

Pathology or Revelation? – The Public Good Index

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Abstract:

This paper sets out from a discussion of the well-known fact that the PGI violates the axiom of local monotonicity (LM). It argues that cases of nonmonotonicity indicate properties of the underlying decision situations which cannot be brought to light by the more popular power measures, i.e., the Banzhaf index and the Shapley-Shubik index, that satisfy LM. The discussion proposes that we can constrain the set of games such that LM also holds for the PGI. A discussion of causality follows. It suggests that the nonmonotonicity can be the result of framing the decision problem in a particular way and perhaps even ask the “wrong question.” Correspondingly, the PGI can be interpreted as an indicator. The probabilistic relationship of Banzhaf index and PGI identifies the factor which is responsible for the formal difference between the two measures and therefore for the violation of LM that characterizes the PGI, but not the Banzhaf.

- 1. The PGI introduction, 2. The pathology, 3. The revelation, 4. Causality and power,**
- 5. Measure or indicator?, 6. On the relationship of Banzhaf index and PGI**

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1. The PGI introduction

This paper focuses on the representation of causality in collective decision making by means of power and power measures and discusses the question whether the Public Good Index (PGI) is a suitable instrument for this representation. The answer this question we relate the power measure to the NESS concept of causality. However, the answer also depends on whether we interpret the PGI as measure or as an indicator.

Section 2 discusses the well-known fact that the PGI violates the axiom of local monotonicity (LM). In section 3, we argue that cases of nonmonotonicity indicate properties of the underlying decision situations which cannot be brought to light by the more popular power measures, i.e., the Banzhaf index and the Shapley-Shubik index, that satisfy LM. The discussion proposes that we can constrain the set of games representing decision situations such that LM also holds for the PGI. This might be a helpful instrument for the design of voting bodies. The discussion of causality in section 4 suggests that the nonmonotonicity can be the result of framing the decision problem in a particular way and perhaps even ask the “wrong question.” However, the core of this section is dedicated to connecting power and responsibility in the case of collective decision making and collective action, i.e., the cause for an outcome cannot directly be assigned to a particular individual agent.

Based on the discussion in the previous sections, section 5 points out that the PGI can be interpreted as an indicator and thus even serve as a valuable instrument in cases where at least some scholars raise serious doubts whether it can be applied as a measure. To conclude, section 6 looks into the probabilistic relationship of Banzhaf index and PGI as elaborated independently by Widgrén (2002) and Brueckner (2002) that identifies the factor which is responsible for the formal difference between the two measures. Can we identify this factor as the cause for the violation of LM that characterizes the PGI, but not the Banzhaf? Can we see from the properties of this factor whether the PGI will indicate a violation for a particular game, or not? However, these are questions that have not been answered as yet.

The normalized Banzhaf index of player i equals the number of coalitions S that have i as a swing player such that S is a winning coalition and $S \setminus \{i\}$ is a losing coalition for all $S \subset N$ if N is the set of all players of game v . For normalization this number is divided by the total number of swing positions that characterize the game v . The PGI differs from the Banzhaf index inasmuch as only minimum winning coalitions (MWCs) are considered. S is a MWC coalition if $S \setminus \{i\}$ is a losing one, for all $i \in S$, i.e., all players of a MWC coalition have a swing position. The PGI of player i , h_i , counts the number of MWCs that have i as a member

and divides this sum by the sum of all swing positions the players have in all MWCs of the game. If m_i is the number of MWCs that have i as a member then i ' PGI value is

$$(1) \quad h_i = \frac{m_i}{\sum_{i \in N} m_i}$$

The following analysis is based on this power measure.

2. The pathology

In 1978, when I first applied the PGI to the study of the power distribution in the Finnish Parliament, I concluded that facing the violation of LM “causes doubt” concerning the validity of this measure. Obviously, I found the “index of Banzhaf-Coleman type” which I used as an alternative “more adequate in the context of this analysis” (Holler 1978: 33).

