Qualified Majority Voting: The Effect of the Quota

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ABSTRACT

We explore the effect of various values of the quota of QMV in an enlarged, 27-member Council of Ministers of the EU. In order to isolate the effect of the quota q, we assume, for all values of q, an 'equitable' distribution of voting power, according to Penrose's Square-Root Rule. For each value of q from $q = 51\%$ to near 100% of the total weight, we compute the system of weights that produces an equitable distribution of voting power. This enables us to examine the effect of q (with an equitable distribution of power) on various quantities, including: the voting power of each member; the blocking power (Coleman's 'power to prevent action') of each member; the sensitivity of the decision rule; Coleman's parameter A ('ability of the collectivity to act'); the mean majority deficit. A particularly interesting phenomenon is the effect of varying q on the weight of each member.

JEL classifications: C63, C71, D71, H77

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1 Introduction

The Council of Ministers (CM) – the main legislative organ of the EU – is a paradigmatic example of a decision-making body whose members act as representatives of constituencies: in this case the citizenries or electorates of the member states. In analysing its decision rule, a representative council of this sort may be viewed in two ways.

First, it can be regarded as separate decision-making body in its own right; and, second, as the upper tier of a two-tier decision-making body.

Accordingly, the various criteria used in assessing a decision rule of the CM fall into two groups: those that address the qualities of the rule in the context of the CM itself, as though it were a free-standing body; and those that address its qualities in the context of the CM acting on behalf of the citizens of the EU (Felsenthal and Machover [1997], [2000], [2001a]; Laruelle and Widgrén [1998]; Leech [2002b]).

The most important criterion of the first group is *efficiency*: how easy it is for the CM to adopt an act – or, conversely, the degree of inertia inherent in the given decision rule.[1] A direct index of efficiency of a binary voting rule – a so-called *simple voting game* (SVG) – is Coleman's *power of the collectivity to act* (Coleman [1971]), given by:

$$A := \frac{\omega}{2^m}, \tag{1}$$

where m is the number of members – in the present case EU member states – and ω is the number of divisions of the members into 'yes' voters and 'no' voters that would result in an act being adopted rather than blocked. Since 2^m is the number of *all* possible divisions of the members into 'yes' voters and 'no' voters, A has a simple interpretation as the a priori probability[2] of a random act being adopted rather than blocked.

Criteria of the second group are concerned with *democratic legitimacy.* The most important desideratum in this group is *equitability* or *equal representation.* Since members of the CM represent their respective constituents – the

[1] Another criterion that belongs to this group is that of *sensitivity*; but we shall not deal with it in this paper.

[2] See Note p. 5f. below.

citizens of their respective states – each EU citizen can be regarded as an indirect decision-maker, acting via his or her representative. A decision rule of the CM is equitable, in the sense of implementing equal representation of all EU citizens, if it endows all citizens of the EU, irrespective of country, with equal indirect voting power.

According to Penrose's Square Root Rule (PSQRR), equitability is achieved if and only if the (direct) voting powers of members of the CM are proportional to the square root of the size of their respective constituencies.[3] Here we assume that each representative on the CM votes on each issue according to the majority view in his or her country.[4]

In passing, let us note that the present qualified majority voting (QMV) rule of the CM is quite close to implementing PSQRR. The new QMV rules prescribed by the TREATY OF NICE [2001] for the current 15 members (which will become effective from the beginning of 2005 if the EU will not have been enlarged by then) and for a CM of an EU enlarged to 27 members are even closer.[5] However, this has not come about by any intention of implementing PSQRR. Rather, it is a fortuitous outcome of bargaining among the practitioners – politicians and officials – who decide these matters.[6]

In what follows, we shall denote by ψ_i the voting power of member i as quantified by the Penrose–Banzhaf (aka 'absolute Banzhaf') measure. This is defined as follows. Let η_i be the number of *swings* of voter i: the number of divisions of all the voters other than i into 'yes' and 'no' voters in which i

[3]See PENROSE [1946], [1952]. For a proof, see FELSENTHAL AND MACHOVER [1998] .

[4]This is called the 'democratic idealization' in FELSENTHAL AND MACHOVER [2000]. While it is an idealization of reality, it is much more realistic than the tacit assumption usually made by EU politicians and officials: that a representative on the CM votes on each issue according to the *unanimous* view in his or her country. This false assumption underpins the widespread fallacy that in order to have equal representation of EU citizens via their representatives on the CM, the voting weights of the member states should be strictly proportional to size of their respective populations.

[5]For details see FELSENTHAL AND MACHOVER [2001a] or LEECH [2002b].

[6]From authoritative accounts (GALLOWAY [2001], MOBERG [2002]) it is clear that the practitioners believe in the fallacy mentioned in footnote 4. On the other hand, they realize that if voting weights were assigned in proportion to population size, the smaller member states would be extremely disempowered in the CM, leading to disaffection on their part. In order to mitigate this effect, they have sought a compromise between the principles of the EU as a 'union of peoples' and a 'union of states'. The former principle prescribes equalizing the indirect voting power of all citizens, which according to the prevailing fallacious view would require assigning weights in proportion to population size. The latter principle prescribes giving equal weights to all member states. The compromise between the two principles has resulted in practice in a fairly close fit with the prescription of PSQRR.

is decisive, so that if i votes 'yes' the outcome is positive (the proposed act is adopted) and if i votes 'no' the outcome is negative (the proposed act is blocked). Then

$$\psi_i := \frac{\eta_i}{2^{m-1}}. \tag{2}$$

We shall denote by β_i the relative voting power of member i, known as the (normalized) Banzhaf index:

$$\beta_i := \frac{\psi_i}{\sum_{j \in N} \psi_j}, \tag{3}$$

where N is the set of voters (in the present case, member states).

