Putting paradoxes into perspective

Looking forward to the UK's referendum on the Alternative Vote

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Abstract

The authors of this paper are engaged in the campaign for a better voting system for UK general elections and they comment from that perspective. They describe the background to the proposal to hold a referendum in the UK on changing the voting system from First-Past-the-Post (i.e. Plurality) to the Alternative Vote (AV), noting the reasons for dissatisfaction with FPTP.

They examine other alternatives to FPTP that could have been chosen. They conclude that other systems that do not involve the ranking of candidates by voters could be as problematic as FPTP if not more so, while systems that involve more than one ballot would be politically unacceptable. It is argued that systems based on pair-wise comparisons are not desirable for public elections: voters may not be as discriminating in the ordering of their lower preferences, yet these rankings can have as much effect on the outcome of an election as a voter's higher preferences. Borda suffers from similar problems.

Although AV may be susceptible to a number of paradoxes, it is argued that these do not appear to be of a serious nature and that consequently, if attention is confined to systems for electing only one of several candidates, the choice of AV is a sound one.

1. Introduction

We approach the subject of this seminar not as academics concerned with the mathematical properties of different voting systems, but as electoral reformers wanting to improve the quality of our democracy. As such, paradoxes are important but, we will argue, they are not the principal characteristics of voting systems that determine whether they are good or bad.

Our paper is written in response to Dan Felsenthal 's "Review of Paradoxes Afflicting Various Voting Procedures" – a paper which provides an interesting analysis of paradoxes. The paper, however, raised questions for us as electoral reformers: 'paradox' is a word that has hardly featured in debates on voting systems in the UK, but has this been a failing of the UK debate or are paradoxes merely interesting curios which have little significance when set against the wider questions of what we want from a voting system?

In the UK there will be, subject to a bill being passed by parliament, a referendum on changing the system for electing MPs from First-Past-the-Post (FPTP) to the Alternative Vote (AV). Dan Felsenthal, however, describes AV as 'highly paradoxical'. Is he right, and if so does it matter? Our paper questions whether a better choice of system could have been made.

We will:

1. Give a brief description of the background to the UK government's proposal for a referendum on the voting system;

¹ In the UK, Plurality is normally referred to as First-Past-the-Post, and STV in single-member constituencies as the Alternative Vote. In this paper we use this more popular terminology. The use of 'AV' for the Alternative Vote should not be confused with Approval Voting, also abbreviated to 'AV' in some literature.

- 2. Examine the reasons why there is much dissatisfaction with the UK's 'first-past-the-post' system and the extent to which the proposed use of AV would overcome the problems of FPTP;
- 3. Argue that no other known non-ranked voting methods would be a satisfactory alternative to FPTP.
- 4. Argue that voting systems based on pair-wise comparisons are not suitable for use in public elections with large electorates and more than a small number of candidates;
- 5. Illustrate why Borda is at least as unacceptable;
- 6. Consider the case for AV for general elections in the UK in the light of the possible alternatives and of the paradoxes which afflict AV.

Firstly, however, we make a general observation. Dan Felsenthal's paper is concerned with voting systems for electing one candidate. No system that elects only one candidate can guarantee a proportional result. For us, the election of a national parliament or local authority requires at least a degree of proportionality, otherwise we risk the election of a body that will not be representative of the diversity of views of the electorate, that may not give any voice to significant but minority opinions, and that consequently may not be effective in providing a body that can hold broad debates and hold executives to account.

There are, of course, situations in which an election will be held for only a single position: in London and a number of local authorities in Britain we elect mayors (unfortunately British democracy has not advanced as far as allowing us elections for our head of state) and it is clearly important that we use the most suitable system we can find for such elections.

Some of the systems considered here have variants for use when several candidates must be elected simultaneously. Of these we consider STV the most superior, but in this paper we confine our attention to elections for single positions.

2. The background to the UK government's proposal for a referendum on the voting system.

Parliamentary debates on how Members of Parliament (MPs) are elected are not new in the UK but, prior to 1997 when Labour was elected after 18 years of Conservative government, they have been exceedingly rare. In 1919 and in 1931, the Commons (the lower house) voted in favour of AV, but in each case the Lords (the upper house) wanted STV and the result was no progress. Until the middle of the last century a small number of MPs were elected in 2-member constituencies and 12 were elected by STV to 'university seats', but since 1951 all MPs have been elected by FPTP (First-Past-the-Post, as single-member plurality is called in the UK).

