

Power Distribution in the Weimar Reichstag in 1919-1933

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Abstract:

We present an analysis of the distribution of voting power in the Reichstag of the Weimar Republic based on the outcome of the nine general elections in the period 1919–1933. The paper contains a brief description of the political and electoral system of the Weimar Republic and a characterization of the main political actors and their political views. The power distributions are evaluated by means of the Banzhaf index and three new indices which take into account the parties' preferences to coalesce. A model is constructed to evaluate the parties' preferences with reference to the closeness of the ideological positions in a one-dimensional political space.

1. Introduction

The Weimar Republic during 1919-1933 represents an example of a parliamentary system based on proportional representation. It was an essential component of the political environment among others that brought Adolf Hitler into power and it is not exaggerated to claim that it thereby was of outstanding relevance for the course of world history. Therefore, to analyze its power structure is more than just another theoretical exercise. However, our work will focus on the theoretical, and not the historical dimension of the problem. Still, we hope to learn about both political history and the properties of formal voting power analysis.

In what follows we use two types of indices to evaluate the power distribution in the Weimar parliament, the Reichstag. For a first evaluation we will use the Banzhaf index. It will be applied in order to measure the a priori voting power of factions within the parliament represented by a so-called voting body. The index assumes that each faction can form a coalition with every other faction and restrictions or preferences do not exist. The second evaluation, following a proposition in Aleskerov (2006), is based on measures that take into account the agents' preferences in coalition formation. Here, of course, ideological closeness and cleavages matter.

To begin with, in Section 2, we give an outline of the socio-economic performance of Weimar Germany. Section 3 contains the characterization of the political system, including brief descriptions of the major parties of the Weimar Republic and of the relevant electoral rules. In Section 4, we introduce a theoretical voting model and the Banzhaf index, and apply these tools to evaluate the a priori voting power of the factions that resulted from the nine elections to the Reichstag in the period of 1919 to 1933. Variations of the Banzhaf index that take care of the factions' preferences to form coalitions with other factions, were presented in Section 5, to be

applied to analyze the power distributions in the Reichstag of January 19, 1919 and March 5, 1933, the “first and the last” of the Weimar Republic, and to give an interpretation. Section 6 concludes. Results of the seven Assemblies between these two cornerstones can be found in the Appendix.

2. The Weimar Germany 1919-1933: A brief history of socio-economic performance

In November 1918, as the result of the German Revolution, the parliamentary republic has replaced the imperial form of government in each state of the Reich. On November 8, 1918 Reichskanzler Max von Baden proclaimed the resignation of Kaiser Wilhelm II, who acknowledged his resignation on November 9, 1918 and emigrated to the Kingdom of the Netherlands. On this very same day Max von Baden handed over his office as a Reichskanzler to Friedrich Ebert, the leader of the Social Democratic Party of Germany (SPD), who became Reichspräsident on February 11, 1919. The republic was proclaimed on November 9, 1918 by Philipp Scheidemann a leading member of the SPD and later prime minister (Reichsministerpräsident) from February through June 1919, as a reaction to Karl Bebel’s attempt to proclaim a “socialist republic” the same day.

The cooperation of the SPD under the leadership of Friedrich Ebert and the Independent Social Democratic Party of Germany (USPD), a pacifist and leftist SPD split-off since 1917, under the leadership of Hugo Haase formed a temporary government (called the Council of People's Commissioners) and acknowledged the military defeat of Germany in the First World War. On November 11, 1918 the warfare stopped.

Other political parties entered the arena such as the Catholic Centre Party (Zentrum). On January 1919, the Communists formally separated from the USPD to create the Communist Party of Germany (KPD). Elections for the National Assembly were scheduled as early as January 19, 1919 also to prevent further revolutionary actions and perhaps even civil war. However, the main goal of these elections was to set the stage for giving the new republic a constitution. On August 11, 1919 the Weimar Constitution was approved.

During its history the Weimar Republic suffered from budget deficits and high rates of unemployment, essentially caused by the consequences of World War I and the harsh conditions of the Versailles Treaty 1919. The years 1920-1923 saw eruption of a devastating hyperinflation that did not only ruin the substantial parts of the economy but also destabilized the social structure, especially the middle classes.⁴ In July 1920, the German Mark – USD exchange rate was 39.5. In February 1923, this ratio was already 28 000, and on November 20, 1923 this ratio

⁴ Seidl (2010) argues that this was to a large extent the consequence of the failure of a planned capital levy.

skyrocketed to 4 200 000 000 000 for 1 \$. At this date a new currency, called Rentenmark, was introduced that implied a substantial depreciation. This at first was no legal tender, however it was widely accepted. As a result, the hyperinflation was curbed. Given the gained monetary stability, on August 30, 1924 the legal tender Reichsmark was introduced.⁵ However, this did not prevent the budget squeeze and unemployment that characterized the following decade.

The strengths of the Weimer Republic lied in a social welfare arrangements: the 8-hours working day for workers, the system of workers' participation, especially the Works Councils, improved social services (e.g., 40% of federal government expenditures went on war-related pensions), and a social insurance policy with a rather extensive coverage (Potthoff and Miller (2006)). Since the time of Bismarck, its pillars were the health insurance, old age pensions insurance, and work accidents insurance. On October 1, 1927 an unemployment insurance was introduced; unemployment was no longer seen as a problem that the individual worker could (or should) solve, but as a social problem and, in modern parlance, a macroeconomic issue. During the first two years, it seems that the major challenge to this insurance was not the economic recession, as this was foreseen, but the seasonable unemployment that resulted from extremely cold winters (see Carroll (1929)). In any case, unemployment was severe, especially since the beginning of the Great Depression in October 1929 which spread quickly over Europe. The leading classes, i.e., many industrialists, generals and even high-ranking state bureaucrats and leading members of the legal profession, supported the Republic even less than before. A huge socio-economic cleavage between the middle-class and the workers and an economy that relied on short-term foreign loans undermined the legitimacy of the democratic regime. Other weaknesses of the Republic were its political fragmentation, the increasing influence of authoritarian-minded bureaucrats, the absence of charismatic leaders, the disappointment about the performance of liberal parliamentary democracy, and the lack of *decisionism*.⁶

3. Political system

The following summary should illustrate the background of our power analysis. It is not meant to give an accurate representation of the historical facts, nor a profound sketch of the governmental system of the Weimar Republic.

3.1 Political institutions

⁵ An alternative date given in the literature is October 11, 1924.

⁶ The term used by Carl Schmitt means the ability to make decision in the state of emergency and the absence of any regulations. (Schmitt (2007)).

According to the Weimar Constitution, the major political institutions of the Reich consisted of the Reichstag, the Reichsrat, the Reichspräsident and the Reichsregierung.

The Reichstag, the national parliament, was the main legislative body. It was meant to serve a four-year period. Its members were elected in a general, equal, immediate and secret election. Elections were held according to the principles of representative democracy. A decision-making procedure in the Reichstag was a simple majority rule, unless the constitution prescribed a different quorum. For constitutional changes, a 2/3 majority rule applied.

In the Reichsrat the German states were represented by members of their respective governments. With some qualifications, the share of votes was proportional to the population shares of the various states. For example, in 1919, Prussia had 25 representatives, Bavaria 7, while Hamburg controlled 1 seat of the 63 seats of this chamber. Each state had a minimum of 1 seat, while an individual state could not have more than 40 per cent of seats. The latter rule gave a cap to the representation of Prussia. The Reichsrat could veto a law that was decided by the Reichstag and the law then needed a 2/3 majority in the Reichstag to pass the legislation. Moreover, the Reichsrat could initiate a referendum if the Reichstag had decided a modification of the Constitution. On the whole, however, the Reichsrat was not a very powerful institution. Symptomatically, sessions of the Reichsrat were convoked by the Reichsregierung, the government.

The Reichspräsident, the president of the Reich, was elected by a popular vote. The term of office lasted seven years and reelection was permitted. The Reichspräsident had the right to dismiss and to appoint the Reichskanzler, the head of the government. He had also the right to dissolve the Reichstag and call for election of a new one, but only once for the same reason. (This of course invites interpretation and a potential for conflicts.) In 1932, President Paul von Hindenburg made use of this privilege when the Reichstag did not decide as expected (and did not produce any majority government). Special attention should be drawn to the Article 48 that provided the President with the power to *govern* by Notverordnungen (presidential emergency decrees) that served as substitutes to regular law-making by the Reichstag. In 1931, the Reichstag passed 34 laws while the President issued 44 Notverordnungen. The right to Notverordnung empowered the President to take the measures to reestablish law and order, if necessary using armed force. In the pursuit of this aim he might even suspend the civil rights of citizens.

The Reichsregierung, the government, consisted of the Reichskanzler, i.e. the Chancellor, and the Reichsminister, i.e. the heads of the ministries. In order to exercise mandates, the government required the confidence of the Reichstag. However, on March 27, 1930 Präsident

Hindenburg appointed Heinrich Brüning as Chancellor, and later Brüning's successors Franz von Papen, Kurt von Schleicher and Adolf Hitler, without the majority support of the Reichstag, but based on the Notverordnung's capacity of the Reichspräsident.

3.2 Electoral system for the Reichstag

Although the power of the Reichstag is constrained by the presidential power, it was still considered the nucleolus of legislation and its division into parliamentary fractions and seat shares was considered important. The electoral law of 27 April, 1920 provided for the election of deputies on a party list system. It divided the Reich into 35 electoral districts (Wahlkreise),⁷ and these districts were combined into 17 groups (Wahlkreisverbände). Votes were cast for the party list at the level of a Wahlkreis and the particular candidates were elected in the order they appeared on the list.

For each sixty thousand votes in a electoral district a party was entitled to a seat in the Reichstag. The rest of the votes were allocated via the Wahlkreisverbände. A party could claim an additional deputy if its remainder in the electoral district was more than thirty thousand votes. Obviously, the total number of seats in the Reichstag varied depending on the turnout of votes; it varied between 423 and 647.

3.3 Political parties

The political system of the Weimar Republic was based on parties. *Table 1* gives a list of those of parties which possessed at least one seat in one of the Reichstags. In the following we will use the English circumscription of the German name plus the German abbreviation (or identification) and give short descriptions of the major parties.

| | |
|--|--|
| 1. Bayerischer Bauernbund (BBB) | Bavarian Farmers' Union |
| 2. Bayrische Volkspartei (BVP) | Bavarian People's Party |
| 3. Christliche Volkspartei (CVP) | Christian People's Party |
| 4. Christlich-Föderalistische Reichswahlliste (CFR) | Christian Federalist National Electoral List |
| 5. Christlich-Nationale Bauern und Landvolkpartei (CNBLVP) | Christian National Farmers' and Rural People's Party |
| 6. Christlich-Sozialer Volksdienst (CVP) | Christian Social People's Service |
| 7. Deutsche Bauernpartei (DBP) | German Farmers' Party |
| 8. Deutsche Demokratische Partei (DDP) | German Democratic Party |

⁷ Reichswahlgesetz. Vom 27. April 1920. Retrieved from: http://www.documentarchiv.de/wr/1920/reichswahlgesetz_1920.html.

| | |
|--|---|
| 9. Deutsche Nationale Volkspartei (DNVP) | German-National People's Party |
| 10. Deutsche Soziale Partei (DSP) | German Social Party |
| 11. Deutsche Staatspartei (DStP) | German State Party |
| 12. Deutsche Volkischen Freiheits-Partei (DVFP) | German Racial Freedom Party |
| 13. Deutsche Volkspartei (DVP) | German People's Party |
| 14. Deutsche Zentrumspartei (Zentrum) | German Centre Party |
| 15. Deutsch-Hannoversche Partei (DHP) | German Hanoverian Party |
| 16. DVP-CZVD-DBP-DHP | Bloc DVP-CZVD-DBP-DHP |
| 17. Kampffront Schwarz-Weiss-Rot (KFSWR) | Battlefront Black-White-Red ⁸ |
| 18. Kommunistische Partei Deutschlands (KPD) | Communist Party of Germany |
| 19. Landbund | Rural Union |
| 20. Landliste | Rural List |
| 21. Landvolkpartei (Landvolk) | Rural People's Party |
| 22. Nationalsozialistische Deutsche Arbeiterpartei (NSDAP) | National Socialist German Workers' Party (Nazi Party) |
| 23. Nationalsozialistische Freiheitsbewegung (NSFB) | National Socialist Freedom Movement (Nazi Party) |
| 24. Reichspartei des deutschen Mittelstandes (Wirtschaftspartei) | National Party of the German Middle Classes |
| 25. Sächsisches Landvolk (SLV) | Saxon Rural People |
| 26. Sozialdemokratische Partei Deutschlands (SPD) | Social Democratic Party of Germany |
| 27. Unabhängige Sozialdemokratische Partei Deutschlands (USPD) | Independent Social Democratic Party of Germany |
| 28. Volksrechtspartei (VP) | People's Law Party |
| 29. Wirtschaftspartei des Deutschen Mittelstandes (WPDM) | Economic Party of the German Middle Classes |

Table 1. Political parties represented in the Weimar Reichstag.

3.3.1 The Social Democratic Party of Germany (SPD)

On May 23, 1863, the Allgemeine Deutsche Arbeiterverein (ADAV, General German Workers' Association) was founded mainly by Ferdinand Lassalle. Six years later, on August 7, 1869 the Social Democratic Workers' Party of Germany (SDAP) was founded by Wilhelm Liebknecht and August Bebel. In 1875 the Socialist Workers' Party of Germany (SAPD), the union of both forerunners (ADAV and SDAP), was founded as a socialist party in legal opposition to the Kaiserreich and exists as the oldest European party till nowadays. It received its final and still valid name Social-Democratic Party of Germany (SPD) in 1890.

In April 1917 the Independent Social Democratic Party of Germany (USPD), including Rosa Luxemburg, Karl Liebknecht and Hugo Haase, split off from the majority SPD (for some

⁸ "Stahlhelm, Bund der Frontsoldaten" plus German-National People's Party (DNVP)

time labeled MSPD). The members of the USPD were no longer prepared to support the German World War I policy. On January 1, 1919 many activists of the USPD became founding members of the KDP, a bitter rival to the SPD as well as to the remaining rest of the USPD.