Using the index A (which quantifies the efficiency of the decision rule) and the voting powers ψ_i, we can obtain another important set of quantities: the blocking powers of the members. The original definition of a member's blocking power was given by Coleman [1971], who called it 'power to prevent action'. It is not difficult to see that Coleman's definition is equivalent to the following: the blocking power of voter i is given by

$$\gamma_i := \frac{\psi_i}{2A}. \tag{4}$$

Similarly, Coleman's measure of a member's 'power to initiate action' is given by

$$\gamma^*{}_i := \frac{\psi_i}{2(1-A)}. \tag{5}$$

For a given decision rule of the CM, the value of A as well as the voting powers ψ_i of the member states (on which the rule's equitability directly depends[7]) are 'hidden' deep-level quantities: they are obtained by high-powered computations underpinned by mathematical theory. The practitioners are not aware of these quantities; what they see – and manipulate – are the distribution of voting weights and the quota, the total weight of 'yes' votes required for adopting an act.

The weights and the quota are the superficial quantities, visible to the 'naked eye'. Jointly they determine the values of the deep-level hidden quantities; but the relationship between the two kinds of quantity is quite complex.

[7]Note that even without PSQRR it ought to be clear that equitability is determined by the voting powers rather than the voting weights of the member states; PSQRR only specifies the exact form of this relationship.

In this paper, which follows on from LEECH [2002b], we consider a 27-member CM, as envisaged in the TREATY OF NICE [2001], consisting of the current 15 members and 12 prospective members. We wish to isolate the effect of the quota q (as a proportion of the total weight) on the efficiency index A, the voting powers ψ_i and blocking powers γ_i. To this end, we compute the data for a one-parameter family,

$$\mathfrak{W} = \left\{ \mathcal{W}(q) : \tfrac{1}{2} < q < 1 \right\},$$

of WVGs for this prospective CM. The members $\mathcal{W}(q)$ of this family differ from one another in the value of q, and hence in the resulting value of A; but they all have the same – optimal – level of equitability, as they all satisfy PSQRR. This means that the values of the Banzhaf index β_i for all the WVGs in the family are invariant – and proportional to the square roots of the respective population sizes. Thus $\mathcal{W}(q)$ may be defined as the WVG with quota q, and with values of β_i determined by PSQRR.

We compute weights for each value of the quota, using a method of successive approximations. We call the resulting weights *equitable* weights for the given value of q and denote them by $w_i(q)$.[8]

These equitable weights $w_i(q)$ for given q also yield the value of A, the ψ_i and γ_i for this q.

Apart from equitability, a second desideratum of democratic legitimacy is *majoritarianism.* In a two-tier decision-making system, consisting of a council of representatives who vote on behalf of their respective constituencies, it is always possible that – although each representative votes in accordance with a majority opinion in his or her own constituency – the outcome may nevertheless be opposed by a majority of the electorate. When this occurs, the difference between the size of the majority opposing the outcome and the minority supporting it is the *majority deficit* of that particular decision. (If the outcome is supported by a majority of the entire electorate, the majority deficit is 0.) The *mean majority deficit (MMD)* of a decision rule is the statistical mean (or a priori expected value) of the majority deficit that may occur under that rule. It is a measure of the degree to which the given rule deviates from majoritarianism.

[8]The accuracy of the algorithm used is discussed in Section 2. As pointed out in LEECH [2003], some 'lumpiness' in the output of the computation is due not to any shortcoming or inaccuracy of the algorithm, but is inherent in the problem itself. For a given number of voters – in our case 27 – there are only a finite number of possible vectors of Banzhaf values β_i. So in general there simply may not exist weights that yield precisely the desired equitable values of the β_i. However, the precise values of the β_i that are obtainable for 27 voters are in practice sufficiently densely distributed to provide very close approximation to the desired values.

We compute the value of the MMD for each value of q. According to Felsenthal and Machover [1998 54–67], the MMD of our two-tier system is given, to an extremely good approximation, by

$$\mathrm{MMD} \approx \frac{\sqrt{n} - \sum_i \psi_i \sqrt{n_i}}{\sqrt{2\pi}}, \tag{6}$$

where n is the size of the entire electorate of the EU and n_i is the size of the electorate of the i-th member state.

We wish to stress that we chose to keep invariant the relative powers β_x rather than the relative weights, because the latter are superficial quantities. If they were to be kept invariant, the resulting WVGs would differ from one another in two deep-level aspects: equitability as well as efficiency. This would defeat our present purpose of isolating efficiency as the key aspect to be examined. What we have here is a family of WVGs that share one deep-level property: optimal equitability. And, keeping this invariant, we study the effect of the quota on the other main deep-level parameter: efficiency and closely related quantities.

Note: Operational meaning of a priori voting power

Throughout, we use the notion of a priori voting power, which assigns equal a priori probability to each of the 2^m possible divisions of a decision-making body of m voters.

This approach has been criticized on various grounds. In particular, it has been claimed on general philosophical grounds that the very notion of a priori probability is incoherent in the present context.

Here we can only deal briefly with this objection.[9] Consider the following thought experiment. You are told that a decision-making council (of which you are not a member) is about to divide on a proposed resolution, which will affect your financial position: if the resolution is passed you will gain €1 million; if the resolution is defeated, you will lose €1 million. You know the decision rule under which the council operates, which is an SVG. *But you have absolutely no information about the preferences of the council's members or any other causes that may affect their voting behaviour.*

Now a, one of the council's members, offers to sell you his voting rights for this particular division at a certain price, P. Should you be prepared to pay this price?