Not even in 1951 and 1974 when FPTP resulted in a 'wrong winner' (a party winning an absolute majority of seats in spite of another party receiving more votes) was there any serious challenge to FPTP. Britain had an essentially two-party system – not until 1997 did Labour and the Conservatives together win less than 90% of the seats – and both of the

main parties were reluctant to consider any reform that could change the nature of the 'two horse race' of British politics.

Prior to the 1997 election, however, Labour, fearful of the consequences of yet another defeat, was anxious to secure Liberal Democrat backing should they fail to win an absolute majority of seats. They agreed with the Liberal Democrats that a new Labour government would set up a commission to recommend an alternative voting system which would be put to the electorate in a referendum.

Following Labour's 1997 victory, the Jenkins Commission was established and it recommended a new system, the Alternative Vote Plus (AV+), a form of Mixed Member Proportional whereby 80 – 85% of MPs would be elected in constituencies using AV and the remainder would be elected from regional party lists to compensate for the disproportionality that might arise from AV. But, finding that 43% of the votes had given them 63% of the seats, Labour had little appetite for changing the system that had rewarded them so generously and consequently the referendum was never held.

Labour must nevertheless be given credit for the reforms it introduced in its first term (1997 – 2001), even if the reforms were driven by reasons of political expediency rather than concerns for the quality of democracy. The Scottish Parliament and the Welsh Assembly, created with powers devolved from central government, were elected by Mixed Member Proportional systems (referred to in the UK as AMS – the Additional Member System) and the Northern Ireland Assembly was established and elected by STV in 6-member constituencies. A regional closed-list system was introduced for European Parliament elections, other than in Northern Ireland where STV was used. Mayors in London and those local authorities that decided to have one (other than mayors with only ceremonial functions) were elected by the Supplementary Vote (a truncated and highly unsatisfactory form of AV). In Scotland, the new Scottish Parliament used its powers to introduce STV in 3-and 4-member wards for local government.

However, not until the second half of 2009, when it was clear that FPTP would not deliver a Labour victory, did Labour return to plans for a referendum on the voting system for the election of MPs. But even at that stage Labour was not prepared to contemplate a change to a proportional system which could have made it difficult for them ever again to hold power on their own, and hence they proposed a referendum on AV (with the hope that Liberal Democrat supporters would use their second preference votes to help Labour defeat Conservatives in marginal seats).

Although Labour lost in May 2010, the result was a hung parliament and the Liberal Democrats extracted from the Conservatives a commitment to a referendum as the price of support in a coalition. The Liberal Democrats were strongly committed to a proportional system, but AV was as much as the Conservative leadership was prepared to offer.

Thus the debate on voting systems has, unsurprisingly, been dominated by the interests of the major parties.

However, there has long been a popular campaign for electoral reform in the UK – the Electoral Reform Society was formed in 1884. But while cross-party /non-party campaigns for reform have no doubt had influence, it seems that change was never likely to happen without political circumstances that made change attractive to a majority of politicians.

3. Dissatisfaction with the UK's 'first-past-the-post' system and the extent to which the proposed use of AV would overcome the problems of FPTP

The arguments of reformers against FPTP have, nevertheless, been powerful. Most, however, relate to problems associated with any non-proportional system based on single-member constituencies, and a degree of proportionality has therefore been the main demand of reformers. These problems are:

- That FPTP produces distorted outcomes
 While FPTP has produced 'wrong winners', more often it produces exaggerated and
 undeserved majorities. In 2005, for example, Labour won 55% of the seats on 35% of
 the votes. That Labour had a majority sufficient to push through parliament almost
 all of its proposals when nearly 2 out of every 3 voters voted for a candidate of a
 party other than Labour raised questions about the legitimacy of government.
- 2. That FPTP makes it difficult for significant minorities to gain representation In 2010 a Green Party candidate won a seat for the first time although the Green Party enjoys considerable popularity. In 2005 the UK Independence Party received nearly 606,000 votes but won no seats.
- 3. That FPTP elections are fought only in marginal seats

