chinese embassy, the US conceded over those protests because, although those protests were mobilised and manipulated, they still constituted strong signals of the general public's anger. After receiving concessions, it was much easier for the Chinese government to claim victory and calm down angry protesters.

Since the foreign opponent is willing to concede even with some doubt, there is space for the home government to manipulate protests for its own benefit. By choosing a higher level of hostility through propaganda and management of private media and activists, the home government can incite more protests, leading to more concessions if protests are sufficiently informative. However higher hostility means the general public is more likely to protest, which can be costly. I call the event of the general public protesting a *crisis*: placating the angry public is costly, even after some foreign concession. The home government, thus, chooses its optimal hostility balancing the benefit of more concessions versus the cost of more crises, with the additional constraint that protests are indeed sufficiently informative.

Two key parameters that shape the home government's choice of propaganda are the degrees of responsiveness to hostility of the different groups. I find the optimal level of propaganda for the home government can be non-monotonic over these *degrees of responsiveness*. I also find the optimal level of hostility can have a discontinuous jump over the relative benefit of hostility for the home government: if getting a concession is intermediately important, a small change in the relative benefit/cost ratio may generate a disproportionate increase in propaganda and thus protests. Therefore, big changes in propaganda are not necessarily associated with big changes in the value of receiving concessions or cost of facing protests. This means that occurrences of protest and strong hostility are most volatile over issues that are intermediately important for a government.

We then look at the welfare implications of the model. We show that the home government would like to promote *restrained patriotism* among the general public: here *restrained patriotism* means an intermediate degree of responsiveness towards higher propaganda.[9] A more responsive general public means it is easier to convince the opponent with a low level of hostility. A more responsive nationalist, however, means it is harder to convince the opponent, so that the home government needs a higher level of hostility. Some responsiveness to hostility from the general public and not too much responsiveness from

[9]Under some parameter conditions, the home government would also prefer that the nationalist has an intermediate level of *degree of responsiveness*

the nationalist[10] are then necessary to avoid the need of generating high levels of costly hostility.

I then show that if the home government faces ex-post temptations to either promote fake protests or suppress protests, then countries with an intermediate level of media freedom and political fragility benefit the most from this mechanism.[11] If it is easy for the home government to generate some 'fake protests' *ex-post*, then to maintain the informativeness of protest, the home government has to choose extreme levels of hostility *ex-ante*. If *ex-post* a government can suppress a protest with some probability, then the foreign opponent will have less incentive to give concessions.[12] For the foreign opponent to concede after a protest, it requires the protest to be more informative. Therefore, both the ability to generate fake protests and the ability to suppress a protest without receiving concession will hurt the home government. This can explain why China often benefits from encouraging protests: China is a country with an intermediate level of media freedom and some political fragility, so a protest is a hard but still manipulable signal for China to seek a concession from a foreign government. Finally, I consider various further extensions to the baseline model.

The structure of the paper is as follows: part 2 reviews the related literature; part 3 describes the model's setup and the equilibrium concept; part 4 solves the model and then characterizes comparative statics and conducts welfare analysis; part 5 explores which kinds of regimes can benefit most from encouraging protests; part 6 considers some further extensions to the original model; part 7 concludes. All formal proofs are in the Appendix.

2 Related Literature

2.1 Bargaining and Strategic Information Transmission

It has long been observed that in bargaining situations being irrational or 'crazy' could actually be a good thing. Schelling (1960) first discussed that being inflexible or irrational can help one side commit to only accepting a good offer. Since the seminal work of Schelling (1960), there has been a huge stream of literature discussing the particular ways

[10]Relative to the responsiveness of the general public
[11]Under some parameter restrictions
[12]This is consistent with the historical evidence that in the bombing crisis the American government believed the situation was under control, when they saw more signs of the Chinese government suppressing the level of protests.

how one side can gain by various commitment tactics (Crawford, 1982; Osborne and Rubinstein, 1990; Muthoo, 1996; Abreu and Gul, 2000; Kambe, 1999). That could be one reason why the government wants to promote hostility in diplomatic crises. By creating a demanding or even fanatical domestic audience behind the government, the government can credibly commit only to accept a better offer. By promoting a higher level of hostility among its people, the government will not accept a bad offer for fear of backlash from a domestic audience of zealots. That can be incorporated in a standard two-person zero-sum bargaining game with complete information.

However, this is far from the whole picture: far from playing a zero-sum game where one side's loss is always the other side's gain, in various crises both sides share a common interest to avoid costly conflict breaking out. Either a hot war or an economic/diplomatic conflict could be detrimental to both sides and burn the potential surplus that comes from cooperation. Therefore, if one side is sure enough that the public from the other side is angry, it has the incentive to make concessions and avoid conflict. The information that the public is angry is socially valuable.

Nevertheless, private incentives may hamper this socially-valuable information being transmitted. Receiving concessions is usually a good thing for a government, either due to actual gain or gain of prestige, while making concessions is usually costly. Hence the government may have an incentive to bluff and exaggerate the probability of crises. This may lead to the other side being less sure whether the public is angry. In the extreme case information transmission may completely break down, and thus the other side will always refuse to make a concession.

That is the now classical problem of *cheap talk* that socially valuable information is lost due to conflict of interests (Crawford and Sobel, 1982; Battaglini, 2002; Green and Stokey, 2007; Lipnowski and Ravid, 2017). It has been shown that a sender can achieve a better outcome for himself if he can persuade by committing to a pre-determined disclosure plan or an experiment (Kamenica and Gentzkow, 2011; Gentzkow and Kamenica, 2014; Alonso and Câmara, 2016, 2018; Bergemann and Morris, 2016). However, the level of commitment power required for the sender to persuade may be unrealistic here: there is no legally mandated commitment or ex-post verifiable independent experiment enabling the government to commit to a disclosure plan.

Given that persuasion is better for the sender, can he generate similar effects as if he is persuading the receiver? This model provides a specific way this can be done: increasing hostility through propaganda will increase the probabilities of protest for both the nationalist and the general public. However, both groups will respond at different speeds when hostility is higher. The foreign government is only concerned about the protest of the general public but cannot identify who is protesting. Therefore, when changing hostility, the home government changes the informativeness of protest as a signal of general public's action.

Thus this paper is related to the literature on signal-jamming and obfuscation (Holmström, 1999; Ellison and Ellison, 2009) This paper is also related to the burgeoning literature about how the commitment assumption in Bayesian Persuasion can be relaxed or micro-founded (Best and Quigley, 2017; Margaria and Smolin, 2017)

2.2 War and Audience Cost

Since the pioneering work of Putnam (1988), there has been a vast literature studying the interaction between domestic politics and international diplomacy. Putnam (1988) coined the concept of *Two-level game* to describe the interconnection between international diplomacy and domestic politics. Domestic politics and diplomacy are usually closely entangled, and governments/leaders take domestic and diplomatic decisions to maximise their own benefits.

Fearon(1994) popularized the concept of *audience cost* and ignited the long literature about *audience cost* and its various implications. Making credible statements about one country's resolution over crises is extremely difficult, and audience cost provides a particular way how this can be done. By making the threat to enter a war known to the public, the government increases its cost to back down, and this increases the credibility of the threat. Despite various studies that apply this concept (Eyerman and Hart, 1996; Mansfield, Milner, and Rosendorff, 2002; Schultz and Weingast, 2003), there are fewer studies discussing the micro-foundations of these concepts. (Smith, 1998; Schultz, 1999; Slantchev, 2006) If leaders bluff for the national interests, it is not clear why domestic voters would like to punish leaders for backing down from their threats. This paper provides a simple *preference channel* to explain why audience cost can work: by increasing hostility or salience over the current dispute through propaganda, the government makes the domestic audience more extreme and thus it would be harder for the government to back down without receiving some concessions, for fear of domestic backlash.

Another question related to this concept would be which countries have high levels of audience cost and are thus more able to commit. Fearon (1994) argued that because leaders in democratic countries face stronger audience cost than leaders in nondemocratic countries, they will be at better positions to commit. Weiss (2013) argues that there can also be big audience cost in non-democratic countries. One important mechanism is through allowing anti-foreign protests. By allowing those risky protests, governments show their resolve and also domestic vulnerability. Weiss (2014) gives detailed discussion and case studies of how non-democratic countries can generate audience cost in the context of China. This paper is also related to this stream of literature. In my model, protest is a noisy signal that conveys some information and may potentially prompt the foreign government to make concessions.

The differences between my paper and previous conceptual and formal models are as follows.

First, this paper focus on *ex-ante* manipulation of protests instead of *ex-post*. In many cases, the government has to decide whether to encourage or hamper protests before knowing whether some particular dispute will become an uncontrollable diplomatic crisis.

Second, my paper also provides some prediction about the optimal strategy of governments' *nation-building* policy: the government would try to promote some form of *restrained patriotism*. It wants its people to be responsive, but not too responsive, over international controversies. That is consistent with governments' actions in real life, for example, the Chinese government's constant call for 'loving the country rationally'.

Last but not least, this paper emphasises the benefit rather than the cost of *obfuscation* for protests to persuade the foreign governments to concede. Previous studies such as Weiss (2014) have emphasised the value of credibility for protests to convince the opponents to concede. In this paper I show the value of at least partial *obfuscation*: in many cases, foreign government's responsive is in discrete levels, and there is no need to perfectly convince her to make her concede. A not perfectly precise but still precise signal can still guarantee concession and makes the probability of getting concession higher than the case of full information.

3 Model Setup

3.1 Players, Actions, States and Payoff Functions

Player Set:

There are two countries, Country 1 and Country 2. This model includes the bargaining problem between Country 1 and Country 2 and the internal politics of Country 1.[13]

There are four players in the game:

$$N = \{H, F, N, P\}$$

H is the home government of Country 1; N is the nationalist group of Country 1 and P is the general public of Country 1; F is the foreign government(government of Country 2).

Action Spaces:

The action spaces of all the players are as follows:

$A_H = \Psi = [0, \bar{\psi}], A_F = \{C, NC\}, A_N = A_P = \{P, NP\}$.

The home government chooses the level of hostility from a closed and bounded interval; the foreign government chooses whether to concede or not; the nationalist and the general public each decides whether to protest or not.

$$\text{Country 1} : \begin{cases} H \to [0, \bar{\psi}] \\ N \to \{P, NP\} \\ G \to \{P, NP\} \end{cases}$$

$$\text{Country 2} : F \to \{C, NC\}$$

Home government chooses ψ from a compact interval, $[0, \bar{\psi}]$. ψ models the level of hostility home government chooses. It captures a government's various methods of generating higher hostility among its people towards a foreign opponent: 1) direct propaganda in government-controlled media; 2) censorship over independent media's coverage; 3) management over activities of the nationalist groups. This paper will just call it propaganda; however ψ can capture much more than direct propaganda.

Here I model the foreign government's action as a discrete choice of whether to concede or not. This reflects the observation that in many cases, diplomatic responses are in discrete levels: instead of choosing from a continuous action space, the opponent usually has different 'steps' of potential responses. Also, many issues' indivisibility can further

[13]For simplicity we don't consider the internal politics of Country 2, so foreign government represents a coherent Country 2

justify this assumption. [14]

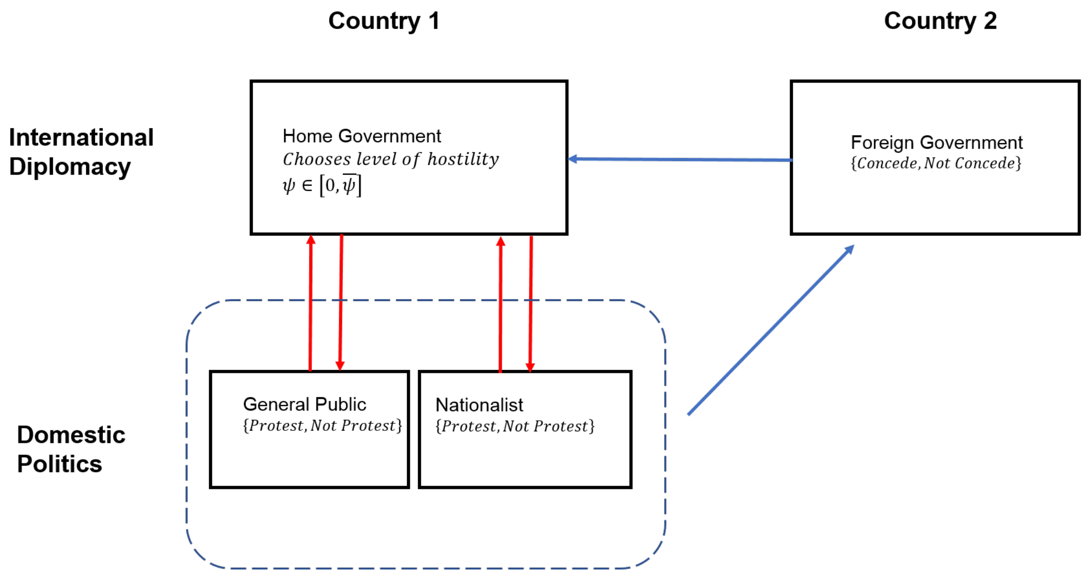

Figure 1: Players, Country Affiliations and Actions

State Space:

The state space, Θ, is a one-dimensional closed and compact set. θ can be thought as the latent 'grievance' that country 1's people feel over some dispute. The higher is θ, the more annoying this event is to the nationalist and the general public. I further assume θ is uniformly distributed among $[0, \bar{\theta}]$ and is statistically independent of any choice of ψ of the home government.

Pay-off Functions

In general, for any player $i, i = H, F, N, P$, its utility function $u_i(\theta, a_H, a_F, a_N, a_G)$ depends on the realization of the state, and the profile of all player's actions. We assume players' payoff functions are as follows:

(Payoffs of Nationalist and General Public)

For $i = N, G$,

$$u_i(\theta, \psi, a_i) = \begin{cases} [w_i(\psi)\theta - c], & \text{if } a_i = P \\ 0, & \text{if } a_i = NP \end{cases}$$

(Payoffs of Home Government and Foreign Government)

For H and F, their payoff functions only depend on the action of general public, a_G, and

[14] In theory, side payments can be possible. However, due to various reasons side payments or transfers cannot always solve this problem. For example, this can be due to the limited attention span or expertise of understanding diplomatic pacts from the public.

the action of foreign government, a_F.

Moreover, we assume the Home Government's payoff function have the following properties:

$$u_H(\cdot, C) > u_H(\cdot, NC)$$

$$u_H(NP, \cdot) > u_H(P, \cdot)$$

We assume the Foreign Government's payoff function have the following properties:

$$u_F(P, NC) < u_F(P, C)$$

$$u_F(NP, NC) > u_F(NP, C)$$

$$u_F(NP, \cdot) > u_F(P, \cdot)$$

Tie-breaking Rules

First, we further assume that when the foreign government is indifferent between conceding or not, it will concede with probability 1. Following Kamenica and Gentzkow(2011), we are looking at the sender-optimal subgame perfect equilibrium.

Second, we assume that when either the nationalist or the general public is indifferent between protesting or not, it will protest with probability 1. That assumption is an innocuous one since for continuous distribution of θ, being indifferent is a measure zero event.

Last, without loss of generality, we assume that when the home government is indifferent between choosing a lower level of ψ or a higher level of ψ, he will choose the lower one with probability 1.[15]

Discussion

I will call the situation of general public protesting as a *crisis*, because general public protesting is always costly, and may get out of control if the foreign opponent does not concede.

There are several assumptions that are worth further discussion.

[15] It can be shown that in this model the home government's optimal level of hatred is in general unique. The home government is only indifferent between more than one optimal level of hatred in rare cases when: 1) under the optimal level of hatred the informativeness constraint is binding; 2) the threshold of doubt equals the benefit/cost ratio. Under this scenario, for any two different randomisations between the optimal levels, the home government and the foreign government get the same payoffs.

First, here we have assumed that the two groups' payoff functions depend on whether they go to protest, but not on the future policy outcome. The event generates anger for both groups, and both groups benefit from expressing anger in the form of protesting in the street. Going to protest is still costly, so both groups are still rational in the sense that they will weigh the benefit and cost of going to the street.

So the two groups are not 'strategic' in the sense that they are not choosing whether to protest in a forward-looking way: protesting to induce the home government to take some particular actions. Instead, they are protesting for 'expressive' reasons. It has been long noticed in the sociology literature and the political science literature that protest can be 'instrumental' and/or 'expressive'. Passarelli and Tabellini(2017) consider a model in which protests are expressive. Protesters protest to express their emotions over the 'unfair' treatment they received. Here, similar to Passarelli and Tabellini(2017), protests are expressive. Two groups protest to react to the unfair treatment their country has received during the crisis, or the humiliation their country has received that needs proper compensation.

Second, we assume that both governments care about the action of the general public and the action of foreign government. It is obvious that both governments are affected by the decision of the foreign government. It is also obvious that the home government cares about whether the public will go to the street. Here we assume the foreign government cares about the public's protest decision, not because we believe that the foreign government cares about the well-being of country 1's public, but as a reduced-form representation: one reason could the direct consequence, like Country 1's public boycotting Country 2's goods. Another reason is more aligned with the audience cost literature: the foreign government cares about the probability of escalation from the home government, and the home government's probability of escalation depends on whether the general public goes to the street. Intuitively, with angry public protesting on the street, the home government is usually facing mounting pressure to be 'tough' and escalate the crisis if not concession is offered.

Third, we've assumed here both home and foreign government do not care about the protest of nationalist intrinsically. Moreover, in the baseline model, θ and ψ do not enter into the utility functions of both governments. We show in later sessions that the conclusions are robust if we relax the first assumption. Intuitively, nationalists are people who care very strongly about national interest and may have very extreme views over diplomatic disputes. Suppose the great majority does not support this kind of extreme views, then nationalists are a thin minority which will not pose serious threats over the home government and will not force the home government into escalation. The opponent only cares about the protest behaviour of country 1's people because it may pressure country 1's government into costly escalation. Therefore, given that only the general public can force the home government into a costly crisis, the foreign government will only care about the action of the general public.

For the assumption of θ and ψ not entering the payoff functions of both governments, those assumptions can be partially relaxed. Essentially, for θ to not affect the payoff functions for both governments, θ needs to only affect both governments indirectly: both governments will not be affected by angry public as long as they stay at home and do not protest. For the ψ to not directly enter the payoff functions of both government, we are essentially assuming: 1) choosing ψ is costless;[16] 2) ψ will only affect the distribution of states, but not both governments' payoffs under each state.

3.2 Timeline and Information Structure

The timeline of the baseline model is as follows:

Period 1: The home government chooses the level of hostility, ψ; the level of ψ is public knowledge to all the players.

Period 2: nature chooses the level of θ. θ is private knowledge to the home government, the nationalist and the general public, but not the foreign government. The nationalist and the general public then decide simultaneously whether to protest or not. The actions of the nationalist and the general public are observable to themselves and the home government, but not to the foreign government. [17]

Period 3: The foreign government can only observe a binary signal, s, $s \in \{P, NP\}$,which

[16]We have assumed that for the home government, choosing any level of $\psi \in \left[0, \bar{\psi}\right]$ has zero cost. This is equivalent to choose $\psi \in [0, +\infty)$ but choosing ψ is costly:

$$C(\psi) = \begin{cases} 0, & \text{if} \psi \in \left[0, \bar{\psi}\right] \\ +\infty, & \text{if} \psi \in \left(\bar{\psi}, +\infty\right) \end{cases}$$

[17]It is not essential to assume that θ is observable to the home government. What is essential here is that the home government can observe *who* is protesting.

Period 1:	Period 2:	Period 3:	Period 4:	Period 5:
H publically chooses ψ	θ realized and revealed only to players in country 1; N and G decides whether to protest simultaneously; their actions are observable to N,G,H, but not F	signal s is observed by F	F decides whether to concede	the state and the pay-offs of all players are realized

Figure 2: Timeline of the Baseline Model

is generated in such a way:

$$s = \begin{cases} P, & \text{if } a_N = P \vee a_G = P \\ NP, & \text{otherwise} \end{cases}$$

This essentially means the foreign government can only observe whether there is a protest but not who is protesting.

Period 4: The foreign government decides whether to concede or not after observing the realisation of signal s.

Period 5: the state and the pay-offs of all players are realised and publicly revealed.

Here we assume the home government chooses ψ before θ is realised. This assumption comes from the observation that the government cannot perfectly predict and control what will happen as the beginning of a potential crisis. When the home government are trying to affect the probability of protests, by either direct propaganda or act towards nationalist leniently, they do not know perfectly how its people will react towards this. Moreover, it is precisely this property of protests being ex-ante 'uncontrollable' and 'unpredictable' that give the opponent incentive to concede and the home government room to manoeuvre the protests.

Here ψ is essentially modelling the ways to *ex-ante* manage protests. Later we will discuss ways for the government to *ex-post* manage protests, such as suppressing a protest or generating some fake protests.

We also assume that θ is unobservable to the foreign government. For the government of Country 2, observing how the people in Country 1 feels about some diplomatic crisis before making the decision is extremely difficult. Observing or inferring what people have in mind is extremely difficult, and it would be even more difficult to observe a rival country's people's real feeling. There is only some limited amount of media information and some secret intelligence reports that the foreign country can rely on, and the home country has strong incentive to misrepresent the information the foreign government can receive. Therefore, it is possible that the foreign government to get some noisy signal over θ, but it is still safe to assume at least some unobservability of θ.

Another crucial assumption here is that the foreign government cannot distinguish between protests of only nationalist and protests of everyone (including both nationalist and the public). The only thing it can observe is whether there are some people on the street protesting. In reality, the foreign government probably can have some noisy signal over some characteristics of a protest, like the number of people appearing in the street. However again, this is usually a very noisy signal, and the home government has the incentive to misrepresent that signal. Therefore, this is a simplifying assumption that captures the fact that foreign government cannot perfectly observe who is protesting on the street. Later we will look at the case when the government can get some noisy and non-revealing signal over the identity of the protesters. That would also allow us to generate additional prediction over what level of media freedom would allow a regime to benefit most from manipulating protests.

3.3 Solution Concept

The equilibrium concept in this model is standard weak Perfect Bayesian Equilibrium.

Definition 1: *The profile of (a^*,μ^*) is weak Perfect Bayesian Equilibrium(PBE as follows) if:*

1) μ^* is consistent: i.e., it is determined by Bayes' Rule according to a^* whenever possible;

1.1) In Period 4, $\mu(\psi, s)$ is foreign government's posterior belief that the general public is protesting, given the level of ψ and realization of s. $\mu(\psi, s)$ is determined by Bayes' Rule whenever possible, according to the signal-generating process and the strategies of the home government, the nationalist and the general public.

2) a^* is sequentially rational:

2.1) In period 4, given$(a_H^*, a_N^*, a_G^*, \mu^*(\psi, s))$, a_F^* is the optimal response of the foreign government.

2.2) In Period 2, given the choice of ψ by the home government in Period 0 and the realisation of θ in period 1, for $i = N, G$, a_i^* is the optimal response of group i given $a^*_{\{N,G\}/i}$ and a_F^*

2.3) In Period 1, the home government chooses the optimal level of ψ, given $(a_N^*, a_G^*, a_F^*, \mu^*)$

4 Equilibrium and Its Properties

In this section, we will solve the equilibrium of the baseline model and characterise its properties. The baseline model is solved by backwards induction. We will then discuss the comparative statics and the welfare analysis.

4.1 The Foreign Government's Optimal Strategy

The foreign government wants to concede if and only if the general public is protesting. Therefore, he will only concede if his posterior of general public protesting, $\mu(\psi, s)$, is higher than some threshold of doubt, $\bar{\mu}$. [18]

When he observes no protest, he knows that the general public is not protesting for sure. Therefore he will not concede in that case.

When he observes a protest, he understands that this can be either a protest where both groups (the Nationalist and the General Public) participate or a protest of only the nationalist. He only wants to concede if the general public protests, but he cannot observe who is protesting. Therefore, he will only concede if the posterior that the general public is protesting is high enough:

$$\mu(\psi, s = P) \equiv Prob(a_G = P|\psi, s = P) = \frac{Prob(a_G = P|\psi)}{Prob(a_G = P|\psi) + Prob(a_G = NP, a_N = P|\psi)} \geq \bar{\mu}$$

[18] *the threshold of doubt* for the foreign government, $\bar{\mu} \in (0, 1)$, is defined as the level of μ such that the foreign government is indifferent between conceding or not conceding. That means $\bar{\mu}[u_F(P, C) - u_F(P, NC)] = (1 - \bar{\mu})[u_F(NP, NC) - u_F(NP, C)]$

We define a protest to be *informative enough* if $\mu(\psi, s = P) \geq \bar{\mu}$. This is the case when a protest is sufficiently informative such that the foreign government will concede when a protest happens.