In their article "Postulates and Paradoxes of Relative Voting Power - A Critical Review", Dan Felsenthal and Moshé Machover (1995: 211) write that “it seems intuitively obvious that if $w_i \leq w_j$ then every voter j has at least as much voting power as voter i , because any contribution that i can make to the passage of a resolution can be equalled or bettered by j .” They conclude that “any reasonable power index” should be required to satisfy local monotonicity, i.e., LM. Even more distinctly, they argue that any a priori measure of power that violates LM is ‘pathological’ and should be disqualified as a valid yardstick for measuring power (Felsenthal and Machover 1998: 221ff). This argument has been repeated again and again when it comes to the evaluation (and application) of the PGI and the Deegan-Packel index.²

A notorious example to illustrate the nonmonotonicity of the PGI is the voting game $v^\circ = (51; 35, 20, 15, 15, 15)$. The corresponding PGI is

$$h(v^\circ) = \left(\frac{4}{15}, \frac{2}{15}, \frac{3}{15}, \frac{3}{15}, \frac{3}{15} \right),$$

² The Deegan-Packel index was introduced in Deegan and Packel (1979). For a recent discussion of this measure, taking a priori unions into account, see Alonso-Mejide et al. (2011).

indicating a violation of LM in the resulting distribution of a priori voting power. (The corresponding Deegan-Packel index, $\rho(v^\circ) = (18/60, 9/60, 11/60, 11/60, 11/60)$, also shows a violation of LM.)

The application of power indices is motivated by the widely shared “hypothesis” that the vote distribution is a poor proxy for a prior voting power. If this is the case, does it make sense to evaluate a power measure by means of a property that refers to the vote distribution as suggested by LM? Of course, our intuition supports LM. However, if we could trust our intuition, do we need the highly sophisticated power measures at all?³

3. The revelation

It has been argued that a larger voter j can be less welcome to join a (non-winning) proto-coalition than a smaller voter i .⁴ The intuitive argument is the following. Let's assume a voting game $v^* = (51; 45, 20, 20, 15)$ and players 2 and 3 form a proto-coalition $S = \{2, 3\}$. The losing coalition S can be “transformed” into a winning coalition if either player 1 or player 4 or both join S . However, if player 1 joins, either individually or together with 4, then neither player 2 nor player 3 is critical to the winning of a majority, i.e., in the coalition $\{1, 2, 3\}$ neither 2 nor 3 is a swinger. If voting power refers to a swing position – and this is, with some modification, the kernel of all standard power measures – and players are interested in power, then it seems likely that players 2 and 3 prefer the “smaller” voter 4 to join S to form a winning coalition. This story tells us that it could well be that a larger player is not always welcome to form a winning coalition if a smaller one does the same job. But does this mean that only minimum winning coalitions will form? Empirical evidence speaks against this conclusion. However, I have repeatedly argued that if (nonminimal) winning coalitions with surplus players form then this is due to luck or ideology (i.e. preferences) and should not be taken into consideration when it comes to represent a priori voting power.⁵ But there are perhaps more straightforward arguments in favor of MWCs and the application of the PGI.

In Holler and Napel (2004a, 2004b), we argued that the PGI shows nonmonotonicity with respect to the vote distribution (and thus confirms that the measure does not satisfy LM) if the game is not decisive, as the above weighted voting game $v^\circ = (51; 35, 20, 15, 15, 15)$, or improper (for an example, see section 4 below) and therefore indicates that perhaps we should

³ See Holler (1997) and Holler and Nurmi (2010) for this argument.

⁴ For a discussion of coalition formation, see e.g. Hardin (1976), Hart and Kurz (1984), Holler (2011), Miller (1984), and Riker (1962).

⁵ See Holler (1982) and Holler and Packel (1983). See also Widgrén (2002).

worry about the design of the decision situation. The more popular power measures, i.e., the Shapley-Shubik index or the Banzhaf index satisfy LM and thus do not indicate any particularity if the game is neither decisive nor proper. Interestingly, these measures also show a violation of LM if we consider a priori unions and the equal probability of permutations and coalitions, respectively, does no longer apply.

The concept of a priori unions or pre-coalition is rather crude because it implies that certain coalitions will not form at all, i.e., have a probability of zero of forming.⁶ Note since the PGI considers MWC only, this is formally equivalent to put a zero weight on coalitions that have surplus players. Is this the (“technical”) reason why the PGI may show nonmonotonicity? We will come back to this question in section 6 below.

Instead of accepting the violation of monotonicity, we may ask for what decision situations the PGI guarantees monotonic results - this may help to design adequate voting bodies. Obviously, the PGI satisfies LM for unanimity games, dictator games and symmetric games. The latter are games that give equal power to each voter; in fact, unanimity games are a subset of symmetric games. In these cases the PGI is identical with the normalized Banzhaf index.