 Most seats are perfectly safe for one party or another. Some have been held by
 candidates of the same party for more than a century. Even in the 2010 election, in
 which there were more uncertainties than usual, the Electoral Reform Society was
 confidently able to send victory congratulations to candidates in a majority of seats
 before the election was even held. In safe seats supporters of parties other than the
 dominant party vote knowing that their vote is only a gesture of defiance. Parties
 focus their campaign resources onto the seats that matter those they need to fight
 to hold or where there is a realistic chance of winning from an opponent. In 2005 it
 was estimated that the two major parties targeted only 2% of the electorate the
 swing voters in swing seats.
- 4. That FPTP produces regionally distorted results The Conservatives, although the dominant coalition partner, won only one seat in Scotland in spite of the fact that they received 16.7% of the votes, while Labour won only 2 of the 58 seats in Eastern Region with 19.6% of the votes (BBC).
- 5. That FPTP is a barrier to increased representation of women and other under-represented groups.
 Parties select one candidate for each constituency. With systems that use multi-member constituencies where parties elect several candidates simultaneously, to maximise their support parties have an incentive to select a group of candidates more representative of society as a whole.

However, FPTP has some defects that could be overcome by a change to AV, such as:

6. FPTP elections result in many 'wasted' votes By 'wasted' votes we mean votes that do not contribute to the election of a candidate, either because they are cast for losing candidates or because they are votes that only add to superfluous majorities. Where no candidate has an absolute majority of first preferences, more votes count in determining the winner.

7. FPTP encourages tactical voting

Rather than voting for their preferred candidate, thoughtful voters may decide to vote for the candidate they think best able to defeat the candidate they do not want. For example, in a Labour-Conservative marginal, a Liberal Democrat supporter may decide to vote, say, Labour to prevent a Conservative victory rather than 'wasting' their vote on the candidate of their own party.

- 8. FPTP allows candidates to win with less than majority support
 In 2010 only 35% of MPs elected had over 50% of the votes in their constituencies. It is quite possible that in some cases the most unpopular candidate won. In FPTP local elections, the extremist British National Party has won seats with less than 30% of the votes² when polling evidence suggests that a large majority of the voters would have preferred any candidate to that of the BNP.
- 9. That FPTP encourages negative campaigning and a form of adversarial politics that is unattractive to voters
 With FPTP it is only necessary to get more votes than any other candidate. It can be easier to do that by attacking opponents in a disparaging way than by promoting one's own ideas positively. With preferential voting, a winner may need transfers of votes from supporters of other candidates, giving candidates an incentive to

Thus while electoral reformers would have wished a referendum on a much more radical change than merely to AV, most recognise that a move to AV would have some merit and therefore see the proposed referendum as something worth campaigning for.

emphasise where they agree with others as well as where they differ.

Of these perceived problems of FPTP, however, only '7' and '8' relate to paradoxes in Dan Felsenthal's list ('7' being a form of strategic voting, although not one that can benefit a voter's first-preference candidate, and '8' being related to FPTP not always electing a Condorcet winner).

4. Are there other non-ranked voting methods would be more satisfactory alternatives to FPTP?

Here we consider:

Plurality with run-off Successive elimination Approval voting

Plurality with Run Off

Plurality with Run Off is simply not a practical one. A system that requires a second public election in most constituencies would be a non-starter because of costs to the state and to the political parties, and it is doubtful that electors would have an appetite for returning to the polling stations for second round. Experience in the US, where the system is used in

² E.g., the BNP won South Oxhey, Hertfordshire, in 2009 with 29.2%,

many local elections, shows that turn outs drop dramatically on the second round. It should be noted that the use of AV is gaining ground in the US under the name Instant Run Off voting (IRV) partly because it saves the expense of a second round.

