Comparative Advantage, Competition, and Firm Heterogeneity

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Abstract

This paper studies how firm heterogeneity shapes comparative advantage. Using matched Chinese firm and Customs data, we find that capital intensive Chinese exporters tend to export fewer products and concentrate more sales on better performing products than labor intensive exporters. Motivated these findings, we set up a theory which embeds Mayer, Melitz and Ottaviano (2014) with Dornbusch, Fischer and Samuelson (1977). We find that exporters face tougher competition in comparative disadvantage industries. Such competition induces exporters to change their product scope and product mix: export product scope is narrower and export sales is more skewed in comparative disadvantage industries. Export selection along the extensive margin generates endogenous Ricardian comparative advantage which is positively correlated with the *ex ante* comparative advantage, as discovered by Bernard, Redding and Schott (2007). However, export selection along the intensive margin generates endogenous Ricardian comparative advantage which is *negatively* correlated with the *ex ante* comparative advantage. We provide sufficient statistics to estimate and decompose comparative advantage. Both channels are found to be quantitatively important determinants of comparative advantage for China.

Key Words: Comparative Advantage, Competition, Multi-product Firm, Sufficient Statistics, Firm Heterogeneity

1 Introduction

Comparative advantage first articulated by David Ricardo in 1817, has been the corner stone of international trade theory. For much of the past 200 years, people have been taking comparative advantage as fixed. In the past two decades, we have more and more realized the importance of firms as central players of trade and their heterogeneity in shaping the aggregate outcomes (Melitz, 2003). In a well-known paper by Bernard, Redding and Schott (2007), they show that firm heterogeneity and endogenous export selection can amplify ex-ante comparative advantage. But is this the end of the story? Is there any other way that firm heterogeneity can change comparative advantage? And are these channels quantitatively important in the data? These are the questions that we tackle in this paper.

We start our analysis with a few stylized facts revealed from a matched Chinese firm and Customs data. First, compared with labor intensive firms, capital intensive Chinese firms are less likely to participate international trade. Second, they not only export a smaller share of output but also a smaller number of products on average if they export. Third, their export sales is more skewed towards the better performing products than labor intensive exporters. Finally, the skewness of domestic sales across firms is higher in labor intensive industries than capital intensive industries. The first two facts which concerned the extensive margin of reallocation within and across firms can be rationalized by extending models such as Arkolakis and Muendler (2010) and Bernard, Redding, and Schott (2011) to multiple industries. However, their assumptions of nested-CES demand and a continuum of firm impose an exogenously fixed mark-up across destinations and industries. The different market condition across industries would have no effects on the export product mix (the relative distribution of sales across products) or the industry variations of skewness of domestic scales across firms. Thus the third and fourth stylized fact which concerned reallocation along the intensive margin cannot be reconciled with these type of models.

Our theory simultaneously explains these facts. We extend the analysis of Mayer, Melitz and Ottaviano (2014) to a continuum of industries by embedding it with Dornbusch, Fischer and Samuelson (1977). The model features heterogeneous firm and variable mark-up as in Melitz and Ottaviano (2008). Firms possess a "core competency" and has access to a multi-product technology. Marginal cost increases as the product moves away from the core competency. Each country produces in multiple industries and the random draw of productivity for the core competency of firms varies across countries and industries. In industries of comparative advan-

tage, firms are more likely to have higher productivity draw than the other country. Export is thus tougher in the comparative disadvantage industries. It shifts down the whole distribution of mark-ups and induces exporters to cut their export product scope and skew export sales towards the better performing products. The relative ease of competition at home in the comparative disadvantage industries also induces firms to sell more at home rather than export. On the other hand, competition of domestic market is tougher in comparative advantage industries. Such tougher competition induces reallocation of scales towards better performing firms and increases the skewness of domestic sales across firms.

Our theory generates new prediction about the effect of endogenous selection on comparative advantage. The Melitz (2003) typed model predicts that opening up to trade reallocates resources towards the more productive firms. Bernard, Redding, and Schott (2007) find that such reallocation differs systematically across industries. Due to higher expected profits, there will be more entries and stronger selections in the comparative advantage industries. It generates endogenous Ricardian comparative advantage which is positively correlated the *ex ante* comparative advantage. Thus export selection amplifies comparative advantage. In our model, there is a new mechanism working on the top of that. The idea is that in industries of comparative disadvantage, tougher competition in the export markets induces more resource reallocation towards the more productive firms and their better performing products after countries opening up to trade. The more competitive the foreign market is, the more that exporters have to toughen up. Such endogenous response reduces relative productivity differences between home and foreign and dampens comparative advantage. The model allows us to theoretically decompose comparative advantage into the *ex ante* exogenous components and *ex post* endogenous components. Productivity measures that only consider selections along the extensive margin only capture the amplifying component. Productivity measures which take into account selections both along the extensive and the intensive margins capture both the amplifying and dampening components.

To quantitatively evaluate the mechanisms of the model, we conduct two empirical analyses. The first analysis is based on the reduced form analysis of Mayer, Melitz and Ottviano (2014). They look at how French exporters vary their export product mix across markets with different size. We add to the regressions new variables which measure the competition firms faced at each market due to comparative advantage. The idea is that capital intensive exporters face tougher competition in capital abundant countries; labor intensive exporters face tougher competition in labor abundant countries. The regressions using our matched data confirm the model's prediction: exporters export fewer products and skew sales more towards the better performing

products in markets with tougher competition due to comparative advantage.

The second analysis takes on a more structural approach to evaluate the different components of comparative advantage. Comparative advantage is not directly observable and hard to measure. We first provide *sufficient statistics* results showing that as long as we observe the trade elasticity, the trade freeness (or iceberg trade costs) and domestic export participations, we can infer home country's comparative advantage against Rest of the World in each industry. The intuition is similar to Balassa's idea of " *revealed comparative advantage* ": conditional on trade costs and trade elasticity, firms' export participations should reveal their relative competitiveness. Our result also allows us to quantify the decomposition of the comparative advantage and evaluate the *ex ante* component, the amplifying component and the dampening component individually. Using these identification results, we estimate our model for the Chinese economy v.s. Rest of the World for year 2000, 2003 and 2006. We show that the dampening component appears to dominate the amplifying component. Ignoring the dampening component would lead to overestimation of comparative advantage.

We contribute to the following streams of literature. Our paper is closely related to the recent literature which studies the interactions of comparative advantage and firm heterogeneity.[1] We show that there is a new channel that firm heterogeneity shapes comparative advantage. That is tougher export market competition in competitive disadvantage industries induces reallocation such that it dampens *ex ante* comparative advantage, which contrasts the amplifying mechanism found in Bernard, Redding and Schott (2007).[2]

We also contribute to the literature on the measurement of comparative advantage. Comparative advantage is the cornerstone concept of classic trade theory. However, it has remained challenging to measure.[3] We provide sufficient statistics results which identify comparative

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[1]Most notable contribution by Bernard, Redding and Schott (2007). Recent contribution includes Lu(2010), Huang et al (2017) and Burstein and Vogel (forthcoming). Gaubert and Itskhoki (2015) also looks a multi-sector Ricardian model with heterogeneous firms but their focus is on the granularity force.

[2]Bernard, Redding and Schott (2011) provides an multi-industry extension and looks at how selection along the extensive margin differ across industries for multi-product firms. Ma et al (2014) builds on Bernard, Redding and Schott (2011) and studies within-firm specialization across products with different factor proportions.

[3]Balassa (1965) formulated the idea of "Revealed comparative advantage" which has been the core for measuring comparative advantage in the last few decades. There has been a renaissance of quantifying the comparative advantage since Eaton and Kortum (2002)'s contribution which provides a tractable multi-country Ricardian model. Costinot *et al* (2011) provide theoretically consistent evaluation of Balassa's idea of "revealed comparative advantage" based on an extension of the Eaton-Kortum model. Levchenko and Zhang (2016) use the gravity equation to infer comparative advantage from trade flow and its evolution over time. Costinot et al (2016) focus on the agriculture sector for which the productivity of fields can be precisely estimated for different crops. Thus comparative advantage can be estimated directly. Huang et al (2017) instead use the two-country Dornbusch, Fischer and Samuelson framework and interact it with the Melitz (2003) model. They explore export participations and production across industries and use micro firm data to back out comparative advantage by structural estimation.

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advantage and decompose it into exogenous and endogenous components. Sufficient statistics approach, as argued in Arkolakis, Costinot and Rodríguez-Clare (2012), saves us from solving all endogenous variables but still provides estimates for the object of interest. As far as we know, this paper is the first to provide sufficient statistics for comparative advantage.[4] We also show that the exact measure used for productivity matters for the measurement of comparative advantage. Measures that only capture extensive margin miss out important determinant of comparative advantage and might bias our estimation.

Finally, the literature on multi-product firm has been booming on both the theoretical and empirical side.[5] Our analysis highlights the role of comparative advantage in the decision of multi-product exporters across industries. The mechanism is similar to Mayer, Melitz and Ottaviano (2014). They focus on competition due to market size while we focus on comparative advantage. This provides us better guideline conducting empirical analysis of multi-product exports in a world with many industries.

The remaining of the paper is arranged as follows. Section 2 presents three stylized facts which motivates our theory. Section 3 presents the model and its implications. Section 4 provides findings from two empirical analysis. Section 5 concludes.

2   Motivating Evidence

2.1   Data

In this section, we present a few basic stylized facts on how export participations and exporters' product mix vary with comparative advantage. These facts are generated using a matched firm and Customs data from China. The first dataset that we use is the Chinese Annual Industrial Survey (CAIS) which covers all State Owned Firms (SOE) and non-SOEs with sales above 5 million Chinese Yuan. This data provides rich information on firms' financial statements and identifications such as name, address, ownership and employment. The other dataset that we

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[4]The sufficient statistics approach has gained its popularity in the field of public finance (Chetty, 2009). Arkolakis *et al* (2012) shows that within a set of trade models which satisfy certain conditions, trade elasticity and the share of expenditure on domestic goods are sufficient statistics for welfare gains from trade. Our sufficient statistics look similar to them but our object of interest is comparative advantage instead of welfare gains from trade.

[5]Feenstra and Ma (2009) and Eckel and Neary (2010) study the effect of competition on the distribution of sales and the cannibalization effect. Arkolakis and Muendler (2010) and Bernard, Redding, and Schott (2011) emphasize selection along the extensive margin while Mayer, Melitz and Ottaviano (2014) emphasize selection along the intensive margin. Manova and Yu (2017) instead focus on quality differentiation and study product selection along the quality margin. Bernard, Redding, and Schott (2010), Iacovone and Javorcik (2010), Mayer, Melitz and Ottaviano (2016) focus on product churning in response to change in market conditions over time.

employ is the Chinese Customs data which covers the universe of Chinese import and export transactions. For each transaction, we know the Chinese importer/exporter, the product (at HS8 level), value, origin, destination etc. There are no common firm identifiers between the two datasets. We match the two datasets based on firms' name, address, telephone number and zip code.[6]

We match the two datasets for year 2000-2006 and focus on the Chinese manufacturers.[7] We use firms' capital intensity to measure the comparative advantage of Chinese firms: given that the abundance in labor endowment, we expect China to have comparative advantage in labor intensive industries. We follow Schott (2004) and Huang et al (2017) to define industries as "Heckscher-Ohlin aggregates" and group Chinese firms into 100 bins according to their capital intensity. [8] Capital intensity is defined as $1 - \frac{LaborCost}{ValueAdded}$ for each firm. For example, firms with capital intensity between 0.99 and 1 are defines as industry 100. Under such industry classification, we find the following stylized facts using data for year 2003.[9]

2.2 Stylized Facts

Stylized fact 1: *Export propensity and export intensity decline with capital intensity.*

This is captured in Figure 1. The left panel plots export propensity for each industry in 2003 while export propensity is defined as the total number of exporters divided by the total number firms in each industry. The right panel plots export intensity while export intensity is defined as the total export divided by the total sales of each industry. As can be seen from the figures, both measures decline with capital intensity. This is consistent with our expectation that China has comparative advantage in labor intensive industries and labor intensity firms are more likely to export, given its abundance in labor.

Stylized fact 2: *Export product scope declines with capital intensity.*

This fact is captured by Figure 2. In left panel we plot the average number of exported

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[6]Such matching method has been used by a few number of papers, including Ma et al (2014), Yu (2015), Manova and Yu (2016).

[7]Thus we exclude firms from the mining and utility sector in CAIS and wholesalers or intermediaries in the Chinese Customs data.

[8]They argue that traditional industry classification defined industries according to the final usage and aggregate products or firms that use different technologies and factor proportions.

[9]Results from other years are qualitatively the same which are presented in the appendix. Labor costs includes payable wage, labor and employment insurance fee, and total employee benefits payable. We exclude those firms with capital intensities that are negative or larger than 1. These are clearly due to misreporting or errors. We also exclude firms with negative value added, employment or assets. Firms with less than 8 employees are also excluded since their under different legal regime. The results for other years qualitatively the same.

products across exporter in each industry. The number of products is counted as the distinctive number of HS8 products exported to all destinations. As we can see, the average number of exported products falls as firms are getting more capital intensive. The right panel instead looks at the share of single-product exporters, firms that export just one HS8 product in a given year. It is obvious that single-product exporters are more prevalent in the capital intensive industries in China.

Stylized fact 3: *Export product mix is more skewed in capital intensive industries.*

This is captured by Figure 3. We plot the average of the log-ratios between the sales of the core product to the second most important products in the left panel. Core product is defined as the product that makes up most of the total export sales. As we can see, the measure is higher in the capital intensive industries. Thus export sales are more concentrated in the better performing products in the capital intensive industries. This measure only captures the skewness of export sales across a few products. To show that the presence of such a relationship across all exported products, we look at measure on skewness of the whole distribution of export sales. The right panel plots the average firm level Theil Index of export across products for each industry. Again, we find the skewness of export sales across products tend to be higher for capital intensive exporters.[10]

Stylized fact 4: *The skewness of domestic sales across firms decreases with capital intensity.*

This is captured in Figure 4. The left panel plots the log ratio of domestic sales between the 75th percentile firm and the 25th percentile firm. Domestic sales is total sales minus export. As can be seen from the figure, such ratio tends to be higher for labor intensive industries. The right panel plots the Theil index of domestic sales across firms within each industry. Still, the Theil index tends to be higher for labor intensive industries.

2.3 Discussion

So far, our results are just graphical evidence for three years. In the Appendix 6.1, we provide further regression evidence using all available data from 2000 to 2006. The stylized facts remain highly significant and robust. Overall, these stylized facts reveal how sales at home and abroad, within and across firms are shaped by comparative advantage. The first two stylized facts focus on the extensive margin while the third and fourth one look at the intensive margins. While the first fact can be explained by Bernard, Redding and Schott (2007) and the second fact can be

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[10]In the appendix, we look at other measures of skewness including Herfindahl index, ratio of the core product v.s. the third largest product etc. The conclusion continues to hold.
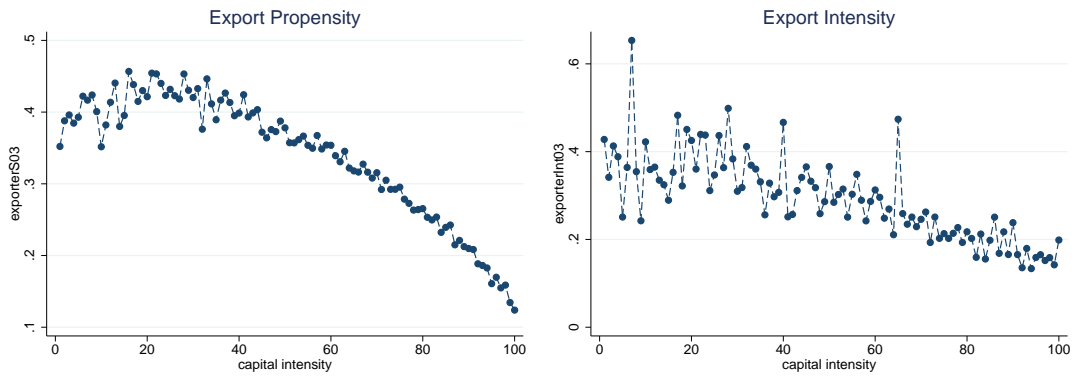
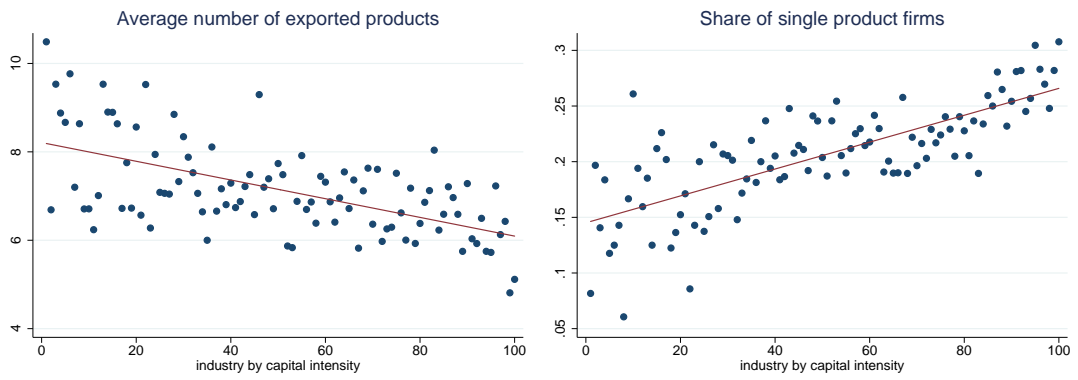Figure 1: Export propensity and Export Intensity
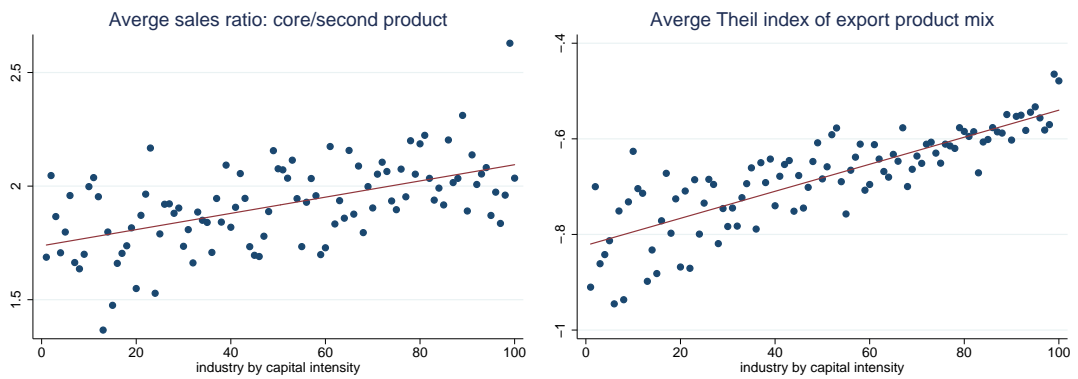