[9] For a detailed discussion see Felsenthal and Machover [1998]. For a reply to other objections to the use of a priori voting power see Felsenthal and Machover [2001b].

Using the theory of a priori voting power, it is not difficult to answer this question. If you don't buy a's voting rights, your payoff (expressed in €millions) is 1 with probability A and -1 with probability $1-A$. Therefore your a priori expected payoff is $2A-1$. On the other hand, it is easy to show if you use a's voting rights to vote for the proposed resolution, your expected payoff (in €million) goes up to $2A-1+\psi_a$, where ψ_a is a's voting power according to the Penrose–Banzhaf measure.[10] Therefore according to our theory you should be prepared to pay for a's voting rights any price smaller than ψ_a, because then you would be increasing your total expected payoff; and you should refuse to pay any price greater than ψ_a, because that would reduce your total expected payoff.

Even if you are one of those philosophers who claim that a priori probability is not a coherent concept in the present context, you must still decide whether or not to buy a's voting rights for the price he demands for them. Whatever you do – buy or refrain from buying – implies in effect a judgment on a priori probabilities. Rejecting the concepts of a priori probability and a priori voting power amounts to rejecting the only possible basis on which you can make a rational decision in the circumstances of our thought experiment.

2 Results

We investigate the effect of the quota q of QMV in the CM for the fully enlarged EU of 27 member states, as envisaged in the Treaty of Nice [2001]. We allow q, expressed as a percentage of the total weight, to vary over its entire feasible range from a simple majority, $q = 51\%$, to near-unanimity, $q = 99\%$.

The analysis here differs from that reported in Leech [2002b] in two important respects. First, only equitable weights are used; so, rather than fixing the respective weights at the values prescribed by the Treaty of Nice [2001], the weights are re-computed for each value of q to ensure that the relative voting powers β_i are proportional to the square root of the respective population. Thus, it is the β_i that are kept fixed. Second, here the quota is changed in steps of one percentage point rather than the five percentage points used in Leech [2002b], yielding a finer-grained analysis. In that study, results for a unanimity rule were included as the limiting case (where each member's normalized power index would be equal to $\frac{1}{27}$), but this limiting case is excluded here, since it would be inconsistent with our requirement regarding the β_i. In other words, a value of $q = 100\%$ is not

[10] For a simple proof, see Felsenthal and Machover [1998 45].

feasible because equitable weights do not exist for it.

In order to define the problem to be analysed clearly, we have assumed a simple QMV decision rule, rather than the triple majority rule prescribed in the Nice Treaty.

The calculations have been carried out using the iterative algorithm described in LEECH [2002a] and [2003], to determine the equitable weights corresponding to given relative power indices for each value of q. At each iteration of the algorithm the power indices themselves were determined using the program LEECH [2001], which implements the direct enumeration algorithm for power indices when the number of voters is small.

The problem of finding the weights $w_i(q)$ for each q is discussed in LEECH [2003]. There are two steps in the calculation where approximation is necessary: first, in choosing the appropriate values of β_i corresponding to population square roots; and second, in calculating the w_i that give rise to these β_i. Some degree of approximation is inherent in the first step and therefore a degree of error cannot be eliminated entirely. The second step involves using a method of successive approximations until convergence is reached relative to an acceptable small error.

Both of these errors are truly negligible in this case: the first, because $n = 27$ gives a sufficiently large WVG; and the second by choice of a sufficiently small convergence criterion for the algorithm, and double-precision arithmetic in the FORTRAN implementation. the convergence criterion used was a sum of squared errors less than 10^{-8}, which meant that the average value of the absolute difference

$$\left| \beta_i - \frac{\sqrt{n_i}}{\sum_{j \in N} \sqrt{n_j}} \right| \tag{7}$$

was of the order of less than 0.00002. Therefore we have taken the computation errors to be truly negligible.

The main results are presented in Table 1. The second column in this table shows the total number of swings,

$$H := \sum_{i \in N} \eta_i, \tag{8}$$

where η_i is the number of swings of member i, which features in the definition (2) of ψ_i. The third column shows the number ω of divisions with positive outcome. The fourth column shows Coleman's A, the power of the CM to act. Recall that according to (1) this is obtained by dividing the value of ω by 2^{27}.

Figure 1 shows the effect of the quota on Coleman's A both for equitable weights and for fixed weights, prescribed in the Treaty of Nice. In this analysis the quota varies in steps of 5% to enable comparability with Leech [2002b]. The quota prescribed in the treaty is marked on the diagram for reference. The results show that there is little difference for values of q below 75% but important differences above this figure. Equitable weights yield slightly smaller efficiency when q is 55% and slightly greater efficiency when q is above 70%. However, the differences are trivial up to $q = 75\%$ – beyond which there is a significant qualitative change in behaviour: the power to act goes to its limiting value of $2^{-27} \approx 7.45 \cdot 10^{-9}$ when the weights are fixed but tends to a much higher limiting value, 0.0153, for equitable weights. This limiting value is attained at $q = 78\%$ and A remains constant for all larger values of q.

