Strategic Disclosure, Primary Market Uncertainty, and Informed Trading

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Abstract

We study the optimal disclosure policy in security issuance using a Bayesian persuasion approach. An issuer designs a signal to persuade an investment bank to underwrite. The bank forms a posterior on the basis of the signal and makes its underwriting and retention decisions. When there is no demand uncertainty, a partially informative disclosure is enough to curb primary market underpricing due to informed sales by the underwriter in the secondary market. When demand is uncertain, the underwriter may shy away because of more retention than his privately optimal level and larger losses due to increased total cost of capital. The optimal disclosure can solve such hold-up problem resulting from weak demand and induce the bank to underwrite. We derive predictions on the effects of the issuer's fundamentals, the underwriter's cost of capital, the demand uncertainty, and the market liquidity on the informativeness of the optimal disclosure. Our model not only captures the adverse selection problem in the originate-to-distribute lending model, but also rationalizes the phenomenon that arrangers may be willing to retain large and costly stakes in leveraged loan syndication. Finally, if viewed as an extant blockholder, we show that the underwriter may exert governance by exit to promote more transparent disclosure by the issuing firm.

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Keywords: Disclosure, Financing Frictions, Originate-to-Distribute, Bayesian Persuasion.

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"In today's aggressive marketplace, listed companies can no longer rely on their numbers to do the talking. If companies can't communicate their achievements and strategy, mounting research evidence suggests, they will be overlooked, their cost of capital will increase and stock price will suffer."

–Westbrook (2014)

1 Introduction

The design and transmission of information plays a vital role in security offering in that it shapes issuers', intermediaries' and investors' expectations of the future, and thus profoundly influences the resulting supply-demand equilibrium. One overarching friction which plagues the well-being of the market participants is information asymmetry: usually one party holds a payoff-relevant informational advantage over another. Issuers have considerable discretion in the disclosure of information to advance their own interests. Intermediaries, by underwriting and investing in the deals, acquire proprietary information which helps them predict future performance but cannot be credibly communicated to other investors. Moreover, they may gain from trading on their private information. Accordingly, understanding such friction and evaluating feasible options for alleviating it is of great importance.

The goal of this paper is to provide a comprehensive theoretical framework to address the following questions. First, how does information disclosure by the issuer potentially affect a financial intermediary's decision to retain and trade the issued securities? Second, can strategic information disclosure help the issuer maximize proceeds from security offering, mitigate adverse selection, and induce the investment bank to underwrite even if some unfavorable market friction (e.g. weak demand) may initially deter the bank from doing so? Third, what are the effects of the issuer's fundamentals, the underwriter's cost of capital, the primary market condition, and the secondary market liquidity on the informativeness of the optimal disclosure policy?

In this paper, we develop a tractable yet comprehensive model that links the issuer's information disclosure in the capital raising process to various primary and secondary market activities by the underwriter and other investors. We model the optimal design of disclosure policy by the issuer as a Bayesian persuasion game *à la* Kamenica and Gentzkow (2011). In their seminal paper, Kamenica and Gentzkow (2011) present a model where a sender chooses a signal to reveal to a receiver, who then takes an action that affects the welfare of both players. They solve for the sender-optimal signal by reframing the problem as maximizing the sender's payoff over distributions of posterior beliefs subject to the Bayesian plausibility condition that the average posteriors should be consistent with the prior. The effectiveness of Bayesian persuasion is that it improves the sender's expected payoff by inducing the receiver to choose a better action. The maximal value is obtained by finding the concave closure of the sender's payoff function for any posterior held by the receiver.

In general, the Bayesian persuasion approach fits the process of security issuance very well. The issuing party (sender) has to first draft a proposal which will be sent to a potential underwriting bank (receiver). Routinely, the issuer possesses marked flexibility in selecting what to disclose and how precise the disclosure is. In effect, issuers usually exercise discretion in reporting forward-looking information which contributes to the valuation of the proposed security. Such information includes but is not limited to forecasts of future sales, earnings, and growth opportunities, which can be either purely qualitative, or quantitative with varying precision – a range or a point estimate. Moreover, issuers often choose to release unique marketing information about business models, corporate strategy, and prospects of the industry to attract potential investors. In sum, the proposal-drafting stage resembles the sender's communication about the optimally designed signal system to the receiver. After seeing the proposal, the investment bank further investigates the realization of the signal through due diligence if it still cannot decide whether it should underwrite. If the bank agrees to underwrite, it engages in information production with the issuer to prepare the information memorandum (for debt) or prospectus (for equity), which is then circulated to potential investors (other receivers). In this sense, the information memorandum or prospectus reflects the informativeness of the issuer's disclosure. The underwriter then prices the security based on the collected information. This stage corresponds to the mapping from the signal realization to the pricing of the security.

Specifically, we consider an issuer who designs an information disclosure system and reveals it to an investment bank to invite it to underwrite the deal. The issuer may represent a borrower in a debt issue, an originator in securitization, or an entrepreneur in an equity issue. The investment bank may serve as a lead bank in loan syndication, an arranger in the sale of asset-backed security (ABS), or an underwriter in equity and bond offering.[1] If the investment bank decides to underwrite, it further helps communicate the signal to potential investors, chooses its stake, and allocates the remaining securities to the participant investors. We assume that the underwriter obtains proprietary information from its underwriting activity and retention. Similar assumptions regarding the generation of private information are commonly used in the literature on banking and blockholders (e.g. Parlour and Plantin, 2008; Edmans and Manso, 2011), and well documented empirically (e.g. Lummer and McConnell, 1989; Edmans, Fang, and Zur, 2013). Nevertheless, the acquisition of material information in our model is an inevitable but adverse consequence of the underwriter's involvement in the issue. As a result, the underwriting bank can profit from insider trading when the secondary market opens. Following Maug (1998), Hennessy and Zechner (2011), and Chemla and Hennessy (2014), who model the secondary markets of equity, bond, and ABS respectively, the market structure is in the sprit of Kyle (1985) where investors submit their market orders to a continuum of deep-pocketed risk-neutral market makers who price the security competitively after observing aggregate demand. If the participant investors anticipate that there is adverse selection in the secondary market, they will demand a discount in the issue price to offset their future losses, a fact widely used in the literature (e.g. Holmström and Tirole, 1993; Maug, 1998; Edmans and Manso, 2011).

In our baseline model, we consider a secondary market where the underwriter is banned

[1]The investment bank can also be viewed as an extant blockholder in the firm who makes decision on whether to support and participate in a seasoned security offering. See more discussion on the corporate governance implication of the blockholder on Page 8.

from selling the security short (or alternatively, short sale is prohibitively costly for him). In reality, it is almost impossible to sell certain assets such as loans short. Furthermore, short sale of securities by underwriters has long been contended as highly controversial and is viewed unfavorably by regulators as well as market participants. Moreover, the SEC has made an effort to restrict short sale of the ABS by securitization participants. For instance, in a proposed rule of "Prohibition against Conflicts of Interests in Certain Securitizations" in September 2011, they prohibit a large group of interested parties including underwriters from engaging in certain transactions, among which a particular one is short sale. Moreover, investors are fiercely opposed to short-selling securities by underwriters, and petitions from institutional investors to urge constraint on short sale in the City of London in recent years are common occurrences. In other financial markets such as the ones in China, short sale of any securities is strictly forbidden. This is why we primarily focus on the case in which there is a short sale constraint for the underwriters.

Like in Aghion, Bolton, and Tirole (2004), we assume that the underwriter's capital is scarce and he incurs an opportunity cost (i.e. cost of capital) proportional to his investment in the security. Consequently, even though the underwriter can free ride on the adverse selection discount, in equilibrium the additional cost due to the retention depresses his stake to the level that is just enough for him to camouflage as liquidity traders and gain from informed trading. Interestingly, a unique equilibrium of informed-sales arises naturally in which the underwriter liquidates his holdings if his private information indicates that the security will subsequently underperform, and he refrains from trading otherwise. Our results speak to the issues associated with the rise of the originate-to-distribute (OTD) lending model in debt markets (Bord and Santos, 2012). Because of the development of active secondary markets, banks' incentives to screen and monitor loans have diminished (Keys, Mukherjee, Seru, and Vig, 2010). Moreover, they tend to sell loans that are of excessively poor quality (Purnanandam, 2010), and underperform their peers by about 9% per year subsequent to the initial sales (Berndt and Gupta, 2009). To this end, our model fully captures the resultant adverse selection problem from OTD.

Working backward, we consider the optimal design of disclosure by the issuer. If she does not disclose additional information, the underwriter will choose to retain a stake only when the *ex ante* uncertainty about the security's payoff is relatively high. Because otherwise his private information has low value and his trading profits are not enough to compensate for his opportunity cost of investment. As a result, underpricing occurs only if the security is more risky. This is consistent with the evidence in Cai, Helwege, and Warga (2007) that find significant underpricing on speculative-grade debt offerings but no significant underpricing on investment-grade bond IPOs. Since the underpricing undermines the issuer's proceeds, she can do better by inducing posteriors beliefs which reduce the uncertainty to the degree that the investment bank is just indifferent between no retention and a positive stake. In this case, the optimal disclosure is partially informative. A sender-preferred equilibrium prescribes that the underwriter should not retain any share, thus no discount will occur in equilibrium.

Next we extend our model by introducing *demand uncertainty* (i.e. demand may fall short of supply) in the primary market. With a positive probability the shares net of the underwriter's planned retention cannot be fully subscribed by the participant investors. In order to complete the deal, the underwriter has to acquire all the remaining shares. Unlike before where the investment bank's decision to underwrite is trivial, the bank will shy away from the deal if his expected payoff is negative. This creates a hold-up problem arising from the possibility of demand shock. Intuitively, the bank will choose to underwrite and hold a stake only if uncertainty about the cash flows from the security is sufficiently high. Then, the underwriter is able to exploit his private information, and his expected trading gain is enough to offset his expected loss from excessive retention. Therefore, the issuer's optimal information design will be as follows. If the *ex ante* uncertainty about the security is so high that the investment bank is always willing underwrite, the issuer will design a signal system inducing posteriors beliefs which reduce the uncertainty to the level that makes the investment bank just indifferent between whether or not to underwrite. This in turn reduces adverse selection and increases the issuer's expected revenue. However, if the *ex ante* uncertainty about the security is relatively low, the investment bank will not underwrite unless the signal changes his prior. The issuer's overriding interest in this scenario is to be able to sell the security and maximize her expected payoff with strategic disclosure. Thanks to the Bayesian plausibility constraint which requires that the average posteriors to be equal to the prior, Bayesian persuasion by the issuer can induce the investment bank to underwrite with positive probability and balances this with a worse belief that leaves the bank's underwriting decision unchanged, which improves the issuer's expected payoff. The optimal disclosure is such that on the one hand it may induce the worst belief which leads to the investment bank's withdrawal from underwriting, but on the other hand it may generate signal that makes the investment bank just willing to underwrite at the relevant beliefs. At the latter belief, the security's uncertainty is in fact increased, and the underwriter's private information thus becomes sufficiently valuable again, although on average the disclosure system still reduces the uncertainty relative to that at the prior belief. Our model features an interesting mechanism where increased payoff uncertainty can mitigate the hold-up problem brought about by demand uncertainty. We contribute to the literature by demonstrating a possible way to avoid security issuance failure due to weak demand, and by offering alternative insight into the "pipeline risk" in Bruche, Malherbe, and Meisenzahl (2018), where they document the successful issuance of leveraged syndicated loans along with the costly excessive retention by the underwriting banks. We argue that it may stem from the fact that the banks are successfully persuaded by the borrowers albeit the presence of high demand uncertainty.

Our model yields novel empirical predictions that relate the informativeness of the optimal disclosure to various aspects of the primary and secondary markets. We show that the effects are not simply monotonic and depend on the *ex ante* uncertainty of the security's payoff. Specifically, when the *ex ante* payoff uncertainty is relatively high, both better growth option of the firm/borrower and more secondary market liquidity lead to more transparent disclosure. Conversely, greater issue size, larger cost of underwriting bank's capital, higher probability of demand shock, and weaker demand are associated with less informative disclosure. Better growth option and more liquidity allow the underwriter to enjoy more profits by trading on his private information. Hence the optimal system only needs to induce less uncertainty at posteriors that make the underwriter just break-even. In contrast, larger issue size and cost of underwriting bank's capital make it more costly for the underwriter to hold a stake in order to gain from informed trading. Thus more uncertainty should be introduced to make the underwriter's private information more valuable. Likewise, higher probability of demand shock and weaker demand make it more costly for the bank to underwrite, thus the optimal system should induce beliefs with higher uncertainty so that his stake carries more trading value in the secondary market. Our result is similar to the model of Pagano and Volpin (2012) which shows that coarse information enhances primary market liquidity at the cost of reducing secondary market liquidity. In contrast, the motivation for the revelation of coarse information in our model is to solve the hold-up problem and promote an active primary market with the underwriter's participation. Moreover, the issuer cannot control over the realizations of the signal, thus the coarse information does not come with certainty.

The results for the security with *ex ante* relatively low payoff uncertainty in the presence of demand uncertainty is just the opposite: better growth option and more liquidity dampen the informativeness of the disclosure, while greater issue size, larger cost of capital, higher probability of demand shock, and weaker improve the informativeness. Especially noteworthy is that in the latter cases although the overall uncertainty is reduced by the optimal disclosure, to attract the bank to underwrite, the inherent uncertainty at the posterior beliefs that make the bank just indifferent actually becomes larger than that at the prior. The uncertainty at these posteriors should vary according to the intuition discussed in the previous paragraph. But the informativeness hinges on how *dispersed* the distribution of the posteriors is.

Finally, we extend our model by relaxing the assumption on the short sale constraint in the secondary market. Without short-sale constraint, it is optimal for the underwriter not to acquire any security in the primary market, but to exploit his private information by selling the asset short in the secondary market. If there is no demand uncertainty, only a fully informative disclosure can deter the underwriter from engaging in informed trading. Nevertheless, when demand is uncertain, all of the results on optimal disclosure we have obtained with short-selling constraint extends to the case without it. Compared with the case where short sale is prohibited, the issuer only needs less transparent disclosure to persuade the investment bank to underwrite when the uncertainty about the security's payoff is relatively low. But she has to design more transparent disclosure to alleviate adverse selection when the payoff uncertainty is relatively high.

Our paper is related to several strands of the literature. First, our work contributes to the theoretical literature that attempts to address the question of how the rapidly evolving debt markets can go awry (e.g. Chemla and Hennessy, 2014; Pagano and Volpin, 2012; Parlour and Plantin, 2008). We model the adverse selection problem in the OTD lending model, and show that strategic disclosure not only benefits the issuer, but also reduces this informational friction.

Second, our theoretical framework enriches the large literature on blockholders' governance by exit (e.g. Aghion, Bolton, and Tirole, 2004; Edmans and Manso, 2011; Faure-Grimaud and Gromb, 2004). Importantly, the applicability of our model naturally goes beyond debt markets and extends to equity markets if we view the underwriter as an extant blockholder in a firm. Under this interpretation, we model the blockholder's decision to support and participate in a security offering (e.g. seasoned equity offering). As long as he participates, the blockholder has an informational advantage over other dispersed investors from holding and learning. As we have explained, he can exert governance by exit to push the firm to *ex ante* disclose more transparent information when the payoff uncertainty of the security is relatively high.

Third, our paper adds to a growing body of literature on information design theory (e.g.

Kamenica and Gentzkow, 2011; Alonso and Câmara, 2016; Bergemann and Morris, 2018; Rayo and Segal, 2010) as well as its application in corporate finance (e.g. Azarmsa, 2017; Azarmsa and Cong, 2018; Boleslavsky, Carlin, and Cotton, 2017; Goldstein and Leitner, 2018; Huang, 2016; Szydlowski, 2016). We extend the basic Bayesian persuasion framework by including a second receiver (the participant investors) who indirectly affects the welfare of both the sender and the first receiver.

Fourth, our theoretical analysis offers new insight to the empirical literature on the effect of disclosure on liquidity (e.g. Balakrishnan, Billings, Kelly, and Ljungqvist, 2014). In contrast with the extant literature, we focus on how firms will design their disclosure in security issuance when faced with varying market liquidity. Our model provides a rationale for whether a liquid secondary market contributes to a better information environment of the issuing firm. To our best knowledge, we are the first to consider the security issuer's optimal design of information disclosure in the presence of both the financing and the trading frictions. We thus call for empirical investigations of the relationship between the informativeness of disclosure (through the lens of the information memoranda and the prospectuses) and the subsequent market activities as predicted in our model.

This paper is organized as follows. Section 2 introduces the setup of the model. Section 3 solves for the secondary market trading equilibrium and the primary market issue price given an active secondary market. Section 4 presents the core results of the model with a secondary market that has short sale constraint. The equilibrium disclosure policies are analyzed both with and without demand uncertainty in the primary market. Section 5 changes the secondary market structure by removing the short sale ban and solve for the optimal disclosure policies. Section 6 conducts welfare analysis for the investment bank and the issuer under different primary market conditions and secondary market structures. Section 7 concludes. All proofs not in the main body of the paper are deferred to the Appendix.

2 The Model

The model has four dates and no discounting. There are three types of players: an issuer, an investment bank, and a group of investors, all of whom are risk-neutral.

2.1 The Issuer

The issuer (also called "she" or "firm") wants to sell claims to cash flows from a productive asset. Examples of such claims include bonds, (syndicated/securitized) loans, or equity stocks. For brevity, we shall simply call them securities. We normalize the number of securities to be issued to 1. The state of the economy ω is binary: it can be Good (G) or Bad (B) with prior probability distribution $\mathbb{P}[\omega = G] = \mu_0$ and $\mathbb{P}[\omega = B] = 1 - \mu_0$ respectively. Cash flows $\tilde{v}$ from state B and state G are $V_H \equiv V_L + \Delta V$ and V_L respectively.

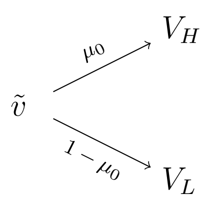


Figure 1: Cash Flows Distribution under the Prior

The issuer designs an experiment which we refer to as a disclosure system π with binary signal $s \in \{h, \ell\}$. The signal realization follows the conditional distribution: $\pi_G \equiv \mathbb{P}[s = h|\omega = G] \geq \pi_B \equiv \mathbb{P}[s = h|\omega = B]$, which also represents the precision of the system. Figure 2 illustrates how the disclosure system maps each state to a signal. Using Bayes' rule, the posteriors μ_s upon observing $s \in \{h, \ell\}$ are

$$\mu_h \equiv \mathbb{P}[\omega = G|s = h] = \frac{\pi_G \mu_0}{\pi_G \mu_0 + \pi_B (1 - \mu_0)},$$

$$\mu_\ell \equiv \mathbb{P}[\omega = G|s = \ell] = \frac{(1 - \pi_G)\mu_0}{(1 - \pi_G)\mu_0 + (1 - \pi_B)(1 - \mu_0)}.$$

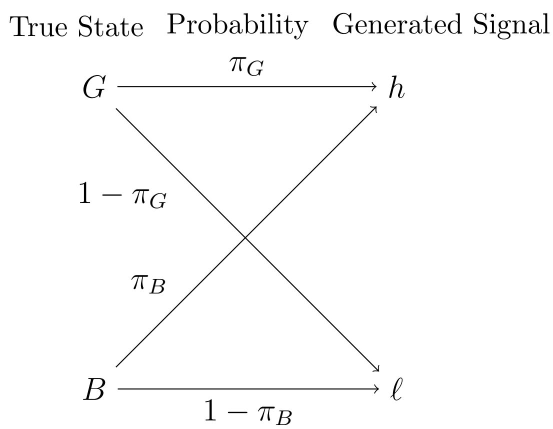


Figure 2: The disclosure system π

Moreover, Bayesian updating requires that the average posterior is consistent with the prior, which gives the Bayesian plausibility condition:

$$\mathbb{P}[s=h]\cdot\mu_h+\mathbb{P}[s=\ell]\cdot\mu_\ell=\mu_0.$$

Therefore, the information design problem for the issuer is equivalent to choosing a pair of posteriors $\{\mu_h,\mu_\ell\}$ whose distribution must satisfy the above constraint.

2.2 Informativeness of the Disclosure System

Following Gentzkow and Kamenica (2014), we use the entropy measure to gauge the uncertainty associated with a given belief. In our binary-state economy, if the belief that the state is G conditional on observing s is μ_s, its entropy is $H(\mu_s)=-\mu_s\ln\mu_s-(1-\mu_s)\ln(1-\mu_s)$. Hence the belief achieves the highest uncertainty when $\mu_s=1/2$, and the closer it is to the endpoints of its support (i.e. 0 or 1), the less uncertain the belief is. And the informativeness of a disclosure system π is measured as the reduction in entropy $L(\pi)=H(\mu_r)-\mathbb{E}_{\langle\pi|\mu_r\rangle}[H(\mu_s)]$, where μ_r is a fixed reference belief independent of the system π, and the subscript $\langle\pi|\mu_r\rangle$ indicates that the expectation is taken under the distribution of posteriors (i.e. the probabilities of $s = h$ and $s = \ell$) given the reference prior μ_r.[2]

The fact that the above $L(\pi)$ function is convex in μ_s implies a simpler yet more intuitive interpretation of the informativeness: the more dispersed the distribution of posteriors, the more informative the disclosure system. Formally, consider two systems π and π' with possible signal realizations $\{h, \ell\}$ and $\{h', \ell'\}$, and induced posteriors $\{\mu_h, \mu_\ell\}$ and $\{\mu_{h'}, \mu_{\ell'}\}$. Suppose that

$$0 \leq \mu_\ell \leq \mu_{\ell'} \leq \mu_{h'} \leq \mu_h \leq 1$$

with either the second or fourth inequality (or both) holding strictly, then we claim that system π is more informative than system π' in the spirit of Blackwell (1951). Furthermore, from the Bayesian updating formulas of the two posteriors, both a higher π_G and a lower π_B imply a more informative signal system. It is because such changes in the precision parameters lead to a higher μ_h and a lower μ_ℓ, which are consistent with our definition of the informativeness above. In this paper we use "informativeness" and "transparency" interchangeably to describe the quality of a disclosure system.

2.3 The Investment Bank and the Participant Investors

In addition to the issuer, there are two other types of players: an investment bank and a group of participant investors. To issue the securities, the issuer has to find an investment bank (also called "underwriter" or "he") to help her underwrite the deal in the primary market. The investment bank can be an underwriting bank in a public offering of bond or equity, a lead bank in loan syndication, or an arranger in securitization. The issuer reveals the disclosure system π to the investment bank. The investment bank then engages in due diligence to find out the realization of the signal s. After observing s the investment bank makes decision on whether to underwrite. If he agrees to underwrite, he further chooses the fraction of securities β to retain. Instead, he can also withdraw from underwriting if he finds

[2] The introduction of this reference belief μ_r ensures that the disclosure informativeness does not vary with the prior μ_0.

it unfavorable, and thus the issue fails.[3] We denote the action set of the investment bank as follow

$$a_{IB} \in \{(\text{Underwrite \& Retain } \beta), (\text{Not Underwrite})\}.$$

Following Aghion, Bolton, and Tirole (2004), we assume that capital is scarce for the investment bank and he incurs an opportunity cost (i.e. cost of capital) $r > 0$ per unit of investment.[4] Moreover, there are a unit mass of participant investors who can also invest in a risk-free asset with zero return. They will invest in the remaining $(1-\beta)$ shares as long as they are break-even.

2.4 Time Line

At $T = 0$, nature determines the prior distribution of the states. The issuer designs a signal system π which will generate a signal s at $T = 1$. She finds an investment bank and reveals this experiment π to him.

At $T = 1$, signal s realizes. The investment bank first engages in due diligence to discover s and then decides if he will underwrite the issuance. If the investment bank chooses to underwrite, he materializes and communicates the signal s to participant investors. He sells $(1-\beta)$ to the participant investors and acquires the remaining β, both at price P_0.

At $T = 2$, a secondary market opens. The market structure is like Kyle (1985). The investment bank and the participant investors submit their market orders to a continuum of deep-pocketed risk-neutral market makers (MM) who price the security competitively after observing the total net order flow y. The market maker sets price $P_1 = \mathbb{E}[\tilde{v}|y]$. The trading episode proceeds with three sub-stages:

[3] In practice when primary market demand for the security is weak and the underwriter is not willing to retain additional shares, he may choose to delay (suspend) the issuance indefinitely, and only to close the deal when the securities can be fully subscribed. For simplicity, we also regard this scenario as failure

[4] We assume throughout the paper that the investment bank will always incur this opportunity cost of his capital expenditure in both the primary and the secondary markets. This helps to eliminate multiple equilibria in the secondary market. Removal of such assumption in the secondary market does not affect the equilibrium we will characterize. Moreover, r cannot be too large as otherwise the investment bank will always find it unfavorable to underwrite. We characterize the exact requirements that r should satisfy in order to ensure the existence of interior solutions of the model in the appendix.