Figure 2: Number of products exported

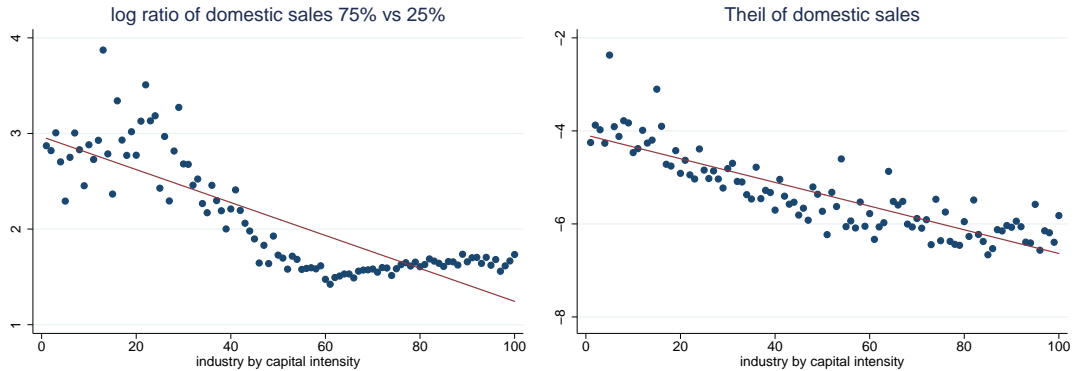Figure 3: Skewness of export product mix

Figure 4: Skewness of domestic sales across firms

explained by Bernard, Redding and Schott (2011) if we extend their model to multiple industries with multi-products.[11] However, the third and fourth stylized facts are not consistent with their model or other models which impose CES demand and the continuum of firm assumptions. This is because the CES demand and the continuum of firm assumptions impose a fixed mark-up across markets and industries. Thus there is no variation in product mix for firms selling to different markets or in different industries.[12] Mayer, Melitz and Ottaviano (2014) presents a multi-product extension of Melitz and Ottaviano (2008) which features variable mark-ups. Their model explains how French exporters vary their sales across markets: firms exporting to tougher markets skew their export towards the better products. Such a mechanism should work across industries and across firms as well and can explain stylized facts 3 and 4. This motivates our theory in the next section.

3   Theory

In this section, we build up a theory which simultaneously explains the three stylized facts discovered in the previous section. Our theory extends the analysis of Mayer, Melitz and Ottaviano (2014) to a continuum of industries as in Dornbusch, Fischer and Samuelson (1977).

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[11]Stylized fact 1 is also in Lu (2010) and Huang *et al* (2017). Both papers interact the Melitz (2003) typed heterogeneous model with the classic comparative advantage. Bernard, Redding and Schott (2011) provide an extension of their model to multiple industries in the appendix.

[12]Relative sales of different products only depends on relative firm/product productivity in these typed models.

3.1 Close Economy

There are two countries, home and foreign. In each country, the preference of the representative consumers is given by

$$U = q_0^c + \int_0^1 [\alpha \int_{i \in \Omega(z)} q_i^c(z)di - \frac{\gamma}{2} \int_{i \in \Omega(z)} (q_i^c(z))^2 di - \frac{\eta}{2} (\int_{i \in \Omega(z)} q_i^c(z)di)^2]dz$$

where $z \in [0,1]$ indexes the continuum of industries and $i \in \Omega(z)$ indexes the varieties available for each industries, with $\Omega(z)$ being the measure of varieties in industry z. q_0^c denotes the consumption for the numeraire good whose price is normalized to be 1. We allow the parameters capturing substitution pattern between the differentiated varieties and the numeraire good are α and η. The parameter capturing the substitution pattern of differentiated varieties within each industry is given by γ. The budget constraint faced by the representative consumer is given by

$$q_0^c + \int_0^1 \int_{i \in \Omega(z)} p_i^c(z)q_i^c(z)didz = y_0^c + I$$

Solving the representative consumer's problem, assuming that the interior solution is always satisfied, we have the following demand for variety i in industry z:

$$p_i(z) = \alpha - \gamma q_i^c(z) - \eta Q^c(z).$$

Then the corresponding market demand for home market is:

$$q_i(z) = Lq_i^c(z) = \frac{L}{\gamma}(p_{max}^z - p_i(z))$$

where L is the number of consumer at home and p_{max}^z is the choke price of industry z at home. Then for a firm with marginal cost c operating in industry z, it is facing the following problem

$$\max_{p(z)}(p(z,c) - c)q(z)$$

Solving the firm's problem, we have

$$
\begin{aligned}
p(z,c) &= \frac{1}{2}(p_{\max}^z + c) \\
\mu(z,c) &= \frac{1}{2}(p_{\max}^z - c) \\
q(z,c) &= \frac{L}{2\gamma}(p_{\max}^z - c) \\
\pi(z,c) &= \frac{L}{4\gamma}(p_{\max}^z - c)^2
\end{aligned}
$$

where $p(z,c), \mu(z,c), q(z,c)$ and $\pi(z,c)$ are price, mark-up, output and profit respectively.

Each industry has a pool of potential entrants. Firms pays a fixed cost of f_E and draw their marginal cost from a common distribution $G(z,c)$ on support $[0, C_M(z)]$ for industry z. Firms with marginal costs higher than certain threshold $C_D(z) = p_{\max}^z$ would exist the market. The free entry condition in industry z implies that

$$
\int_0^{C_D(z)} \pi(z,c) dG(z,c) = f_E.
$$

Under Pareto distribution assumption such that

$$
G(z,c) = \left(\frac{c}{C_M(z)}\right)^k, c \in [0, C_M(z)],
$$

Thus the cut-off marginal cost under autarky is given by

$$
C_D(z)^A = \left[\frac{2(k+1)(k+2)\gamma C_M(z)^k f_E}{L}\right]^{1/(k+2)} \tag{3.1}
$$

Similarly, for the foreign countries, we have

$$
C_D(z)^{*A} = \left[\frac{2(k+1)(k+2)\gamma C_M(z)^{*k} f_E}{L^*}\right]^{1/(k+2)} \tag{3.2}
$$

3.2 Open economy with single-product firms

Suppose now countries are open to trade and firms are single product firm. To export to foreign country, there is an iceberg cost of τ which is the same for all domestic firms operating in all industries. The iceberg cost faced by foreign exporters is τ^*.

Free entry implies that the sum of expected profits from both markets equals the fixed cost.

Thus the free entry condition becomes

$$\int_0^{C_D(z)} \pi_D(z,c)dG(z,c) + \int_0^{C_D(z)^*/\tau} \pi_X(z,c)dG(z,c) = f_E$$

which can be simplified as

$$LC_D(z)^{k+2} + \rho L^* C_D^*(z)^{k+2} = \beta C_M(z)^k \tag{3.3}$$

where $\rho = \tau^{-k} \in [0,1]$ is the freeness of trade and $\beta = 2\gamma(k+1)(k+2)f_E$ is a constant. Similarly for the foreign, we have

$$L^* C_D^*(z)^{k+2} + \rho^* L C_D(z)^{k+2} = \beta C_M^*(z)^k \tag{3.4}$$

where $\rho^* = \tau^{*-k}$. Combining the two equations above, we have[13]

$$C_D(z)^{k+2} = \frac{\beta[C_M(z)^k - \rho C_M^*(z)^k]}{L(1 - \rho\rho^*)} \tag{3.5}$$

$$C_D^*(z)^{k+2} = \frac{\beta[C_M^*(z)^k - \rho^* C_M(z)^k]}{L^*(1 - \rho\rho^*)} \tag{3.6}$$

Following Dornbusch, Fischer and Samuelson (1977), we rank industries such that $\frac{\partial C_M(z)}{\partial z} > 0$ and $\frac{\partial C_M^*(z)}{\partial z} < 0$. That is domestic firms in industries with higher z are drawing their marginal costs from a wider support, *vice versa* for the foreign. Under such assumptions, home country would have comparative advantage in industries with lower z. There are different ways that these assumptions can be micro-founded. For example, it can be generated by the Heckscher-Ohlin force. Suppose that firms uses a Cobb-Douglas production function and z indexes the capital intensity. Moreover, the supports are given by $C_M(z) = w^{1-z}r^z$ and $C_M^*(z) = w^{*1-z}r^{*z}$. Then $\frac{\partial C_M(z)}{\partial z} = C_M(z)\ln\frac{r}{w}$ and $\frac{\partial C_M^*(z)}{\partial z} = C_M^*(z)\ln\frac{r^*}{w^*}$. If home country is labor abundant relative to foreign such that $\frac{r^*}{w^*} < 1 < \frac{r}{w}$. Then we have $\frac{\partial C_M(z)}{\partial z} > 0$ and $\frac{\partial C_M^*(z)}{\partial z} < 0$. Under this interpretation, home country has comparative advantage in labor intensive industries while foreign countries have comparative advantages in capital intensive industries.

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[13]To insure that the two equations have real solutions, we assume that $\rho \leq \frac{C_M(z)^k}{C_M(z)^{*k}} \leq \frac{1}{\rho^*}$ for all industries.

Under our assumption that $\frac{\partial C_M(z)}{\partial z} > 0$ and $\frac{\partial C_M^*(z)}{\partial z} < 0$, it is easy to show that

$$\frac{\partial C_D(z)}{\partial z} > 0 \text{ and } \frac{\partial C_D^*(z)}{\partial z} < 0$$

So the cut-off margin costs are lower in the comparative advantageous industries. Then immediately we have the following proposition.

Proposition 1. *Export propensity $\chi(z) \equiv (\frac{C_X(z)}{C_D(z)})^k$ and export intensity $\lambda(z) \equiv \frac{EXP(z)}{TotalSales(z)}$ increase with comparative advantage.*

Proof. See Appendix. $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\square$

This proposition implies that export participation as measured by the share of firms that export and the sales that are exported should be higher in comparative advantage industries. This is consistent with Stylized fact 1 if we believe China has comparative advantage in labor intensive industries.

The number of entrants in each industries are given by

$$N_E = \frac{2C_M(z)^k(k+1)\gamma}{\eta(1-\rho\rho^*)}(\frac{\alpha - C_D(z)}{C_D(z)^{k+1}} - \rho^* \frac{\alpha - C_D^*(z)}{C_D^*(z)^{k+1}})$$
$$N_E^* = \frac{2C_M^*(z)^k(k+1)\gamma}{\eta(1-\rho\rho^*)}(\frac{\alpha - C_D^*(z)}{C_D^*(z)^{k+1}} - \rho \frac{\alpha - C_D(z)}{C_D(z)^{k+1}})$$

Home country would specialize in industries where $\frac{\alpha - C_D^*(z)}{C_D^*(z)^{k+1}} \leq \rho \frac{\alpha_z - C_D(z)}{C_D(z)^{k+1}}$ such that $N_E^* = 0$ or there is no foeign entrants. This might happen if ρ is sufficiently large or $C_D^*(z)$ is large relative to $C_D(z)$. Intuitively, in such cases, foreign firms are facing tough competition from home country and get eliminated from the market. Similarly, foreign countries would specialize in industries such that $\frac{\alpha - C_D(z)}{C_D(z)^{k+1}} \leq \rho^* \frac{\alpha - C_D^*(z)}{C_D^*(z)^{k+1}}$. [14]

3.3 Open economy with multi-product firms

Now we extend the model to allow firms producing multiple products. The multi-product firm technology is the same as Mayer, Melitz and Ottaviano (2014). Firms marginal cost to produce their core competency is given by c. For the same firm, varieties are ranked in increasing order of distance from their core competency and indexed by m. The marginal cost of producing

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[14]Given that China imports and exporters in every industries, we assume the no-specialization conditions are always satisfied.

variety m is given by $v(m, c) = \varpi^{-m} c$ where $\varpi \in (0, 1)$. So the marginal cost increases as we move away from the core competency. Thus firms will keep adding products until the marginal the costs is higher than the choke price. Then the number of varieties produced by each firm is then given by

$$M_D(z, c) = \begin{cases} 0, \ if \ c > C_D(z) \\ \max\{m | v(m, c) \leq C_D(z)\} + 1, \ if \ c \leq C_D(z) \end{cases}$$

And the number of varieties exported to foreign by domestic firms is given by

$$M_X(z, c) = \begin{cases} 0, \ if \ c > C_X(z) \\ \max\{m | v(m, c) \leq C_X(z) = \frac{C_D^*(z)}{\tau}\} + 1, \ if \ c \leq C_X(z) \end{cases}$$

Now the free entry condition becomes

$$\int_0^{C_D(z)} \Pi_D(z, v(m, c)) dG(z, c) + \int_0^{C_D(z)^*/\tau} \Pi_X(z, v(m, c)) dG(z, c) = f_E \qquad (3.7)$$

where the profits of each firm is the sum of the profits from each product it sells:

$$\Pi_D(z, c) = \sum_{m=0}^{M_D(z,c)-1} \pi_D(z, v(m, c))$$

$$\Pi_X(z, v(m, c)) = \sum_{m=0}^{M_X(z,c)-1} \pi_X(z, v(m, c)).$$

Using the results from Mayer, Melitz and Ottaviano (2014), the free entry condition equation (3.7) can be simplified as

$$L C_D(z)^{k+2} + \rho L^* C_D^*(z)^{k+2} = \frac{\beta C_M(z)^k}{\Psi} \qquad (3.8)$$

where $\Psi = (1 - \varpi^k)^{-1}$ is an index of multi-product flexibility. Similarly, for the foreign countries, we have

$$L^* C_D^*(z)^{k+2} + \rho^* L C_D(z)^{k+2} = \frac{\beta C_M^*(z)^k}{\Psi}$$

Again, we can solve the two equations for the choke prices:

$$C_D(z)^{k+2} = \frac{\beta [C_M(z)^k - \rho C_M^*(z)^k]}{\Psi L (1 - \rho \rho^*)} \qquad (3.9)$$

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$$C_D^*(z)^{k+2} = \frac{\beta[C_M^*(z)^k - \rho^* C_M(z)^k]}{\Psi L^*(1 - \rho\rho^*)} \tag{3.10}$$

It is easy to see that $\frac{\partial C_D(z)}{\partial z} > 0$ and $\frac{\partial C_D^*(z)}{\partial z} < 0$ under our assumptions that $\frac{\partial C_M(z)}{\partial z} > 0$ and $\frac{\partial C_M^*(z)}{\partial z} < 0$. This implies Propositions 1 still hold under the multi-product extension.

The next two propositions look at how product scope and product mix varies across industries.

Proposition 2. *The export product scope is weakly increasing in comparative advantage.*

Proof. See Appendix. □

Proposition 2 implies that the export product scope tends to lower in the comparative disadvantage industries. For firms with the same marginal costs, the firm in the comparative disadvantage industries are more likely to be single product exporter. This is consistent with Stylized Fact 2.

Proposition 3. *Export sales is skewed towards the better products in the industries with comparative disadvantage.*

Proof. See Appendix. □

In industries of comparative disadvantage, the export market is more competitive. Such tougher competition would induce exporters to reallocate sales towards better products. If we agree that capital intensive industries are the comparative disadvantage industries of China, we should expect capital intensive exporters would have skewer export product mix which is consistent with Stylized Fact 3.