The Social Democrats had been represented in the Reichstag since 1871. In 1917 they joined the 'Inter-factional Committee' in the Reichstag with the Catholic Centre Party and the Progressive People's Party. As a result of the National Assembly election on January 19, 1919, the SPD formed so called Weimar coalition with the catholic Centre Party and the left-liberal German Democratic Party (DDP), the most committed to Germany's new parliamentary system. The Social Democrats accepted the Treaty of Versailles signed in France on June 28, 1919.

After the Kapp-Putsch, a right-wing uprising, on March 13-17, 1920 and a general strike against this putsch,⁹ both the SPD and especially the DDP lost a considerable share of their votes on the Reichstag election on June 6, 1920. The Social Democrats were held responsible of all the troubles of the new republic by both left- and right-wing extremists. The SPD abandoned the Weimar coalition and hence let the Centre Party (Zentrum) to play a major role. The party spent the whole period of a relative stability for the Weimar Republic, the Golden Era 1923–1928, in opposition, providing moderate support for the government policy in the Reichstag.

A SPD comeback happened in 1928 when Chancellor Hermann Müller (of the SPD) formed the government based on 'the Grand Coalition' of the Social Democrats, the Centre Party, the German Democratic Party and the German People's Party. This coalition fell apart on the onset of the Great Depression. The main reason was the constant disputes between the Social Democrats and German People's Party over budget, growing inflation and unemployment.

In 1930, when the Grand coalition collapsed, the government was functioning through Notverordnungen stipulated in Article 48 of the Constitution. The reason for tolerance towards such practice was an informal coalition of parties against 'the Nazi threat'. After the installation of the Nazi government, on March 23, 1933, when the Enabling Act¹⁰ submitted by the Nazi government was passed by the Reichstag, all SPD deputies who had not yet been arrested voted against the bill.

3.3.2 The Communist Party of Germany (Kommunistische Partei Deutschlands – KPD)

⁹ On March 13, with the help of the *Freikorps* brigades Ehrhardt, Wolfgang Kapp and his allies took power in the German capital Berlin and forced the elected Government to flee to southern Germany. However, a general strike called by the left-wing parties and the refusal by civil servants to follow Kapp's orders led to the collapse of the coup.

¹⁰ Ermächtigungsgesetz, entitled 'Law to Remedy the Distress of the People and the Nation'

The Communist Party of Germany (KPD) was founded on January 1, 1919 on the base of the Spartacus League. The party was headed by Rosa Luxemburg and Karl Liebknecht, separated from the Independent Social Democratic Party of Germany (USPD). Struggling for the worker's votes the KPD opposed both the Independent Social Democratic Party of Germany and the Social Democratic Party of Germany. The Communists stood for the republic of councils (or soviets), disregarded the National Assembly election of January 19, 1919 and were committed to a violent revolution in Germany. Despite such a commitment to radicalism, in 1920 the KPD was represented the Reichstag by two party members.

In December 1920 at the 6th congress held in Berlin the KPD merged with a substantial (majority) section of the USPD. After the failed attempt of *the coup d'état* the KPD went underground till spring of 1924. In 1924 the goal of immediate revolution was put off by its leadership (headed by Ernst Thälmann) and the party began to contest Reichstag elections, usually polling more than 10% of the vote.

Since the mid-1920s the KPD followed the directions of the Communist International – e.g., it accused the SPD of the commitment to 'social fascism'. Tensions between socialist and communist parties foiled the attempts to set up a united opposition to the rising Nazi party.

The KDP influence was steadily growing throughout the course of the Weimar republic history and the party received its maximum (6 millions votes) in the Reichstag election of November 6, 1932. On February 28, 1933 any public activity of the KDP was banned by the Notverordnungen of the Reichspräsident under the pretext of the arson of the Reichstag Building the day before. All 81 mandates of the party were nullified after the Reichstag election of March 5, 1933 and communist deputies were arrested and jailed. The KDP stopped its legal existence on March 8, 1933.

3.3.3 The German Democratic Party (Deutsche Demokratische Partei – DDP)

The German Democratic Party (DDP) was founded on November 20, 1918 by leaders of the former Progressive People's Party: Max Weber and Hugo Preuss. It was committed to individual freedom, private property and social responsibility. The party was a member of the Weimar coalition and entered particularly each ruling coalition till May 1932.

In 1927 during the period of economic growth of Germany the DDP began to lose votes in a dramatic way. In May 1930 the Democrats attempted to transform themselves into the German State Party (Deutsche Staatspartei), but the effect was miserable and soon the party

became insignificant in political life of the declining Weimar republic. The DDP dissolved itself on 28 June 1933 under pressure by the Nazi government.

3.3.4 The Centre Party (Deutsche Zentrumspartei – Zentrum)

The Centre Party was founded in 1871 as Catholic political party on Germany. In the Kaiserreich the Catholic Centre Party protected the rights of the Catholics, a minority of the total population, and the rights of national minorities such as Poles and Alsatians. The party stood for the representative government. In 1918, the Bavarian People's Party (BVP) split off from the Centre Party and began to advocate for the autonomy of the Reich states.

The Zentrum was a member of the Weimar coalition and played the major role there after the SPD went out. The main reason was its ideological blurriness. The Centre Party cooperated with both liberals and nationalists, except radicals, and entered each governing coalition.

In 1928, Ludwig Kaas, a Roman Catholic prelate, became a chairman of the Centre Party. He stood for the incorporation of the NSDAP into the political system through parliament, and soon the conservative component in party's ideology gained strength. In 1932, the Centre Party supported Paul von Hindenburg in his campaign to get reelected as Reichspräsident with Adolf Hitler with his main rival. This created tensions with the Nazi party. However, after the appointment of Adolf Hitler to the Reich Chancellor position the Centre Party was ready to support him and his government in order to become partners and to pursue religion-oriented policy, but Hindenburg dismissed the Reichstag, obviously at the suggestion of Hitler. The elections of March 5, 1933 brought only 44% of the seats for the NSDAP; it could gain the majority only by a coalition with Kampffront Black-White-Red. The Centre Party split up on the question of the support to the Enabling Act on March 23, 1933, but its members contributed the necessary two thirds of votes to the Act's adoption by Reichstag. The party dissolved itself on 5 July 1933 under Nazi pressure.

3.3.5 The German People's Party (Deutsche Volkspartei – DVP)

The German People's Party was founded on November 22, 1918 by Gustav Stresemann after the reorganization of the National Liberal Party, and was the representative of corporations and big industrial capital interests. The party proclaimed the commitment to national liberalism and

aspiration for restoration of the former greatness of the Kaiserreich. The DVP did not cooperate with radical nationalists and blamed anti-Semites. The party held the Social Democrats responsible for the November revolution of 1918 and considered them as the main rivals. The DVP opposed the Weimar coalition but later cooperated with centre- and left-centre parties.

In 1929, after the death of Gustav Stresemann, the DVP returned back to its right-oriented platform. The main party goals were the struggle against parliamentary democracy and the propaganda of the nationalism. The party dissolved itself on June 27, 1933.

3.3.6 German-National People's Party (Deutsche Nationale Volkspartei – DNVP)

The German-National People's Party was founded on November 24, 1918. The party stood for monarchical form of government and the restitution of colonies, and she advocated interests of the privileged strata of German society, industrialists and big landed classes. The DNVP took the anti-Semitic and xenophobic position. The party argued against the republic constitution and presented the opposition.

In 1925-1928 the DNVP restrained its radicalism and attempted to enter the ruling coalition. The sharp shift back to ardent nationalism happened after Alfred Hugenberg became the chairman of the DNVP in 1928. Soon the less radical faction, headed by the former chairman Magnus Freiherr von Braun, left the party. In 1933, the DNVP united with 'Stahlhelm' to form 'the Battlefront Black-White-Red' in order to help the NSDAP to obtain the absolute majority in parliament. The party dissolved itself on June 28, 1933 under Nazi pressure.

3.3.7 National Socialist German Workers' Party (Nationalsozialistische Deutsche Arbeiterpartei – NSDAP)

The National Socialist German Workers' Party (NSDAP) was founded on February 24, 1920 in Munich on the basis of the German Workers' Party (DAP) founded January 5, 1919 and headed by Anton Drexler. Its high-priority goals were the abrogation of the Treaty of Versailles of 1919 and the consolidation of the Aryan race against enemies included the Jews and the Slavs. After the Beer Hall Putsch on November 9, 1923 and the imprisonment of Adolf Hitler the NSDAP was legally banned. Many *völkisch* movements continued promoting German nationalism, anti-communism and anti-Semitism. They fought against the Weimar Republic and left-wing political

opponents accusing them of the failures and weaknesses of the democracy and the communism. In 1924 many Nazi merged with other rightist factions to form a united organization. The NSDAP joined with the National Socialist Freedom Movement for the Reichstag election of this year.

In February 1925, after Hitler had come back from his imprisonment, he started to reorganize the NSDAP according to the Führer principle tailored to his person. This led to bitter struggles between the left and the right wings of the Nazi party concerning the degree of nationalism and socialism in early 1926. In the end, Adolf Hitler was unanimously elected as a leader of the NSDAP on May 22, 1926.

At the onset of the Great Depression in 1929, the Nazi party started to draw mass support, mainly from the impoverished urban and rural lower middle classes. In 1932, the Nazis became the largest faction in the Reichstag. The only remaining alternatives to the NSDAP were the Centre Party, the SPD and the Communists. In spite of the great successes of the NSDAP at several parliamentary elections, in the presidential election in March 1932 Adolf Hitler experienced a severe set-back: he was defeated by the incumbent, Reichspäsident Paul von Hindenburg, in the second round of the race for presidency.

Since 1930 the weak minority governments exercised their authorities through the *Notverordnungen*. On January 30, 1933 Adolf Hitler was assigned by Reichspäsident Paul von Hindenburg to be the Chancellor and to succeed in forming the ruling coalition with the German-National People's Party and the Stahlhelm organization. The Enabling Act, also known as 'the Law to Remedy the Distress of the People and the Nation', was passed by the Reichstag on March 23, 1933 by a two thirds majority of the remaining members (the 81 KPD seats were nullified on the basis of the Reichstag Fire *Notverordnungen*). During 1933 the authoritarian regime dismissed or enforced to dissolve themselves all political parties of the Weimar republic. On July 14, it was ruled that the forming of any new party was forbidden by law. On September 12, the NSDAP was the only party admitted to the list for the election to the Reichstag.

4. A priori power in the Weimar Reichstag 1919-1933

4.1 Formal model of parliamentary decision making and Banzhaf index

Consider a parliament $N = \{1, \dots, m\}$ with faction $i \in N$. The number of seats in the parliament is n . Faction i has v_i seats, $\sum_i v_i = n$. Faction i often does not have enough seats to pass a decision and has to enter a coalition with other factions. A coalition ω is a set of factions (a subset of N , $\omega \subseteq N$) voting together *for* or *against* a motion proposed to the parliament.

A decision is made and is a winning coalition ω if the quota q is satisfied such that

$$\sum_{i \in \omega} v_i \geq q.$$

That is, coalition ω is *winning* if the sum of votes in the coalition is not fewer than q . A faction $i \in \omega$ is said to be *critical* in a coalition ω if ω is winning while $\omega \setminus \{i\}$ is losing. Then faction i is called a *swinger*. The regular decision-making procedure in the Reichstag was simple majority rule (The Weimar Constitution, Article 32), and therefore the quota was $q = n/2 + 1$. For constitutional changes the minimum of votes was $q_{const} = 2n/3 + 1$ to get accepted, i.e. a 2/3 rule applied.

More general, the seat distribution with respect to factions, (v_i) , and the relevant quota (decision rule), q , describe a voting body, i.e., a voting game. This constitutes a formal model of parliamentary decision making.

In general, the seat distribution (v_i) is a poor proxy of the power distribution in the voting body, i.e., the potential to form winning coalitions. To get the first evaluation of the power distribution in the Reichstag we use the Banzhaf index.¹¹ The index examines the number of winning coalitions where faction i is critical and therefore has a swing position. Let us denote this number as b_i .

Given b_i , the normalized Banzhaf index of faction i is calculated as the ratio of coalitions that faction i can make to win, so that i a swing player, and the sum of all swing positions, i.e.

$$\beta_{(i)} = b_i / \sum b_j. \quad (1)$$

The values of normalized Banzhaf index range from 0 to 1, i.e., $0 \leq \beta_{(i)} \leq 1$. This measure assumes that all deputies in the faction ‘solidify’ to vote and that each faction can coalesce with any other faction without any restrictions.

¹¹ The reference paper is Banzhaf (1965). However, this measure coincides with a measure presented in Coleman (1971) and, early, in Penrose (1946). Penrose-Banzhaf, Banzhaf-Coleman, and permutations are often used to label the measure that we use in this text.

In the following we will refer to the normalized form $\beta_{(i)}$ whenever we discuss the Banzhaf index. It is well known that the normalized form has properties that are different from the properties of its non-normalized form (see, e.g., Felsenthal and Machover (1995), and Turnovec (1998)), and a probability interpretation of the normalized version is not straightforward. The normalized form, however, has the advantage that its values are directly comparable to the seat shares.

Of course, there is still the question why to take the Banzhaf index instead of the Shapley-Shubik index, the Deegan-Packel index or the Public Good Index just to mention the perhaps most prominent alternatives. A series of arguments in favor of this choice are offered in Felsenthal and Machover (1998). However, the fact that we have selected the Banzhaf index does not mean that we subscribe to all of these arguments - in fact, we submit that the application of other indices could be justifiable and produce very interesting results as well.