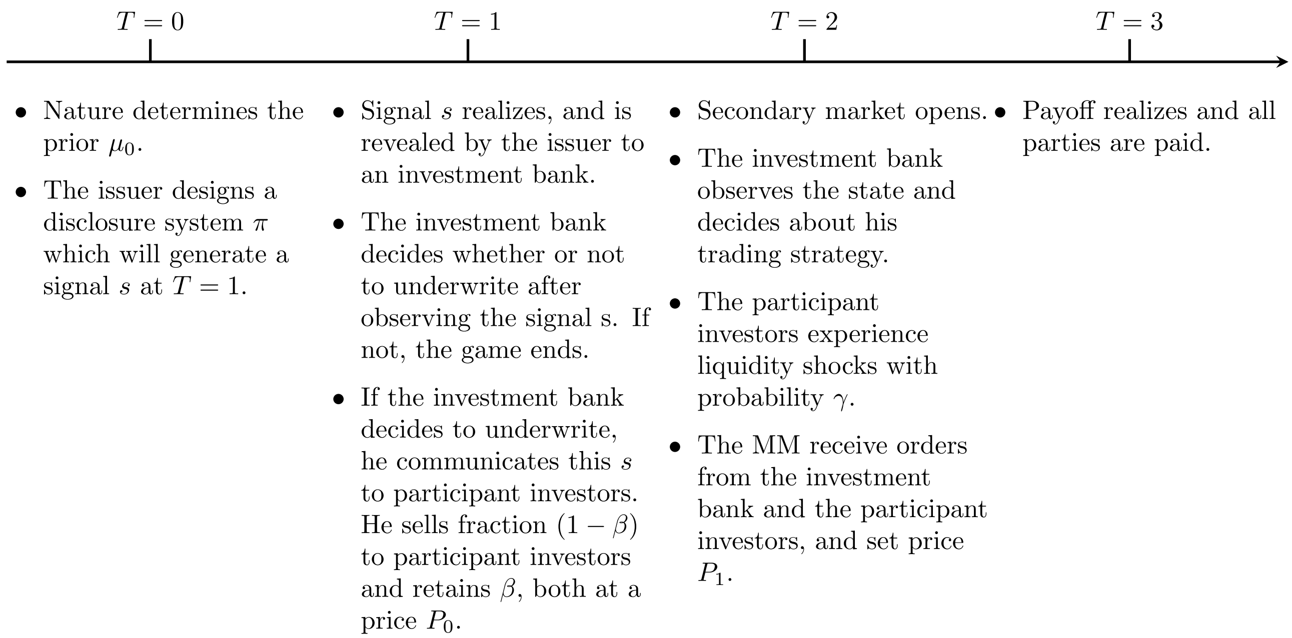


Figure 3: Time line

1. The investment bank observes the true state ω and determines his trading strategy, i.e. the amount of securities $\{x_{IB}\}$ to trade.

2. Liquidity shocks happen with probability $\gamma \in (0, 1)$. The participant investors submit their aggregate market order $\{x_{PI}\}$, whereby

   a. with probability γ a fraction $\phi \in (0, \frac{1}{2})$ of the participant investors experience liquidity shocks and have to liquidate their holdings;

   b. with probability $(1-\gamma)$, there is no liquidity shock and these participant investors don't sell.

3. The MM receive the net order flow from the investment bank and the participant investors $y \equiv x_{IB} + x_{PI}$, and set P_1.

At $T = 3$, payoffs of the underlying securities are realized, and all parties get paid.

The time line is summarized in Figure 3.

2.5 Payoff Functions

We next define the expected payoff functions of the issuer, the investment bank, and the participant investors at $T = 1$ in the primary market. Consider the situation after the signal s has realized. The issuer's expected payoff is

$$U_E(\beta, \mu_s) = \mathbb{1}_{a_{IB}=\{\text{Underwrite, } \beta\}} P_0.$$

$\mathbb{1}_{a_{IB}=\{\text{Underwrite, } \beta\}}$ is an indicator function which takes value 1 if $a_{IB} = \{\text{Underwrite, } \beta\}$ (i.e. the investment bank underwrites and acquires β), and 0 otherwise. Since the investment bank will make his underwriting and retention decisions after observing s, it follows that a_{IB} will be a function of posterior belief μ_s. P_0 is the price of the securities and the money she will obtain in the primary market conditional on the investment bank choosing to underwrite. We follow the Bayesian persuasion literature (e.g. Kamenica and Gentzkow, 2011; Huang, 2016; Szydlowski, 2016) by assuming that information design incurs no cost, and when the issuer is indifferent between two disclosure systems, she always selects the one that is less informative.[5]

Back to $T = 0$ when the issuer designs the disclosure system π, she rationally anticipates the best response by the investment bank conditional on induced posterior belief. Her expected payoff is therefore

$$\mathbb{E}_\pi[U_E(a_{IB}, \mu_s)].$$

[5]This assumption ensures the tractability of our model as well as the uniqueness of the equilibrium. Alternatively, we can define the issuer's expected payoff as

$$U_E(\beta, \mu_s) = \mathbb{1}_{a_{IB}=\{\text{Underwrite, } \beta\}} P_0 - C,$$

where C represents a sunk cost of disclosure which varies with the informativeness of the disclosure system π as in Gentzkow and Kamenica (2014):

$$C(\pi) \equiv kL(\pi) = k\{H(\mu_r) - \mathbb{E}_{\langle\pi|\mu_r\rangle}[H(\mu_s)]\}.$$

Note that $k > 0$ is the cost of a one-unit reduction in entropy. Therefore, at $T = 0$ when two disclosure systems deliver the issuer the same expected proceeds, she prefers the one that is less informative and thus less costly. When the unit cost $k \to 0^+$, the optimal disclosure policies converge to the ones in our paper. And for small k our main intuitions still go through and thus our results are robust to costly information disclosure.

And the subscript π implies that the expectation is taken under the distribution of signal realizations (posteriors).

The investment bank's expected payoff after observing s depends on whether he becomes an underwriter as well as his retention β if he chooses to underwrite:

$$U_{IB}(a_{IB}) = \mathbb{1}_{a_{IB}=\{\text{Underwrite, }\beta\}} \times \{\beta[(\mu_s \Delta V + V_L) - (1+r)P_0] + \mathbb{E}_s[\Pi]\},$$

where $\beta[(\mu_s \Delta V + V_L) - (1+r)P_0]$ is his net payoff from retaining β shares in the primary market, and $\mathbb{E}_s[\Pi]$ is his expected trading profits in the secondary if there is any at $T = 2$. Here the subscript s implies that we take the expectation under the distribution of underlying states induced by signal s.

Finally, for the participant investors to acquire the remaining $(1 - \beta)$ shares, they will demand a price P_0 which makes them at least break even. Therefore the issuer will offer a price such that their expected payoff is $U_{PI}(\beta, \mu_s) = 0$.

3 Secondary Market Trading and Primary Market Discount

In this section, we solve for the subgame perfect equilibrium of the game by backward induction. Suppose that the investment bank chooses to underwrite at $T = 1$. Then at $T = 2$, the disclosure system π, the signal realization s, the share price P_0 in the primary market, and the investment bank's retention β are all taken as given.

Now that the investment bank has observed the true underlying state at $T = 2$, he decides about the optimal market order x_{IB} he should submit. We characterize the unique informed-sale equilibrium where the investment bank do not trade in state G and sell $(1-\beta)\phi$ in state B as follows.

In state G, the true value of the security is V_H. The investment bank has no incentive to sell simply because the secondary market price cannot exceed the security's intrinsic value, i.e. $P_1 \leq V_H$. Moreover, the investment bank has no incentive to purchase additional shares in this state too. This is because if he buys, the aggregate order flow $y > -u$ if liquidity shocks happen, and $y > 0$ if there is no liquidity shock. In order to pool in state B, he may need to buy shares too. Yet he could lose money because the cash flow in state B is only V_L but the price $P_1 \geq V_L$, and buying in bad state is thus sequentially irrational. Therefore, although he could gain in state G he would suffer a loss in state B. Such cross-subsidization renders him at most the same expected net trading profits as in the informed-sale equilibrium while his purchases incur additional opportunity cost.[6] And such trading strategy is obviously sub-optimal. Accordingly, in state G when there are liquidity shocks the net order flow will be $y = -u$, yet it will be $y = 0$ if there is no liquidity shock.

In state B, since the price is always at least as much as the security's intrinsic value (i.e. $P_1 \geq V_L$), the investment bank can potentially benefit from sale. The maximal amount that can be sold in order to at least partially conceal his private information is u. In this case the aggregate order flow will be $y = -2u$ if participant investors are hit by liquidity shocks, and $y = -u$ otherwise. Therefore, the MM cannot tell which state the economy is in when the net order flow is $-u$, and the investment bank enjoys informed trading profits if the true state happens to be bad.

In sum, to best exploit his private information, the investment bank refrains from trading in good state and liquidates $(1-\beta)\phi$ in bad state to maximize his expected informed trading profits while not fully reveal his identity.

We tabulate the equilibrium in the secondary market in Table 1, and summarize in the following proposition.

Proposition 1 *(Secondary market equilibrium):*

1. *The investment bank's optimal trading strategy is to submit an order $x_{IB} = 0$ in state*

[6] Recall that the investment bank will also incur opportunity cost as long as he acquires shares in the secondary market, although the informed-sale equilibrium is robust to the removal of the assumption about the investment bank's opportunity cost in the secondary market.

Table 1: Secondary Market Trading and Pricing

State	Liquidity Shocks	$\tilde{v}$	Probability	x_{PI}	x_{IB}	y	P_1
G	Yes	V_H	$\mu_s\gamma$	$-u$	0	$-u$	$\frac{\mu_s\gamma\Delta V}{\mu_s\gamma+(1-\mu_s)(1-\gamma)}+V_L$
G	No	V_H	$\mu_s(1-\gamma)$	0	0	0	V_H
B	Yes	V_L	$(1-\mu_s)\gamma$	$-u$	$-u$	$-2u$	V_L
B	No	V_L	$(1-\mu_s)(1-\gamma)$	0	$-u$	$-u$	$\frac{\mu_s\gamma\Delta V}{\mu_s\gamma+(1-\mu_s)(1-\gamma)}+V_L$

Note: $u \equiv (1-\beta)\phi$.

G, and an order $x_{IB} = -(1-\beta)\phi$ in state B.

2. *The MM's posterior belief about the probability of state G is*

$$\mu_{MM} = \begin{cases} 1 & \text{if } y = 0, \\ \frac{\mu_s\gamma}{\mu_s\gamma+(1-\mu_s)(1-\gamma)} & \text{if } y = -(1-\beta)\phi, \\ 0 & \text{if } y = -2(1-\beta)\phi. \end{cases}$$

3. *The MM set price*

$$P_1 = \begin{cases} V_H & \text{if } y = 0, \\ \frac{\mu_s\gamma\Delta V}{\mu_s\gamma+(1-\mu_s)(1-\gamma)} + V_L & \text{if } y = -(1-\beta)\phi, \\ V_L & \text{if } y = -2(1-\beta)\phi. \end{cases}$$

Having obtained the trading equilibrium, we now derive the primary market issue price taking into account the adverse selection in the secondary market. Recall from Table 1 that the investment bank's trading strategy mixes case {State G, Liquidity Shocks} with case {State B, No Liquidity Shocks}, and he only makes profits in the second case where he manages to camouflage as liquidity traders. His informed-sale profits per share are

$$G \equiv P_1 - V_L = \frac{\mu_s \gamma \Delta V}{\mu_s \gamma + (1-\mu_s)(1-\gamma)}.$$

The next proposition derives the investment bank's total expected trading profits and the primary market issue price when he observes signal s at $T = 1$.

Proposition 2 *(Expected trading profits, and Primary market underpricing):*

1. *The investment bank's total expected trading profits are*

$$\mathbb{E}_s[\Pi] = (1-\beta)\phi\ \mathbb{E}_s[G] = \frac{(1-\beta)\phi(1-\mu_s)(1-\gamma)\mu_s\gamma\Delta V}{\mu_s\gamma + (1-\mu_s)(1-\gamma)}.$$

2. *Since the investment bank's gain per share is just the participant investors' loss per share, in order for these investors to purchase at $T = 1$,*

$$\begin{aligned} P_0 &\equiv \mathbb{E}_s[\tilde{v}] - \Delta P \\ &= (\mu_s \Delta V + V_L) - \frac{\mathbb{E}_s[\Pi]}{1-\beta} \\ &= (\mu_s \Delta V + V_L) - \frac{(1-\mu_s)\mu_s(1-\gamma)\gamma\phi\Delta V}{\mu_s\gamma + (1-\mu_s)(1-\gamma)}. \end{aligned}$$

The fact that securities are issued with a discount due to adverse selection in the secondary market has been commonly in the literature (e.g. Holmström and Tirole, 1993; Maug, 1998; Edmans and Manso, 2011).

4 Short Sale Constraint (SS)

As we will see, whether short sale by the underwriter is allowed in the secondary market has somewhat different implications for the equilibrium in the primary market at $T = 1$ as well as the issuer's choice of optimal disclosure policy at $T = 0$. Note that whether there is short sale constraint in the secondary market does not affect the equilibrium strategies we have characterized in the previous section. We first consider the baseline model where the investment bank cannot sell the security short. Then we proceed with the model in which there is no short sale constraint.

The next lemma establishes the condition under which strategic trading by the investment bank is feasible when there is short sale constraint in the secondary market.

Lemma 1 *(Minimal stake): When selling the security short is not allowed in the secondary market, the investment bank can engage in strategic informed trading iff* $\frac{\phi}{1+\phi} \leq \beta < 1$.

Suppose that part of the participant investors are hit by liquidity shocks. They will liquidate a fraction of $u \equiv (1-\beta)\phi$ shares in total. To gain informed trading profits, the investment bank has to camouflage as liquidity traders. Because he cannot short sell, to achieve this goal his holdings β should not be too samll, i.e. no less than $(1-\beta)\phi$. Also note that β should be strictly less than 1 because otherwise the market is completely illiquid and there will be no liquidity traders.

4.1 No Demand Uncertainty (NDU)

In this section we first consider the benchmark model where there is no demand uncertainty in the primary market, i.e. all the shares can be fully subscribed by the participant investors even if the investment bank does not acquire any.

At $T = 1$, from Lemma 1 we have already established that when $\beta \in [0, \frac{\phi}{1+\phi})$ or $\beta = 1$, the investment bank cannot gain from trading on his private information, because either his stake is not enough or the secondary market is completely illiquid. Thus the issue price will not include the adverse selection discount. The following proposition characterizes the price in the primary market for different levels of retention by the investment bank.

Proposition 3 *(Primary market issue price): The issue price in the primary market is*

$$P_0(\beta, \mu_s) = \begin{cases} \mu_s \Delta V + V_L - \frac{(1-\mu_s)\mu_s(1-\gamma)\gamma\phi\Delta V}{\mu_s\gamma+(1-\mu_s)(1-\gamma)} & \text{if } \beta \in [\frac{\phi}{1+\phi}, 1), \\ \mu_s \Delta + V_L & \text{if } \beta \in [0, \frac{\phi}{1+\phi}) \text{ or } \beta = 1. \end{cases}$$

4.1.1 Investment Bank's Optimal Decision I

Absent any demand uncertainty, the investment bank can always stay break-even by choosing to underwrite yet retaining no shares. Therefore, the investment bank's decision to underwrite is trivial in our benchmark model here.

At $T = 1$ after signal s has realized and posterior belief μ_s has been formed, the investment bank decides on his stake β to maximize his expected payoff, denoted $U^1_{IB}(\beta, \mu_s)$:

$$\max_{\beta\in[0,1]} \left\{ \begin{array}{c} \beta \cdot [(\mu_s \Delta V + V_L) - (1+r)P_0(\beta, \mu_s)] \\ + \mathbb{1}_{\{\beta \geq (1-\beta)\phi\}} \cdot (1-\beta)\phi \cdot \frac{(1-\mu_s)\mu_s(1-\gamma)\gamma\Delta V}{\mu_s\gamma+(1-\mu_s)(1-\gamma)} \end{array} \right\}.$$

The first term above represents the investment bank's expected payoff in the primary market which is the intrinsic value of the β shares net of his capital expenditure and opportunity cost. The second term is his expected trading profits as we have shown in Proposition 2 if he has acquired adequate stake in the primary market. Observe that the above expected utility function $U^1_{IB}(\beta, \mu_s)$ is in fact piece-wise linear in β. Hence its maximum must be attained at $\beta^* = 0$, or 1, or $\frac{\phi}{1+\phi}$, or $\beta^* \uparrow 1$ (i.e. $\beta^* = 1^-$). The investment bank's optimal retention problem thus becomes

$$\beta^* = \underset{\beta\in\{0,1,\frac{\phi}{1+\phi},1^-\}}{\arg\max} \left\{ U^1_{IB}(0, \mu_s), U^1_{IB}(1, \mu_s), U^1_{IB}(\frac{\phi}{1+\phi}, \mu_s), U^1_{IB}(1^-, \mu_s) \right\}.$$

The investment bank's expected payoff $U^1_{IB}(\beta, \mu_s)$ is calculated as follows:

(i). If $\beta^* = 0$, there will be no informed trading in the secondary market and no price discount in the primary market, $U^1_{IB}(0, \mu_s) = 0$.

(ii). If $\beta^* = 1$, the secondary market is completely illiquid and the issue price has no discount,

$$U^1_{IB}(1, \mu_s) = (\mu_s \Delta V + V_L) - (1+r)P_0(1, \mu_s) = -r(\mu_s \Delta V + V_L).$$

(iii). If $\beta^* = \frac{\phi}{1+\phi}$, informed trading is feasible and thus issue price must be discounted,

$$U^1_{IB}(\frac{\phi}{1+\phi}, \mu_s) = \frac{\phi}{1+\phi}\left[(\mu_s \Delta V + V_L) - (1+r)P_0(\frac{\phi}{1+\phi}, \mu_s)\right]$$
$$+\frac{1}{1+\phi} \cdot \frac{(1-\mu_s)\mu_s(1-\gamma)\gamma\phi\Delta V}{\mu_s\gamma + (1-\mu_s)(1-\gamma)}.$$

(iv). Finally, if $\beta^* = 1^-$, there is (infinitesimal) informed trading profit yet still a relatively sizable adverse selection discount,

$$\begin{aligned} U^1_{IB}(1^-, \mu_s) &= 1^- \cdot [(\mu_s \Delta V + V_L) - (1+r)P_0(1^-, \mu_s)] + 0^+ \cdot \frac{(1-\mu_s)\mu_s(1-\gamma)\gamma\phi\Delta V}{\mu_s\gamma + (1-\mu_s)(1-\gamma)} \\ &\doteq (\mu_s \Delta V + V_L) - (1+r)P_0(1^-, \mu_s) \\ &= (\mu_s \Delta V + V_L) - (1+r)\left[(\mu_s \Delta V + V_L) - \frac{(1-\mu_s)\mu_s(1-\gamma)\gamma\phi\Delta V}{\mu_s\gamma + (1-\mu_s)(1-\gamma)}\right]. \end{aligned}$$

To pin down the optimal retention by the investment bank in response to the observed signal s, it suffices to show for different μ_s which of the above U^1_{IB}'s achieve the largest value. The lemma below provides some important properties of the investment bank's expected payoff function if he chooses to retain $\beta = \frac{\phi}{1+\phi}$.

Lemma 2 *(Indifference cut-off posteriors I):*

1. *There exists a pair* $\{\underline{\mu}, \overline{\mu}\}$ *with* $0 < \underline{\mu} < \frac{1}{2} < \overline{\mu} < 1$ *such that* $U^1_{IB}(\frac{\phi}{1+\phi}, \underline{\mu}) = U^1_{IB}(\frac{\phi}{1+\phi}, \overline{\mu}) = 0$.

2. $U^1_{IB}(\frac{\phi}{1+\phi}, \mu_s) > 0$ *if* $\mu_s \in (\underline{\mu}, \overline{\mu})$*, and* $U^1_{IB}(\frac{\phi}{1+\phi}, \mu_s) < 0$ *if* $\mu_s \in [0, \underline{\mu})$ *or* $\mu_s \in (\overline{\mu}, 1]$*.*

Therefore, at posteriors $\mu_s = \underline{\mu}$ and $\overline{\mu}$, the investment bank is indifferent between holding $\beta = \frac{\phi}{1+\phi}$ and $\beta = 0$. Furthermore, the investment bank will only consider purchasing a fraction of the shares when uncertainty about the security is large (i.e. the posterior belief μ_s lies in an intermediate range).

The following proposition characterizes the investment bank's optimal strategy and the relevant equilibrium payoffs under different posterior beliefs.

Proposition 4 *(Investment bank's optimal strategy and relevant payoffs I): The investment bank's optimal stake is*

$$\beta^* = \begin{cases} \frac{\phi}{1+\phi} & \text{if } \mu_s \in (\underline{\mu}, \overline{\mu}), \\ 0 & \text{if } \mu_s \in [0, \underline{\mu}] \cup [\overline{\mu}, 1]. \end{cases}$$

His equilibrium payoff is

$$\hat{U}^1_{IB}(\mu_s) = \begin{cases} U^1_{IB}(\frac{\phi}{1+\phi}, \mu_s) & \text{if } \mu_s \in (\underline{\mu}, \overline{\mu}), \\ 0 & \text{if } \mu_s \in [0, \underline{\mu}] \cup [\overline{\mu}, 1]. \end{cases}$$

In Figure 4 the blue line shows the payoff of the investment bank if he chooses to retain $\beta = \frac{\phi}{1+\phi}$, i.e. $U^1_{IB}(\frac{\phi}{1+\phi}, \mu_s)$. The red dashed line depicts his equilibrium payoff under his optimal retention strategy, denoted by $\hat{U}^1_{IB}(\mu_s)$. In equilibrium when $\mu_s \in [0, \underline{\mu}] \cup [\overline{\mu}, 1]$, the investment bank does not retain any share, and his payoff is zero. Yet when $\mu_s \in (\underline{\mu}, \overline{\mu})$, he chooses his retention $\beta = \frac{\phi}{1+\phi}$ and his payoff is $U^1_{IB}(\frac{\phi}{1+\phi}, \mu_s)$, which corresponds to the hump-shaped part of the red dashed line. So in equilibrium both the investment bank's optimal stake and his expected payoff depend only on his belief μ_s.

The intuition of Proposition 4 is straightforward: when uncertainty about the security's payoff is relatively small, the investment bank's informed trading profits in the secondary market is not enough to cover his cost of capital in the primary market, even though he

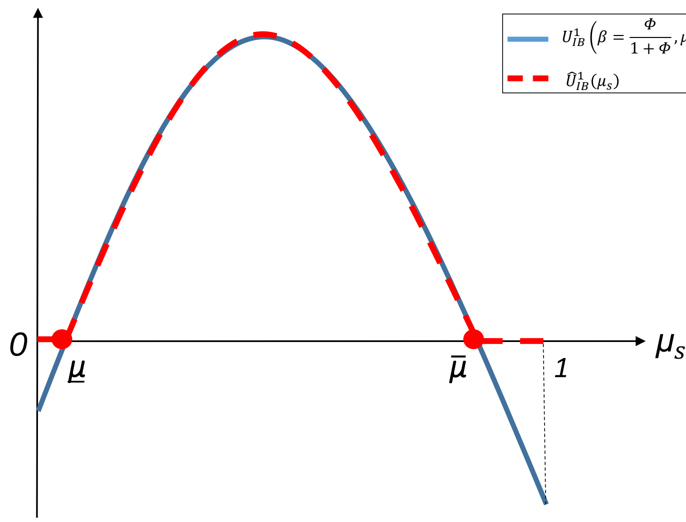


Figure 4: The investment bank's payoff (i)

free rides on the discounted issue price. This results in zero retention by the bank. When the uncertainty about the security is relatively large, it is profitable for the investment bank to acquire some shares in order to later trade on his private information strategically. Yet such gain in the secondary market trades off against the opportunity cost incurred from his primary market capital expenditure. In equilibrium the investment bank optimally chooses his retention such that it is just enough for him to camouflage as liquidity traders in the secondary market. This minimizes his total cost of capital while maximizes his expected trading profits. Our result contrasts with the retention equilibrium in Leland and Pyle (1977) where a firm holds a large fraction of its shares to have some skin in the game and signal to the market its quality when information asymmetry problem is severe. In our model, the investment bank acquires a stake to later gain from informed sales in the secondary market when the security's cash flows are relatively more uncertain. In this regard, such retention exacerbates the adverse selection problem.