Proposition 4. *Domestic sales tend to skew towards more productive firms in comparative advantage industries.*

Proof. See Appendix. □

In industries of comparative advantage, domestic market is more competitive. Such tougher competition would induce reallocation sales towards products that are produced with lower marginal costs. Since such products are more likely to be produced by firms with higher core efficiencies, outputs are reallocated towards these firms. In the end, sales are skewed towards these firms and it would rationalize our stylized fact 4.

3.4 Comparative advantage

Our model has new implications on comparative advantage. Bernard, Redding and Schott (2007) shows that the different extent of selection across industries generates endogenous Ricardian comparative advantage which amplifies ex-ante comparative advantage. In this subsection, we show that variable mark-up allows for selection along the intensive margin which generates an endogenous Ricardian comparative advantage dampening ex-ante comparative advantage. If we use productivity measure which only depends on the extensive margin, we only capture the amplifying effect of selection. If we use productivity measure that include both selections along the extensive and intensive margin, we capture both the amplifying and dampening effect of selection.

3.4.1 The extensive margin only

Comparative advantage is defined as the relative productivity between home and foreign for each industry. If we only care the extensive margin and measure productivity as the inverse of the simple average marginal cost within each industry:

$$\bar{c}(z) = \int_0^{C_D(z)} c \, dG(z,c) = \frac{k}{k+1} C_D(z)$$

For the foreign country, it is $\bar{c}(z)^* = \frac{k}{k+1} C_D(z)^*$. Then using equation (3.1) and (3.2), the relative average marginal cost under autarky is given by:

$$
\begin{aligned}
\frac{\bar{c}(z)}{\bar{c}(z)^*} &= \frac{C_D(z)^A}{C_D(z)^{*A}} \\
&= \left(\frac{L^*}{L} \frac{C_M(z)^k}{C_M^*(z)^k} \right)^{1/(k+2)}.
\end{aligned}
$$

In the case of open economy, using equations (3.9) and (3.10) the relative marginal cost between home and foreign is given by[15]

$$
\begin{aligned}
\frac{\bar{c}(z)}{\bar{c}(z)^*} &= \frac{C_D(z)^T}{C_D(z)^{*T}} \\
&= \left(\frac{L^*}{L} \frac{C_M(z)^k - \rho C_M^*(z)^k}{C_M^*(z)^k - \rho^* C_M(z)^k} \right)^{1/(k+2)}
\end{aligned}
$$

---

[15]The single-product economy gives the same results since equation (3.5) and (3.6) only differ by a constant.

Proposition 5. *Comparative advantage as measured by the relative simple average of margin costs between home and foreign* $\frac{\overline{c}(z)}{\overline{c}(z)^*}$ *is amplified after opening up to trade as*

$$\frac{C_D(z)^T}{C_D(z)^{*T}} = \underbrace{\frac{C_D(z)^A}{C_D(z)^{*A}}}_{ex\ ante} \underbrace{\left[\frac{1 - \rho\frac{C_M^{*\ k}}{C_M^{\ k}}}{1 - \rho^*\frac{C_M^{\ k}}{C_M^{*\ k}}}\right]^{\frac{1}{k+2}}}_{amplifying}. \tag{3.11}$$

Proof. See Appendix. □

3.4.2 Both the intensive and extensive margins

The productivity measure in the previous subsection only takes into account the extensive margin. Now we consider a quantity-based TFP as used by Mayer, Melitz and Ottaviano (2014). It captures both the intensive and extensive margins since incorporates the fact that firms have different amount of outputs and inputs and only a subset of firms exports. In the close economy, we have

$$\overline{\Phi}(z)^A = \frac{\int_0^{C_D(z)} Q(z,c)dG(z,c)}{\int_0^{C_D(z)} C(z,c)dG(z,c)} = \frac{k+2}{k}\frac{1}{C_D(z)}$$

as shown in Mayer, Melitz and Ottaviano (2014) where $Q(z,c) = \sum_{m=0}^{M_D(z,c)-1} q(z,v(m,c))$ and $C(z,c) = \sum_{m=0}^{M_D(z,c)-1} v(m,c)q(z,v(m,c))$. Then the relative productivity under autarky is given by

$$\frac{\overline{\Phi}(z)^A}{\overline{\Phi}(z)^{*A}} = \frac{C_D(z)^*}{C_D(z)} = \left(\frac{L}{L^*}\frac{C_M^*(z)^k}{C_M(z)^k}\right)^{1/(k+2)}.$$

which is the *ex ante* comparative advantage before countries open to trade and is the same as the measure which only captures extensive margin. In the open economy

$$\overline{\Phi}(z)^T = \frac{\int_0^{C_D(z)} Q_D(z,c)dG(z,c) + \int_0^{C_X(z)} Q_X(z,c)dG(z,c)}{\int_0^{C_D(z)} C_D(z,c)dG(z,c) + \int_0^{C_X(z)} C_X(z,c)dG(z,c)}$$

It can be shown that

$$\int_0^{C_D(z)} Q_D(z,c)dG(z,c) = \frac{LC_D(z)^{k+1}}{2\gamma C_M^k(k+1)}\frac{1}{1-\varpi^k}$$

$$\int_0^{C_X(z)} Q_X(z,c)dG(z,c) = \frac{\rho L^* C_D^*(z)^{k+1}}{2\gamma C_M^k(k+1)}\frac{1}{1-\varpi^k}$$

$$\int_0^{C_D(z)} C_D(z,c)dG(z,c) = \frac{kLC_D(z)^{k+2}}{2\gamma C_M^k(k+1)(k+2)}\frac{1}{1-\varpi^k}$$

$$\int_0^{C_X(z)} C_X(z,c)dG(z,c) = \frac{\rho k L^* C_D^*(z)^{k+2}}{2\gamma C_M^k(k+1)(k+2)}\frac{1}{1-\varpi^k}$$

So we have

$$\overline{\Phi}(z)^T = \frac{k+2}{k}[\frac{LC_D(z)^{k+2}}{LC_D(z)^{k+2} + \rho L^* C_D^*(z)^{k+2}}\frac{1}{C_D(z)} + \frac{\rho L^* C_D^*(z)^{k+2}}{LC_D(z)^{k+2} + \rho L^* C_D^*(z)^{k+2}}\frac{1}{C_D^*(z)}]$$

which is a weighted average of the competitiveness of each market. The weight is given by the total costs for the goods sold in that market. If $\rho = 0$, we go back to the close economy case. Using the free entry condition (3.8), it can be further simplified as

$$\overline{\Phi}(z))^T = \frac{(k+2)\Psi}{k\beta C_M(z)^k}(LC_D(z)^{k+1} + \rho L^* C_D^*(z)^{k+1}).$$

There is a similar equation for the foreign country. Then the relative productivity between home and foreign for each industry is given by

$$\frac{\overline{\Phi}(z)^T}{\overline{\Phi}^*(z)^T} = \frac{C_M^*(z)^k}{C_M(z)^k}\frac{LC_D(z)^{k+1} + \rho L^* C_D^*(z)^{k+1}}{L^* C_D^*(z)^{k+1} + \rho^* LC_D(z)^{k+1}}.$$

Proposition 6. *Comparative advantage as measured by the relative quantify-based TFP between home and foreign $\frac{\overline{\Phi}(z)}{\overline{\Phi}^*(z)}$ can be decomposed into three components after opening up to trade: an ex ante component, an amplifying component and a dampening component as:*

$$\frac{\overline{\Phi}(z)^T}{\overline{\Phi}^*(z)^T} = \underbrace{\frac{\overline{\Phi}(z)^A}{\overline{\Phi}(z)^{*A}}}_{ex\ ante}\underbrace{(\frac{\overline{\Phi}(z)^A}{\overline{\Phi}(z)^{*A}})^{k+1}}_{amplifying}\frac{L^*}{L}\underbrace{\frac{\frac{L}{L^*}(\frac{C_D(z)}{C_D^*(z)})^{k+1} + \rho}{1 + \rho^*\frac{L}{L^*}(\frac{C_D(z)}{C_D^*(z)})^{k+1}}}_{dampening}. \tag{3.12}$$

Proof. See Appendix. □

As pointed out by Bernard, Redding and Schott (2007), the different extent of selection in each market and industry gives rise to an endogenous Ricardian comparative advantage. Given the higher expected profit of export, there will be more entrants and more intense selection in the comparative advantage industries. This would enlarge the relative productivity difference across industries and amplify the *ex ante* comparative advantage. That is to say, the endogenous Ricardian comparative advantage magnifies ex ante cross-country differences in comparative advantage and is positively correlated with *ex ante* comparative advantage. This channel is preserved in our model and is given by the term $(\frac{\overline{\Phi}(z)^A}{\overline{\Phi}(z)^{*A}})^{k+1}$ which is positively correlated with the ex ante component $\frac{\overline{\Phi}(z)^A}{\overline{\Phi}(z)^{*A}}$.[16]

However, their assumptions of CES demand and a continuum of firm impose constant exogenous mark-up. This implies the relative revenue in each market between firms only depends on relative TFP and has nothing to do with market condition. So the selection along the intensive margin is constant across markets and industries. Our model with variable mark-up has different implications. Tougher competition would induce reallocation of resources towards the more productive firms and better performing products, as evident from our Proposition 4. In other words, firms toughen up in tougher markets or industries. This channel tends to dampen their comparative disadvantage.

We can also look at revenue-based TFP as define in Mayer, Melitz and Ottaviano (2014). Using the same price index as the way they define:

$$\overline{P} = \frac{\int_0^{C_D(z)} R_D(z,c)dG(z,c) + \int_0^{C_X(z)} R_X(z,c)dG(z,c)}{\int_0^{C_D(z)} Q_D(z,c)dG(z,c) + \int_0^{C_X(z)} Q_X(z,c)dG(z,c)}.$$

It would deliver the same result as the quantify based TFP since:

$$\begin{aligned} \overline{\Phi}_R(z) &= \frac{(\int_0^{C_D(z)} R_D(z,c)dG(z,c) + \int_0^{C_X(z)} R_X(z,c)dG(z,c))/\overline{P}}{\int_0^{C_D(z)} C_D(z,c)dG(z,c) + \int_0^{C_X(z)} C_X(z,c)dG(z,c)} \\ &= \overline{\Phi}(z). \end{aligned}$$

[16]In the Appendix 10.2 we extend their analysis to multiple industries. We show the decomposition of comparative advantage measured by the relative industry productivity which is a quantify-weighted average of firm TFP. It has two components: an ex ante component and an amplifying component.

4   Empirical Analysis

We provide two empirical tests on our theory in this section. The first one is a reduced form exercise which shows that exporters skew their export towards better products in markets that they are competitive disadvantageous, conditioning on the market size effects studied by Mayer, Melitz and Ottaviano (2014). The second test is a structural exercise in which we calibrate our model to Chinese economy. The model allows us to quantify the relative importance of the different components of comparative advantage. We find that the dampening component of endogenous Ricardian comparative advantage is quantitatively as important as the amplifying component.

4.1   Comparative advantage and export product mix

Exporter face different level of competition across destinations. This can be due to the market size difference: exporters face tougher competition in larger markets (Mayer, Melitz and Ottaviano, 2014). It can also be the variation of competitive advantage across markets, as emphasized in our theory above. Capital intensive firms would face tougher competition in capital abundant countries while labor intensive firms would face tougher competition in labor abundant countries, conditioning on the market size. To capture this channel, we need to measure the competition due to comparative advantage that firms face in each markets. We propose two measures. The first one is given by:

$$CA1_{ij} = (z_i - \bar{z})(\frac{K_j}{L_j} - \frac{\overline{K}}{L})$$

where $\frac{K_j}{L_j}$ is the capital labor ratio of country j and $\frac{\overline{K}}{L}$ is the average capital ratio of all countries (other than China); z_i is the capital intensity of firm i and $\bar{z}$ is the average capital intensity of all Chinese manufacturing firms. The larger that $CA1_{ij}$ is, the tougher the competition that exporter i would face in market j. This is because $CA1_{ij}$ would get larger if z_i is high above $\bar{z}$ and $\frac{K_j}{L_j}$ is also high above $\frac{\overline{K}}{L}$, or if z_i is far below $\bar{z}$ and $\frac{K_j}{L_j}$ is far below $\frac{\overline{K}}{L}$. In either cases, firms i would face tougher competition in country j since the country is abundant in the factor that firm i uses intensively. Alternatively, we use the firms' capital labor ratio instead of capital intensity and have the following measure:

$$CA2_{ij} = (\frac{k_i}{l_i} - \frac{\overline{k}}{l})(\frac{K_j}{L_j} - \frac{\overline{K}}{L})$$

where $\frac{k_i}{l_i}$ is the capital labor ratio of firm i and $\frac{\overline{k}}{l}$ is the average capital labor ratio of all Chinese firms.

To construct these measures, we first need to estimate the capital labor ratio for each country. We use the Penn World Table 9.0 which the estimate of capital stock (at constant price) and employment are available. [17] The capital ratio of each country is then computed as the ratio of capital stock to employment. The world average capital labor ratio $\frac{\overline{K}}{L}$ is computed as the average of capital labor ratio across all countries except China.[18] For the firm level measure, firstly the capital intensity is measure as $z = 1 - \frac{LaborCost}{ValueAdded}$ as explained above in the motivating evidence. To measure the capital labor ratio of each firm, we first estimate the capital stock for each firm using the perpetual inventory method following Brandt et al (2012). Labor is measured as the total number of employees. The average capital intensity $\overline{z}$ or capital labor ratio $\frac{\overline{k}}{l}$ computed as the simple average across all Chinese firms.

To compare our results with Mayer, Melitz and Ottaviano (2014, hereafter MMO), we use the data from year 2003. Table 5 extends the basic empirical results of MMO(2014) by including the two measures of competition due to comparative advantage faced by firm at each market as additional controls.[19] The dependent variable is the logarithm of the ratio of export sales of the core product(m=0) and second product (m=1) in each market for each firm.[20] We include the GDP of each export market to capture competition due to market size effect. We include the supply potential to capture competition due to geography following MMO. As we can see, the market size effects highlighted in MMO remains highly significant. The supply potential is positive but not precisely estimate. And the two different measures of comparative disadvantage faced are positive and significant. That is to say, in markets that firms face tougher competition, export sales are skewed towards the core products.

Table 6 looks at the skewness across all products that firms export to each market, as measured by the Herfindhal index and Theil index, controlling the market fixed effect and firm fixed effect. The market fixed effects would capture the market size and geography of the destination markets. As we can see, the skewness measures are higher in markets firms faced more comparative disadvantages. That is to say the export sales is more skewed in export markets that exporters faced tougher competition due to comparative advantage.

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[17]The data is downloaded from http://www.rug.nl/ggdc/

[18]We exclude China from the sample to make the measure more exogenous. But adding China would make little difference.

[19]We follow their empirical approach to use demean all variables at firm level and estimate the model using random effects with robustness standard errors.

[20]The product rank is the local rank.

Table 7 examines the effect on product scope, that is the number products exported by each firm to each market. Again, we control the market fixed effect and firm fixed effect in all regressions. Column (1) and (2) estimates using OLS with ln(product count) as the dependent variable. Column (3) to (6) estimate using product count as the dependent variable. Column (3) and (4) use Poisson model while (5) and (6) use negative binomial model. In all cases, firms tend to export fewer number of products in markets they face tougher competition due to comparative advantage.

To sum up, the evidence are consistent with Proposition 3 and 4 that firms facing tougher competition due to comparative advantage would have narrower export product scope and more skewed export product mix.

4.2 Quantification of comparative advantage

We have shown in our theoretical sections that different measures of comparative advantage capture different margins of reallocation in actions. Measure which only captures the extensive margin would miss out the dampening forces due to tougher competition in the export market. Does such a distinction quantitatively make a difference? To answer such a question, we need to quantify the different ways of decomposition for comparative advantage. However, there are a few challenges in doing that. First, we do not observe the *ex ante* comparative advantage. It is the relatively productivity for each industry between home and foreign under close economy. But we only observe the open economy. Second, even for the open economy that we can observe, measuring the relative productivity between home country and foreign country remains difficult. We do not have the firm level data for production or export for Rest of the World. These are needed either for the productivity measure just capturing the extensive margin or the quantify-based TFP which also captures the intensive margin. Finally, even if we have the data to measure the different components of the decompositions directly, they either depend on the relative upper bounds of the Pareto distribution or relative choke prices between the two countries. Again, these are not directly observable and difficult to estimate.

In this subsection, we first provide an identification result which show that we only need to know the share of exporters χ_z and export intensity λ_z for *home country*, trade elasticity k and trade freeness ρ, ρ^* in order to measure comparative advantage and the decompositions. In other words, $(\chi_z, \lambda_z, k, \rho, \rho^*)$ are *sufficient statistics* for comparative advantage itself and the different components of comparative advantage.