Figure 2 shows the relationships between the equitable weight and the quota for certain members. That for Germany is shown in Figure 2(a). The equitable weight calculated is such that Germany's normalized (relative) power index is always equal to 0.0954. This weight varies around approximately 10% of the total voting weight for values of q up to $q = 76\%$ and thereafter it increases steadily towards 100%. The equitable weight increases as the quota increases in order to maintain the constant voting power for Germany. The corresponding graphs for selected other member countries are shown in Figures 2(b)–(d). They all show a generally similar pattern with some variation in the equitable weights as q increases but then a strong fall towards zero beyond $q = 76\%$. The equitable weight for the UK, for example, shown in Figure 2(b), varies around approximately 8% (to give a normalized power index of 0.081). Then it falls strongly for values of q above 76%. The pattern for all remaining member states is the same. Figure 2(e) shows comparative graphs for the top four members, Germany, UK, France and Italy.

Figure 3 shows the MMD as a function of q. It increases from its lowest value of $1,789$ at $q = 51\%$ to a maximum level of $8,100$ at $q = 77\%$, at which it remains.

Figure 4 shows how the quota affects the power of the member countries. Three measures of voting power: the blocking power or power to prevent action (PPA), the positive power to initiate action (PIA) and the Penrose power (also known as the absolute or un-normalized Banzhaf index, Banzhaf power index, BZNN) measuring the overall power to swing a decision. The pattern is very similar for all member states. Figure 4(a) shows that for Germany. As the quota increases, the Germany's blocking power increases towards 1. On the other hand, the power to initiate action falls to a low but positive value; this is a different result from that in Leech [2003], where (assuming fixed weights) it falls to zero as q increases. Likewise the Penrose power of Germany falls to a positive value and then remains constant. The results for Denmark in Figure 4(b) show the power to prevent action reaches a maximum when q is greater than 76%, and – as in the case of Germany – the power to initiate action and the Penrose measure fall to a constant positive value.

Finally, Figure 5 shows the relations between the power measures for the same countries and the power to act, Coleman's, A. These graphically show the trade-offs facing the members in choosing the quota, since the vertical axis shows the power of the member while on the horizontal axis is a measure of the efficiency of the EUCM as a whole.

3 Some observations

First we would like to point out an interesting and somewhat surprising fact concerning the behaviour of the equitable weights as functions of q.

If the relative weights were to remain unchanged when q approaches 1, then the decision rule would approach the unanimity rule, which has $q = 1$, and in which all voters with positive weight have equal relative voting power (in our case, $\frac{1}{27}$). Therefore when q gets close to 1, in order to compensate for this 'near-unanimity tendency' and produce the prescribed equitable relative powers β_i rather than nearly equal ones, the equitable weights of the larger member states must become larger than they were for values of q near $\frac{1}{2}$, and the opposite holds for the smaller member states. Our results confirm this expectation.

What is somewhat surprising is that this effect begins to operate rather abruptly from about $q = 0.76$, and from that point on is linear. Thus we have two distinct regimes. Up to about $q = 0.75$, the equitable relative weights $w_i(q)$ are relatively stable (their graphs are nearly flat in this range). From about $q = 0.77$, the equitable relative weight of Germany shoots up as a linear function of q and approaches 1 (ie Germany takes up nearly the entire total weight) as q approaches 1. Consequently, the equitable relative weights of all other members decrease in a virtually linear fashion and approach 0 as q approaches 1. However, the relative weights of the smaller members decrease more sharply than those of the larger ones.

Providing an explanation for this behaviour, and especially the linear form of the graphs from about $q = 0.77$, is an open theoretical problem.

Our results regarding the behaviour of the voting and blocking powers of the members – the ψ_i and γ_i – also display two distinct regimes, with a marked transition at around $q = 0.76$. Here the general pattern is the same for all members. As q increases from 0.51 to about 0.75, the blocking powers of all members increases steadily, while their voting powers decrease. The blocking powers get close to their maximal values and the voting powers get close to their minimal values – which are of course different for different members. From about $q = 0.76$ the blocking and voting powers change very little.

Also for these values of the quota the total number H of swings is constant.[11] The behaviour of A also changes at around $q = 0.76$. As q increases from 0.51 to about 0.75, A decreases quite sharply (although not in linear fashion). From that point on, having reached a very low value, it hardly changes at all.

Finally, the behaviour of the MMD also displays two regimes: as q increases from 0.51 to about 0.75, the MMD increases steadily. It then stabilizes and from $q = 0.78$ it remains virtually constant.

[11] In fact, the small variation of the computed values of H in this range are due to the approximation error. This is evident also in the graph for Luxembourg where the errors in calculating the equitable weights, although just as small as those for the other members, are large in relation to the small weight of that country.

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Figure 1: Coleman's *A* (the power to act) versus the Quota *q* assuming both Fixed Weights (Nice) and Equitable Weights