4.2 Voting power analyses of the Reichstag

Table 2 shows the percentage of voters, faction size (i.e., number of seats controlled by a party), share of seats¹² and Banzhaf index values of the parties in the Reichstag as a consequence of the National Assembly election of January 19, 1919.

| Party | % | Seats | Share s | Banzhaf β |
|---|------|-------|---------|-----------------|
| German Centre Party (Zentrum) | 21.5 | 91 | 0.22 | 0.154 |
| German Democratic Party (DDP) | 17.7 | 75 | 0.18 | 0.154 |
| German National People's Party (DNVP) | 10.4 | 44 | 0.10 | 0.115 |
| German People's Party (DVP) | 4.5 | 19 | 0.04 | 0.038 |
| Social Democratic Party of Germany (SPD) | 39.0 | 165 | 0.39 | 0.462 |
| Independent Social Democratic Party of Germany (USPD) | 5.2 | 22 | 0.05 | 0.038 |
| Others | 1.7 | 7 | 0.02 | 0.038 |
| Total | | 423 | | |

Table 2. National Assembly election of January 19, 1919

The Reichstag composition in Table 2 confirms the Weimar constitution proposed and supported by the Weimar coalition SPD, the DDP, and the Zentrum. Given a quota of $q = 212$, the Weimar coalition with its total of 331 seats had a power value of 1. However, this presupposes that the coalition behaves like a single actor. It empowered the coalition parties to pass bills without any difficulties and no efforts to involve other supporters were needed.

¹² These data as well as similar data in the following tables have been taken from Falten (1986).

| Party | Seats s | Banzhaf β |
|---|----------------|-----------------------------------|
| Weimar coalition | 331 | 1 |
| German National People's Party (DNVP) | 44 | 0 |
| German People's Party (DVP) | 19 | 0 |
| Independent Social Democratic Party of Germany (USPD) | 22 | 0 |
| Others | 7 | 0 |

Table 2a. National Assembly election of January 19, 1919

In fact, the DNVP was the only significant force to oppose the Weimar coalition in parliament. It rejected the regime of the republic and advocated the return of the monarchy.

On the other hand, the results of this election turned out to be the best ones for the SPD during the whole interwar period. A seat share of $s = 0.39$ corresponds to a power share of $\beta = 0.462$, if we consider all possible coalitions of equal likelihood, which, however, was not the case. The differences in the s - and β -values indicate that seat shares are only a poor proxy for a priori voting power as measured by the Banzhaf index.

| Party | % | Seats | Share s | Banzhaf β |
|---|----------|--------------|----------------|-----------------------------------|
| Bavarian People's Party (BVP) | 4.6 | 21 | 0.05 | 0.047 |
| German Democratic Party (DDP) | 8.5 | 39 | 0.08 | 0.062 |
| German National People's Party (DNVP) | 15.5 | 71 | 0.15 | 0.155 |
| German People's Party (DVP) | 14.2 | 65 | 0.14 | 0.147 |
| German Centre Party (Zentrum) | 13.9 | 64 | 0.14 | 0.147 |
| Communist Party of Germany (KPD) | 0.9 | 4 | 0.01 | 0.008 |
| Social Democratic Party of Germany (SPD) | 22.4 | 103 | 0.22 | 0.248 |
| Independent Social Democratic Party of Germany (USPD) | 18.1 | 83 | 0.18 | 0.171 |
| Others | 2.2 | 10 | 0.02 | 0.016 |
| Total | | 459 | | |

Table 3. Reichstag election of June 6, 1920

The main outcome of elections on June 6, 1920 can be found in the approximately equal power of the representation of the Left and the Right. Despite the unsuccessful and unpopular right-wing Kapp-Putsch in March 1920, the SPD suffered from the dramatic loss of votes and potential influence. Its power measure of $\beta = 0.462$ dropped down to a mere $\beta = 0.248$ in 1920. The Social Democrats were held responsible for all the troubles of the new republic by both left- and right-wing extremists, but also by a large share of its traditional turf of voters. The popularity of the coalition partner DDP divided in half. With a control of 206 seats (of total of 459) the Weimar coalition maintained its ability to pass bills and prevent undesirable legislation,

having a power value $\beta = 0.574$ (assuming the coalition is the agent with 206 seats). However, the coalition was unstable.

| Party | Seats | Banzhaf β |
|---|--------------|-----------------------------------|
| Bavarian People's Party (BVP) | 21 | 0.064 |
| Weimar coalition | 206 | 0.574 |
| German National People's Party (DNVP) | 71 | 0.106 |
| German People's Party (DVP) | 65 | 0.106 |
| Communist Party of Germany (KPD) | 4 | 0.021 |
| Independent Social Democratic Party of Germany (USPD) | 83 | 0.106 |
| Others | 10 | 0.021 |

Table 3a. Reichstag election of June 6, 1920

In 1919, the Bavarian People's Party (BVP) formed an a priori union with the Zentrum in the Reichstag that resulted from the election of 19 January, 1919. This union broke up due to political objectives and perspectives. The latter became obvious in the election of Paul von Hindenburg for Reichspräsident, the most important political position in the Republic. His election was supported by BVP but objected by the Zentrum.

This time period was the peak of popularity for the USPD, a pacifist and mainly left-wing split-off of the SPD. The party sharply increased its a priori power share from $\beta = 0.038$ in the Reichstag of 1919 to $\beta = 0.171$ in the Reichstag of USPD. But a too narrow 'ideological space' between two major socialist parties: the SPD and the KDP – was a reason for merging first with the KPD (in 1920) and later with the SPD (in 1922). We can notice that the Communists began their successful contest with the election of 1920 with a rather moderate $\beta = 0.008$. Moreover, we can note that the DNVP, the DVP and Zentrum possessed approximately equal Banzhaf index values: 0.155 by the DNVP and 0.147 by the DVP and Zentrum.

| Party | Seats | Banzhaf β |
|---|--------------|-----------------------------------|
| Bavarian People's Party (BVP) | 21 | 0.000 |
| DDP + DVP + Zentrum | 168 | 0.500 |
| German National People's Party (DNVP) | 71 | 0.167 |
| Communist Party of Germany (KPD) | 4 | 0.000 |
| Social Democratic Party of Germany (SPD) | 103 | 0.167 |
| Independent Social Democratic Party of Germany (USPD) | 83 | 0.167 |
| Others | 10 | 0.000 |

Table 3b. Reichstag election of November 30, 1923

In November 30, 1923, a new government succeeded the Weimar coalition. The new government coalition did not include the SPD, but was formed by Zentrum, DVP, and DDP. It was headed by Wilhelm Marx of the Zentrum party and lasted until May 26, 1924. It controlled 168 seats only and was therefore far away from a stable majority. However, the Zentrum-DVP-DDP coalition managed to achieve a compromise about war reparations of the Dawes Plan.

| Party | % | Seats | Share s | Banzhaf β |
|--|------|-------|---------|-----------------|
| Bavarian People's Party (BVP) | 3.4 | 16 | 0.03 | 0.043 |
| German Democratic Party (DDP) | 5.9 | 28 | 0.06 | 0.05 |
| German-National People's Party (DNVP) | 20.1 | 95 | 0.20 | 0.206 |
| German People's Party (DVP) | 9.5 | 45 | 0.10 | 0.091 |
| German Centre Party (Zentrum) | 13.8 | 65 | 0.14 | 0.139 |
| Communist Party of Germany (KPD) | 13.1 | 62 | 0.13 | 0.121 |
| Social Democratic Party of Germany (SPD) | 21.2 | 100 | 0.21 | 0.232 |
| Deutsch-Völkische Freiheitspartei (DVFP) and NSDAP | 6.8 | 32 | 0.07 | 0.065 |
| Others | 6.1 | 29 | 0.06 | 0.054 |
| Total | | 472 | | |

Table 4. Reichstag election of May 4, 1924

The results from Table 4 indicate that the popularity of liberal and social parties declined. The background for the more successful performance of “anti-system parties” was the economic crisis of 1923, the corresponding decline in the living standard and dramatic restructuring of the income and wealth distribution, the rise of the separatism and the sense of national infringement. The Communist's faction extremely raised its relative power from the earlier $\beta = 0.008$ to a $\beta = 0.121$. Within the alliance of nationalistic movements and *völkische* organizations, the NSDAP possessed some seats in the Reichstag and enjoyed a relative voting power $\beta = 0.065$. The DDP with $\beta = 0.05$ lagged behind the nationalists. Maximum relative powers in this Reichstag belonged to the SPD and the nationalistic party, the DNVP. The DNVP had the second largest value of Banzhaf index ($\beta = 0.206$), which is just a bit smaller result than the SPD ($\beta = 0.232$). This somehow reflects the closeness of the seats the two parties control: the SDP and DNVP control 100 and 95, respectively, out of a total of 472 seats.

The government headed by Wilhelm Marx continued its policy aimed at economic and political stabilization by means of receiving foreign loans and was moderately successful in it. This success inspired Reichspräsident Friedrich Ebert, who had his roots in the SPD, to dismiss parliament hoping for a better electoral performance of the Weimar coalition parties.

| Party | % | Seats | Share s | Banzhaf β |
|-------------------------------|-----|-------|---------|-----------------|
| Bavarian People's Party (BVP) | 3.9 | 19 | 0.04 | 0.040 |

| | | | | |
|---|------|-----|------|-------|
| German Democratic Party (DDP) | 6.5 | 32 | 0.06 | 0.063 |
| German National People's Party (DNVP) | 20.9 | 103 | 0.21 | 0.206 |
| German People's Party (DVP) | 10.3 | 51 | 0.10 | 0.095 |
| German Centre Party(Zentrum) | 14.0 | 69 | 0.14 | 0.143 |
| Communist Party of Germany (KPD) | 9.1 | 45 | 0.09 | 0.079 |
| National Socialist Freedom Movement (NSDAP) | 2.8 | 14 | 0.03 | 0.024 |
| Social Democratic Party of Germany (SPD) | 26.6 | 131 | 0.27 | 0.294 |
| Others | 5.9 | 29 | 0.06 | 0.056 |
| Total | | 493 | | |

Table 5. Reichstag election of December 7, 1924

Friedrich Ebert's expectations were justified. Given the favourable economic and international situation, the SPD-DDP-Zentrum-DVP group gained a total of 283 seats while the radical parties of the left and right ended up with 162, only. The SPD's influence substantially increased from $\beta = 0.232$ to $\beta = 0.294$, while voting power of the KPD and the NSDAP dropped by 0.042 and 0.041, respectively.

When the new government formed, it was the first time that representatives of the DNVP were invited to join the Cabinet along with the DVP, Zentrum, the BVP and the DDP. The sum of seats of this coalition was 274 which is a majority given a total of 493 seats. In addition, the SPD, although in opposition, was loyal to the governing coalition under Hans Luther (the former non-partisan Minister of Finance). But despite its clear majority, the ruling coalition, due to different ideological attitudes of the coalition partners, was unstable. So it came as no surprise when in May 1926 the coalition collapsed when the common position of the DDP ($\beta = 0.063$) and Zentrum ($\beta = 0.143$) prevented changes in the Constitution that included the return of the old Reich's national flag. Because of this resistance, it was impossible to gain 2/3 majority of deputies' votes that was necessary to change the Constitution. The relative power of the coalition that followed, formed by Zentrum, DVP and DDP, controlled 152 seats only. Its power value was $\beta = 0.333$ against SPD's and DNVP's $\beta = 0.294$.

| Party | Seats s | Banzhaf β |
|---|----------------|-----------------------------------|
| Bavarian People's Party (BVP) | 19 | 0.020 |
| DDP + DVP + Zentrum | 152 | 0.333 |
| German National People's Party (DNVP) | 103 | 0.294 |
| Communist Party of Germany (KPD) | 45 | 0.020 |
| National Socialist Freedom Movement (NSDAP) | 14 | 0.020 |
| Social Democratic Party of Germany (SPD) | 131 | 0.294 |
| Others | 29 | 0.020 |

Table 5a. Reichstag election of December 7, 1924

The new coalition of May 1926, again under Wilhelm Marx (Zentrum), still tried to involve the DNVP, and in fact succeeded in the beginning of 1927, but now it did not enjoy SPD's support. Right-of-centre governments stood for the policy of budget consolidation and substantial cuts in social allowances. However, military expenditures increased. In this atmosphere of discontent the popularity of governing parties began to decrease rapidly.

| Party | % | Seats | Share s | Banzhaf β |
|--|----------|--------------|----------------|-----------------------------------|
| Bavarian People's Party (BVP) | 3.3 | 16 | 0.03 | 0.025 |
| German Democratic Party (DDP) | 5.1 | 25 | 0.05 | 0.042 |
| German National People's Party (DNVP) | 14.9 | 73 | 0.15 | 0.127 |
| German People's Party (DVP) | 9.2 | 45 | 0.09 | 0.093 |
| German Centre Party(Zentrum) | 12.6 | 62 | 0.13 | 0.11 |
| Communist Party of Germany (KPD) | 11.0 | 54 | 0.11 | 0.102 |
| National Socialist German Workers' Party (NSDAP) | 2.4 | 12 | 0.02 | 0.017 |
| Social Democratic Party of Germany (SPD) | 31.2 | 153 | 0.31 | 0.39 |
| Others | 10.4 | 51 | 0.10 | 0.093 |
| Total | | 491 | | |

Table 6. Reichstag election of May 20, 1928

After the election of May 20, 1928 a total of fourteen political parties were represented in the Reichstag. We can see that the SPD significantly raised its relative voting power from $\beta = 0.294$ on the last elections to $\beta = 0.390$ now. On the other hand, the voting power of DDP further decreased. The absence of a clear ideological commitment influenced the attitude of the DDP voters to 'their' party. As a result the DDP's Banzhaf index value dropped to $\beta = 0.042$. The "Parties of the Right" have lost some of their electorate support and impact in the Reichstag. The DNVP faction ended up with $\beta = 0.127$, the BVP with $\beta = 0.025$ and NSDAP with $\beta = 0.017$, while the DVP preserved its position $\beta = 0.093$, The Communist Party KPD gained additional seat and its impact in the Reichstag extended to a relative power $\beta = 0.11$.