Proposition 7. *As long as we know the trade elasticity k, the trade freeness ρ and ρ^*, and observe the percentage of exporters $\chi(z)$ and export intensity $\lambda(z)$ for each industry, we can quantify the decomposition of comparative advantage in Proposition 5 and 6.*

Proof. See Appendix. $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\square$

To quantify comparative advantage using these theoretical results, we calibrate the the model to the Chinese economy. We set the Pareto shape parameter $k = 3.43$ which is the estimated median trade elasticity by Broda et al (2006) for China.[21] The trade freeness ρ and ρ^* are estimated using the Head-Ries Index (Head and Ries, 2001) and the World Input Output Database. The details of the estimation are in the Appendix 7. The results are presented in Table 10. According to the estimated results, the trade freeness between China and the Rest of World has been increasing over time. The average trade freeness was 0.043 in 2000 and rose to 0.058 in 2003 and 0.071 in 2006. Given the trade elasticity k=3.43, the implied average iceberg trade costs dropped from 2.50 in 2000 to 2.16 in 2006. The final piece of data we need is measures for export propensity $\chi(z)$ and export intensity $\lambda(z)$. We take the corresponding data underlining Figure 1 which are the share of exporters and the fraction of sales exported for each industry respectively.

Armed with these parameters and data, we are ready to quantify and decompose the comparative advantage of China relative to the rest of world for year 2000, 2003 and 2006 using the results from Proposition 7. Before showing the results, we first validate the estimation by evaluating the model's prediction on moments that have not been used. Our sufficient statics results only rely on information of exports. We would like to evaluate its prediction on imports. According to the model, export from China to the RoW in industry z is given by $EXP(z) = \frac{1}{2\gamma(k+2)C_M(z)^k}N_E(z)C_D^*(z)^{k+2}L^*\rho$ and import from the RoW to China is $IMP(z) = \frac{1}{2\gamma(k+2)C_M^*(z)^k}N_E^*(z)C_D(z)^{k+2}L\rho^*$. Thus import relative to export in industry z is given by

$$\frac{IMP(z)}{EXP(z)} = \frac{L}{L^*}\frac{\rho^*}{\rho}\frac{N_E^*(z)}{N_E(z)}\frac{C_M(z)^k}{C_M^*(z)^k}\frac{C_D(z)^{k+2}}{C_D^*(z)^{k+2}}$$

which is determined by the relative market size $\frac{L}{L^*}$, relative trade freeness $\frac{\rho^*}{\rho}$, the relative number of entrants $\frac{N_E^*(z)}{N_E(z)}$, the exogenous relative competitiveness $\frac{C_M(z)^k}{C_M^*(z)^k}$ and endogenous relative

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[21] As proved by Melitz and Ottviano (2008) and synthesized in Head and Mayer(2014), the Pareto shape parameter correspond to the trade elasticity under our current model. For robustness, we also check our results using the median trade elasticity of 5.03 from the literature (Head and Mayer, 2014) and experiment with a relative low and high elasticity of 2.5 and 7.5. The results are qualitatively the same.

competitiveness $\frac{C_D(z)^{k+2}}{C_D^*(z)^{k+2}}$. Our model provides sufficient statistics to estimate $\frac{C_D(z)^{k+2}}{C_D^*(z)^{k+2}}$ and $\frac{C_M(z)^k}{C_M^*(z)^k}$. How well do they explain the variation of $\frac{IMP(z)}{EXP(z)}$ in the data? Answering such a question would give us a validation of the performance of the model.

Our matched Chinese firm-customs data contains import volume for the importers. We assume that import of industry z from RoW is the total import of importers from industry z in China. [22] Under such an assumption, we find $\frac{IMP(z)}{EXP(z)}$ tend to increase with capital intensity z for China as Figure 5 shows for year 2000, 2003 and 2006. For the most capital intensive industries, China runs trade deficits since $\frac{IMP(z)}{EXP(z)} > 1$.
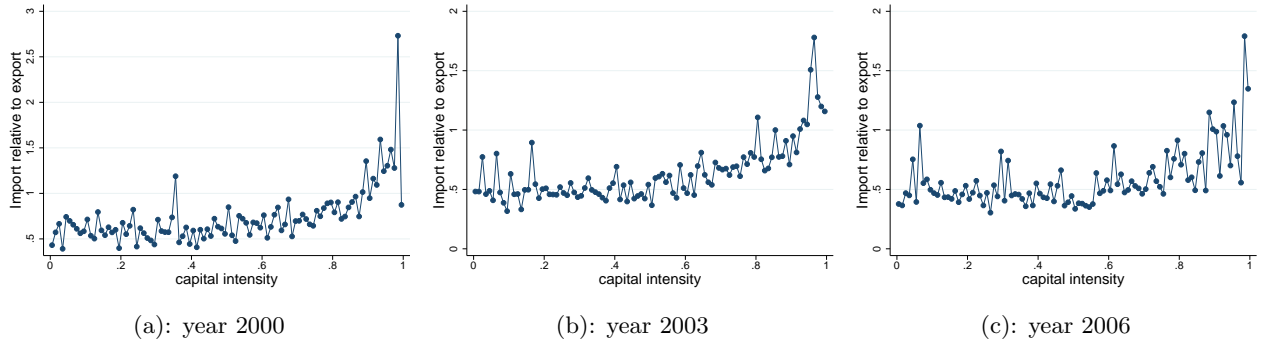

(a): year 2000   (b): year 2003   (c): year 2006

Figure 5: Import relative to export by industries

How well does the model explain such variations across industries? On the Figure 6, we plot $\ln(\frac{IMP(z)}{EXP(z)})$ against $\ln(\frac{C_M(z)^k}{C_M^*(z)^k}\frac{C_D(z)^{k+2}}{C_D^*(z)^{k+2}})$ for each year. As can be seen, there is a very strong positive correlation. That is in industries that China is relatively unproductive to the RoW, China tends to run trade deficit. We confirm this by regression with $\ln(\frac{IMP(z)}{EXP(z)})$ as the dependent variable and $\ln(\frac{C_M(z)^k}{C_M^*(z)^k}\frac{C_D(z)^{k+2}}{C_D^*(z)^{k+2}})$ as the explanatory variables, we evaluate the explanatory power of $\ln(\frac{C_M(z)^k}{C_M^*(z)^k}\frac{C_D(z)^{k+2}}{C_D^*(z)^{k+2}})$. [23] The results are show in Table 8. As we can see, the coefficients for $\ln(\frac{C_M(z)^k}{C_M^*(z)^k}\frac{C_D(z)^{k+2}}{C_D^*(z)^{k+2}})$ is highly significant and precisely estimated. It explains about half of the variations in imports relative to export as we can see in columns (1), (3) and (5). It remains highly significant after we control the capital intensity of each industry in columns (2), (4) and (6).

Now we are ready to show the quantification results of comparative advantage. First, the

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[22]Ideally, we would like the China-equivalent data for the RoW which allows to compute its export in each industry to China. However, this is not available to us.

[23]Ideally, we would like to run the following regression: $ln(\frac{IMP(z)}{EXP(z)}) = a_0 + a_1 \ln(\frac{C_M(z)^k}{C_M^*(z)^k}\frac{C_D(z)^{k+2}}{C_D^*(z)^{k+2}}) + a_2 \ln\frac{N_E^*(z)}{N_E(z)} + \varepsilon_z$ where our theory predicts that $a_0 = \frac{L}{L^*}\frac{\rho^*}{\rho}$, $a_1 = a_2 = 1$. However, we don't observe $\frac{N_E^*(z)}{N_E(z)}$.
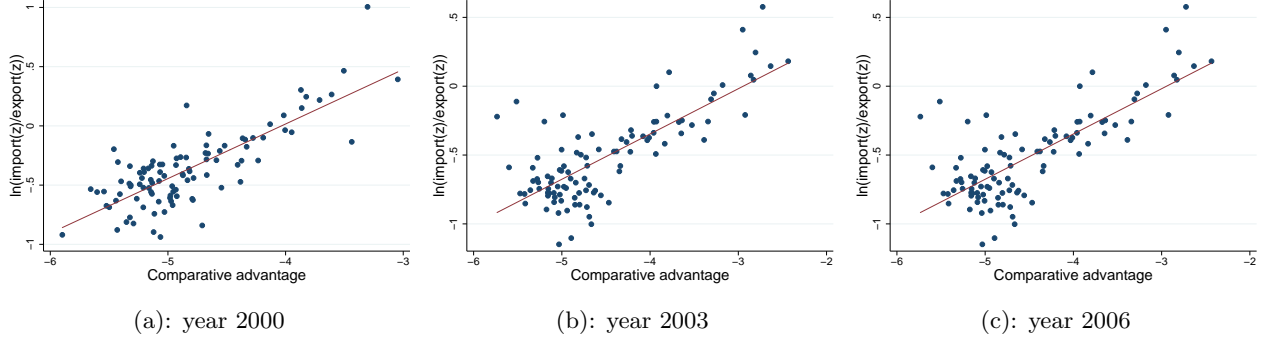
24

| (a): year 2000 | (b): year 2003 | (c): year 2006 |

Figure 6: Import relative to export and comparative advantage

ex ante components are the same for the two measures of comparative advantage according to the proof of proposition 7. They are plotted in Figure 7. To filter out the noise in the data, we use local polynomials to fit the data, with confidence intervals indicated. According to the results, China has *ex ante* comparative advantage in the labor intensive industries. Over time, the *ex ante* component favors labour intensive industries more and more.[24] This is in-line with Huang et al (2017) in which they find firm level productivities grew faster for labor intensive Chinese firms and the exogenous Ricardian comparative favored labor intensive industries over time from 1999-2007.

To single out the endogenous components of comparative advantage, we divide the total comparative advantage by the *ex ante* component and get:

$$\frac{C_D(z)^T}{C_D(z)^{*T}} / \frac{C_D(z)^A}{C_D(z)^{*A}} = \underbrace{\left[\frac{1 - \rho \frac{C_M^{*\,k}}{C_M^{\,k}}}{1 - \rho^* \frac{C_M^{\,k}}{C_M^{*\,k}}}\right]^{\frac{1}{k+2}}}_{amplifying} \qquad (4.13)$$

$$\frac{\overline{\Phi}(z)^T}{\overline{\Phi}^*(z)^T} / \frac{\overline{\Phi}(z)^A}{\overline{\Phi}(z)^{*A}} = \underbrace{\left(\frac{\overline{\Phi}(z)^A}{\overline{\Phi}(z)^{*A}}\right)^{k+1}}_{amplifying} \frac{L^*}{L} \underbrace{\frac{\frac{L}{L^*}(\frac{C_D(z)}{C_D^*(z)})^{k+1} + \rho}{1 + \rho^* \frac{L}{L^*}(\frac{C_D(z)}{C_D^*(z)})^{k+1}}}_{dampening}. \qquad (4.14)$$

The right hand side of the equations now are only left with the endogenous components. These are plotted in Figure 8 for equation (4.13) and Figure 9 for equation (4.14). As we can

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[24]It looks as if there is a "wave" travelling to the labor intensive industries over the three figures.

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see from Figure 8, the endogenous component of the measure which only captures the endogenous component tend to favor labor intensive industries. This is not surprising given that the theory predicts that it is positively correlated with the *ex ante* component and the *ex ante* component favors the labor intensive industries. However, as evident from Figure 9, the endogenous margin of the measure capturing both margins tend to favor capital intensive industries. Given that our theory predicts that the dampening component is negatively correlated with the *ex ante* component, the dampening component should tend to favor capital intensive industries. This implies that the dampening component of comparative advantage must have dominated the amplifying component such that it determines how the endogenous component vary with capital intensity.

To examine the effect of the endogenous components on comparative advantage, we plot the inferred overall comparative advantage in Figure 10 and 11. Figure 10 plots the overall comparative advantage which only captures the extensive margin, i.e., $\frac{C_D(z)^{*T}}{C_D(z)^T}$. Figure 11 plots the overall comparative advantage which captures both the extensive and intensive margin, i.e., $\frac{\overline{\Phi}(z)^T}{\overline{\Phi}^*(z)^T}$. Both measures tend to favor the labor intensive industries. Given that the endogenous component of the measure capturing only extensive margin amplifies the *ex ante* component as we have seen in Figure 8, it is more variant than the *ex ante* component since all the lines are steeper in Figure 10 than in Figure 7. However, due to the dampening effect of the endogenous component, the measure captures both margins is less variant than the *ex ante* component as all the lines in Figure 11 are flatter than in Figure 7.

This is also illustrated in Table 9. Column (1) reports the regression coefficients of capital intensity out of an OLS regression which regresses the *ex ante* comparative advantage on capital intensity. Indeed given the negative coefficients, home country tend to be less productive in the capital intensive industries *ex ante*. These coefficients become even more negative in Column (2) when we replace the dependent variable by the measure of comparative advantage which captures only the extensive margin. This shows the effect of the amplifying endogenous component. However, the coefficients become less negative in Column (3) when we replace the dependent variable by the measure of comparative advantage which captures both margins. This shows that the dampening component dominates the amplifying component.
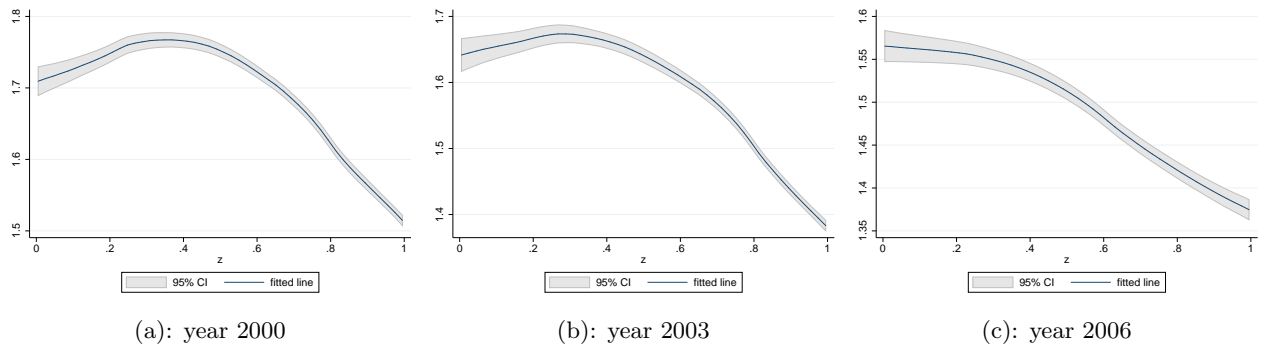
(a): year 2000   (b): year 2003   (c): year 2006

Figure 7: The ex ante component of comparative advantage
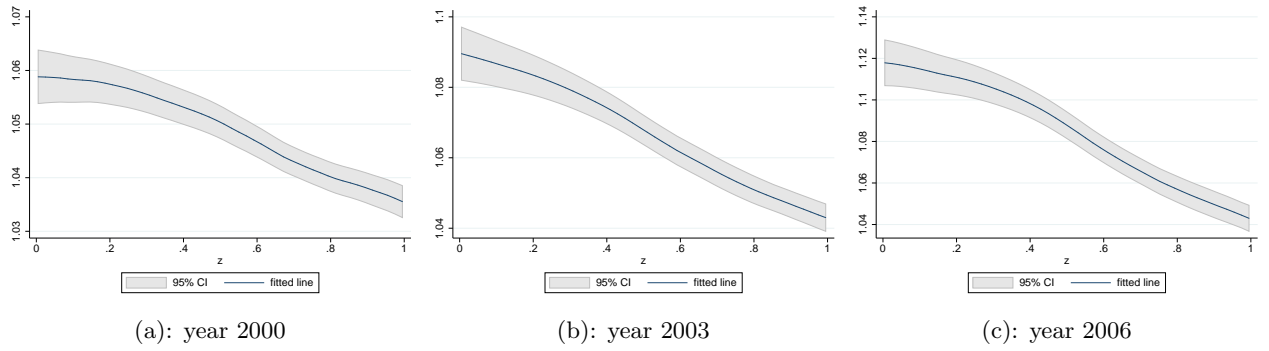
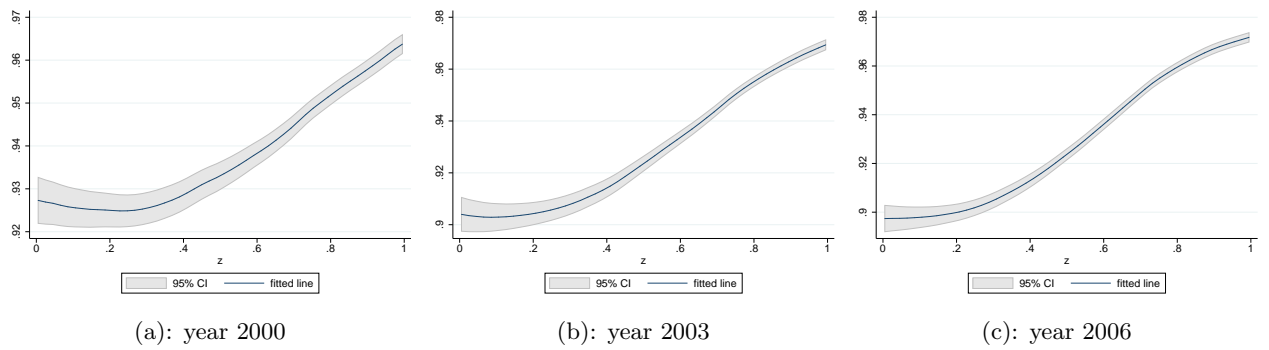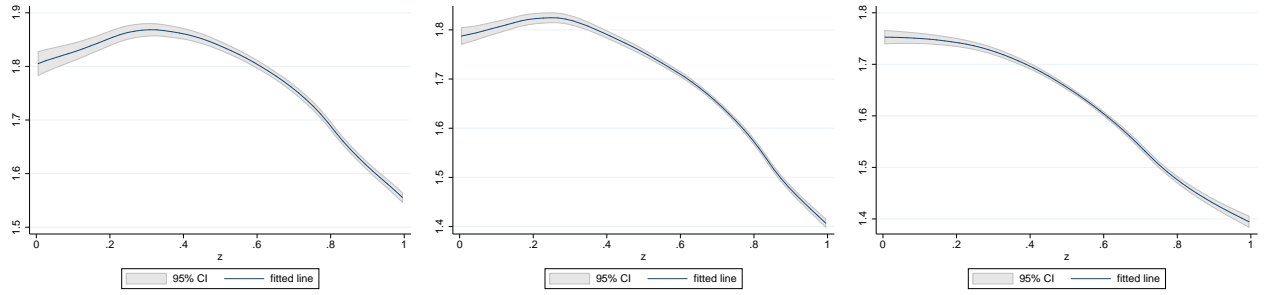(a): year 2000   (b): year 2003   (c): year 2006