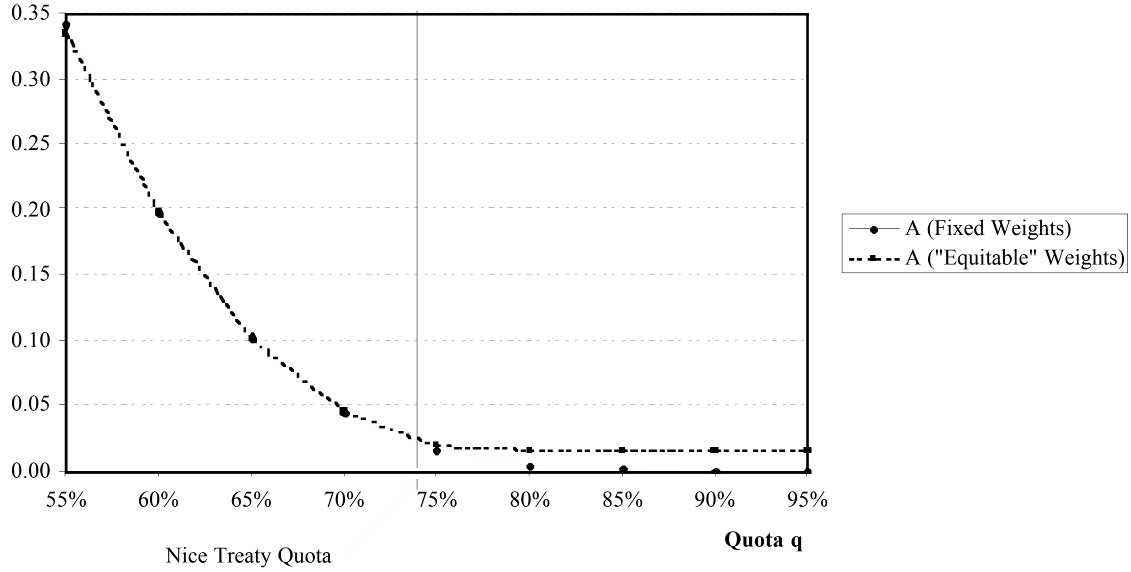

Figure 2a: Equitable Weights and the Quota: Germany

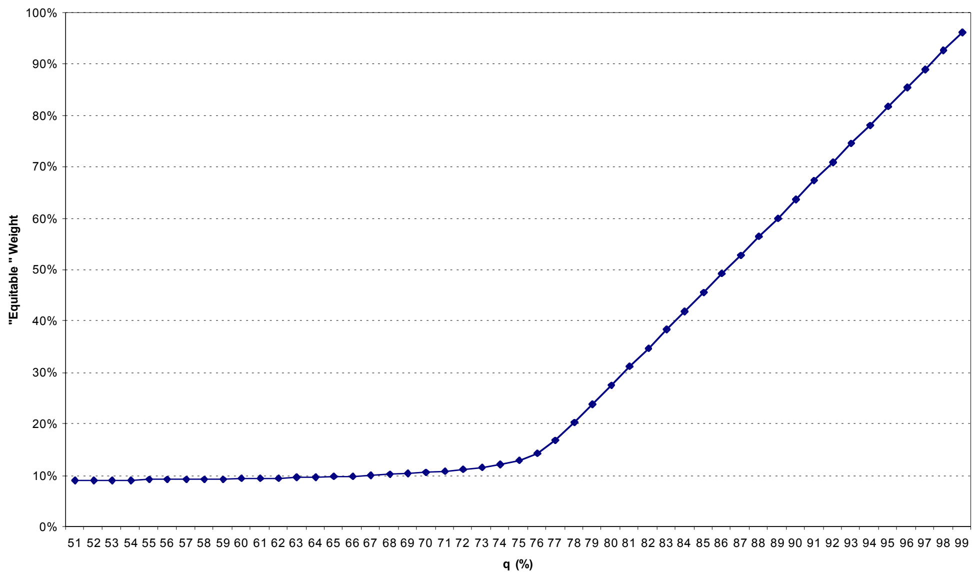

Figure 2b: Equitable Weights and the Quota: UK

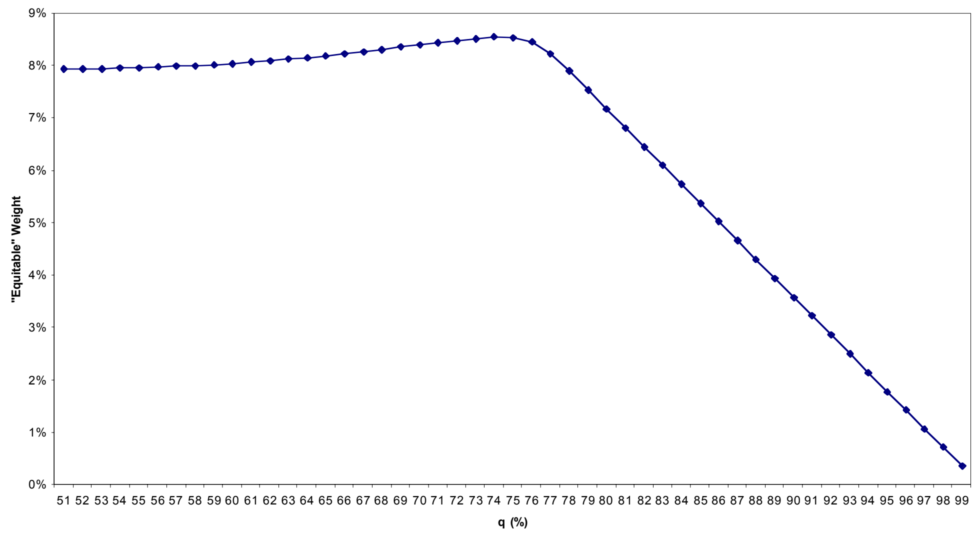

Figure 2c: Equitable Weights and the Quota: Netherlands

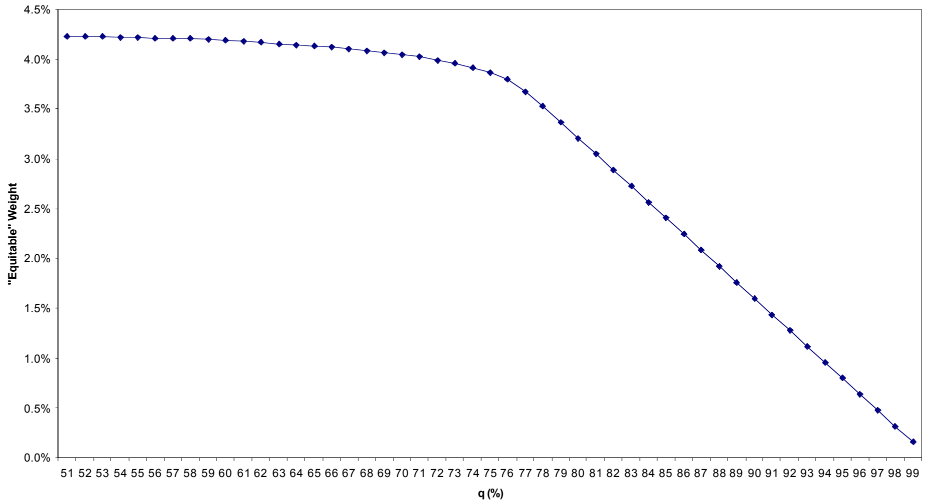

Figure 2d: Equitable Weights and the Quota: Luxembourg

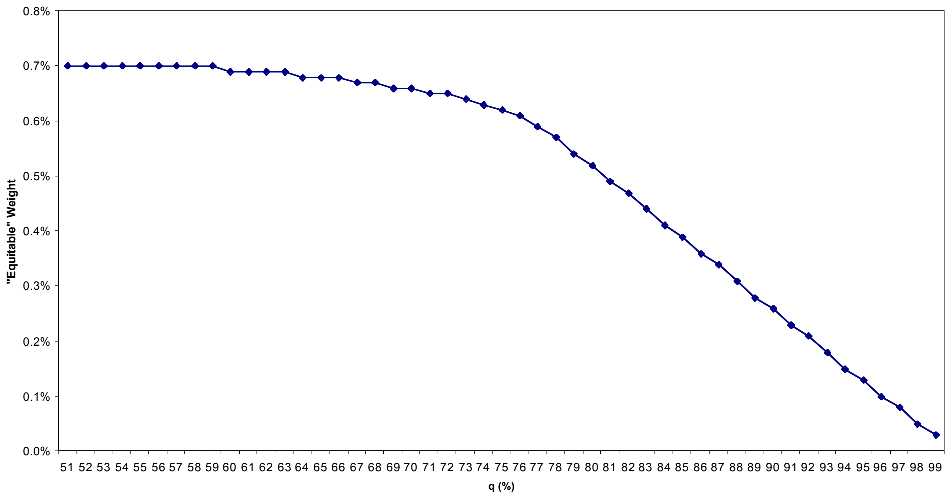