4.2 The Nationalist and the General Public's Optimal Strategy

For both the nationalist and the general public, their costs of protesting are constant but benefits of going to the street are higher if the grievance, θ, is higher. Therefore, for both groups, their best responses are cut-off strategies.

For group $i, i = N, G$, it will protest if and only if $(w_0 + w_{1i}\psi)\theta - c \geq 0$. This means there exist a cut-off level, $\hat{\theta}_i(\psi)$, such that it will protest if and only if $\theta \geq \hat{\theta}_i(\psi)$.

Figure 3 describes the best response of both groups given the level of hostility, ψ. Because we have assume that the Nationalist is more responsive to propaganda, it is obvious that it has a lower cut-off level than the general public. That also means the we can divides the whole line of θ into three parts: 1) $\theta \in \left[\hat{\theta}_G, \bar{\theta}\right]$. When θ is high, i.e., the event is very severe, both groups will protest. 2) $\theta \in \left[\hat{\theta}_N, \hat{\theta}_G\right)$. When θ is in intermediate level, only the Nationalist will protest. 3)$\theta \in \left[0, \hat{\theta}_N\right)$. When θ is low, no one will protest.

Define the ex-ante probability of protesting [19] for group i as $H_i(\psi) \equiv Prob(\text{Player i protests}) = Prob(\theta \geq \hat{\theta}_i)$. Because of the assumptions of our model, $H_N(\psi) \geq H_G(\psi)$, i.e., the nationalist will be more likely to protest. Given we have defined *crisis* as the General Public protesting, $H_G(\psi)$ is also the probaility of *crises*. Moreover, given that the foreign government will observes a protesting whenever the nationalist protests, $H_N(\psi)$ is also the probaility of the foreign government *observing* a protest. We know from the model set up that higher hostility, ψ, will increase the benefit of protesting for both groups at any given θ. Figure 4 shows that this would reduce the cutoff levels of both groups and make them more likely to protest.

Now we can also characterise how the foreign government's posterior when observing a protest, $\mu(\psi, s = P)$, changes as ψ changes:

$$\mu(\psi, s = P) = \frac{Prob(a_G = P|\psi)}{Prob(a_G = P|\psi) + Prob(a_G = NP, a_N = P|\psi)} = \frac{H_G(\psi)}{H_N(\psi)}$$

Figure 5 shows that foreign government posterior is actually *U-shape* over ψ. That

[19] Here ex-ante probability of protest means the *interim* ex-ante expected probability of protest. That means the probability of protesting after ψ is chosen but before θ is realized

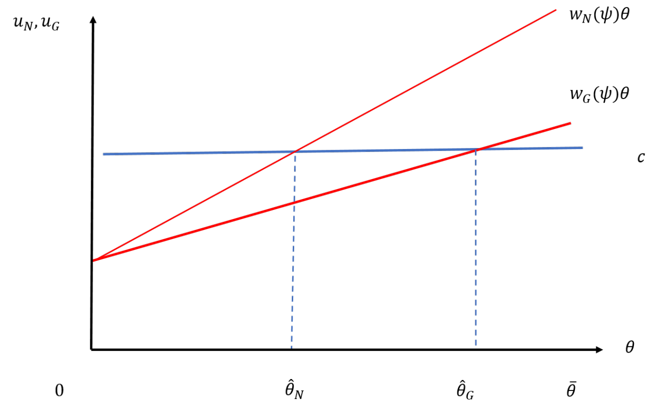

Figure 3: Nationalist and General Public's Best Responses

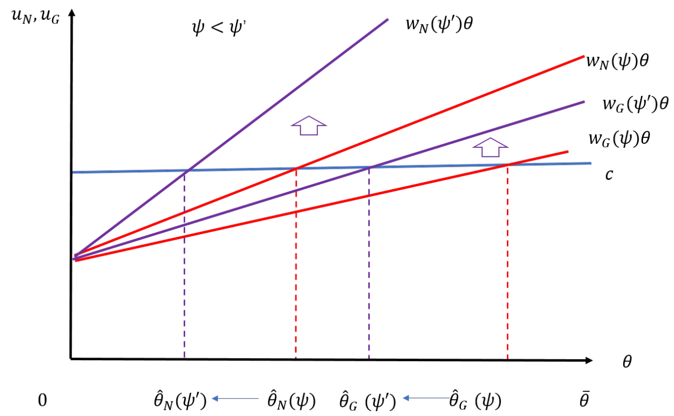

Figure 4: Nationalist and General Public's Best Responses

means for a high $\bar{\mu}$, a protest will only be informative enough if it is easy very low or very high.

The intuition of this U-shape is as follows: when the ψ is low so there is not much manipulation of protests, the opponent knows that the nationalist and the public are both unlikely to be on the street, and their differences over probabilities of protesting are low. Thus when the opponent observes a protest, he knows with high probability both groups are protesting. When ψ is very high, the opponent knows the nationalist is almost always protesting on the street. However, since ψ is very high, the general public is also very likely to be protesting, so in this case, the difference over protest probabilities are also low. In the case that ψ is at intermediate level, $H_N(\psi)$ is much larger than $H_G(\psi)$. There would be many protests of only the nationalist, and this would make foreign government

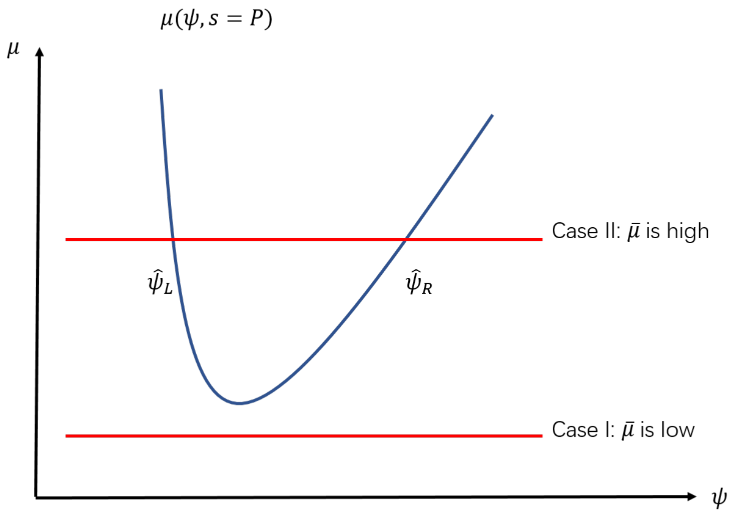

Figure 5: Foreign Government's posterior when observing Protest, as function of ψ

less sure that the general public supports a protest when it observes one. [20]

4.3 The Home Government

The home government's choice of ψ will affect the home government's pay-off through: 1) the actions(protest or not) for both groups(nationalists and the general public); 2)the strategy of the foreign receiver.

First, it can be shown that the home government will never choose a level of ψ such that the $\mu(\psi, P) < \bar{\mu}$. The intuition is simple: the home government will only increase ψ to increase its probability of getting concessions, and the cost of doing so is a higher probability of crises. When ψ is at a level such that $\mu(\psi, P) < \bar{\mu}$, the opponent will not make a concession, and positive ψ will only generate cost but without benefit. Thus home government can deviate and choose $\psi = 0$ instead, which secures concession when there is a protest and generates a lower probability of crises.

This simplifies the home government's problem substantially: we only need to consider the home government's optimal decision assuming that the foreign government will concede

[20]Mathematically, the reason of U-shape we observe here comes from the elasticity of $H_N(\psi)$, $\frac{H_N'(\psi)}{H_N(\psi)}$, single-crossing the elasticity of $H_G(\psi)$, $\frac{H_G'(\psi)}{H_G(\psi)}$. Roughly speaking, when ψ is small, $H_N(\psi)$ is growing much faster than $H_G(\psi)$, and the levels of $H_N(\psi)$ and $H_G(\psi)$ are both small. Thus $H_N'(\psi)$ is substantially larger than $H_G'(\psi)$, while the difference between $H_N(\psi)$ and $H_G(\psi)$ is small. When ψ becomes large, $H_N(\psi)$ and $H_G(\psi)$ are both growing very slowly so $H_N'(\psi)$ is very near to $H_G'(\psi)$, while the difference between $H_N(\psi)$ and $H_G(\psi)$ are also small but still in larger magnitude.

after observing a protest. Then to make sure it is indeed incentive compatible for the foreign government to concede, we just need to put a constraint such that the level of ψ chosen by the home government makes it incentive-compatible for the opponent to concede. We will call it the *informativeness constraint.*

The home government's optimisation problem is equivalent to:

$$\max_{\psi\in[0,\bar{\psi}]} H_G(\psi)u_H(P,C) + [H_N(\psi) - H_G(\psi)]u_H(NP,C) \\ + [1 - H_N(\psi)]u_H(NP,NC)$$

$$s.t.\ \mu(\psi,P) = \frac{H_G(\psi)}{H_N(\psi)} \geq \bar{\mu}$$

[21]

We will first characterise the objective function and the informativeness constraint:

Lemma 1:

- *The objective function is quasi-concave on the interval* $[0,\bar{\psi}]$. *It maximal point on this region,* ψ^I, *is a weakly increasing function over the relative benefit/cost ratio,* $\frac{t_H}{\tau_H}$.
- *The posterior belief that the general public is protesting when there is a protest,* $\mu(\psi,P)$, *is a quasi-convex function over* ψ. *It is first decreasing and then increasing over* ψ.
- *define* $\mu(\hat{\psi},P) = \min_{\psi\in[0,\bar{\psi}]} \mu(\psi,P)$. *If* $\bar{\mu} \in (\mu(\hat{\psi},P), \mu(\bar{\psi},P))$, *there exist two levels of* ψ, $\hat{\psi}_L(\bar{\mu})$ *and* $\hat{\psi}_R(\bar{\mu})$, $0 < \hat{\psi}_L(\bar{\mu}) < \hat{\psi}_R(\bar{\mu}) < \bar{\psi}$, *such that* $\mu(\hat{\psi}_L(\bar{\mu}),P) = \mu(\hat{\psi}_R(\bar{\mu}),P) = \bar{\mu}$. *Moreover, for any* $\psi \in [0,\bar{\psi}], \mu(\psi,P) \geq \bar{\mu}$ *iff* $\psi \in [0,\hat{\psi}_L(\bar{\mu})] \cup [\hat{\psi}_R(\bar{\mu}),\bar{\psi}]$

The analytical forms of $\hat{\psi}_L(\bar{\mu})$ and $\hat{\psi}_R(\bar{\mu})$ can be find in the appendix. So there exists a unique ideal point of ψ , ψ^I, that maximises the objective function. The informative constraint is U-shape over ψ, so if the threshold of $\bar{\mu}$ is high, then a protest is only informative if ψ is very high or very low. So the solution to the home government's

[21] The objective function is continuous. The set of ψ that satisfies the constraint is non-empty and is a finite union of disjoint compact intervals. Therefore, the optimal solution and optimal value to this question both exist.

optimisation problem, ψ^*, depends on whether the *informativeness constraint* is violated at ψ^I.

Now let us define two parameters of the home government's payoff function:

Definition 2

- $t_H \equiv u_H(NP, C) - u_H(NP, NC)$
- $\tau_H \equiv u_H(P, C) - u_H(NP, C)$

t_H can be thought as the benefit of higher propaganda: a higher level of propaganda increases the probability of protest from only nationalist. When protests of only the nationalist happen, there is no crisis, but the foreign opponent will still concede. From our assumption, getting concession without a crisis happening is beneficial to the home government.

τ_H can be thought as the cost of higher propaganda: higher propaganda also increases the probability of general public protesting. This is costly because general public protesting is costly. Therefore the ratio of t_H over τ_H, $\frac{t_H}{\tau_H}$, can be thought of as a measure of the relative benefit of propaganda.

Now we are ready to characterise the optimal feasible level of ψ:

Proposition 1:
Assume $\bar{\psi} \leq \frac{w_0}{\sqrt{w_{1N} w_{1G}}} \sqrt{\frac{w_0 - \tilde{c}}{w_0}}$.[22] *Then:*

1) *If* $\frac{t_H}{\tau_H} \geq \frac{w_{1G}}{w_{1N}}$, *then* $\psi^* = \psi^I = 0$ *for any* $\bar{\mu} \in (0, 1)$*;*

2) *If* $\frac{t_H}{\tau_H} \geq \frac{w_{1G}}{w_{1N}} (\frac{w_0 + w_{1N}\bar{\psi}}{w_0 + w_{1G}\bar{\psi}})^2$, *then*

- *If* $\bar{\mu} \in \Big(0, \mu(\bar{\psi}, P)\Big]$, *then* $\psi^* = \psi^I = \bar{\psi}$
- *If* $\bar{\mu} \in \Big(\mu(\bar{\psi}, P), 1\Big)$, *then there exists* $\hat{\psi}_L(\bar{\mu})$, *such that:*
  - $\mu(\hat{\psi}_L(\bar{\mu}), P) = \bar{\mu}$
  - $\psi^* = \mu(\hat{\psi}_L(\bar{\mu}), P)$

3) *If* $\frac{t_H}{\tau_H} \in (\frac{w_{1G}}{w_{1N}}, \frac{w_{1G}}{w_{1N}} (\frac{w_0 + w_{1N}\bar{\psi}}{w_0 + w_{1G}\bar{\psi}})^2)$ *There exist a cut-off level* $\mu_1 \in (0, 1)$, *such that:*

- *If* $\bar{\mu} \in (0, \mu_1]$, *then* $\psi^* = \psi^I$

[22] The other case leads to similar results. The complete results can be find in the appendix.

- *If* $\bar{\mu} \in \left(\mu_1, \mu(\bar{\psi}, P)\right]$*, then there exist* $\hat{\psi}_L(\bar{\mu})$ *and* $\hat{\psi}_R(\bar{\mu})$*, such that:*
  - $\mu(\hat{\psi}_L(\bar{\mu}), P) = \mu(\hat{\psi}_R(\bar{\mu}), P) = \bar{\mu}$
  - $$\psi^* = \begin{cases} \hat{\psi}_L, \textit{if} & \frac{t_H}{\tau_H} \leq \bar{\mu} \\ \hat{\psi}_R, \textit{if} & \frac{t_H}{\tau_H} > \bar{\mu} \end{cases}$$
- *If*$\bar{\mu} \in \left(\mu(\bar{\psi}, P), 1\right)$*, then there exists* $\hat{\psi}_L(\bar{\mu})$*, such that:*
  - $\mu(\hat{\psi}_L(\bar{\mu}), P) = \bar{\mu}$
  - $\psi^* = \hat{\psi}_L(\bar{\mu})$

Figure 6 and 7 illustrate the intuition of Proposition 1 under the case that $\bar{\psi}$ is large and $\frac{t_H}{\tau_H}$ is in intermediate levels.[23] Proposition 1 says, when the threshold of doubt is so low enough such $\mu(\psi, P) \geq \bar{\mu}$ for any $\psi \in [0, \bar{\mu}]$, then it is not a concern. The optimal level of ψ equals to the ideal point of ψ.

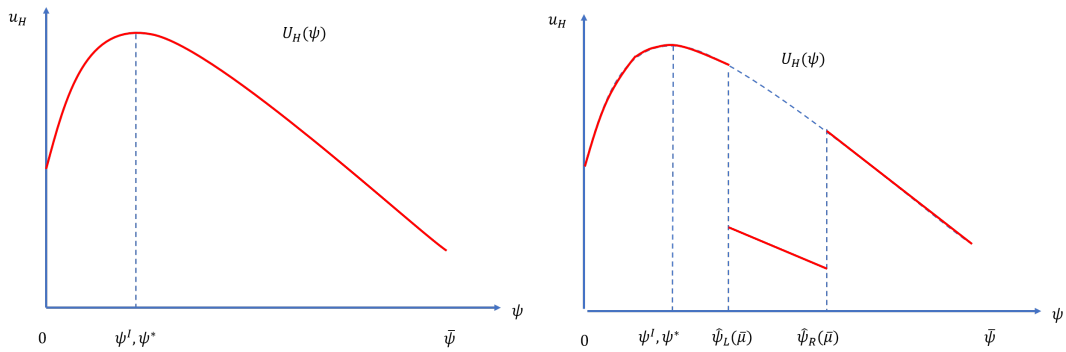

Figure 6: Optimal propaganda, when $\bar{\mu}$ is low or middle level

When the threshold of doubt is higher, then protest is only informative enough if $\psi \leq \hat{\psi}_L(\bar{\mu})$ or $\psi \geq \hat{\psi}_R(\bar{\mu})$. Now there are three cases: First, if $\bar{\mu}$ is in an intermediate range, then the interval that a protest is not informative enough, $\left(\hat{\psi}_L(\bar{\mu}), q\hat{\psi}_R(\bar{\mu})\right)$, is small. Then the ideal point ψ^I lies out of this range and thus the optimal level ψ^* is the same as the previous case. Second, if $\bar{\mu}$ is larger, then the interval that a protest is not informative enough, $\left(\hat{\psi}_L(\bar{\mu}), q\hat{\psi}_R(\bar{\mu})\right)$, is larger. Then the ideal point ψ^I lies out of this range and cannot be chosen. Since the objective function is single-peaked,the constrained optimal must be either $\hat{\psi}_L(\bar{\mu})$ or $\hat{\psi}_R(\bar{\mu})$. The relative benefit of $\hat{\psi}_L(\bar{\mu})$ over $\hat{\psi}_R(\bar{\mu})$ would be a lower

[23]For other cases the intuition is the same

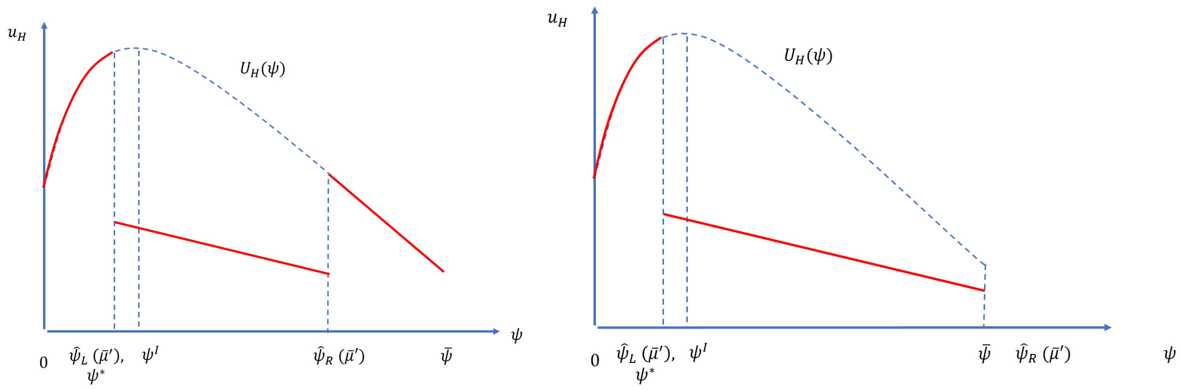

Figure 7: Optimal propaganda, when $\bar{\mu}$ becomes higher and higher

probability of bad state because of lower ψ. However, the relative cost would be a lower probability of a 'Protest' and thus less concession. Which effect dominates depends on the comparison between $\frac{t_H}{\tau_H}$ and $\bar{\mu}$. Last, when ψ^I is even higher, $\hat{\psi}_R(\bar{\mu})$ is too high so the only levels of propaganda that make a protest informativeness are in the range $\left[0, \hat{\psi}_L(\bar{\mu})\right]$.

4.4 Comparative Statics

So the next set of interesting questions would be: (1) How the optimal level of ψ, ψ^*, changes with the parameters[24] of the model? Moreover, how does ψ^* transit from unconstrained optimal point(home government's ideal point) to constrained optimal point($\hat{\psi}_L(\bar{\mu})$ or $\hat{\psi}_L(\bar{\mu})$)? (2) How the equilibrium probability of protest from the general public, $H_G(\psi^*)$, and the probability of protest from the nationalist, $H_N(\psi^*)$, changes with the parameters of the model ?

The complete comparative statics analysis of ψ^*, $H_G(\psi^*)$ and $H_N(\psi^*)$ over various parameters can be found in the appendix. Some interest results can be found from the comparative static analysis:

First, ψ^* may have non-monotonic transitions over some parameters. In some cases, this is due to the non-monotonicity of the interior solution ψ^I itself. However, in many cases this non-monotonic transition comes from the transition between the interior solution ψ^I and the boundary solutions $\hat{\psi}_L(\bar{\mu}, P)$ and $\hat{\psi}_R(\bar{\mu}, P)$.

Second, ψ^* may have discontinuous jump over $\frac{t_H}{\tau_H}$ at point $\frac{t_H}{\tau_H} = \bar{\mu}$. A very small change

[24]Parameters of the model: w_0; w_{1G} and w_{1N}; c; $\bar{\theta}$; $\bar{\mu}$; $u_H(P,C) - u_H(NP,C)$ and $u_H(NP,C) - u_H(NP,NC)$. We will look at last two parameters' effects jointly thorough $\frac{t_H}{\tau_H} \equiv \frac{u_H(NP,C)-u_H(NP,NC)}{u_H(P,C)-u_H(NP,C)}$, because that is the only way those two parameters affect ψ^*.

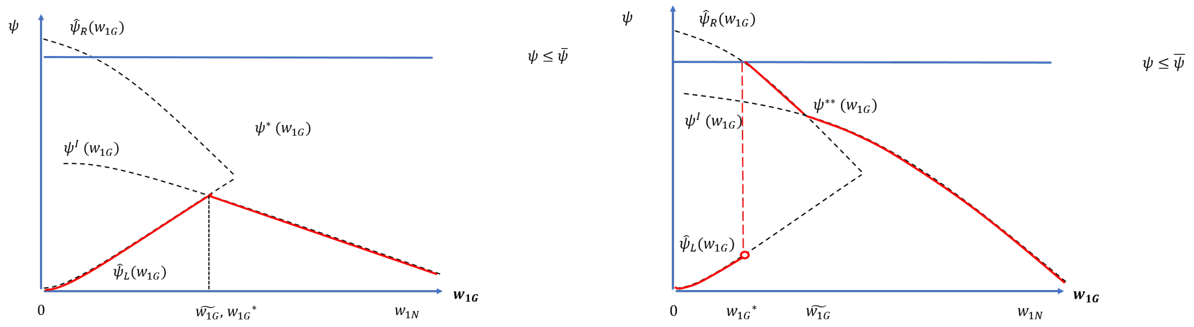

Figure 8: Comparative Statics of ψ^* over w_{1G}: when $\frac{t_H}{\tau_H} < 1$ and $\bar{\mu}$ is high

over $\frac{t_H}{\tau_H}$ would lead to discrete jump from $\hat{\psi}_L(\bar{\mu}, P)$ to $\hat{\psi}_R(\bar{\mu}, P)$ (or *vice versa*). This could possibly explain that when the fundamentals of some dispute remain the same, with some small change of benefit/cost for the home government, it will dramatically escalate or de-escalate the tension. Moreover, this only happens when the relative benefit of driving up tensions, $\frac{t_H}{t_H}$, is in an intermediate range. This would predict that we would see the nationalism tension to be most volatile when the dispute is intermediately important for the home government. In matters that are either too important or too trivial, we would not expect to see big shifts in the level of hostility unless there are some big changes in the fundamentals of the dispute.

The comparative statics of $H_G(\psi^*)$ and $H_N(\psi^*)$ also have similar patterns.

Here we will look at one example, the comparative statics of optimal hostility, ψ^*, over w_{1G}, the degree of responsiveness for the general public.

Corollary 1

Assume $\frac{t_H}{\tau_H} < 1$ *and* $\bar{\mu}$ *is large. There exists a level of* w_{1G}, w_{1G}^*, *such that*

- ψ^* *is increasing in* $w_{1G} \in (0, w_{1G}^*)$
- ψ^* *is decreasing in* $w_{1G} \in (w_{1G}^*, w_{1N})$

This corollary has an intuitive explanation: when w_{1G} is low compared to w_{1N}, as the home government increases ψ, the nationalist will increase its probability of protest in a much faster speed than the general public. In this case, the informativeness of the protest is problematic and to keep informativeness, the home government cannot choose a level of hatred that is too high.

However, when w_{1G} is very high, as ψ increases, the nationalist will not increase its probability of protest much faster than the general public. In this case, informativeness is not a concern. However, as w_{1G} becomes higher, costly crisis is more likely to occur and thus choosing a high level of ψ becomes more and more costly.