4.1.2 Optimal Disclosure System I

Given the optimal retention scheme by the investment bank described in Proposition 4, it follows naturally that the issuer's expected revenue conditional on signal s at $T = 1$ will be either the intrinsic value of the security if the bank does not acquire any share, or the expected cash flows from the security net of an adverse selection discount if the bank holds a positive stake $\frac{\phi}{1+\phi}$. This gives the following proposition.

Proposition 5 *(Issuer's payoff after information design I): At $T = 1$ the issuer's expected payoff conditional on signal s is:*

$$U_E^1(\mu_s) = \begin{cases} \mu_s \Delta V + V_L - \frac{(1-\mu_s)\mu_s(1-\gamma)\gamma\phi\Delta V}{\mu_s\gamma+(1-\mu_s)(1-\gamma)} & \text{if } \mu_s \in (\underline{\mu}, \overline{\mu}), \\ \mu_s \Delta V + V_L & \text{if } \mu_s \in [0, \underline{\mu}] \cup [\overline{\mu}, 1]. \end{cases}$$

Note that at the two posteriors $\underline{\mu}$ and $\overline{\mu}$, the investment bank is actually indifferent between retaining 0 and a positive stake $\frac{\phi}{1+\phi}$. Following the convention of information disclosure literature, we select the sender-preferred equilibrium in which the investment bank does not acquire any share in the primary market when he is indifferent, and thus there will be no discount. In reality, given the high cost of bank capital, we have reason to believe that if the issuer is not opposed to it, investment banks are more prone to no retention although a positive stake gives him the same expected payoff.

At $T = 0$ the issuer designs the optimal disclosure policy to maximize her expected proceeds from issuing the security. She has to choose the precision of her signal π_G and π_B for the disclosure system π. By Bayes' rule, essentially her problem is equivalent to the optimal choice of two posteriors μ_h and μ_ℓ.

Because we have assumed that demand never falls short of the supply in the primary market, the investment bank does not have to worry about the risk of retaining more shares than his privately optimal level. Thus he will always underwrite, and his decision problem is reduced to the choice of stake β. And we can write the issuer's payoff at $T = 1$ as

$$U_E^1(\beta, \mu_s) = \mathbb{1}_{a_{IB}=\{\text{Underwrite, } \beta\}} P_0(\beta, \mu_s).$$

Since we already know from Proposition 4 that the investment bank's optimal retention β^* depends on μ_s, the issuer's expected proceeds will only depend on μ_s in equilibrium, which we denote by $U_E^1(\mu_s) \equiv U_E^1(\beta^*, \mu_s) = P_0(\beta^*, \mu_s)$. So the issuer solves the following maximization problem:

$$\hat{U}_E^1(\mu_0) \equiv \max_{\{\mu_\ell, \mu_h\}} \mathbb{E}_\pi[U_E^1(\mu_s)]$$

$$\text{s.t.} \qquad \beta^*(\mu_s) = \arg\max_{\beta \in [0,1]} U_{IB}^1(\beta, \mu_s),$$

$$\mathbb{P}[s = h] \cdot \mu_h + \mathbb{P}[s = \ell] \cdot \mu_\ell = \mu_0,$$

$$\mathbb{P}[s = h] + \mathbb{P}[s = \ell] = 1.$$

The first constraint states that the investment bank will choose the stake that maximizes his expected payoff based on his posterior belief. The second constraint is the Bayesian plausibility condition in which the expectation of posteriors must equal the prior. The last constraint requires that the probabilities of signal realizations should sum to one.

To solve this problem, we use the concavification technique in Kamenica and Gentzkow (2011). In particular, the issuer's *ex ante* optimal design of disclosure system can be derived by finding the concave closure of $U_E^1(\mu_s)$, which we define as $\hat{U}_E^1(\mu_s)$. A graphic representation is given in Figure 5. The black line depicts the issuer's expected payoff conditional on different posteriors. When the uncertainty is relatively large, the investment bank retains a stake and there is underpricing. Thus we observe a dent from the graph when $\mu_s \in (\underline{\mu}, \overline{\mu})$. The blue dashed line illustrates $\hat{U}_E^1(\mu_s)$ – the issuer's maximized expected payoff from the optimal disclosure system.

Intuitively, for any given prior μ_0, it must be equal to some convex combination of two posteriors μ_ℓ and μ_h induced by the optimal system due to the Bayesian plausibility condition (i.e. $\mu_0 = \lambda\mu_\ell + (1-\lambda)\mu_h$ for some $\lambda \in [0, 1]$). So the issuer's *ex ante* expected payoff under the distribution of posteriors must be a convex combination of two expected payoffs conditional on relevant signal realizations too (i.e. $\mathbb{E}_\pi[U_E^1(\mu_s)] = \lambda U_E^1(\mu_\ell) + (1-\lambda)U_E^1(\mu_h)$). Obviously,

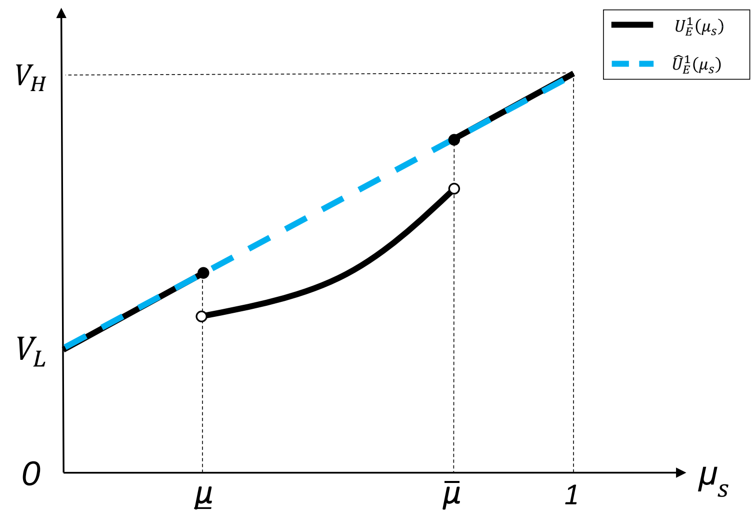


Figure 5: The issuer's payoff (i)

the optimal $\mathbb{E}_\pi[U_E^1(\mu_s)]$ is attained on the concave closure of $U_E^1(\mu_s)$. The optimal μ_ℓ and μ_h are obtained at the intersections of $U_E^1(\mu_s)$ and its concave closure, which are to the left and right of μ_0 respectively.[7] λ and $(1-\lambda)$ are the probabilities of posteriors μ_ℓ and μ_h. The proposition below characterizes the optimal disclosure policy employed by the issuer at $T = 0$.

Proposition 6 *(Optimal information design I): At $T = 0$ the issuer's optimal disclosure policy is:*

1. *If $\mu_0 \in [0, \underline{\mu}] \cup [\overline{\mu}, 1]$, the optimal disclosure system has $\pi_G = \pi_B \in (0, 1)$, and is therefore completely uninformative, yielding posteriors $\mu_\ell = \mu_h = \mu_0$.*

2. *If $\mu_0 \in (\underline{\mu}, \overline{\mu})$, the optimal disclosure system has $\pi_G = \frac{\overline{\mu}(\mu_0 - \underline{\mu})}{\mu_0(\overline{\mu} - \underline{\mu})}$ and $\pi_B = \frac{(1-\overline{\mu})(\mu_0 - \underline{\mu})}{(1-\mu_0)(\overline{\mu} - \underline{\mu})}$, yielding posteriors $\mu_\ell = \underline{\mu}$ and $\mu_h = \overline{\mu}$.*

One caveat is worth some discussion here. When $\mu_0 \in (\underline{\mu}, \overline{\mu})$, there are multiple disclosure systems which gives the issuer the same expected payoff. In fact she can set any arbitrary π_G and π_B, as long as they induce posteriors $\mu_\ell \in [0, \underline{\mu}]$ and $\mu_h \in [\overline{\mu}, 1]$ subject to $\mathbb{P}[s = h] \cdot \mu_h + \mathbb{P}[s = \ell] \cdot \mu_\ell = \mu_0$. But since we have assumed before that if multiple disclosure policies

[7] In a completely uninformative system, $\mu_\ell = \mu_h = \mu_0$.

give the issuer the same expected payoff, she selects the one that is the least informative (and thus the least costly if we assume an infinitesimal cost of reduction in entropy due to the disclosure that varies with the informativeness of the system). Accordingly, Proposition 6 characterizes the least informative optimal disclosure system at $T = 0$.

From Figure 5 it is clear that if the issuer does not release information, underpricing happens when uncertainty about the firm is relatively large. This is consistent with Cai, Helwege, and Warga (2007) that find significant underpricing on speculative-grade debt IPOs but no significant underpricing on investment-grade bond IPOs. We take a further step by showing that in fact issuer can strategically design her disclosure policy to curb underpricing even if *ex ante* the uncertainty about the security is relatively large. This is achieved by designing a system which decreases the uncertainty associated with the security to the degree that the investment bank is just indifferent between holding either zero or a positive stake. And a security with its payoff uncertainty below some thresholds will in turn have no discount. In practice, because of other possible frictions such as issuer's limited capability in reducing uncertainty, we will still observe some underpricing. Later we will show that when there is demand uncertainty in the primary market, underpricing always arises in equilibrium, but strategic disclosure can reduce it on average.

Since we have derived the optimal disclosure policy, it is natural to ask what factors may potentially affect the informativeness of the optimal system. Moreover, how do firms with different levels of uncertainty alter their optimal strategies in response to changes in those factors? We address these important questions in Proposition 7.

Proposition 7 *(Comparative statics I):*

(1) $\frac{\partial \underline{\mu}}{\partial V_L} > 0$ *and* $\frac{\partial \overline{\mu}}{\partial V_L} < 0$.

(2) Define $\eta \equiv \frac{\Delta V}{V_L}$, *then* $\frac{\partial \underline{\mu}}{\partial \eta} < 0$ *and* $\frac{\partial \overline{\mu}}{\partial \eta} > 0$.

(3) $\frac{\partial \underline{\mu}}{\partial r} > 0$ *and* $\frac{\partial \overline{\mu}}{\partial r} < 0$.

(4) $\frac{\partial \underline{\mu}}{\partial \phi} < 0$ *and* $\frac{\partial \overline{\mu}}{\partial \phi} > 0$.

Result (1) states that as V_L increases, the lower-bound cut-off posterior $\underline{\mu}$, at which the investment bank is indifferent between holding 0 and $\frac{\phi}{1+\phi}$, becomes larger and the similar upper-bound cut-off posterior $\overline{\mu}$ becomes smaller. This implies that the range $(\underline{\mu}, \overline{\mu})$ shrinks inward. V_L is the reservation value of the security, and can be viewed as a proxy for the issue size. We first discuss the implications of the comparative statics if the system is completely uninformative. In this case the posterior belief is simply the prior. A larger V_L makes it more costly for the underwriter to retain a stake. So at the cut-off posterior beliefs, only marginally higher uncertainty will induce the underwriter to have a positive retention and stay break-even. The enhanced uncertainty makes the bank's private information more valuable in the secondary market trading, hence offsetting the additional cost brought about by the larger V_L.

Turning to the optimal disclosure, a larger V_L means that only firms that are relatively more uncertain (i.e. $\mu_0 \in (\underline{\mu}, \overline{\mu})$) will employ a system which induces a pair of posteriors $\{\underline{\mu}, \overline{\mu}\}$. Yet as V_L becomes larger, the resulting optimal system will be less transparent because of the inward-shrunken $(\underline{\mu}, \overline{\mu})$, (i.e. less dispersed distribution of posteriors).[8] Therefore, for firms whose security payoffs are *ex ante* highly uncertain, larger issue size allows them to use less transparent disclosure to curb underpricing in the primary market.

Result (2) concerns the effect of the firm's growth option η on the optimal disclosure policy used by the issuer. Better growth option is potentially beneficial to the underwriting bank because it makes his informed trading more profitable. Consequently, at the cut-off beliefs, even marginally lower uncertainty still ensures a non-negative payoff from his retention and subsequent informed trading. As a result, the range $(\underline{\mu}, \overline{\mu})$ expands, and the issuer will use more transparent system as the growth option improves if the security's *ex ante* payoff uncertainty is high.

Result (3) shows that the greater cost of capital of the investment bank will push the two cut-off posteriors inward. Similar to Result (1), at the cut-off beliefs, only marginally

[8] Recall from our definition of informativeness in Section 2.2, an inward (outward) shrunken range of posteriors $(\underline{\mu}, \overline{\mu})$ indicates less (more) informativeness of the system.

higher uncertainty will compensate the underwriter's increased cost of capital by making his private information more valuable in the secondary market trading. Therefore, greater cost of capital of the investment bank results in less transparent disclosure by the issuer with high *ex ante* payoff uncertainty.

Finally, result (4) relates disclosure to market liquidity. A more liquid secondary market pushes the two threshold posteriors outward. In effect, higher liquidity is beneficial to the underwriter as it improves his trading profits. Hence at the margins, cut-off beliefs with relatively lower uncertainty are sufficient to make the underwriter just break-even by holding a stake. And the optimal disclosure reduces more uncertainty, rendering it more transparent if the prior is associated with high uncertainty. Result (4) implies a benefit of the market liquidity in that potentially a more liquid secondary market can push the issuer to design a more transparent disclosure system when issuing securities although this is not the complete story as we will see in the next section.

4.2 Demand Uncertainty (DU)

In this section, we extend the model by introducing the possibility of negative demand shock in the primary market. When demand shock happens, the securities are under-subscribed and the underwriting bank has to acquire additional shares to close the deal if he chooses to underwrite the issue. Note that the demand shock does not affect our secondary market equilibrium as well as the discounted issue price due to informed trading discussed in Section 3. We thus proceed with our analysis from $T = 1$ and then work backward to determine the optimal disclosure policy at $T = 0$.

Formally, we assume that if demand shock happens in the primary market, the demand for the issuer's security is only ψ which satisfies the following inequality:

$$0 < \psi < 1 - \frac{\phi}{1 + \phi}.$$

Therefore, if the investment bank plans to retain a fraction $\beta \leq \frac{\phi}{1+\phi}$, the aggregate demand for the security will fall short of the supply (i.e. $\beta + \psi < 1$). We further assume that if initially the investment bank has entered into an agreement to underwrite the issue, he has to acquire all of the remaining $(1-\psi)$ shares. Also, recall from Lemma 1 that with short sale constraint informed trading is feasible for the investment bank if and only if the fraction of his retention is at least $\frac{\phi}{1+\phi}$ yet strictly less than 1, and the pricing of shares in the primary still follows Proposition 3.

More specifically, suppose that at $T = 1$ after the investment bank has agreed to underwrite and makes his initial retention plan $\hat{\beta}$,

a. with probability $\epsilon \in (0,1)$, the total demand of shares by the participant investors is only ψ. So the investment bank has to acquire $\beta = 1 - \psi$. And the issue price is $P_0(1-\psi, \mu_s)$;

b. with probability $(1-\epsilon)$, there is no demand shock. The investment bank's ultimate retention is $\beta = \hat{\beta}$ and the issue price is $P_0(\hat{\beta}, \mu_s)$.[9]

4.2.1 Investment Bank's Optimal Decision II

In this scenarior, even if the investment bank initially decides to retain only $\hat{\beta} = 0$, the possible demand shock may force him to acquire more than he plans and depress his expected payoff below zero. Nevertheless, the investment bank has an exit option "Not Underwrite" to stay break-even. So the decision to underwrite is no longer trivial, and it depends crucially on the posteriors induced by the issuer's disclosure. We denote the investment bank's payoff by $U^2_{IB}(\hat{\beta}, \mu_s)$ if he enters into the underwriting contract and makes his initial retention plan $\hat{\beta}$.

Consider the situation in which the investment bank chooses to underwrite. He needs to determine his initial retention plan $\hat{\beta}$ to maximize his expected payoff before the demand

[9]To avoid confusion, we use β and $\hat{\beta}$ respectively to distinguish between the issuer's planned and ultimate retention.

uncertainty is resolved. With probability ϵ, the demand shock happens and the investment bank has to buy $(1-\psi)$. His expected payoff is:

$$A(1-\psi,\mu_s) \equiv (1-\psi)[(\mu_s \Delta V + V_L) - (1+r)P_0(1-\psi,\mu_s)] + \frac{\psi(1-\mu_s)\mu_s(1-\gamma)\gamma\phi\Delta V}{\mu_s\gamma + (1-\mu_s)(1-\gamma)}.$$

With probability $(1-\epsilon)$, the demand shock does not occur, and the underwriter's payoff is the same as in the no demand uncertainty case:

$$B(\hat{\beta},\mu_s) \equiv \hat{\beta} \cdot [(\mu_s \Delta V + V_L) - (1+r)P_0(\hat{\beta},\mu_s)]$$
$$+ \mathbb{1}_{\{\hat{\beta} \geq (1-\hat{\beta})\phi\}} \cdot \frac{(1-\hat{\beta})(1-\mu_s)\mu_s(1-\gamma)\gamma\phi\Delta V}{\mu_s\gamma + (1-\mu_s)(1-\gamma)}.$$

Therefore, after observing signal s, the investment bank has to first decide whether he will underwrite. If he underwrites, he further chooses a planned retention $\hat{\beta}$ to maximize his expected payoff. Formally, he chooses his optimal action a^*_{IB} to solve the following maximization problem

$$\max_{a_{IB} \in \{\{\text{NU}\},\{\text{U},\hat{\beta}\}\}} \mathbb{1}_{a_{IB}=\{\text{U},\hat{\beta}\}} \cdot [\epsilon A(1-\psi,\mu_s) + (1-\epsilon)B(\hat{\beta},\mu_s)].$$

To derive the investment bank's optimal action, we first characterize the investment bank's optimal planned retention $\hat{\beta}$ if he chooses to underwrite based on the observed signal in the proposition below.

Proposition 8 *(Investment bank's optimal planned retention): If the investment bank decides to underwrite, it is a dominant strategy for him to choose an initial retention* $\hat{\beta} = \frac{\phi}{1+\phi}$ *before demand uncertainty is unraveled.*

Proposition 8 implies that the investment bank's planned retention is independent of the issuer's disclosure. And such planned purchase serves as an insurance scheme against the demand uncertainty. The result can be understood in the following way. If demand shock happens, the investment bank is forced to complete the deal by acquiring all the remaining $(1-\psi)$ shares. In this case any *ex ante* planned retention $\hat{\beta} \leq 1-\psi$ will not affect his expected payoff. Meanwhile, any initial stake that is larger than $(1-\psi)$ is never optimal. As we have seen in Proposition 4, any stake β that is larger than $\frac{\phi}{1+\phi}$ for the range of more uncertain beliefs $(\underline{\mu}, \overline{\mu})$ is sub-optimal in that it incurs more cost of capital while the informed trading profits become less owing to lower liquidity $\phi(1-\beta)$. Therefore, acquiring a stake that is larger than $(1-\psi)$ is even less desirable. When there is no demand shock, a retention which is just enough for the investment bank to camouflage as liquidity traders, i.e. $\frac{\phi}{1+\phi}$, is optimal as we have shown before. Consequently, it is optimal for the investment bank to choose an initial retention $\hat{\beta} = \frac{\phi}{1+\phi}$. In order for the investment bank to underwrite, his expected payoff should be at least zero. Compared with the cut-off posteriors $\underline{\mu}$ and $\overline{\mu}$ before, it is obvious that the new thresholds satisfy $\underline{\mu}^* > \underline{\mu}$ and $\overline{\mu}^* < \overline{\mu}$. It is because at the old posteriors the investment bank's expected payoff when demand shock happens, i.e. $A(\mu_s)$, will be strictly negative as a result of the higher-than-optimum retention $(1-\psi)$. Thus only a larger lower bound $\underline{\mu}^*$ and a smaller upper bound $\overline{\mu}^*$ will suffice to make the investment bank just break-even by accepting to underwrite.

Recall that $U^2_{IB}(\hat{\beta}, \mu_s) \equiv \epsilon A(\mu_s) + (1-\epsilon)B(\hat{\beta}, \mu_s)$ is the investment bank's expected payoff conditional on posterior μ_s if he accepts to underwrite. And $\hat{\beta}$ represents his planned retention before demand uncertainty is resolved. We summarize our discussion above in Lemma 3.

Lemma 3 *(Indifference cut-off posteriors II):*

1. *There exists a pair* $\{\underline{\mu}^*, \overline{\mu}^*\}$ *with* $0 < \underline{\mu} < \underline{\mu}^* < \frac{1}{2} < \overline{\mu}^* < \overline{\mu} < 1$ *such that* $U^2_{IB}(\frac{\phi}{1+\phi}, \underline{\mu}^*) = U^2_{IB}(\frac{\phi}{1+\phi}, \overline{\mu}^*) = 0$.

2. $U^2_{IB}(\frac{\phi}{1+\phi}, \mu_s) > 0$ *if* $\mu_s \in (\underline{\mu}^*, \overline{\mu}^*)$, *and* $U^2_{IB}(\frac{\phi}{1+\phi}, \mu_s) < 0$ *if* $\mu_s \in [0, \underline{\mu}^*)$ *or* $\mu_s \in (\overline{\mu}^*, 1]$.

Unlike before, if the investment bank's expected payoff is negative conditional on the observed signal s, he will choose not to underwrite. This happens when the induced posterior

μ_s lies in either $[0, \underline{\mu}^*)$ or $(\overline{\mu}^*, 1]$. In general, the bank will not always underwrite, and he withdraws from underwriting when $\mu_s \in [0, \underline{\mu}^*) \cup (\overline{\mu}^*, 1]$. Proposition 9 summarizes the investment bank's best response to different posteriors induced by the issuer's disclosure system and his equilibrium payoff given his optimal action.

Proposition 9 *(Investment bank's optimal strategy and relevant payoffs II): The investment bank's optimal action is*

$$a^*_{IB}(\mu_s) = \begin{cases} \text{Underwrite and } \hat{\beta}^* = \frac{\phi}{1+\phi} & \text{if } \mu_s \in [\underline{\mu}^*, \overline{\mu}^*], \\ \text{Not Underwrite} & \text{if } \mu_s \in [0, \underline{\mu}^*) \cup (\overline{\mu}^*, 1]. \end{cases}$$

His equilibrium payoff is

$$\hat{U}^2_{IB}(\mu_s) = \begin{cases} U^2_{IB}(\frac{\phi}{1+\phi}, \mu_s) & \text{if } \mu_s \in [\underline{\mu}^*, \overline{\mu}^*], \\ 0 & \text{if } \mu_s \in [0, \underline{\mu}^*) \cup (\overline{\mu}^*, 1]. \end{cases}$$

Since $\hat{\beta}^*$ in equilibrium depends on the posterior μ_s only, we can simply write the investment bank's expected payoff as $\hat{U}^2_{IB}(\mu_s)$, a function of μ_s too. In Figure 6, the red dashed line depicts the investment bank's expected payoff given his optimal action a^*_{IB}, while the yellow solid line is his expected payoff if he sticks to a planned retention $\hat{\beta} = \frac{\phi}{1+\phi}$ regardless of his posterior. For comparison, we also draw the investment bank's expected payoff if he always retains $\frac{\phi}{1+\phi}$ shares when there is no demand uncertainty (i.e. the blue dashed line, which corresponds to the blue solid line in Figure 4). The yellow line is beneath the blue dashed one in that the presence of possible demand shock extracts a rent from the investment bank thus decreases its expected payoff in general. In this case the two cut-off posteriors are less dispersed. Indeed, to induce the investment bank to underwrite, higher uncertainty in the primary market is needed. Then the losses due to unfortunate retention can be offset by larger trading profits from the underwriter's private information in the secondary market.

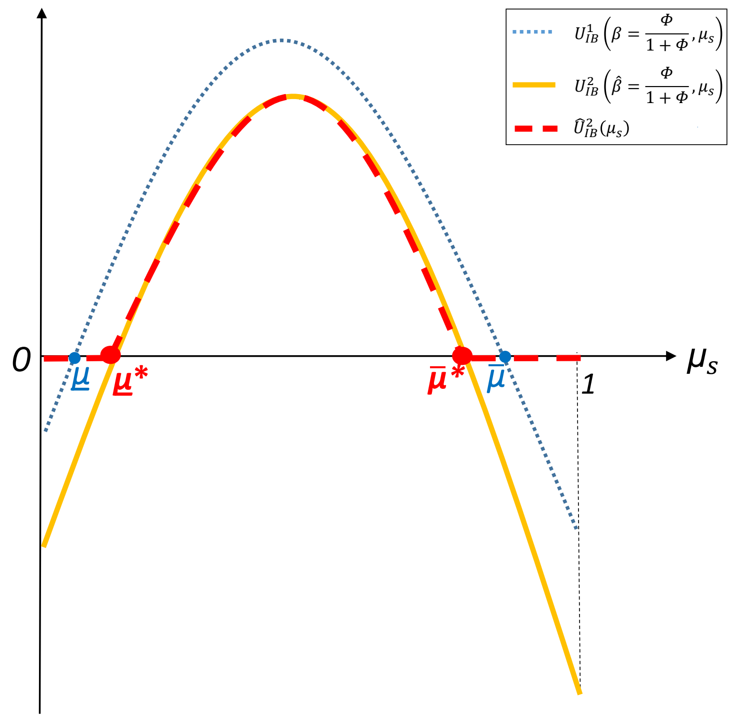


Figure 6: The investment bank's payoff (ii)

Accordingly, when the uncertainty in the primary market is relatively small (i.e. $\mu_s \in [0, \underline{\mu}^*) \cup (\overline{\mu}^*, 1]$), the investment bank's private information is less valuable and on average he expects to suffer a loss from accepting to underwrite. His optimal strategy is to withdraw from underwriting the issue. Only when the uncertainty is relatively large (i.e. $\mu_s \in [\underline{\mu}^*, \overline{\mu}^*]$) can the investment bank's expected loss from unfortunate retention be compensated by his informed trading profits owing to more valuable private information. In this case, he will agree to underwrite even though he may end up with more retention than he originally plans.