The policy of budget consolidation, the ignorance of growing social problems and the absence of effective mechanism to solve them in the relatively stable economic situation have led to strengthening of the left parties. The "Grand Coalition" headed by Hermann Müller (SPD), that included the SPD, the DDP, Zentrum and the DVP controlled a total sum of 285 seats and thus a majority of votes.

The outlook of political stability was promising. However, the onset of the Great Depression seriously damaged the financial system of Germany. The unemployment rates began to grow in 1928 substantially, and foreign loans flow tended to stop. On December 29, 1929 the Reichsregierung decided to solidify the budget cutting expenditures. In 1930 the Reichstag approved the Young Plan that restructured Germany's guarantee to pay reparations debts. The

acceptance of this scheme aroused nationalist passions and resentment. In March 1930, the SPD did not accept further cuts of unemployment benefits. This led to the withdrawal of the SPD from government and Cabinet collapsed.

The follow-up Cabinet headed by Heinrich Brüning (Zentrum) attempted to reduce government expenditures but faced strong opposition from the SPD, the KPD, and the DNVP. Obviously, the voting power of this opposition, which controlled 280 seats, was much bigger than the government's influence that derived from 132 seats, only. However, the opposition parties could not form a coalition because of the ideological diversity between the parties was too high. The Reichstag was dismissed.

| Party | % | Seats | Share s | Banzhaf β |
|--|----------|--------------|----------------|-----------------------------------|
| Bavarian People's Party (BVP) | 3.3 | 19 | 0.03 | 0.038 |
| German National People's Party (DNVP) | 7.1 | 41 | 0.07 | 0.069 |
| German State Party (former DDP) | 3.5 | 20 | 0.03 | 0.038 |
| German People's Party (DVP) | 5.2 | 30 | 0.05 | 0.046 |
| German Centre Party (Zentrum) | 11.8 | 68 | 0.12 | 0.107 |
| Communist Party of Germany (KPD) | 13.3 | 77 | 0.13 | 0.13 |
| National Socialist German Workers' Party (NSDAP) | 18.5 | 107 | 0.19 | 0.183 |
| Social Democratic Party of Germany (SPD) | 24.8 | 143 | 0.25 | 0.275 |
| Others | 12.5 | 72 | 0.12 | 0.115 |
| Total | | 577 | | |

Table 7. Reichstag election of September 14, 1930

The election of September 14, 1930 resulted in substantial gains for the extremist parties, the right-wing NSDAP and the communist KPD. These parties proposed very diverse but new policies to fight the immense problems given by the economic and political situation. The relative power of the NSDAP faction increased rapidly to $\beta = 0.183$; this gave the party the second place in the Reichstag after the SPD. On the contrary, the German State Party (the former DDP) and the DNVP lost their influence due to the blurriness of positions. Their Banzhaf index values were less than 0.05 each. Because of its specific catholic electorate the Zentrum's relative power in the Reichstag decreased slightly, $\beta = 0.107$.

The SPD significantly lost power in comparison to the last elections but still had the value of $\beta = 0.275$. It announced its willingness to negotiate about a participation in a coalition government, but this did not led to positive results. The same happened to the DNVP and its ambition to take part in government. It seemed that the NSDAP left no place to relatively moderate nationalists. The political situation was truly problematic. The NSDAP constantly ignored the parliamentary work; both the Communists and the Socialists opposed the Cabinet

policy. In 1931 the national financial system collapsed. The cases of violence, clashes and open brutality became more frequent. After a vote of no-confidence for Chancellor Franz von Papen (Zentrum) an election was called in the hope to solve the political stalemate.

| Party | % | Seats | Share s | Banzhaf β |
|--|----------|--------------|----------------|-----------------------------------|
| Bavarian People's Party (BVP) | 3.6 | 22 | 0.04 | 0.013 |
| German National People's Party (DNVP) | 6.1 | 37 | 0.06 | 0.013 |
| German State Party (former DDP) | 0.7 | 4 | 0.01 | 0.003 |
| German People's Party (DVP) | 1.2 | 7 | 0.01 | 0.008 |
| German Centre Party (Zentrum) | 12.3 | 75 | 0.12 | 0.15 |
| Communist Party of Germany (KPD) | 14.6 | 89 | 0.15 | 0.15 |
| National Socialist German Workers' Party (NSDAP) | 37.8 | 230 | 0.38 | 0.501 |
| Social Democratic Party of Germany (SPD) | 21.9 | 133 | 0.22 | 0.15 |
| Others | 1.8 | 11 | 0.02 | 0.013 |
| Total | | 608 | | |

Table 8. Reichstag election of July 31, 1932

After Reichstag election of July 31, 1932, the NSDAP and the KPD controlled 319 seats in total and thus a majority of seats. Both parties had radical views and rejected democracy, but were bitter rivals to each other. Their election results were truly impressive. The KPD caught up with the 'respectable' Zentrum and the SPD ($\beta = 0.15$). Even more impressive was the performance of the NSDAP: it was the first party that achieved a Banzhaf index value in the history of Weimar republic that exceeded the level of 0.5. But the NSDAP was not able to form the government by itself as it did not enjoy the absolute majority of seats in the Reichstag. As a consequence of the ongoing stalemate, governing was accomplished by the president's decrees and the Reichstag became a secondary agent ready for being dismissed again. As a consequence, the year 1932 saw a second Reichstag election in November.

| Party | % | Seats | Share s | Banzhaf β |
|--|----------|--------------|----------------|-----------------------------------|
| Bavarian People's Party (BVP) | 3.4 | 20 | 0.03 | 0.033 |
| German National People's Party (DNVP) | 8.9 | 52 | 0.09 | 0.051 |
| German State Party (former DDP) | 0.3 | 2 | 0.00 | 0.005 |
| German People's Party (DVP) | 1.9 | 11 | 0.02 | 0.014 |
| German Centre Party (Zentrum) | 12.0 | 70 | 0.12 | 0.098 |
| Communist Party of Germany (KPD) | 17.1 | 100 | 0.17 | 0.191 |
| National Socialist German Workers' Party (NSDAP) | 33.6 | 196 | 0.34 | 0.405 |
| Social Democratic Party of Germany (SPD) | 20.7 | 121 | 0.21 | 0.191 |
| Others | 2.1 | 12 | 0.02 | 0.014 |
| Total | | 584 | | |

Table 9. Reichstag election of November 6, 1932

The composition of the Reichstag that resulted from the November election was not very promising either. The Communists (i.e., the KPD) further strengthened their positions ($\beta = 0.191$) and became as powerful as the SPD. The NSDAP lost some power as its Banzhaf value dropped from $\beta = 0.501$ to $\beta = 0.405$. It is said this was due to the fact that Adolf Hitler had challenged the very popular Reichspräsident Paul von Hindenburg in the presidential elections of April 10, 1932 and was defeated. Hindenburg was supported by all major democratic parties although he clearly did not think highly of democracy and a republican constitution.

After the November election there was no majority coalition that supported the election of a Chancellor. As a consequence, on November 17, 1932 the government under Reichskanzler von Papen resigned, Hindenburg nominated Kurt von Schleicher as Reichskanzler on December 3, 1932. Von Schleicher wanted to get the Socialists and the left-wing Nazi faction under Gregor Strasser to join him to form a government. However, Hitler repelled Strasser's plan and gained full control. The Socialists distrusted von Schleicher, a general. Hence, von Schleicher had to resign on January 28, 1933. Only then Hindenburg appointed Hitler as the head of the government. The Reichstag was again dismissed and new elections were called to gain the parliamentary majority through democratic procedure. The arson of the Reichstag building not only influenced the election, it gave also reason to arrest communist delegates and thereby reducing the number of representatives in the Reichstag in favor of the share of votes of the NSDAP and its allies.

| Party | % | Seats | Share s | Banzhaf β |
|--|----------|--------------|----------------|-----------------------------------|
| Battlefront Black-White-Red (KFSWR) | 8.0 | 52 | 0.08 | 0.091 |
| Bavarian People's Party (BVP) | 2.8 | 18 | 0.03 | 0 |
| Communist Party of Germany (KPD) | 12.5 | 81 | 0.13 | 0.091 |
| German Centre Party (Zentrum) | 11.4 | 74 | 0.11 | 0.091 |
| German People's Party (Bloc DVP-CZVD-DBP-DHP) | 0.3 | 2 | 0.00 | 0 |
| German State Party (former DDP) | 0.8 | 5 | 0.01 | 0 |
| National Socialist German Workers' Party (NSDAP) | 44.5 | 288 | 0.45 | 0.636 |
| Social Democratic Party of Germany (SPD) | 18.5 | 120 | 0.19 | 0.091 |
| Others | 1.1 | 7 | 0.01 | 0 |
| Total | | 647 | | |

Table 10. Reichstag election of March 5, 1933

Again, the NSDAP obtained a relative majority in the Reichstag but it did not gain an absolute majority despite the control it extended to the police, radio and the press in its favor. As a result of the latter the freedom to assemble was restricted by law. The relative power of the

NSDAP faction in the Reichstag was equal to 0.636. Together with the support of the Battlefront Black-White-Red they could control legislation as the size of quota to pass ordinary laws was 324 votes and the coalition had 340 votes. However, this coalition could not implement constitutional changes. In the following, the position of Zentrum with 74 seats became crucial.

After the Reichstag Fire on February 27, 1933 all communist deputies were arrested and their mandates were nullified. On March 8 the KPD was legally forbidden, and there were no more Communists admitted to Reichstag. *Table 10a* shows the new power distribution. The quota size dropped to 284 votes ($566/2+1$) and the NSDAP had an absolute majority and thus the power to pass ordinary bills. The quota required by constitutional bills was 378 votes ($2/3$). Given this quota, the Banzhaf index value of NSDAP was $\beta = 0.524$. At the same time, the SPD's relative influence in giving acceptance to constitutional legislation was $\beta = 0.238$. This was the reason for Adolf Hitler to get the Zentrum involved in changing the constitution and to pass the so-called *Ermächtigungsgesetz*.¹³ On March 23, 1933 the Reichstag accepted the constitutional law 'to remedy the distress of the people and the nation' and gave Hitler's government dictatorial power. Soon all political parties were forbidden, of course with exception of the NSDAP.

| Party | % | Seats | Shares | Banzhaf β after 27 Feb 1933 | Banzhaf β on 23 March 1933 |
|--|------|-------|--------|-----------------------------------|----------------------------------|
| Battlefront Black-White-Red (KFSWR) | 9.2 | 52 | 0.09 | 0 | 0.048 |
| Bavarian People's Party (BVP) | 3.2 | 18 | 0.03 | 0 | 0.048 |
| German Centre Party (Zentrum) | 13.1 | 74 | 0.13 | 0 | 0.143 |
| German People's Party (Bloc DVP-CZVD-DBP-DHP) | 0.4 | 2 | 0.00 | 0 | 0 |
| German State Party (former DDP) | 0.9 | 5 | 0.01 | 0 | 0 |
| National Socialist German Workers' Party (NSDAP) | 50.9 | 288 | 0.51 | 1 | 0.524 |
| Social Democratic Party of Germany (SPD) | 21.2 | 120 | 0.21 | 0 | 0.238 |
| Others | 1.2 | 7 | 0.01 | 0 | 0 |
| Total | | 566 | | | |

Table 10a. Reichstag election of March 5, 1933

5. Coalitional preferences and coalition formation

¹³ In fact, this law was meant to be temporary; it did not necessitate a change of the constitution but a $2/3$ majority as necessary for changing the constitution.

In order to evaluate the Banzhaf index we have assumed that each faction can coalesce with any other faction without any restrictions. A set of new power indices introduced in Aleskerov (2006) takes into account the extent to which players prepared to coalesce, i.e., taking into consideration the preferences of the political decision makers as defined by ideologies, party programs, historical records, etc. Of course, the political personnel also matters as the history of the Weimar Republic excessively demonstrates.

5.1 A model of preference-based power index

There are many ways to take political preferences into account in order to analyze the relationship between political agents and thereby try to explain or forecast a particular outcome. But we have to acknowledge that we thereby add information to the power analysis proper that deserves quite a different level of interpretation than the decision rule (i.e., the quota q) and the vote distribution v that constitute the material of the analysis of a priori voting power. In terms of Barry (1982) by taking care of preferences we add some “luck” to our analysis.¹⁴

Banzhaf index examines a number of winning coalitions where faction i is a swing player, assuming that each coalition is equally likely. Aleskerov (2006) introduces two modifications of this measure which evaluate the intensity of i 's connections within the winning coalitions. One index is based on the assumption that the agents' preferences are linear orders, the second uses cardinal information about the intensity of preferences. In what follows we will use only ordinal measures.

Example 1.1 Let $N = \{A, B, C, D\}$, $v_A = 45$, $v_B = 10$, $v_C = 6$, $v_D = 39$ such that the total number of votes $\sum_i v_i = 100$. Let's assume a quota $q = 51$.

In ordinal indices agents' preferences are presented as linear orders. (See coalitional preferences P_i in *Table 11*.) Each faction i has a willingness to coalesce with faction j denoted as p_{ij} . In the preference order of faction i a higher value p_{ij} means a larger willingness to form a coalition with j , such that $p_{ij} = |N|-1$ holds if faction j is in the highest rank in i 's preference order. It is not assumed that i and j have the same preferences to coalesce with each other, i.e., in general, $p_{ij} \neq p_{ji}$.

Table 11 implies that faction A wants to coalesce with faction B only, and intensity $p_{AB} = 3$. It does not wish to coalesce with C and D , so $p_{AC} = p_{AD} = 0$. Faction D wants to coalesce only with B , however with a lower intensity, and the corresponding intensities for D $p_{DB} = 2$, $p_{DA} =$

¹⁴ For an extensive discussion of the use of preferences in power analyses, see Braham and Holler (2005a,b), Holler and Nurmi (2005), and the references given in this material.