4.5 Welfare Analysis

In this part, we will look at how home government's equilibrium utility changes as different parameters change. [25]

Home government's equilibrium utility can be non-monotonic over various parameters. Especially, home government's equilibrium utility is single-peaked over the general public's *responsiveness to hostility.* Therefore, if a government can in the long-run cultivate its people's *responsiveness to hostility* through *nation-building* process, it would try to foster *restrained patriotism*—intermediate level of responsiveness to international controversies. The intuition is simple: a government would hope its people to be responsive enough to controversies, so it does not need to exert huge and very costly effort to convince the foreign opponent to concede. However, if the people are already responsive enough so informativeness of protest is not a concern for the foreign government, then a more responsive general public means more crises for the home government, which is costly. This prediction is consistent with what we observe in real life: for example, in Chinese textbooks and official media, the government often emphasise the importance of 'loving the country' rationally. This is consistent with the prediction of our model.

4.5.1 Baseline Welfare Analysis

In this section, we will look at how the Home Government's payoff depends on the various parameters of the model. We will focus on two parameters, the General Public's *degree of responsiveness*, w_{1G}, and the Nationalist's *degree of responsiveness*, w_{1N}. The complete welfare analysis over all the parameters in the model can be found in the Appendix.

Proposition 2

If $\frac{t_H}{\tau_H} \leq \bar{\mu}$, $U_H(\psi^*)$ *is:*

- *increasing in* w_{1N}*;*

[25] The model is a stylized model and is not designed to describe the total social welfare. An interesting future research direction would be how social welfare, measured under different possible welfare functions, changes as various parameters change. In the appendix, we will briefly discuss how the general public, the nationalist and the foreign government's welfares depend on various parameters

- *decreasing in* w_{1G}*;*

If $\frac{t_H}{\tau_H} > \bar{\mu}$, $U_H(\psi^*)$ *is:*

- *single-peaked over some interior level of* w_{1G}*;*
- *Increasing or single-peaked over* w_{1N}*;*

Therefore, when the relative benefit of hostility ($\frac{t_H}{\tau_H}$) is higher than the threshold of doubt ($\bar{\mu}$), the home government's utility is single-peaked over general public's *responsiveness to hostility*, w_{1G}. This result is robust even if we consider more general convex cost functions of propaganda. Essentially, this result comes from the two effects of w_{1G}: first, higher w_{1G} means the public is more likely to go to the street, and this would make it easier to convince the foreign government to concede. Second, higher w_{1G} and a higher probability of general public protesting also means that there will be more costly crises. Thus having a responsive or irritable general public can be a *double-edged sword*: it will bring costs to both the foreign government and home government.

Home Government's equilibrium utility can also be non-monotonic over other parameters. For example, $U_H(\psi^*)$ is non-monotonic over w_{1N}, nationalist's responsiveness to hostility, if $\bar{\psi}$ is relatively small compared to $\hat{\psi}_R(w_{1N} \to \infty)$. This happens, for example, when w_{1G} is small. Having a very active nationalist fraction as noisemakers is not always a good thing; without an also active general public, the home government would find it hard to convince the opponent to concede.

4.5.2 Nation-building and *Restrained Patriotism*

It has been widely known that the coherent nations we observe today are very recent phenomena. According to Alesina and Reich (2013), 'In 1860 French was still a foreign language to half of all French children' and 'In 1860 at most 10% of the Italian population spoke what would become the Italian language'. In the long run, the state can invest in systematic and gradual education and indoctrination that affects people's language, believes, identity, and preference.

Therefore, in the long run, it has some considerable flexibility over choosing how people will respond to hostility and diplomatic issues. By emphasising the importance of things like national pride and the importance of national interest, the government can affect how the public values the importance of going to the street. If in the long run, some parameters in the model can be changed by the home government, how would it change it?

Ideally, the home government will hope to make $C(\psi) = 0$ for any ψ. However this may not be feasible: there is a limited amount of hostility promotion the government can do, and any amount higher than that may be extremely costly or just impossible. Even with complete deregulation over the nationalist's activities, and putting all the available media resource on covering the issue in a provocative way, essentially the government needs the people to be responsive enough over propaganda and go to the street.

If it is always at least somehow costly to generate ψ, what would be the optimal level of w_{1G}? From previous analysis, we know that some intermediate range of w_{1G} is optimal. Therefore, the government hopes its people to have 'restrained patriotism'. They hope them to care about national interest and go to the street if the event is serious, so there will be protests, and this can be used as hard evidence for the need of getting concessions. However, the government does not hope its people to not care too much over these issues. Otherwise, they are very sensitive and easy to be angry and protesting, which is very costly for the home government. This is consistent with the policies of governments in real life: for example, in Chinese textbooks and official media, there are often discussions about 'loving China rationally'. [26]

Similarly, the government may hope its nationalist to have intermediate responsiveness to international controversies. Systematic education and indoctrination can potentially also affect w_{1N}, although probably in less degree: people usually self-select into being nationalists. However other government policies and regulations can still affect w_{1N}. For example, the government can set various regulations over the activities of nationalist groups.

5 Which Regimes Benefit most from Encouraging Protests? Role of Media and Political Fragility

In this section, we will try to answer the following questions: will every type of regime benefit at the same degree from the ability to generate propaganda and protests? If not, which kind of regimes would benefit from this technology of generating hostility? More specifically, this paper look at how media freedom and political fragility affect regimes benefit from the technology of generating hostility and protests.

[26] **Wang, Yankun and Ye Su.**"Rational Emotion of Loving China is needed in the path Of Rejuvenation of China (Chinese)", People.cn. http://theory.people.com.cn/n1/2016/0818/c40531-28646272.html

More precisely, in this paper media freedom is treated as the level of media capture by the home government. With less media capture, there will be better information over protests that the opponent will be able to observe. This can be rationalized in classical models of media capture like Besley and Prat (2006). Political fragility here is modelled as the probability home government can successfully suppress a protest even without receiving a concession.

To explore these questions, I made a few extensions to the baseline model.

First, I consider that the foreign government can have better information than just observing whether there is a protest. The better the information foreign government has, the harder would be for the home government to manipulate protests to generate concessions. Related to the issue of media freedom, a freer media in the home country would make it for the foreign government to have better knowledge about a protest. This means stable democracies like US or UK will be hard to obtain concessions from this mechanism. However, this does not mean that completely totalitarian countries like North Korea would benefit most from this mechanism. Countries like North Korea may be hard to generate concessions.

Second, if the home government can likely suppress a protest even without even receiving the concession of a foreign government, this would reduce the incentive for the foreign government to make concessions. Protests from the general public can be costly for various reasons, and governments usually have the incentive to suppress them or calm them done. Therefore, if the foreign government believe the home government can successfully suppress a protest with high probability, it would have less incentive to make concessions. This would increase the threshold of the foreign government and make it harder for the foreign government to make concessions. If this probability becomes high enough, foreign government will never concede even it knows for sure that the general public is protesting. Related the probability of successfully suppressing a protest to political fragility, this means at least some fragility is needed for the foreign government to be willing to help.

Third, I consider the possibility of home government generating a 'fake' protest if no one is actually protesting. We show this ability to generate 'fake' protests is not always a blessing: if the foreign government always expects to see a protest, it would be very hard to convince her to concede. Therefore, countries like North Korea where generating a 'fake' protest is easy, and no free media can be relied on the identify them may have a severe problem convincing their opponent to concede.

From those extensions, we can better answer the questions previously asked: neither stable democracies nor completely totalitarian regimes can benefit most from this mechanism. On the contrary, countries with intermediately level of media freedom and at least some political fragility may benefit from this mechanism, especially those with strong nationalism mood and historical grievance.

5.1 Additional Information for the Foreign Government

In the canonical model, we assumed a simple signal structure: the foreign government can observe whether there is protest happening but not from which group. Now we will relax this assumption.

We still assume the foreign government can observe whether there is protest happening. Moreover, if there is a protest, it can now observe an additional signal, s, distributed in the following way:

We assume

$$f(s|a_G = P, a_N = P) = \begin{cases} \lambda_H \exp(-\lambda_H s), \text{if} s \geq 0 \\ 0, \quad \text{if} s < 0 \end{cases}$$

$$f(s|a_G = NP, a_N = P) = \begin{cases} \lambda_L \exp(-\lambda_L s), \text{if} s \geq 0 \\ 0, \quad \text{if} s < 0 \end{cases}$$

We assume $0 < \lambda_H < \lambda_L$, and thus $E(s|a_G = P, a_N = P) > E(s|a_G = NP, a_N = P)$. We can think of s as the number of people protesting in the street.

In expectation, if the public is protesting, the number of people protesting on the street should be higher than the number of people protesting when there are only nationalists protesting. When no one is protesting, thus surely there are no people in the street. When at least one group is protesting, then the higher the number of people appearing in the street, the more likely the protest comes from the general public.

These assumptions essentially mean: when no one is protesting, the foreign government can be perfectly sure about that. When there is at least one group protesting, the government will be surer that the general public is angry and protesting, the higher is the realisation of s.

Then the posterior of the foreign government that the general public is protesting, when observing a protest with signal realization s, is:

$$\begin{aligned}Prob(a_N = P|\psi, P, s) =& \frac{H_G(\psi)f(s|a_N = P, a_G = P)}{H_G(\psi)f(s|a_N = P, a_G = P) + [H_N(\psi) - H_G(\psi)]f(s|a_N = P, a_G = NP)} \\ =& \frac{1}{1 + \frac{H_N(\psi)-H_G(\psi)}{H_G(\psi)} \frac{f(s|a_N=P,a_G=NP)}{f(s|a_N=P,a_G=P)}}\end{aligned}$$

Then the foreign government will only concede iff $Prob(a_N = P|\psi, P, s) \geq \bar{\mu}$ Therefore, there exists a cutoff level $\hat{s}(\psi)$ such that

$$a_F^*(s) = \begin{cases} C, & \text{if} s \geq \hat{s}(\psi) \\ NC, & \text{if} s < \hat{s}(\psi) \end{cases} \tag{1}$$

It can be shown easily that $\hat{s}(\psi)$ decreases over $\frac{H_G(\psi)}{H_N(\psi)}$. That just means that the surer the foreign government thinks the general public is protesting, the more tolerate it will be over a low level of s.

We can now characterize the optimal solution with noisy signal.

Lemma 2

1) For any ψ, there exist a cut-off level $\bar{s}$, such that F will only concede iff $s \geq \bar{s}$.

$$s(\bar{\psi}) = \begin{cases} \log([(\frac{1-\bar{\mu}}{\bar{\mu}})(\frac{\lambda_H}{\lambda_L})(\frac{H_G(\psi)}{H_N(\psi)-H_G(\psi)})]^{\frac{-1}{\lambda_L-\lambda_H}}), & if \quad (\frac{H_N(\psi)-H_G(\psi)}{H_G(\psi)}) \geq (\frac{1-\bar{\mu}}{\bar{\mu}})(\frac{\lambda_H}{\lambda_L}) \\ 0, & otherwise \end{cases}$$

2) Home Government's utility function:

- *If* $(\frac{H_N(\psi)-H_G(\psi)}{H_G(\psi)}) > (\frac{1-\bar{\mu}}{\bar{\mu}})(\frac{\lambda_H}{\lambda_L})$,

  $$U_H(\psi) = H_G(\psi)\{B[\frac{H_G(\psi)}{H_N(\psi)-H_G(\psi)}]^{\frac{\lambda_H}{\lambda_L-\lambda_H}} - A\} + u_H(NP, NC),$$

  in which
  $A \equiv u_H(NP, C) - u_H(NP, NC)$,
  and $B \equiv \{[(\frac{1-\bar{\mu}}{\bar{\mu}})(\frac{\lambda_H}{\lambda_L})]^{\frac{\lambda_H}{\lambda_L-\lambda_H}}[U_H(P, C) - U_H(P, NC)]$

$+ [(\frac{1-\bar{\mu}}{\bar{\mu}})(\frac{\lambda_H}{\lambda_L})]^{\frac{\lambda_L}{\lambda_L - \lambda_H}} [U_H(NP, C) - U_H(NP, NC)]\},$
$A, B > 0$

- *If* $(\frac{H_N(\psi) - H_G(\psi)}{H_G(\psi)}) \leq (\frac{1-\bar{\mu}}{\bar{\mu}})(\frac{\lambda_H}{\lambda_L})$,

$$U_H(\psi) = H_G(\psi) u_H(P, C) + [H_N(\psi) - H_G(\psi)] u_H(NP, C) + [1 - H_N(\psi)] u_H(NP, NC).$$

It can be shown that $U_H(\psi)$ under any ψ such that $(\frac{H_N(\psi) - H_G(\psi)}{H_G(\psi)}) > (\frac{1-\bar{\mu}}{\bar{\mu}})(\frac{\lambda_H}{\lambda_L})$ will be strictly dominated by $U_H(\psi)$ under ψ that $(\frac{H_N(\psi) - H_G(\psi)}{H_G(\psi)}) = (\frac{1-\bar{\mu}}{\bar{\mu}})(\frac{\lambda_H}{\lambda_L})$, which means at optimal the home government chooses a level of ψ such that $(\frac{H_G(\psi)}{H_N(\psi) - H_G(\psi)})$ is so high that the number of people s is obsolete as a signal.

Corollary 2

1) The optimal solution to the home government's optimization problem with noisy signal is also the solution to the following optimization problem:

$$\max_{\psi \in [0, \bar{\psi}]} H_G(\psi) u_H(P, C) + [H_N(\psi) - H_G(\psi)] u_H(NP, C)$$
$$+ [1 - H_N(\psi)] u_H(NP, NC)$$

$$s.t.\ \mu(\psi, P) = \frac{H_G(\psi)}{H_N(\psi)} \geq \frac{1}{1 + \frac{\lambda_H}{\lambda_L} \frac{1-\bar{\mu}}{\bar{\mu}}}$$

2) Denote The optimal value to this optimization problem as $u_H(\psi^{**})$. Then $u_H(\psi^{**})$ is non-increasing over $\frac{\lambda_L}{\lambda_H}$

3) $\lim_{\frac{\lambda_L}{\lambda_H} \to \infty} \psi^{**} = 0$

So it turns out that at least in the case of exponential distributions over people protesting on the strict, it is optimal for the home government to choose a level of ψ such that s becomes useless: home government would make the fact there is someone protesting so informative, so for any s, the opponent will always make concession.

Here the effect of a more precise signal is essentially increasing the threshold of doubt of the opponent. More specifically, the higher is $\frac{\lambda_L}{\lambda_H}$, the easier it would be to distinguish two types of protest from each other by looking at the amount of people on the street. Therefore, the foreign government would be more demanding and has a higher level of threshold of doubt for it to concede whenever observing a protest.

A prediction from this section would be free and informative media in the home country would harm the home government in this issue, because it limits the ability for the home government to affect the inference of the opponent. If we think that media freedom affect the quality of the signal, then countries with free media like UK would be hard to benefit from this mechanism.

Another prediction would be that if the home government is in a country with a very free and informative media, the optimal level of hostility it will choose will be very small. This also means the probability of protest and the probability of crisis would also be small. This implies that probabilities of disputes between stable democratic countries would be small and this could be thought as a natural extension to the concept of 'democratic peace': there are not only less wars and conflicts, but also less disputes and anti-foreign protests between democratic countries. However the reason for this phenomenon comes from democratic countries usually have free media instead of their representative governments.

5.2 Ex-post Incentive and Ability to Suppress

We have shown in the last session that free media could hurt home government's ability to manipulate protests. Does this mean totalitarian regimes like North Korea would be the best country to use this approach?

The answer to this question is probably no. One reason could be North Korea can easily crush every protest even without receiving a concession from the opponent, and it will have every incentive to suppress it. Knowing that the opponent will not be willing to giving a concession, knowing that North Korea will not be forced to escalate if there is a protest and no concession is given to it. In this session, we will formalise this intuition.

It is frequently observed that many protests, even massive ones, are suppressed by governments. In our environment, one fundamental reason for the opponent to concede is that if the protestors' don't see concessions the home government may be replaced or toppled, which is also bad for the opponent as well. However, given that protests are threats to the regime's survival, the regime would have every incentive to suppress it if possible.

So assume that if the opponent chooses not to concede, the protest by only nationalist

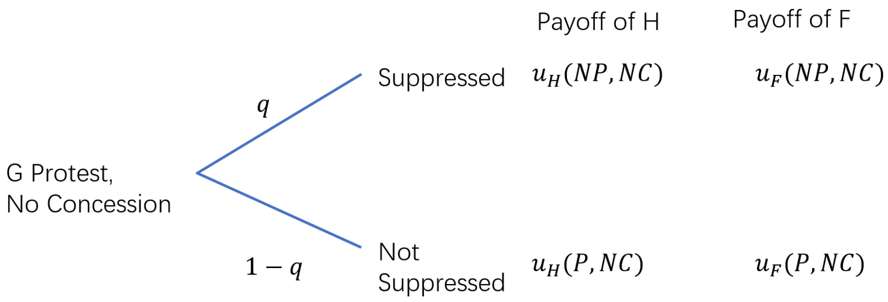

Figure 9: General Public Protests but No Concession is Made

is suppressed with probability 1[27], while the protest attended by the general public is suppressed with probability $q \in [0, 1]$. We assume that when the general public's protest is suppressed, it is equivalent to the general public not protesting to both governments.

We'll look at the threshold of doubt, $\bar{\mu}$, as a function of q, and for notation simplicity we defined $\bar{\mu}$ in the following way: $\bar{\mu}$ is defined as the level of posterior such that the foreign government will concede iff $\mu \geq \bar{\mu}$, and now $\bar{\mu}$ can be greater than 1. $\bar{\mu}$ greater than 1 just means the foreign government will not concede when observing a protest.

So we can show that:

Proposition 3

1) $\bar{\mu} = \frac{u_F(NP,NC) - u_F(P,NC)}{[u_F(P,C) - (1-q)u_F(P,NC) - qu_F(NP,NC)] + [u_F(NP,NC) - u_F(P,NC)]}$*;*

2) the threshold of doubt $\bar{\mu}$ is weakly increasing over q;

3) there exist a cut-level $\bar{q}$ such that $\bar{\mu} > 1$ iff $q > \bar{q}$

The logic is simple: if the home government is more likely to suppress the protest anyway even without concession, then the foreign government's benefit of making concession is smaller. So the foreign government would have to be more certain that the general public is protesting, for her to be willing to concede. In the extreme case that the home government can suppress the protest by himself very easily even without the opponent's concession, the opponent will have no incentive to give concessions, even if he is perfectly sure that the general public is protesting. Another prediction from this result would be that if one government or regime has stabilised its control over the country, the opponent should be more demanding regarding the level of evidence to grant a concession.

So if a regime is more totalitarian and is less immune to street protesters, it would

[27] Although any probability between $[0, 1]$ will do

increase the threshold of doubt for the foreign government. This would reduce the utility of the home government. Moreover, if the threshold of doubt is high enough, the optimal level of hostility it will choose will be minimal.

5.3 Ex-post Ability to Bias the Signal

Another reason why North Korea would not be able to make use of this mechanism could be that ex-post, then it can always generate some 'fake' protests, and the opponent cannot distinguish the 'fake' protests and real protests. But if North Korea is willing to generate fake protests ex-post, this will harm the informativeness of protest. In the extreme case that it will always generate protests and the opponent cannot distinguish the difference between the two, then the opponent will not make any concession after observing a protest.

When facing nationalist protests, the foreign governments are often suspicious whether the home governments have designed fake protests by themselves just to get more concessions. Ex-post, the home government does have the incentive to generate fake protests if there has been no protest because protest brings concession. For example, the government can send their own people or hire people to go to the street as if they are angry and protesting.

The new timeline is as follows:

Period 1: The home government chooses the level of hostility, ψ; the level of ψ is public knowledge to all the players.

Period 2: The nature chooses the level of θ. θ is private knowledge to the home government, the nationalist and the general public, but not the foreign government. The nationalist and the general public then decide simultaneously whether to protest or not. The actions of the nationalist and the general public are observable to themselves and the home government, but not to the foreign government. [28]

Period 3: A binary signal, s, $s \in \{P, NP\}$ is generated and observed to only players in country 1. s is generated in such a way:

$$s = \begin{cases} P, & \text{if } a_N = P \vee a_G = P \\ NP, & \text{otherwise} \end{cases}$$

[28] It is not essential to assume that θ is observable to the home government. What is essential here is that the home government can observe *who* is protesting.

Home government can then take a decision $b_H \in \{T, NT\}$, whether to tamper with the signal s.

Period 4: The signal $\tilde{s}$ is generated according to s and b_H in the following way:

$$\tilde{s} = \begin{cases} P, & \text{if} s = P \\ P \quad \text{with prob} \quad p, & \text{if} s = NP \wedge b_H = T \\ NP \quad \text{with prob} \quad 1-p, & \text{if } s = NP \wedge b_H = T \\ NP, & \text{if} s = NP \wedge b_H = NT \end{cases}$$

$\tilde{s}$ is revealed to every player and the foreign government then decides whether to concede or not after observing the realization of signal $\tilde{s}$.

Period 5: the state and the pay-offs of all players are realised and publicly revealed.

Here we consider a case that after the nationalist and the public make the decision of protesting or not, and before the foreign government observes the protest outcome, the government can choose whether to arrange a fake protest. Suppose the foreign can distinguish a real protest and a fake protest with probability $1-p$. Then a higher p would reduce the informativeness of protest as a signal.

Proposition 4 shows that sometimes home government's payoff could be maximised over an intermediate level of p:

Proposition 4:

1) If $\frac{t_H}{\tau_H} \geq \bar{\mu} \vee \bar{\psi} \geq \frac{\frac{\tilde{c}}{1-\bar{\mu}} - w_0}{w_{1G}}$, $U_H(\psi^*)$ *is increasing over* p

2) If $\frac{t_H}{\tau_H} \geq \bar{\mu}$ & $\bar{\psi} < \frac{\frac{\tilde{c}}{1-\bar{\mu}} - w_0}{w_{1G}}$, $U_H(\psi^*)$ *is single-peaked over some interior level of* p, p^*;

A lower p would allow the home government to generate more protests and thus more concessions. However, a lower p would make protest less informative and requires a higher level of hostility to make protests convincing to the opponent. If this is higher than the maximal possible hostility, the home government would be forced to choose a very low level of hostility.

Therefore, if p is very high, i.e., the ability to generate fake protest without being identified is too power, the home government could actually benefit if it can decrease p. This happens when $\bar{\psi} < \frac{\frac{\tilde{c}}{1-\bar{\mu}} - w_0}{w_{1G}}$, which is likely to be true if: 1)the threshold of doubt, $\bar{\mu}$, is high; 2)the public's responsiveness to hostility, w_{1G}, is low; 3) People's cost of protesting, c, is high; 4) people's initial weight on the dispute, w_0, is low; the maximal possible hostility,

$\bar{\psi}$, is low.

In totalitarian countries like North Korea, as we discussed in the last subsection, μ could be very high due to its ability to suppress protests easily.

Moreover, for those protests to convince the foreign opponent to concede, they have to be *independent* instead of *state-sponsored*. Participating in *state-sponsored* protests usually will not bring any risk to individuals, and potential benefits. However those orchestrated protests will not put any pressure over the home government, and the home government will not have the incentive to give concessions. *Independent* protests can potentially force the home government to escalate, and thus could prompt the foreign government to concede. However, those *Independent* protests can easily become anti-regime and would bear very high risks for individuals in totalitarian countries. Therefore we would expect people's cost to participate in those independent protests, c, is high, and their initial weight on this issue,w_0 is low. In summary, people's potential low willingness to participate in those independent protests, and the inability to distinguish real versus fake protests make totalitarian countries very hard to convince the opponent to give in and concede.

In later sections this paper will further discuss the difference between *Independent* and *state-sponsored* protests, especially the different roles they serve for governments.

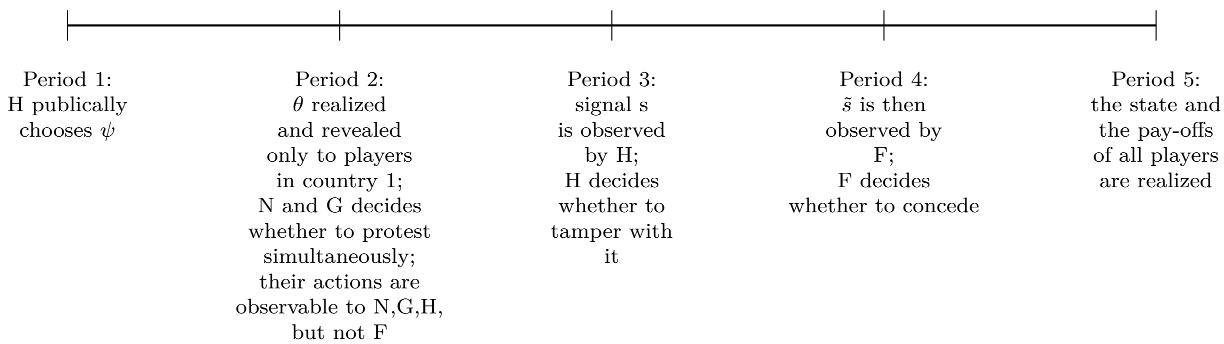

Figure 10: Timeline: when Home Government can generate fake protests

Related to media freedom, high media freedom may reduce p, as real rather than orchestrated protests may get more coverage from independent media. Also, free media may want to reveal the fake protest to get more readership for its services. This can be rationalized by classical media capture models such as Besley and Prat (2006)

So to make the signal to be informative enough and not completely useless, at least some level of media freedom is needed to retain credibility.