Figure 8: The endogenous component of comparative advantage: extensive margin only

(a): year 2000   (b): year 2003   (c): year 2006

Figure 9: The endogenous component of comparative advantage: both margins
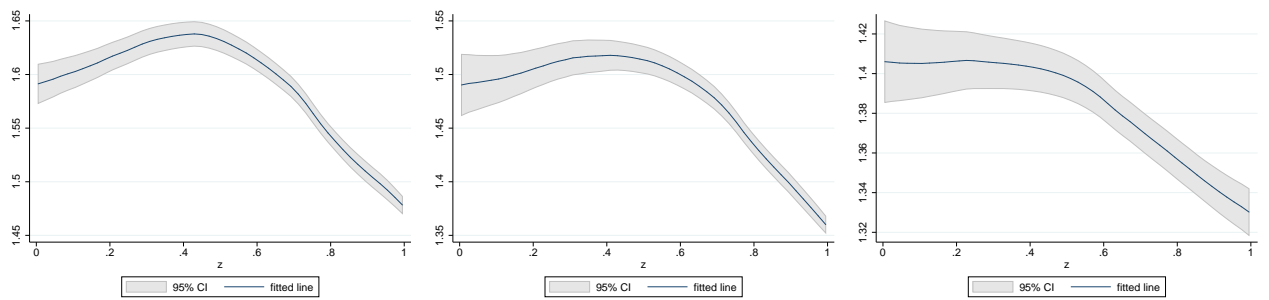
(a): year 2000   (b): year 2003   (c): year 2006

Figure 10: The overall comparative advantage: extensive margin only

(a): year 2000   (b): year 2003   (c): year 2006

Figure 11: The overall comparative advantage: both margins

5 Conclusion

Our paper find stylized facts on how comparative advantage matters for multi-product exporter. Not all of these facts can be reconciled with models with constant mark-ups. We embed Mayer, Melitz and Ottaviano (2014) with Dornbusch, Fischer and Samuelson (1977) in our model. The model simultaneously explains all these facts. We find that firms facing tougher competition in comparative disadvantage industries. Such competition induces exporters to change their product scope and product mix. Export product scope is narrower and export sales is more skewed in the comparative disadvantage industries. This is confirmed in our reduced form regression analysis. We also find that export selection along the intensive margin generates endogenous Ricardian comparative advantage which is *negatively* correlated with the *ex ante* comparative advantage. This is in contrast with Bernard, Redding and Scott (2007) in which endogenous selection amplifies *ex ante* comparative advantage. We calibrate the model to the Chinese economy and find both the dampening force and amplifying force are important determinants of comparative advantage.

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6 Regression Tables

6.1 Motivation

Table 1: Export propensity and intensity: 2000-2006

	(1) export propensity	(2) export intensity
capital intensity	-0.247***	-0.247***
	(0.0140)	(0.0103)
year FE	Y	Y
R^2	0.793	0.648
No. of observations	700	700

Notes: Each observation is a year-industry (100 industries × 7 years) while industry is defined as "HO aggregate". Column (1) and (2) regress the export propensity and export intensity of each industries on capital intensity respectively, controlling year fixed effects. OLS is used. Standard errors clustered at each industries are reported in the parentheses. Significance levels are indicated by *, **, *** at 0.1, 0.05 and 0.01 respectively.

Table 2: Export product scope: 2000-2006

	(1) mean export product scope	(2) share of single product firm
capital intensity	-1.926***	0.0973***
	(0.159)	(0.00608)
year FE	Y	Y
R^2	0.269	0.505
No. of observations	700	700

Notes: Each observation is a year-industry (100 industries × 7 years) while industry is defined as "HO aggregate". Column (1) and (2) regress the average export product scope and share of single product exporter of each industry on capital intensity respectively, controlling year fixed effects. OLS is used. Standard errors clustered at each industries are reported in the parentheses. Significance levels are indicated by *, **, *** at 0.1, 0.05 and 0.01 respectively.

Table 3: Export product mix: 2000-2006

	(1) core product share	(2) m0/m1	(3) m0/m2	(4) mean Herfindhal	(5) mean Theil
capital intensity	0.0759***	0.332***	0.474***	0.0941***	0.240***
	(0.00354)	(0.0266)	(0.0316)	(0.00418)	(0.00935)
year FE	Y	Y	Y	Y	Y
R^2	0.584	0.291	0.328	0.607	0.639
No. of observations	700	700	700	700	700

Notes:Each observation is a year-industry (100 industries × 7 years) while industry is defined as "HO aggregate". Column (1) regress the average sales share of the core product on capital intensity, column (2) looks at the average log sales ratio of the core product to the second most important product while column (3) looks at the third. Column (5) and (6) regress the average Herfindhal index and Theil Index of export sales on capital intensity. Standard errors clustered at each industries are reported in the parentheses. Significance levels are indicated by *, **, *** at 0.1, 0.05 and 0.01 respectively.

Table 4: Comparative advantage and skewness of sales within industry 2000-2006

	(1) HHI	(2) Theil	(3) inter quartile range of log sales	(4) Ratio of largest vs 2nd largest firm	(5) Ratio of largest vs 3rd largest firm
capital intensity	-0.0358***	-2.270***	-1.633***	-0.217***	-0.265***
	(0.00621)	(0.0958)	(0.0997)	(0.0784)	(0.0853)
year FE	Y	Y	Y	Y	Y
R^2	0.0905	0.712	0.601	0.0296	0.0287
No. of observations	700	700	700	700	700

Notes: Robust standard errors clustered at industry level are reported in the parentheses. The constants are absorbed by the year fixed effects. Significance levels are indicated by *, **, *** at 0.1, 0.05 and 0.01 respectively.

6.2 Comparative advantage and multi-product

Table 5: Comparative Advantage and local ratio of core and second product-2003

	(1)	(2)	(3)	(4)
ln GDP	0.0157***	0.0138***	0.0147***	0.0146***
	(0.00381)	(0.00390)	(0.00399)	(0.00402)
ln supply potential		0.0127	0.0128	0.0136
		(0.00823)	(0.00863)	(0.00871)
CA1			0.0695***	
			(0.0244)	
CA2				0.00744*
				(0.00383)
Constant	-0.000603	-0.000367	0.000748	-0.000163
	(0.00808)	(0.00815)	(0.00868)	(0.00849)
Within R^2	0.000113	0.000124	0.000229	0.000165
No. of observations	92904	92904	85293	92637

Notes: Country specific random effects on firm-demanded data. Robustness standard error are reported in the parentheses. The number of observations is significantly less the following two tables since the dependent variable can only be constructed if the firms export at least two products at the destination. Significance levels are indicated by *, **, *** at 0.1, 0.05 and 0.01 respectively.

Table 6: Comparative Advantage and skewness of sales: 2003

	(1)	(2)	(3)	(4)
	Herfindahl	Herfindahl	Theil	Theil
CA1	0.0141***		0.0261***	
	(0.00271)		(0.00536)	
CA2		0.00208***		0.00381***
		(0.000488)		(0.00108)
country fixed effect	Y	Y	Y	Y
firm fixed effect	Y	Y	Y	Y
No. of observations	187693	202779	187693	202779

Notes: Standard error cluster at firm level. Significance levels are indicated by *, **, *** at 0.1, 0.05 and 0.01 respectively.

Table 7: Comparative Advantage and product scope: 2003

	(1) ln product #	(2) ln product #	(3) product #	(4) product #	(5) product #	(6) product #
CA1	-0.0340***		-0.146***		-0.113***	
	(0.00866)		(0.0226)		(0.00919)	
CA2		-0.00358**		-0.0340***		-0.0277***
		(0.00167)		(0.00674)		(0.00151)
Constant					1.547***	1.543***
					(0.00763)	(0.00734)
country fixed effect	Y	Y	Y	Y	Y	Y
firm fixed effect	Y	Y	Y	Y	Y	Y
R^2	0.451	0.454				
No. of observations	187693	202779	187694	202779	187694	202779

Notes: Column (1) and (2) use OLS estimation and ln(product count) as the dependent variable. Column (3) to (6) use product count as the dependent variable. Column (3) and (4) use Poisson estimation while (5) and (6) use negative binomial estimation. Significance levels are indicated by *, **, *** at 0.1, 0.05 and 0.01 respectively.

6.3 Quantifying Comparative advantage

Table 8: Import relative export and comparative advantage

dependent variable: import relative to export $\ln(\frac{IMP(z)}{EXP(z)})$	year 2000		year 2003		year 2006	
	(1)	(2)	(3)	(4)	(5)	(6)
$\ln(\frac{C_M(z)^k}{C_M^*(z)^k}\frac{C_D(z)^{k+2}}{C_D^*(z)^{k+2}})$	0.461***	0.404***	0.329***	0.257***	.247***	0.317***
	(.0460)	(.071)	(.0322)	(.032)	(.037)	(.087)
capital intensity z		0.143		.214		-.228
		(.115)		(.168)		(.245)
constant	1.86***	1.51***	0.968***	.540	.432***	0.837*
	(.227)	(.396)	(.141)	(.372)	(.156)	(.485)
Adjusted R^2	0.635	0.641	0.589	0.596	0.413	0.420
N	100	100	100	100	100	100

Notes: The dependent variable Robust standard errors are in the parentheses. Significance levels are indicated by *, **, *** at 0.1, 0.05 and 0.01 respectively.

7 Head-Reis Index

We estimate the trade freeness between China and the Rest of World using the Head-Reis Index (Head and Ries, 2001) as summarized by Head and Mayer (2014). According to Head and Mayer

Table 9: Comparing Measures of Comparative Advantage

regressor:	dependent variable: Comparative Advantage		
capital intensity	(1)	(2)	(3)
year	*ex ante* CA	extensive margin only	both margins
2000	-0.223	-0.285	-0.135
2003	-0.292	-0.428	-0.142
2006	-0.267	-0.451	-0.106

Notes: The table reports the coefficients of capital intensity out of regressions which regress the different measures of comparative advantage on capital intensity. The dependent variable of column (1) is the ex ante comparative advantage. For column (2), it is the measure of comparative advantage which only captures the extensive margin. For column (3), it is the measure which captures both the intensive and extensive margin. All coefficients are significant at 0.1% level.

(2014), if we assume symmetric trade costs $\rho_{ij} = \rho_{ji}$ and zero domestic trade costs, then

$$\rho_{ij} = \sqrt{\frac{X_{ij}X_{ji}}{X_{ii}X_{jj}}}$$

where X_{ij} is the aggregate export from region i to region j which follows the gravity equation.[25] So if let $i = China$ and $j = RoW$, we can infer the trade freeness between China and RoW. However, to implement this equation, we need data on local absorption X_{ii} and X_{jj}. These are information not available from our firm survey or customs data but available from the World Input Output Database (WIOD).[26] Local absorption is computed as the total output minus total export. We estimate trade freeness using the formula above for each sector. The summary statistics for the manufacturing sectors are displayed in Table 10.[27] The estimated average trade freeness between China and RoW increased from 0.043 in 2000 to 0.071 in 2006. If we assume that the trade elasticity $k = 3.43$ which is the median trade elasticity estimated for China by Broda et al (2006), then the implied iceberg trade costs $\tau = \rho^{-\frac{1}{k}}$ dropped from around 2.50 in 2000 to 2.16 in 2006. If we use the median trade elasticity 5.03 from the literature (Head and Mayer, 2014), the implied iceberg trade costs are lower but still falling over time.

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[25]Our model generates gravity equation for the sectoral trade flow which satisfies the general gravity equations classified by Head and Mayer(2014) even firms produce multiple products.

[26]We use the 2013 release at http://www.wiod.org/database/wiots13. The details of the data can be found in Timmer, Dietzenbacher, Los and Vries (2015).

[27]There are 15 sectors for goods and 20 sectors for services. Manufacturing sectors include all the 15 goods sector except the sector of "Agriculture, hunting, forestry and fishing" and the sector of " Mining and quarrying". For brevity, we do not report the trade freeness for the service sectors. But consistent with out expectation, the trade freeness for services between China and the RoW is much lower but also rising over time.

Table 10: Trade Costs between China and RoW: Head and Ries Index

| year | trade freeness ρ | | | implied iceberg trade costs τ | |
	average	min	max	k=3.43	k=5.03
2000	0.043	0.012	0.116	2.50	1.87
2001	0.045	0.012	0.129	2.47	1.85
2002	0.051	0.012	0.165	2.38	1.81
2003	0.058	0.013	0.218	2.29	1.76
2004	0.070	0.015	0.282	2.17	1.70
2005	0.073	0.015	0.323	2.15	1.68
2006	0.071	0.015	0.313	2.16	1.69

Notes: Trade freeness ρ is estimated using the Head and Ries (2001) method and the World Input Output Data(WIOD) for manufacturing industries. The columns titled "average", "min" and "max" are the average, minimum and maximum of the Head-Ries Index across 13 manufacturing sectors. The iceberg trade costs τ are inferred using the average trade freeness according to $\rho = \tau^{-k}$ where k is the trade elasticity.

8 Appendix: Proofs

8.1 Proof of Proposition 1

Proof. Let's define the share of exporters from home to foreign for industry z as

$$\chi(z) = (\frac{C_X(z)}{C_D(z)})^k$$

where $C_X(z)$ is the cut-off marginal cost of exporting to foreign and $\tau C_X(z) = C_D^*(z)$. So we have

$$\chi(z) = \rho(\frac{C_D^*(z)}{C_D(z)})^k.$$

Given that $\frac{\partial C_D(z)}{\partial z} > 0$ and $\frac{\partial C_D^*(z)}{\partial z} < 0$, it is easy to see that $\frac{\partial \chi(z)}{\partial z} < 0$. Similarly, we can prove that $\frac{\partial \chi^*(z)}{\partial z} > 0$.