$p_{DC} = 0$. Faction B wants to form a coalition with faction A and less with faction D , i.e. $p_{BA} = 3$ and $p_{BD} = 2$. She does not want to join a coalition with C , thus $p_{BC} = 0$. Faction C does not want to enter any coalition, so all factions obtain coalitional preferences equal to zero, $p_{CA} = p_{CB} = p_{CD} = 0$.

| | P_A | P_B | P_C | P_D |
|---|-------|-------|---------|-------|
| 3 | B | A | - | - |
| 2 | - | D | - | B |
| 1 | - | - | - | - |
| 0 | C, D | C | A, B, D | A, C |

Table 11. Coalitional preference profile

The coalitional preferences, given in Table 11, can be summaries by the matrix given in Table 12.

| $i \setminus j$ | A | B | C | D |
|-----------------|---|---|---|---|
| A | 0 | 3 | 0 | 0 |
| B | 3 | 0 | 0 | 2 |
| C | 0 | 0 | 0 | 0 |
| D | 0 | 2 | 0 | 0 |

Table 12. Preference matrix, $\|p_{ij}\|$

Next, denote the intensity function between faction i and each winning coalition ω as $f(i, \omega)$. Intensity functions $f(i, \omega)$ can be constructed on the following basis:

(a) Intensity of i 's coalitional preferences. We only consider the i 's direct preferences towards each faction j , i.e. (p_{ij}) , in winning coalition ω ,

$$f^+(i, \omega) = \sum_{j \in \omega} p_{ij} / |\omega|; \quad (2)$$

(b) Intensity of preferences for i . In this case we summarize backward preferences p_{ji} given by other members of ω coalition to i :

$$f^-(i, \omega) = \sum_{j \in \omega} p_{ji} / |\omega|; \quad (3)$$

(c) Average intensity with respect to faction i . This intensity function is constructed by summarizing direct intensity functions (2) and backward intensity functions (3). The sum is divided by 2, i.e.,

$$f(i, \omega) = \frac{f^+(i, \omega) + f^-(i, \omega)}{2}; \quad (4)$$

(d) Total average intensity of coalition ω is defined as

$$f(\omega) = \frac{\sum_{i \in \omega} f(i, \omega)}{|\omega|}. \quad (5)$$

i.e., we sum average intensity functions for $i \in \omega$ in accordance with (4). The corresponding intensity is the same for any i .¹⁵

Let us define the set of winning coalitions for factions A, B, C, D for the above example and calculate intensity function values according to the formula $f^+(i, \omega) = \sum_{j \in \omega} p_{ij} / |\omega|$. The result is presented in *Table 13*.

Example 1.1 (continued):

For coalition $\omega = \{A, B\}$

$$f^+(A, \omega) = p_{AB} / |\omega| = (3 + 0) / 2 = 3/2; f^+(B, \omega) = p_{BA} / |\omega| = (3 + 0) / 2 = 3/2;$$

For coalition $\omega = \{A, B, D\}$

$$f^+(A, \omega) = p_{AB} + p_{AD} / |\omega| = (3 + 0) / 3 = 1; f^+(B, \omega) = p_{BA} + p_{BD} / |\omega| = (3 + 2) / 3 = 1;$$

$$f^+(D, \omega) = p_{DA} + p_{DB} / |\omega| = (0 + 2) / 3 = 2/3.$$

| Coalition ω | A | B | C | D |
|--------------------|-----|-----|---|-----|
| A, B | 3/2 | 3/2 | - | - |
| A, C | 0 | - | 0 | - |
| A, D | 0 | - | - | 0 |
| A, B, C | 1 | 1 | 0 | - |
| A, B, D | 1 | 5/3 | - | 2/3 |
| A, C, D | 0 | - | 0 | 0 |
| B, C, D | - | 2/3 | 0 | 2/3 |

Table 13. Intensity functions values $f^+(i, \omega)$

Using intensity function values from *Table 12* Aleskerov (2006) defines the corresponding power χ_i for each faction i , which is evaluated as

$$\chi_i = \sum_{\omega} f(i, \omega), \quad (6)$$

that is, we sum i 's intensity functions for all winning coalitions ω that have i as a swing player.

In the above example, χ_i is evaluated as the sum of i 's direct coalitional intensities towards any faction j ,

$$\chi_i = \sum_{\omega} f^+(i, \omega).$$

¹⁵ In Aleskerov (2006) other alternatives of constructing a intensity function $f(i, \omega)$ are also proposed.

Example 1.2: $\chi_A = f^+(A, A+B) + f^+(A, A+C) + f^+(A, A+D) + f^+(A, A+B+C) + f^+(A, A+B+D) + f^+(A, A+C+D) + f^+(A, A+B+C+D) = 3/2 + 1 + 1 = 7/5 = 3,5$. Similarly, $\chi_B = 2,17$, $\chi_C = 0$ and $\chi_D = 0,11$.

The normalized form of these voting power indices, $\alpha(i)$, is defined as the ratio between faction i 's power and all factions' powers, i.e. their sum. The relative power of faction i is defined as

$$\alpha_{(i)} = \chi_i / \sum_j \chi_j. \quad (7)$$

Normalized power indices add up to one. This gives us clear understanding of the existing power balance.

Example 1.3: $\alpha_{(A)} = \alpha_{(A)} / (\alpha_{(A)} + \alpha_{(B)} + \alpha_{(C)} + \alpha_{(D)}) = 3,5 / (3,5 + 2,17 + 0 + 0,67) = 0,55$.

| | <i>A</i> | <i>B</i> | <i>C</i> | <i>D</i> | $N = \{A+B+C+D\}$ |
|----------------|----------|----------|----------|----------|-------------------|
| $\alpha_{(i)}$ | 0,55 | 0,34 | 0 | 0,11 | 1 |

Table 14. Normalized power index values

5.2 Coalitional preferences and power in the Reichstag

The coalitional preferences of the parties in the Reichstag varied substantially, in fact, to a large extent these preferences were decisive of what Germany and the World experienced in the last century. Our analysis can only outline some of the aspects of this issue. Its focus is not on the results but on the discussion and exemplification of the analytical tools. To evaluate the relative power relations among the factions of the Reichstag under the assumption of coalitional preferences we will use following ordinal α -indices:

- $\alpha_{1) i}$ – based on the intensity of i 's preferences, i.e. on the functions $f^+(i, \omega)$,
- $\alpha_{2) i}$ – based on the intensity of preferences for i , i.e. on the functions $f^-(i, \omega)$,
- α_3 – based on total average intensity in coalition ω , i.e. on the functions $f(\omega)$.

To construct the preference profile of the factions of the Reichstag, we have to represent the preferences of each faction i as a linear order. As a first approach, we can use the closeness of the parties' positions on a one-dimension 'left-right' ideological scale. For instance, one can

order parties according to the economic policy dimension they proclaim as it is done in the list below in *Table 15*. We define r_i as the position of party i . In order to simplify the analysis, let us take a set of small regional nationalistic parties as an entity and give them the same position. Similarly, some forerunners to the NSDAP itself will be assigned a common position. We identify ten possible positions, r_i , with $i = 1, 2, \dots, 10$, the first one being the most *left*, i.e. socially-oriented position. How does this apply to the parties in the Reichstag?

The Communists stood for expropriation of private property, while the Nazi party presented itself as guarantor of private property and especially of “Big Business.” The most radical parties on this issue, the KPD and the NSDAP, receive opposite marginal positions of 1 and 10, respectively. The German Centre Party (Zentrum) is located in the middle of the scale, $r_{CP} = 5$. On the left side from the Centre Party there are the KPD, the USPD, the SPD, the DDP and the DStP (former DDP). On the right side – the DVP, a set of small regional parties, the DNVP, the Battlefront Black-White-Red (KFSWR) and the NSDAP itself. This linear order was constructed in accordance with the analysis in Vatlin (2002).

| Party (i) | position (r) |
|--|------------------|
| 1. Communist Party of Germany (KPD) | 1 |
| 2. Independent Social Democratic Party of Germany (USPD) | 2 |
| 3. Social Democratic Party of Germany (SPD) | 3 |
| 4. German Democratic Party (DDP) | 4 |
| 5. German States Party (DStP) | 4 |
| 6. German Centre Party (Zentrum) | 5 |
| 7. Christian People's Party (CVP) | 5 |
| 8. German People's Party (DVP) | 6 |
| 9. Bloc DVP-CZVD-DBP-DHP | 7 |
| 10. Bavarian Farmers' Union (BBB) | 7 |
| 11. Bavarian People's Party (BVP) | 7 |
| 12. German National People's Party (DNVP) | 8 |
| 13. Battlefront Black-White-Red (KFSWR) | 9 |
| 14. German Racial Freedom Party (DVFP) | 10 |
| 15. National Socialist Freedom Movement (NSFB) | 10 |
| 16. National Socialist German Workers' Party (NSDAP) | 10 |

Table 15. Parties' positions

Next we construct the coalitional preferences p_{ij} and then intensity functions $f(i, \omega)$. Consider parties' possibilities to coalesce as presented in *Table 16*. Position r_i on the *left-right* ideological scale serve to detect systematic preferences. To simplify the table we do not include all parties j but only those that have different positions r_j . The column ‘possibility to coalesce’ shows whether parties ever formed coalitions. Of course, in parliament with many ideologically polarized factions, such as the Reichstag, not all coalitions are likely to be possible. For example,

the SPD did not form any coalition with the KPD as well as with the Bavarian Farmers' Union or the DNVP. Instead, the alliances formed with the USPD, the DDP, Zentrum and the DVP. In the last column, we give a measure $\lambda = |r_i - r_j|$ based on the difference between two parties' positions.

In order to achieve a majority status, i.e. to pass a quota, all parties had to enter a coalition with other parties. Usually, it was possible to obtain support from ideologically close factions from the left or right. However, ideologically distant parties could hardly ever support the same policy. It is assumed that the preferences to coalesce decrease when λ increases, i.e.

- if $\lambda = 0$ or $\lambda = 1$, then $p_{ij} = 3$;
- if $\lambda = 2$, then $p_{ij} = 2$;
- if $\lambda = 3$, then $p_{ij} = 1$;
- if $\lambda > 3$, then parties are assumed not to coalesce, $p_{ij} = 0$.

| Party <i>i</i> | Party <i>j</i> | position r_j | possibility to coalesce | λ |
|---|---|----------------|-------------------------|-----------|
| Social Democratic Party of Germany (SPD), $r_i = 3$ | Communist Party of Germany (KPD) | 1 | - | 0 |
| | Independent Social Democratic Party of Germany (USPD) | 2 | + | 3 |
| | German Democratic Party (DDP) | 4 | + | 3 |
| | German Centre Party (Zentrum) | 5 | + | 2 |
| | German People's Party (DVP) | 6 | + | 1 |
| | Bavarian Farmers' Union (BBB) | 7 | - | 0 |
| | German National People's Party (DNVP) | 8 | - | 0 |
| | Battlefront Black-White-Red (KFSRW) | 9 | - | 0 |
| | National Socialist German Workers' Party (NSDAP) | 10 | - | 0 |

| Party <i>i</i> | Party <i>j</i> | position r_j | possibility to coalesce | λ |
|--|---|----------------|-------------------------|-----------|
| German National People's Party (DNVP), $r_i = 8$ | Communist Party of Germany (KPD) | 1 | - | 0 |
| | Independent Social Democratic Party of Germany (USPD) | 2 | - | 0 |
| | Social Democratic Party of Germany (SPD) | 3 | - | 0 |
| | German Democratic Party (DDP) | 4 | - | 0 |
| | German Centre Party (Zentrum) | 5 | + | 1 |
| | German People's Party (DVP) | 6 | + | 2 |
| | Bavarian Farmers' Union (BBB) | 7 | + | 3 |
| | Battlefront Black-White-Red (KFSRW) | 9 | + | 3 |
| | National Socialist German Workers' Party (NSDAP) | 10 | + | 2 |

Table 16. Possibility to coalesce between parties (continuation is given in *Appendix*)

It seems reasonable, and by and large confirmed by the historical data, to assume a threshold $\lambda = 3$ to coalition formation. Examples to justify the threshold λ are presented in *Appendix*. Some deviations from the threshold $\lambda = 3$ can be explained by a careful analysis of coalitions that emerged in the Reichstag 1919-1933. Consider some of these cases.

Case 1. The first ‘Grand coalition’ of August 13 to November 23, 1923, headed by Gustav Stresemann (member of the DVP), involved the SPD, the DDP, the Centre Party and the DVP. Their main goal was to stabilize the economic and political situation. The Munich Putsch was suppressed on November 9, 1923. In order to stop the hyperinflation, on November 16, the government issued a new currency, the Rentenmark, to substitute the Mark. The Reichstag did not approve these strict measures and the government was dissolved, but the Rentenmark policy was successful.

Case 2. The second ‘Grand coalition’ formed by Hermann Müller (member of the SPD) in 1928 was the last government formed in compliance with parliamentary procedure. Because of conflicting views on economic policy and measures for supporting the unemployed, on March 23, 1930 this government collapsed. For these parties the biggest value of λ was 3, i.e. $\lambda = |r_{SPD} - r_{DVP}| = |3 - 6| = 3$, and their preferences can be presented as follows: $p_{SPD-DVP} = 1$ and $p_{DVP-SPD} = 1$.

Case 3. The governing coalition formed on January 15, 1925 under Hans Luther consisted of the DVP, Zentrum, the BVP, the DDP and the DNVP. The SPD was not represented in the Cabinet, but the Social Democrats were loyal to Luther’s policy. The coalition existed until May 1926. Nationalistic forces in the DNVP tried to restore imperial symbols of the Reich but the DDP and the Zentrum strongly opposed this project. There was a huge gap between the Social Democrats (SPD) and the Nationalists, dominating the DNVP, so $\lambda = |r_{SPD} - r_{DNVP}| = |3 - 8| = 5$ can be assumed, and between the Democrats (DDP) and the Nationalists, so that $\lambda = |r_{DDP} - r_{DNVP}| = |4 - 8| = 4$ applies.