5.4 Comparative Politics: Who would benefit Most from Encouraging Protests?

After considering the previous extensions, now we can answer the previous question, i.e., which regimes benefit most from the technology of manipulating protests.

Stable democratic countries, like the UK and the US, have high levels of media freedom and the rule of law and thus may not be able to benefit much from this technology of generating more animosity. Free media will leave not much space for the home government to affect the opponent's inference and cost of generating hostility may be too high.

Completely totalitarian regimes like North Korea may also not be able to benefit much from this technology of generating more animosity. State control over the society is too strong, so it is unlikely to be forced into escalation. Also, there is no free media at all, so the government can easily produce fake protests ex-post and the opponent cannot distinguish the fake protests and real protests.

Countries with intermediate level of media freedom and regimes that are not too strong may be the best candidate to benefit from this technology. They have the necessary levels of media freedom to keep the signal informative enough, and also they are to some extent fragile to angry protesters. Among those countries, those with strong nationalism mood and colonial history may be in even better positions to encourage protests to receive concessions. People will be responsive to the government's promotion of hostility, so it is not too costly to generate protests that are informative enough. Good candidates for this set of countries include China, Turkey and so on. That can possibly explain why the Chinese government can often generate concession by encouraging protests.

6 Further Extensions and Discussions

In this section, we will consider various further extensions to the canonical model that will relax various assumptions of the canonical model. We show that many conclusions of the baseline model are robust to changes of some assumptions of the canonical model.

We also considered whether it is possible to apply the model we have developed under International Relation contexts to other situations. Currently, there are two cases that we find that may be particularly fitting to our model: 1) foreign aid and domestic reform; 2) anti-terrorism aid and anti-terrorism efforts. We explained in the incoming sessions why these two cases may fit with our model, and we show in both cases, the incentive to help actually generate adverse incentive that makes the socially inefficient states happening more often.

6.1 Protest of Nationalists also Costly

We also make another substantial assumption that both home government and foreign government do not care intrinsically about the protest of nationalists. That assumption can be substantially relaxed. We show in this section that if nationalist's protest also costly, it is either the case that the protest of nationalist is very costly that the results are trivial, or the protest of the nationalist is not so costly, and hence all the previous results remain qualitatively the same.

We first looked at the optimal response of the foreign government. We still assume that $u_F(a_G = P, a_N, \psi, a_F = C) \geq u_F(a_G = P, a_N, \psi, a_F = NC)$, so regardless of the action of the nationalist, the foreign government would prefer conceding if the general public is protesting.

We can also safely assume that $u_F(a_G = NP, a_N = NP, \psi, a_F = C) < u_F(a_G = NP, a_N = NP, \psi, a_F = NC)$. This assumption means that if no one is protesting, the foreign government won't concede for sure.

Now consider two cases:

Case1 $u_F(a_G = NP, a_N = P, \psi, a_F = C) \geq u_F(a_G = P, a_N = P, \psi, a_F = NC)$

Case2 $u_F(a_G = NP, a_N = P, \psi, a_F = C) < u_F(a_G = P, a_N = P, \psi, a_F = NC)$

In Case 1, the cost of nationalist protest is also high, hence regardless of who is protesting, the foreign government will always concede when there is a protest. Then informativeness is never a problem. Then $\psi^* = \psi^I$

In Case 2, the cost of nationalist protest is low, then the foreign government will not concede if she is sure only nationalists are protesting. In this case, when the foreign government observes a protest, it will concede if and only if it thinks the general public participates in this protest with high probability. Otherwise, it will not concede. Therefore, there exists a 'threshold of doubt', μ, the same as the canonical model.

Assuming Case 2, we can now write down the optimal decision of the home government. We can safely assume that a protest of nationalist is also costly for the home government: for any a_G, a_H, a_F, if $a_N = P$, $u_H(a_G, a_N = P, a_H = \psi, a_F) < u_H(a_G, a_N = NP, a_H = \psi, a_F)$.

Now the home government's optimization problem:

$$\max_{\psi \in [0,\bar{\psi}]} H_G(\psi) u_H(a_G = P, a_N = P, C) + [H_N(\psi) - H_G(\psi)] u_H(a_G = NP, a_N = P, C) \\ + [1 - H_N(\psi)] u_H(a_G = NP, a_N = NP, NC)$$

$$s.t.\ \mu(\psi, P) = \frac{H_G(\psi)}{H_N(\psi)} \geq \bar{\mu}$$

The objective function is equivalent to:

$$\max_{\psi \in [0,\bar{\psi}]} H_G(\psi)[u_H(a_G = P, a_N = P, C) - u_H(a_G = NP, a_N = P, C)] \\ + H_N(\psi)[u_H(a_G = NP, a_N = P, C) - u_H(a_G = NP, a_N = NP, NC)] \\ + u_H(a_G = NP, a_N = NP, NC)$$

Define $t_H \equiv u_H(a_G = NP, a_N = P, C) - u_H(a_G = NP, a_N = NP, NC)$ and $\tau_H \equiv u_H(a_G = NP, a_N = P, C) - u_H(a_G = P, a_N = P, C)$. Again, we can safely assume τ_H is positive. t_H can be either positive or negative. The difference, $u_H(a_G = NP, a_N = P, C) - u_H(a_G = NP, a_N = NP, NC)$, now depends on two factors:1) the benefit of concession 2)the cost of having nationalists protesting. As long as the cost of nationalist protesting is smaller compared to the benefit of getting a concession, then t_H is also positive. Then all the previous results will carry through.

If t_H is negative, then the result is trivial. There is no benefit but only benefit of increasing hostility, and thus the optimal level of ψ would be zero. $\psi^* = \psi^I = 0$.

So in summary, as long as the protest of nationalists is not so costly, our main results carry through even when a protest of nationalists does affect the payoff functions of home government and foreign government intrinsically.

6.2 Alternative Distribution of Crisis

In the canonical model, we have assumed the seriousness of the event, θ is uniformly distributed. One possible concern could be that some important conclusions of the model are specific the assumption of uniform distribution.

Here we will show that two important patterns are to some extent robust to the distribution of θ: first, we will show here that the fact the informativeness of a protest, $\frac{H_G(\psi)}{H_N(\psi)}$, is first decreasing and then increasing is not specific to the assumption of uniform distribution. Second, we will also show that the objective function has a single optimal solution under general assumptions.

Assume θ is distributed with a distribution, $F(\theta)$, with the following properties:

- $F(\theta)$ is independent of ψ
- $F(\theta)$ is non-degenerate continuous distribution. Denote $f(\theta)$ as the pdf of the distribution
- $F(\theta)$ is second-order continuously differentiable

Therefore, we have

$$\frac{H_G(\psi)}{H_N(\psi)} = \frac{Prob(\theta \geq \hat{\theta}_G)}{Prob(\theta \geq \hat{\theta}_N)} = \frac{Prob(\theta \geq \frac{c}{w_0+w_{1G}\psi})}{Prob(\theta \geq \frac{c}{w_0+w_{1N}\psi})} = \frac{1-F(\frac{c}{w_0+w_{1G}\psi})}{1-F(\frac{c}{w_0+w_{1N}\psi})}$$

6.2.1 U-shape Informativeness Curve

$$\begin{aligned}
\frac{\partial \frac{H_G(\psi)}{H_N(\psi)}}{\partial \psi} \geq 0 \quad & \textit{iff} \quad \frac{f(\frac{c}{w_0+w_{1G}\psi})\frac{cw_{1G}}{(w_0+w_{1G}\psi)^2}}{1-F(\frac{c}{w_0+w_{1G}\psi})} / \frac{f(\frac{c}{w_0+w_{1N}\psi})\frac{cw_{1N}}{(w_0+w_{1N}\psi)^2}}{1-F(\frac{c}{w_0+w_{1N}\psi})} \geq 0 \\
& \textit{iff} \quad \left[h(\frac{c}{w_0+w_{1G}\psi})\frac{cw_{1G}}{(w_0+w_{1G}\psi)^2}\right] / \left[h(\frac{c}{w_0+w_{1N}\psi})\frac{cw_{1N}}{(w_0+w_{1N}\psi)^2}\right] \geq 0
\end{aligned}$$

, in which $h(\theta) \equiv \frac{f(\theta)}{1-F(\theta)}$ is the hazard rate function.

$\frac{H_G(\psi)}{H_N(\psi)}$ to be first decreasing and then increasing is equivalent to the following condition:

(Quasi-convexity condition) there exist a $\hat{\psi}$ such that:

$$sgn(\frac{\partial \frac{H_G(\psi)}{H_N(\psi)}}{\partial \psi}) = \begin{cases} 1, & \text{if } \psi > \hat{\psi} \\ 0, & \text{if } \psi = \hat{\psi} \\ -1, & \text{if } \psi < \hat{\psi} \end{cases}$$

Two examples that satisfy this condition are Exponential Distribution and Weibull Distribution (with some constraints on parameters).

More generally, here are some sufficient conditions for this to be true:

- $\frac{f(\frac{c}{w_0+w_{1G}\psi})\frac{cw_{1G}}{(w_0+w_{1G}\psi)^2}}{1-F(\frac{c}{w_0+w_{1G}\psi})} / \frac{f(\frac{c}{w_0+w_{1N}\psi})\frac{cw_{1N}}{(w_0+w_{1N}\psi)^2}}{1-F(\frac{c}{w_0+w_{1N}\psi})}$ is increasing over ψ
- $\frac{f(\frac{c}{w_0+w_{1G}\psi})\frac{cw_{1G}}{(w_0+w_{1G}\psi)^2}}{1-F(\frac{c}{w_0+w_{1G}\psi})} / \frac{f(\frac{c}{w_0+w_{1N}\psi})\frac{cw_{1N}}{(w_0+w_{1N}\psi)^2}}{1-F(\frac{c}{w_0+w_{1N}\psi})}|_{\psi=0} < 1$
- $\frac{f(\frac{c}{w_0+w_{1G}\psi})\frac{cw_{1G}}{(w_0+w_{1G}\psi)^2}}{1-F(\frac{c}{w_0+w_{1G}\psi})} / \frac{f(\frac{c}{w_0+w_{1N}\psi})\frac{cw_{1N}}{(w_0+w_{1N}\psi)^2}}{1-F(\frac{c}{w_0+w_{1N}\psi})}|_{\psi=\bar{\psi}} > 1$

The condition above essentially requires the hazard rate to be 'flat' enough and bounded.

6.2.2 Single-peaked Objective Function

For the objective function to be single-peaked, we need the propensity density function to be flat enough and bounded. Intuitively, this requires the distribution to be not too volatile such that the effect of $\frac{cw_{1G}}{(w_0+w_{1G}\psi)^2} / \frac{cw_{1N}}{(w_0+w_{1N}\psi)^2}$ would dominates in determining $\frac{H_G(\psi)}{H_N(\psi)}$. Again, exponential distribution can work as well, although it requires the parameter λ to be not too small. With some constraints on parameters, Weibull distribution also satisfies this condition.

6.3 Other Roles of Anti-Foreign Protests

We have shown that in countries where regimes' control over information and society are too strong, it would be hard for anti-foreign protests to generate concessions. However, we still often observe those protests in autocracies, for example, anti-US demonstrations in North Korea and Syria. Moreover, even in countries where anti-foreign or patriotic can potentially generate concessions from foreign governments, those protests can still potentially serve more roles than just bargaining chips for international disputes. In this subsection, we will look the other roles for anti-foreign protests.

One potential role for anti-foreign protests would be domestic legitimacy issues and shifting domestic political focus. Many regimes' political legitimacies are based on acting as the defender of national interests. Therefore they may have the incentive to use those protests to emphasise that they care about national interests. Also, those protests can shift people's focus on other domestic issues. Those domestic issues can be very important reasons for the home government to encourage protests, however as long as those protests will generate pressure for the home government to escalate, most predictions of our model will still carry through.

Another role for anti-foreign protests would be for domestic mobilisation purpose. Those foreign rivals provide useful 'straw man' that allow regimes to mobilise its domestic audience for various reasons. Again, our model points out the international concerns regime need to take into account when deciding whether to mobilise protests.

Moreover, as discussed before, anti-foreign protests can be *independent*, *state-organized*, or a mixture of two with various weights. *State-organized* protests are usually pro-government and under government controls, thus less threatening to the government. They could potentially boost government legitimacy or other domestic agendas but are unlikely to convince foreign opponents to concede. On the contrary, *independent* protests can be dangerous to regimes but are useful signals that could convince foreign governments the necessity to concede to avoid costly escalation. Our model mainly applies to cases where protests are *independent*. In cases where protests are mixtures of both, our model would predict that the higher the level of regime participation, the harder it would be for those protests to generate concessions from foreign governments.

6.4 Other applications: Two other stories and a general framework

Although this paper considers a model of how the sender tries to affect the receiver's inference problem of the receiver under the setting of crisis diplomacy, some conclusion of the model can be extended to other settings.

One possible environment this model may apply to would be foreign aid and institutional reform. There is vast literature over the effects of foreign aids on the donated country. Especially, various studies have discussed how foreign aid can either promote or hamper necessary domestic reforms. This paper can easily fit into this discussion: suppose in some underdeveloped country, some serious disasters may have significant impacts over itself and other countries, so some developed country may have the incentive to provide foreign aid to help relieve the disaster. However, providing foreign aid is costly. There may be some other disasters that are also severe but not destabilising enough, so the donor would not want to provide foreign aid for that issue. However, the problem is that the donor may find it hard to distinguish different kinds of disasters perfectly. Then one implication from the previous model would be that the foreign aid would actually reduce the incentive to implement crucial institutional reforms and may actually make the humanitarian crisis perpetual.

Another environment this model applies to would be anti-terrorism. Some country fighting remote terrorist groups may have the incentive to support its local partner (can be country or local groups) with aids, especially if the terrorism level has been very high and destabilising the local community. By providing aids, that country may stop the area to be further radicalized. However, providing aid is very costly, and the country would not provide aid to some non-destabilizing events. Again, the donor country may not perfectly observe the type of terrorism the local group is facing. Then one implication from the previous model would be that the ex-post socially benefit action of providing aid in the local area may reduce the local group's incentive to fight terrorism and thus actually destabilising the area.[29]

More general, this paper can be applied to the standard sender-receiver environment, where the state can be endogenously affected by the sender. The receiver may want to take some action that is preferred by the sender if the receiver is sure enough about some states happening. However the receiver cannot perfectly observe the states of the world, and this gives the sender incentive to change the state to affect the receiver's inference problem. Sometimes this could create an incentive for the sender to spend effort on increasing socially inefficient states.

[29] One newspaper report described Pakistan's incentive to keep terrorists active: "Over the years successive governments - both military and civilian - have developed the fine art of using the begging bowl as an extortion racket. Zia played the Soviets off against the US after the invasion of Afghanistan. The US wrote the bigger cheque. Musharraf adroitly squeezed money out of Bush for his war on terror, capturing a few al Qaeda operatives, whilst at the same time letting the Taliban off the hook (proving that the Pakistani Army is the largest mercenary force in the world). Nor have civilian administrations been any better. We have become particularly good of late of warning the international community of dire consequences of militants getting hold of our nukes if they don,t pay up. Kerry-Lugar Zindabad!". **Fulton, George.** "Cry Wolf". The Express Tribune, August 18 2010. https://tribune.com.pk/story/40220/cry-wolf/

7 Conclusion

We have considered a model exploring how a government can manipulate anti-foreign protests of its people to get more concessions from foreign countries. By promoting a high level of hostility through propaganda, that government generates more protests and thus more concessions. However, this comes with more crises, which are costly for the home government. Moreover, the government has to choose a level of hostility that makes a protest informative enough about a real crisis. These factors jointly determine the optimal and feasible level of animosity the home government would choose and the equilibrium probabilities of protests and crises.

We characterised the comparative statics and found some interesting patterns that are consistent with real-life examples. We also show that due to the cost of generating animosity, governments may hope their people to be responsive but not too responsive over international disputes. Moreover, government's *ex-post* ability to tamper with protest can sometimes harm its *ex-ante* welfare. Therefore, regimes that are in some sense 'weak' can be those that can benefit more from the mechanism we mentioned here: in our context, this means countries with intermediate levels of media freedom and at least some fragility to political protests.

Our paper suggests some possible directions for future research: First, it would be interesting to see what will happen in a repeated environment where reputations and dynamic interactions kick in. Second, combining the model with models of political competitions in democratic environment may generate additional insights over how protests will have effects over democratic countries' domestic politics and international affairs. Third, a crisis bargaining model with domestic politics and protests on both sides may generate additional insights over whether promoting hostility can act as 'defence' over the opponent's investment in hostility. That model may also help us understand better why inefficient deadlocks like World War I may occur in equilibrium. Last, this paper focuses on protest as 'street diplomacy' instead of formal diplomacy. A richer model may be needed to explore the interaction of 'street diplomacy' and 'formal diplomacy'.

References

Abreu, Dilip and Faruk Gul, "Bargaining and reputation," *Econometrica*, 2000, *68* (1), 85–117.

Alesina, Alberto and Bryony Reich, "Nation building," 2013.

Alonso, Ricardo and Odilon Câmara, "Persuading voters," *American Economic Review*, 2016, *106* (11), 3590–3605.

_ **and** _ , "On the value of persuasion by experts," *Journal of Economic Theory*, 2018, *174*, 103–123.

Battaglini, Marco, "Multiple referrals and multidimensional cheap talk," *Econometrica*, 2002, *70* (4), 1379–1401.

Bergemann, Dirk and Stephen Morris, "Bayes correlated equilibrium and the comparison of information structures in games," *Theoretical Economics*, 2016, *11* (2), 487–522.

Besley, Timothy and Andrea Prat, "Handcuffs for the grabbing hand? Media capture and government accountability," *American economic review*, 2006, *96* (3), 720–736.

Best, James and Daniel Quigley, "Persuasion for the long run," 2017.

Che, Yeon-Koo, Wouter Dessein, and Navin Kartik, "Pandering to persuade," *American Economic Review*, 2013, *103* (1), 47–79.

Ciorciari, John D and Jessica Chen Weiss, "Nationalist Protests, Government Responses, and the Risk of Escalation in Interstate Disputes," *Security Studies*, 2016, *25* (3), 546–583.

Crawford, Vincent P, "A theory of disagreement in bargaining," *Econometrica: Journal of the Econometric Society*, 1982, pp. 607–637.

_ **and Joel Sobel**, "Strategic information transmission," *Econometrica: Journal of the Econometric Society*, 1982, pp. 1431–1451.

Debs, Alexandre and Jessica Chen Weiss, "Circumstances, domestic audiences, and reputational incentives in international crisis bargaining," *Journal of Conflict Resolution*, 2016, *60* (3), 403–433.

Ellison, Glenn and Sara Fisher Ellison, "Search, obfuscation, and price elasticities on the internet," *Econometrica*, 2009, *77* (2), 427–452.

Ely, Jeffrey C, "Beeps," *American Economic Review*, 2017, *107* (1), 31–53.

Eyerman, Joe and Robert A Hart Jr, "An empirical test of the audience cost proposition: Democracy speaks louder than words," *Journal of Conflict Resolution*, 1996, *40* (4), 597–616.

Fearon, James D, "Domestic political audiences and the escalation of international disputes," *American political science review*, 1994, *88* (3), 577–592.

_ , "Rationalist explanations for war," *International organization*, 1995, *49* (3), 379–414.

Gentzkow, Matthew and Emir Kamenica, "Costly persuasion," *American Economic Review*, 2014, *104* (5), 457–62.

Green, Jerry R and Nancy L Stokey, "A two-person game of information transmission," *Journal of economic theory*, 2007, *135* (1), 90–104.

Holmström, Bengt, "Managerial incentive problems: A dynamic perspective," *The review of Economic studies*, 1999, *66* (1), 169–182.

Kambe, Shinsuke, "Bargaining with imperfect commitment," *Games and Economic Behavior*, 1999, *28* (2), 217–237.

Kamenica, Emir and Matthew Gentzkow, "Bayesian persuasion," *American Economic Review*, 2011, *101* (6), 2590–2615.

Lipnowski, Elliot and Doron Ravid, "Cheap talk with transparent motives," 2017.

Mansfield, Edward D, Helen V Milner, and B Peter Rosendorff, "Why democracies cooperate more: Electoral control and international trade agreements," *International Organization*, 2002, *56* (3), 477–513.

Muthoo, Abhinay, "A bargaining model based on the commitment tactic," *Journal of Economic theory*, 1996, *69* (1), 134–152.

Osborne, Martin J and Ariel Rubinstein, *Bargaining and markets*, Academic press, 1990.

Passarelli, Francesco and Guido Tabellini, "Emotions and political unrest," *Journal of Political Economy*, 2017, *125* (3), 903–946.

Putnam, Robert D, "Diplomacy and domestic politics: the logic of two-level games," *International organization*, 1988, *42* (3), 427–460.

Schelling, Thomas C, "The strategy of conflict," *Harvard University*, 1960, *1980*, 13.

Schultz, Kenneth A, "Do democratic institutions constrain or inform? Contrasting two institutional perspectives on democracy and war," *International Organization*, 1999, *53* (2), 233–266.

_ **and Barry R Weingast**, "The democratic advantage: institutional foundations of financial power in international competition," *International Organization*, 2003, *57* (1), 3–42.

Slantchev, Branislav L, "Politicians, the media, and domestic audience costs," *International Studies Quarterly*, 2006, *50* (2), 445–477.

Smith, Alastair, "International crises and domestic politics," *American Political Science Review*, 1998, *92* (3), 623–638.

Weiss, Jessica Chen, "Authoritarian signaling, mass audiences, and nationalist protest in China," *International Organization*, 2013, *67* (1), 1–35.

_ , *Powerful patriots: nationalist protest in China's foreign relations*, Oxford University Press, 2014.

Appendix

The foreign government's optimal strategy

The foreign government's optimal choice is simple:

based on its information(level of ψ, the public signal s) in Period 3, denote its posterior belief that the general public of home country is protesting as $\mu(\psi, s)$. Then by Bayes' Rule,

$$\begin{aligned}\mu(\psi, s=S) \equiv & Prob(a_G = P|\psi, s=S) = \frac{Prob(a_G = P, s=S|\psi)}{Prob(s=S|\psi)} \\ &= \begin{cases} \frac{Prob(a_G=P, s=NP|\psi)}{Prob(s=NP|\psi)} = 0, \text{if} \quad S = NP \\ \frac{Prob(a_G=P|\psi)}{Prob(a_G=P|\psi)+Prob(a_G=NP, a_N=P|\psi)} \quad \text{,if} \quad S = P \end{cases}\end{aligned}$$

The previous result comes from the signal structure of s:

$$Prob(s = P|a_G, a_N, \psi, \theta) = \begin{cases} 1, \text{ if } a_G = P \vee a_N = P \\ 0, \qquad \text{otherwise} \end{cases}$$

The optimization problem of the foreign government:

$$\max_{a_F \in \{C, NC\}} I(a_F = C)\{\mu(\psi, s)u_F(P, C) + (1 - \mu(\psi, s))u_F(NP, C)\} + \\ I(a_F = NC)\{\mu(\psi, s)u_F(P, NC) + (1 - \mu(\psi, s))u_F(NP, NC)\}$$

The optimal solution is simple:

$$a_F^* = \begin{cases} NC, \text{ if } \{\mu(\psi, s)u_F(P, NC) + (1 - \mu(\psi, s))u_F(NP, NC)\} > \{\mu(\psi, s)u_F(P, C) + (1 - \mu(\psi, s))u_F(NP, C)\} \\ C, \qquad \text{otherwise} \end{cases}$$

This just means that:

$$a_F^* = \begin{cases} NC, \text{ if } \mu(\psi, s) < \bar{\mu} \\ C, \text{ if } \mu(\psi, s) \geq \bar{\mu} \end{cases}$$

in which $\bar{\mu} \in (0, 1)$, *the threshold of doubt* for foreign government, is defined as the level of μ such that $\bar{\mu}[u_F(P, C) - u_F(P, NC)] = (1 - \bar{\mu})[u_F(NP, NC) - u_F(NP, C)]$

From the signal structure, $s = NP$ means neither group is protesting so the general public is not protesting for sure.

Therefore, when $s = NP$, $a_F^* = NC$, since $\mu(\psi, NP) = 0 < \bar{\mu}$.

When $s = P$, $a_F^* = C$ if and only if $\mu(\psi, P) \geq \bar{\mu}$.