4.2.2 Optimal Disclosure System II

Since we have solved for the optimal strategy of the investment bank, it is easy to derive the issuer's expected proceeds from security issuance conditional on different signal realizations at $T = 1$.

Proposition 10 *(Issuer's payoff after information design II):*

1. *When $\mu_s \in [0, \underline{\mu}^*) \cup (\overline{\mu}^*, 1]$, the investment bank does not underwrite, and $U_E^2(\mu_s) = 0$.*

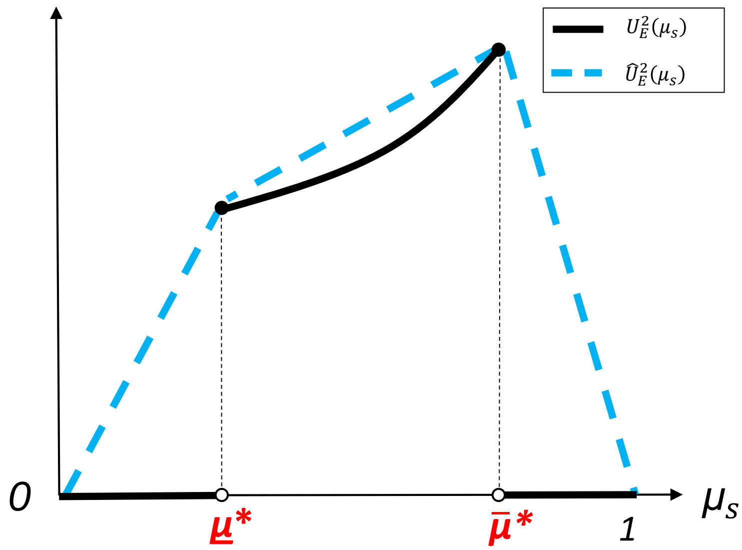


Figure 7: The issuer's payoff (ii)

2. *When* $\mu_s \in [\underline{\mu}^*, \overline{\mu}^*]$, $U_E^2(\mu_s) \equiv U_E^2(\frac{\phi}{1+\phi}, \mu_s) = \epsilon P_0(1-\psi, \mu_s) + (1-\epsilon)P_0(\frac{\phi}{1+\phi}, \mu_s)$

$$= \mu_s \Delta V + V_L - \frac{(1-\mu_s)\mu_s(1-\gamma)\gamma\phi\Delta V}{\mu_s\gamma + (1-\mu_s)(1-\gamma)}.$$

The second part of Proposition 10 implies that the issue prices are the same under two different levels of retention by the investment bank, $(1-\psi)$ and $\frac{\phi}{1+\phi}$. This is because as long as the bank acquires a stake of at least $\frac{\phi}{1+\phi}$, the issue price will always have an adverse selection discount. Yet such discount does not vary with the investment bank's retention in that each participant investor's expected loss per share from trading in the secondary market is independent of the investment bank's ultimate stake β, a result that has already been shown in Proposition 2. From Proposition 10 it is easy to see that conditional on signal s, the issuer's expected revenue $U_E^2(\hat{\beta}^*, \mu_s)$ depends on posterior μ_s only, thus we denote it by $U_E^2(\mu_s)$.

At $T = 0$, taking into account the optimal action that will be taken by the investment bank at different posteriors, the issuer designs the disclosure system to maximize her expected payoff. In particular, she chooses a distribution of posteriors to solve

$$\hat{U}_E^2(\mu_0) \equiv \max_{\{\mu_\ell,\mu_h\}} \mathbb{E}_\pi[U_E^2(\mu_s)]$$

$$\text{s.t. } a_{IB}^*(\mu_s) = \underset{a_{IB}\in\{\{\text{U},\hat{\beta}\},\{\text{NU}\}\}}{\arg\max} \mathbb{1}_{\{a_{IB}=\{\text{U},\hat{\beta}\}\}} \cdot U_{IB}^2(\hat{\beta},\mu_s),$$

$$\mathbb{P}[s=h]\cdot\mu_h + \mathbb{P}[s=\ell]\cdot\mu_\ell = \mu_0,$$

$$\mathbb{P}[s=h] + \mathbb{P}[s=\ell] = 1.$$

The first constraint concerns the investment bank's optimal underwriting decision, and his planned retention if he chooses to underwrite. The second constraint is the Bayesian plausibility condition. The third constraint ensures that the sum of probabilities of high signal h and low signal ℓ equals 1. We solve this constrained maximization problem by finding the concave closure of $U_E^2(\mu_s)$. In Figure 7 the black solid line depicts the issuer's expected payoff $U_E^2(\mu_s)$ as characterized in Proposition 10. The blue dashed line is the concave closure of $U_E^2(\mu_s)$, which is denoted by $\hat{U}_E^2(\mu_s)$. Hence we can read off the optimal disclosure system directly from the graph.

Proposition 11 *(Optimal information design II): At $T=0$, the issuer's optimal disclosure policy is:*

1. *If $\mu_s \in [0,\underline{\mu}^*)$, the optimal disclosure system has $\pi_B = \frac{\mu_0(1-\underline{\mu}^*)}{\underline{\mu}^*(1-\mu_0)}$ and $\pi_G = 1$, yielding posteriors $\mu_\ell = 0$ and $\mu_h = \underline{\mu}^*$.*

2. *If $\mu_s \in (\underline{\mu}^*,\overline{\mu}^*)$, the optimal disclosure system has $\pi_G = \frac{\overline{\mu}^*(\mu_0-\underline{\mu}^*)}{\mu_0(\overline{\mu}^*-\underline{\mu}^*)}$ and $\pi_B = \frac{(1-\overline{\mu}^*)(\mu_0-\underline{\mu}^*)}{(1-\mu_0)(\overline{\mu}^*-\underline{\mu}^*)}$, yielding posteriors $\mu_\ell = \underline{\mu}^*$ and $\mu_h = \overline{\mu}^*$.*

3. *If $\mu_s \in (\overline{\mu}^*,1]$, the optimal disclosure system has $\pi_B = \frac{\mu_0-\overline{\mu}^*}{\mu_0(1-\overline{\mu}^*)}$ and $\pi_G = 0$, yielding posteriors $\mu_\ell = \overline{\mu}^*$ and $\mu_h = 1$.*

4. *If $\mu_0 = \underline{\mu}^*$ or $\overline{\mu}^*$, the optimal disclosure system has $\pi_G = \pi_B \in (0,1)$, and is therefore completely uninformative, yielding posteriors $\mu_\ell = \mu_h = \mu_0$.*

Again, we have characterized the sender-preferred equilibrium. At the two cut-off posteriors $\underline{\mu}^*$ and $\overline{\mu}^*$, the investment bank is indifferent between declining and underwriting with a planned retention $\frac{\phi}{1+\phi}$. Yet the latter is strictly preferred by the issuer in that she would otherwise fail to issue the security. So we assume that for the sake of the issuer's interest, the investment bank will underwrite when he is indifferent. Here the merit of strategic disclosure lies in that even though an *ex ante* prior $\mu_0 \in (0, \underline{\mu}^*) \cup (\overline{\mu}^*, 1)$ implies failure of issuance owing to the investment bank's unwillingness to underwrite, the optimal disclosure policy is still able to induce the investment bank to underwrite with strictly positive probability. In this sense, strategic disclosure may solve the hold-up problem introduced by the demand shock in the primary market. The other advantage of this disclosure policy manifests in that when uncertainty is higher $\mu_0 \in (\underline{\mu}^*, \overline{\mu}^*)$, the expected issue-price discount is reduced compared with that under no informative disclosure, as is clear from the wedge between the blue dashed line and the black line in Figure 7.

Moreover, although the optimal disclosure reduces payoff uncertainty on average, with some particular signal realization, the uncertainty is actually enhanced. For instance, if the prior $\mu_0 \in (0, \underline{\mu}^*)$, an h signal leads to a posterior belief of $\underline{\mu}^*$. And $\underline{\mu}^*$ is more uncertain than μ_0 as it has higher entropy. When the signal realization is ℓ, the disclosure is fully revealing and the underlying state is B. The same logic applies to posterior $\overline{\mu}^*$ induced by signal ℓ as it has higher entropy than μ_0 when $\mu_0 \in (\overline{\mu}^*, 1)$. And an h signal indicates that the state is G. Thanks to the Bayesian plausibility constraint, the strategic disclosure by the issuer can induce the investment bank to underwrite with positive probability and balances this with a worse belief that leaves the bank's underwriting decision unchanged, which generally improves the issuer's expected payoff. The optimal disclosure is such that on the one hand it induces the worst beliefs which lead to the investment bank's withdrawal from underwriting, and on the other hand it generates signals that make the investment bank just willing to underwrite at the other beliefs. At these beliefs that the underwrite chooses to underwrite, the security's uncertainty is in fact enhanced, and the underwriter's private information becomes sufficiently valuable, although on average the disclosure system reduces the uncertainty compared with the situation at the prior belief. In the meantime the issuer's expected proceeds from the issue is maximized. In this regard, the optimal disclosure features a mechanism in which the increased payoff uncertainty can offset the loss brought about by the demand uncertainty so that the investment bank will change to the better action that is favored by the issuer.

Nevertheless, a posterior of either $\underline{\mu}^*$ or $\overline{\mu}^*$ does not necessarily mean that the demand risk is alleviated. In fact it is entirely possible that the investment bank will acquire more than his planned retention eventually. Our result sheds some light on the empirically documented "Pipeline Risk" (or "Unfortunate Retention") in leveraged loan syndication by Bruche, Malherbe, and Meisenzahl (2018). We have shown that because of the issuer's disclosure policy, even in the presence of demand uncertainty a fully rational investment bank will still agree to underwrite. But when demand shock happens, the investment bank will suffer large losses as a result of excessive retention.

The next proposition provides some empirical predictions that relate the optimal disclosure to various aspects of the primary and secondary markets.

Proposition 12 *(Comparative statics II):*

(1) $\frac{\partial \underline{\mu}^*}{\partial \epsilon} > 0$ *and* $\frac{\partial \overline{\mu}^*}{\partial \epsilon} < 0$.

(2) $\frac{\partial \underline{\mu}^*}{\partial \psi} < 0$ *and* $\frac{\partial \overline{\mu}^*}{\partial \psi} > 0$.

(3) $\frac{\partial \underline{\mu}^*}{\partial V_L} > 0$ *and* $\frac{\partial \overline{\mu}^*}{\partial V_L} < 0$.

(4) Recall that $\eta = \frac{\Delta V}{V_L}$*, then* $\frac{\partial \underline{\mu}^*}{\partial \eta} < 0$ *and* $\frac{\partial \overline{\mu}^*}{\partial \eta} > 0$.

(5) $\frac{\partial \underline{\mu}^*}{\partial r} > 0$ *and* $\frac{\partial \overline{\mu}^*}{\partial r} < 0$.

(6) $\frac{\partial \underline{\mu}^*}{\partial \phi} < 0$ *and* $\frac{\partial \overline{\mu}^*}{\partial \phi} > 0$.

From result (1), it is easy to see that as the probability of demand shock in the primary market becomes higher, the two cut-off posteriors shrink inward. So when the prior belief about the security's cash flow is relatively more uncertain (i.e. $\mu_0 \in (\underline{\mu}^*, \overline{\mu}^*)$), higher likelihood of under-subscription results in less transparent disclosure designed by the issuer. Indeed, since the demand shock is more likely to occur in the primary market, in order for the investment bank to at least stay break-even from underwriting the issue, the disclosure should bring in more uncertainty so that his stake carries more trading value with his private information in the secondary market. And the additional informed trading profits can offset his expected loss from "unfortunate retention" due to demand shock. Anticipating this, the issuer will employ a relatively more opaque disclosure *ex ante*.

However, when the uncertainty about the security's payoff is relatively low (i.e. $\mu_0 \in (0, \underline{\mu}^*)$ or $\mu_0 \in (\overline{\mu}^*, 1)$), larger ϵ leads to more transparent disclosure. In this case, π_B is smaller, suggesting that the h signal is more indicative of the good state and the ℓ signal is more indicative of the bad state. As in this case, only a marginally higher payoff uncertainty will be enough to compensate for the additional expected loss due to higher probability of demand shock and make the unwilling bank to accept the deal again at the high-uncertainty posterior belief.

Result (2) contrasts with result (1) above: if demand shock happens, a stronger demand (larger ψ), or equivalently, a smaller unfortunate retention (smaller $(1-\psi)$) by the underwriter, expands the two cut-off posteriors outward. Therefore, if demand shock happens, this in turn reduces the additional cost of capital incurred from the investment bank's unfortunate retention and increases his future trading profits thanks to more liquidity traders. As a result, when the *ex ante* payoff uncertainty is relatively large, a more transparent disclosure will be employed in equilibrium as in this case marginally less uncertain cut-off posteriors are enough to make the investment bank indifferent between whether or not to underwrite. Nevertheless, when *ex ante* uncertainty is relatively small, a higher ψ result in less transparent disclosure. Both lower $\underline{\mu}$ and higher $\overline{\mu}$ bring about less transparent disclosure systems for $\mu_0 \in (0, \underline{\mu}^*)$ and $\mu_0 \in (\overline{\mu}, 1)$ respectively. In both cases, due to Bayesian plausibility condition, the probabilities of full revelation will be smaller, and the probabilities of the more uncertain posteriors will be higher, making the systems less informative.

The dichotomy remains valid regarding result (3). Higher V_L (issue size or reservation value of the firm) expands the range of posterior beliefs $(\underline{\mu}, \overline{\mu})$. As V_L grows, it is more costly for the investment bank to underwrite and retain a positive stake. So when prior belief about the uncertainty of the security's payoff is relatively large, marginally more uncertain cut-off posteriors (i.e. higher $\underline{\mu}$ and lower $\overline{\mu}$) should be generated for the system so that the investment bank will be just willing to underwrite. Yet when the *ex ante* payoff uncertainty is relatively small, both higher $\underline{\mu}$ and lower $\overline{\mu}$ result in more transparent disclosure systems for $\mu_0 \in (0, \underline{\mu}^*)$ and $\mu_0 \in (\overline{\mu}, 1)$ respectively. So the probabilities of fully revealing states will be higher, and the probabilities of the more uncertain posteriors will be lower, rendering the systems more informative.

Result (4) asserts that higher growth option (η) gives rises to the expansion of $(\underline{\mu}, \overline{\mu})$. When prior belief about uncertainty is relatively large, as growth option improves, the investment bank will benefit more from his informed sales in the secondary market. Hence the optimal disclosure will be more informative as now marginally less uncertain cut-off posteriors are still able to induce the investment bank to underwrite. When the *ex ante* payoff uncertainty is relatively small, better growth option leads to less informative disclosure. The reasoning is similar to what we have discussed in result (3): less uncertain cut-off posteriors give rise to higher probabilities of high-uncertainty posteriors and lower probability of fully revealing signals. In addition, a less informative information disclosure arises naturally in equilibrium.

With the same token, result (5) states that higher r makes the disclosure system less informative when *ex ante* uncertainty is relatively high, but it leads to less informative disclosure when the uncertainty is relatively low. Higher opportunity cost per unit of investment by the bank makes him less willing to retain a positive stake at the old cut-off posteriors. To induce him to underwrite and compensate his additional cost of capital, posteriors with higher uncertainty must be generated from the optimal system.

Likewise, the implications of result (6) depend on the prior μ_0. When the *ex ante* payoff uncertainty is relatively high, a more liquid secondary market leads to more transparent disclosure. This is because better liquidity in the secondary market allows the underwriter to gain more from trading on his private information. Therefore, a more transparent system, although decreases the value of the investment bank's private information, is still able to make the investment bank just break-even by underwriting the deal. Yet when uncertainty about the firm is relatively low, the disclosure becomes less transparent as the secondary market liquidity increases. Recall that in order to change the investment bank's decision of not underwriting, the system should produce one particular signal which increases the payoff uncertainty to the extent that the investment bank is just willing to serve as an underwriter. As liquidity pumps up, the optimal disclosure only needs to generate a marginally less uncertain high-uncertainty posterior (higher $\underline{\mu}^*$ or lower $\overline{\mu}^*$) such that the bank still wants to underwrite. As a result the disclosure becomes less informative than before.

5 No Short Sale Constraint (NSS)

In this section, we briefly layout the equilibria by relaxing the previous assumption that the underwriter is not allowed to sell the security short in the secondary market. We also assume that short sale does not incur any other cost to the underwriter. As before, we divide into two scenarios: 1. the security can always be fully subscribed by the participant investors even in the absence of underwriter retention; and 2. there is demand uncertainty in the primary market. In face, in case 2, the results on the optimal disclosure we have obtained with short-selling constraint extend to the scenario without the ban on short sale.

5.1 No Demand Uncertainty (NDU)

We first consider the case in which there is neither demand uncertainty in the primary market nor ban on short sale in the secondary market. Since the demand for the security will never

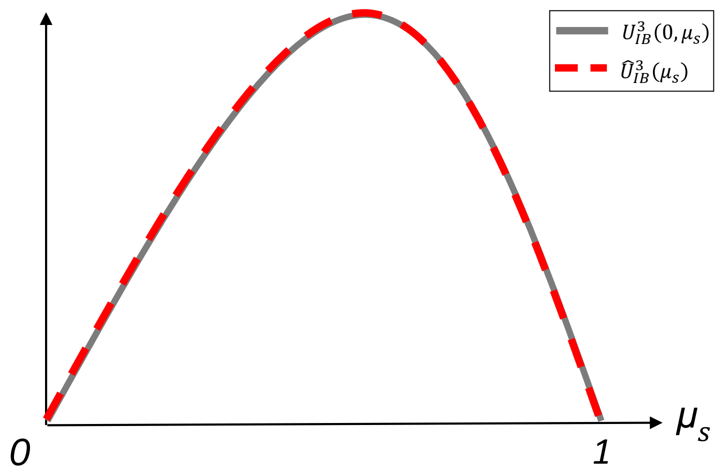


Figure 8: The investment bank's payoff (iii)

fall short of the supply, the investment bank is always willing to underwrite.

Proposition 13 *(Investment bank's optimal retention): It is optimal for the investment bank to retain zero stake in the primary market regardless of the signal realization (i.e. $\beta^*(\mu_s) = 0$).*

The intuition is fairly straightforward: recall from Part 1 of Proposition 2, the underwriting bank's informed trading profits are proportional to the fraction of liquidity traders $(1-\beta)\phi$. Hence such profits are maximized at $\beta = 0$ when the liquidity in the secondary market is maximized. Since now the underwriter can sell the security short, he no longer has to hold a stake, but is still able to camouflage as liquidity traders. Meanwhile, zero retention is optimal in the primary market in that any positive retention in the primary market would incur an opportunity cost for the investment bank while his gain per share from primary market underpricing is the same as his informed trading profit per share in the secondary market. Hence the investment bank's expected payoff is just his expected trading profits from the secondary market:

$$\hat{U}_{IB}^3(\mu_s) = U_{IB}^3(0, \mu_s) \equiv \frac{(1-\mu_s)\mu_s(1-\gamma)\gamma\phi\Delta V}{\mu_s\gamma + (1-\mu_s)(1-\gamma)}.$$

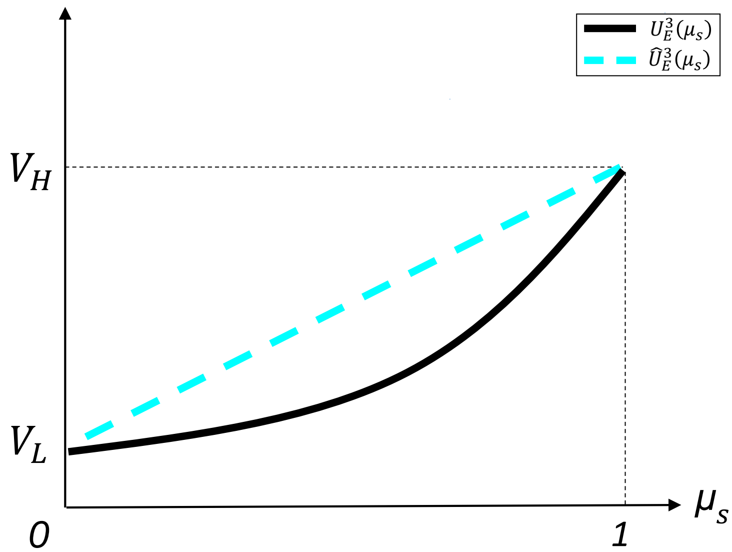


Figure 9: The issuer's payoff (iii)

Figure 8 depicts the investment bank's expected payoff as a function of the posterior belief μ_s. And given the investment bank's zero retention and short-sale trading strategy, from Part 2 of Proposition 2 the issuer's expected proceeds conditional on signal s at $T = 1$ is

$$U_E^3(\mu_s) \equiv (\mu_s \Delta V + V_L) - \frac{(1-\mu_s)\mu_s(1-\gamma)\gamma\phi\Delta V}{\mu_s\gamma + (1-\mu_s)(1-\gamma)}.$$

To solve the optimal information design problem faced by the issuer at $T = 0$, it suffices to find the concave closure of $U_E^3(\mu_s)$, which we denote by $\hat{U}_E^3(\mu_s)$. In Figure 9, the black line represents $U_E^3(\mu_s)$ and the blue dashed line is its concave closure $\hat{U}_E^3(\mu_s)$. Since $U_E^3(\mu_s)$ is concave on the support of μ_s, the optimal disclosure system is fully revealing.

Proposition 14 *(Optimal information design III): At $T = 0$, the issuer's optimal disclosure policy is completely informative, i.e. $\pi_G = 1$ and $\pi_B = 0$, yielding posteriors $\mu_\ell = 0$ and $\mu_h = 1$.*

5.2 Demand Uncertainty (DU)

We next explore the scenario where there is demand uncertainty in the primary market.

First suppose that the investment bank chooses to underwrite. Then if demand shock does not happen, the investment bank's optimal underwriting, retention and short selling strategy coincides with what we have obtained in the previous subsection. Yet if demand shock happens, the investment bank is forced to acquire a stake of $(1-\psi)$. As he is able to short sell in the secondary market, his planned retention should still be zero before the demand uncertainty is unraveled. His expected payoff from underwriting with zero planned retention is

$$U_{IB}^{4}(0,\mu_s) \equiv \epsilon \cdot \left\{ (1-\psi) \cdot [(\mu_s \Delta V + V_L) - (1+r)P_0(1-\psi,\mu_s)] + \psi \cdot \frac{(1-\mu_s)\mu_s(1-\gamma)\gamma\phi\Delta V}{\mu_s\gamma + (1-\mu_s)(1-\gamma)} \right\}$$

$$+(1-\epsilon) \cdot \frac{(1-\mu_s)\mu_s(1-\gamma)\gamma\phi\Delta V}{\mu_s\gamma + (1-\mu_s)(1-\gamma)},$$

where $P_0(1-\psi,\mu_s)$ is the issue price defined in Part 2 of Proposition 2. The first term above represents the investment bank's expected payoff if demand shock happens while the second is his expected payoff if the demand shock does not occur, both at posterior belief μ_s. The second term is always strictly positive while the first one can be negative for some set of beliefs which are associated with low uncertainty.

Consequently, choosing to underwrite regardless of his posterior belief is not a best response for the investment bank. This is because when the *ex ante* uncertainty about the security's payoff is relatively small, the expected profits from trading on his private information are far from enough to cover the investment bank's opportunity cost of unfortunate retention. Although the bank can always enjoy a strictly positive payoff from short selling when the demand shock does not occur, the investment bank's expected payoff before the resolution of the demand uncertainty under these low-uncertainty beliefs will still be negative. As a result, the investment bank will shy away from underwriting the deal.