Case 4. Beginning from July 1932 the Zentrum and the BVP tried to form a government with the NSDAP, but the attempt failed. The Centre Party did not want to approve of Adolf Hitler’s political ambitions attacking the substance of the constitution. The NSDAP, on the other hand, used non-parliamentary means in order to put pressure on the administration. After the Reichstag election in November 1932, Adolf Hitler rejected a coalition with the Centre Party. After the Reichstag election of March 5, 1933, and some constitutional guarantees and the promise not to dissolve the party, the Zentrum supported the NSDAP in order to secure a 2/3 majority and to pass the Enabling Act on March 23, 1933. The historical data suggest $\lambda = |r_{NSDAP} - r_{Zentrum}| = |10 - 5| = 5$ for the relative positions of the two parties.

If $\lambda > 3$ and coalitions formed as described above, then we assume intensities of $p_{ij} = 1$ for the involved ideologically distant factions, i.e.

- $p_{SPD-DNVP} = 1$ and $p_{DNVP-SPD} = 1$;
- $p_{DDP-DNVP} = 1$ and $p_{DNVP-DDP} = 1$;
- $p_{Zentrum-NSDAP} = 1$ and $p_{NSDAP-Zentrum} = 1$.

In the other cases if $\lambda > 3$ and the factions did not form coalitions, then we assume $p_{ij} = 0$.

For an application, consider the results of the elections for the National Assembly on January 19, 1919 (Table 17). There are seven parties with different positions r_i that have seats in the Assembly. The total number of seats in this Reichstag was $n = 423$, and the decision-making procedure implied a simple majority rule. Hence the quota $q = 423/2 + 1 = 212$. No faction enjoyed an absolute majority in the Reichstag, thus they had to form coalitions. The total number of winning coalitions was 64.

| Party i | position r_i | seats ($n = 423$) | Share of seats |
|--|----------------|------------------------|-------------------|
| Christian People's Party (CVP) (Zentrum) | $r_{CVP} = 5$ | 91 | 0.215 |
| German Democratic Party (DDP) | $r_{DDP} = 4$ | 75 | 0.177 |
| German National People's Party (DNVP) | $r_{DNVP} = 8$ | 44 | 0.104 |
| German People's Party (DVP) | $r_{DVP} = 6$ | 19 | 0.045 |
| Social Democratic Party of Germany (SPD) | $r_{SPD} = 3$ | 165 | 0.390 |
| Independent Social Democratic Party of Germany (USPD) | $r_{USPD} = 2$ | 22 | 0.052 |
| Others | | 7 | 0.017 |

Table 17. Reichstag election of January 19, 1919

We model parties' preferences to coalesce as given in Table 18. We assume that if $\lambda = 0$ or $\lambda = 1$, then the preferences are the highest, i.e. $p_{ij} = 3$. If $\lambda = 2$, then $p_{ij} = 2$, and if $\lambda = 3$, then $p_{ij} = 1$. Parties are assumed not to coalesce if $\lambda > 3$, $p_{ij} = 0$. The intensity of p_{ij} is given in the first column of Table 18, other columns describe i 's preferences.

| p_{ij} | P_{CVP} | P_{DDP} | P_{DNVP} | P_{DVP} | P_{SPD} | P_{USPD} |
|----------|---------------|--------------|----------------------|--------------|--------------|--------------|
| 3 | DDP, DVP | SPD, CVP | | CVP | USPD, DDP | SPD |
| 2 | SPD | USPD, DVP | DVP | DDP, DNVP | CVP | DDP |
| 1 | DNVP, USPD | | CVP | SPD | DVP | CVP |
| 0 | | DNVP | DDP, SPD, USPD | USPD | DNVP | DVP, DNVP |

Table 18. Preference profile and intensities

The coalitional preferences from Table 17 can be summarized in the matrix given in Table 19.

| $i \setminus j$ | <i>CVP</i> | <i>DDP</i> | <i>DNVP</i> | <i>DVP</i> | <i>SPD</i> | <i>USPD</i> |
|-----------------|------------|------------|-------------|------------|------------|-------------|
| <i>CVP</i> | 0 | 3 | 1 | 3 | 2 | 1 |
| <i>DDP</i> | 3 | 0 | 0 | 2 | 3 | 2 |
| <i>DNVP</i> | 1 | 0 | 0 | 2 | 0 | 0 |
| <i>DVP</i> | 3 | 2 | 2 | 0 | 1 | 0 |
| <i>SPD</i> | 2 | 3 | 0 | 1 | 0 | 3 |
| <i>USPD</i> | 1 | 2 | 0 | 0 | 3 | 0 |

Table 19. Preference matrix, $\|p_{ij}\|$

Next we construct intensity functions $f(i, \omega)$ according to the formulae (2) – (4) introduced above. We obtain the intensity function values for each i . The corresponding power value χ_i is calculated by summarizing all intensity function values for the winning coalitions that have i as a swing player. Then we normalize these power values and calculate the relative power of faction i , $\alpha_{(i)}$, as defined above in (7). The results of this procedure, given in Table 20, represent the power balance in the Reichstag after January 19, 1919.

| Party | share of seats ($n = 423$) | Banzhaf | α_1 | α_2 | α_3 |
|---|---------------------------------|---------|------------|------------|------------|
| Christian People's Party (CVP) | 0.215 | 0.154 | 0.186 | 0.186 | 0.159 |
| German Democratic Party (DDP) | 0.177 | 0.154 | 0.193 | 0.193 | 0.171 |
| German National People's Party (DNVP) | 0.104 | 0.115 | 0.041 | 0.041 | 0.094 |
| German People's Party (DVP) | 0.045 | 0.038 | 0.038 | 0.038 | 0.039 |
| Social Democratic Party of Germany (SPD) | 0.390 | 0.462 | 0.512 | 0.512 | 0.482 |
| Independent Social Democratic Party of Germany (USPD) | 0.052 | 0.038 | 0.03 | 0.03 | 0.033 |
| Others | 0.017 | 0.038 | 0 | 0 | 0.023 |

Table 20. Power index values for National Assembly election on January 19, 1919

The Banzhaf index is constructed on the number of winning coalitions in which faction i is a swing player. The ordinal indices α_1 and α_2 are based on the intensity of direct and backward preferences for i , respectively, and index α_3 is based on total average intensity of winning coalition ω . For an interpretation of the results in Table 20, let us first compare the share of seats and the values of Banzhaf index. Obviously, there are considerable differences. The Christian

People's Party (DVP) and the German Democratic Party (DDP) had quite different shares of seats, but $\beta_{CVP} = \beta_{DDP} = 0.154$. These values seem to indicate a rather low influence of both factions, because they are smaller than the corresponding shares of seats in the Reichstag. On the contrary, the potential power of the Social Democratic Party of Germany, $\beta_{SPD} = 0.462$, exceeded its shares of seats of 0.390 by 18%. The German People's Party (DVP) and the Independent Social Democratic Party of Germany (USPD) had the shares of seats of 0.045 and 0.052, respectively, but the same relative power evaluated by Banzhaf index, $\beta_{DVP} = \beta_{USPD} = 0.038$.¹⁶

The Banzhaf index does not take into account agents' preferences in coalition formation, while ordinal indices allow us to differentiate factions according to their ability to coalesce. Note that the indices α_1 and α_2 give the same power values to factions in this Reichstag, because the agents' preferences are symmetric, e.g., $p_{SPD-DVP} = 1$ and $p_{DVP-SPD} = 1$. The SPD possessed almost 39% of seats in parliament, and both α -indices and Banzhaf index values are high, expressing the party's dominating position, $\alpha_{1SPD} = 0.512$, $\alpha_{3SPD} = 0.482$ and $\beta_{SPD} = 0.462$. The party provided a relatively moderate policy, and thereby caused irritations to conservative and nationalistic forces. The latter accused the Social Democrats of having been operational to the defeat in the World War and therefore did not want to join the Weimar coalition headed by the SPD. Obviously, this fact did not strongly influence the relative power of the Social Democrats as α_{3SPD} is higher than α_{1SPD} only by 6%.

Opposite results we get from the analysis of the performance of the German National People's Party (DNVP). The high value of Banzhaf index, $\beta_{DNVP} = 0.115$, does not reflect possibilities to coalesce if parties take their ideological positions and platforms into account. The DNVP did not support the policy provided by the ruling coalition and positioned itself as a challenge to the Weimar Republic. The value $\alpha_{1DNVP} = 0.041$ is by 64% smaller than the corresponding value of Banzhaf index. For the USPD there is an $\alpha_{1USPD} = 0.03$ which is smaller than its value of the Banzhaf index (which is $\beta_{USPD} = 0.038$) but the difference does not explain much in absolute terms. The USPD was rather instable as different wings struggled for the party's policy direction. Radical activists, the majority of party members, showed no willingness to coalesce with the other parliamentary parties, but intended to form a Soviet republic out of Germany.

The power values of the Christian People's Party (CVP, the forerunner to the Zentrum party) and the German Democratic Party (DDP) were relatively high. The CVP's α_{1i} index of 0.186 is more than 20% larger than its value of the Banzhaf index ($\beta_{CVP} = 0.154$). Similar results

¹⁶ The often observed phenomenon that different seat shares imply identical values of the power indices is a result of the discrete nature of the power indices. In the extreme, a vector of seat shares is a point in a continuous space, while the power indices are based on the *counting* membership in winning coalitions. (See Holler, 1985, and Berg and Holler, 1986.)

we can observe for the DDP. It is interesting to note that there is a small advantage of DDP with respect to the power based on the intensity of i 's preferences, as $\alpha_{1)CVP} = 0.186$ and $\alpha_{1)DDP} = 0.193$, although the seat shares of CVP and DDP are 0.215 and 0.177, respectively¹⁷.

The ordinal α -indices show that in the Reichstag of January 19, 1919, three larger players, were represented; they were the members of the Weimar coalition, They were together with three smaller players with different power values revealing their potential to participate in a winning coalition. This result fits the evaluation of that period given in Vatlin (2002).

Let us consider now the case with asymmetric preferences in coalition formation, i.e. $p_{ij} \neq p_{ji}$. The Reichstag after the fatal election on March 5, 1933, seems to be a good example. Results and preferences are presented in *Tables 21* and *Table 22*, respectively.

| Party | position r_i | seats ($n = 647$) | share |
|--|-------------------|---------------------|-------|
| Bavarian People's Party (BVP) | $r_{BVP} = 7$ | 18 | 0.028 |
| Bloc DVP-CZVD-DBP-DHP | $R_{DVP} = 7$ | 2 | 0.003 |
| German Centre Party (Zentrum) | $r_{Zentrum} = 5$ | 74 | 0.114 |
| Battlefront Black-White-Red (KFSWR) | $r_{KFSWR} = 9$ | 52 | 0.080 |
| Communist Party of Germany (KPD) | $r_{KDP} = 1$ | 81 | 0.125 |
| German States Party (former DDP) | $r_{DSIP} = 4$ | 5 | 0.008 |
| National Socialist German Workers' Party (NSDAP) | $r_{NSDAP} = 10$ | 288 | 0.445 |
| Social Democratic Party of Germany (SPD) | $r_{SPD} = 3$ | 120 | 0.185 |
| Others | | 7 | 0.010 |

Table 21. Reichstag election on March 5, 1933

Table 22 shows asymmetric preferences for the Zentrum and the Social Democrats (SDP), i.e. $p_{Zentrum-SPD} \neq p_{SPD-Zentrum}$, $2 \neq 3$. After most of the communist deputies were arrested or murdered and none of them was allowed to take a seat, the number of seats in the Reichstag was decreased to $n = 566$, the NSDAP urged the Zentrum to join its coalition to provide the two-thirds majority in the parliament, $q_{const} = 2 \cdot 566 / 3 + 1 = 377$ seats. This would help the Nazis to pass the Ermächtigungsgesetz that gave dictatorial power to Adolf Hitler, in compliance with the democratic procedures. Assume $p_{NSDAP-Zentrum} = 1$ and $p_{Zentrum-NSDAP} = 1$. The SPD was opposing the coalition in favor of Ermächtigungsgesetz.¹⁸ Still we can assume a strong preference of the

¹⁷ We can conclude that $\alpha_{1)i}$ does not satisfy local monotonicity. (For a discussion of local monotonicity, see Holler and Napel, 2004).

¹⁸ It is said that on the occasion of the parliamentary discussion of the Ermächtigungsgesetz, Otto Wels, the head of SPD party, did the last free speech at the Reichstag. All 91 representatives of the SDP present voted against the Law. 26 elected SPD were already imprisoned or on the run. Of course, Otto Wels had to leave the country and died in his exile at Paris. He is not widely remembered although there are some memorials and some streets named after him.