The model predicts the trade flow from home to foreign in industry z is

$$EXP(z) = \frac{1}{2\gamma(k+2)C_M(z)^k} N_E(z) C_D^*(z)^{k+2} L^* \rho$$

On the other hand, the sales of industry z at home is

$$S(z) = \frac{1}{2\gamma(k+2)C_M(z)^k} N_E(z) C_D(z)^{k+2} L$$

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Thus the export intensity of industry z is given by

$$
\begin{aligned}
\lambda(z) &\equiv \frac{EXP(z)}{EXP(z) + S(z)} \\
&= \frac{L^* \rho}{L^* \rho + L(\frac{C_D(z)}{C_D^*(z)})^{k+2}} \\
&= \frac{L^* \rho}{L^* \rho + L\rho^{\frac{k+2}{k}} \chi(z)^{-\frac{k+2}{k}}}
\end{aligned}
$$

Since we have $\frac{\partial \chi(z)}{\partial z} < 0$, it is easy to see that $\frac{\partial \lambda(z)}{\partial \chi(z)} > 0$, thus $\frac{\partial \lambda(z)}{\partial z} < 0$. Similar results holds for foreign countries. $\square$

8.2 Proof of Proposition 2

Proof. The export product scope is given by $M_X(z, c)$. For firms that do export, i.e., their core marginal cost $c \leq C_X(z)$. Then $M_X(z, c) = \max\{m | v(m, c) \leq \frac{C_D^*(z)}{\tau}\} + 1$. Since $v(m, c) = \varpi^{-m} c$ and $\varpi \in (0, 1)$, we have

$$
M_X(z, c) = \max\{m | \ln \tau + \ln c + m \ln(\frac{1}{\varpi}) \leq \ln C_D^*(z)\} + 1
$$

Since $\frac{\partial C_D^*(z)}{\partial z} < 0$, for two industries $z\prime > z$, we have $C_D^*(z') < C_D^*(z)$, we should have

$$
M_X(z', c) \leq M_X(z, c)
$$

That is the export product scope is non-increasing with comparative disadvantage. $\square$

8.3 Proof of Proposition 3

Proof. The sales ratio of product m and m' for an exporter to foreign country in industry z can be written as

$$
\frac{r(z, v(m, c))}{r(z, v(m', c))} = \frac{C_D^*(z)^2 - (\tau \varpi^{-m} c)^2}{C_D^*(z)^2 - (\tau \varpi^{-m'} c)^2}
$$

Suppose m'>m, so product m is closer to core $\tau \varpi^{-m} c < \tau \varpi^{-m'} c$. Since $\frac{\partial C_D^*(z)}{\partial z} < 0$, it can be shown that

$$
\frac{\partial \frac{r(z, v(m, c))}{r(z, v(m', c))}}{\partial z} > 0
$$

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So the sales of home country's export to foreign country would be more concentrated in the more comparative disadvantage industries. □

8.4 Proof of Proposition 4

Proof. For any two single-product firms within industry z such that $c_1 < c_2$, the ratio of sales in domestic market is given by

$$\frac{r_d(z, c_1)}{r_d(z, c_2)} = \frac{C_D^2(z) - c_1^2}{C_D^2(z) - c_2^2}$$

Thus we have

$$\frac{\partial \frac{r_d(z, c_1)}{r_d(z, c_2)}}{\partial C_D(z)} = 2C_D(z)\frac{c_1^2 - c_2^2}{(C_D^2(z) - c_2^2)^2} < 0$$

That is in an industry with more competition, we should see sales more skewed towards the better performing firms. For multi-product firms, within industry z such that $c_1 < c_2$, the ratio of sales in domestic market is given by

$$\frac{R_d(z, c_1)}{R_d(z, c_2)} = \frac{\displaystyle\sum_{m=0}^{M_1-1} r_d(z, v(m, c_1))}{\displaystyle\sum_{m=0}^{M_2-1} r_d(z, v(m, c_2))}$$

$$= \frac{\displaystyle\sum_{m=0}^{M_1-1} \frac{L}{4\gamma}(C_D^2(z) - v(m, c_1)^2)}{\displaystyle\sum_{m=0}^{M_2-1} \frac{L}{4\gamma}(C_D^2(z) - v(m, c_2)^2)}$$

$$= \frac{C_D^2(z)M_1 - c_1^2 \frac{w^2}{w^{2M_1}} \frac{1 - w^{2M_1}}{1 - w^2}}{C_D^2(z)M_2 - c_2^2 \frac{w^2}{w^{2M_2}} \frac{1 - w^{2M_2}}{1 - w^2}}$$

where M_1 and M_2 are the product scope of the two firms respectively. Since $c_1 < c_2$, we have $M_2 \leq M_1$. If $M_1 = M_2$ are fixed, it is easy to see that $\frac{\partial \frac{R_d(z, c_1)}{R_d(z, c_2)}}{\partial C_D(z)} = 2C_D(z)M_1 \frac{w^2}{w^{2M_1}} \frac{1 - w^{2M_1}}{1 - w^2} \frac{c_1^2 - c_2^2}{(C_D^2(z) - c_2^2)^2} < 0$ as the single-product case. In case that $M_1 < M_2$, we cannot sign the partial derivative clearly. We claim that if the reallocation within firm is dominated by cross firm reallocation, our result is still true. First, we write

$$\frac{R_d(z, c_1)}{R_d(z, c_2)} = \frac{r_0 + r_1 + r_2 + \ldots + r_{M_1-1}}{r_0' + r_1' + r_2' + \ldots + r_{M_2-1}'}$$

where $r_0 = r_d(z, v(0, c_1)), r_1 = r_d(z, v(1, c_1)), ..., r_{M_1-1} = r_d(z, v(M_1 - 1, c_1))$;
$r_0' = r_d(z, v(0, c_2)), r_1' = r_d(z, v(1, c_2)), ..., r_{M_2-1}' = r_d(z, v(M_1 - 1, c_2))$. It can be further rearranged as

$$\frac{R_d(z, c_1)}{R_d(z, c_2)} = \frac{\sum_{i=0}^{M_2-1} r_i}{R_d(z, c_2)} + \frac{R_d(z, c_1) - \sum_{i=0}^{M_2-1} r_i}{R_d(z, c_1)} \frac{R_d(z, c_1)}{R_d(z, c_2)}$$

Moving the second term of the equation to the left, we have

$$(1 - \frac{R_d(z, c_1) - \sum_{i=0}^{M_2-1} r_i}{R_d(z, c_1)}) \frac{R_d(z, c_1)}{R_d(z, c_2)} = \frac{\sum_{i=0}^{M_2-1} r_i}{R_d(z, c_2)}$$

or

$$\frac{R_d(z, c_1)}{R_d(z, c_2)} \frac{\sum_{i=0}^{M_2-1} r_i}{R_d(z, c_1)} = \frac{\sum_{i=0}^{M_2-1} r_i}{\sum_{i=0}^{M_2-1} r_i'}.$$

Now we will make two claims. First, the term on the right hand side $\frac{\sum_{i=0}^{M_2-1} r_i}{R_d(z,c_2)}$, which captures

reallocation across firms, decreases with $C_D(z)$. This is because $\frac{\sum_{i=0}^{M_2-1} r_i}{R_d(z,c_2)} = \frac{C_D^2(z)M_2 - c_1^2 \frac{w^2}{w^{2M_2}} \frac{1-w^{2M_2}}{1-w^2}}{C_D^2(z)M_2 - c_2^2 \frac{w^2}{w^{2M_2}} \frac{1-w^{2M_2}}{1-w^2}}$ and

$$\frac{\partial \frac{\sum_{i=0}^{M_2-1} r_i}{R_d(z,c_2)}}{\partial z} = 2C_D(z)M_2 \frac{w^2}{w^{2M_2}} \frac{1-w^{2M_2}}{1-w^2} \frac{c_1^2 - c_2^2}{(C_D^2(z)M_2 - c_2^2 \frac{w^2}{w^{2M_2}} \frac{1-w^{2M_2}}{1-w^2})^2} < 0.$$

Second, the within-firm reallocation component $\frac{\sum_{i=0}^{M_2-1} r_i}{R_d(z,c_1)}$ decreases with $C_D(z)$. To show that, first we know that any $0 \le i \le M_1 - 1$,

$$\begin{aligned}
\frac{r_i}{R_d(z, c_1)} &= \frac{\frac{L}{4\gamma}[C_D^2(z) - (c_1 w^{-i})^2]}{\frac{L}{4\gamma}[C_D^2(z)M_1 - c_1^2 \frac{w^2}{w^{2M_1}} \frac{1-w^{2M_1}}{1-w^2}]} \\
&= \frac{C_D^2(z) - (c_1 w^{-i})^2}{C_D^2(z)M_1 - c_1^2 \frac{w^2}{w^{2M_1}} \frac{1-w^{2M_1}}{1-w^2}}
\end{aligned}$$

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so

$$\frac{\partial \frac{r_i}{R_d(z,c_2)}}{\partial C_D(z)} = \frac{2C_D c_1^2 (M_1 w^{-2i} - \frac{w^2}{w^{2M_1}} \frac{1-w^{2M_1}}{1-w^2})}{(C_D^2(z)M_1 - c_1^2 \frac{w^2}{w^{2M_1}} \frac{1-w^{2M_1}}{1-w^2})^2}$$

For $i = 0$, we have $M_1 w^{-2i} = M_1 < \frac{w^2}{w^{2M_1}} \frac{1-w^{2M_1}}{1-w^2} = \sum_{i=0}^{M_1-1} w^{-2i}$ given that $0 < w < 1$. Thus $\frac{\partial \frac{r_0}{R_d(z,c_1)}}{\partial C_D(z)} < 0$. For $i = M_1 - 1$, we have $M_1 w^{-2i} = M_1 w^{-2(M_1-1)} > \frac{w^2}{w^{2M_1}} \frac{1-w^{2M_1}}{1-w^2}$ since it is equivalent to $M_1 > \frac{1-w^{2M_1}}{1-w^2} = \sum_{i=0}^{M_1-1} w^{2i}$. Thus we $\frac{\partial \frac{r_{M_1-1}}{R_d(z,c_1)}}{\partial C_D(z)} > 0$. Since $M_1 w^{-2i}$ is decreasing with i, there exist a m^* such that for $i \leq m^*$, we have $\frac{\partial \frac{r_i}{R_d(z,c_1)}}{\partial C_D(z)} \leq 0$ and $i \geq m^*$, $\frac{\partial \frac{r_i}{R_d(z,c_1)}}{\partial C_D(z)} \geq 0$. Then for the first M_2 products, when $C_D(z)$ increases, their total share must decline.

So as long as within firm reallocation is smaller than the cross reallocation, our result is still true.

$\square$

8.5   Proof of Proposition 5

Proof. Comparing the relative average marginal costs between home and foreign under autarky and open economy, we have:

$$\frac{C_D(z)^T}{C_D(z)^{*T}} = \underbrace{\frac{C_D(z)^A}{C_D(z)^{*A}}}_{ex\ ante} \underbrace{\left[\frac{1 - \rho \frac{C_M^{*k}}{C_M^k}}{1 - \rho^* \frac{C_M^k}{C_M^{*k}}}\right]^{\frac{1}{k+2}}}_{amplifying} \tag{8.1}$$

where the first term is *ex ante* comparative advantage and the second term is only present when countries open to trade. It is easy to see that the second term increases with capital intensity z. Depending on the relative size of $C_M(z)$ and $C_M(z)^*$ and the trade freeness[28], the relationship between $\frac{C_D(z)^T}{C_D(z)^{*T}}$ and $\frac{C_D(z)^A}{C_D(z)^{*A}}$ is illustrated by Figure 12. Panel (a) is when $\rho^* C_M^{2k}$ is always larger than ρC_M^{*2k} so that $\frac{1-\rho \frac{C_M^{*k}}{C_M^k}}{1-\rho^* \frac{C_M^k}{C_M^{*k}}} > 1$, *vice versa* for panel (c). Panel (b) is when there exists an industry such that $\rho^* C_M^{2k} = \rho C_M^{*2k}$. In all 3 cases, the differences in the relative average marginal costs across industries enlarge under the trade equilibrium. Hence comparative advantage is amplified by the second component. $\square$

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[28]In the Appendix 10.1, we study how trade liberalization affect comparative advantage.
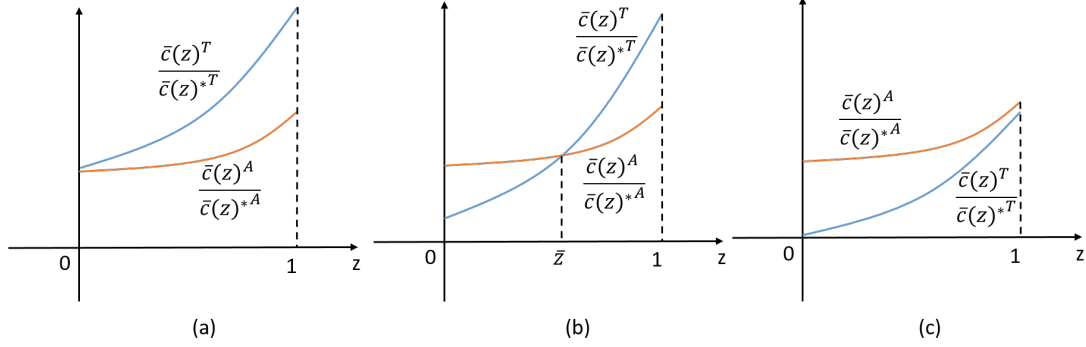
Figure 12: Relative average marginal costs: Autarky v.s Trade

8.6 Proof of Proposition 6

Proof. The relative quantify-based TFP between home and foreign under open economy can be rewritten as

$$
\frac{\overline{\Phi}(z)^T}{\overline{\Phi}^*(z)^T} = (\frac{L}{L^*}\frac{C_M^*(z)^k}{C_M(z)^k})^{\frac{1}{k+2}}(\frac{L}{L^*}\frac{C_M^*(z)^k}{C_M(z)^k})^{\frac{k+1}{k+2}}\frac{L^*}{L}\frac{\frac{L}{L^*}(\frac{C_D(z)}{C_D^*(z)})^{k+1}+\rho}{1+\rho^*\frac{L}{L^*}(\frac{C_D(z)}{C_D^*(z)})^{k+1}} \tag{8.2}
$$

$$
= \underbrace{\frac{\overline{\Phi}(z)^A}{\overline{\Phi}(z)^{*A}}}_{ex\ ante}\underbrace{(\frac{\overline{\Phi}(z)^A}{\overline{\Phi}(z)^{*A}})^{k+1}}_{amplifying}\underbrace{\frac{L^*}{L}\frac{\frac{L}{L^*}(\frac{C_D(z)}{C_D^*(z)})^{k+1}+\rho}{1+\rho^*\frac{L}{L^*}(\frac{C_D(z)}{C_D^*(z)})^{k+1}}}_{dampening}. \tag{8.3}
$$

Given that $k > 0$, it is obvious that the second component is amplifying the effect of the first component, the ex ante relative quantify-based TFP between home and foreign. For the third component, if we define as $f(z) \equiv \frac{L^*}{L}\frac{\frac{L}{L^*}(\frac{C_D(z)}{C_D^*(z)})^{k+1}+\rho}{1+\rho^*\frac{L}{L^*}(\frac{C_D(z)}{C_D^*(z)})^{k+1}}$, we have

$$
\frac{\partial f(z)}{\partial z} = \frac{(1-\rho\rho^*)(k+1)(\frac{C_D(z)}{C_D^*(z)})^k}{(1+\rho^*\frac{L}{L^*}(\frac{C_D(z)}{C_D^*(z)})^{k+1})^2}\frac{\partial(\frac{C_D(z)}{C_D^*(z)})}{\partial z} > 0.
$$

Given our assumptions that $\frac{\partial C_M(z)}{\partial z} > 0$ and $\frac{\partial C_M^*(z)}{\partial z} < 0$, we have $\frac{\partial(\frac{\overline{\Phi}(z)^A}{\overline{\Phi}(z)^{*A}})}{\partial z} < 0$. That is to say the third component is negatively correlated with the first two components. Hence, it is dampening the ex ante comparative advantage. □

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8.7 Proof of Proposition 7

Proof. According to proposition 1, the export intensity

$$
\begin{aligned}
\lambda(z) &= \frac{L^*\rho}{L^*\rho + L\rho^{\frac{k+2}{k}}\chi(z)^{-\frac{k+2}{k}}} \\
&= \frac{1}{1 + \frac{L}{L^*}\rho^{\frac{2}{k}}\chi(z)^{-\frac{k+2}{k}}}
\end{aligned}
$$

Thus we can infer the relative market size $\frac{L}{L^*}$ as:

$$
\frac{L}{L^*} = \frac{1-\lambda(z)}{\lambda(z)}\frac{\chi(z)^{\frac{k+2}{k}}}{\rho^{\frac{2}{k}}}. \tag{8.4}
$$

Again, according to Proposition 1, the export probability of export in each industry is given by

$$
\begin{aligned}
\chi(z) &= \rho(\frac{C_D^*(z)}{C_D(z)})^k \tag{8.5} \\
&= \rho(\frac{L}{L^*}\frac{C_M^*(z)^k - \rho^*C_M(z)^k}{C_M(z)^k - \rho C_M^*(z)^k})^{\frac{k}{k+2}} \tag{8.6}
\end{aligned}
$$

Immediately, ratio of average costs between home country and the foreign is given by

$$
\frac{C_D^*(z)}{C_D(z)} = (\frac{\chi(z)}{\rho})^{1/k} \tag{8.7}
$$

which is the measure of comparative advantage. Moreover the relative upper bound of marginal cost can solved be solved out of equation (8.6) as

$$
\frac{C_M^*(z)^k}{C_M(z)^k} = \frac{\rho^* + \frac{L^*}{L}(\frac{\chi_z}{\rho})^{\frac{k+2}{k}}}{1 + \frac{L^*}{L}\rho^{-\frac{2}{k}}\chi(z)^{\frac{k+2}{k}}}, \tag{8.8}
$$

substituting the relative size of $\frac{L}{L^*}$ using equation (8.4), it can be computed. Then the endogenous component of the comparative advantage given by $\frac{1-\rho\frac{C_M^*{}^k}{C_M{}^k}}{1-\rho^*\frac{C_M{}^k}{C_M^*{}^k}}$ is also known. Finally, the *ex ante* comparative advantage $\frac{C_D(z)^{*A}}{C_D(z)^A} = (\frac{L}{L^*}\frac{C_M^*(z)^k}{C_M(z)^k})^{1/(k+2)}$ can be inferred.