Lemma 4 *(Indifference cut-off posteriors III):*

1. *There exists a pair* $\{\underline{\mu}^{**}, \overline{\mu}^{**}\}$ *with* $0 < \underline{\mu}^{**} < \underline{\mu}^{*} < \frac{1}{2} < \overline{\mu}^{*} < \overline{\mu}^{**} < 1$ *such that*

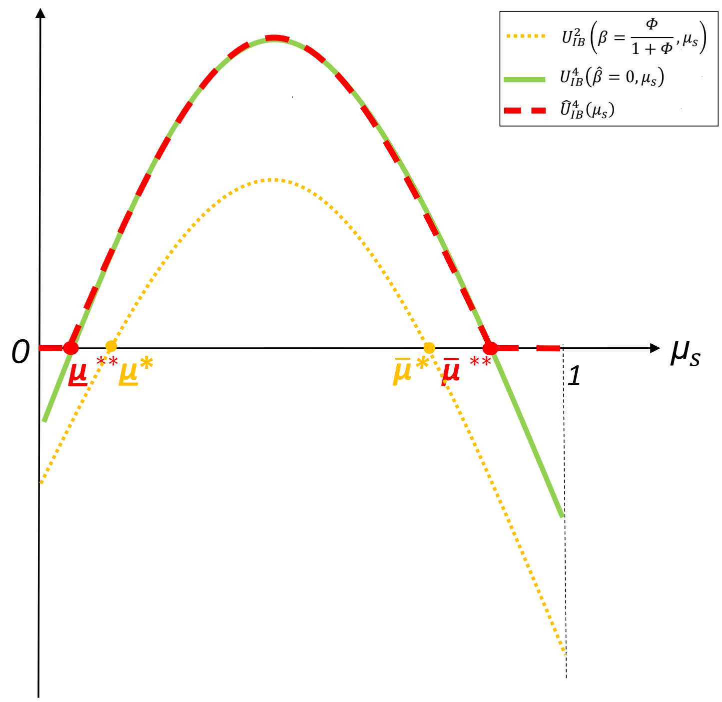


Figure 10: The investment bank's payoff (iv)

$U^4_{IB}(0, \underline{\mu}^*) = U^4_{IB}(0, \overline{\mu}^{**}) = 0.$

2. $U^4_{IB}(0, \mu_s) > 0$ *if* $\mu_s \in (\underline{\mu}^{**}, \overline{\mu}^{**})$, *and* $\tilde{U}_{IB}(0, \mu_s) < 0$ *if* $\mu_s \in [0, \underline{\mu}^{**})$ *or* $\mu_s \in (\overline{\mu}^{**}, 1]$.

Proposition 15 *(Investment bank's optimal strategy and relevant payoffs III): The investment bank's optimal action is*

$$a^*_{IB}(\mu_s) = \begin{cases} \textit{Underwrite and } \hat{\beta}^* = 0 & \textit{if } \mu_s \in [\underline{\mu}^{**}, \overline{\mu}^{**}], \\ \textit{Not Underwrite} & \textit{if } \mu_s \in [0, \underline{\mu}^{**}) \cup (\overline{\mu}^{**}, 1]. \end{cases}$$

His equilibrium payoff is

$$\hat{U}^4_{IB}(\mu_s) = \begin{cases} U^4_{IB}(0, \mu_s) & \textit{if } \mu_s \in [\underline{\mu}^{**}, \overline{\mu}^{**}], \\ 0 & \textit{if } \mu_s \in [0, \underline{\mu}^{**}) \cup (\overline{\mu}^{**}, 1]. \end{cases}$$

In Figure 10, the green line depicts $U^4_{IB}(0, \mu_s)$ (i.e. the investment bank's expected payoff from underwriting with zero planned retention) while the red dashed line depicts the investment bank's expected payoff under his optimal underwriting and retention strategy. For comparison, the yellow dashed line is the investment bank's expected payoff by underwriting and retaining $\frac{\phi}{1+\phi}$ when there is demand uncertainty yet short sale is not allowed, the scenario that we have discussed in Section 4.2. An interesting observation is that compared with before, even if the issuer does not disclosure additional information, there is a wider range of beliefs under which the investment bank is willing to underwrite. This is because the feasibility of short sale by underwriter enables the investment bank to enjoy positive expected payoffs under two sets of relatively less uncertainty beliefs $(\underline{\mu}^{**}, \underline{\mu}^{*})$ and $(\overline{\mu}^{*}, \overline{\mu}^{**})$. The removal of short sale constraint reduces the total cost of capital due to primary market retention to zero, yet allows the underwriter to trade more intensively on his private information. In turn the indifference cut-off posteriors only need to involve less uncertainty.

Given the optimal strategy of the investment bank, the next proposition follows naturally.

Proposition 16 *(Issuer's payoff after information design III):*

1. *When $\mu_s \in [0, \underline{\mu}^{**}) \cup (\overline{\mu}^{**}, 1]$, the investment bank does not underwrite, and $U^4_E(\mu_s) = 0$.*

2. *When $\mu_s \in [\underline{\mu}^{**}, \overline{\mu}^{**}]$, $U^4_E(\mu_s) \equiv U^4_E(\hat{\beta}^* = 0, \mu_s) = (\mu_s \Delta V + V_L) - \frac{(1-\mu_s)\mu_s(1-\gamma)\gamma\phi\Delta V}{\mu_s\gamma + (1-\mu_s)(1-\gamma)}$.*

Concavification of $U^4_E(\mu_s)$ gives us the optimal disclosure system designed by the issuer at $T = 0$, as illustrated in Figure 11.

Proposition 17 *(Optimal information design III): At $T = 0$, the issuer's optimal disclosure policy is:*

1. *If $\mu_s \in [0, \underline{\mu}^{**})$, the optimal disclosure system has $\pi_B = \frac{\mu_0(1-\underline{\mu}^{**})}{\underline{\mu}^{**}(1-\mu_0)}$ and $\pi_G = 1$, yielding posteriors $\mu_\ell = 0$ and $\mu_h = \underline{\mu}^{**}$.*

2. *If $\mu_s \in (\underline{\mu}^{**}, \overline{\mu}^{**})$, the optimal disclosure system has $\pi_G = \frac{\overline{\mu}^{**}(\mu_0 - \underline{\mu}^{**})}{\mu_0(\overline{\mu}^{**} - \underline{\mu}^{**})}$ and $\pi_B = \frac{(1-\overline{\mu}^{**})(\mu_0 - \underline{\mu}^{**})}{(1-\mu_0)(\overline{\mu}^{**} - \underline{\mu}^{**})}$, yielding posteriors $\mu_\ell = \underline{\mu}^{**}$ and $\mu_h = \overline{\mu}^{**}$.*

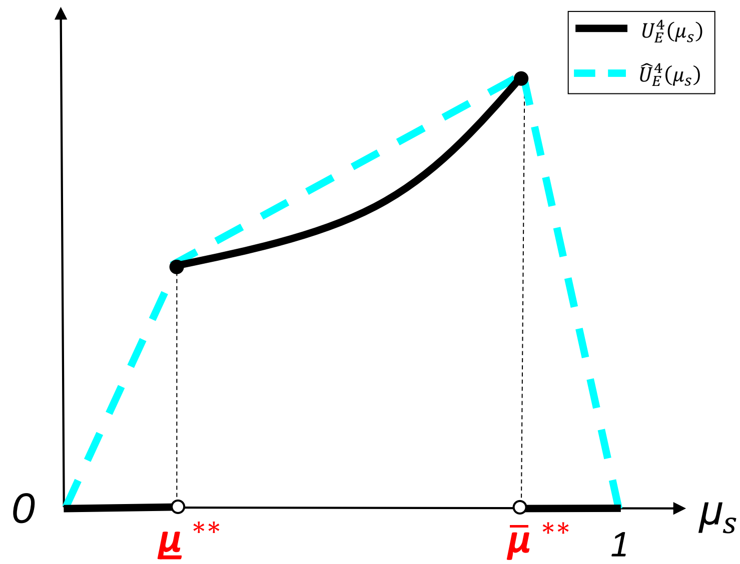


Figure 11: The issuer's payoff (iv)

3. *If $\mu_s \in (\overline{\mu}^{**}, 1]$, the optimal disclosure system has $\pi_B = \frac{\mu_0 - \overline{\mu}^{**}}{\mu_0(1-\overline{\mu}^{**})}$ and $\pi_G = 0$, yielding posteriors $\mu_\ell = \overline{\mu}^{**}$ and $\mu_h = 1$.*

4. *If $\mu_0 = \underline{\mu}^{**}$ or $\overline{\mu}^{**}$, the optimal disclosure system has $\pi_G = \pi_B \in (0,1)$, and is therefore completely uninformative, yielding posteriors $\mu_\ell = \mu_h = \mu_0$.*

Proposition 18 *(Comparative statics III):*

(1) $\frac{\partial \underline{\mu}^{**}}{\partial \epsilon} > 0$ *and* $\frac{\partial \overline{\mu}^{**}}{\partial \epsilon} < 0$.

(2) $\frac{\partial \underline{\mu}^{**}}{\partial \psi} < 0$ *and* $\frac{\partial \overline{\mu}^{**}}{\partial \psi} > 0$.

(3) $\frac{\partial \underline{\mu}^{**}}{\partial V_L} > 0$ *and* $\frac{\partial \overline{\mu}^{**}}{\partial V_L} < 0$.

(4) Recall that $\eta = \frac{\Delta V}{V_L}$, *then* $\frac{\partial \underline{\mu}^{**}}{\partial \eta} < 0$ *and* $\frac{\partial \overline{\mu}^{**}}{\partial \eta} > 0$.

(5) $\frac{\partial \underline{\mu}^{**}}{\partial r} > 0$ *and* $\frac{\partial \overline{\mu}^{**}}{\partial r} < 0$.

(6) $\frac{\partial \underline{\mu}^{**}}{\partial \phi} < 0$ *and* $\frac{\partial \overline{\mu}^{**}}{\partial \phi} > 0$.

Note that Proposition 17 and 18 are identical to what we have obtained in Proposition 11 and 12. Therefore, all the intuitions go through.

6 Welfare Analysis

We have explored the four possible scenarios: 1. (No Short Sale, No Demand Uncertainty), 2. (No Short Sale, Demand Uncertainty), 3. (Short Sale, No Demand Uncertainty), and 4. (Short Sale, Demand Uncertainty). Now suppose that the economy is populated with a continuum of mass 1 issuers with their types μ_0 drawn from a uniform distribution $U[0,1]$, and each issuer invites an investment bank to underwrite.[10]

Let $i \in \{1,2,3,4\}$ denote one of the above four scenarios. Recall that $U_E^i(\mu_0)$ is a type-μ_0 issuer's expected payoff and $\hat{U}_{IB}^i(\mu_0)$ is the relevant investment bank's expected payoff conditional on his prior (or equivalently if the issuer does not disclose additional information). Moreover, $\hat{U}_E^i(\mu_0)$ is the type-μ_0 issuer's maximized expected payoff under optimal disclosure system in scenario i.[11] Since the optimal disclosure always makes the investment bank just break-even at any of the posteriors induced by the signal generated from the optimal system, the investment bank's expected utility will be zero given the issuer's optimal disclosure strategy.

Therefore, if the issuers do not disclose additional information at $T = 0$, their welfare in scenario i is

$$W_E(i) \equiv \int_0^1 U_E^i(\mu_0)\, d\mu_0,$$

and the investment banks' welfare in scenario i is

$$W_{IB}(i) \equiv \int_0^1 \hat{U}_{IB}^i(\mu_0)\, d\mu_0.$$

[10] Alternatively, assume that a generic issuer has type $\mu_0 \sim U[0,1]$. Hence the welfare is just the issuer's expected payoff.

[11] Note that we have already characterized $U_E^i(\mu_0)$, $U_{IB}^i(\mu_0)$, and $\hat{U}_E^i(\mu_0)$, each corresponds to the issuer's expected payoff at $T = 1$ given the investment bank's best response (the black solid line in Figure 5, 7, 9, and 11), the investment bank's expected payoff at $T = 1$ with his optimal underwriting and retention decision (the red dashed line in Figure 4, 6, 8, and 10), and the issuer's expected payoff at $T = 0$ under the optimal disclosure system (the blue dashed line in Figure 5, 7, 9, and 11).

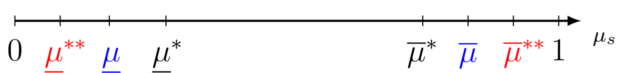
The issuers' welfare with their optimal disclosure policies in scenario i is

$$\hat{W}_E(i) \equiv \int_0^1 \hat{U}_E^i(\mu_0)\, d\mu_0.$$

We first look at the investment banks' welfare if the issuers do not disclose any informative signal. The ranking of their welfare in the four scenarios depends on the probability of demand shocks ϵ.

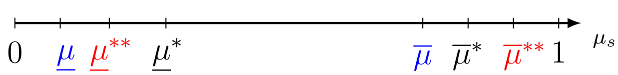
Proposition 19 *(Investment banks' welfare):*

(1) When $0 < \epsilon < \frac{\phi}{(1-\psi)(1+\phi)}$,



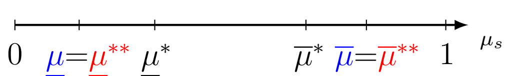
$$W_{IB}(SS, NDU) > W_{IB}(SS, DU) > W_{IB}(NSS, NDU) > W_{IB}(NSS, DU).$$

(2) When $\frac{\phi}{(1-\psi)(1+\phi)} < \epsilon < 1$,



$$W_{IB}(SS, NDU) > W_{IB}(NSS, NDU) > W_{IB}(SS, DU) > W_{IB}(NSS, DU).$$

(3) When $\epsilon = \frac{\phi}{(1-\psi)(1+\phi)}$,



$$W_{IB}(SS, NDU) > W_{IB}(NSS, NDU) = W_{IB}(SS, DU) > W_{IB}(NSS, DU).$$

The red, blue, and black cut-offs posteriors represent the threshold beliefs that make the investment bank just break-even as an underwriter in scenarios (SS, DU), (NSS,NDU), and (NSS, DU) respectively. And $\{0, 1\}$ are relevant beliefs in scenario (SS,NDU). In general, the more dispersed the cut-off posteriors, the better off the investment banks as a whole. (NSS,DU) is the least desirable. This is because demand uncertainty gives rise to possible unfortunate retention by the investment banks. Furthermore, the ban on short sale forces the investment bank to retain a stake so that he can trade strategically. Yet his stake incurs additional cost of bank capital. In contrast, (SS,NDU) renders the investment banks the highest welfare in that they can always sell the security short to gain informed trading profits in the secondary market while they do not have to acquire any stake in the primary market. The comparison between the welfare of the remaining two scenarios is more involved. When ϵ is small (Case (1)), the investment banks' welfare is still higher if short sale is allowed compared to the scenario where there is no demand uncertainty but short selling is banned. Yet when ϵ is large (Case (2)), the investment banks are strictly better off without demand uncertainty even if short sale is prohibited. The trade-off hinges on whether the gain brought about by short sale is able to compensate for the loss due to the demand shock.

Finally, we summarize the rankings of the issuers' welfare in the next proposition.

Proposition 20 *(Issuers' Welfare): If the issuers do not disclosure additional information, their welfare have the following ranking:*

$$W_E(NSS, NDU) > W_E(SS, NDU) > W_E(SS, DU) > W_E(NSS, DU).$$

Yet if they use Bayesian persuasion to maximize their expected proceeds,

$$\hat{W}_E(NSS, NDU) = \hat{W}_E(SS, NDU) > \hat{W}_E(SS, DU) > \hat{W}_E(NSS, DU).$$

A graphical illustration of Proposition 20 is given in Figure 12. The proposition asserts that if issuers do not reveal informative signals, they achieve the highest welfare when there is no demand uncertainty in the primary market and short sale is not allowed in the secondary

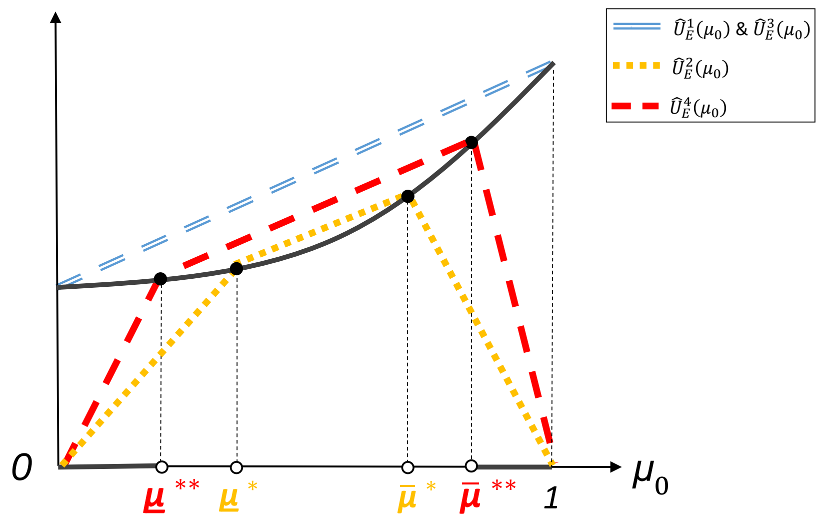


Figure 12: The Entrepreneur's Expected Payoffs

market. A primary market without demand uncertainty along with a short selling ban in the secondary market delivers the issuers the second highest welfare. They are worse off if demand shocks may happen in the primary market and underwriters are allowed to sell the security short. Their welfare is the lowest if it is probable that the security will be under-subscribed by participant investors in the primary market and there is short sale constraint in the secondary market. From the perspective of the issuers, they strictly prefer a primary market that has no demand uncertainty. Then the investment banks are always willing to underwrite, and the issuers can sell off their securities with certainty. Absent any possibility of demand shocks, they prefer a secondary market where underwriters are prohibited from short selling the securities. However, if demand is uncertain, the option of short sale allows the investment banks to reduce the opportunity cost associated with primary market retention and gain more from informed trading when demand shocks do not happen. This induces more banks to underwrite and thus enables more issuers to successfully issue their securities.

Under the issuers' optimal persuasion mechanisms, most parts of the ranking remain the same. They still dislike demand uncertainty in the primary market. However, with strategic disclosure the issuers will be indifferent between whether or not there is short sale constraint if there is no demand uncertainty. In both scenarios, the aim of the optimal disclosure is to discourage the investment bank from trading on his private information in the secondary market. To achieve this goal the optimal disclosure needs to be fully informative if short sale is allowed in the secondary market while a partially informative disclosure suffices to do the job if there is the short-sale ban.

7 Conclusion

This paper presents a novel Bayesian persuasion model of security offering and trading with issuer's strategic disclosure. We show that disclosure can be used to boost the issue's expected revenue, mitigate underpricing resulting from underwriter's informed trading, and increase the likelihood of security issue even when demand is weak and underwriters may shy away. On average, the optimal disclosure reduces the uncertainty of the security's payoff. Nevertheless, full transparency is not always optimal. Signal realizations that introduce more uncertainty can potentially solve the hold-up problem brought about by demand uncertainty. In general, the optimal information design depends crucially on the *ex ante* level of payoff uncertainty. We provide new empirical predictions which relate the informativeness of the optimal disclosure to the issue size and the issuer's growth option, the underwriter's cost of capital, the uncertainty about demand, and the secondary market liquidity. Moreover, the underwriter in our model can be viewed as an existing blockholder in the firm who makes decision on whether to support and participate in a security issue (e.g. seasoned debt/equity offering). We show that the blockholder, by participating, may exert governance by exit to push the firm to disclose more transparent information. In sum, corporate finance application of information design theory appears to be a promising topic to work on. Future work can be done by extending our model with issuer's moral hazard and signal manipulation as well as investors' information acquisition. Empirical side, textually analysis of the information memoranda and the prospectuses in both debt and equity issuance can be performed to test the new empirical predictions generated from our model.

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Appendix

Proof of Proposition 1. Suppose that the investment bank trades x when the state is G, and z when the state is B. He incurs additional cost of capital if he further acquires shares in the secondary market (i.e. either $x > 0$ or $z > 0$). And recall that $u \equiv (1-\beta)\phi$.

	State	Liquidity Shocks	$\tilde{v}$	Probability	x_{PI}	x_{IB}	y
(I).	G	Yes	V_H	$\mu_s\gamma$	$-u$	x	$y_I \equiv -u+x$
(II).	G	No	V_H	$\mu_s(1-\gamma)$	0	x	$y_{II} \equiv x$
(III).	B	Yes	V_L	$(1-\mu_s)\gamma$	$-u$	z	$y_{III} \equiv -u+z$
(IV).	B	No	V_L	$(1-\mu_s)(1-\gamma)$	0	z	$y_{IV} \equiv z$

To camouflage as liquidity traders, the investment bank has to design his trading strategy such that two of the above four scenarios have the same aggregate order flows. This gives four possibilities: $y_I = y_{III}$ (i.e. $-u+x=-u+z$), $y_I = y_{IV}$ (i.e. $-u+x=z$), $y_{II} = y_{III}$ (i.e. $x=-u+z$) or $y_{II} = y_{IV}$ (i.e. $x=z$). Note that the first and the last coincide. Hence we investigate the following three cases: 1. $x=z$, 2. $z=-u+x$, and 3. $z=u+x$.

Case 1. $x=z$:

	State	Liquidity Shocks	Probability	x_{PI}	x_{IB}	y	P_1
(I).	G	Yes	$\mu_s\gamma$	$-u$	x	$-u+x$	$\frac{\mu_s\gamma\Delta V}{\mu_s\gamma+(1-\mu_s)\gamma}+V_L$
(II).	G	No	$\mu_s(1-\gamma)$	0	x	x	$\frac{\mu_s(1-\gamma)\Delta V}{\mu_s(1-\gamma)+(1-\mu_s)(1-\gamma)}+V_L$
(III).	B	Yes	$(1-\mu_s)\gamma$	$-u$	x	$-u+x$	$\frac{\mu_s\gamma\Delta V}{\mu_s\gamma+(1-\mu_s)\gamma}+V_L$
(IV).	B	No	$(1-\mu_s)(1-\gamma)$	0	x	x	$\frac{\mu_s(1-\gamma)\Delta V}{\mu_s(1-\gamma)+(1-\mu_s)(1-\gamma)}+V_L$

It is easy to see that $P_1 = \mu_s\Delta V + V_L$ since the net order flows are only indicative of whether or not there is liquidity shock, but reveals no information concerning the underlying state due to the investment bank's consistent trading strategy regardless of his private information. So the market maker will set a price to the intrinsic value of the security conditional on the posterior belief μ_s. The investment bank's expected payoff from this trading strategy is

$$\begin{aligned}\mathbb{E}_s[\Pi_1] = \quad & [V_H - (\mu_s \Delta V + V_L)][\mu_s\gamma + \mu_s(1-\gamma)]x \\ & +[V_L - (\mu_s \Delta V + V_L)][(1-\mu_s)\gamma + (1-\mu_s)(1-\gamma)]x - \mathbb{1}_{\{x>0\}}\, r(\mu_s\Delta V + V_L)x \\ = \quad & -\mathbb{1}_{\{x>0\}}\, r(\mu_s \Delta V + V_L)x \\ \leq \quad & 0.\end{aligned}$$

Case 2. $z = -u + x$:

	State	Liquidity Shocks	Probability	x_{PI}	x_{IB}	y	P_1
(I).	G	Yes	$\mu_s\gamma$	$-u$	x	$-u+x$	$\frac{\mu_s\gamma\Delta V}{\mu_s\gamma+(1-\mu_s)(1-\gamma)} + V_L$
(II).	G	No	$\mu_s(1-\gamma)$	0	x	x	V_H
(III).	B	Yes	$(1-\mu_s)\gamma$	$-u$	$-u+x$	$-2u+x$	V_L
(IV).	B	No	$(1-\mu_s)(1-\gamma)$	0	$-u+x$	$-u+x$	$\frac{\mu_s\gamma\Delta V}{\mu_s\gamma+(1-\mu_s)(1-\gamma)} + V_L$

The investment bank's expected trading profits from this trading strategy are

$$\begin{aligned}\mathbb{E}_s[\Pi_2] = \quad & \left(V_H - \frac{\mu_s\gamma\Delta V}{\mu_s\gamma + (1-\mu_s)(1-\gamma)} - V_L\right)\mu_s\gamma x \\ & + \left(V_L - \frac{\mu_s\gamma\Delta V}{\mu_s\gamma + (1-\mu_s)(1-\gamma)} - V_L\right)(1-\mu_s)(1-\gamma)(-u+x) \\ & -\mathbb{1}_{\{x>0\}}\, rx\left[\mu_s\gamma\frac{\mu_s\gamma\Delta V}{\mu_s\gamma + (1-\mu_s)(1-\gamma)} + V_L + \mu_s(1-\gamma)V_H\right] \\ & -\mathbb{1}_{\{-u+x>0\}}\, r(-u+x)\left[(1-\mu_s)\gamma V_L + (1-\mu_s)(1-\gamma)\left(\frac{\mu_s\gamma\Delta V}{\mu_s\gamma + (1-\mu_s)(1-\gamma)} + V_L\right)\right] \\ = \quad & \frac{\mu_s\gamma(1-\mu_s)(1-\gamma)\Delta V}{\mu_s\gamma + (1-\mu_s)(1-\gamma)} \cdot u - \mathbb{1}_{\{x>0\}}\, rx\left[\mu_s\gamma\frac{\mu_s\gamma\Delta V}{\mu_s\gamma + (1-\mu_s)(1-\gamma)} + V_L + \mu_s(1-\gamma)V_H\right] \\ & -\mathbb{1}_{\{-u+x>0\}}\, r(-u+x)\left[(1-\mu_s)\gamma V_L + (1-\mu_s)(1-\gamma)\left(\frac{\mu_s\gamma\Delta V}{\mu_s\gamma + (1-\mu_s)(1-\gamma)} + V_L\right)\right] \\ \leq \quad & \frac{\mu_s\gamma(1-\mu_s)(1-\gamma)\Delta V}{\mu_s\gamma + (1-\mu_s)(1-\gamma)} \cdot u.\end{aligned}$$

In this case, it is optimal to set $x = 0$ and $z = -u$ such that the investment bank can achieve the maximal expected trading profits $\frac{\mu_s\gamma(1-\mu_s)(1-\gamma)\Delta V}{\mu_s\gamma+(1-\mu_s)(1-\gamma)} \cdot u$ while do not incur additional cost of capital from acquiring shares in the secondary market. It is an informed sales equilibrium where the investment bank only sell his stake when his private information is unfavorable. Moreover, such trading strategy is sequentially rational as well.