The foreign government will never concede if there is no protest. Moreover, it will concede after seeing a protest, if and only if it thinks protest is an informative enough signal.

the Nationalist and the Public's Best Response

In the baseline model, the decision problems for the nationalist and the public are straightforward. Their actions depend only on ψ and θ, and there is no strategic interaction between them.

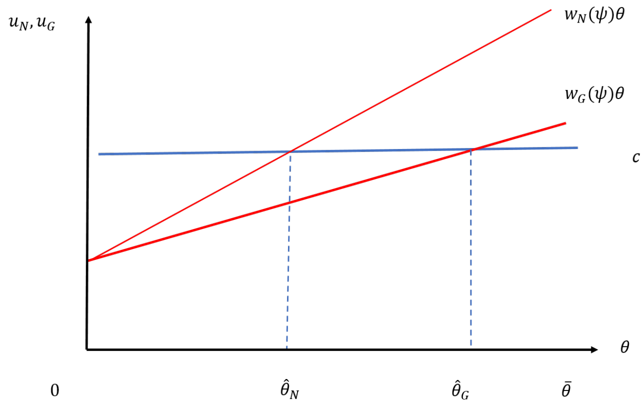

Figure 11: Nationalist and General Public's Best Responses

For i=N,G, the optimization problem is:

$$\max_{a_i \in \{P, NP\}} I(a_i = P)\{w_i(\psi)\theta - c\},$$

in which $w_i(\psi) = w_0 + w_{1i}\psi$.

The optimal strategy of player i, $i = N, G$ is: $a_i^* = P$, iff $\theta \geq \hat{\theta}_i(\psi)$, in which $\hat{\theta}_i(\psi) \equiv \frac{c}{w_0 + w_{1i}\psi}$.

$\hat{\theta}_i$ increases with ψ, and the tuple of both types' cut-off levels, $(\hat{\theta_N}(\psi), \hat{\theta_G}(\psi))$ are function of ψ in the following way:

$$(\hat{\theta_N}(\psi), \hat{\theta_G}(\psi)) = (\frac{c}{w_0 + w_1 N\psi}, \frac{c}{w_0 + w_1 G\psi})$$

Because w_0 is the same among the two groups and $w_{1N} > w_{1G}$, $\hat{\theta_N} \leq \hat{\theta_G}$ for any ψ.

The interim ex-ante expected probability of protest (after ψ is chosen but before θ is realized) for group i is $H_i(\psi) \equiv Prob(\text{Player i protests}) = Prob(\theta \geq \hat{\theta}_i)$, and $H_N(\psi) \geq H_G(\psi)$.

$H_i(\psi)$ decreases with ψ, and the tuple of both types' ex-ante probability of protesting, $(H_N(\psi), H_G(\psi))$ are increasing functions of ψ:

$$(H_N(\psi), H_G(\psi)) = (\frac{\bar{\theta} - \frac{c}{w_0 + w_1 N\psi}}{\bar{\theta}}, \frac{\bar{\theta} - \frac{c}{w_0 + w_1 G\psi}}{\bar{\theta}})$$

From last section we know: $\mu(\psi, P) = \frac{H_G(\psi)}{H_N(\psi)}$.

And $\mu(\psi, P)$ is a function of ψ:

$$\mu(\psi, P) = \frac{\bar{\theta} - \frac{c}{w_0 + w_1 G\psi}}{\bar{\theta} - \frac{c}{w_0 + w_1 N\psi}}$$

We show that there is an inverse-U relationship between the level of animosity, ψ, and the posterior of the foreign government when observing a protest, $\mu(\psi, P)$. This means that for $\bar{\mu}$ large enough, there exists two levels of ψ, $\hat{\psi}_L$ and $\hat{\psi}_R$, such that $\mu(\hat{\psi}_L, P) = \mu(\hat{\psi}_R, P) = \bar{\mu}$. Moreover, $\mu(\psi, P) \geq \bar{\mu}$ if and only if $\psi \in \left[0, \hat{\psi}_L\right] \cup \left[\hat{\psi}_R, \bar{\psi}\right]$

Mathematically, the reason of inverse-U shape we observe here comes from the elasticity of $H_N(\psi)$, $\frac{H_N'(\psi)}{H_N(\psi)}$, single cross the elasticity of $H_G(\psi)$, $\frac{H_G'(\psi)}{H_G(\psi)}$. Roughly speaking, when ψ is small, $H_N(\psi)$ is growing much faster than $H_G(\psi)$, and the levels of $H_N(\psi)$ and $H_G(\psi)$ are both small. Thus $H_N'(\psi)$ is substantially larger than $H_G'(\psi)$, while the difference between $H_N(\psi)$ and $H_G(\psi)$ is small. When ψ becomes large, $H_N(\psi)$ and $H_G(\psi)$ are both growing very slowly so $H_N'(\psi)$ is very near to $H_G'(\psi)$, while the difference between $H_N(\psi)$ and $H_G(\psi)$ are also small but still in larger magnitude.

Intuitively, when the ψ is low so there is not much manipulation of protests, the opponent knows that the nationalist and the public are both unlikely to be on the street, and their differences over protest probabilities are low. Thus when the opponent observes a protest, he knows with high probability both groups are protesting. When ψ is very high, the opponent knows the nationalist is almost always protesting on the street. However, since ψ is very high, the general public is also very likely to be protesting, so in this case the difference over protest probabilities are also low. In the case that ψ is at intermediate level, $H_N(\psi)$ is much larger than $H_G(\psi)$. There would be many protests of only nationalist, and this would make foreign government less sure that general public supports a protest when it observe one.

The home government's general optimization problem

The home government's most general optimization problem is:

Def A1: General Optimization Problem:

$$\max_{\psi \in [0, \bar{\psi}]} \sum_{a_G} \sum_{a_N} \sum_{S} \sum_{a_F} [Prob(a_F^* = a_F | \psi, S) Prob(s = S | \psi, a_G, a_N) Prob(a_G^* = a_G, a_N^* = a_N | \psi)$$
$$u_H(a_G^*, a_F^*)]$$

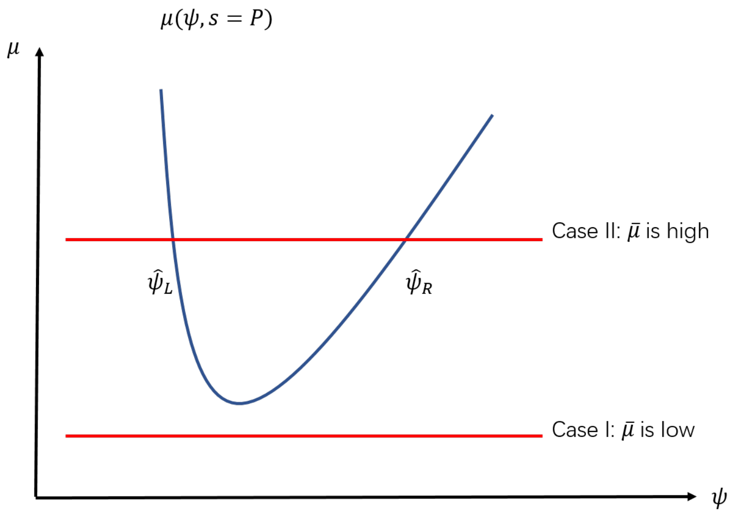

Figure 12: Foreign Government's posterior when observing Protest, as function of ψ

$a_i, i = N, G, F$ is the optimal strategy of player i defined in previous subsections.

However, this problem can be substantially simplified: In the canonical model $Prob(s = S|\psi, a_G, a_N)$ and $Prob(a_F^* = a_F|\psi, S)$ are both degenerate.

In the baseline model the general optimization problem becomes:

$$\max_{\psi \in [0,\bar{\psi}]} H_G(\psi)u_H(P, a_F^*(\psi, P)) + [H_N(\psi) - H_G(\psi)]u_H(NP, a_F^*(\psi, P))$$
$$+ [1 - H_N(\psi)]u_H(NP, a_F^*(\psi, P))$$

, in which $a_F^*(\psi, P) = C$ iff $\mu(\psi, P) \geq \bar{\mu}$

It can be show that it is never optimal to choose some level of ψ such that $a_F^*(\psi, P) = NC$ (We know already that $a_F^*(\psi, NP) = NC$)

Def A2: Simplified Optimization Problem:

$$\max_{\psi \in [0,\bar{\psi}]} H_G(\psi)u_H(P, C) + [H_N(\psi) - H_G(\psi)]u_H(NP, C)$$
$$+ [1 - H_N(\psi)]u_H(NP, NC)$$

$$s.t.\ \mu(\psi, P) = \frac{H_G(\psi)}{H_N(\psi)} \geq \bar{\mu}$$

Lemma A1:

1 Denote ψ^S as the optimal solution to the Simplified Optimization Problem.ψ^S exists.

2 Denote ψ^* as the optimal solution to the General Optimization Problem. ψ^* exists. $\psi^S = \psi^*$.

3 $a_F^*(\psi^*, s = P) = C$

Proof

1) Suppose H chooses a level $\psi > 0$ such that $a_F^*(\psi, s = P) = NC$.
Then H's payoff is
$H_G(\psi)u_H(P, NC) + [H_N(\psi) - H_G(\psi)]u_H(NP, NC) + [1 - H_N(\psi)]u_H(NP, NC)$.
If deviating to $\psi = 0$, $\mu(\psi = 0, s = P) = 1 \geq \bar{\mu}$ for any $\bar{\mu}$
Then H's payoff is
$H_G(0)u_H(P, C) + [H_N(0) - H_G(0)]u_H(NP, C) + [1 - H_N(0)]u_H(NP, NC) >$
$H_G(0)u_H(P, NC) + [H_N(0) - H_G(0)]u_H(NP, NC) + [1 - H_N(0)]u_H(NP, NC) >$
$H_G(\psi)u_H(P, NC) + [H_N(\psi) - H_G(\psi)]u_H(NP, NC) + [1 - H_N(\psi)]u_H(NP, NC)$ for any $\psi > 0$

2)For the simplified Optimization Problem:
The objective function is continuous. The set of ψ that satisfies the constraint is non-empty and is a finite union of disjoint compact intervals. Therefore, the optimal solution and optimal value to this question both exists.

3)It is obvious that $\psi^S = \psi^*$. □

The intuition is simple: The potential benefits of higher ψ are: 1) More concessions; 2) keeping the signal(Protest) informative enough. At the same time, the cost of higher ψ is higher probability of bad states.

When $a_F^*(\psi, P) = NC$ and ψ has no benefit, the home government can always improve by reducing ψ which reduces the cost. Therefore a level of ψ such that $a_F^*(\psi, P) = NC$ can never be optimal.

Therefore we'll just look at the simplified problem from now on.

Therefore we can solve for ψ^* just by solving the simplified optimization problem.

Def A3 *Define $\tilde{c} \equiv c/\bar{\theta}$*

Now let's look at the posterior of Foreign government when observing a protest, $\mu(\psi, P)$:

We will make the following assumption over

Lemma A2:

1 $\mu(\psi,P)$ is quasi-convex; it is decreasing on $[0,\frac{w_0}{\sqrt{w_{1N}w_{1G}}}\sqrt{\frac{w_0-\tilde{c}}{w_0}}]$ and increasing on $[\frac{w_0}{\sqrt{w_{1N}w_{1G}}}\sqrt{\frac{w_0-\tilde{c}}{w_0}},\bar{\psi}]$

2 Define $\hat{\psi}\equiv\frac{w_0}{\sqrt{w_{1N}w_{1G}}}\sqrt{\frac{\bar{\theta}w_0-c}{\bar{\theta}w_0}}$. Then $\mu(\hat{\psi},P)=\min_{\psi\in[0,\bar{\psi}]}\mu(\psi,P)=\frac{w_{1G}}{w_{1N}}\left(\frac{1+\sqrt{\frac{w_{1N}}{w_{1G}}}\sqrt{\frac{w_0-\tilde{c}}{w_0}}}{1+\sqrt{\frac{w_{1G}}{w_{1N}}}\sqrt{\frac{w_0-\tilde{c}}{w_0}}}\right)^2$.

2.1 If $\bar{\psi}>\frac{w_0}{\sqrt{w_{1N}w_{1G}}}\sqrt{\frac{w_0-\tilde{c}}{w_0}}$,

2.1.1 If $\bar{\mu}\in\left(0,\mu(\hat{\psi},P)\right]$, then for any $\psi\in[0,\bar{\psi}]$, $\mu(\psi,P)\geq\bar{\mu}$;

2.1.2 If $\bar{\mu}\in(\mu(\hat{\psi},P),\mu(\bar{\psi},P))$, there exist two levels of ψ, $\hat{\psi}_L(\bar{\mu})$ and $\hat{\psi}_R(\bar{\mu})$, $0<\hat{\psi}_L(\bar{\mu})<\hat{\psi}_R(\bar{\mu})<\bar{\psi}$, such that $\mu(\hat{\psi}_L(\bar{\mu}),P)=\mu(\hat{\psi}_R(\bar{\mu}),P)=\bar{\mu}$. Moreover, for any $\psi[0,\bar{\psi}],\mu(\psi,P)\geq\bar{\mu}$ iff $\psi\in[0,\hat{\psi}_L(\bar{\mu})]\cup[\hat{\psi}_R(\bar{\mu}),\bar{\psi}]$

2.1.3 For any $\bar{\mu}\in\left(\mu(\bar{\psi},P),1\right)$, there exists a level of ψ, $\hat{\psi}_L(\bar{\mu})$, $0<\hat{\psi}_L(\bar{\mu})<\bar{\psi}$, such that $\mu(\hat{\psi}_L(\bar{\mu}),P)=\bar{\mu}$. Moreover, for any $\psi[0,\bar{\psi}],\mu(\psi,P)\geq\bar{\mu}$ iff $\psi\in[0,\hat{\psi}_L(\bar{\mu})]$

2.1.4 $\hat{\psi}_L(\bar{\mu})=\frac{-\left[w_{1N}(w_0-\frac{\tilde{c}}{1-\bar{\mu}})+w_{1G}(w_0+\frac{\tilde{c}\bar{\mu}}{1-\bar{\mu}})\right]-\sqrt{(\left[w_{1N}(w_0-\frac{\tilde{c}}{1-\bar{\mu}})+w_{1G}(w_0+\frac{\tilde{c}\bar{\mu}}{1-\bar{\mu}})\right])^2-4w_{1G}w_{1N}w_0(w_0-\tilde{c})}}{2w_{1G}w_{1N}}$

$\hat{\psi}_R(\bar{\mu})=\frac{-\left[w_{1N}(w_0-\frac{\tilde{c}}{1-\bar{\mu}})+w_{1G}(w_0+\frac{\tilde{c}\bar{\mu}}{1-\bar{\mu}})\right]+\sqrt{(\left[w_{1N}(w_0-\frac{\tilde{c}}{1-\bar{\mu}})+w_{1G}(w_0+\frac{\tilde{c}\bar{\mu}}{1-\bar{\mu}})\right])^2-4w_{1G}w_{1N}w_0(w_0-\tilde{c})}}{2w_{1G}w_{1N}}$

2.2 If $\bar{\psi}\leq\frac{w_0}{\sqrt{w_{1N}w_{1G}}}\sqrt{\frac{w_0-\tilde{c}}{w_0}}$,

2.2.1 For any $\bar{\mu}\in\left(0,\mu(\bar{\psi},P)\right]$, then for any $\psi\in[0,\bar{\psi}]$, $\mu(\psi,P)\geq\bar{\mu}$;

2.2.2 For any $\bar{\mu}\in\left(\mu(\bar{\psi},P),1\right)$, there exists a level of ψ, $\hat{\psi}_L(\bar{\mu})$, $0<\hat{\psi}_L(\bar{\mu})<\bar{\psi}$, such that $\mu(\hat{\psi}_L(\bar{\mu}),P)=\bar{\mu}$. Moreover, for any $\psi[0,\bar{\psi}],\mu(\psi,P)\geq\bar{\mu}$ iff $\psi\in[0,\hat{\psi}_L(\bar{\mu})]$

2.2.3 $\hat{\psi}_L(\bar{\mu})=\frac{-\left[w_{1N}(w_0-\frac{\tilde{c}}{1-\bar{\mu}})+w_{1G}(w_0+\frac{\tilde{c}\bar{\mu}}{1-\bar{\mu}})\right]-\sqrt{(\left[w_{1N}(w_0-\frac{\tilde{c}}{1-\bar{\mu}})+w_{1G}(w_0+\frac{\tilde{c}\bar{\mu}}{1-\bar{\mu}})\right])^2-4w_{1G}w_{1N}w_0(w_0-\tilde{c})}}{2w_{1G}w_{1N}}$

Proof 1) $\mu(\psi,P)=\frac{H_G(\psi)}{H_N(\psi)}$

then $\frac{\partial\mu(\psi,P)}{\partial\psi}=\frac{H_G(\psi)}{H_N(\psi)}\left[\frac{\frac{\partial H_G(\psi)}{\partial\psi}}{H_G(\psi)}-\frac{\frac{\partial H_N(\psi)}{\partial\psi}}{H_N(\psi)}\right]$

$=\frac{H_G(\psi)}{H_N(\psi)}\left[\frac{\tilde{c}w_{1G}}{(w_0+w_{1G}\psi)^2}\frac{w_0+w_{1G}\psi}{(w_0-\tilde{c})+w_{1G}\psi}-\frac{\tilde{c}w_{1N}}{(w_0+w_{1N}\psi)^2}\frac{w_0+w_{1N}\psi}{(w_0-\tilde{c})+w_{1N}\psi}\right]$

$=\frac{H_G(\psi)}{H_N(\psi)}\left[\frac{\tilde{c}w_{1G}}{(w_0+w_{1G}\psi)[(w_0-\tilde{c})+w_{1G}\psi]}-\frac{\tilde{c}w_{1N}}{(w_0+w_{1N}\psi)[(w_0-\tilde{c})+w_{1N}\psi]}\right]$

Therefore, $\frac{\partial\mu(\psi,P)}{\partial\psi}\geq0$ *iff* $\psi\geq\frac{w_0}{\sqrt{w_{1N}w_{1G}}}\sqrt{\frac{\bar{\theta}w_0-c}{\theta w_0}}$

2) $\hat{\psi}_L(\bar{\mu})$ and $\hat{\psi}_R(\bar{\mu})$ are two solutions(if exist) to $\mu(\psi,P)=\bar{\mu}$

This is equivalent to: $w_{1G}w_{1N}\psi^2+\left[w_{1N}(w_0-\frac{\tilde{c}}{1-\bar{\mu}})+w_{1G}(w_0+\frac{\tilde{c}\bar{\mu}}{1-\bar{\mu}})\right]\psi+w_0(w_0-\tilde{c})=0$

Therefore $\hat{\psi}_L(\bar{\mu}) = \frac{-\left[w_{1N}(w_0-\frac{\tilde{c}}{1-\bar{\mu}})+w_{1G}(w_0+\frac{\tilde{c}\bar{\mu}}{1-\bar{\mu}})\right]-\sqrt{(\left[w_{1N}(w_0-\frac{\tilde{c}}{1-\bar{\mu}})+w_{1G}(w_0+\frac{\tilde{c}\bar{\mu}}{1-\bar{\mu}})\right])^2-4w_{1G}w_{1N}w_0(w_0-\tilde{c})}}{2w_{1G}w_{1N}}$

and $\hat{\psi}_R(\bar{\mu}) = \frac{-\left[w_{1N}(w_0-\frac{\tilde{c}}{1-\bar{\mu}})+w_{1G}(w_0+\frac{\tilde{c}\bar{\mu}}{1-\bar{\mu}})\right]+\sqrt{(\left[w_{1N}(w_0-\frac{\tilde{c}}{1-\bar{\mu}})+w_{1G}(w_0+\frac{\tilde{c}\bar{\mu}}{1-\bar{\mu}})\right])^2-4w_{1G}w_{1N}w_0(w_0-\tilde{c})}}{2w_{1G}w_{1N}}$

The other conclusions of *Lemma 3* can be proved obviously.

□

Now let's look at the objective function:

The expected utility of home government for choosing some level of ψ:

$$\begin{aligned} OB(\psi) \equiv & H_G(\psi)[u_H(P,C) - u_H(NP,C)] + H_N(\psi)[u_H(NP,C) - u_H(NP,NC)] \\ & + u_H(NP,NC) \end{aligned}$$

Now we can give some basic characterizations of the objective function:

Def A4 $t_H \equiv u_H(NP,C) - u_H(NP,NC)$;
$\tau_H \equiv u_H(NP,C) - u_H(P,C)$;
$\frac{t_H}{\tau_H} \equiv \frac{u_H(NP,C)-u_H(NP,NC)}{u_H(NP,C)-u_H(P,C)}$

Lemma A3

1 *The objective function $OB(\psi)$ is quasi-concave on the interval $[0,\bar{\psi}]$. There exist a unique maximum point for the objective function in that region. Denote ψ^I as the maximum point of the objective function on $[0,\bar{\psi}]$, i.e., $\psi^I = argmax_{[0,\psi]}OB(\psi)$. Then:*

1.1 If $\frac{t_H}{\tau_H} \le \frac{w_{1G}}{w_{1N}}$, the objective function is decreasing on $[0,\bar{\psi}]$ and $\psi^I = 0$

1.2 If $\frac{t_H}{\tau_H} \ge \frac{w_{1G}}{w_{1N}}(\frac{w_0+w_{1N}\bar{\psi}}{w_0+w_{1G}\bar{\psi}})^2$, the objective function is decreasing on $[0,\bar{\psi}]$ and $\psi^I = \bar{\psi}$

1.3 If $\frac{t_H}{\tau_H} \in (\frac{w_{1G}}{w_{1N}}, \frac{w_{1G}}{w_{1N}}(\frac{w_0+w_{1N}\bar{\psi}}{w_0+w_{1G}\bar{\psi}})^2)$, $\psi^I = \frac{w_0}{\sqrt{w_{1N}w_{1G}}}\frac{\sqrt{\frac{w_{1N}}{w_{1G}}}\sqrt{\frac{t_H}{\tau_H}}-1}{\sqrt{\frac{w_{1N}}{w_{1G}}}-\sqrt{\frac{t_H}{\tau_H}}}$, the objection function is increasing over ψ on $[0, \frac{w_0}{\sqrt{w_{1N}w_{1G}}}\frac{\sqrt{\frac{w_{1N}}{w_{1G}}}\sqrt{\frac{t_H}{\tau_H}}-1}{\sqrt{\frac{w_{1N}}{w_{1G}}}-\sqrt{\frac{t_H}{\tau_H}}}]$ and decreasing over ψ on $[\frac{w_0}{\sqrt{w_{1N}w_{1G}}}\frac{\sqrt{\frac{w_{1N}}{w_{1G}}}\sqrt{\frac{t_H}{\tau_H}}-1}{\sqrt{\frac{w_{1N}}{w_{1G}}}-\sqrt{\frac{t_H}{\tau_H}}}, \bar{\psi}]$. Moreover, $\psi^I = \frac{w_0}{\sqrt{w_{1N}w_{1G}}}\frac{\sqrt{\frac{w_{1N}}{w_{1G}}}\sqrt{\frac{t_H}{\tau_H}}-1}{\sqrt{\frac{w_{1N}}{w_{1G}}}-\sqrt{\frac{t_H}{\tau_H}}}$ is strictly increasing over $\frac{t_H}{\tau_H}$

2 *the utility of the home government at $\hat{\psi}_L(\bar{\mu})$, $u_H(\hat{\psi}_L(\bar{\mu}))$, is greater or equal to the its utility at $\hat{\psi}_R(\bar{\mu})$,$u_H(\hat{\psi}_R(\bar{\mu}))$,if and only if $\frac{t_H}{\tau_H} \le \bar{\mu}$*

Proof

1) $\frac{\partial OB(\psi)}{\partial\psi} = \frac{\partial H_N(\psi)}{\partial\psi}t_H - \frac{\partial H_G(\psi)}{\partial\psi}\tau_H = \frac{cw_{1N}}{(w_0+w_{1N}\psi)^2}t_H - \frac{cw_{1G}}{(w_0+w_{1G}\psi)^2}\tau_H$
$= \frac{cw_{1G}}{(w_0+w_{1N}\psi)^2}\left[\frac{t_H}{\tau_H}\frac{w_{1N}}{w_{1G}} - \frac{(w_0+w_{1N}\psi)^2}{(w_0+w_{1G}\psi)^2}\right]$,
which is decreasing over ψ
Therefore, either 1) $\frac{\partial OB(\psi)}{\partial\psi} > 0$ for any $\psi \in [0,\bar{\psi}]$; 2) $\frac{\partial OB(\psi)}{\partial\psi} < 0$ for any $\psi \in [0,\bar{\psi}]$; or 3) $\frac{\partial OB(\psi)}{\partial\psi}$ is first positive and then negative.