The *ex ante* components of comparative advantage are the same for the two measures of

comparative advantage since

$$\frac{\overline{\Phi}(z)^A}{\overline{\Phi}(z)^{*A}} = (\frac{L}{L^*}\frac{C_M^*(z)^k}{C_M(z)^k})^{1/(k+2)} \tag{8.9}$$

$$= \frac{C_D(z)^{*A}}{C_D(z)^A} \tag{8.10}$$

Thus the way to quantify $\frac{\overline{\Phi}(z)^A}{\overline{\Phi}(z)^{*A}}$ is the same as quantifying $\frac{C_D(z)^{*A}}{C_D(z)^A}$ which we have shown in Proposition 7. Then the amplifying component $(\frac{\overline{\Phi}(z)^A}{\overline{\Phi}(z)^{*A}})^{k+1}$ is also known. On the other hand, the dampening component is given by

$$\frac{L^*}{L}\frac{\frac{L}{L^*}(\frac{C_D(z)}{C_D^*(z)})^{k+1} + \rho}{1 + \rho^*\frac{L}{L^*}(\frac{C_D(z)}{C_D^*(z)})^{k+1}} = \frac{(\frac{\chi(z)}{\rho})^{-\frac{k+1}{k}} + \rho^{1+\frac{2}{k}}\frac{\lambda(z)}{1-\lambda(z)}\chi(z)^{-\frac{k+2}{k}}}{1 + \rho^*\frac{1-\lambda(z)}{\lambda(z)}\frac{\chi(z)^{\frac{k+2}{k}}}{\rho^{\frac{2}{k}}}(\frac{\chi(z)}{\rho})^{-\frac{k+1}{k}}}$$

$$= (\frac{\chi(z)}{\rho})^{-\frac{k+1}{k}}\frac{1 + \rho^{\frac{1}{k}}\frac{\lambda(z)}{1-\lambda(z)}\chi(z)^{-\frac{1}{k}}}{1 + \rho^*\frac{1-\lambda(z)}{\lambda(z)}\chi(z)^{\frac{1}{k}}\rho^{\frac{k-1}{k}}}$$

which can also be inferred as long as we know $\{\rho, \rho^*, k\}$ and observe $\{\chi(z), \lambda(z)\}$. □

9 Additional Figures for motivating facts

Our benchmark results only used data from year 2003. Here we also show results using data for 2000 and 2006.

Our stylized fact 3 states that export product mix is more skewed in capital intensive industries. Other than the measures of skewness used in the main text, we present results using other measures. Figure 17 plots the average sales share of the core product. Core product is defined as the product that makes up most of the total export sales. As evident from the figure, the average share of sales from the core product is higher for the capital intensive industries. Figure 19 plot the average of the log-ratios between the sales of the core product to the third most important product. Figure 20 plots the average Herfindhal Index of exporters for each industry.

Similarly, we also include additional evidence for stylized fact 4 using alternative measures. They are: Figure 21 which plots the Herfindahl Index of domestic sales across firms; Figure 23 which plots the log ratios of domestic sales between the largest firm and 2nd largest firm; and Figure 24 plots the log ratios of domestic sales between the largest firm and 3rd largest firm.
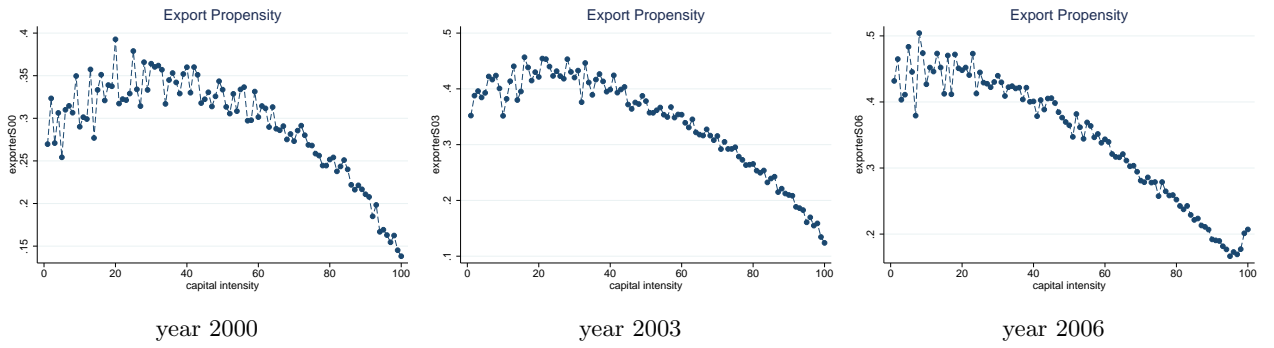
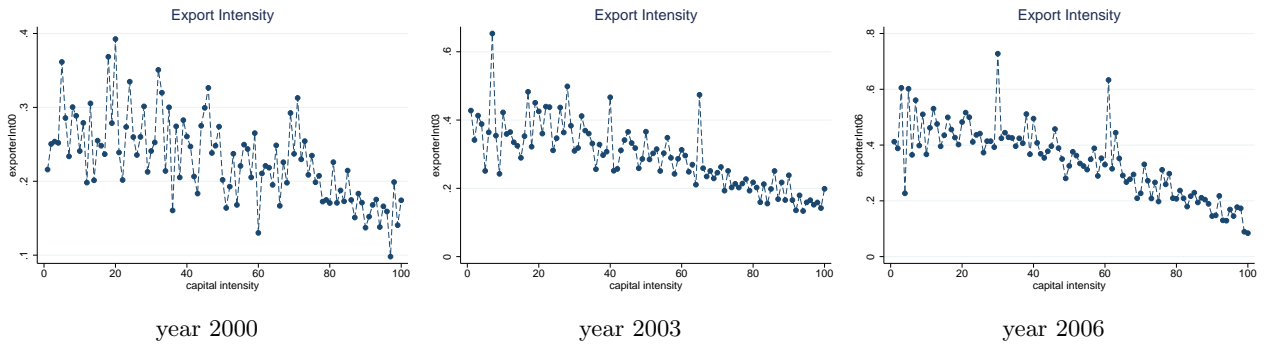Figure 13: Export propensity by industry

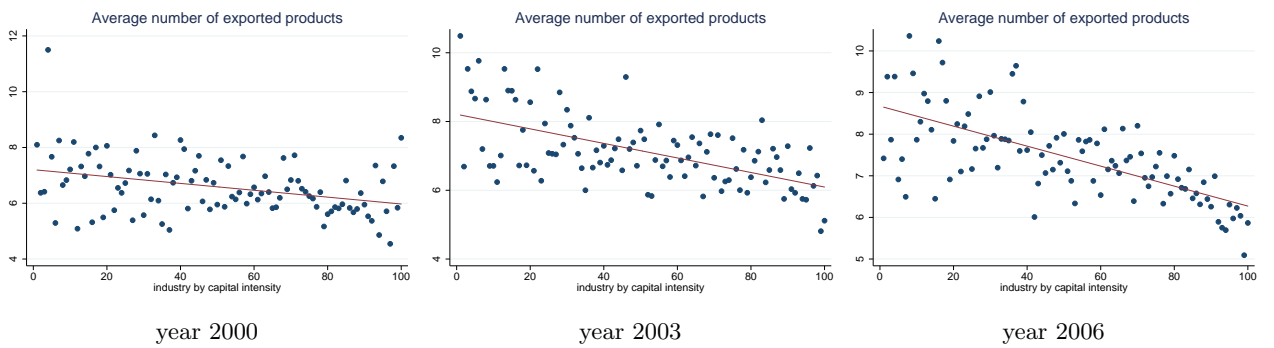Figure 14: Export intensity by industry

Figure 15: Number of products exported

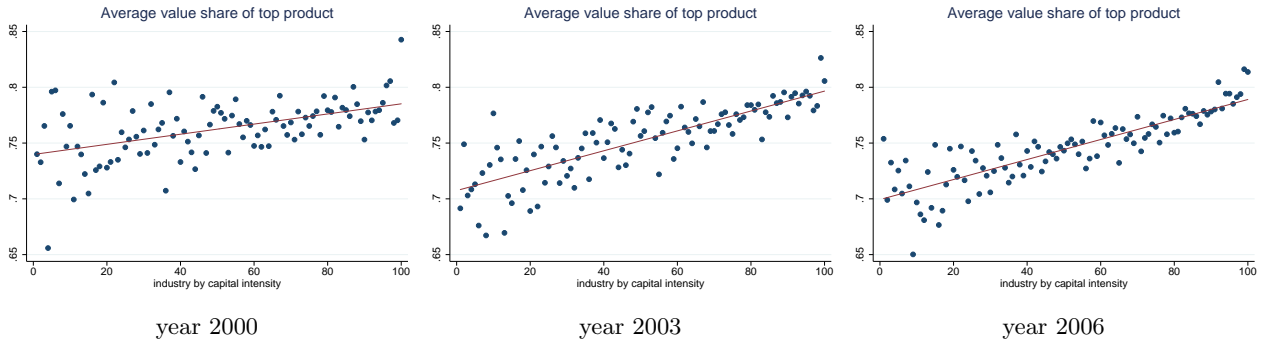Figure 16: Share of single product exporter

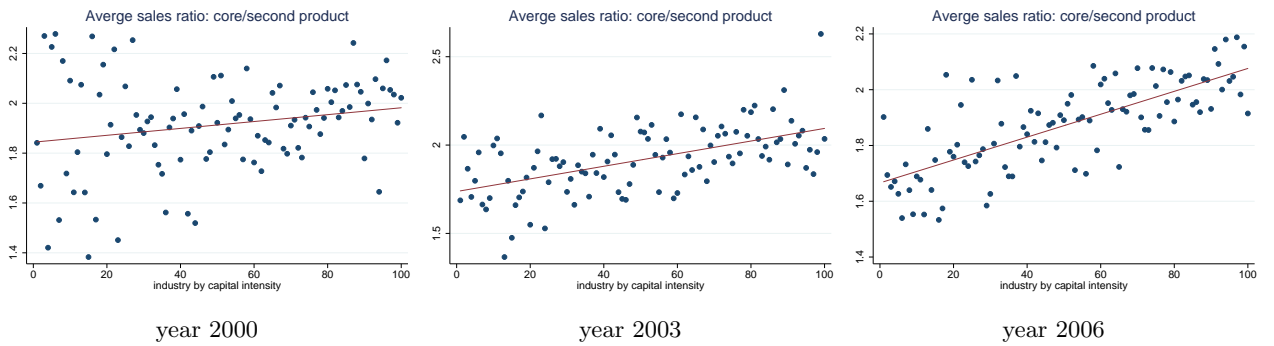Figure 17: Average value share of core product for exporters

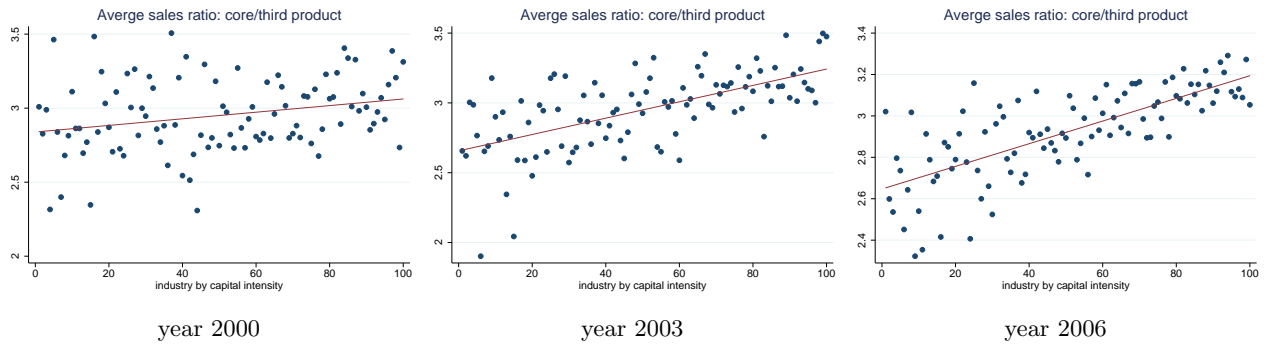Figure 18: ln(core product export sales/second product export sales)

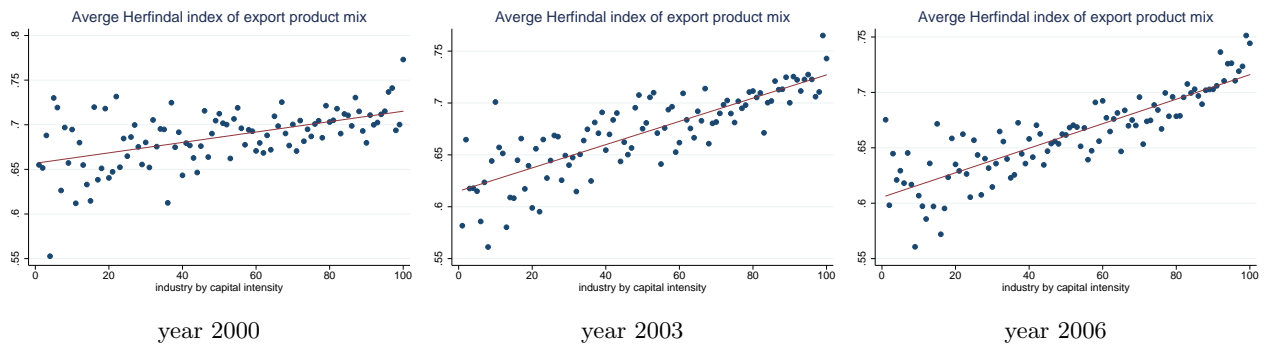Figure 19: ln(core product export sales/third product export sales)

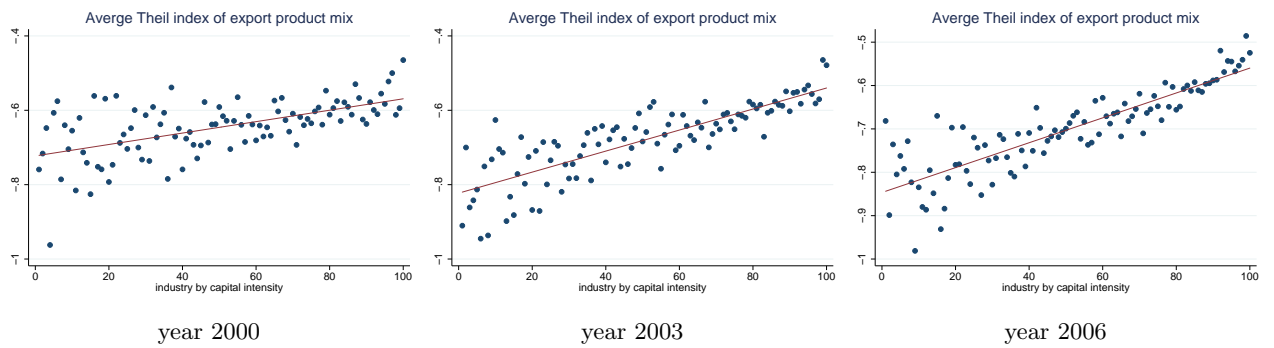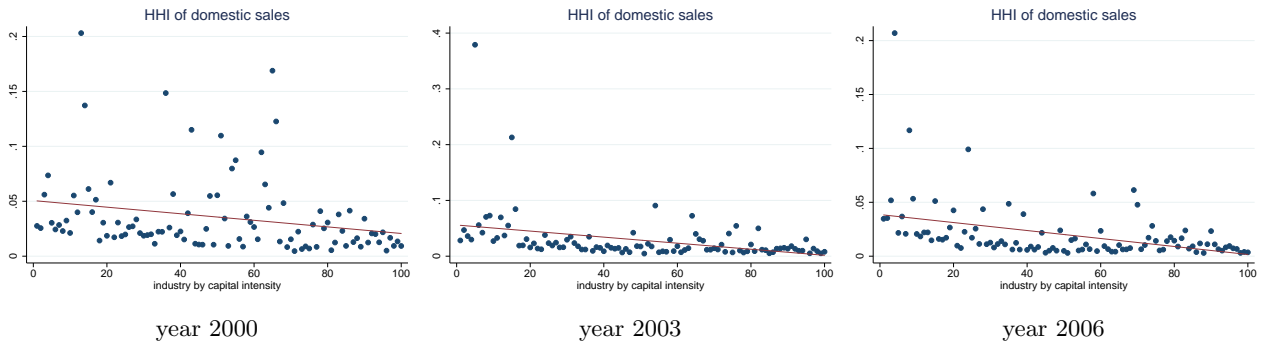Figure 20: Average Herfindhal Index of export across products

Figure 21: Average Theil index of export across products

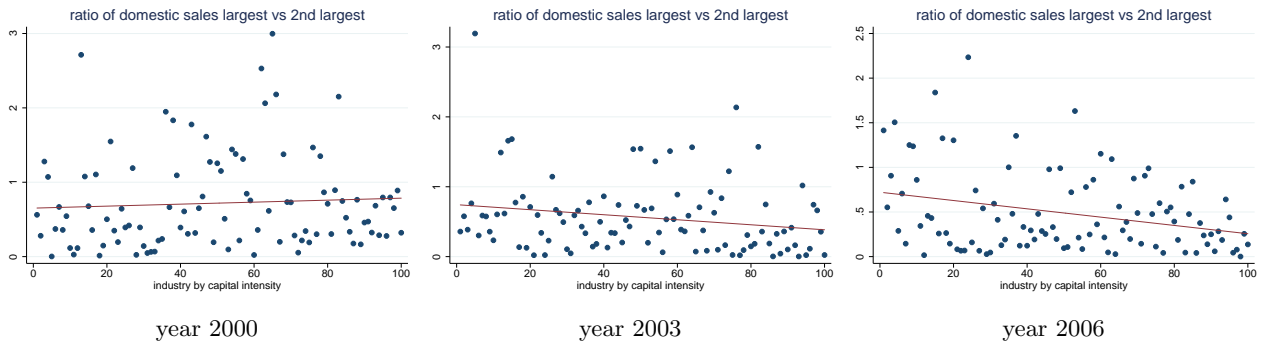Figure 22: Herfidhal index of domestic sales across firms