Finally, we consider Case 3. ($z = u + x$):

	State	Liquidity Shocks	Probability	x_{PI}	x_{IB}	y	P_1
(I).	G	Yes	$\mu_s\gamma$	$-u$	x	$-u+x$	V_H
(II).	G	No	$\mu_s(1-\gamma)$	0	x	x	$\frac{\mu_s(1-\gamma)\Delta V}{\mu_s(1-\gamma)+(1-\mu_s)\gamma} + V_L$
(III).	B	Yes	$(1-\mu_s)\gamma$	$-u$	$u+x$	x	$\frac{\mu_s(1-\gamma)\Delta V}{\mu_s(1-\gamma)+(1-\mu_s)\gamma} + V_L$
(IV).	B	No	$(1-\mu_s)(1-\gamma)$	0	$u+x$	$u+x$	V_L

His relevant expected trading profits are

$$\begin{aligned}
\mathbb{E}_s[\Pi_3] = \quad & \left[V_H - \frac{\mu_s(1-\gamma)\Delta V}{\mu_s(1-\gamma)+(1-\mu_s)\gamma} - V_L\right]\mu_s(1-\gamma)x \\
& + \left[V_L - \frac{\mu_s(1-\gamma)\Delta V}{\mu_s(1-\gamma)+(1-\mu_s)\gamma} - V_L\right](1-\mu_s)\gamma x \\
& - \mathbb{1}_{\{x>0\}}\, rx \left[\mu_s\gamma V_H + \mu_s(1-\gamma)\left(\frac{\mu_s(1-\gamma)\Delta V}{\mu_s(1-\gamma)+(1-\mu_s)\gamma} + V_L\right)\right] \\
& - \mathbb{1}_{\{x+u>0\}}\, r(x+u)\left[(1-\mu_s)\gamma\left(\frac{\mu_s(1-\gamma)\Delta V}{\mu_s(1-\gamma)+(1-\mu_s)(1-\gamma)} + V_L\right) + (1-\mu_s)\gamma V_L\right] \\
\leq \quad & - \mathbb{1}_{\{x>0\}}\, rx \left[\mu_s\gamma V_H + \mu_s(1-\gamma)\left(\frac{\mu_s(1-\gamma)\Delta V}{\mu_s(1-\gamma)+(1-\mu_s)\gamma} + V_L\right)\right] \\
& - \mathbb{1}_{\{x+u>0\}}\, r(x+u)\left[(1-\mu_s)\gamma\left(\frac{\mu_s(1-\gamma)\Delta V}{\mu_s(1-\gamma)+(1-\mu_s)(1-\gamma)} + V_L\right) + (1-\mu_s)\gamma V_L\right] \\
\leq \quad & 0.
\end{aligned}$$

This strategy is obviously suboptimal.

In sum, the investment bank's optimal trading strategy is $x_{IB} = 0$ in state G and $x_{IB} = -u$ in state B. This gives the equilibrium characterized in Proposition 1. ■

Proof of Lemma 2.

$$
\begin{aligned}
U_{IB}^1(\frac{\phi}{1+\phi},\mu_s) &= \frac{\phi}{1+\phi}\left[(\mu_s\Delta V+V_L)-(1+r)P_0(\frac{\phi}{1+\phi},\mu_s)\right]+\frac{1}{1+\phi}\cdot\frac{(1-\mu_s)\mu_s(1-\gamma)\gamma\phi\Delta V}{\mu_s\gamma+(1-\mu_s)(1-\gamma)}\\
&= \frac{\phi}{1+\phi}\cdot\left[-r(\mu_s\Delta V+V_L)+(1+r)\cdot\frac{\mu_s(1-\mu_s)\gamma(1-\gamma)\phi\Delta V}{\mu_s\gamma+(1-\mu_s)(1-\gamma)}\right]\\
&\quad+\frac{1}{1+\phi}\cdot\frac{\mu_s(1-\mu_s)\gamma(1-\gamma)\phi\Delta V}{\mu_s\gamma+(1-\mu_s)(1-\gamma)}\\
&= \frac{\phi}{1+\phi}\cdot\{-r\,\mathbb{E}_s[\tilde v]+(1+r)\Delta P\}+\frac{1}{1+\phi}\cdot\Delta P\\
&= -\frac{r\phi}{1+\phi}\cdot\mathbb{E}_s[\tilde v]+\left(1+\frac{r\phi}{1+\phi}\right)\cdot\Delta P.
\end{aligned}
$$

Note that

$$
\frac{\partial\,\mathbb{E}_s[\tilde v]}{\partial\mu_s}=\frac{\partial(\mu_s\Delta V+V_L)}{\partial\mu_s}=\Delta V,\qquad \frac{\partial^2\,\mathbb{E}_s[\tilde v]}{\partial\mu_s^2}=0,
$$

and

$$
\begin{aligned}
\frac{\partial\,\Delta P}{\partial\mu_s} &= \frac{\partial}{\partial\mu_s}\left(\frac{\mu_s(1-\mu_s)\gamma(1-\gamma)\phi\Delta V}{\mu_s\gamma+(1-\mu_s)(1-\gamma)}\right)\\
&= \gamma(1-\gamma)\phi\Delta V\cdot\frac{(1-2\gamma)\mu_s^2-2(1-\gamma)\mu_s+(1-\gamma)}{[\mu_s\gamma+(1-\mu_s)(1-\gamma)]^2}.
\end{aligned}
$$

Moreover,

$$
\frac{\partial^2\,\Delta P}{\partial\mu_s^2}=\gamma(1-\gamma)\phi\Delta V\cdot\frac{-2\gamma(1-\gamma)}{[\mu_s\gamma+(1-\mu_s)(1-\gamma)]^3}<0.
$$

Therefore

$$
\frac{\partial\,U_{IB}^1}{\partial\mu_s}=-\frac{r\phi\Delta V}{1+\phi}+\left(1+\frac{r\phi}{1+\phi}\right)\cdot\frac{\gamma(1-\gamma)\phi\Delta V[(1-2\gamma)\mu_s^2-2(1-\gamma)\mu_s+(1-\gamma)]}{[\mu_s\gamma+(1-\mu_s)(1-\gamma)]^2},
$$

and

$$
\frac{\partial^2\,U_{IB}^1}{\partial\mu_s^2}=\left(1+\frac{r\phi}{1+\phi}\right)\cdot\left(\frac{\partial^2\,\Delta P}{\partial\mu_s^2}\right)<0,
$$

i.e. U_{IB}^1 is concave and $\partial U_{IB}^1/\partial\mu_s$ is decreasing in $\mu_s \in (0,1)$.

To ensure that the interior optimum is attained at some $\mu^* \in (0,1)$, the following must be satisfied:

$$\left.\frac{\partial U_{IB}^1}{\partial\mu_s}\right|_{\mu_s=0} = -\frac{r\phi\Delta V}{1+\phi} + \left(1+\frac{r\phi}{1+\phi}\right)\gamma\phi\Delta V > 0;$$

$$\left.\frac{\partial U_{IB}^1}{\partial\mu_s}\right|_{\mu_s=1} = -\frac{r\phi\Delta V}{1+\phi} - \left(1+\frac{r\phi}{1+\phi}\right)(1-\gamma)\phi\Delta V < 0.$$

The first implies that $r < \frac{\gamma(1+\phi)}{1-\gamma\phi}$ while the second is always satisfied. Then $\partial U_{IB}^1/\partial\mu_s = 0$ when $\mu_s = \mu^*$. And for $\mu_s \in [0,\mu^*)$, $\partial U_{IB}^1/\partial\mu_s > 0$ yet $\partial U_{IB}^1/\partial\mu_s < 0$ for $\mu_s \in (\mu^*,1]$. Therefore, U_{IB}^1 is single-peaked and has a hump shape on $[0,1]$.

Since from above we know that

$$U_{IB}^1(\frac{\phi}{1+\phi},\mu_s) = -\frac{r\phi}{1+\phi}\cdot(\mu_s\Delta V + V_L) + \left(1+\frac{r\phi}{1+\phi}\right)\cdot\frac{(1-\mu_s)\mu_s(1-\gamma)\gamma\phi\Delta V}{\mu_s\gamma+(1-\mu_s)(1-\gamma)},$$

it is obvious that there always exists a set of $\mu_s \in (0,1)$ such that $U_{IB}^1(\frac{\phi}{1+\phi},\mu_s) > 0$ as long as r is not too large. In particular, we impose that for $\mu_s = \frac{1}{2}$, $U_{IB}^1(\frac{\phi}{1+\phi},\frac{1}{2}) > 0$. This implies $r < \frac{\gamma(1-\gamma)(1+\phi)\Delta V}{\Delta V-\gamma(1-\gamma)\phi\Delta V+2V_L}$. Therefore, $0 < r < \min\{\frac{\gamma(1+\phi)}{1-\gamma\phi}, \frac{\gamma(1-\gamma)(1+\phi)\Delta V}{\Delta V-\gamma(1-\gamma)\phi\Delta V+2V_L}\}$, i.e. $r \in (0, \frac{\gamma(1-\gamma)(1+\phi)\Delta V}{\Delta V-\gamma(1-\gamma)\phi\Delta V+2V_L})$.

In the meantime, $U_{IB}^1(\frac{\phi}{1+\phi},0) = -\frac{\phi r V_L}{1+\phi} < 0$ and $U_{IB}^1(\frac{\phi}{1+\phi},1) = -\frac{\phi r(\Delta V+V_L)}{1+\phi} < 0$. Hence there must be a pair of $\{\underline{\mu},\overline{\mu}\}$ with $0 < \underline{\mu} < \frac{1}{2} < \overline{\mu} < 1$ such that $U_{IB}^1(\frac{\phi}{1+\phi},\underline{\mu}) = U_{IB}^1(\frac{\phi}{1+\phi},\overline{\mu}) = 0$. In addition, $U_{IB}^1(\frac{\phi}{1+\phi},\mu_s) > 0$ if $\mu_s \in (\underline{\mu},\overline{\mu})$, and $U_{IB}^1(\frac{\phi}{1+\phi},\mu_s) < 0$ if $\mu_s \in [0,\underline{\mu}) \cup (\overline{\mu},1]$.

Last but not least, it follows naturally that $\partial U_{IB}^1/\partial\mu_s > 0$ at $\mu_s = \underline{\mu}$ but $\partial U_{IB}^1/\partial\mu_s < 0$ at $\mu_s = \overline{\mu}$, an important observation that will be useful to calculate the comparative statics of the optimal disclosure later. ■

Proof of Proposition 4. When $\beta \in [\frac{\phi}{1+\phi},1)$, there will be discount in the issue price. And

$$U_{IB}^1(\beta,\mu_s) = \beta\{-r\,\mathbb{E}_s[\tilde{v}] + (1+r)\Delta P\} + (1-\beta)\Delta P.$$

Note that

$$\begin{aligned}
\{-r\,\mathbb{E}_s[\tilde{v}] + (1+r)\Delta P\} - \Delta P &= -r(\mathbb{E}_s[\tilde{v}] - \Delta P) \\
&= -r\left[V_L + \mu_s \Delta V \cdot \frac{\mu_s\gamma + (1-\mu_s)(1-\gamma)(1-\gamma\phi)}{\mu_s\gamma + (1-\mu_s)(1-\gamma)}\right] \\
&< 0.
\end{aligned}$$

Hence to maximize $U^1_{IB}(\beta,\mu_s)$, we want $(1-\beta)$ to be as large as possible. This is achieved by choosing the smallest $\beta = \frac{\phi}{1+\phi}$ such that informed trading is still feasible. And it is easy to see that $U^1_{IB}(\frac{\phi}{1+\phi},\mu_s) > U^1_{IB}(1^-,\mu_s)$ for all $\mu_s \in (0,1)$. So in equilibrium, stake $\frac{\phi}{1+\phi}$ strictly dominates stake 1^-. Moreover, we know that for $\beta = 0$, $U^1_{IB}(0,\mu_s) = 0$, and for $\beta = 1$, $U^1_{IB}(1,\mu_s) < 0$. So $\beta = 0$ strictly dominates $\beta = 1$. To characterize the investment bank's optimal retention at posterior belief μ_s, it suffices to compare $U^1_{IB}(0,\mu_s)$ with $U^1_{IB}(\frac{\phi}{1+\phi},\mu_s)$. From Lemma 2, it follows that the investment bank's optimal stake is

$$\beta^* = \begin{cases} \frac{\phi}{1+\phi} & \text{if } \mu_s \in (\underline{\mu},\overline{\mu}), \\ 0 & \text{if } \mu_s \in [0,\underline{\mu}] \cup [\overline{\mu},1]. \end{cases}$$

And his equilibrium payoff is

$$\hat{U}^1_{IB}(\mu_s) = \begin{cases} U^1_{IB}(\frac{\phi}{1+\phi},\mu_s) & \text{if } \mu_s \in (\underline{\mu},\overline{\mu}), \\ 0 & \text{if } \mu_s \in [0,\underline{\mu}] \cup [\overline{\mu},1]. \end{cases}$$

Q.E.D. ■

Proof of Proposition 5. From Proposition 4 we know that the investment bank will hold a positive stake $\frac{\phi}{1+\phi}$ only when $\mu_s \in (\underline{\mu},\overline{\mu})$. So for this set of posterior beliefs, there will be informed trading by the bank and thus an adverse selection discount in the issue price. And the issuer's expected proceeds are

$$U_E^1(\mu_s) = \mu_s \Delta V + V_L - \frac{\mu_s(1-\mu_s)\gamma(1-\gamma)\phi\Delta V}{\mu_s\gamma + (1-\mu_s)(1-\gamma)}.$$

At any other posterior belief, the investment bank retains zero stake and cannot engage in informed trading. The issue price will just be the intrinsic value of the security, i.e.

$$U_E^1(\mu_s) = \mu_s \Delta V + V_L.$$

Q.E.D. ■

Proof of Proposition 6. At any prior belief $\mu_0 \in [0, \underline{\mu}] \cup [\overline{\mu}, 1]$, a sender-preferred equilibrium prescribes that the investment bank should not retain any shares. In this case, the issue price will be the expected value of the cash flows from the security with no discount. Thus the issuer does not benefit from persuasion and the optimal disclosure system should be completely uninformative, i.e $\pi_G = \pi_B \in (0,1)$, yielding posteriors $\mu_\ell = \mu_h = \mu_0$.

At prior belief $\mu_0 \in (\underline{\mu}, \overline{\mu})$, the investment bank holds a strictly positive stake, and there will be a discounted associated with the issue price. The issuer's expected payoff under any Bayesian plausible posteriors μ_h and μ_ℓ is

$$\begin{aligned}
\mathbb{E}_\pi[U_E^1(\mu_s)] &= \mathbb{E}_\pi[\mathbb{1}_{\{\mu_0 \in [0,\underline{\mu}] \cup [\overline{\mu},1]\}} \cdot (\mu_s \Delta V + V_L) + \mathbb{1}_{\{\mu_0 \in (\underline{\mu}, \overline{\mu})\}}(\mu_s \Delta V + V_L - \Delta P)] \\
&= \mathbb{P}[\mu_h] \cdot [\mathbb{1}_{\{\mu_h \in [0,\underline{\mu}] \cup [\overline{\mu},1]\}} \cdot (\mu_h \Delta V + V_L) + \mathbb{1}_{\{\mu_h \in (\underline{\mu}, \overline{\mu})\}}(\mu_h \Delta V + V_L - \Delta P)] \\
&\quad + \mathbb{P}[\mu_\ell] \cdot [\mathbb{1}_{\{\mu_\ell \in [0,\underline{\mu}] \cup [\overline{\mu},1]\}} \cdot (\mu_\ell \Delta V + V_L) + \mathbb{1}_{\{\mu_\ell \in (\underline{\mu}, \overline{\mu})\}}(\mu_\ell \Delta V + V_L - \Delta P)] \\
&\leq \mathbb{P}(\mu_h)(\mu_h \Delta V + V_L) + \mathbb{P}(\mu_\ell)(\mu_\ell \Delta V + V_L),
\end{aligned}$$

where the last inequality is satisfied with if $\mu_\ell \in [0, \underline{\mu}]$, $\mu_h \in [\overline{\mu}, 1]$ and $\mathbb{P}(\mu_h)\mu_h + \mathbb{P}(\mu_\ell) = \mu_0$. Hence the least informative optimal disclosure yields posteriors $\mu_\ell = \underline{\mu}$ and $\mu_h = \overline{\mu}$. In this case $\hat{U}_E^1(\mu_0) = \max \mathbb{E}_\pi[U_E^1(\mu_s)] = \mu_0 \Delta V + V_L$. Using Bayes' theorem, simple algebra gives

$\pi_B = \frac{(1-\overline{\mu})(\mu_0-\underline{\mu})}{(1-\mu_0)(\overline{\mu}-\underline{\mu})}$ and $\pi_G = \frac{\overline{\mu}(\mu_0-\underline{\mu})}{\mu_0(\overline{\mu}-\underline{\mu})}$. ■

Proof of Proposition 7. Recall that $\overline{\mu}$ and $\underline{\mu}$ are two roots to the equation $U^1_{IB}(\frac{\phi}{1+\phi}, \mu_s) = 0$. Write explicitly,

$$\begin{aligned} U^1_{IB}(\frac{\phi}{1+\phi}, \mu_s) &= \frac{\phi}{1+\phi} \cdot \left[-r(\mu_s \Delta V + V_L) + (1+r) \cdot \frac{\mu_s(1-\mu_s)\gamma(1-\gamma)\phi\Delta V}{\mu_s\gamma + (1-\mu_s)(1-\gamma)} \right] \\ &\quad + \frac{1}{1+\phi} \cdot \frac{\mu_s(1-\mu_s)\gamma(1-\gamma)\phi\Delta V}{\mu_s\gamma + (1-\mu_s)(1-\gamma)} \\ &= 0. \end{aligned}$$

Multiply both sides by $\frac{1+\phi}{\phi}$, and define

$$\begin{aligned} F(\mu_s, \theta) &\equiv \frac{1+\phi}{\phi} \cdot U^1_{IB}(\frac{\phi}{1+\phi}, \mu_s) \\ &= -r(\mu_s \Delta V + V_L) + [1 + (1+r)\phi] \cdot \frac{\mu_s(1-\mu_s)\gamma(1-\gamma)\Delta V}{\mu_s\gamma + (1-\mu_s)(1-\gamma)}, \end{aligned}$$

where $\theta \in \{V_L, \frac{\Delta}{V_L}, r, \phi\}$. By the implicit function theorem, at $\mu_s = \underline{\mu}$ or $\overline{\mu}$,

$$\frac{\partial F}{\partial \mu_s} \cdot \frac{\partial \mu_s}{\partial \theta} + \frac{\partial F}{\partial \theta} = 0.$$

This gives

$$\text{sign}\left(\frac{\partial \mu_s}{\partial \theta}\right) = -\text{sign}\left(\frac{\partial F}{\partial \mu_s} \cdot \frac{\partial F}{\partial \theta}\right).$$

Next we calculate $F(\mu_s, \theta)$'s partial derivatives with respect to different $\theta \in \{V_L, \eta, r, \phi\}$:

$$\begin{aligned} \frac{\partial F}{\partial V_L} &= -r < 0; \\ \frac{\partial F}{\partial r} &= -V_L - \mu_s \Delta V \left[1 - \frac{(1-\mu_s)(1-\gamma)\gamma\phi}{\mu_s\gamma + (1-\mu_s)(1-\gamma)} \right] < 0; \\ \frac{\partial F}{\partial \phi} &= (1+r) \cdot \frac{\mu_s(1-\mu_s)\gamma(1-\gamma)\Delta V}{\mu_s\gamma + (1-\mu_s)(1-\gamma)} > 0. \end{aligned}$$

Define $\eta \equiv \frac{\Delta V}{V_L}$, and

$$f \equiv \frac{F}{\Delta V} = -r(\mu_s \Delta V + \frac{1}{\eta}) + [1 + (1+r)\phi] \cdot \frac{\mu_s(1-\mu_s)\gamma(1-\gamma)}{\mu_s\gamma + (1-\mu_s)(1-\gamma)}.$$

So

$$\frac{\partial f}{\partial \eta} = \frac{r}{\eta^2} > 0 \Rightarrow \frac{\partial F}{\partial \eta} = \frac{r\Delta V}{\eta^2} > 0.$$

Moreover, in the proof of Lemma 2 we have shown that $\frac{\partial F}{\partial \mu_s} > 0$ at $\mu_s = \underline{\mu}$ but $\frac{\partial F}{\partial \mu_s} < 0$ at $\mu_s = \overline{\mu}$. Consequently, we have (1) $\frac{\partial \underline{\mu}}{\partial V_L} > 0$ and $\frac{\partial \overline{\mu}}{\partial V_L} < 0$; (2) $\frac{\partial \underline{\mu}}{\partial \eta} < 0$ and $\frac{\partial \overline{\mu}}{\partial \eta} > 0$; (3) $\frac{\partial \underline{\mu}}{\partial r} > 0$ and $\frac{\partial \overline{\mu}}{\partial r} < 0$; (4) $\frac{\partial \underline{\mu}}{\partial \phi} < 0$ and $\frac{\partial \overline{\mu}}{\partial \phi} > 0$. ■

Proof of Proposition 8. If the investment bank chooses to underwrite and his planned retention is $\hat{\beta}$, we can write his expected payoff as

$$\begin{aligned} U_{IB}^2(\hat{\beta}, \mu_s) = \epsilon A(1-\psi, \mu_s) + (1-\epsilon)B(\hat{\beta}, \mu_s) &= [\epsilon(1-\psi) + (1-\epsilon)\hat{\beta}] \cdot [-r\,\mathbb{E}_s[\tilde{v}] + (1+r)\Delta P] \\ &\quad +[\epsilon\psi + (1-\epsilon)(1-\hat{\beta})] \cdot \Delta P. \end{aligned}$$

Recall from the proof of Proposition 4 that $-r\,\mathbb{E}_s[\tilde{v}] + (1+r)\Delta P < \Delta P$, thus we want $\hat{\beta}$ to be as small as possible yet such stake still allows the underwriter to engage in informed trading if demand shock does not happen. The optimal planned retention is $\hat{\beta} = \frac{\phi}{1+\phi}$, the stake that is just enough for the bank to camouflage as liquidity traders. ■

Proof of Lemma 3. The proof resembles that of Lemma 2. Specifically, the equation now becomes

$$\begin{aligned} U_{IB}^2(\frac{\phi}{1+\phi}, \mu_s) &= [\epsilon(1-\psi) + (1-\epsilon)\hat{\beta}][-r\,\mathbb{E}_s[\tilde{v}] + (1+r)\Delta P] + [\epsilon\psi + (1-\epsilon)(1-\hat{\beta})]\Delta P \\ &= 0. \end{aligned}$$

$\frac{\partial^2 U_{IB}^2}{\partial \mu_s^2} < 0$ because $\frac{\partial^2 \Delta P}{\partial \mu_s^2} < 0$. So U_{IB}^2 is concave in μ_s. To ensure that the interior optimum is attained at some $\mu^* \in (0,1)$, the following must be satisfied:

$$\left.\frac{\partial U_{IB}^2}{\partial \mu_s}\right|_{\mu_s=0} = -Kr\Delta V + [(1+r)K + (1-K)]\gamma\phi\Delta V > 0;$$

$$\left.\frac{\partial U_{IB}^2}{\partial \mu_s}\right|_{\mu_s=1} = -Kr\Delta V - [(1+r)K + (1-K)](1-\gamma)\phi\Delta V < 0,$$

where $K \equiv \epsilon(1-\psi) + (1-\epsilon)(\frac{\phi}{1+\phi})$ and $1-K = \epsilon\psi + (1-\epsilon)(\frac{1}{1+\phi})$. The first inequality implies

$$r < \frac{\gamma\phi(1+\phi)}{[(1+\phi)(1-\psi)\epsilon + \phi(1-\epsilon)](1-\gamma\phi)},$$

while the second is always satisfied.