2)

$$\begin{aligned}
&u_H(\hat{\psi}_R(\bar{\mu}))\\
&= H_G(\hat{\psi}_R(\bar{\mu}))[u_H(P,C) - u_H(NP,C)] + H_N(\hat{\psi}_R(\bar{\mu}))[u_H(NP,C) - u_H(NP,NC)] + u_H(NP,NC)\\
&= H_N(\hat{\psi}_R(\bar{\mu}))\tau_H[\frac{t_H}{\tau_H} - \frac{H_G(\hat{\psi}_R(\bar{\mu}))}{H_N(\hat{\psi}_R(\bar{\mu}))}] + u_H(NP,NC)\\
&= H_N(\hat{\psi}_R(\bar{\mu}))\tau_H[\frac{t_H}{\tau_H} - \bar{\mu}] + u_H(NP,NC)
\end{aligned}$$

Because $\mu(\hat{\psi}_R(\bar{\mu}),P) = \frac{H_G(\hat{\psi}_R(\bar{\mu}))}{H_N(\hat{\psi}_R(\bar{\mu}))} = \bar{\mu}$
Similarly,

$$u_H(\hat{\psi}_L(\bar{\mu})) = H_N(\hat{\psi}_L(\bar{\mu}))\tau_H[\frac{t_H}{\tau_H} - \bar{\mu}] + u_H(NP,NC)$$

Because $H_N(\hat{\psi}_R(\bar{\mu})) > H_N(\hat{\psi}_L(\bar{\mu}))$,
$u_H(\hat{\psi}_R(\bar{\mu})) > u_H(\hat{\psi}_L(\bar{\mu}))$ iff $\frac{t_H}{\tau_H} - \bar{\mu} > 0$

□

So if $\frac{t_H}{\tau_H}$ is small enough, ψ^I will be constrained to the left boundary 0; Then after $\frac{t_H}{\tau_H}$ becomes large enough, ψ^I starts to strictly increase over $\frac{t_H}{\tau_H}$, until $\frac{t_H}{\tau_H}$ becomes so high that ψ^I hits the right boundary $\bar{\psi}$.

The second part of the lemma says that if the home government is forced to choose between $\hat{\psi}_L(\bar{\mu})$ and $\hat{\psi}_R(\bar{\mu})$, it will prefer $\hat{\psi}_L(\bar{\mu})$ iff $\frac{t_H}{\tau_H} \leq \bar{\mu}$, i.e., the relative benefit/cost ratio is smaller than the threshold of doubt.

Now we are ready to characterize the optimal feasible level of ψ:

Proposition A1:

1. Suppose $\bar{\psi} \geq \frac{w_0}{\sqrt{w_{1N}w_{1G}}}\sqrt{\frac{w_0-\bar{c}}{w_0}}$. Then:

1.1 If $\frac{t_H}{\tau_H} \leq \frac{w_{1G}}{w_{1N}}$, then $\psi^* = \psi^I = 0$ for any $\bar{\mu} \in (0,1)$;

1.2 If $\frac{t_H}{\tau_H} \geq \frac{w_{1G}}{w_{1N}}(\frac{w_0+w_{1N}\bar{\psi}}{w_0+w_{1G}\bar{\psi}})^2$*, then*

- *If* $\bar{\mu} \in \left(0, \mu(\bar{\psi}, P)\right]$*, then* $\psi^*{=}\psi^I = \bar{\mu}$
- *If* $\bar{\mu} \in \left(\mu(\bar{\psi}, P), 1\right)$*, then there exists* $\hat{\psi}_L(\bar{\mu})$*, such that:*
  - $\mu(\hat{\psi}_L(\bar{\mu}), P) = \bar{\mu}$
  - $\psi^* = \mu(\hat{\psi}_L(\bar{\mu}), P)$

1.3 If $\frac{t_H}{\tau_H} \in (\frac{w_{1G}}{w_{1N}}, \frac{w_{1G}}{w_{1N}}(\frac{w_0+w_{1N}\bar{\psi}}{w_0+w_{1G}\bar{\psi}})^2)$ *There exist a cut-off level* $\mu_1 \in (0,1)$*, such that:*

- *If* $\bar{\mu} \in (0, \mu_1]$ *, then* $\psi^*{=}\psi^I$
- *If* $\bar{\mu} \in \left(\mu_1, \mu(\bar{\psi}, P)\right]$*, then there exist* $\hat{\psi}_L(\bar{\mu})$ *and* $\hat{\psi}_R(\bar{\mu})$*, such that:*
  - $\mu(\hat{\psi}_L(\bar{\mu}), P) = \mu(\hat{\psi}_R(\bar{\mu}), P) = \bar{\mu}$
  - $$\psi^* = \begin{cases} \hat{\psi}_L, \textit{if} & \frac{t_H}{\tau_H} \leq \bar{\mu} \\ \hat{\psi}_R, \textit{if} & \frac{t_H}{\tau_H} > \bar{\mu} \end{cases}$$
- *If* $\bar{\mu} \in \left(\mu(\bar{\psi}, P), 1\right)$*, then there exists* $\hat{\psi}_L(\bar{\mu})$*, such that:*
  - $\mu(\hat{\psi}_L(\bar{\mu}), P) = \bar{\mu}$
  - $\psi^* = \hat{\psi}_L(\bar{\mu})$

2. Suppose $\bar{\psi} < \frac{w_0}{\sqrt{w_{1N}w_{1G}}}\sqrt{\frac{w_0-\tilde{c}}{w_0}}$*. Then there exist a cut-off level* $\mu_2 \in (0,1)$*, such that:*

2.1 If $\frac{t_H}{\tau_H} \leq \frac{w_{1G}}{w_{1N}}$*, then* $\psi^*{=}\psi^I = 0$ *for any* $\bar{\mu} \in (0,1)$*;*

2.2 If $\frac{t_H}{\tau_H} \geq \frac{w_{1G}}{w_{1N}}(\frac{w_0+w_{1N}\bar{\psi}}{w_0+w_{1G}\bar{\psi}})^2$*, then*

- *If* $\bar{\mu} \in \left(0, \mu(\bar{\psi}, P)\right]$*, then* $\psi^*{=}\psi^I = \bar{\mu}$
- *If* $\bar{\mu} \in \left(\mu(\bar{\psi}, P), 1\right)$*, then there exists* $\hat{\psi}_L(\bar{\mu})$*, such that:*
  - $\mu(\hat{\psi}_L(\bar{\mu}), P) = \bar{\mu}$
  - $\psi^* = \hat{\psi}_L(\bar{\mu})$

2.3 If $\frac{t_H}{\tau_H} \in (\frac{w_{1G}}{w_{1N}}, \frac{w_{1G}}{w_{1N}}(\frac{w_0+w_{1N}\bar{\psi}}{w_0+w_{1G}\bar{\psi}})^2)$ *There exist a cut-off level* $\mu_2 \in (0,1)$*, such that:*

- *If* $\bar{\mu} \in (0, \mu_2]$ *, then* $\psi^*{=}\psi^I$
- *If* $\bar{\mu} \in (\mu_2, 1)$*, then there exists* $\hat{\psi}_L(\bar{\mu})$*, such that:*
  - $\mu(\hat{\psi}_L(\bar{\mu}), P) = \bar{\mu}$
  - $\psi^* = \hat{\psi}_L(\bar{\mu})$

Characterisation of optimal level of ψ

We'll first look at the marginal effects of changes of the parameters on the ideal point and then how marginal changes of parameters affect $\hat{\psi}_L(\bar{\mu})$ and $\hat{\psi}_L(\bar{\mu})$ respectively.

Lemma A4 *:*

- $\hat{\psi}_L$ *is marginally:*
  - *increasing in* $\bar{\theta}$ *;*
  - *decreasing in* w_{1N}*;*
  - *increasing in* w_{1G}*;*
  - *increasing in* w_0
  - *decreasing of* c*;*
  - *independent of the utility function of the sender;*
  - *decreasing in* $\bar{\mu}$*, the threshold of doubt;*
- *the signs of* $\hat{\psi}_R$ *over the respective parameters are exactly the opposite to the signs of* $\hat{\psi}_L$*.*

Proof For any ψ, it is obvious that $\mu(\psi, P)$ is:

- decreasing in c
- increasing in $\bar{\theta}$
- increasing in w_0
- decreasing in w_{1N}
- increasing in w_{1G}

Also, $\frac{\partial \mu(\psi,P)}{\partial \psi}|_{\psi=\hat{\psi}_L} < 0$ and $\frac{\partial \mu(\psi,P)}{\partial \psi}|_{\psi=\hat{\psi}_R} > 0$

By Implicit Function Theorem the results follow through. □

- **Lemma A5**:

  ψ^I*, is marginally:*

- *independent of* $\bar{\theta}$ *;*
- *if* $\frac{t_H}{\tau_H} \geq 1$*,* ψ^I *is weakly decreasing in* w_{1N}*; if* $\frac{t_H}{\tau_H} < 1$*,* ψ^I *is weakly decreasing in* w_{1N} *if* $\sqrt{\frac{w_{1N}}{w_{1G}}} > \sqrt{\frac{\tau_H}{t_H}} + \sqrt{\frac{\tau_H - t_H}{t_H}}$ *and weakly increasing in* w_{1N} *if* $1 < \sqrt{\frac{w_{1N}}{w_{1G}}} < \sqrt{\frac{\tau_H}{t_H}} + \sqrt{\frac{\tau_H - t_H}{t_H}}$
- *if* $\frac{t_H}{\tau_H} \leq 1$*,* ψ^I *is weakly decreasing in* w_{1G}*; if* $\frac{t_H}{\tau_H} > 1$*,* ψ^I *is weakly decreasing in* w_{1G} *if* $\sqrt{\frac{w_{1N}}{w_{1G}}} > \sqrt{\frac{t_H}{\tau_H}} + \sqrt{\frac{t_H - \tau_H}{\tau_H}}$ *and weakly increasing in* w_{1G} *if* $\sqrt{\frac{w_{1N}}{w_{1G}}} < \sqrt{\frac{t_H}{\tau_H}} + \sqrt{\frac{t_H - \tau_H}{\tau_H}}$
- *increasing in* w_0*;*
- *independent of* c*;*
- *increasing in* $\frac{t_H}{\tau_H}$*, the benefit/cost ratio;*
- *independent of the pay-off function of the foreign government.*

Proof:

$$\psi^I = \min\left\{\max\left\{\frac{w_0}{\sqrt{w_{1N}w_{1G}}}\frac{\sqrt{\frac{w_{1N}}{w_{1G}}}\sqrt{\frac{t_H}{\tau_H}}-1}{\sqrt{\frac{w_{1N}}{w_{1G}}}-\sqrt{\frac{t_H}{\tau_H}}},0\right\},\bar{\psi}\right\}$$

It is obvious that ψ^I is:

- independent over $\bar{\theta}$ and c
- increasing over w_0
- increasing in $\frac{t_H}{\tau_H}$
- independent of the pay-off function of the foreign government

Moreover, If $\sqrt{\frac{w_{1N}}{w_{1G}}} < \max\left\{\sqrt{\frac{t_H}{\tau_H}}, \sqrt{\frac{\tau_H}{t_H}}\right\}$, $\psi^I = 0$. in the interior case,

$$\frac{\partial \psi^I}{\partial \sqrt{w_{1N}}} = -\frac{w_0\sqrt{\frac{t_H}{\tau_H}}}{w_{1N}\sqrt{w_{1G}}(\sqrt{\frac{w_{1N}}{w_{1G}}}-\sqrt{\frac{t_H}{\tau_H}})^2}\left[\left(\sqrt{\frac{w_{1N}}{w_{1G}}}-\sqrt{\frac{\tau_H}{t_H}}\right)^2+\left(1-\frac{\tau_H}{t_H}\right)\right]$$

$$\frac{\partial \psi^I}{\partial \sqrt{w_{1G}}} = -\frac{w_0\sqrt{\frac{t_H}{\tau_H}}}{w_{1G}\sqrt{w_{1N}}(\sqrt{\frac{w_{1N}}{w_{1G}}}-\sqrt{\frac{t_H}{\tau_H}})^2}\left[\left(\sqrt{\frac{w_{1N}}{w_{1G}}}-\sqrt{\frac{t_H}{\tau_H}}\right)^2+\left(1-\frac{t_H}{\tau_H}\right)\right] \qquad \square$$

Now given that we know how ψ^I, $\hat{\psi}_L$ and $\hat{\psi}_R$ change as different parameters change, and how ψ^* depends on ψ^I, $\hat{\psi}_L$ and $\hat{\psi}_R$, we'll look at how ψ^* transits between ψ^O, $\bar{\psi}_L$ and $\bar{\psi}_R$ when parameters change. After then, we can give a complete characterization about how ψ^* changes as different parameters change.

Lemma A6 $\mu(\psi^I, P)$ *is:*

- *continuously decreasing over* $w_{1N} \in (w_{1G}, +\infty)$*, and* $\mu(\psi^I, P) \in (\frac{\theta w_0 - c}{\theta w_0}, 1)$
- *continuously increasing over* $w_{1G} \in (0, w_{1N})$*, and* $\mu(\psi^I, P) \in (\frac{\theta w_0 - c}{\theta w_0}, 1)$
- *continuously increasing over* $w_0 \in \left(c/\bar{\theta}, +\infty\right)$*, and* $\mu(\psi^I, P) \in \left(\frac{w_{1G}}{w_{1N}} \frac{1+\sqrt{\frac{w_{1N}}{w_{1G}}}\left(\frac{\sqrt{\frac{t_H}{\tau_H}}\sqrt{\frac{w_{1N}}{w_{1G}}}-1}{\sqrt{\frac{t_H}{\tau_H}}-\sqrt{\frac{w_{1N}}{w_{1G}}}}\right)}{1+\sqrt{\frac{w_{1G}}{w_{1N}}}\left(\frac{\sqrt{\frac{t_H}{\tau_H}}\sqrt{\frac{w_{1N}}{w_{1G}}}-1}{\sqrt{\frac{t_H}{\tau_H}}-\sqrt{\frac{w_{1N}}{w_{1G}}}}\right)}, 1 \right)$*;*
- *continuously decreasing over* $c \in (0, \bar{\theta} w_0)$*, and* $\mu(\psi^I, P) \in \left(\frac{w_{1G}}{w_{1N}} \frac{1+\sqrt{\frac{w_{1N}}{w_{1G}}}\left(\frac{\sqrt{\frac{t_H}{\tau_H}}\sqrt{\frac{w_{1N}}{w_{1G}}}-1}{\sqrt{\frac{t_H}{\tau_H}}-\sqrt{\frac{w_{1N}}{w_{1G}}}}\right)}{1+\sqrt{\frac{w_{1G}}{w_{1N}}}\left(\frac{\sqrt{\frac{t_H}{\tau_H}}\sqrt{\frac{w_{1N}}{w_{1G}}}-1}{\sqrt{\frac{t_H}{\tau_H}}-\sqrt{\frac{w_{1N}}{w_{1G}}}}\right)}, 1 \right)$*;*
- *continuously increasing over* $\bar{\theta} \in \left(\frac{c}{w_0}, +\infty\right)$*, and* $\mu(\psi^I, P) \in \left(\frac{w_{1G}}{w_{1N}} \frac{1+\sqrt{\frac{w_{1N}}{w_{1G}}}\left(\frac{\sqrt{\frac{t_H}{\tau_H}}\sqrt{\frac{w_{1N}}{w_{1G}}}-1}{\sqrt{\frac{t_H}{\tau_H}}-\sqrt{\frac{w_{1N}}{w_{1G}}}}\right)}{1+\sqrt{\frac{w_{1G}}{w_{1N}}}\left(\frac{\sqrt{\frac{t_H}{\tau_H}}\sqrt{\frac{w_{1N}}{w_{1G}}}-1}{\sqrt{\frac{t_H}{\tau_H}}-\sqrt{\frac{w_{1N}}{w_{1G}}}}\right)}, 1 \right)$*;*
- *continuously decreasing over* $\frac{t_H}{\tau_H}$ *when* $0 \leq \frac{t_H}{\tau_H} < \mu(\hat{\psi}, P)$ *and increase over* $\frac{t_H}{\tau_H}$ *when* $\frac{t_H}{\tau_H} > \mu(\hat{\psi}, P)$*,* $\mu(\psi^O, P) \in \left[\mu(\hat{\psi}, P), 1\right]$

Proof We will show the case when ψ^I is interior. The case when ψ^I is zero or $\bar{\psi}^I$ is trivial.

$\psi^I = \frac{w_0}{\sqrt{w_{1N} w_{1G}}} \frac{\sqrt{\frac{w_{1N}}{w_{1G}}}\sqrt{\frac{t_H}{\tau_H}}-1}{\sqrt{\frac{w_{1N}}{w_{1G}}}-\sqrt{\frac{t_H}{\tau_H}}}$,

Therefore $H_G(\psi^I) = 1 - \frac{\tilde{c}}{w_0 + w_{1G}\psi^I} = 1 - \frac{\tilde{c}}{w_0 + w_{1G}\frac{w_0}{\sqrt{w_{1N}w_{1G}}}\frac{\sqrt{\frac{w_{1N}}{w_{1G}}}\sqrt{\frac{t_H}{\tau_H}}-1}{\sqrt{\frac{w_{1N}}{w_{1G}}}-\sqrt{\frac{t_H}{\tau_H}}}} =$

$1 - \frac{\tilde{c}}{w_0} \frac{\sqrt{\frac{w_{1N}}{w_{1G}}}-\sqrt{\frac{t_H}{\tau_H}}}{\sqrt{\frac{w_{1N}}{w_{1G}}}-\sqrt{\frac{w_{1G}}{w_{1N}}}} = 1 - \frac{\tilde{c}}{w_0} \frac{\sqrt{\frac{w_{1N}}{w_{1G}}}-\sqrt{\frac{t_H}{\tau_H}}}{\sqrt{\frac{w_{1G}}{w_{1N}}}(\frac{w_{1N}}{w_{1G}}-1)}$

and $H_N(\psi^I) = 1 - \frac{\tilde{c}}{w_0 + w_{1N}\psi^I} = 1 - \frac{\tilde{c}}{w_0 + w_{1N}\frac{w_0}{\sqrt{w_{1N}w_{1G}}}\frac{\sqrt{\frac{w_{1N}}{w_{1G}}}\sqrt{\frac{t_H}{\tau_H}}-1}{\sqrt{\frac{w_{1N}}{w_{1G}}}-\sqrt{\frac{t_H}{\tau_H}}}}$

$= 1 - \frac{\tilde{c}}{w_0} \frac{\sqrt{\frac{w_{1N}}{w_{1G}}}-\sqrt{\frac{t_H}{\tau_H}}}{\sqrt{\frac{t_H}{\tau_H}(\frac{w_{1N}}{w_{1G}}-1)}}$

Therefore,

$$\mu(\psi^I) = \frac{H_G(\psi^I)}{H_N(\psi^I)} = \frac{1 - \frac{\tilde{c}}{w_0}\frac{\sqrt{\frac{w_{1N}}{w_{1G}}}-\sqrt{\frac{t_H}{\tau_H}}}{\sqrt{\frac{w_{1G}}{w_{1N}}}(\frac{w_{1N}}{w_{1G}}-1)}}{1 - \frac{\tilde{c}}{w_0}\frac{\sqrt{\frac{w_{1N}}{w_{1G}}}-\sqrt{\frac{t_H}{\tau_H}}}{\sqrt{\frac{t_H}{\tau_H}}(\frac{w_{1N}}{w_{1G}}-1)}}$$

, which is obviously decreasing over $\frac{\tilde{c}}{w_0}$. $\frac{\tilde{c}}{w_0}$ is increasing over c, and decreasing over w_0 and $\bar{\theta}$.

Therefore $\mu(\psi^I)$ is decreasing over c, and increasing over w_0 and $\bar{\theta}$.

For $\frac{t_H}{\tau_H}$:

When $\frac{t_H}{\tau_H} < 1$,

$$\mu(\psi^I) = \frac{H_G(\psi^I)}{H_N(\psi^I)} = \frac{1 - \frac{\tilde{c}}{w_0}\frac{\sqrt{\frac{w_{1N}}{w_{1G}}}-\sqrt{\frac{t_H}{\tau_H}}}{\sqrt{\frac{w_{1G}}{w_{1N}}}(\frac{w_{1N}}{w_{1G}}-1)}}{1 - \frac{\tilde{c}}{w_0}\frac{\sqrt{\frac{w_{1N}}{w_{1G}}}-\sqrt{\frac{t_H}{\tau_H}}}{\sqrt{\frac{t_H}{\tau_H}}(\frac{w_{1N}}{w_{1G}}-1)}} = \frac{1-\sqrt{\frac{t_H}{\tau_H}}\sqrt{\frac{w_{1N}}{w_{1G}}}}{1 - \frac{\tilde{c}}{w_0}\frac{\sqrt{\frac{w_{1N}}{w_{1G}}}-\sqrt{\frac{t_H}{\tau_H}}}{\sqrt{\frac{t_H}{\tau_H}}(\frac{w_{1N}}{w_{1G}}-1)}} + \sqrt{\frac{t_H}{\tau_H}}\sqrt{\frac{w_{1N}}{w_{1G}}}$$

$$\frac{d\mu(\psi^I)}{d\sqrt{\frac{w_{1N}}{w_{1G}}}} = \left[\frac{1-\sqrt{\frac{t_H}{\tau_H}}}{1 - \frac{\tilde{c}}{w_0}\frac{\sqrt{\frac{w_{1N}}{w_{1G}}}-\sqrt{\frac{t_H}{\tau_H}}}{\sqrt{\frac{t_H}{\tau_H}}(\frac{w_{1N}}{w_{1G}}-1)}} + \sqrt{\frac{t_H}{\tau_H}}\right] - \frac{1-\sqrt{\frac{t_H}{\tau_H}}\sqrt{\frac{w_{1N}}{w_{1G}}}}{\left(1 - \frac{\tilde{c}}{w_0}\frac{\sqrt{\frac{w_{1N}}{w_{1G}}}-\sqrt{\frac{t_H}{\tau_H}}}{\sqrt{\frac{t_H}{\tau_H}}(\frac{w_{1N}}{w_{1G}}-1)}\right)^2}\frac{\partial\left(1 - \frac{\tilde{c}}{w_0}\frac{\sqrt{\frac{w_{1N}}{w_{1G}}}-\sqrt{\frac{t_H}{\tau_H}}}{\sqrt{\frac{t_H}{\tau_H}}(\frac{w_{1N}}{w_{1G}}-1)}\right)}{\partial\sqrt{\frac{w_{1N}}{w_{1G}}}} =$$

$$\frac{-\sqrt{\frac{t_H}{\tau_H}}}{H_N(\psi^I)}\left[\frac{\tilde{c}}{w_0}\frac{\sqrt{\frac{\tau_H}{t_H}}}{\frac{w_{1N}}{w_{1G}}-1}\right]\left[[\sqrt{\frac{w_{1N}}{w_{1G}}}-\sqrt{\frac{t_H}{\tau_H}}] - \frac{\frac{\frac{w_{1N}}{w_{1G}}-2\sqrt{\frac{t_H}{\tau_H}}\sqrt{\frac{w_{1N}}{w_{1G}}}+1}{\frac{w_{1N}}{w_{1G}}-1}}{1 - \frac{\tilde{c}}{w_0}\frac{\sqrt{\frac{w_{1N}}{w_{1G}}}-\sqrt{\frac{t_H}{\tau_H}}}{\sqrt{\frac{t_H}{\tau_H}}(\frac{w_{1N}}{w_{1G}}-1)}}\frac{\sqrt{\frac{t_H}{\tau_H}}\sqrt{\frac{w_{1N}}{w_{1G}}}-1}{\sqrt{\frac{t_H}{\tau_H}}}\right]$$

$\left[[\sqrt{\frac{w_{1N}}{w_{1G}}}-\sqrt{\frac{t_H}{\tau_H}}] - \frac{\frac{\frac{w_{1N}}{w_{1G}}-2\sqrt{\frac{t_H}{\tau_H}}\sqrt{\frac{w_{1N}}{w_{1G}}}+1}{\frac{w_{1N}}{w_{1G}}-1}}{1 - \frac{\tilde{c}}{w_0}\frac{\sqrt{\frac{w_{1N}}{w_{1G}}}-\sqrt{\frac{t_H}{\tau_H}}}{\sqrt{\frac{t_H}{\tau_H}}(\frac{w_{1N}}{w_{1G}}-1)}}\frac{\sqrt{\frac{t_H}{\tau_H}}\sqrt{\frac{w_{1N}}{w_{1G}}}-1}{\sqrt{\frac{t_H}{\tau_H}}}\right]$ is decreasing over $\frac{c}{w_0}$, and $\left[[\sqrt{\frac{w_{1N}}{w_{1G}}} - \sqrt{\frac{t_H}{\tau_H}}] - \frac{\frac{\frac{w_{1N}}{w_{1G}} - 2\sqrt{\frac{t_H}{\tau_H}}\sqrt{\frac{w_{1N}}{w_{1G}}}+1}{\frac{w_{1N}}{w_{1G}}-1}}{1-\frac{\tilde{c}}{w_0}\frac{\sqrt{\frac{w_{1N}}{w_{1G}}}-\sqrt{\frac{t_H}{\tau_H}}}{\sqrt{\frac{t_H}{\tau_H}(\frac{w_{1N}}{w_{1G}}-1)}}} \frac{\sqrt{\frac{t_H}{\tau_H}}\sqrt{\frac{w_{1N}}{w_{1G}}}-1}{\sqrt{\frac{t_H}{\tau_H}}} \right] |_{\frac{c}{w_0}=1} = \sqrt{\frac{t_H}{\tau_H}} - \sqrt{\frac{w_{1G}}{w_{1N}}} > 0$

Therefore $\left[[\sqrt{\frac{w_{1N}}{w_{1G}}} - \sqrt{\frac{t_H}{\tau_H}}] - \frac{\frac{\frac{w_{1N}}{w_{1G}} - 2\sqrt{\frac{t_H}{\tau_H}}\sqrt{\frac{w_{1N}}{w_{1G}}}+1}{\frac{w_{1N}}{w_{1G}}-1}}{1-\frac{\tilde{c}}{w_0}\frac{\sqrt{\frac{w_{1N}}{w_{1G}}}-\sqrt{\frac{t_H}{\tau_H}}}{\sqrt{\frac{t_H}{\tau_H}(\frac{w_{1N}}{w_{1G}}-1)}}} \frac{\sqrt{\frac{t_H}{\tau_H}}\sqrt{\frac{w_{1N}}{w_{1G}}}-1}{\sqrt{\frac{t_H}{\tau_H}}} \right]$ is always positive and $\frac{d\mu(\psi^I)}{d\sqrt{\frac{w_{1N}}{w_{1G}}}}$ is always negative.