SPD in favour of the Centre Party, and thus $p_{SPD-Zentrum} = 3$, but a more moderate preference of the Zentrum with respect to the SPD, expressed by $p_{Zentrum-SPD} = 2$.

| <i>i</i> \ <i>j</i> | <i>BVP</i> | <i>DVP-CZVD-DBP-DHP</i> | <i>Zentrum</i> | <i>KFSWR</i> | <i>KPD</i> | <i>DStP</i> | <i>NSDAP</i> | <i>SPD</i> |
|-------------------------|------------|-------------------------|----------------|--------------|------------|-------------|--------------|------------|
| <i>BVP</i> | | 3 | 2 | 2 | 0 | 1 | 1 | 0 |
| <i>DVP-CZVD-DBP-DHP</i> | 3 | | 2 | 2 | 0 | 1 | 1 | 0 |
| <i>Zentrum</i> | 2 | 2 | | 1 | 0 | 3 | 1 | 2 |
| <i>KFSWR</i> | 2 | 2 | 1 | | 0 | 0 | 3 | 0 |
| <i>KPD</i> | 0 | 0 | 0 | 0 | | 0 | 0 | 0 |
| <i>DStP</i> | 1 | 1 | 3 | 0 | 0 | | 0 | 3 |
| <i>NSDAP</i> | 1 | 1 | 1 | 3 | 0 | 0 | | 0 |
| <i>SPD</i> | 0 | 0 | 3 | 0 | 0 | 3 | 0 | |

Table 22. Preference matrix, $\|p_{ij}\|$

The ordinal indices α_1 and α_2 are not the same for the Zentrum and for the SPD, i.e. $\alpha_1 \neq \alpha_2$. This implication is evident from Table 23.

| Party | share of seats (<i>n</i> = 647) | Banzhaf | α_1 | α_2 | α_3 |
|--|-------------------------------------|---------|------------|------------|------------|
| Bavarian People's Party (BVP) | 0.028 | 0 | 0 | 0 | 0 |
| Bloc DVP-CZVD-DBP-DHP | 0.003 | 0 | 0 | 0 | 0 |
| German Centre Party (Zentrum) | 0.114 | 0.091 | 0.161 | 0.174 | 0.101 |
| Battlefront Black-White-Red (KFSWR) | 0.080 | 0.091 | 0.132 | 0.132 | 0.106 |
| Communist Party of Germany (KPD) | 0.125 | 0.091 | 0 | 0 | 0.069 |
| German States Party | 0.008 | 0 | 0 | 0 | 0 |
| National Socialist German Workers' Party (NSDAP) | 0.445 | 0.636 | 0.625 | 0.625 | 0.645 |
| Social Democratic Party of Germany (SPD) | 0.185 | 0.091 | 0.082 | 0.070 | 0.079 |
| Others | 0.010 | 0 | 0 | 0 | 0 |

Table 23. Power index values for Reichstag election on March 5, 1933

The last elections in the Weimar Germany took place under overwhelming influence of the NSDAP on country's political life, particularly in the Reichstag: the Banzhaf index value for the NSDAP was equal to 0.636. Obviously, both Banzhaf and α -indices (α_1 , α_2 , α_3) values exceed the share of seats. The NSDAP was inevitable but not the most desirable player in all real

winning coalitions. This is reflected by the values $\alpha_{1)NSDAP} = \alpha_{2)NSDAP} = 0.625$ which are a bit lower than the value of the Banzhaf index. The NSDAP and its closest supporter, the KFSWR, formed a winning coalition. The quota was 324 and the joint number of seats for these parties was 340. The KFSWR had substantially strengthened its position by its ideological proximity to the NSDAP. This is confirmed by the values $\alpha_{1)KFSWR} = \alpha_{2)KFSWR} = 0.132$, both larger than the vote share of 0.08 and the value of the Banzhaf index, i.e., $\beta_{KFSWR} = 0.091$.

The relative power of the Zentrum within the Reichstag by both $\alpha_{1)Zentrum} = 0.161$ and $\alpha_{2)Zentrum} = 0.174$ was much higher than by Banzhaf index ($\beta_{Zentrum} = 0.091$), this seems to be reasonable because of Zentrum's decisive position on the issue of the Enabling Act and other laws that needed a 2/3 majority to pass. Note that $\alpha_{1)Zentrum} \neq \alpha_{2)Zentrum}$ due to our assumption denote above $p_{Zentrum-SPD} \neq p_{SPD-Zentrum}$.

In 1933, the SPD and the KPD were the only factions not to coalesce with the Nazi faction. They are evaluated as equal by Banzhaf index, but their $\alpha_{1)}$ were rather different, i.e., 0.082 and 0, respectively. There were no factions that wanted to form a coalition with the Social Democrats except, to some extent, the Zentrum and, perhaps, but rather restricted the DDP. As result power value of the SPD that takes into consideration "reciprocal preferences" ($\alpha_{2)SPD} = 0.070$) was smaller than the power value based on direct preferences, $\alpha_{1)SPD}$, and the value of the Banzhaf index. The absence of any preferences of and for the KPD gives zero values to both $\alpha_{1)KPD}$ and $\alpha_{2)KPD}$. Note that it was impossible to form a coalition with the KPD because, after election of March 5, 1933, the party's elected representatives were not allowed to take their seats in the Reichstag. The KPD was accused of having supported the Reichstag Fire on February 27 and on March 8, 1933 the party lost its legal status and became non-existing from an official point of view.

More detailed analysis of the Reichstag after each of the elections during the period under study is given in Appendix.

6. Conclusions

Our study was motivated by an interest in the historical case as well as in the theoretical tools. We want to get a better understanding of the historical facts and, more general, of how a parliamentary system *in nascendi* works and why it might fail. We also want to learn about the properties of the theoretical instruments we applied, i.e. the Banzhaf index proper and some of

its variations which take into account the preferences of parties to coalesce. However, there is also the expectation that this study will be of help to future designers of parliamentary systems.

The analysis of power distribution using preference-based indices shows substantial differences in comparison with the values of Banzhaf index. This results from the fact that some major parties in the Reichstag did not coalesce, and were not able to do so, even when the Weimar Republic was threatened by political collapse. This paved the way to Hitler's regime.

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Appendix

Table 16. Possibility to coalesce between parties (continuation)

| Party (<i>i</i>) | Party (<i>j</i>) | position (r_j) | possibility to coalesce | λ |
|--|---|--------------------|-------------------------|-----------|
| Communist Party of Germany (KPD) $r_i = 1$ | Independent Social Democratic Party of Germany (USDP) | 2 | - | 0 |
| | Social Democratic Party of Germany (SPD) | 3 | - | 0 |
| | German Democratic Party (DDP) | 4 | - | 0 |
| | German Centre Party (Zentrum) | 5 | - | 0 |
| | German People's Party (DVP) | 6 | - | 0 |
| | Bavarian Farmers' Union (BBB) | 7 | - | 0 |
| | German National People's Party (DNVP) | 8 | - | 0 |
| | Battlefront Black-White-Red (KFSWR) | 9 | - | 0 |
| | National Socialist German Workers' Party (NSDAP) | 10 | - | 0 |
| Independent Social Democratic Party of Germany (USDP) $r_i = 2$ | Communist Party of Germany (KPD) | 1 | - | 0 |
| | Social Democratic Party of Germany (SPD) | 3 | + | 3 |
| | German Democratic Party (DDP) | 4 | + | 2 |
| | German Centre Party (Zentrum) | 5 | + | 1 |
| | German People's Party (DVP) | 6 | - | 0 |
| | Bavarian Farmers' Union (BBB) | 7 | - | 0 |
| | German National People's Party (DNVP) | 8 | - | 0 |
| | Battlefront Black-White-Red (KFSWR) | 9 | - | 0 |
| | National Socialist German Workers' Party (NSDAP) | 10 | - | 0 |
| German Democratic Party (DDP) $r_i = 4$ | Communist Party of Germany (KPD) | 1 | - | 0 |
| | Independent Social Democratic Party of Germany (USDP) | 2 | + | 2 |
| | Social Democratic Party of Germany (SPD) | 3 | + | 3 |
| | German Centre Party (Zentrum) | 5 | + | 3 |
| | German People's Party (DVP) | 6 | + | 2 |
| | Bavarian Farmers' Union (BBB) | 7 | + | 1 |
| | German National People's Party (DNVP) | 8 | - | 0 |
| | Battlefront Black-White-Red (KFSWR) | 9 | - | 0 |
| | National Socialist German Workers' Party (NSDAP) | 10 | - | 0 |
| German Centre Party (Zentrum) $r_i = 5$ | Communist Party of Germany (KPD) | 1 | - | 0 |
| | Independent Social Democratic Party of Germany (USDP) | 2 | + | 1 |
| | Social Democratic Party of Germany (SPD) | 3 | + | 2 |
| | German Democratic Party (DDP) | 4 | + | 3 |
| | German People's Party (DVP) | 6 | + | 3 |
| | Bavarian Farmers' Union (BBB) | 7 | + | 2 |
| | German National People's Party (DNVP) | 8 | + | 1 |
| | Battlefront Black-White-Red (KFSWR) | 9 | - | 0 |
| | National Socialist German Workers' Party (NSDAP) | 10 | - | 0 |
| German People's | Communist Party of Germany (KPD) | 1 | - | 0 |

| | | | | |
|--------------------------|---|----|---|---|
| Party (DVP) $r_i = 6$ | Independent Social Democratic Party of Germany (USDP) | 2 | - | 0 |
| | Social Democratic Party of Germany (SPD) | 3 | + | 1 |
| | German Democratic Party (DDP) | 4 | + | 2 |
| | German Centre Party (Zentrum) | 5 | + | 3 |
| | Bavarian Farmers' Union (BBB) | 7 | + | 3 |
| | German National People's Party (DNVP) | 8 | + | 2 |
| | Battlefront Black-White-Red (KFSWR) | 9 | + | 1 |
| | National Socialist German Workers' Party (NSDAP) | 10 | - | 0 |

| Party (i) | Party (j) | position (r_j) | possibility to coalesce | λ |
|--|---|--------------------|-------------------------|-----------|
| Bavarian Farmers' Union $r_i = 7$ | Communist Party of Germany (KPD) | 1 | - | 0 |
| | Independent Social Democratic Party of Germany (USDP) | 2 | - | 0 |
| | Social Democratic Party of Germany (SPD) | 3 | - | 0 |
| | German Democratic Party (DDP) | 4 | + | 1 |
| | German Centre Party (Zentrum) | 5 | + | 2 |
| | German People's Party (DVP) | 6 | + | 3 |
| | German National People's Party (DNVP) | 8 | + | 3 |
| | Battlefront Black-White-Red (KFSWR) | 9 | + | 2 |
| National Socialist German Workers' Party (NSDAP) | 10 | + | 1 | |

| Party (i) | Party (j) | position (r_j) | possibility to coalesce | λ |
|--|---|--------------------|-------------------------|-----------|
| Battlefront Black-White-Red (KFSWR) $r_i = 9$ | Communist Party of Germany (KPD) | 1 | - | 0 |
| | Independent Social Democratic Party of Germany (USDP) | 2 | - | 0 |
| | Social Democratic Party of Germany (SPD) | 3 | - | 0 |
| | German Democratic Party (DDP) | 4 | - | 0 |
| | German Centre Party (Zentrum) | 5 | - | 0 |
| | German People's Party (DVP) | 6 | + | 1 |
| | Bavarian Farmers' Union (BBB) | 7 | + | 2 |
| | German National People's Party (DNVP) | 8 | + | 3 |
| National Socialist German Workers' Party (NSDAP) | 10 | + | 3 | |

| Party (i) | Party (j) | position (r_j) | possibility to coalesce | λ |
|--|---|--------------------|-------------------------|-----------|
| National Socialist German Workers' Party (NSDAP) $r_i = 10$ | Communist Party of Germany (KPD) | 1 | - | 0 |
| | Independent Social Democratic Party of Germany (USDP) | 2 | - | 0 |
| | Social Democratic Party of Germany (SPD) | 3 | - | 0 |
| | German Democratic Party (DDP) | 4 | - | 0 |
| | German Centre Party (Zentrum) | 5 | - | 0 |
| | German People's Party (DVP) | 6 | - | 0 |
| | Bavarian Farmers' Union (BBB) | 7 | + | 1 |
| | German National People's Party (DNVP) | 8 | + | 2 |
| Battlefront Black-White-Red (KFSWR) | 9 | + | 3 | |

Power index values

Reichstag election of June 6, 1920

Preference matrix

| | BVP | DDP | DNVP | DVP | Zentrum | KPD | SPD | USPD | Others |
|---------|-----|-----|------|-----|---------|-----|-----|------|--------|
| BVP | | 1 | 3 | 3 | 2 | 0 | 0 | 0 | 0 |
| DDP | 1 | | 0 | 2 | 3 | 0 | 3 | 2 | 0 |
| DNVP | 3 | 0 | | 2 | 1 | 0 | 0 | 0 | 0 |
| DVP | 3 | 2 | 2 | | 3 | 0 | 1 | 0 | 0 |
| Zentrum | 2 | 3 | 1 | 3 | | 0 | 2 | 1 | 0 |
| KPD | 0 | 0 | 0 | 0 | 0 | | 0 | 0 | 0 |
| SPD | 0 | 3 | 0 | 1 | 2 | 0 | | 3 | 0 |
| USPD | 0 | 2 | 0 | 0 | 1 | 0 | 3 | | 0 |
| Others | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |

| Party | Banzhaf β | α_1 | α_2 | α_3 |
|---------|-----------------|------------|------------|------------|
| BVP | 0.047 | 0.047 | 0.053 | 0.046 |
| DDP | 0.062 | 0.077 | 0.079 | 0.066 |
| DNVP | 0.155 | 0.102 | 0.104 | 0.138 |
| DVP | 0.147 | 0.180 | 0.183 | 0.155 |
| Zentrum | 0.147 | 0.198 | 0.202 | 0.160 |
| KPD | 0.008 | 0 | 0 | 0.005 |
| SPD | 0.248 | 0.277 | 0.257 | 0.254 |
| USPD | 0.171 | 0.119 | 0.121 | 0.162 |
| Others | 0.016 | 0 | 0 | 0.014 |

Reichstag election of May 4, 1924

Preference matrix

| | BVP | DDP | DNVP | DVP | Zentrum | KPD | SPD | DVFP and NSDAP | Others |
|----------------|-----|-----|------|-----|---------|-----|-----|----------------|--------|
| BVP | | 1 | 3 | 3 | 2 | 0 | 0 | 1 | 0 |
| DDP | 1 | | 0 | 2 | 3 | 0 | 3 | 0 | 0 |
| DNVP | 3 | 0 | | 2 | 1 | 0 | 0 | 2 | 0 |
| DVP | 3 | 2 | 2 | | 3 | 0 | 1 | 0 | 0 |
| Zentrum | 2 | 3 | 1 | 3 | | 0 | 2 | 0 | 0 |
| KPD | 0 | 0 | 0 | 0 | 0 | | 0 | 0 | 0 |
| SPD | 0 | 3 | 0 | 1 | 2 | 0 | | 0 | 0 |
| DVFP and NSDAP | 1 | 0 | 2 | 0 | 0 | 0 | 0 | | 0 |
| Others | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |

| Party | Banzhaf β | α_1 | α_2 | α_3 |
|-------|-----------------|------------|------------|------------|
| BVP | 0.043 | 0.064 | 0.064 | 0.047 |
| DDP | 0.050 | 0.070 | 0.070 | 0.055 |

| | | | | |
|----------------|-------|-------|-------|-------|
| DNVP | 0.206 | 0.243 | 0.243 | 0.209 |
| DVP | 0.091 | 0.153 | 0.153 | 0.108 |
| Zentrum | 0.139 | 0.231 | 0.231 | 0.162 |
| KPD | 0.121 | 0 | 0 | 0.093 |
| SPD | 0.232 | 0.210 | 0.210 | 0.227 |
| DVFP and NSDAP | 0.065 | 0.029 | 0.029 | 0.055 |
| Others | 0.054 | 0 | 0 | 0.044 |