In Holler et al. (2001), the authors analyze alternative constraints on the number of players and other properties of the decision situations. For example, it is obvious that local monotonicity will not be violated by any of the known power measures, including PGI, if there are n voters and $n-2$ voters are dummies. It is, however, less obvious that local monotonicity is also satisfied for the PGI if one constrains the set of games so that there are only $n-4$ dummies. A hypothesis that needs further research is that the PGI does not show nonmonotonicity if the voting game is decisive and proper and the number of decision makers is lower than 6. (Perhaps this result also holds for a larger number of decision makers but I do not know of any proof?) The idea of restricting the set of games such that LM applies for PGI has been further elaborated in Alonso-Mejide and Holler (2009) in the form of “weighted monotonicity of power”. It seems that these considerations are relevant for all power indices if we drop the equal probability assumption and, for example, take the possibility of a priori unions into account.

The elaboration of various power measures and their discussion is meant to increase our understanding of power in collectivities and also to be of help in the design of voting bodies. A relatively new application of these measures results from their formal equivalence

⁶ See Alonso-Mejide and Bowles (2005) for examples of voting games with a priori unions and Alonso-Mejide and Holler (2009) and Holler and Nurmi (1010) for a discussion.

with representations of causality in collective decision making. Given this, it seems a short step to equate power and responsibility.

4. Causality and power

The specification of causality in the case of collective decision making with respect to the individual agent cannot be derived from the action and the result as both are determined by the collectivity. They have to be traced back to decision making and, in general, the decision making process. However, collective decision making has a quality that substantially differs from individual decision making. For instance, an agent may support his favored alternative by voting for another alternative or by not voting at all. Nurmi (1999, 2006) contains a collection of such “paradoxes”. These paradoxes tell us that we cannot derive the contribution of an individual to a particular collective action from the individual’s voting behavior. Trivially, a vote is not a contribution, but a decision. Resources such as power, money, etc. are potential contributions and causality might be traced back to them if a collective action results. As a consequence causality follows even from votes that do not support the collective action. This is reflected by everyday language that simply states that the Parliament has decided when in fact a decision was made by a majority smaller than 100 per cent. But how can we allocate causality if it is not derived from decisions?

Alternatively, we may assume in what follows that the vote (even in committees) is secret and we do not know who voted “yes” or “no”. Moreover, in general, there are more than two alternatives and the fact that a voter votes “yes” for A in a last pairwise voting only means that he/she prefers A to B or does not want to abstain, but this vote does not tell us why and how alternatives C, D, etc. were excluded.⁷ That is causality (and responsibility) do not derive from a particular known voting result that indicates who said “yes” and who said “no” and therefore differs from the approach discussed in Felsenthal and Machover (2009).

Imagine a five-person committee $N = \{1,2,3,4,5\}$ that makes a choice between the two alternatives x und y .⁸ The voting rule specifies that x is chosen if either (i) 1 votes for x , or (ii) at least three of the players 2-5 vote for x . Let’s assume all individuals vote for x . What can be said about causality? Clearly, this is a case of over-determination and the allocation of causation is not straightforward. The action of agent 1 is a member of only one minimally sufficient condition, i.e., decisive set, while the actions of each of the other four members are

⁷ See “The Fatal Vote: Berlin versus Bonn” (Leininger 1993) for an illustration.

⁸ The rest of this section derives from Holler (2011).

in three decisive sets each. If we take the membership in decisive sets as a proxy for causation, and standardize such that the shares of causation adds up to one, then vector

$$h^\circ = \left(\frac{1}{13}, \frac{3}{13}, \frac{3}{13}, \frac{3}{13}, \frac{3}{13} \right)$$

represents the degrees of causation. Braham and van Hees (2009: 334), who introduced and discussed the above case, conclude that “this is a questionable allocation of causality.” They add that “by focusing on minimally sufficient conditions, the measure ignores the fact that anything that players 2-5 can do to achieve x , player 1 can do, and in fact more – he can do it alone.”

Let’s review the above example. Imagine that x stands for polluting a lake. Now the lake is polluted, and all five members of N are under suspicion of having polluted. Then h° implies that the share of causation for 1 is significantly smaller than the shares of causation of each of the other four members of N . If responsibility and perhaps punishment follow from causation then the allocation h° seems highly pathological. As a consequence Braham and van Hees propose to apply the weak NESS instead of the strong one, i.e., not to refer to decisive sets, but to consider sufficient sets instead and count how often an element i of N is a necessary element of a sufficient set (i.e., a NESS). Taking care of an adequate standardization so that the shares add up to 1, we get the following allocation of causation:

$$b^\circ = \left(\frac{11}{23}, \frac{3}{23}, \frac{3}{23}, \frac{3}{23}, \frac{3}{23} \right).$$

The result expressed by b° looks much more convincing than the result proposed by h° , doesn’t it? Note that the b -measure and h -measure correspond to the Banzhaf index and the PGI, respectively, and can be calculated accordingly.