When $\frac{t_H}{\tau_H} \geq 1$,

$$\left[[\sqrt{\frac{w_{1N}}{w_{1G}}} - \sqrt{\frac{t_H}{\tau_H}}] - \frac{\frac{\frac{w_{1N}}{w_{1G}} - 2\sqrt{\frac{t_H}{\tau_H}}\sqrt{\frac{w_{1N}}{w_{1G}}}+1}{\frac{w_{1N}}{w_{1G}}-1}}{1-\frac{\tilde{c}}{w_0}\frac{\sqrt{\frac{w_{1N}}{w_{1G}}}-\sqrt{\frac{t_H}{\tau_H}}}{\sqrt{\frac{t_H}{\tau_H}(\frac{w_{1N}}{w_{1G}}-1)}}} \frac{\sqrt{\frac{t_H}{\tau_H}}\sqrt{\frac{w_{1N}}{w_{1G}}}-1}{\sqrt{\frac{t_H}{\tau_H}}} \right] \text{ is always positive}$$

and $\frac{d\mu(\psi^I)}{d\sqrt{\frac{w_{1N}}{w_{1G}}}} < 0$

When ψ^I is either 0 or $\bar{\psi}$, $\mu(\psi^I, P)$ is increasing in w_{1G} and decreasing in w_{1N}. Moreover,there is no discontinuous jump of $\mu(\psi^I, P)$ with respect to w_{1G} or w_{1N}.

□

From this lemma we know that if we are interested in the transition of ψ^* between unconstrained case (ψ^I, information constraint not binding) and constrained case ($\hat{\psi}_L$ or $\hat{\psi}_R$, information constraint binding), the transition would happen *at most once* with respect to various parameters.

Therefore if we draw a graph of ψ^I, $\hat{\psi}_L$ and $\hat{\psi}_R$ with respect to various parameters, there is at most one intersection between 1) ψ^I and 2) $\hat{\psi}_L$ or $\hat{\psi}_R$.

Now we can give a complete characterization of ψ^*:

Proposition A2

- $\bar{\theta}$:
  - *if* $\bar{\mu} \in \left(0, \frac{w_{1G}}{w_{1N}}\right]$, *then for any level of* $\bar{\theta} \in (\frac{c}{w_0}, +\infty)$, $\mu(\psi, P) \geq \bar{\mu}$ *for any* $\psi \in [0, \bar{\psi}]$. $\psi^* = \psi^I$ *is independent of* $\bar{\theta}$.

– *if* $\bar{\mu} \in \left(\frac{w_{1G}}{w_{1N}}, \frac{w_{1G}}{w_{1N}} \frac{1+\sqrt{\frac{w_{1N}}{w_{1G}}}\left(\frac{\sqrt{\frac{t_H}{\tau_H}}\sqrt{\frac{w_{1N}}{w_{1G}}}-1}{\sqrt{\frac{t_H}{\tau_H}}-\sqrt{\frac{w_{1N}}{w_{1G}}}}\right)}{1+\sqrt{\frac{w_{1G}}{w_{1N}}}\left(\frac{\sqrt{\frac{t_H}{\tau_H}}\sqrt{\frac{w_{1N}}{w_{1G}}}-1}{\sqrt{\frac{t_H}{\tau_H}}-\sqrt{\frac{w_{1N}}{w_{1G}}}}\right)} \right]$, *then there exists a cutoff level* $\hat{\hat{\theta}}$, *such that for* $\bar{\theta} \leq \hat{\hat{\theta}}$, *there exist two levels of* ψ, $\hat{\psi}_L(\bar{\mu}, \bar{\theta})$ *and* $\hat{\psi}_R(\bar{\mu}, \bar{\theta})$, $0 < \hat{\psi}_L(\bar{\mu}, \bar{\theta}) < \hat{\psi}_R(\bar{\mu}, \bar{\theta})$, *such that* $\mu(\hat{\psi}_L(\bar{\mu}), P) = \mu(\hat{\psi}_R(\bar{\mu}), P) = \bar{\mu}$. $\psi^* = \psi^I$ *is independent of* $\bar{\theta}$.

– *if* $\bar{\mu} \in \left(\frac{w_{1G}}{w_{1N}} \frac{1+\sqrt{\frac{w_{1N}}{w_{1G}}}\left(\frac{\sqrt{\frac{t_H}{\tau_H}}\sqrt{\frac{w_{1N}}{w_{1G}}}-1}{\sqrt{\frac{t_H}{\tau_H}}-\sqrt{\frac{w_{1N}}{w_{1G}}}}\right)}{1+\sqrt{\frac{w_{1G}}{w_{1N}}}\left(\frac{\sqrt{\frac{t_H}{\tau_H}}\sqrt{\frac{w_{1N}}{w_{1G}}}-1}{\sqrt{\frac{t_H}{\tau_H}}-\sqrt{\frac{w_{1N}}{w_{1G}}}}\right)}, \mu(\bar{\psi}, P) \right]$, *then there exists a cutoff level* $\hat{\hat{\theta}}$, *such that for* $\bar{\theta} \leq \hat{\hat{\theta}}$, *there exist two levels of* ψ, $\hat{\psi}_L(\bar{\mu}, \bar{\theta})$ *and* $\hat{\psi}_R(\bar{\mu}, \bar{\theta})$, $0 < \hat{\psi}_L(\bar{\mu}, \bar{\theta}) < \hat{\psi}_R(\bar{\mu}, \bar{\theta})$, *such that* $\mu(\hat{\psi}_L(\bar{\mu}), P) = \mu(\hat{\psi}_R(\bar{\mu}), P) = \bar{\mu}$. *Then there exists a level of* $\tilde{\tilde{\theta}}$, *such that*

$$\psi^* = \begin{cases} \psi^I, & \text{if } \bar{\theta} \geq \tilde{\tilde{\theta}} \\ \hat{\psi}_L(\bar{\mu}, P), & \text{if } \bar{\theta} < \tilde{\tilde{\theta}} \text{ and} \frac{t_H}{\tau_H} \leq \bar{\mu} \\ \hat{\psi}_R(\bar{\mu}, P), & \text{if } \bar{\theta} < \tilde{\tilde{\theta}} \text{ and} \frac{t_H}{\tau_H} > \bar{\mu} \end{cases}$$

If $\frac{t_H}{\tau_H} \leq \bar{\mu}$, ψ^* *is increasing over* $\bar{\theta}$ *if* $\bar{\theta} < \tilde{\tilde{\theta}}$ *and flat over* $\bar{\theta} \geq \tilde{\tilde{\theta}}$; *If* $\frac{t_H}{\tau_H} > \bar{\mu}$, ψ^* *is decreasing over* $\bar{\theta}$ *if* $\bar{\theta} < \tilde{\tilde{\theta}}$ *and flat over* $\bar{\theta} \geq \tilde{\tilde{\theta}}$.

- *c:*

  – *if* $\bar{\mu} \in \left(0, \frac{w_{1G}}{w_{1N}}\right]$, *then for any level of* $c \in (0, \bar{\theta} w_0)$, $\mu(\psi, P) \geq \bar{\mu}$ *for any* $\psi \in [0, \bar{\psi}]$. $\psi^* = \psi^I$ *is independent of* c.

  – *if* $\bar{\mu} \in \left(\frac{w_{1G}}{w_{1N}}, \frac{w_{1G}}{w_{1N}} \frac{1+\sqrt{\frac{w_{1N}}{w_{1G}}}\left(\frac{\sqrt{\frac{t_H}{\tau_H}}\sqrt{\frac{w_{1N}}{w_{1G}}}-1}{\sqrt{\frac{t_H}{\tau_H}}-\sqrt{\frac{w_{1N}}{w_{1G}}}}\right)}{1+\sqrt{\frac{w_{1G}}{w_{1N}}}\left(\frac{\sqrt{\frac{t_H}{\tau_H}}\sqrt{\frac{w_{1N}}{w_{1G}}}-1}{\sqrt{\frac{t_H}{\tau_H}}-\sqrt{\frac{w_{1N}}{w_{1G}}}}\right)} \right]$, *then there exists a cut-off level* $\hat{c}$, *such that for* $c \geq \hat{c}$, *there exist two levels of* ψ, $\hat{\psi}_L(\bar{\mu}, c)$ *and* $\hat{\psi}_R(\bar{\mu}, c)$, $0 < \hat{\psi}_L(\bar{\mu}, c) < \hat{\psi}_R(\bar{\mu}, c)$, *such that* $\mu(\hat{\psi}_L(\bar{\mu}), P) = \mu(\hat{\psi}_R(\bar{\mu}), P) = \bar{\mu}$. $\psi^* = \psi^I$ *is independent of* c.

– *if* $\bar{\mu} \in \left(\frac{w_{1G}}{w_{1N}} \frac{1+\sqrt{\frac{w_{1N}}{w_{1G}}}\left(\frac{\sqrt{\frac{t_H}{\tau_H}}\sqrt{\frac{w_{1N}}{w_{1G}}}-1}{\sqrt{\frac{t_H}{\tau_H}}-\sqrt{\frac{w_{1N}}{w_{1G}}}}\right)}{1+\sqrt{\frac{w_{1G}}{w_{1N}}}\left(\frac{\sqrt{\frac{t_H}{\tau_H}}\sqrt{\frac{w_{1N}}{w_{1G}}}-1}{\sqrt{\frac{t_H}{\tau_H}}-\sqrt{\frac{w_{1N}}{w_{1G}}}}\right)}, \mu(\bar{\psi}, P)\right]$, *then there exists a cutoff level* $\hat{c}$, *such that for* $c \geq \hat{c}$, *there exist two levels of* ψ, $\hat{\psi}_L(\bar{\mu}, c)$ *and* $\hat{\psi}_R(\bar{\mu}, c)$, $0 < \hat{\psi}_L(\bar{\mu}, c) < \hat{\psi}_R(\bar{\mu}, c)$, *such that* $\mu(\hat{\psi}_L(\bar{\mu}), P) = \mu(\hat{\psi}_R(\bar{\mu}), P) = \bar{\mu}$. *Then there exists a level of* $\tilde{\tilde{\theta}}$, *such that*

$$\psi^* = \begin{cases} \psi^I, & \text{if } c \leq \hat{c} \\ \hat{\psi}_L(\bar{\mu}, P), & \text{if } c > \hat{c} \text{ and} \frac{t_H}{\tau_H} \leq \bar{\mu} \\ \hat{\psi}_R(\bar{\mu}, P), & \text{if } c > \hat{c} \text{ and} \frac{t_H}{\tau_H} > \bar{\mu} \end{cases}$$

If $\frac{t_H}{\tau_H} \leq \bar{\mu}$, ψ^* *is decreasing over c if* $c > \hat{c}$ *and flat over c if* $c \leq \hat{c}$; *If* $\frac{t_H}{\tau_H} > \bar{\mu}$, ψ^* *is increasing over c if* $c > \hat{c}$ *and flat over* $c \leq \hat{c}$.

- w_0:

– *if* $\bar{\mu} \in \left(0, \frac{w_{1G}}{w_{1N}}\right]$, *then for any level of* $w_0 \in (\frac{c}{\theta}, +\infty)$, $\mu(\psi, P) \geq \bar{\mu}$ *for any* $\psi \in [0, \bar{\psi}]$. $\psi^* = \psi^I$ *is increasing over* w_0.

– *if* $\bar{\mu} \in \left(\frac{w_{1G}}{w_{1N}}, \frac{w_{1G}}{w_{1N}} \frac{1+\sqrt{\frac{w_{1N}}{w_{1G}}}\left(\frac{\sqrt{\frac{t_H}{\tau_H}}\sqrt{\frac{w_{1N}}{w_{1G}}}-1}{\sqrt{\frac{t_H}{\tau_H}}-\sqrt{\frac{w_{1N}}{w_{1G}}}}\right)}{1+\sqrt{\frac{w_{1G}}{w_{1N}}}\left(\frac{\sqrt{\frac{t_H}{\tau_H}}\sqrt{\frac{w_{1N}}{w_{1G}}}-1}{\sqrt{\frac{t_H}{\tau_H}}-\sqrt{\frac{w_{1N}}{w_{1G}}}}\right)}\right]$, *then there exists a cutoff level* $\hat{w}_0$, *such that for* $w_0 \leq \hat{w}_0$, *there exist two levels of* ψ, $\hat{\psi}_L(\bar{\mu}, w_0)$ *and* $\hat{\psi}_R(\bar{\mu}, w_0)$, $0 < \hat{\psi}_L(\bar{\mu}, w_0) < \hat{\psi}_R(\bar{\mu}, w_0)$, *such that* $\mu(\hat{\psi}_L(\bar{\mu}), P) = \mu(\hat{\psi}_R(\bar{\mu}), P) = \bar{\mu}$. $\psi^* = \psi^I$ *is increasing over* w_0.

– *if* $\bar{\mu} \in \left(\frac{w_{1G}}{w_{1N}} \frac{1+\sqrt{\frac{w_{1N}}{w_{1G}}}\left(\frac{\sqrt{\frac{t_H}{\tau_H}}\sqrt{\frac{w_{1N}}{w_{1G}}}-1}{\sqrt{\frac{t_H}{\tau_H}}-\sqrt{\frac{w_{1N}}{w_{1G}}}}\right)}{1+\sqrt{\frac{w_{1G}}{w_{1N}}}\left(\frac{\sqrt{\frac{t_H}{\tau_H}}\sqrt{\frac{w_{1N}}{w_{1G}}}-1}{\sqrt{\frac{t_H}{\tau_H}}-\sqrt{\frac{w_{1N}}{w_{1G}}}}\right)}, \mu(\bar{\psi}, P)\right]$, *then there exists a cutoff level* $\hat{w}_0$, *such that for* $w_0 \leq \hat{w}_0$, *there exist two levels of* ψ, $\hat{\psi}_L(\bar{\mu}, w_0)$ *and* $\hat{\psi}_R(\bar{\mu}, w_0)$, $0 < \hat{\psi}_L(\bar{\mu}, w_0) < \hat{\psi}_R(\bar{\mu}, w_0)$, *such that* $\mu(\hat{\psi}_L(\bar{\mu}), P) = \mu(\hat{\psi}_R(\bar{\mu}), P) = \bar{\mu}$. *Then* *there exists a level of* $\tilde{w_0}$*, such that*

$$\psi^* = \begin{cases} \psi^I, & \text{if } w_0 \geq \tilde{w_0} \\ \hat{\psi}_L(\bar{\mu}, P), & \text{if } w_0 < \tilde{w_0} \text{ and} \frac{t_H}{\tau_H} \leq \bar{\mu} \\ \hat{\psi}_R(\bar{\mu}, P), & \text{if } w_0 < \tilde{w_0} \text{ and} \frac{t_H}{\tau_H} > \bar{\mu} \end{cases}$$

If $\frac{t_H}{\tau_H} \leq \bar{\mu}$*,* ψ^* *is increasing over* w_0*; If* $\frac{t_H}{\tau_H} > \bar{\mu}$*,* ψ^* *is decreasing over* w_0 *if* $w_0 < \tilde{w_0}$ *and increasing* w_0 *if* $w_0 \geq \tilde{w_0}$*.*

- w_{1N}*:*
  - *if* $\bar{\mu} \in \left(0, \frac{\bar{\theta} w_0 - c}{\bar{\theta} w_0}\right]$*, then for any level of* $w_{1N} \in (w_{1G}, +\infty)$*,*$\mu(\psi, P) \geq \bar{\mu}$ *for any* $\psi \in [0, \bar{\psi}]$*.* $\psi^* = \psi^I$*.*
    - *if* $\frac{t_H}{\tau_H} \geq 1$*,* ψ^* *is decreasing in* w_{1N}*;*
    - *if* $\frac{t_H}{\tau_H} < 1$*,* ψ^* *is decreasing in* w_{1N} *if* $\sqrt{\frac{w_{1N}}{w_{1G}}} > \sqrt{\frac{\tau_H}{t_H}} + \sqrt{\frac{\tau_H - t_H}{t_H}}$ *and increasing in* w_{1N} *if* $1 < \sqrt{\frac{w_{1N}}{w_{1G}}} < \sqrt{\frac{\tau_H}{t_H}} + \sqrt{\frac{\tau_H - t_H}{t_H}}$
  - *if* $\bar{\mu} \in \left(\frac{\bar{\theta} w_0 - c}{\bar{\theta} w_0}, \mu(\bar{\psi}, P)\right)$*, then exist a level of* $\hat{w_{1N}}$ *such that for any* $w_{1N} \geq \hat{w_{1N}}$*, there exist two levels of* ψ*,* $\hat{\psi}_L(\bar{\mu}, w_{1N})$ *and* $\hat{\psi}_R(\bar{\mu}, w_{1N})$*,* $0 < \hat{\psi}_L(\bar{\mu}, w_{1N}) < \hat{\psi}_R(\bar{\mu}, w_{1N})$*, such that* $\mu(\hat{\psi}_L(\bar{\mu}), P) = \mu(\hat{\psi}_R(\bar{\mu}), P) = \bar{\mu}$*. Then there exists a level of* $\tilde{w_{1N}} \in [\hat{w_{1N}}, +\infty)$*, such that*

$$\psi^* = \begin{cases} \psi^I, & \text{if } w_{1N} \leq \tilde{w_{1N}} \\ \hat{\psi}_L(\bar{\mu}, P), & \text{if } w_{1N} > \tilde{w_{1N}} \text{ and} \frac{t_H}{\tau_H} \leq \bar{\mu} \\ \hat{\psi}_R(\bar{\mu}, P), & \text{if } w_{1N} > \tilde{w_{1N}} \text{ and} \frac{t_H}{\tau_H} > \bar{\mu} \end{cases}$$

    - *if* $\frac{t_H}{\tau_H} \geq 1$*, then* ψ^* *is decreasing in* w_{1N} *if* $w_{1N} \in (w_{1G}, \tilde{w_{1N}})$ *and increasing in* w_{1N} *if* $w_{1N} \in (\tilde{w_{1N}}, +\infty)$
    - *if* $\frac{t_H}{\tau_H} < 1$*, then there exist a cut-off level* $\tilde{\bar{\mu}} \in (\frac{\bar{\theta} w_0 - c}{\bar{\theta} w_0}, 1)$*, and for any* $\bar{\mu}$ *a level of* $\frac{\bar{t_H}}{\tau_H}(\bar{\mu}) \in (0, 1)$ *such that:*
      - *if* $\bar{\mu} \in \left(\frac{\bar{\theta} w_0 - c}{\bar{\theta} w_0}, \tilde{\bar{\mu}}\right]$*, then* $\frac{\bar{t_H}}{\tau_H}(\bar{\mu}) \leq \bar{\mu}$*. Moreover:*
        (1) if $\frac{t_H}{\tau_H} \in \left(0, \frac{\bar{t_H}}{\tau_H}\right]$*,* ψ^* *is increasing over* w_{1N} *if* $w_{1N} \in (w_{1G}, \tilde{w_{1N}})$ *and decreasing over* w_{1N} *if* $w_{1N} \in (\tilde{w_{1N}}, +\infty)$
        (2) if $\frac{t_H}{\tau_H} \in \left(\frac{\bar{t_H}}{\tau_H}, \bar{\mu}\right]$*, then* ψ *is increasing over* w_{1N} *if* $w_{1N} \in (w_{1G}, \sqrt{\frac{\tau_H}{t_H}} + \sqrt{\frac{\tau_H - t_H}{t_H}})$ *and decreasing over* w_{1N} *if* $w_{1N} \in (\sqrt{\frac{\tau_H}{t_H}} + \sqrt{\frac{\tau_H - t_H}{t_H}}, +\infty)$

(3) if $\frac{t_H}{\tau_H} \in (\bar{\mu}, 1)$, then ψ^* is increasing over w_{1N} if $w_{1N} \in (w_{1G}, \sqrt{\frac{\tau_H}{t_H}} + \sqrt{\frac{\tau_H - t_H}{t_H}})$ and decreasing over w_{1N} if $w_{1N} \in (\sqrt{\frac{\tau_H}{t_H}} + \sqrt{\frac{\tau_H - t_H}{t_H}}, \tilde{w_{1N}})$ and then increasing over w_{1N} if $w_{1N} \in (\tilde{w_{1N}}, +\infty)$

· if $\bar{\mu} \in \left(\frac{\bar{\theta}w_0 - c}{\theta w_0}, \tilde{\tilde{\mu}}\right]$, then $\frac{\bar{t_H}}{\tau_H}(\bar{\mu}) > \bar{\mu}$. Moreover:
(1) if $\frac{t_H}{\tau_H} \in (0, \bar{\mu}]$, ψ^* is increasing over w_{1N} for $w_{1N} \in (w_{1G}, \tilde{w_{1N}})$ and decreasing over w_{1N} if $w_{1N} \in (\tilde{w_{1N}}, +\infty)$
(2) if $\frac{t_H}{\tau_H} \in \left(\bar{\mu}, \frac{\bar{t_H}}{\tau_H}\right]$, then ψ is increasing over w_{1N} for $w_{1N} \in (w_{1G}, +\infty)$
(3) if $\frac{t_H}{\tau_H} \in (\bar{\mu}, 1)$, then ψ^* is increasing over w_{1N} if $w_{1N} \in (w_{1G}, \sqrt{\frac{\tau_H}{t_H}} + \sqrt{\frac{\tau_H - t_H}{t_H}})$ and decreasing over w_{1N} if $w_{1N} \in (\sqrt{\frac{\tau_H}{t_H}} + \sqrt{\frac{\tau_H - t_H}{t_H}}, \tilde{w_{1N}})$ and then increasing over w_{1N} if $w_{1N} \in (\tilde{w_{1N}}, +\infty)$