Figure 2e: Equitable Weights for the "Big Four" Countries

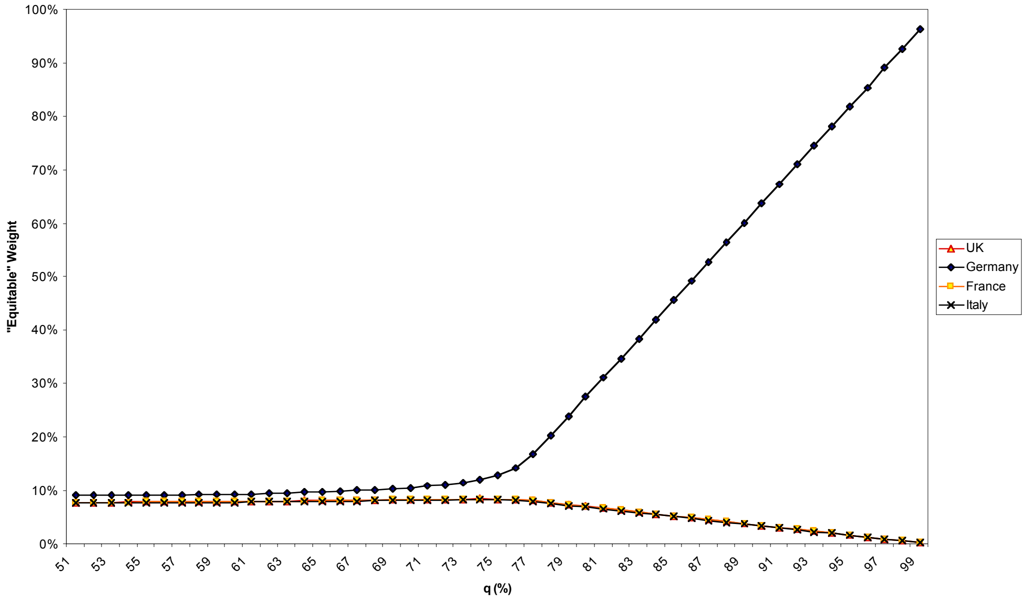

Figure 3: the Mean Majority Deficit

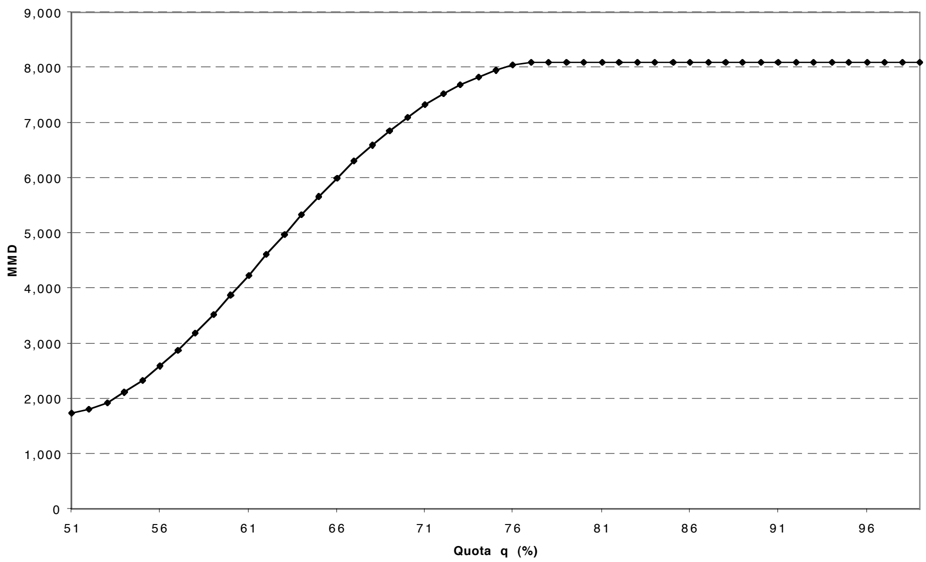

Figure 4(a): Penrose Measure (BZNN), Blocking Power (Coleman's Power to Prevent Action, PPA), and Power to Initiate Action (PIA): Germany

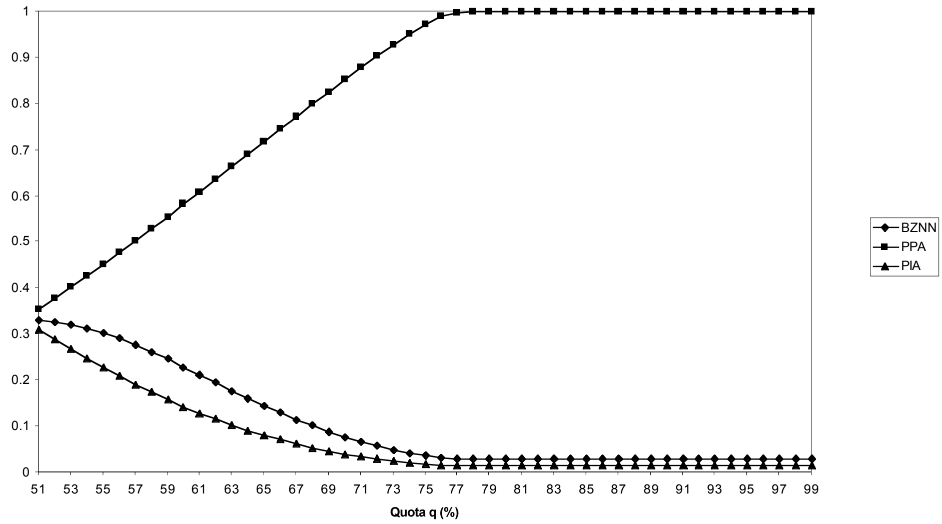