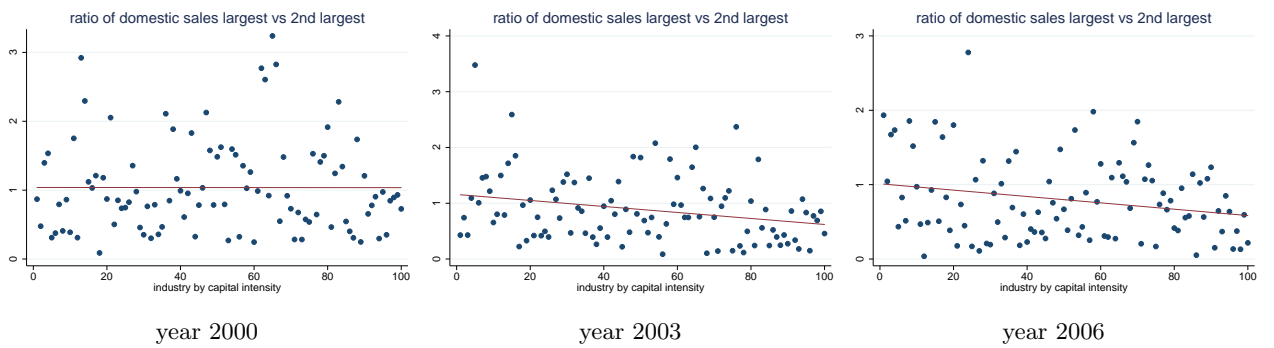Figure 23: ln ratios of domestic sales: largest/2nd largest

Figure 24: ln ratios of domestic sales: largest/3rd largest

10   Additional theoretical results

10.1   Trade liberalization and comparative advantage

This appendix section studies how trade liberalization affects comparative advantage. The first scenario is a unilateral liberalization: suppose the iceberg trade costs of exporting to the foreign country drops and ρ increases. We find that

$$\frac{\partial(\frac{\bar{c}(z)}{\bar{c}(z)^*})^{k+2}}{\partial\rho} = \frac{L^*}{L}\frac{-C_M^*(z)^k}{C_M^*(z)^k - \rho^* C_M(z)^k} < 0$$

which implies that:

$$\begin{aligned}
\frac{\partial\frac{\bar{c}(z)}{\bar{c}(z)^*}}{\partial\rho} &= \frac{\partial\frac{\bar{c}(z)}{\bar{c}(z)^*}}{\partial(\frac{\bar{c}(z)}{\bar{c}(z)^*})^{k+2}}\frac{\partial(\frac{\bar{c}(z)}{\bar{c}(z)^*})^{k+2}}{\partial\rho} \\
&= \frac{(\frac{\bar{c}(z)}{\bar{c}(z)^*})^{-(k+1)}}{k+2}\frac{\partial(\frac{\bar{c}(z)}{\bar{c}(z)^*})^{k+2}}{\partial\rho} < 0
\end{aligned}$$

That is home country is getting more competitive in every industry. The intuition is that the expected profit for domestic firms increases as ρ increases. This leads to more entry and induce tougher competition at home. This case is illustrated in panel (a) of Figure 25.

The other scenario is multilateral liberalization. Suppose $\rho^* = \rho$ and both countries cut their trade barriers with the same magnitude. In this case, we find that

$$\frac{\partial(\frac{\bar{c}(z)}{\bar{c}(z)^*})^{k+2}}{\partial\rho} = \frac{L^*}{L}\frac{C_M(z)^{2k} - C_M^*(z)^{2k}}{(C_M^*(z)^k - \rho^* C_M(z)^k)^2}$$

which implies

$$\frac{\partial\frac{\bar{c}(z)}{\bar{c}(z)^*}}{\partial\rho} = \frac{(\frac{\bar{c}(z)}{\bar{c}(z)^*})^{-(k+1)}}{k+2}\frac{L^*}{L}\frac{C_M(z)^{2k} - C_M^*(z)^{2k}}{(C_M^*(z)^k - \rho^* C_M(z)^k)^2}$$

The sign of $\frac{\partial\frac{\bar{c}(z)}{\bar{c}(z)^*}}{\partial\rho}$ depends on the absolute advantage $C_M(z)$ and $C_M^*(z)$. If $C_M(z) = C_M^*(z)$, then $\frac{\partial\frac{\bar{c}(z)}{\bar{c}(z)^*}}{\partial\rho} = 0$ trade liberalization would not affect comparative advantage. On the other hand, if $C_M(z) > C_M^*(z)$, then $\frac{\partial\frac{\bar{c}(z)}{\bar{c}(z)^*}}{\partial\rho} > 0$ and home country would become relatively more competitive. And if $\exists\ \bar{z} \in (0,1)$ such that $C_M(\bar{z}) = C_M^*(\bar{z})$ and for $z \in [0,\bar{z})$ we have $C_M(z) < C_M^*(z)$, for $z \in (\bar{z},1]$ we have $C_M(z) > C_M^*(z)$. Then we have

$$\frac{\partial \frac{\bar{c}(z)}{\bar{c}(z)^*}}{\partial \rho} \begin{cases} < 0, \ z \in [0, \bar{z}) \\ = 0, \ z = \bar{z} \\ > 0, \ z \in (\bar{z}, 1] \end{cases}.$$

This implies that for the industries $z \in [0, \bar{z})$ that home has comparative advantage, the comparative advantage is even greater since $\frac{\partial \frac{\bar{c}(z)}{\bar{c}(z)^*}}{\partial \rho} < 0$ and home cost advantage is getting higher. For $z \in (\bar{z}, 1]$, the comparative advantage of the foreign country is getting larger since $\frac{\partial \frac{\bar{c}(z)}{\bar{c}(z)^*}}{\partial \rho} > 0$. Thus comparative advantage is further amplified by multilateral trade liberalization. This is illustrated by panel (b) of Figure 25.
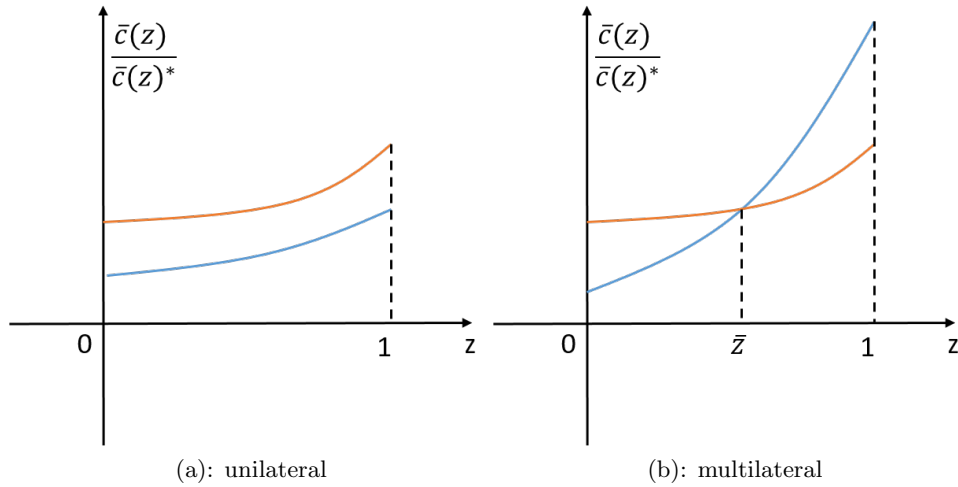

(a): unilateral $\qquad\qquad$ (b): multilateral

Figure 25: Trade liberalization and comparative advantage

10.2 Comparative advantage and with constant mark-up model

This appendix section shows how to do decomposition in the Melitz typed model *a la* Bernard, Redding and Schott (2007). Suppose the demand is given by the following quasi-CES preference[29]

$$U = q_0^c - \gamma \int_0^1 \ln Q(z) dz$$

Under such a preference, solving the consumer's problem we have

---

[29]We get rid of the income effect to simplify the algebra. Huang et al (2017) include the income effect and arrive at similar results.

$$q_i^c = -\gamma \frac{p_i^{-\sigma}}{P(z)^{1-\sigma}}$$

where $P(z) = (\int_{i \in \Omega(z)} p_i(z)^{1-\sigma} di)^{\frac{1}{1-\sigma}}$ is the price index such that $P(z)Q(z) = \int p_i(z)q_i^c(z)di = -\gamma$.

On the supply side, we take the standard Melitz(2003) set up in the case of open economy: the entry cost is f_E and fixed cost of serving domestic market and foreign market is f_d and f_x respectively. On top of that, we assume that firms draw their marginal cost from the Pareto distribution $G(z,c) = (\frac{c}{C_M(z)})^k$, where $C_M(z)$ is the upper bound of the marginal cost at home. Given the market demand faced by firm at home and foreign and iceberg cost assumption:

$$q_i(z) = -\gamma L \frac{p_i^{-\sigma}}{P(z)^{1-\sigma}}$$

$$q_i^*(z) = -\gamma L^* \frac{p_i^{-\sigma}}{P^*(z)^{1-\sigma}}$$

the optimal pricing for each market is given by

$$p_d(z,c) = \frac{\sigma}{\sigma - 1} c$$

$$p_x(z,c) = \frac{\sigma}{\sigma - 1} \tau c$$

Then firm's profit functions for each market are given by

$$\pi_d(z,c) = \frac{r_d(z,c)}{\sigma} - f_d = \frac{-\gamma L}{\sigma} (\frac{p_d(z,c)}{P(z)})^{1-\sigma} - f_d$$

$$\pi_x(z,c) = \frac{r_x(z,c)}{\sigma} - f_d = \frac{-\gamma L^*}{\sigma} (\frac{p_x(z,c)}{P^*(z)})^{1-\sigma} - f_x$$

Then the zero-profit conditions implies that we have

$$\frac{-\gamma L}{\sigma} (\frac{\frac{\sigma}{\sigma-1} c_D(z)}{P(z)})^{1-\sigma} = f_d$$

$$\frac{-\gamma L^*}{\sigma} (\frac{\frac{\sigma}{\sigma-1} \tau c_X(z)}{P^*(z)})^{1-\sigma} = f_x$$

where $c_D(z)$ and $c_X(z)$ are the cut-off marginal costs. Dividing the two equations, we have

$$\frac{c_X(z)}{c_D(z)} = \frac{P^*(z)}{\tau P(z)}(\frac{f_d L^*}{f_x L})^{\frac{1}{\sigma-1}} \tag{10.11}$$

To determine how $\frac{c_X(z)}{c_D(z)}$ varies across industries, we need to know how $\frac{P^*(z)}{P(z)}$ varies with z. To do that, we follow Bernard, Redding and Schott (2007) to consider two extreme cases: free trade and autarky. Then the costly trade case would fall between.

In the case of free trade, every surviving firms from every country export. The number of varieties and the price charged by each firm in each market is the same. Thus the price index $P(z) = (\int_{i\in\Omega(z)} p_i(z)^{1-\sigma}di)^{\frac{1}{1-\sigma}} = P(z)^* = (\int_{i\in\Omega^*(z)} p_i(z)^{1-\sigma}di)^{\frac{1}{1-\sigma}}$ under free trade and $\frac{P^*(z)}{P(z)}$ is constant.

The case of autarky is a bit more complicated. Under autarky, $P(z) = (\int_{i\in\Omega(z)} p_i(z)^{1-\sigma}di)^{\frac{1}{1-\sigma}} = M_z^{\frac{1}{1-\sigma}}p_d(\widehat{c}_d(z))$ where M_z is the domestic firm mass and $\widehat{c}_d(z)^{-1} = (\frac{1}{G(c_D(z))}\int_0^{c_D(z)} c^{1-\sigma}g(c)dc)^{\frac{1}{\sigma-1}}$ is the average marginal cost of industry z. Similarly $P(z)^* = M_z^{*\frac{1}{1-\sigma}}p_d^*(\widehat{c}_d^*(z))$. For the firm mass, using the market clearing condition, we have $M_z = \frac{P(z)Q(z)}{r(\widehat{c}_d(z))} = \frac{-\gamma}{r(\widehat{c}_d(z))}$. Under the CES demand, we have $\frac{r(\widehat{c}_d(z))}{r(c_D(z))} = (\frac{c_D(z)}{\widehat{c}_d(z)})^{\sigma-1}$. Thus combined the zero profit condition, we have $r(\widehat{c}_d(z)) = r(c_D(z))(\frac{c_D(z)}{\widehat{c}_d(z)})^{\sigma-1} = \sigma f_d(\frac{c_D(z)}{\widehat{c}_d(z)})^{\sigma-1}$. This implies that the firm mass is given by

$$M_z = \frac{-\gamma}{\sigma f_d}(\frac{\widehat{c}_d(z)}{c_D(z)})^{\sigma-1}.$$

So the price index under autarky at home is given by

$$P(z) = [\frac{-\gamma}{\sigma f_d}(\frac{\widehat{c}_d(z)}{c_D(z)})^{\sigma-1}]^{\frac{1}{1-\sigma}}\frac{\sigma}{\sigma-1}\widehat{c}_d(z)$$

If we impose the Pareto distribution assumption, we have $\frac{\widehat{c}_d(z)}{c_D(z)} = (\frac{k-\sigma+1}{k})^{\frac{1}{\sigma-1}}$. Then the price index

$$P(z) = [\frac{-\gamma}{\sigma f_d}\frac{k-\sigma+1}{k}]^{\frac{1}{1-\sigma}}\frac{\sigma}{\sigma-1}(\frac{k-\sigma+1}{k})^{\frac{1}{\sigma-1}}c_D(z)$$

varies one-to-one with $c_D(z)$. To determine $c_D(z)$, we use the free entry condition under autarky which says the probability of survival times the expected profit equals to the fixed cost of entry:

$$G(c_D(z))\pi(\widehat{c}_d(z)) = f_e$$

where $G(c_D(z)) = (\frac{c_D(z)}{C_M(z)})^k$. And since $\pi(\hat{c}_d(z)) = \frac{r(\hat{c}_d(z))}{\sigma} = \frac{r(c_D(z))}{\sigma}(\frac{c_D(z)}{\hat{c}_d(z)})^{\sigma-1} = f_d(\frac{k}{k-\sigma+1})^{\frac{1}{\sigma-1}}$,

it is easy to find that

$$c_D(z) = (\frac{f_e}{f_d}\frac{k-\sigma+1}{k})^{1/k}C_M(z)$$

which varies one-to-one with the upper bound of the marginal cost. Thus under autarky, we have

$$\frac{P^*(z)}{P(z)} = \frac{C_M^*(z)}{C_M(z)}.$$

which declines with z given our assumption that $\frac{\partial C_M(z)}{\partial z} > 0$ and $\frac{\partial C_M^*(z)}{\partial z} < 0$.

That is to say, if we have $z\prime > z$, then we have

$$
\begin{aligned}
\frac{c_X(z)}{c_D(z)} &= \frac{P^*(z)}{\tau P(z)}(\frac{f_d L^*}{f_x L})^{\frac{1}{\sigma-1}} \\
&= \frac{P^*(z')}{\tau P(z')}(\frac{f_d L^*}{f_x L})^{\frac{1}{\sigma-1}} \\
&= \frac{c_X(z')}{c_D(z')}
\end{aligned}
$$

under free trade. And

$$\frac{c_X(z)}{c_D(z)} > \frac{c_X(z')}{c_D(z')}$$

under autarky. Then given the continuity of trade costs, it must be the case that under costly trade, we have

$$\frac{\partial \chi(z)}{\partial z} < 0$$

where $\chi(z) = (\frac{c_X(z)}{c_D(z)})^k$ is the probability of export. Similarly, we prove that $\frac{\partial \chi(z)^*}{\partial z} > 0$ for foreign.

Combining the zero profit condition and free entry condition under costly trade, we have

$$f_d \int_0^{C_D(z)} (\frac{\pi_x(z,c)}{\pi_x(z,C_D(z))} - 1)dG(z,c) + f_x \int_0^{C_X(z)} (\frac{\pi_x(z,c)}{\pi_x(z,C_X(z))} - 1)dG(z,c) = f_e$$

for home country. It can be simplified as

$$f_d C_D(z)^k + f_x C_X(z)^k = \frac{k - \sigma + 1}{\sigma - 1} f_e C_M(z)^k$$

Similarly for foreign country, we have

$$f_d C_D^*(z)^k + f_x C_X^*(z)^k = \frac{k - \sigma + 1}{\sigma - 1} f_E C_M^*(z)^k.$$

These two equations implies

$$\frac{f_D C_D^*(z)^k + f_X C_X^*(z)^k}{f_D C_D(z)^k + f_X C_X(z)^k} = \frac{C_M^*(z)^k}{C_M(z)^k}$$

or

$$\left(\frac{C_D^*(z)}{C_D(z)}\right)^k = \underbrace{\frac{C_M^*(z)^k}{C_M(z)^k}}_{exogenous} \underbrace{\frac{1 + \frac{f_X}{f_D}\chi(z)}{1 + \frac{f_X}{f_D}\chi(z)^*}}_{endogenous}.$$

where the exogenous and endogenous components are positively correlated.