However, even leaving the practicalities aside, it is an unsatisfactory system. If it is accepted that FPTP does not necessarily produce the right winner, why should it be assumed that one of the first two past the post should be the right winner?³ Plurality with Run-Off is highly disproportional and favours large parties. Unlike AV, the first round still encourages a certain amount of tactical voting because of risk of a compromise choice not reaching the second round. If no compromise candidate reaches the second round it can lead to surprising outcomes. To illustrate, Jean-Marie Le Pen of the French National Front qualified for the second round in the French Presidential election in 2002 with only 16.8% of the vote, defeating Jospin because the left vote was fragmented amongst a number of minor candidates. This ultimately gave Jacques Chirac one of the biggest landslides in French history.

Successive elimination

This is an even more unsatisfactory than Plurality with Run-Off. The need for a whole series of eliminating ballots makes it a non-starter for public elections.

Approval Voting

Voters can list whom they would tolerate, but cannot express preferences between them. As a result it can give strange results and is very dependent on voter tactics rather than voter preferences, as voters can harm candidates they strongly favour by voting for one that you merely don't think is going to eat your children. As a result, it is very prone to tactical/strategic voting: parties have an interest in ensuring that their supporters vote only for their candidate and for no others, and consequently it is a system in which it can be difficult for voters to decide how best to vote to achieve an outcome they want. Approval Voting would lead to the election of "lowest common denominator" candidates disliked by few, and strongly liked by few.

(Range Voting and Majority Judgement, although they involve a form of ranking, also suffer from strategic voting in a serious way and we therefore do not give them further consideration.)

We thus conclude that any single-member constituency method to replace FPTP must be based on preferential voting.

5. Voting systems based on pair-wise comparisons (and "the most worthless votes of the most worthless candidates"?)

Such systems attach an equal weighting to all rankings. We doubt, however, that this provides an accurate picture of voters'views. Most voters will know who they want to win, and many will have a clear idea which candidate is their second choice, and even their third. But with lower preferences there is less guarantee that voters will be as discriminating in their choices. Having completed long STV ballot papers ourselves, we are aware that once we have ranked our favoured candidates there is an element of arbitrariness about how we

³ We are grateful to D Hill for pointing out this absurdity.

rank the remaining candidates. If a voter ranks candidates A, B, ... H (say) in that order, can it be right that we regard A as being placed above B as being no more important that G being above H? We think not.

There is the added risk of 'donkey voting' – having decided on their first few preferences, some voters might complete the ballot paper by numbering other candidates in the order shown on the ballot paper.

Systems that require voters to rank all candidates will be particularly susceptible to these risks. With such systems we can expect many voters to leave some candidates unranked. In the Scottish STV elections of 2007, although voters were electing either 3 or 4 candidates, the median voter only expressed 3 preferences and many (perhaps this being the first election with preferential voting) only marked the ballot paper with a cross (taken to mean a first preference with no other preferences expressed).

When AV was proposed for UK elections in 1931, Winston Churchill, later to become British Prime Minister, described AV as taking account of "the most worthless votes of the most worthless candidates". He went on to describe AV as containing an element of blind chance and accident. His observation that some preference orderings might be worthless cannot be ignored when it comes to methods that use pair-wise comparisons.

His remark has less validity, however, when applied to AV. AV only uses lower preferences when they are needed to decide a winner, and most elections will be decided by voters' first few preferences. An analysis of UK's 2010 general election suggests that in seats where there was not an outright winner, the winner would be determined by second preferences – very rarely would lower preferences have any bearing on the result.

While it can be argued that AV's disregard for some lower preferences may prevent a candidate with broad popularity but few higher preferences from winning, there is a risk that methods based on pair-wise comparisons may result in victories for candidates who are least objectionable rather than for candidates with stronger merits. In electing members of a national parliament, that does not seem desirable.

Some examples

We illustrate our concerns by use of examples.

Example 1

Suppose we have an election with 3 candidates, A, B and C and the votes are

Α	40
BA	15
BC	20
CA	11
СВ	14

In an AV election, C is eliminated and A wins with 51 votes. 'A' is also the Condorcet winner.

The following examples illustrate how this result can be affected by the entry of an additional candidate, D, who has no first preference support whatsoever. In each case first preference votes remain the same and A would win under AV.

Example 2

Α	40
BDA	15
BCD	10
BDC	10
CDA	11
CBD	3
CDB	11

Here supporters of B and C have include D in their preferences, perhaps assuming D to be less threatening than those with first preference support whom they assume are their main rivals.