Some simple algebra reveals that $U_{IB}^2(\frac{\phi}{1+\phi}, 0) < 0$ and $U_{IB}^2(\frac{\phi}{1+\phi}, 1) < 0$. Moreover, we need $U_{IB}^2(\frac{\phi}{1+\phi}, \frac{1}{2}) > 0$. This implies

$$r < \frac{\gamma\phi(1+\phi)(1-\gamma)\Delta V}{[(1+\phi)(1-\psi)\epsilon + \phi(1-\epsilon)][\Delta V - \gamma\phi(1-\gamma)\Delta V + 2V_L]}.$$

Therefore, $r < \min\left\{\frac{\gamma\phi(1+\phi)}{[(1+\phi)(1-\psi)\epsilon+\phi(1-\epsilon)](1-\gamma\phi)}, \frac{\gamma\phi(1+\phi)(1-\gamma)\Delta V}{[(1+\phi)(1-\psi)\epsilon+\phi(1-\epsilon)][\Delta V-\gamma\phi(1-\gamma)\Delta V+2V_L]}\right\}$, i.e. $r < \frac{\gamma\phi(1+\phi)(1-\gamma)\Delta V}{[(1+\phi)(1-\psi)\epsilon+\phi(1-\epsilon)][\Delta V-\gamma\phi(1-\gamma)\Delta V+2V_L]}$. Note that both U_{IB}^1 and U_{IB}^2 are convex combinations of two ingredients $-r\,\mathbb{E}_s[\tilde{v}] + (1+r)\Delta P$ and ΔP with the latter strictly larger than the former. And it is easy to see that U_{IB}^1 puts more weight on ΔP and thus less weight on $-r\,\mathbb{E}_s[\tilde{v}] + (1+r)\Delta P$ than U_{IB}^2. Hence $U_{IB}^1 > U_{IB}^2$, $\forall \mu_s \in [0,1]$.

With the same logic used in the proof of Lemma 2, it follows naturally:

1. There exists a pair $\{\underline{\mu}^*, \overline{\mu}^*\}$ with $0 < \underline{\mu} < \underline{\mu}^* < \frac{1}{2} < \overline{\mu}^* < \overline{\mu} < 1$ such that $U_{IB}^2(\frac{\phi}{1+\phi}, \underline{\mu}^*) = U_{IB}^2(\frac{\phi}{1+\phi}, \overline{\mu}^*) = 0$.

2. $U_{IB}^2(\frac{\phi}{1+\phi}, \mu_s) > 0$ if $\mu_s \in (\underline{\mu}^*, \overline{\mu}^*)$, and $\tilde{U}_{IB}(\frac{\phi}{1+\phi}, \mu_s) < 0$ if $\mu_s \in [0, \underline{\mu}^*)$ or $\mu_s \in (\overline{\mu}^*, 1]$.

Q.E.D. ■

Proof of Proposition 9 and Proposition 10. It follows naturally from Proposition 8 and Lemma 3 that at $T = 1$, the investment bank will agree to underwrite if his planned retention $\frac{\phi}{1+\phi}$ gives him a non-negative expected payoff. So he chooses to underwrite if $\mu_s \in [\underline{\mu}^*, \overline{\mu}^*]$, and not underwrite otherwise. And the issuer is only able to issue the security when $\mu_s \in [\underline{\mu}^*, \overline{\mu}^*]$, and get an expected payoff of $\mathbb{E}_s[\tilde{v}] - \Delta P$. ■

Proof of Proposition 11. The optimal information design depends on the prior μ_0.

1. First we investigate the optimal system when prior $\mu_0 \in [0, \underline{\mu}^*)$. In this case the investment bank does not underwrite if no additional information is disclosed. Consider any two arbitrary posteriors μ_ℓ and μ_h with $0 \le \mu_\ell \le \mu_0 \le \mu_h \le 1$ and $\mathbb{P}[s = \ell]\mu_\ell + \mathbb{P}[s = h]\mu_h = \mu_0$. To maximize her expected proceeds, the issuer will set $\mu_\ell = 0$ to have the maximal $\mathbb{P}[s = h]\mu_h$ which is μ_0. And the issuer will set a $\mu_h \in [\underline{\mu}^*, \overline{\mu}^*]$ so that the investment bank is willing to underwrite. Her expected payoff is therefore $\mathbb{P}[s = h]P_0(\mu_h) = \frac{\mu_0 P_0(\mu_h)}{\mu_h}$. Recall that

$$U_{IB}^2(\frac{\phi}{1+\phi}, \mu_s) = K[-r\,\mathbb{E}_s[\tilde{v}] + (1+r)\Delta P] + (1-K)\Delta P = -rKP_0(\mu_s) + \Delta P(\mu_s),$$

where $K = \epsilon(1-\psi) + (1-\epsilon)(\frac{\phi}{1+\phi})$, and $\Delta P(\mu_s)$ means ΔP is a function of μ_s.

At $\mu_s = \underline{\mu}^*$ or $\overline{\mu}^*$, $U_{IB}^2(\frac{\phi}{1+\phi}, \mu_s) = 0$. This implies

$$\begin{aligned} & -rKP_0(\mu_s) + \Delta P(\mu_s) = 0 \\ \Rightarrow \quad & \frac{P_0(\mu_s)}{\mu_s} = \frac{\Delta P(\mu_s)}{rK\mu_s} = \frac{(1-\mu_s)\gamma(1-\gamma)\phi\Delta V}{rK[\mu_s\gamma + (1-\mu_s)(1-\gamma)]}. \end{aligned}$$

The last term is decreasing in $\mu_s \in [\underline{\mu}^*, \overline{\mu}^*]$. Since $\underline{\mu}^* < \overline{\mu}^*$, we have $\frac{P_0(\underline{\mu}^*)}{\underline{\mu}^*} > \frac{P_0(\overline{\mu}^*)}{\overline{\mu}^*}$.

Moreover, at $\mu_s \in [\underline{\mu}^*, \overline{\mu}^*]$, $U_{IB}^2(\frac{\phi}{1+\phi}, \mu_s) \ge 0$. This implies

$$\begin{aligned} & -rKP_0(\mu_s) + \Delta P(\mu_s) \ge 0 \\ \Rightarrow \quad & \frac{P_0(\mu_s)}{\mu_s} \le \frac{\Delta P(\mu_s)}{rK\mu_s} \le \frac{\Delta P(\underline{\mu}^*)}{rK\underline{\mu}^*} = \frac{P_0(\underline{\mu}^*)}{\underline{\mu}^*}. \end{aligned}$$

Therefore, the optimal system will induce two posteriors $\mu_\ell = 0$ and $\mu_h = \underline{\mu}^*$. The relevant precision parameters are $\pi_B = \frac{\mu_0(1-\underline{\mu}^*)}{\underline{\mu}^*(1-\mu_0)}$ and $\pi_G = 1$.

2. Second, we derive the optimal system when $\mu_0 \in (\overline{\mu}^*, 1]$. Consider any two arbitrary posteriors μ_ℓ and μ_h with $0 \le \mu_\ell \le \mu_0 \le \mu_h \le 1$ and $\mathbb{P}[s=\ell]\mu_\ell + \mathbb{P}[s=h]\mu_h = \mu_0$. To maximize her expected proceeds, the issuer will set $\mu_h = 1$. This ensures that for any fixed μ_ℓ, the probability of achieving this posterior $\mathbb{P}[s=\ell] = \frac{\mu_h - \mu_0}{\mu_h - \mu_l}$ will be maximized, i.e. the probability of underwriting will be the highest. Her expected payoff is therefore $\mathbb{P}[s=\ell]P_0(\mu_\ell) = \frac{1-\mu_0}{1-\mu_\ell} \cdot P_0(\mu_\ell)$. Since both $\frac{1-\mu_0}{1-\mu_\ell}$ and $P_0(\mu_\ell)$ are increasing in μ_ℓ, it is optimal to set $\mu_\ell = \overline{\mu}^*$. Hence the optimal system yields two posteriors $\mu_\ell = \underline{\mu}^*$ and $\mu_h = 1$. This gives $\pi_B = \frac{\mu_0 - \overline{\mu}^*}{\mu_0(1-\overline{\mu}^*)}$ and $\pi_G = 0$.

3. Third, when $\mu_0 = \underline{\mu}^*$ or $\overline{\mu}^*$, the investment bank is break-even by underwriting the deal. In this case, a completely uninformative disclosure system is optimal. It has $\pi_G = \pi_B \in (0,1)$, yielding posteriors $\mu_\ell = \mu_h = \mu_0$.

4. Finally, we find the optimal system when $\mu_0 \in (\underline{\mu}^*, \overline{\mu}^*)$. Since $\Delta P(\mu_s)$ is concave in μ_s, $P_0(\mu_s) = \mathbb{E}_s[\tilde{v}] - \Delta P$ is convex and increases in μ_s. First consider any arbitrary posteriors μ_ℓ and μ_h such that $\underline{\mu}^* \le \mu_\ell \le \mu_0 \le \mu_h \le \overline{\mu}^*$.

   In order for the two pairs of posteriors $\{\underline{\mu}^*, \overline{\mu}^*\}$ and $\{\mu_\ell, \mu_h\}$ to be Bayesian plausible, they should satisfy

$$\begin{aligned}\mu_0 &= \lambda\underline{\mu}^* + (1-\lambda)\overline{\mu}^*,\\ \mu_0 &= \bar{\lambda}\mu_\ell + (1-\bar{\lambda})\mu_h.\end{aligned}$$

   Moreover, we can write

$$\begin{aligned}\mu_\ell &= \lambda_\ell\underline{\mu}^* + (1-\lambda_\ell)\overline{\mu}^*,\\ \mu_h &= \lambda_h\underline{\mu}^* + (1-\lambda_h)\overline{\mu}^*.\end{aligned}$$

Here λ, λ_ℓ, λ_h, and $\bar{\lambda}$ all lie in $[0,1]$.

So we have

$$\begin{aligned}\mu_0 &= \bar{\lambda}[\lambda_\ell \underline{\mu}^* + (1-\lambda_\ell)\overline{\mu}^*] + (1-\bar{\lambda})[\lambda_h \underline{\mu}^* + (1-\lambda_h)\overline{\mu}^*] \\ &= [\bar{\lambda}\lambda_\ell + (1-\bar{\lambda})\lambda_h]\underline{\mu}^* + [\bar{\lambda}(1-\lambda_\ell) + (1-\bar{\lambda})(1-\lambda_h)]\overline{\mu}^* \\ &= \lambda\underline{\mu}^* + (1-\lambda)\overline{\mu}^*.\end{aligned}$$

By Jensen's inequality,

$$\begin{aligned}U_E^2(\frac{\phi}{1+\phi}, \mu_0) &= P_0(\mu_0) \\ &\leq \bar{\lambda}P_0(\mu_\ell) + (1-\bar{\lambda})P_0(\mu_h) \\ &\leq \bar{\lambda}[\lambda_\ell P_0(\underline{\mu}^*) + (1-\lambda_\ell)P_0(\overline{\mu}^*)] + (1-\bar{\lambda})[\lambda_h P_0(\underline{\mu}^*) + (1-\lambda_h)P_0(\overline{\mu}^*)] \\ &= [\bar{\lambda}\lambda_\ell + (1-\bar{\lambda})\lambda_h]P_0(\underline{\mu}^*) + [\bar{\lambda}(1-\lambda_\ell) + (1-\bar{\lambda})(1-\lambda_h)]P_0(\overline{\mu}^*) \\ &= \lambda P_0(\underline{\mu}^*) + (1-\lambda)P_0(\overline{\mu}^*) \\ &= \hat{U}_E^2(\mu_0),\end{aligned}$$

where $\lambda = \frac{\overline{\mu}^* - \mu_0}{\overline{\mu}^* - \underline{\mu}^*} = \mathbb{P}[s=\ell]$. And the issuer achieves expected payoff $\hat{U}_E^2(\mu_0)$ by setting $\mu_\ell = \underline{\mu}^*$ and $\mu_h = \overline{\mu}^*$.

We further consider two other possibilities. If we set $\mu_\ell = 0$, then the issuer's expected payoff upon observing $s = \ell$ is zero. Her expected payoff is thus $\frac{\mu_0}{\mu_h} \cdot P_0(\mu_h) < P_0(\mu_h)$. Since $P_0(\mu_s)$ is convex in μ_s, we have $P_0(\mu_h) \leq \lambda P_0(\underline{\mu}^*) + (1-\lambda)P_0(\overline{\mu}^*) = \hat{U}_E^2(\mu_0)$. Hence $\frac{\mu_0}{\mu_h} \cdot P_0(\mu_h) < P_0(\mu_h) \leq \hat{U}_E^2(\mu_0)$, rendering this strategy suboptimal. If we set $\mu_h = 1$, under this system, the issuer's expected payoff is $\frac{1-\mu_0}{1-\mu_\ell} \cdot P_0(\mu_\ell) < P_0(\mu_0) < \hat{U}_E^2(\mu_0)$ because $P_0(\mu_s)$ is convex and increasing in μ_s. Again, such system is not optimal too.

In sum, the optimal system will induce two posteriors $\mu_\ell = \underline{\mu}^*$ and $\mu_h = \overline{\mu}^*$. By Bayes' theorem, $\pi_G = \frac{\overline{\mu}^*(\mu_0 - \underline{\mu}^*)}{\mu_0(\overline{\mu}^* - \underline{\mu}^*)}$ and $\pi_B = \frac{(1-\overline{\mu}^*)(\mu_0 - \underline{\mu}^*)}{(1-\mu_0)(\overline{\mu}^* - \underline{\mu}^*)}$. ■

Proof of Proposition 12. Recall that if the investment bank chooses to underwrite and his planned retention is $\frac{\phi}{1+\phi}$, then

$$\begin{aligned} U_{IB}^2(\frac{\phi}{1+\phi}, \mu_s) &= -r\left[\epsilon(1-\psi) + (1-\epsilon)\frac{\phi}{1+\phi}\right](\mu_s \Delta V + V_L) \\ &\quad + \left\{1 + r\left[\epsilon(1-\psi) + (1-\epsilon)\frac{\phi}{1+\phi}\right]\right\} \cdot \frac{(1-\mu_s)\mu_s(1-\gamma)\gamma\phi\Delta V}{\mu_s\gamma + (1-\mu_s)(1-\gamma)} \end{aligned}$$

Define $G(\mu_s, \theta_1) \equiv U_{IB}^2(\frac{\phi}{1+\phi}, \mu_s) = 0$ where $\theta_1 \in \{\epsilon, \psi, V_L, \frac{\Delta V}{V_L}, r, \phi\}$. By the implicit function theorem, at $\mu_s = \underline{\mu}^*$ or $\overline{\mu}^*$,

$$\frac{\partial G}{\partial \mu_s} \cdot \frac{\partial \mu_s}{\partial \theta_1} + \frac{\partial G}{\partial \theta_1} = 0.$$

Like before,

$$\text{sign}\left(\frac{\partial \mu_s}{\partial \theta_1}\right) = -\text{sign}\left(\frac{\partial G}{\partial \mu_s} \cdot \frac{\partial G}{\partial \theta_1}\right).$$

Moreover,

$$\begin{aligned} \frac{\partial G}{\partial \epsilon} &= -r\left[(1-\psi) - \frac{\phi}{1+\phi}\right](\mathbb{E}_s[\tilde{v}] - \Delta P) < 0; \\ \frac{\partial G}{\partial \psi} &= r\epsilon(\mathbb{E}_s[\tilde{v}] - \Delta P) > 0; \\ \frac{\partial G}{\partial V_L} &= -r\left[\epsilon(1-\psi) + (1-\epsilon)\frac{\phi}{1+\phi}\right] < 0; \\ \frac{\partial G}{\partial r} &= -\left[\epsilon(1-\psi) + (1-\epsilon)\frac{\phi}{1+\phi}\right](\mathbb{E}_s[\tilde{v}] - \Delta P) < 0. \end{aligned}$$

Multiply $G(\mu_s, \phi)$ by $(1+\phi)$ we obtain

$$g_1(\mu_s, \phi) \equiv (1+\phi)G(\mu_s, \phi) = -rK_1\,\mathbb{E}_s[\tilde{v}] + \{1 + rK_1\}\Delta P = 0,$$

where $K_1 \equiv \epsilon(1-\psi)(1+\phi) + (1-\epsilon)\phi$. This implies

$$\mathbb{E}_s[\tilde{v}] = \frac{(1+rK_1)\Delta P}{rK_1}.$$

Note that

$$\frac{\partial K_1}{\partial \phi} = 1 - \psi\epsilon = \frac{K_1 - \epsilon(1-\psi)}{\phi}.$$

Therefore,

$$\begin{aligned}
\frac{\partial g_1}{\partial \phi} &= -r \cdot \frac{K_1 - \epsilon(1-\psi)}{\phi} \cdot \frac{(1+rK_1)\Delta P}{rK_1} \\
&\quad + r \cdot \frac{K_1 - \epsilon(1-\psi)}{\phi} \cdot \Delta P + (1+rK_1) \cdot \frac{\Delta P}{\phi} \\
&= \frac{\Delta P}{\phi} \cdot \left\{ -r[K_1 - \epsilon(1-\psi)] \cdot \frac{1}{rK_1} + (1+rK_1) \right\} \\
&= \frac{\Delta P}{\phi} \cdot \left[\frac{\epsilon(1-\psi)}{K_1} + rK_1 \right] \\
&> 0.
\end{aligned}$$

We then divide $g_1(\mu_s, \phi)$ by ΔV, and obtain

$$g_2 \equiv -rK_1(\mu_s + \frac{1}{\eta}) + (1+rK_1) \cdot \frac{\mu_s(1-\mu_s)\gamma(1-\gamma)}{\mu_s\gamma + (1-\mu_s)(1-\gamma)},$$

where $\eta = \frac{\Delta V}{V_L}$. So

$$\frac{\partial g_2}{\partial \eta} = \frac{rK_1}{\eta^2} > 0.$$

Recall from the proof of Lemma 3, we know that $\partial U_{IB}^2/\partial \mu_s > 0$ at $\mu_s = \underline{\mu}^*$ yet $\partial U_{IB}^2/\partial \mu_s < 0$ at $\mu_s = \overline{\mu}^*$. Hence at $\mu_s = \underline{\mu}^*$, $\partial G/\partial \mu_s > 0$, $\partial g_1/\partial \mu_s > 0$, and $\partial g_2/\partial \mu_s > 0$. Meanwhile at $\mu_s = \overline{\mu}^*$, $\partial G/\partial \mu_s < 0$, $\partial g_1/\partial \mu_s < 0$, and $\partial g_2/\partial \mu_s < 0$.

Accordingly, by the implicit function theorem, (1) $\frac{\partial \underline{\mu}^*}{\partial \epsilon} > 0$ and $\frac{\partial \overline{\mu}^*}{\partial \epsilon} < 0$; (2) $\frac{\partial \underline{\mu}^*}{\partial \psi} < 0$ and $\frac{\partial \overline{\mu}^*}{\partial \psi} > 0$; (3) $\frac{\partial \underline{\mu}^*}{\partial V_L} > 0$ and $\frac{\partial \overline{\mu}^*}{\partial V_L} < 0$; (4) Recall that $\eta = \frac{\Delta V}{V_L}$, then $\frac{\partial \underline{\mu}^*}{\partial \eta} < 0$ and $\frac{\partial \overline{\mu}^*}{\partial \eta} > 0$; (5) $\frac{\partial \underline{\mu}^*}{\partial r} > 0$ and $\frac{\partial \overline{\mu}^*}{\partial r} < 0$; (6) $\frac{\partial \underline{\mu}^*}{\partial \phi} < 0$ and $\frac{\partial \overline{\mu}^*}{\partial \phi} > 0$. ∎

Proof of Proposition 13. If there is no demand uncertainty, the investment bank chooses his optimal retention β to maximize his expected payoff:

$$U_{IB}^3(\beta, \mu_s) = \beta\{-r\,\mathbb{E}_s[\tilde{v}] + (1+r)\Delta P\} + (1-\beta)\Delta P.$$

Because we know that $\{-r\,\mathbb{E}_s[\tilde{v}] + (1+r)\Delta P\} < \Delta P$, it is optimal to choose the largest possible $(1-\beta)$. Since the underwrite can sell the security short in the secondary market, he no longer has to retain any share in the primary market. Thus he chooses the optimal $\beta^*(\mu_s) = 0$, and his maximal expected payoff is just

$$\hat{U}^3_{IB}(\mu_s) = U^3_{IB}(0, \mu_s) = \frac{\mu_s(1-\mu_s)\gamma(1-\gamma)\phi\Delta V}{\mu_s\gamma + (1-\mu_s)(1-\gamma)}.$$

Q.E.D. ■

Proof of Proposition 14. Given the investment bank's best response in the primary market, the issuer's expected payoff conditional on posterior belief is

$$U^3_E(\mu_s) = (\mu_s\Delta V) - \frac{\mu_s(1-\mu_s)\gamma(1-\gamma)\phi\Delta V}{\mu_s\gamma + (1-\mu_s)(1-\gamma)} = P_0(\mu_s).$$

As we have shown before, this function is convex in $\mu_s \in [0,1]$. For any posteriors μ_ℓ and μ_h that are Bayesian plausible,

$$\begin{aligned} U^3_E(\mu_0) &\leq \mathbb{P}[s=\ell]P_0(\mu_\ell) + \mathbb{P}[s=h]P_0(\mu_h) \\ &\leq \mathbb{P}[s=\ell]P_0(0) + \mathbb{P}[s=h]P_0(1). \end{aligned}$$

The last inequality follows form the convexity of the function, and it holds with strict inequality if $\mu_0 \in (0,1)$. And the optimal system generates a low posterior $\mu_\ell = 0$ and a high posterior $\mu_h = 1$. The system is fully informative in that $\pi_G = 1$ and $\pi_B = 0$. ■

Proof of Lemma 4. If the investment bank agrees to underwrite and chooses a planed retention $\hat{\beta} = 0$, his expected payoff is

$$\begin{aligned}
U_{IB}^4(\hat{\beta} = 0, \mu_s) &= \epsilon\{(1-\psi)[\mathbb{E}_s[\tilde{v}] - (1+r)(\mathbb{E}_s[\tilde{v}] - \Delta P)] + \psi\Delta P\} + (1-\epsilon)\Delta P \\
&= \epsilon(1-\psi)\{-r\,\mathbb{E}_s[\tilde{v}] + (1+r)\Delta P\} + [\epsilon\psi + (1-\epsilon)]\Delta P \\
&= -\epsilon(1-\psi)r\,\mathbb{E}_s[\tilde{v}] + [(1+r)\epsilon(1-\psi) + \epsilon\psi + (1-\epsilon)]\Delta P \\
&> U_{IB}^2(\hat{\beta} = \frac{\phi}{1+\phi}, \mu_s).
\end{aligned}$$

The last inequality holds because when demand shock does not happen and there is short sale constraint, the underwriter has to retain a positive stake to engage in informed trading, which incurs cost of capital and undermines the informed trading profits.

It is easy to see that $U_{IB}^4(\hat{\beta} = 0, \mu_s)$ is concave in μ_s because of the concavity of ΔP. Like in the proofs of Lemma 2 and Lemma 3, to ensure its optimum appears at some $\mu^{**} \in (0,1)$, we require

$$\left.\frac{\partial U_{IB}^4}{\partial \mu_s}\right|_{\mu_s=0} = -\epsilon(1-\psi)r\Delta V + [(1+r)\epsilon(1-\psi) + \epsilon\psi + (1-\epsilon)]\gamma\phi\Delta V > 0;$$

$$\left.\frac{\partial U_{IB}^4}{\partial \mu_s}\right|_{\mu_s=1} = -\epsilon(1-\psi)r\Delta V - [(1+r)\epsilon(1-\psi) + \epsilon\psi + (1-\epsilon)](1-\gamma)\phi\Delta V < 0.$$

The first requires that $r < \frac{\gamma\phi}{\epsilon(1-\psi)(1-\gamma\phi)}$, while the second always holds.