So far the numerical results propose the weak NESS test and thus the application of the normalized Banzhaf index. However, what happened to alternative y ? If y represents “no pollution” then the set of decisive sets consists of all subsets of N that are formed of the actions of agent 1 and the actions of two out of agents 2, 3, 4, and 5. Thus, the actions of 1 are members of six decisive sets while the actions of 2, 3, 4, and 5 are members of three decisive sets each. The corresponding shares are given by the vector

$$h^* = \left(\frac{1}{3}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6} \right)$$

Obviously, h^* looks much more convincing than h° and the critical interpretation of Braham and van Hees does no longer apply: agent 1 cannot bring about y on its own, but can cooperate with six different pairs of two other agents to achieve this goal.

Note that the actions (votes) bringing about x represent an improper game – two “winning” subsets can exist at the same time - while the determination of y can be described by a proper game. However, if there are only two alternatives x and y then “not x ” necessarily implies y , irrespective of whether the (social) result is determined by voting or by polluting. The h -values indicate that it seems to matter what issue we analyze and what questions we raise while the Banzhaf index with respect to y is identical to the one for x : $b^\circ = b^*$.

Whether we should apply h and b , or a third alternative, to measure causation seems still an open question, and this paper will not answer this question. However, if we want to relate responsibility to power then the nonmonotonicity, i.e., the violation of LM, that represents the strong NESS test of the PGI is quite a challenge: If the collective choice is made through voting then it is not guaranteed that a voter with a larger share of votes has at least as much responsibility for the collectively determined outcome as a voter with a smaller share.

To conclude, the PGI and thus the strong NESS concept may produce results that are counter-intuitive at first glance. However, in some decision situations they seem to reveal more about the power structure and corresponding causality allocation than the Banzhaf index and the corresponding weak NESS concept.

5. Measure or indicator?

From the example above we can learn that nonmonotonicity might indicate that perhaps we asked the wrong question: Is the responsibility with respect to keeping the lake clean or is it with polluting the lake? Both alternatives may imply the sharing of the costs of cleaning it? Of course, there is no quantitative answer to this question, but the quantification by the index showed us that there might be a problem with the specification of the game model. A possible answer of whether the PGI represents a pathology or not, might be found in this quality-quantity duality: the use of quantity measures to indicate qualitative properties of (voting) games. Whether a game is improper or non-decisive is not a matter of degree.

Indicators show red lights or make strange noises when an event happens that has some meaning in a particular context. This does not necessarily mean that the corresponding indicator functions as a measure, but often it does and when it does it summarizes the measured values in the form of signals. What is a relevant and appropriate signal of course depends on the context and the recipient. Red lights are not very helpful for blind people. What are the relevant and appropriate signals that correspond to power measures? What are the problems that should be uncovered and perhaps even be solved? What are the properties a power measure has to satisfy when it should serve as a signal? These are questions that I cannot answer in a systematic way right now, so I better leave them for future research.

6. On the relationship of Banzhaf index and PGI

Widgrén (2002) proved the following linear relationship that relates the normalized Banzhaf index (β_i) and the PGI.⁹

$$\beta_i = (1 - \pi) h_i + \pi \varepsilon_i$$

$$\text{where } \varepsilon_i = \frac{\bar{c}_i}{\sum_{i \in N} \bar{c}_i} \text{ and } \pi = \frac{\sum_{i \in N} \bar{c}_i}{\sum_{i \in N} c_i}$$

Here, c_i represents the number of (crucial) coalitions that contain player i as a swing player and \bar{c}_i represents the number of coalitions which are crucial with respect to i , i.e., have a swing player i , but are not minimum winning. Loosely speaking, the coalitions represented by \bar{c}_i are the source of the difference between the normalized Banzhaf index, β_i , and the PGI, h_i . Can we identify the corresponding factors in (2) as the cause for the violation of LM that characterizes the PGI, but not the Banzhaf? Can we see from the properties of this factor whether the PGI will indicate a violation for a particular game, or not?

However, these are questions that have not been answered so far, but it is immediate from (2) that the PGI satisfies LM for unanimity games, dictator games and symmetric games. For these games $\pi = 0$ and the PGI equals the normalized Banzhaf index (which satisfies LM for all voting games).

⁹ Widgrén uses the symbols θ_i for the PGI and C_i for the set of crucial coalitions that contain i as a swing player.

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