Reichstag election of December 7, 1924

Preference matrix

| | BVP | DDP | DNVP | DVP | Zentrum | KPD | NSDAP | SPD | Others |
|---------|-----|-----|------|-----|---------|-----|-------|-----|--------|
| BVP | | 1 | 3 | 3 | 2 | 0 | 1 | 1 | 0 |
| DDP | 1 | | 0 | 2 | 3 | 0 | 0 | 3 | 0 |
| DNVP | 3 | 0 | | 2 | 1 | 0 | 2 | 1 | 0 |
| DVP | 3 | 2 | 2 | | 3 | 0 | 0 | 1 | 0 |
| Zentrum | 2 | 3 | 1 | 3 | | 0 | 0 | 2 | 0 |
| KPD | 0 | 0 | 3 | 0 | 0 | | 0 | 0 | 0 |
| NSDAP | 1 | 0 | 2 | 0 | 0 | 0 | | 0 | 0 |
| SPD | 1 | 3 | 1 | 1 | 2 | 0 | 0 | | 0 |
| Others | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |

| Party | Banzhaf β | α_1 | α_2 | α_3 |
|---------|-----------------|------------|------------|------------|
| BVP | 0.040 | 0.053 | 0.051 | 0.043 |
| DDP | 0.063 | 0.072 | 0.069 | 0.065 |
| DNVP | 0.206 | 0.224 | 0.285 | 0.218 |
| DVP | 0.095 | 0.129 | 0.124 | 0.103 |
| Zentrum | 0.143 | 0.195 | 0.187 | 0.156 |
| KPD | 0.079 | 0.029 | 0 | 0.062 |
| NSDAP | 0.024 | 0.009 | 0.009 | 0.020 |
| SPD | 0.294 | 0.288 | 0.275 | 0.292 |
| Others | 0.056 | 0 | 0 | 0.041 |

Reichstag election of May 20, 1928

Preference matrix

| | BVP | DDP | DNVP | DVP | Zentrum | KPD | NSDAP | SPD | Others |
|------|-----|-----|------|-----|---------|-----|-------|-----|--------|
| BVP | | 1 | 3 | 3 | 2 | 0 | 1 | 0 | 0 |
| DDP | 1 | | 0 | 2 | 3 | 0 | 0 | 3 | 0 |
| DNVP | 3 | 0 | | 2 | 1 | 0 | 2 | 0 | 0 |
| DVP | 3 | 2 | 2 | | 3 | 0 | 0 | 1 | 0 |

| | | | | | | | | | |
|---------|---|---|---|---|---|---|---|---|---|
| Zentrum | 2 | 3 | 1 | 3 | | 0 | 0 | 2 | 0 |
| KPD | 0 | 0 | 0 | 0 | 0 | | 0 | 0 | 0 |
| NSDAP | 1 | 0 | 2 | 0 | 0 | 0 | | 0 | 0 |
| SPD | 0 | 3 | 0 | 1 | 2 | 0 | 0 | | 0 |
| Others | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |

| Party | Banzhaf β | α_1 | α_2 | α_3 |
|---------|-----------------|------------|------------|------------|
| BVP | 0.025 | 0.040 | 0.040 | 0.031 |
| DDP | 0.042 | 0.064 | 0.064 | 0.044 |
| DNVP | 0.127 | 0.156 | 0.156 | 0.132 |
| DVP | 0.093 | 0.161 | 0.161 | 0.106 |
| Zentrum | 0.110 | 0.197 | 0.197 | 0.132 |
| KPD | 0.102 | 0 | 0 | 0.080 |
| NSDAP | 0.017 | 0.008 | 0.008 | 0.016 |
| SPD | 0.390 | 0.373 | 0.373 | 0.385 |
| Others | 0.093 | 0 | 0 | 0.074 |

Reichstag election of September 14, 1930

Preference matrix

| | BVP | DNVP | GStP | DVP | Zentrum | KPD | NSDAP | SPD | Others |
|---------|-----|------|------|-----|---------|-----|-------|-----|--------|
| BVP | | 3 | 1 | 3 | 2 | 0 | 1 | 0 | 0 |
| DNVP | 3 | | 0 | 3 | 1 | 0 | 2 | 0 | 0 |
| GStP | 1 | 0 | | 2 | 3 | 0 | 0 | 3 | 3 |
| DVP | 3 | 2 | 2 | | 3 | 0 | 0 | 1 | 1 |
| Zentrum | 2 | 1 | 3 | 3 | | 0 | 0 | 2 | 2 |
| KPD | 0 | 0 | 0 | 0 | 0 | | 0 | 0 | 0 |
| NSDAP | 1 | 2 | 0 | 0 | 0 | 0 | | 0 | 0 |
| SPD | 0 | 0 | 3 | 1 | 2 | 0 | 0 | | |
| Others | 0 | 0 | 3 | 1 | 2 | 0 | 0 | | |

| Party | Banzhaf β | α_1 | α_2 | α_3 |
|---------|-----------------|------------|------------|------------|
| BVP | 0,038 | 0,074 | 0,074 | 0,042 |
| DNVP | 0,069 | 0,107 | 0,107 | 0,076 |
| GStP | 0,038 | 0,070 | 0,070 | 0,042 |
| DVP | 0,046 | 0,098 | 0,098 | 0,053 |
| Zentrum | 0,107 | 0,229 | 0,229 | 0,129 |
| KPD | 0,130 | 0,000 | 0,000 | 0,108 |
| NSDAP | 0,183 | 0,106 | 0,106 | 0,168 |
| SPD | 0,275 | 0,316 | 0,316 | 0,284 |
| Others | 0,115 | 0,000 | 0,000 | 0,097 |

Reichstag election of July 31, 1932

Preference matrix

| | BVP | DNVP | GStP | DVP | Zentrum | KPD | NSDAP | SPD | Others |
|---------|-----|------|------|-----|---------|-----|-------|-----|--------|
| BVP | | 3 | 0 | 1 | 2 | 0 | 1 | 0 | 0 |
| DNVP | 3 | | 0 | 0 | 1 | 0 | 2 | 0 | 0 |
| GStP | 1 | 0 | | 0 | 3 | 0 | 0 | 3 | 0 |
| DVP | 3 | 3 | 2 | | 1 | 0 | 0 | 1 | 0 |
| Zentrum | 2 | 1 | 3 | 3 | | 0 | 0 | 2 | 0 |
| KPD | 0 | 0 | 0 | 0 | 0 | | 0 | 0 | 0 |
| NSDAP | 1 | 2 | 0 | 0 | 0 | 0 | | 0 | 0 |
| SPD | 0 | 0 | 3 | 1 | 2 | 0 | 0 | | 0 |
| Others | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |

| Party | Banzhaf β | α_1 | α_2 | α_3 |
|---------|-----------------|------------|------------|------------|
| BVP | 0.013 | 0.021 | 0.032 | 0.016 |
| DNVP | 0.013 | 0.015 | 0.028 | 0.015 |
| GStP | 0.003 | 0.007 | 0.011 | 0.005 |
| DVP | 0.008 | 0.021 | 0.008 | 0.012 |
| Zentrum | 0.150 | 0.382 | 0.334 | 0.181 |
| KPD | 0.150 | 0 | 0 | 0.133 |
| NSDAP | 0.501 | 0.347 | 0.370 | 0.468 |
| SPD | 0.150 | 0.206 | 0.219 | 0.157 |
| Others | 0.013 | 0 | 0 | 0.013 |

Preference matrix (if we assume $r_{Others} = 7$)

| | BVP | DNVP | GStP | DVP | Zentrum | KPD | NSDAP | SPD | Others |
|---------|-----|------|------|-----|---------|-----|-------|-----|--------|
| BVP | | 3 | 0 | 1 | 2 | 0 | 1 | 0 | 3 |
| DNVP | 3 | | 0 | 0 | 1 | 0 | 2 | 0 | 2 |
| GStP | 1 | 0 | | 0 | 3 | 0 | 0 | 3 | 1 |
| DVP | 3 | 3 | 2 | | 1 | 0 | 0 | 1 | 3 |
| Zentrum | 2 | 1 | 3 | 3 | | 0 | 0 | 2 | 2 |
| KPD | 0 | 0 | 0 | 0 | 0 | | 0 | 0 | 0 |
| NSDAP | 1 | 2 | 0 | 0 | 0 | 0 | | 0 | 1 |
| SPD | 0 | 0 | 3 | 1 | 2 | 0 | 0 | | 0 |
| Others | 3 | 2 | 1 | 3 | 2 | 0 | 1 | 0 | |

| Party | Banzhaf β | α_1 | α_2 | α_3 |
|-------|-----------------|------------|------------|------------|
| BVP | 0.038 | 0.067 | 0.067 | 0.042 |

| | | | | |
|---------|-------|-------|-------|-------|
| DNVP | 0.069 | 0.094 | 0.094 | 0.074 |
| GStP | 0.038 | 0.055 | 0.055 | 0.040 |
| DVP | 0.046 | 0.088 | 0.088 | 0.053 |
| Zentrum | 0.107 | 0.191 | 0.191 | 0.124 |
| KPD | 0.130 | 0.000 | 0.000 | 0.106 |
| NSDAP | 0.183 | 0.100 | 0.100 | 0.168 |
| SPD | 0.275 | 0.223 | 0.223 | 0.265 |
| Others | 0.115 | 0.183 | 0.183 | 0.128 |

Reichstag election of July 31, 1932

Preference matrix

| | BVP | DNVP | GStP | DVP | Zentrum | KPD | NSDAP | SPD | Others |
|---------|-----|------|------|-----|---------|-----|-------|-----|--------|
| BVP | | 3 | 1 | 3 | 2 | 0 | 1 | 0 | 0 |
| DNVP | 3 | | 0 | 2 | 1 | 0 | 2 | 0 | 0 |
| GStP | 1 | 0 | | 2 | 3 | 0 | 0 | 3 | 0 |
| DVP | 3 | 2 | 2 | | 3 | 0 | 0 | 1 | 0 |
| Zentrum | 2 | 1 | 3 | 3 | | 0 | 0 | 2 | 0 |
| KPD | 0 | 0 | 0 | 0 | 0 | | 0 | 0 | 0 |
| NSDAP | 1 | 2 | 0 | 0 | 0 | 0 | | 0 | 0 |
| SPD | 0 | 0 | 3 | 1 | 2 | 0 | 0 | | 0 |
| Others | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |

| Party | Banzhaf β | α_1 | α_2 | α_3 |
|---------|-----------------|------------|------------|------------|
| BVP | 0.013 | 0.029 | 0.029 | 0.017 |
| DNVP | 0.013 | 0.022 | 0.022 | 0.015 |
| GStP | 0.003 | 0.010 | 0.010 | 0.005 |
| DVP | 0.008 | 0.021 | 0.021 | 0.012 |
| Zentrum | 0.150 | 0.375 | 0.375 | 0.181 |
| KPD | 0.150 | 0.000 | 0.000 | 0.134 |
| NSDAP | 0.501 | 0.341 | 0.341 | 0.469 |
| SPD | 0.150 | 0.202 | 0.202 | 0.155 |
| Others | 0.013 | 0.000 | 0.000 | 0.013 |

Reichstag election of November 6, 1932

Preference matrix

| | BVP | DNVP | GStP | DVP | Zentrum | KPD | NSDAP | SPD | Others |
|-----|-----|------|------|-----|---------|-----|-------|-----|--------|
| BVP | | 3 | 1 | 3 | 2 | 0 | 1 | 0 | 0 |

| | | | | | | | | | |
|---------|---|---|---|---|---|---|---|---|---|
| DNVP | 3 | | 0 | 2 | 1 | 0 | 2 | 0 | 0 |
| GStP | 1 | 0 | | 2 | 3 | 0 | 0 | 3 | 0 |
| DVP | 3 | 2 | 2 | | 3 | 0 | 0 | 1 | 0 |
| Zentrum | 2 | 1 | 3 | 3 | | 0 | 0 | 2 | 0 |
| KPD | 0 | 0 | 0 | 0 | 0 | | 0 | 0 | 0 |
| NSDAP | 1 | 2 | 0 | 0 | 0 | 0 | | 0 | 0 |
| SPD | 0 | 0 | 3 | 1 | 2 | 0 | 0 | | 0 |
| Others | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |

| Party | Banzhaf β | α_1 | α_2 | α_3 |
|--------------|-----------------------------------|------------------------------|------------------------------|------------------------------|
| BVP | 0.033 | 0.076 | 0.076 | 0.036 |
| DNVP | 0.051 | 0.098 | 0.098 | 0.054 |
| GStP | 0.005 | 0.011 | 0.011 | 0.007 |
| DVP | 0.014 | 0.037 | 0.037 | 0.017 |
| Zentrum | 0.098 | 0.245 | 0.245 | 0.125 |
| KPD | 0.191 | 0.000 | 0.000 | 0.166 |
| NSDAP | 0.405 | 0.275 | 0.275 | 0.386 |
| SPD | 0.191 | 0.259 | 0.259 | 0.199 |
| Others | 0.014 | 0.000 | 0.000 | 0.011 |