With AV the result is unchanged. But if we now consider pairwise comparisons, there is no Condorcet winner:

Neither is there a Copeland winner, but Kemeny would give B victory (BCDA and BDAC being the strongest scoring sequences).

Example 3

22
18
15
10
10
11
3
11

Here we assume that some of A's supporters had given a second preference vote to D. Now D becomes the Condorcet winner. We cannot imagine the electorate understanding the election of a candidate with no first-preference support⁴, and we would argue that such a candidate is unlikely to be a good choice.

This example also illustrates a major failing of Condorcet, namely that a voter's lower preferences can upset higher preferences. In this case, voting AD benefited D over A. With AV (and, of course, STV when multi-member constituencies are used) a voter cannot disadvantage a preferred candidate by expressing subsequent preferences. Here that is not the case, and voters would be right to be wary about ranking all candidates in an honest way.

⁴ David Hill has pointed out to us that results would make more sense to the ordinary elector if reported as "D beats A by 60-40, D beats B by 40-38, D beats C by 43-35", but it would be surprising if the fact that D had no first-preference votes did not emerge from the reporting of the count.

Example 4

А	28
AD	12
BDA	15
BCD	10
BDC	10
CDA	11
CBD	3
CDB	11

If A's supporters had sensed that their candidate enjoyed strong support, they might not have felt the same need to include D on their preference lists. Suppose only 12 rather than 18 included D.

Again the AV result remains unchanged, but now there is no longer a Condorcet winner. We find:

If Copeland had been used, we would have had a tie with A, B and D each winning two pairwise comparisons.

With Kemeny, B and D would have tied, the highest scoring sequences being BDCA and DABC. It is, however, hard to image that more than a fraction of 1% of the voters would have understood why this should have been the result.

Example 5

А	12
ACB	10
AD	18
BDA	15
BCD	10
BDC	10
CDA	11
CBD	3
CDB	11

There is still no Condorcet winner, but in this case with Copeland, A would have won (defeating B and C, but with B, C and D each winning only one pair-wise contest and B tying with C). With Kemeny, D would have won.

With real elections, it is important that the results are understandable and that they appear sensible. That small changes in the lower preferences of some voters and the choice of system for deriving a winner from pair-wise comparisons can yield such varied results can hardly inspire confidence in the approach.

6. Why Borda won't do

The Borda Count does not use pair-wise comparisons but it suffers from similar problems.

In all of the examples in section 5 above, however, it would elect A. Although D defeats A in a pair-wise contest, A benefits from higher preferences.

However, consider again example 3:

Example 6

А	22
AD	18
BDA	15
BCD	10
BDC	10
CDA	11
CBD	3
CDB	11

Here A wins. But if the 22 who voted just for A had voted:

- AB then B would have won;
- AC then C would have won; and
- AD then D would have won,

thus illustrating that Borda also suffers from the fault that expressing a lower preference can upset a higher one. In most circumstances a party would advise its supporters only to vote for that party's candidate and to show no other preferences, thereby defeating the purpose of the system.

Borda can also be criticised for the way it assigns values to preferences. With AV and systems based on pair-wise comparisons, we are only concerned with voter's preferences for one candidate over another. But there is no reason for assuming that the strength of voters' preferences shouldbe valued as 4, 3, 2, 1 rather than, say, 4, 2, 1, 0.5.

7. The case for AV in the light of the possible alternatives and of the paradoxes which afflict AV.

The primary reason that UK has chosen AV as its alternative to FPTP is probably that other single-member constituency systems have simply never been considered. There has been support for AV over a long period (as noted above, parliament voted for AV in both 1919 and 1931) and AV is seen as a stepping stone by those reformers who seek STV in multi-member constituencies. Politically, it is relatively easy to project what the outcome of an AV election might be (assumptions can be made about likely vote transfers from the political positions of the parties as well as opinion polling), and for politicians it is not such a step into the dark as would be the case for a change to a system that uses a quite different approach to vote counting.

However, should the paradoxes that afflict AV be a serious concern?