Figure 4(b): Banzhaf index (BZNN), Blocking Power (Coleman's Power to Prevent Action, PPA), and Power to Initiate Action (PIA): Denmark

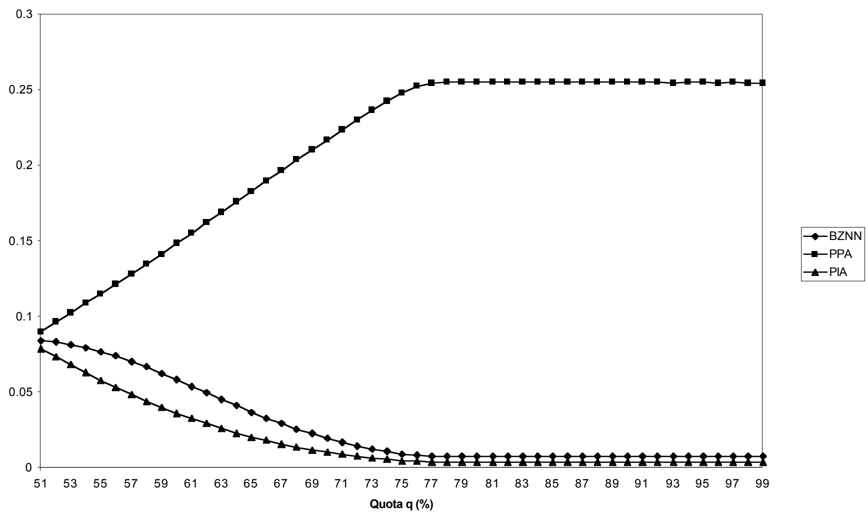

Figure 5(a): Penrose Measure (BZNN), Blocking Power (PPA) and Power to Initiate Action (PIA) v A: Germany

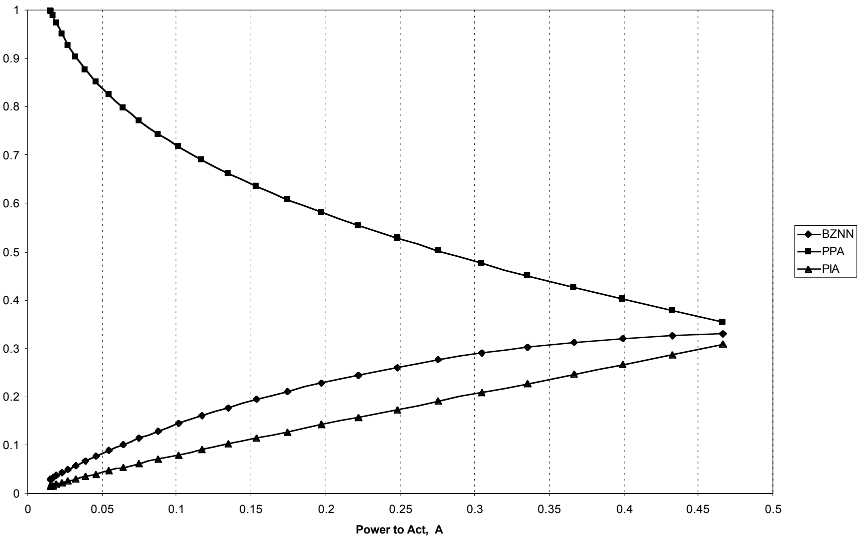

Figure 5(b): Penrose Measure (BZNN), Blocking Power (PPA) and Power to Initiate Action (PIA) v A: Denmark