- w_{1G}:
  - if $\bar{\mu} \in \left(0, \frac{\bar{\theta}w_0 - c}{\theta w_0}\right]$, then for any level of $w_{1G} \in (0, w_{1N})$,$\mu(\psi, P) \geq \bar{\mu}$ for any $\psi \in [0, \bar{\psi}]$. $\psi^* = \psi^I$.
    * if $\frac{t_H}{\tau_H} \leq 1$, ψ^* is decreasing in w_{1G};
    * if $\frac{t_H}{\tau_H} > 1$, ψ^* is decreasing in w_{1G} if $\sqrt{\frac{w_{1N}}{w_{1G}}} > \sqrt{\frac{t_H}{\tau_H}} + \sqrt{\frac{t_H - \tau_H}{\tau_H}}$ and increasing in w_{1G} if $\sqrt{\frac{w_{1N}}{w_{1G}}} < \sqrt{\frac{t_H}{\tau_H}} + \sqrt{\frac{t_H - \tau_H}{\tau_H}}$
  - if $\bar{\mu} \in \left(\frac{\bar{\theta}w_0 - c}{\theta w_0}, \mu(\bar{\psi}, P)\right)$, then exist a level of $\hat{w_{1G}}$ such that for any $w_{1G} \leq \hat{w_{1G}}$, there exist two levels of ψ, $\hat{\psi}_L(\bar{\mu}, w_{1G})$ and $\hat{\psi}_R(\bar{\mu}, w_{1G})$, $0 < \hat{\psi}_L(\bar{\mu}, w_{1G}) < \hat{\psi}_R(\bar{\mu}, w_{1G})$, such that $\mu(\hat{\psi}_L(\bar{\mu}), P) = \mu(\hat{\psi}_R(\bar{\mu}), P) = \bar{\mu}$. Then there exists a level of $\tilde{w_{1G}} \in (0, \hat{w_{1G}}]$, such that

$$\psi^* = \begin{cases} \psi^I, & \text{if } w_{1G} \geq \tilde{w_{1G}} \\ \hat{\psi}_L(\bar{\mu}, P), & \text{if } w_{1G} < \tilde{w_{1G}} \text{ and} \frac{t_H}{\tau_H} \leq \bar{\mu} \\ \hat{\psi}_R(\bar{\mu}, P), & \text{if } w_{1G} < \tilde{w_{1G}} \text{ and} \frac{t_H}{\tau_H} > \bar{\mu} \end{cases}$$

    * if $\frac{t_H}{\tau_H} \in (0, \bar{\mu}]$, then ψ^* is increasing in w_{1G} if $w_{1G} \in (0, \tilde{w_{1G}})$ and decreasing in w_{1G} if $w_{1G} \in (\tilde{w_{1G}}, w_{1N})$
    * if $\frac{t_H}{\tau_H} \in (\bar{\mu}, 1]$, then ψ^* is decreasing in w_{1G} for $w_{1G} \in (0, w_{1N})$
    * if $\frac{t_H}{\tau_H} \in (1, +\infty)$, then there exist a cut-off point $\frac{\bar{t_H}}{\tau_H}$ such that:
      · if $\frac{t_H}{\tau_H} \geq \frac{\bar{t_H}}{\tau_H}$, then ψ^* is decreasing over w_{1G} if $w_{1G} \in (0, \tilde{w_{1G}})$ and increasing in w_{1G} if $w_{1G} \in (\tilde{w_{1G}}, w_{1N})$
      · if $\frac{t_H}{\tau_H} < \frac{\bar{t_H}}{\tau_H}$, then ψ^* is decreasing over w_{1G} if $w_{1G} \in (0, \sqrt{\frac{t_H}{\tau_H}} + \sqrt{\frac{t_H}{\tau_H} - 1})$ and increasing in w_{1G} if $w_{1G} \in (\sqrt{\frac{t_H}{\tau_H}} + \sqrt{\frac{t_H}{\tau_H} - 1}, w_{1N})$

- $\bar{\mu}$:
  - *if* $\bar{\mu} \in \left(0, \mu(\hat{\psi}, P)\right]$, $\psi^* = \psi^I$ *is independent of* $\bar{\mu}$.
  - *if* $\bar{\mu} \in (\mu(\hat{\psi}, P), \mu(\bar{\psi}, P))$, *there exist a level of cutoff point* $\tilde{\bar{\mu}}$ *such that:*

$$\psi^* = \begin{cases} \psi^I, & \text{if } \bar{\mu} \leq \tilde{\bar{\mu}} \\ \hat{\psi}_L(\bar{\mu}, P), & \text{if } \bar{\mu} > \tilde{\bar{\mu}} \text{ and} \frac{t_H}{\tau_H} \leq \bar{\mu} \\ \hat{\psi}_R(\bar{\mu}, P), & \text{if } \bar{\mu} > \tilde{\bar{\mu}} \text{ and} \frac{t_H}{\tau_H} > \bar{\mu} \end{cases}$$

    - *if* $\frac{t_H}{\tau_H} \in (0, \bar{\mu}]$, ψ^* *is flat over* $\bar{\mu}$ *if* $\bar{\mu} \leq \tilde{\bar{\mu}}$ *and decreasing over* $\bar{\mu}$ *if* $\bar{\mu} > \tilde{\bar{\mu}}$
    - *if* $\frac{t_H}{\tau_H} \in (\bar{\mu}, +\infty)$, ψ^* *is flat over* $\bar{\mu}$ *if* $\bar{\mu} \leq \tilde{\bar{\mu}}$ *and increasing over* $\bar{\mu}$ *if* $\bar{\mu} > \tilde{\bar{\mu}}$

- $\frac{t_H}{\tau_H}$
  - *if* $\bar{\mu} \in (0, \mu(\hat{\mu})]$, $\psi^* = \psi^I$. ψ^* *is weakly increasing over* $\frac{t_H}{\tau_H}$
  - *if* $\bar{\mu} \in \left(\mu(\hat{\mu}), \mu(\hat{\psi}, P)\right]$, *then there exists two cut-off points,* $\hat{\frac{t_H}{\tau_H}}_1$ *and* $\hat{\frac{t_H}{\tau_H}}_2$ *such that*

$$\psi^* = \begin{cases} \psi^I, & \text{if } \frac{t_H}{\tau_H} \in \left(0, \hat{\frac{t_H}{\tau_H}}_1\right] \\ \hat{\psi}_L(\bar{\mu}, P), & \text{if } \frac{t_H}{\tau_H} \in \left(\hat{\frac{t_H}{\tau_H}}_1, \bar{\mu}\right] \\ \hat{\psi}_R(\bar{\mu}, P), & \text{if } \frac{t_H}{\tau_H} \in \left(\bar{\mu}, \hat{\frac{t_H}{\tau_H}}_2\right) \\ \psi^I, & \text{if } \frac{t_H}{\tau_H} \in \left(\hat{\frac{t_H}{\tau_H}}_2, +\infty\right] \end{cases}$$

  ψ^* *is increasing over* $\frac{t_H}{\tau_H}$ *if* $\frac{t_H}{\tau_H} \in \left(0, \hat{\frac{t_H}{\tau_H}}_1\right] \cup \left(\hat{\frac{t_H}{\tau_H}}_2, +\infty\right]$, *flat over* $\frac{t_H}{\tau_H}$ *if* $\left(\hat{\frac{t_H}{\tau_H}}_1, \bar{\mu}\right]$ *and* $\left(\bar{\mu}, \hat{\frac{t_H}{\tau_H}}_2\right)$. *There is discrete jump at the point* $\frac{t_H}{\tau_H} = \bar{\mu}$

Proof 1) We first proof that ψ^I intersects with $\hat{\psi}_L$ iff $\frac{t_H}{\tau_H} \leq \bar{\mu}$, and intersects with $\hat{\psi}_R$ iff $\frac{t_H}{\tau_H} > \bar{\mu}$.

From previous lemma we know that as a function of any parameter, ψ^I can only cross either $\hat{\psi}_L$ or $\hat{\psi}_R$ for once.

2) We have shown that

$$\hat{\psi}_L(\bar{\mu}) = \frac{-\left[w_{1N}(w_0 - \frac{\tilde{c}}{1-\bar{\mu}}) + w_{1G}(w_0 + \frac{\tilde{c}\bar{\mu}}{1-\bar{\mu}})\right] - \sqrt{(\left[w_{1N}(w_0 - \frac{\tilde{c}}{1-\bar{\mu}}) + w_{1G}(w_0 + \frac{\tilde{c}\bar{\mu}}{1-\bar{\mu}})\right])^2 - 4w_{1G}w_{1N}w_0(w_0 - \tilde{c})}}{2w_{1G}w_{1N}}$$

and

$$\hat{\psi}_R(\bar{\mu}) = \frac{-\left[w_{1N}(w_0-\frac{\tilde{c}}{1-\bar{\mu}})+w_{1G}(w_0+\frac{\tilde{c}\bar{\mu}}{1-\bar{\mu}})\right]+\sqrt{(\left[w_{1N}(w_0-\frac{\tilde{c}}{1-\bar{\mu}})+w_{1G}(w_0+\frac{\tilde{c}\bar{\mu}}{1-\bar{\mu}})\right])^2-4w_{1G}w_{1N}w_0(w_0-\tilde{c})}}{2w_{1G}w_{1N}}$$

This is equivalent to:

$$\hat{\psi}_L(\bar{\mu}) = \frac{-\left[\frac{1}{w_{1G}}(w_0-\frac{\tilde{c}}{1-\bar{\mu}})+\frac{1}{w_{1N}}(w_0+\frac{\tilde{c}\bar{\mu}}{1-\bar{\mu}})\right]-\sqrt{(\left[\frac{1}{w_{1G}}(w_0-\frac{\tilde{c}}{1-\bar{\mu}})+\frac{1}{w_{1N}}(w_0+\frac{\tilde{c}\bar{\mu}}{1-\bar{\mu}})\right])^2-4\frac{1}{w_{1G}w_{1N}}w_0(w_0-\tilde{c})}}{2}$$

$$= \frac{\frac{2w_0(w_0-\tilde{c})}{w_{1G}w_{1N}}}{-\left[\frac{1}{w_{1G}}(w_0-\frac{\tilde{c}}{1-\bar{\mu}})+\frac{1}{w_{1N}}(w_0+\frac{\tilde{c}\bar{\mu}}{1-\bar{\mu}})\right]+\sqrt{(\left[\frac{1}{w_{1G}}(w_0-\frac{\tilde{c}}{1-\bar{\mu}})+\frac{1}{w_{1N}}(w_0+\frac{\tilde{c}\bar{\mu}}{1-\bar{\mu}})\right])^2-4\frac{1}{w_{1G}w_{1N}}w_0(w_0-\tilde{c})}}$$

and $\hat{\psi}_R(\bar{\mu}) = \frac{-\left[\frac{1}{w_{1G}}(w_0-\frac{\tilde{c}}{1-\bar{\mu}})+\frac{1}{w_{1N}}(w_0+\frac{\tilde{c}\bar{\mu}}{1-\bar{\mu}})\right]+\sqrt{(\left[\frac{1}{w_{1G}}(w_0-\frac{\tilde{c}}{1-\bar{\mu}})+\frac{1}{w_{1N}}(w_0+\frac{\tilde{c}\bar{\mu}}{1-\bar{\mu}})\right])^2-4\frac{1}{w_{1G}w_{1N}}w_0(w_0-\tilde{c})}}{2}$

First some intermediate results:

Comparative Statics for $H_N(\psi^*)$ and $H_G(\psi^*)$

We've looked at the comparative statics of ψ^*, i.e., the home government's optimal effort of driving up tension. However, the government has no obvious reason to reveal its effort and it could be hard or impossible to measure\observe its effort. A variable that are easier to observe could be $H_N(\psi)$, the probability of a protest.

Also $H_N(\psi)$ and $H_G(\psi)$ would be interesting for their own sake: $H_N(\psi)$ is the probability of nationalist protest and also the probability of home government receiving a concession. $H_G(\psi)$ is the probability of general public protest and also the probability of a real crisis.

We can similarly look at how $H_N(\psi^*)$ and $H_G(\psi^*)$ change according to the parameters:

First some intermediate results:

Lemma A7

- $H_N(\psi^I)$, $H_G(\psi^I)$

$$H_N(\psi^I) = \begin{cases} 1-\frac{\tilde{c}}{w_0}, & if \frac{t_H}{\tau_H} \leq \frac{w_{1G}}{w_{1N}} \\ 1-\frac{\tilde{c}}{w_0}\frac{\sqrt{\frac{w_{1N}}{w_{1G}}}-\sqrt{\frac{t_H}{\tau_H}}}{\sqrt{\frac{t_H}{\tau_H}[\frac{w_{1N}}{w_{1G}}-1]}}, & if \frac{t_H}{\tau_H} \in (\frac{w_{1G}}{w_{1N}}, \frac{w_{1G}}{w_{1N}}(\frac{w_0+w_{1N}\bar{\psi}}{w_0+w_{1G}\bar{\psi}})^2) \\ 1-\frac{\tilde{c}}{w_0+w_{1N}\bar{\psi}}, & if \frac{t_H}{\tau_H} \geq \frac{w_{1G}}{w_{1N}}(\frac{w_0+w_{1N}\bar{\psi}}{w_0+w_{1G}\bar{\psi}})^2 \end{cases}$$

$$H_G(\psi^I) = \begin{cases} 1 - \frac{\tilde{c}}{w_0}, & if \frac{t_H}{\tau_H} \le \frac{w_{1G}}{w_{1N}} \\ 1 - \frac{\tilde{c}}{w_0} \frac{\sqrt{\frac{w_{1N}}{w_{1G}}} - \sqrt{\frac{t_H}{\tau_H}}}{\sqrt{\frac{w_{1N}}{w_{1G}}} - \sqrt{\frac{w_{1G}}{w_{1N}}}}, & if \frac{t_H}{\tau_H} \in (\frac{w_{1G}}{w_{1N}}, \frac{w_{1G}}{w_{1N}}(\frac{w_0 + w_{1N}\bar{\psi}}{w_0 + w_{1G}\hat{\psi}})^2) \\ 1 - \frac{\tilde{c}}{w_0 + w_{1G}\hat{\psi}}, & if \frac{t_H}{\tau_H} \ge \frac{w_{1G}}{w_{1N}}(\frac{w_0 + w_{1N}\bar{\psi}}{w_0 + w_{1G}\hat{\psi}})^2 \end{cases}$$

- *$H_N(\psi^I)$ and $H_G(\psi^I)$ are increasing over w_0 and $\bar{\theta}$, and decreasing over c*
- *If $\frac{t_H}{\tau_H} < 1$, then $H_N(\psi^I)$ is increasing over $\sqrt{\frac{w_{1N}}{w_{1G}}}$; If $\frac{t_H}{\tau_H} > 1$ $H_N(\psi^I)$ is decreasing over $\sqrt{\frac{w_{1N}}{w_{1G}}}$ if $\sqrt{\frac{w_{1N}}{w_{1G}}} < \sqrt{\frac{t_H}{\tau_H}} + \sqrt{\frac{t_H - \tau_H}{\tau_H}}$ and increasing over $\sqrt{\frac{w_{1N}}{w_{1G}}}$ if $\sqrt{\frac{w_{1N}}{w_{1G}}} > \sqrt{\frac{t_H}{\tau_H}} + \sqrt{\frac{t_H - \tau_H}{\tau_H}}$;*
- *If $\frac{t_H}{\tau_H} > 1$, $H_G(\psi^I)$ is decreasing over $\sqrt{\frac{w_{1N}}{w_{1G}}}$; If $\frac{t_H}{\tau_H} \le 1$, $H_G(\psi^I)$ is decreasing over $\sqrt{\frac{w_{1N}}{w_{1G}}}$ if $\sqrt{\frac{w_{1N}}{w_{1G}}} > \frac{1}{\sqrt{\frac{\tau_H}{t_H}} - \sqrt{\frac{\tau_H - t_H}{t_H}}}$, and increasing over $\sqrt{\frac{w_{1N}}{w_{1G}}}$ if $\sqrt{\frac{w_{1N}}{w_{1G}}} < \frac{1}{\sqrt{\frac{\tau_H}{t_H}} - \sqrt{\frac{\tau_H - t_H}{t_H}}}$,*
- *$H_N(\psi^I)$ and $H_G(\psi^I)$ are weakly increasing over $\frac{t_H}{\tau_H}$*
- *$H_N(\psi^I)$ and $H_G(\psi^I)$ are independent over $\bar{\mu}$*

- *$H_N(\hat{\psi}_L(\bar{\mu}))$,$H_G(\hat{\psi}_L(\bar{\mu}))$*
  - *$H_N(\hat{\psi}_L(\bar{\mu}))$ and $H_G(\hat{\psi}_L(\bar{\mu}))$ decrease over $\frac{w_{1N}}{w_{1G}}$*
  - *$H_N(\hat{\psi}_L(\bar{\mu}))$ and $H_G(\hat{\psi}_L(\bar{\mu}))$ increase over $\bar{\theta}$*
  - *$H_N(\hat{\psi}_L(\bar{\mu}))$ and $H_G(\hat{\psi}_L(\bar{\mu}))$ increase over w_0*
  - *$H_N(\hat{\psi}_L(\bar{\mu}))$ and $H_G(\hat{\psi}_L(\bar{\mu}))$ decrease over c*
  - *$H_N(\hat{\psi}_L(\bar{\mu}))$ and $H_G(\hat{\psi}_L(\bar{\mu}))$ are independent of $\frac{t_H}{\tau_H}$*
  - *$H_N(\hat{\psi}_L(\bar{\mu}))$ and $H_G(\hat{\psi}_L(\bar{\mu}))$ are decreasing over $\bar{\mu}$*

- *$H_N(\hat{\psi}_R(\bar{\mu}))$,$H_G(\hat{\psi}_R(\bar{\mu}))$*
  - *$H_N(\hat{\psi}_R(\bar{\mu}))$ and $H_G(\hat{\psi}_R(\bar{\mu}))$ increase over $\frac{w_{1N}}{w_{1G}}$*
  - *$H_N(\hat{\psi}_R(\bar{\mu}))$ and $H_G(\hat{\psi}_R(\bar{\mu}))$ decrease over $\bar{\theta}$*
  - *$H_N(\hat{\psi}_R(\bar{\mu}))$ and $H_G(\hat{\psi}_R(\bar{\mu}))$ decrease over w_0*
  - *$H_N(\hat{\psi}_R(\bar{\mu}))$ and $H_G(\hat{\psi}_R(\bar{\mu}))$ increase over c*

- *$H_N(\hat{\psi}_R(\bar{\mu}))$ and $H_G(\hat{\psi}_R(\bar{\mu}))$ are independent of $\frac{t_H}{\tau_H}$*
- *$H_N(\hat{\psi}_R(\bar{\mu}))$ and $H_G(\hat{\psi}_R(\bar{\mu}))$ are increasing over $\bar{\mu}$*

Proof:

If ψ^I is interior solution, then $\frac{\partial H_G(\psi^I)}{\partial\sqrt{\frac{w_{1N}}{w_{1G}}}} = \frac{-\tilde{c}\sqrt{\frac{t_H}{\tau_H}}}{w_0(\sqrt{\frac{w_{1N}}{w_{1G}}}-\sqrt{\frac{w_{1G}}{w_{1N}}})^2}\left[(\sqrt{\frac{w_{1G}}{w_{1N}}}-\sqrt{\frac{\tau_H}{t_H}})^2+1-\frac{\tau_H}{t_H}\right]$

$\frac{\partial H_N(\psi^I)}{\partial\sqrt{\frac{w_{1N}}{w_{1G}}}} = \frac{\tilde{c}}{w_0(\sqrt{\frac{w_{1N}}{w_{1G}}}-\sqrt{\frac{w_{1G}}{w_{1N}}})^2}\left[(\sqrt{\frac{w_{1N}}{w_{1G}}}-\sqrt{\frac{t_H}{\tau_H}})^2+1-\frac{t_H}{\tau_H}\right]$ □

Corrollary 1

- If $\frac{t_H}{\tau_H} \leq \bar{\mu}$, $H_N(\psi^*)$ and $H_G(\psi^*)$ are
  - increasing in w_0 and $\bar{\theta}$ and decreasing in c
  - first increasing and then decreasing over $\frac{w_{1N}}{W_{1G}}$
- If $\frac{t_H}{\tau_H} > \bar{\mu}$,
  - $H_N(\psi^*)$ and $H_G(\psi^*)$ are:
    - first decreasing and then increasing over $\bar{\theta}$
    - first decreasing and then increasing over w_0
    - first decreasing and then increasing over c
  - If $\frac{t_H}{\tau_H} \in (\bar{\mu}, 1]$, $H_N(\psi^*)$ is increasing over $\frac{w_{1N}}{w_{1G}}$, and $H_G(\psi^*)$ is increasing and then decreasing and then increasing again over $\frac{w_{1N}}{w_{1G}}$
  - if $\frac{t_H}{\tau_H} > 1$, $H_N(\psi^*)$ and $H_G(\psi^*)$ are first decreasing and then increasing over $\frac{w_{1N}}{w_{1G}}$
- $H_N(\psi^*)$ and $H_G(\psi^*)$ are weakly increasing over $\frac{t_H}{\tau_H}$. $H_N(\psi^*)$ and $H_G(\psi^*)$ both jump discontinuously over $\frac{t_H}{\tau_H}$ at point $\frac{t_H}{\tau_H} = \bar{\mu}$
- $\bar{\mu}$:
  - if $\frac{t_H}{\tau_H}$ is small, $H_N(\psi^*)$ and $H_G(\psi^*)$ are first constant over $\bar{\mu}$ and then decreasing over $\bar{\mu}$;
  - if $\frac{t_H}{\tau_H}$ is in intermediate range, $H_N(\psi^*)$ and $H_G(\psi^*)$ are first constant over $\bar{\mu}$,then increasing over $\bar{\mu}$, and then decreasing over $\bar{\mu}$
  - if $\frac{t_H}{\tau_H}$ is large enough, $H_N(\psi^*)$ and $H_G(\psi^*)$ are first constant over $\bar{\mu}$ and then increasing over $\bar{\mu}$

So the probability of protest, $H_N(\psi^*)$, and the probability of crisis, $H_G(\psi^*)$ have similar patterns as the equilibrium level of animosity, ψ^*.

Welfare Analysis

Lemma A8

$U_H(\psi^I)$ is:

- *increasing in w_{1N};*
- *decreasing in w_{1G};*
- *increasing in c if $\frac{t_H}{\tau_H} \in (0, \frac{w_0+w_{1N}\bar{\psi}}{w_0+w_{1G}\bar{\psi}})$ and decreasing in c if $\frac{t_H}{\tau_H} \in (\frac{w_0+w_{1N}\bar{\psi}}{w_0+w_{1G}\bar{\psi}}, +\infty)$*
- *decreasing in w_0 if $\frac{t_H}{\tau_H} \in (0, (\frac{w_0+w_{1N}\bar{\psi}}{w_0+w_{1G}\bar{\psi}})^2)$ and increasing in w_0 if $\frac{t_H}{\tau_H} \in ((\frac{w_0+w_{1N}\bar{\psi}}{w_0+w_{1G}\bar{\psi}})^2, +\infty)$*
- *decreasing in $\bar{\theta}$ if $\frac{t_H}{\tau_H} \in (0, \frac{w_0+w_{1N}\bar{\psi}}{w_0+w_{1G}\bar{\psi}})$ and increasing in $\bar{\theta}$ if $\frac{t_H}{\tau_H} \in (\frac{w_0+w_{1N}\bar{\psi}}{w_0+w_{1G}\bar{\psi}}, +\infty)$;*

Lemma A9

$U_H(\hat{\psi}_L)$ and $U_H(\hat{\psi}_R)$ is:

- *increasing in w_{1N};*
- *decreasing in w_{1G};*
- *increasing in c;*
- *decreasing in w_0;*
- *decrease in $\bar{\theta}$*

- **Proposition A3**

  If $\frac{t_H}{\tau_H} \leq \bar{\mu}$, $U_H(\psi^*)$ is:

  * *increasing in w_{1N};*
  * *decreasing in w_{1G};*
  * *increasing in c ;*
  * *decreasing in w_0*
  * *decreasing in $\bar{\theta}$*
  * *weakly decreasing in $\bar{\mu}$*

  If $\frac{t_H}{\tau_H} > \bar{\mu}$, $U_H(\psi^*)$ is:

  * *single-peaked over some interior level of w_{1G};*
  * *Increasing or single-peaked over w_{1N};*

- *If* $\frac{t_H}{\tau_H} \in (\frac{w_0+w_{1N}\bar{\psi}}{w_0+w_{1G}\bar{\psi}}, +\infty)$*,decreasing over* c*; If* $\frac{t_H}{\tau_H} \in (0, \frac{w_0+w_{1N}\bar{\psi}}{w_0+w_{1G}\bar{\psi}})$*, increasing or single-peaked over* c*;*
- *If* $\frac{t_H}{\tau_H} \in (\frac{w_0+w_{1N}\bar{\psi}}{w_0+w_{1G}\bar{\psi}}, +\infty)$*,increasing over* $\bar{\theta}$*; If* $\frac{t_H}{\tau_H} \in (0, \frac{w_0+w_{1N}\bar{\psi}}{w_0+w_{1G}\bar{\psi}})$*, decreasing or single-peaked over* $\bar{\theta}$*;*
- *If* $\frac{t_H}{\tau_H} \in ((\frac{w_0+w_{1N}\bar{\psi}}{w_0+w_{1G}\bar{\psi}})^2, +\infty)$*, increasing over* w_0*, if* $\frac{t_H}{\tau_H} \in (0, (\frac{w_0+w_{1N}\bar{\psi}}{w_0+w_{1G}\bar{\psi}})^2)$*, decreasing or single-peaked over* w_0