AV may not select a Condorcet winner, for example in an election in which votes cast are:

Table 7.1

AC	40
BC	35
С	25

In the UK, in which the Conservatives and Labour are the largest parties in terms of first-preference support, this may disadvantage the Liberal Democrats (C in this example). However, whether this should be viewed as a paradox or a matter of political choice is a matter for debate.

Coombs method has a problem when voters do not give full preference lists. If supporters of C had shown other preferences, then depending on these preferences, either A or B would have been eliminated and C would win. However, we have reservations about using a system based as much on who voters do not want as on who they support. In the UK our elections suffer from much negative campaigning: with Coombs method there is a risk that this problem would be accentuated. We do not therefore favour it.

AV's vulnerability to strategic voting is more theoretical than real. Indeed, a strong argument for AV over FPTP is that it allows voters to express true preferences (e.g. a Green Party supporter can give a first-preference for a Green candidate who has very little chance of winning without fear of the vote being wasted).

There are, of course, situations in which AV might be non-monotonic and in which strategic voting might be needed to achieve a voter's desired result. In English⁵ elections we could have the following situation:

Table 7.2

Labour	38
Conservative	30
Liberal Democrat	32

Most Labour and Conservative voters may give the Liberal Democrats their second preference, but more Liberal Democrats will favour Labour over the Conservatives. Here transfers from the Conservatives may give victory to the Liberal Democrats. However, if 3% of Labour voters were to switch their votes to the Conservatives, the Liberal Democrat candidate would be eliminated and Labour would win.

In practice, however, voters do not know how their parties have performed until all votes have been cast and the result counted. It is highly unlikely that Labour would feel sufficiently confident to ask some of its supporters to vote for their arch-enemy, the Conservatives.

We can also look at real elections to get an idea of how often situations in which such strategic voting is theoretically possible. We have examined the 531 constituency results in England in the 2010 general election looking for constituencies in which:

- 1. No candidate had an absolute majority;
- 2. Labour would not have been the winner under AV (using preference data from opinion polls)

⁵ Here we consider only English constituencies and not constituencies in Scotland and Wales where the nationalist parties must be considered, and not Northern Ireland which has its own political parties.

- 3. Under FPTP, the Liberal Democrats had fewer votes than the Conservatives;
- 4. Labour's vote less the Liberal Democrats' lead over the Conservatives was not less than the Liberal Democrat vote (if it were not, the switch in Labour votes would result in Labour's elimination).

We found no constituencies that satisfied all of these criteria. Thus, even if we were to suppose that some voters knew how all other voters had used their votes, the opportunity for strategic voting was not there. While we have only examined 531 constituencies in one election, we conclude that non-monotonicity and the opportunity it might give for tactical voting is unlikely to be a problem that need concern us in practice.

7 Conclusion: Has the UK made the right choice?

If we were to choose an electoral system on the basis of the number of associated paradoxes, then FPTP would be one of the front-runners. It is affiliated by only 6 of the paradoxes listed by Dan Felsenthal. We have argued, however, that FPTP is a highly unsatisfactory system, and much more unsatisfactory than AV which is afflicted by 9 of the listed paradoxes.

While it is important that paradoxes are considered, any assessment of a voting system must weigh up:

- The extent to which any paradox presents a problem is a paradoxical result merely a reflection of wider political choices and values (e.g. failure to select a Condorcet winner) or is it plainly perverse (e.g. non-monotonicity)?
- How frequently will a paradox manifest itself (e.g. under FPTP in those UK constituencies which are not safe for one party of another (about 40% of the total in 2010) strategic voting can make a difference, but under AV the opportunities for strategic voting would be rare and probably more theoretical than real);
- Other characteristics of voting systems (including issues relating to the conduct and costs of elections, transparency and the extent to which results will be understandable and acceptable) which might be considered of equal or greater importance.

In this paper we have compared AV with all other systems listed in Dan Felsenthal's paper and have found no other system for single-member elections better suited for UK general elections.

However, even if we had formed a preference for another system, unless that preference were a strong one we would not argue for opposition to proposed referendum on AV. Electoral reform is a political – not a mathematical – process and politics is the art of the possible. If the UK is to move to a better electoral system than FPTP, then AV appears to be the only possible first step.