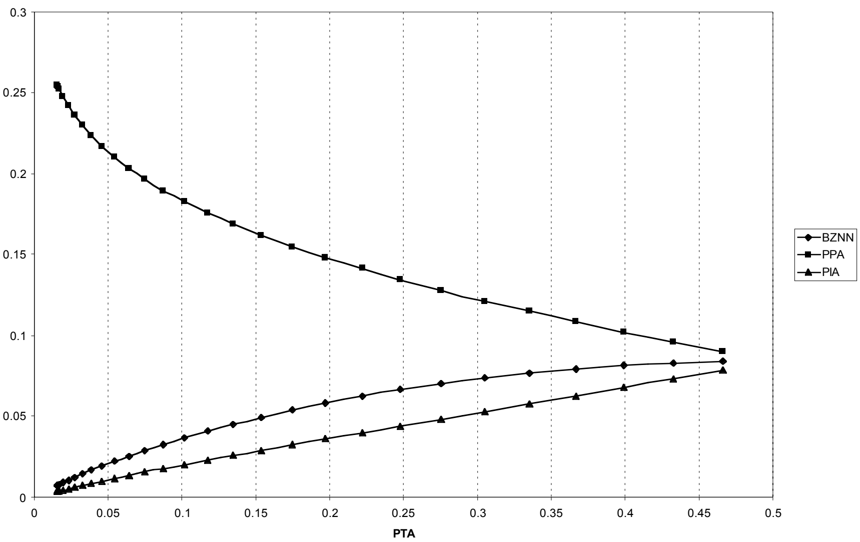

Table 1

Quota q	Total number of Swings, H	Number of Positive Decisions	Power to Act, A	Mean Majority Deficit
0.51	232,789,241	62,543,353	0.4660	1,739
0.52	230,264,904	58,010,078	0.4322	1,816
0.53	226,046,732	53,543,016	0.3989	1,943
0.54	220,275,759	49,174,967	0.3664	2,116
0.55	213,071,102	44,932,872	0.3348	2,333
0.56	204,540,612	40,853,034	0.3044	2,590
0.57	194,995,838	36,934,456	0.2752	2,878
0.58	184,492,488	33,221,276	0.2475	3,194
0.59	173,287,842	29,718,912	0.2214	3,532
0.60	161,582,727	26,441,019	0.1970	3,884
0.61	149,580,352	23,399,126	0.1743	4,246
0.62	137,507,431	20,597,991	0.1535	4,609
0.63	125,461,374	18,021,470	0.1343	4,972
0.64	113,687,144	15,685,668	0.1169	5,327
0.65	102,310,687	13,581,213	0.1012	5,670
0.66	91,437,556	11,693,762	0.0871	5,997
0.67	81,183,422	10,017,220	0.0746	6,306
0.68	71,613,481	8,538,503	0.0636	6,594
0.69	62,789,510	7,243,920	0.0540	6,860
0.70	54,733,919	6,118,733	0.0456	7,103
0.71	47,462,843	5,148,761	0.0384	7,322
0.72	40,975,275	4,319,867	0.0322	7,517
0.73	35,281,320	3,621,068	0.0270	7,688
0.74	30,372,155	3,040,723	0.0227	7,836
0.75	26,307,301	2,576,231	0.0192	7,959
0.76	23,281,152	2,240,568	0.0167	8,050
0.77	21,791,627	2,078,895	0.0155	8,095
0.78	21,587,817	2,057,077	0.0153	8,101
0.79	21,582,466	2,056,518	0.0153	8,101
0.80	21,586,697	2,056,959	0.0153	8,101
0.81	21,586,485	2,056,939	0.0153	8,101
0.82	21,586,509	2,056,945	0.0153	8,101
0.83	21,586,830	2,056,980	0.0153	8,101
0.84	21,587,613	2,057,061	0.0153	8,101
0.85	21,588,943	2,057,205	0.0153	8,101
0.86	21,583,702	2,056,654	0.0153	8,101
0.87	21,586,458	2,056,956	0.0153	8,101
0.88	21,590,017	2,057,323	0.0153	8,101
0.89	21,586,010	2,056,914	0.0153	8,101

Table 1 (continued)

Quota q	Total number of Swings, H	Number of Positive Decisions	Power to Act, A	Mean Majority Deficit
0.90	21,592,188	2,057,558	0.0153	8,101
0.91	21,590,202	2,057,368	0.0153	8,101
0.92	21,586,149	2,056,897	0.0153	8,101
0.93	21,595,293	2,057,909	0.0153	8,101
0.94	21,621,625	2,060,891	0.0154	8,100
0.95	21,613,921	2,059,897	0.0153	8,100
0.96	21,606,869	2,059,363	0.0153	8,100
0.97	21,633,586	2,062,346	0.0154	8,100
0.98	21,605,191	2,058,977	0.0153	8,100
0.99	21,599,579	2,058,475	0.0153	8,101