It's easy to see that $U_{IB}^4(\hat{\beta} = 0, 0) < 0$ and $U_{IB}^4(\hat{\beta} = 0, 1) < 0$. We further require that $U_{IB}^4(\hat{\beta} = 0, \frac{1}{2}) > 0$. This implies that $r < \frac{\gamma(1-\gamma)\phi\Delta V}{\epsilon(1-\psi)[\Delta V - \gamma(1-\gamma)\phi\Delta V + 2V_L]}$. So $r < \min\{\frac{\gamma\phi}{\epsilon(1-\psi)(1-\gamma\phi)}, \frac{\gamma(1-\gamma)\phi\Delta V}{\epsilon(1-\psi)[\Delta V - \gamma(1-\gamma)\phi\Delta V + 2V_L]}\}$, i.e. $r < \frac{\gamma(1-\gamma)\phi\Delta V}{\epsilon(1-\psi)[\Delta V - \gamma(1-\gamma)\phi\Delta V + 2V_L]}$.

As long as all of the above are satisfied, it follows naturally that:

1. There exists a pair $\{\underline{\mu}^{**}, \overline{\mu}^{**}\}$ with $0 < \underline{\mu}^{**} < \underline{\mu}^{*} < \frac{1}{2} < \overline{\mu}^{*} < \overline{\mu}^{**} < 1$ such that $U_{IB}^4(0, \underline{\mu}^{*}) = U_{IB}^4(0, \overline{\mu}^{**}) = 0$.

2. $U_{IB}^4(0, \mu_s) > 0$ if $\mu_s \in (\underline{\mu}^{**}, \overline{\mu}^{**})$, and $\tilde{U}_{IB}(0, \mu_s) < 0$ if $\mu_s \in [0, \underline{\mu}^{**})$ or $\mu_s \in (\overline{\mu}^{**}, 1]$.

Q.E.D. ■

Proof of Proposition 15. From Proposition 13 we know that if demand shock does not happen, it is optimal for the investment bank not to retain any share in the primary market. If demand shock happens, he is forced to retain $(1-\psi)$. Therefore, his optimal planned retention should always be zero if the bank decides to underwrite. And from Lemma 4 we know that the investment bank will choose to underwrite only at posteriors $\mu_s \in [\underline{\mu}^{**}, \overline{\mu}^{**}]$, otherwise he will withdraw from underwriting. This gives his expected payoff

$$\hat{U}^4_{IB}(\mu_s) = \begin{cases} U^4_{IB}(\hat{\beta}=0, \mu_s) & \text{if } \mu_s \in [\underline{\mu}^{**}, \overline{\mu}^{**}], \\ 0 & \text{if } \mu_s \in [0, \underline{\mu}^{**}) \cup (\overline{\mu}^{**}, 1]. \end{cases}$$

Q.E.D. ■

Proof of Proposition 16. Proposition 16 follows naturally from Proposition 15. ■

Proof of Proposition 17. Much of the proof resembles that of Proposition 11. Likewise, we consider four cases respectively.

1. If $\mu_0 \in [0, \underline{\mu}^{**})$, like part 1 of Proposition 11's proof, it is optimal to set $\mu_\ell = 0$ and the issuer's expected payoff is $\frac{\mu_0 P_0(\mu_h)}{\mu_h}$. Define $K_2 = \epsilon(1-\psi)$, so

$$U^4_{IB}(0, \mu_s) = -rK_2\, \mathbb{E}_s[\tilde{v}] + (1 + rK_2)\Delta P \geq 0$$

$$\Rightarrow rK_2(\mathbb{E}_s[\tilde{v}] - \Delta P) \leq \Delta P$$

$$\Rightarrow \frac{\mu_0 P_0(\mu_s)}{\mu_s} \leq \frac{\mu_0 \Delta P}{rK_2 \mu_s}.$$

The last holds with equality when $\mu_s = \underline{\mu}^{**}$ or $\overline{\mu}^{**}$. Since

$$\frac{\Delta P}{\mu_s} = \frac{(1-\mu_s)(1-\gamma)\gamma\phi\Delta V}{\mu_s\gamma + (1-\mu_s)(1-\gamma)}$$

which is decreasing in μ_s and achieves the maximum at $\mu_s = \underline{\mu}^{**}$. Therefore it is optimal for the issuer to set $\mu_h = \underline{\mu}^{**}$ so that she gets the highest expected payoff $\frac{\mu_0 P_0(\underline{\mu}^{**})}{\underline{\mu}^{**}}$. In sum, the optimal system will induce two posteriors $\mu_\ell = 0$ and $\mu_h = \underline{\mu}^{**}$. The relevant precision parameters are $\pi_B = \frac{\mu_0(1-\underline{\mu}^{**})}{\underline{\mu}^{**}(1-\mu_0)}$ and $\pi_G = 1$.

2. If $\mu_0 \in (\overline{\mu}^{**}, 1]$, with the same reasoning as part 2 of Proposition 11's proof, it is optimal to set $\mu_h = 1$. Her expected payoff is therefore $\mathbb{P}[s = \ell]P_0(\mu_\ell) = \frac{1-\mu_0}{1-\mu_\ell} \cdot P_0(\mu_\ell)$. Since both $\frac{1-\mu_0}{1-\mu_\ell}$ and $P_0(\mu_\ell)$ are increasing in μ_ℓ, it is optimal to set $\mu_\ell = \overline{\mu}^{**}$. Hence the optimal system yields two posteriors $\mu_\ell = \underline{\mu}^{**}$ and $\mu_h = 1$. This gives $\pi_B = \frac{\mu_0 - \overline{\mu}^{**}}{\mu_0(1-\overline{\mu}^{**})}$ and $\pi_G = 0$.

3. Third, when $\mu_0 = \underline{\mu}^{**}$ or $\overline{\mu}^{**}$, the investment bank is break-even by underwriting the deal. In this case, a completely uninformative disclosure system is optimal. It has $\pi_G = \pi_B \in (0, 1)$, yielding posteriors $\mu_\ell = \mu_h = \mu_0$.

4. Finally, we explore the cae when $\mu_0 \in (\underline{\mu}^{**}, \overline{\mu}^{**})$. Using a similar argument as in part 3 of Proposition 11's proof, we have $\mu_\ell = \underline{\mu}^{**}$ and $\mu_h = \overline{\mu}^{**}$ due to the convexity of $U^4_{IB}(0, \mu_s)$ in μ_s on $[\underline{\mu}^{**}, \overline{\mu}^{**}]$. And again, setting either $\mu_\ell = 0$ or $\mu_h = 1$ is suboptimal. Hence the optimal system has $\pi_G = \frac{\overline{\mu}^{**}(\mu_0 - \underline{\mu}^{**})}{\mu_0(\overline{\mu}^{**} - \underline{\mu}^{**})}$ and $\pi_B = \frac{(1-\overline{\mu}^{**})(\mu_0 - \underline{\mu}^{**})}{(1-\mu_0)(\overline{\mu}^{**} - \underline{\mu}^{**})}$. ■

Proof of Proposition 18. Note that $\overline{\mu}^{**}$ and $\underline{\mu}^{**}$ are two roots of the following equation:

$$U^4_{IB}(0, \mu_s) = -rK_2\,\mathbb{E}_s[\tilde{v}] + (1 + rK_2)\Delta P = 0.$$

Define

$$J(\mu_s, \theta_1) = -rK_2\,\mathbb{E}_s[\tilde{v}] + (1 + rK_2)\Delta P,$$

where $K_2 = \epsilon(1 - \psi)$ and $\theta_1 \in \{\epsilon, \psi, V_L, \frac{\Delta V}{V_L}, r, \phi\}$. Some simple algebra gives

$$\begin{aligned} \frac{\partial J}{\partial \epsilon} &= -r(1-\psi)(\mathbb{E}_s[\tilde{v}] - \Delta P) < 0; \\ \frac{\partial J}{\partial \psi} &= r\epsilon(\mathbb{E}_s[\tilde{v}] - \Delta P) > 0; \end{aligned}$$

$$\begin{aligned} \frac{\partial J}{\partial V_L} &= -r\epsilon(1-\psi) < 0; \\ \frac{\partial J}{\partial r} &= -\epsilon(1-\psi)(\mathbb{E}_s[\tilde{v}] - \Delta P) < 0; \\ \frac{\partial J}{\partial \phi} &= [1 + r\epsilon(1-\psi)] \cdot \frac{\Delta P}{\phi} > 0 \end{aligned}$$

Let $j = J/\Delta V$, we obtain

$$\frac{\partial j}{\partial \eta} = \frac{r\epsilon(1-\psi)}{\eta^2} > 0.$$

Moreover, from the proof of Lemma 4, at $\mu_s = \underline{\mu}^{**}$, $\frac{\partial J}{\partial \mu_s} > 0$, while at $\mu_s = \overline{\mu}^{**}$, $\frac{\partial J}{\partial \mu_s} < 0$.

So by the implicit function theorem, we have (1) $\frac{\partial \underline{\mu}^{**}}{\partial \epsilon} > 0$ and $\frac{\partial \overline{\mu}^{**}}{\partial \epsilon} < 0$; (2) $\frac{\partial \underline{\mu}^{**}}{\partial \psi} < 0$ and $\frac{\partial \overline{\mu}^{**}}{\partial \psi} > 0$; (3) $\frac{\partial \underline{\mu}^{**}}{\partial V_L} > 0$ and $\frac{\partial \overline{\mu}^{**}}{\partial V_L} < 0$; (4) Recall that $\eta = \frac{\Delta V}{V_L}$, then $\frac{\partial \underline{\mu}^{**}}{\partial \eta} < 0$ and $\frac{\partial \overline{\mu}^{**}}{\partial \eta} > 0$; (5) $\frac{\partial \underline{\mu}^{**}}{\partial r} > 0$ and $\frac{\partial \overline{\mu}^{**}}{\partial r} < 0$; (6) $\frac{\partial \underline{\mu}^{**}}{\partial \phi} < 0$ and $\frac{\partial \overline{\mu}^{**}}{\partial \phi} > 0$. ■

Proof of Proposition 19. Recall that $i \in \{1, 2, 3, 4\}$ represents one of the following four scenarios: 1. (No Short Sale, No Demand Uncertainty), 2. (No Short Sale, Demand Uncertainty), 3. (Short Sale, No Demand Uncertainty), and 4. (Short Sale, Demand Uncertainty).

We have already shown that $U_{IB}^1(\beta = \frac{\phi}{1+\phi}, \mu_s) > U_{IB}^2(\hat{\beta} = \frac{\phi}{1+\phi}, \mu_s)$ and $0 < \underline{\mu} < \underline{\mu}^* < \frac{1}{2} < \overline{\mu}^* < \overline{\mu} < 1$, as well as $U_{IB}^4(\hat{\beta} = 0, \mu_s) > U_{IB}^2(\hat{\beta} = \frac{\phi}{1+\phi}, \mu_s)$ and $0 < \underline{\mu}^{**} < \underline{\mu}^* < \frac{1}{2} < \overline{\mu}^* < \overline{\mu}^{**} < 1$. Thus it remains to compare $U_{IB}^1(\beta = \frac{\phi}{1+\phi}, \mu_s)$ and $U_{IB}^4(\hat{\beta} = 0, \mu_s)$ to rank the welfare of the investment banks. Recall that

$$\begin{aligned} U_{IB}^4(\hat{\beta} = 0, \mu_s) &= \epsilon\{(1-\psi)\{\mathbb{E}_s[\tilde{v}] - (1+r)(\mathbb{E}_s[\tilde{v}] - \Delta P)\} + \psi\Delta P\} + (1-\epsilon)\Delta P, \\ &= (\epsilon - \epsilon\psi)\{-r\,\mathbb{E}_s[\tilde{v}] + (1+r)\Delta P\} + (\psi\epsilon + 1 - \epsilon)\Delta P, \end{aligned}$$

and

$$U_{IB}^1(\beta = \frac{\phi}{1+\phi}, \mu_s) = \frac{\phi}{1+\phi} \cdot \{-r\,\mathbb{E}_s[\tilde{v}] + (1+r)\Delta P\} + \frac{1}{1+\phi} \cdot \Delta P.$$

Since we have shown that $\{-r\,\mathbb{E}_s[\tilde{v}] + (1+r)\Delta P\} < \Delta P$, it is easy to see:

(1) If $\epsilon - \epsilon\psi < \frac{\phi}{1+\phi}$, i.e. $\epsilon < \frac{\phi}{(1-\psi)(1+\phi)}$, then $U_{IB}^1(\beta = \frac{\phi}{1+\phi}, \mu_s) < U_{IB}^4(\hat{\beta} = 0, \mu_s)$, and

$0 < \underline{\mu}^{**} < \underline{\mu} < \underline{\mu}^{*} < \frac{1}{2} < \overline{\mu}^{*} < \overline{\mu} < \overline{\mu}^{**} < 1$. Note that the investment banks' welfare is

$$W_{IB}(i) = \int_0^1 \hat{U}^i_{IB}(\mu_0)\, d\mu_0 = \int_{\underline{\mu}_{(i)}}^{\overline{\mu}_{(i)}} U^i_{IB}(\,\cdot\,, \mu_0)\, d\mu_0.$$

where $\underline{\mu}_{(i)}$ and $\overline{\mu}_{(i)}$ denote the relevant cut-offs in scenario i, and "$\cdot$" denotes the investment banks' relevant retention in $U^i_{IB}(\,\cdot\,, \mu_s)$. Hence we obtain the following ranking:

$$W_{IB}(SS, NDU) > W_{IB}(SS, DU) > W_{IB}(NSS, NDU) > W_{IB}(NSS, DU).$$

(2) Similarly, if $\epsilon > \frac{\phi}{(1-\psi)(1+\phi)}$, then $0 < \underline{\mu} < \underline{\mu}^{**} < \underline{\mu}^{*} < \frac{1}{2} < \overline{\mu}^{*} < \overline{\mu}^{**} < \overline{\mu} < 1$ and

$$W_{IB}(SS, NDU) > W_{IB}(NSS, NDU) > W_{IB}(SS, DU) > W_{IB}(NSS, DU).$$

(3) Finally, if $\epsilon = \frac{\phi}{(1-\psi)(1+\phi)}$, then $0 < \underline{\mu} = \underline{\mu}^{**} < \underline{\mu}^{*} < \frac{1}{2} < \overline{\mu}^{*} < \overline{\mu}^{**} = \overline{\mu} < 1$ and

$$W_{IB}(SS, NDU) > W_{IB}(NSS, NDU) = W_{IB}(SS, DU) > W_{IB}(NSS, DU).$$

Q.E.D. ■

Proof of Proposition 20. If the issuers do not disclose additional information, the investment banks' decisions to underwrite and the issuers' expected payoffs will depend directly on μ_0. And

$$\begin{aligned} W_E(1) &= \int_0^{\underline{\mu}} (\mu_0 \Delta V + V_L)\, d\mu_0 + \int_{\underline{\mu}}^{\overline{\mu}} \left[(\mu_0 \Delta V + V_L) - \frac{(1-\mu_0)\mu_0(1-\gamma)\gamma\phi\Delta V}{\mu_0\gamma + (1-\mu_0)(1-\gamma)} \right] d\mu_0 \\ &\quad + \int_{\overline{\mu}}^{1} (\mu_0 \Delta V + V_L)\, d\mu_0, \\ W_E(2) &= \int_0^{\underline{\mu}^*} 0\, d\mu_0 + \int_{\underline{\mu}^*}^{\overline{\mu}^*} \left[(\mu_0 \Delta V + V_L) - \frac{(1-\mu_0)\mu_0(1-\gamma)\gamma\phi\Delta V}{\mu_0\gamma + (1-\mu_0)(1-\gamma)} \right] d\mu_0 + \int_{\overline{\mu}^*}^{1} 0\, d\mu_0, \end{aligned}$$

$$W_E(3) = \int_0^1 \left[(\mu_0\Delta V + V_L) - \frac{(1-\mu_0)\mu_0(1-\gamma)\gamma\phi\Delta V}{\mu_0\gamma + (1-\mu_0)(1-\gamma)}\right] d\mu_0,$$

$$W_E(4) = \int_0^{\underline{\mu}^{**}} 0\, d\mu_0 + \int_{\underline{\mu}^{**}}^{\overline{\mu}^{**}} \left[(\mu_0\Delta V + V_L) - \frac{(1-\mu_0)\mu_0(1-\gamma)\gamma\phi\Delta V}{\mu_0\gamma + (1-\mu_0)(1-\gamma)}\right] d\mu_0 + \int_{\overline{\mu}^{**}}^1 0\, d\mu_0.$$

Therefore, the ranking is as follow,

$$W_E(NSS, NDU) > W_E(SS, NDU) > W_E(SS, DU) > W_E(NSS, DU).$$

We can write

$$P_0(\mu) = (\mu\Delta V + V_L) - \frac{(1-\mu)\mu(1-\gamma)\gamma\phi\Delta V}{\mu\gamma + (1-\mu)(1-\gamma)},$$

which is increasing in μ and does not exceed $(\mu\Delta V + V_L)$. Then if all of the issuers design their disclosure policies optimally, their welfare under four different scenarios are

$$\hat{W}_E(1) = \int_0^1 (\mu_0\Delta V + V_L)\, d\mu_0,$$

$$\hat{W}_E(2) = \int_0^{\underline{\mu}^*} P_0(\underline{\mu}^*) \cdot \frac{\mu_0}{\underline{\mu}^*}\, d\mu_0 + \int_{\underline{\mu}^*}^{\overline{\mu}^*} \left[P_0(\underline{\mu}^*) + \frac{P_0(\overline{\mu}^*) - P_0(\underline{\mu}^*)}{\overline{\mu}^* - \underline{\mu}^*}\right] d\mu_0 + \int_{\overline{\mu}^*}^1 \left[P_0(\overline{\mu}^*) - \frac{P_0(\overline{\mu}^*)}{1-\overline{\mu}^*} \cdot (\mu_0 - \overline{\mu}^*)\right] d\mu_0,$$

$$\hat{W}_E(3) = \int_0^1 (\mu_0\Delta V + V_L)\, d\mu_0,$$

$$\hat{W}_E(4) = \int_0^{\underline{\mu}^{**}} P_0(\underline{\mu}^{**}) \cdot \frac{\mu_0}{\underline{\mu}^{**}}\, d\mu_0 + \int_{\underline{\mu}^{**}}^{\overline{\mu}^{**}} \left[P_0(\underline{\mu}^{**}) + \frac{P_0(\overline{\mu}^{**}) - P_0(\underline{\mu}^{**})}{\overline{\mu}^{**} - \underline{\mu}^{**}}\right] d\mu_0 + \int_{\overline{\mu}^{**}}^1 \left[P_0(\overline{\mu}^{**}) - \frac{P_0(\overline{\mu}^{**})}{1-\overline{\mu}^{**}} \cdot (\mu_0 - \overline{\mu}^{**})\right] d\mu_0.$$

It is easy to see that $\hat{W}_E(1) = \hat{W}_E(3)$, and both achieve the highest possible welfare. It suffices to show that $\hat{W}_E(4) > \hat{W}_E(2)$. Intuitively, this is because the graph of $\hat{U}_E^2(\mu)$ is beneath that of $\hat{U}_E^4(\mu)$ for $\forall \mu \in (0,1)$ due to the convexity of $P_0(\mu)$.

Next we formally show that indeed $\hat{U}_E^4(\mu_0)$ is piece-wise larger than $\hat{U}_E^2(\mu_0)$ for any prior

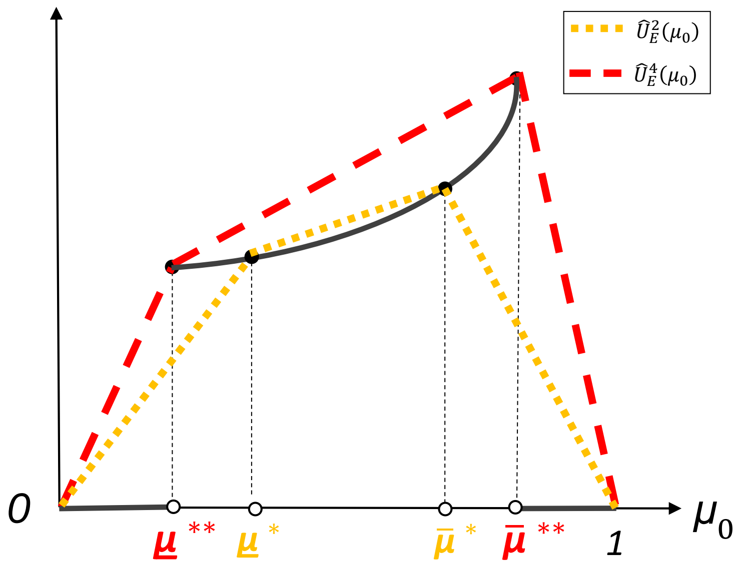

Figure 13: Welfare Comparison

belief $\mu_0 \in (0,1)$. A graphical illustration is given in Figure 13.

1. When $\mu_0 \in (0, \underline{\mu}^{**}]$, we have shown in the proofs of Proposition 11 and 17 that because $\underline{\mu}^{**} < \underline{\mu}^{*}$, we have $\frac{P_0(\underline{\mu}^{**})}{\underline{\mu}^{**}} > \frac{P_0(\underline{\mu}^{*})}{\underline{\mu}^{*}}$. Hence $\frac{P_0(\underline{\mu}^{**})\mu_0}{\underline{\mu}^{**}} > \frac{P_0(\underline{\mu}^{*})\mu_0}{\underline{\mu}^{*}}$, i.e. $\hat{U}_E^4(\mu_0) > \hat{U}_E^2(\mu_0)$.

2. When $\mu_0 \in (\underline{\mu}^{**}, \underline{\mu}^{*})$, $\hat{U}_E^4(\mu_0)$ is a convex combination of $P_0(\underline{\mu}^{**})$ and $P_0(\overline{\mu}^{**})$, which is strictly larger than $P_0(\underline{\mu}^{*})$ due to convexity of $P_0(\mu)$. Since $\hat{U}_E^2(\mu_0) = \frac{P_0(\underline{\mu}^{*})\mu_0}{\underline{\mu}^{*}} < P_0(\underline{\mu}^{*})$, we have $\hat{U}_E^4(\mu_0) > \hat{U}_E^2(\mu_0)$.

3. When $\mu_0 \in [\underline{\mu}^{*}, \overline{\mu}^{*}]$, the convexity of $P_0(\mu)$ implies that the convex combination of $P_0(\underline{\mu}^{**})$ and $P_0(\overline{\mu}^{**})$ strictly dominates the convex combination of $P_0(\underline{\mu}^{*})$ and $P_0(\overline{\mu}^{*})$. This implies $\hat{U}_E^4(\mu_0) > \hat{U}_E^2(\mu_0)$.

4. When $\mu_0 \in (\overline{\mu}^{*}, \overline{\mu}^{**})$, $\hat{U}_E^2(\mu_0) = P_0(\overline{\mu}^{*}) - \frac{P_0(\overline{\mu}^{*})}{1-\overline{\mu}^{*}} \cdot (\mu_0 - \overline{\mu}^{*}) < P_0(\overline{\mu}^{*})$. And $P_0(\overline{\mu}^{*})$ is strictly smaller than the convex combination of $P_0(\underline{\mu}^{**})$ and $P_0(\overline{\mu}^{**})$. Hence $\hat{U}_E^4(\mu_0) > \hat{U}_E^2(\mu_0)$.

5. When $\mu_0 \in [\overline{\mu}^{**}, 1)$, we define

$$\Delta U(\mu_0) \equiv \hat{U}_E^2(\mu_0) - \hat{U}_E^4 = [(\mu_0) = P_0(\overline{\mu}^{*}) - \frac{P_0(\overline{\mu}^{*})}{1-\overline{\mu}^{*}} \cdot (\mu_0 - \overline{\mu}^{*})] - [P_0(\overline{\mu}^{**}) - \frac{P_0(\overline{\mu}^{**})}{1-\overline{\mu}^{**}} \cdot (\mu_0 - \overline{\mu}^{**})].$$

It is easy to see that $\frac{\partial \Delta U}{\partial \mu_0} = \frac{P_0(\overline{\mu}^{**})}{1-\overline{\mu}^{**}} - \frac{P_0(\overline{\mu}^{*})}{1-\overline{\mu}^{*}} > 0$ and $\Delta U(\mu_0) = 0$ if $\mu_0 = 1$. Hence at $\mu_0 \in [\overline{\mu}^{**}, 1)$, $\Delta U(\mu_0) < 0$, i.e. $\hat{U}_E^4(\mu_0) > \hat{U}_E^2(\mu_0)$.

Therefore, it follows naturally that

$$\hat{W}_E(NSS, NDU) = \hat{W}_E(SS, NDU) > \hat{W}_E(SS, DU) > \hat{W}_E(NSS, DU).$$

Q.E.D. ■